PHYSICS (1 ST YEAR) IIT MATERIAL

1. PHYSICAL WORLD

2. UNITS & MEASUREMENTS

Physical Quantity:

- Any quantity which can be measured directly (or) indirectly (or) interms of which the laws of physics can be expressed is called physical quantity.
- There are two types of physical quantities
  1) Fundamental quantities 2) Derived quantities
  Fundamental Quantities: Physical quantities which cannot be expressed interms of any other physical quantities are called fundamental physical quantities.
  Ex. length, mass, time, temperature etc..
  Derived Quantities: Physical Quantities which are derived from fundamental quantities are called derived quantities.
  Ex. Area, density, force etc...

Unit of physical qantity:

- A unit of measurement of a physical quantity is the standard reference of the same physical quantity which is used for comparison of the given physical quantity.
  Fundamental unit : The unit used to measure the fundamental quantity is called fundamental unit.
  Ex: metre for length, kilogram for mass etc..
  Derived unit : The unit used to measure the derived quantity is called derived unit.
  Ex: m^2 for area, gm cm^{-3} for density etc...
- The numerical value obtained on measuring a physical quantity is inversely proportional to the magnitude of the unit chosen.

$$\boxed{n \; \alpha \frac{1}{U}} \Rightarrow nU = \text{constant}$$

$$\Rightarrow \boxed{n_1 U_1 = n_2 U_2}$$

Where n_1 and n_2 are the numerical values and U_1 and U_2 are the units of same physical quantity in different systems.

System of units

- There are four systems of units
  1) F.P.S 2) C.G.S
  3) M.K.S 4) SI
- Based on SI system there are three categories of physical quantities.
  1) fundamental quantities
  2) supplementary quantities and
  3) derived quantities

Fundamental Quantities and their SI Units

- There are seven fundamental quantities and two supplementary quantities in S. I. system. These quantities along with their unit and symbols are given below:

S.No	Physical Quantity	SI unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Thermo dynamic temperature	kelvin	K (or) θ
5.	Luminous intensity	candela	Cd
6.	Electric current	ampere	A
7.	Amount of substance (or) quantity of matter	mole	mol
	Supplementary quantities		
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

Measurement of length

- The length of an object can be measured by using different units. Some particle units of length are

  angstrom(A^o)= 10^{-10} m= 10^{-8} cm

  nanometre(nm) $= 10^{-9} m = 10 A^0$

  fermi $= 10^{-15} m$

  micron $= 10^{-6} m$

  X-ray unit $= 10^{-13} m$

  1 A.U. = distance between sun & earth = 1.496×10^{11} m

- One light year is the distance travelled by light in one year in vacuum . This unit is used in astronomy.

  Light year $= 9.46 \times 10^{15} m$

  parsec $= 3.26$ light years $= 30.84 \times 10^{15} m$

  Bohr radius $= 0.5 \times 10^{-10} m$

  Mile=1.6 km

Measurement of mass:

The mass of an object can be measured by using different units.Some practical units of mass are

Quintal = 100 kg   Metric ton = 1000 kg

Atomic mass unit (a.m.u) = $= 1.67 \times 10^{-27} kg$

Measurement of time:

One day = 86400 second

Shake $= 10^{-8} second$

Abbreviations for multiples and sub multiples:

MACRO Prefixes

Multiplier	Symbol	Prefix
10^1	da	Deca
10^2	h	Hecto
10^3	k	Kilo
10^6	M	Mega
10^9	G	Giga
10^{12}	T	Tera
10^{15}	P	Peta
10^{18}	E	Exa
10^{21}	Z	Zetta
10^{24}	Y	Yotta

MICRO Prefixes

Multiplier	Symbol	Prefix
10^{-1}	d	deci
10^{-2}	c	centi
10^{-3}	m	milli
10^{-6}	μ	micro
10^{-9}	n	nano
10^{-12}	p	pico
10^{-15}	f	femto
10^{-18}	a	atto
10^{-21}	z	zepto
10^{-24}	y	yocto

Some important conversions:

$1kmph = \frac{5}{18} ms^{-1}$

1 newton=10^5 dyne

1 joule=10^7 erg

1 calorie=4.18 J

1eV= $1.6 \times 10^{-19} J$

$1gcm^{-3} = 1000kgm^{-3}$

1 lit=$1000cm^3 = 10^{-3} m^3$

1KWH$= 36 \times 10^5 J$

1 HP=746 W

1 degree=0.017 rad

1cal $g^{-1} = 4180JKg^{-1}$

1kgwt= 9.8 N

1 telsa=10^4 gauss

$1Am^{-1} = 4\pi \times 10^{-3} oersted$

1 weber=10^8 maxwell

Some physical constants and their values:

- 1 amu $=1.67\times10^{-27}kg=931.5MeV$

  1 atm pressure = pressure exerted by 76cm of Hg column $=1.013\times10^{5}\,Pa$

  Avagadro number (N)= 6.023×10^{23}

  Permittivity of free space= $8.854\times10^{-12}\,Fm^{-1}$ or C^2/Nm^2

  Permeability of free space $(\mu_0)=4\pi\times10^{-7}\,Hm^{-1}$

  Joule's constant (J)= $4.186Jcal^{-1}$

  Planck's constant(h)= $6.62\times10^{-34}\,Js$

  Rydberg's constant(R)= $1.0974\times10^{7}\,m^{-1}$

  Boltzmann's constant(K_B)=$1.38\times10^{-23}\,JK^{-1}$

  Stefan's constant $(\sigma)=5.67\times10^{-8}\,Wm^{-2}K^{-4}$

  Universal gas constant(R)=$8.314Jmol^{-1}K^{-1}$

  $=1.98cal\,mol^{-1}K^{-1}$

  Wien's constant(b)=$2.93\times10^{-3}\,metre\ kelvin$

Accuracy and precision of instruments :

- The numerical values obtained on measuring physical quantities depend upon the measuring instruments, methods of measurement.
- Accuracy refers to how closely a measured value agrees with the true value.
- Precision refers to what limit or resolution the given physical quantity can be measured.
- Precision refers to closeness between the different observed values of the same quantity .
- High precision does not mean high accuracy.
- The difference between accuracy and precision can be understood and by the following example: Suppose three students are asked to find the length of a rod whose length is known to be 2.250cm.The observations are given in the table .

Student	Measurement-1	Measurement-2	Measurement-3	Average length
A	2.25cm	2.27cm	2.26cm	2.26cm
B	2.252cm	2.250cm	2.251cm	2.251cm
C	2.250cm	2.250cm	2.251cm	2.250cm

It is clear from the above table , that the observations taken by a student A are neither precise nor accurate. The observations of student B are more precise . The observations of student C are precise as well as accurate.

Error:

- The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty in measurement is called error.

  Mathematically

- Error = True value - Measured value
  Correction =-error
- True value means, standard value free of errors.
- Errors are broadly classified into 3 types :
  i) Systematic errors
  ii) Random errors
  iii) Gross errors

Systematic Errors

- The errors due to a definite cause and which follow a particular rule are called systematic errors. They always occur in one direction (either +ve or -ve)
- Systematic errors with a constant magnitude are called constant errors.
  The constant arised due to imperfect design, zero error in the instrument or any other such defects. These are also called instrumental errors.
- Example for the error due to improper designing and construction.
  If a screw gauge has a zero error of -4 head scale divisions, then every reading will be 0.004cm less than the true value.
- The error arised due to external conditions like changes in environment, changes in temperature, pressure, humidity etc.
  Ex: Due to rise in temperature, a scale gets expanded and this results in error in measurement of length.

Imperfection in Experimental technique or Procedure:

- The error due to experimental arrangement, procedure followed and experimental technique is called imperfection error.
  Ex: In calorimetric experiments, the loss of heat due to radiation, the effect on weighing due to buoyancy of air cannot be avoided.

Personal errors or observational errors:

- These are entirely due to the personal peculiarities of the experimenter. Individual bias, lack of proper setting of the apparatus, carelessness in taking observations (without taking the required necessary precautions.) etc. are the causes for these type of errors. A person may be habituated to hold his eyes (head) always a bit too far to the right (or left) while taking the reading with a scale. This will give rise to parallax error.
- If a person keeps his eye-level below the level of mercury in a barometer all the time, his readings will have systematic error.
  These errors can be minimised by obtaining several readings carefully and then taking their arithmetic mean..

$$probable\ error\ \alpha \frac{1}{\text{no. of readings}}$$

Ex: Parallax error

Random Errors:

- They are due to uncontrolled disturbances which influence the physical quantity and the instrument. these errors are estimated by statistical methods.

$$\text{Random } error\ \alpha \frac{1}{\text{no. of observations}}$$

Ex-: The errors due to line voltage changes and backlash error.
Backlash errors are due to screw and nut.

Gross Errors

- The cause for gross errors are improper recording, neglecting the sources of the error, reading the instrument incorrectly, sheer carelessness
  Ex: In a tangent galvanometer experiment, the coil is to be placed exactly in the magnetic meridian and care should be taken to see that no any other magnetic material is present in the vicinity.
- No correction can be applied to these gross errors.
- When the errors are minimised, the accuracy increases.
  The systematic errors can be estimated and observations can be corrected.
- Random errors are compensating type. A physical quantity is measured number of times and these values lie on either side of mean value. These errors are estimated by statistical methods and accuracy is achieved.
- Personal errors like parallax error can be avoided by taking proper care.
- The instrumental errors are avoided by calibrating the instrument with a standard reference and by applying proper corrections.

Errors in measurement.

True Value :

- In the measurement of a physical quantity the arithmetic mean of all readings which is found to be very close to the most accurate reading is to be taken as True value of the quantities.

If $a_1, a_2, a_3 .... a_n$ are readings then true value $a_{mean} = \frac{1}{n}\sum_{i=1}^{n} a_i$

Absolute Error :

- The magnitude of the difference between the true value of the measured physical quantity and the value of individual measurement is called absolute error.
  Absolute error =|True value - measured values|

$$\Delta a_i = |a_{mean} - a_i|$$

The absolute error is always positive.

Mean absolute error:

- The arithmetic mean of all the absolute errors is considered as the mean absolute error of the physical quantity concerned.

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + --- + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n}|\Delta a_i|$$

The mean absolute error is always positive.

Relative error:

- The relative error of a measured physical quantity is the ratio of the mean absolute error to the mean value of the quantity measured.

Relative error= $\frac{\Delta a_{mean}}{a_{mean}}$

It is a pure number having no units.

Percentage error:

$$\delta a = \left[\frac{\Delta a_{mean}}{a_{mean}} \times 100\right]\%$$

Relative error and percentage error give a measure of accuracy i.e. if percentage error increases accuracy decreases.

EX. 1: Repetition in the measurements of a certain quantity in an experiment gave the following values: 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, and 1.33. Calculate the mean value, mean absolute error, relative error and percentage error.

Sol. Here, mean value

$$x_m = \frac{1.29+1.33+1.34+1.35+1.32+1.36+1.30+1.33}{8}$$

= 1.3275=1.33 (rounded off to two places of decimal)

Absolute errors in measurement are

$$\Delta x_1 = |1.33-1.29| = 0.04;\ \Delta x_2 = |1.33-1.33| = 0.00;$$

$$\Delta x_3 = |1.33-1.34| = 0.01;\ \Delta x_4 = |1.33-1.35| = 0.02;$$

$$\Delta x_5 = |1.33-1.32| = 0.01;\ \Delta x_6 = |1.33-1.36| = 0.03;$$

$$\Delta x_7 = |1.33-1.30| = 0.03;\ \Delta x_8 = |1.33-1.33| = 0.00;$$

mean absolute error

$$\overline{\Delta x_m} = \frac{0.04+0.00+0.01+0.02+0.01+0.03+0.03+0.00}{8}$$

$= 0.0175$

$= 0.02$ (rounded off to two places of decimal)

Relative error $= \pm\frac{\overline{\Delta x_m}}{x_m} = \pm\frac{0.02}{1.33} = \pm 0.01503 = \pm 0.02$

(rounded off to two places of decimal)

Percentage error $= \pm 0.01503 \times 100 = \pm 1.503 = \pm 1.5\%$

EX.2 : The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate the area of the rectangle with error limits.

Sol. Here $l = (5.7 \pm 0.1)$ cm, $b = (3.4 \pm 0.2)$ cm

Area : $A = l \times b = 5.7 \times 3.4 = 19.38\,cm^2 = 19\,cm^2$

(rounding off to two significant figures)

$$\therefore \frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right)$$

$$= \pm\left(\frac{0.34+1.14}{5.7 \times 3.4}\right) = \pm\frac{1.48}{19.38}$$

$$\Rightarrow \Delta A = \pm\frac{1.48}{19.38} \times A = \pm\frac{1.48}{19.38} \times 19.38 = \pm 1.48 = \pm 1.5$$

(rounding off to two significant figures)

So, Area $=(19.0\pm1.5)cm^2$

EX.3: The distance covered by a body in time (5.0 ± 0.6) s is (40.0 ± 0.4) m. Calculate the speed of the body. Also determine the percentage error in the speed.

Sol. Here, $s=(40.0\pm0.4)m$ and $t=(5.0\pm0.6)s$

$\therefore$ Speed $v=\frac{s}{t}=\frac{40.0}{5.0}=8.0\,ms^{-1}$ As $v=\frac{s}{t}$

$\therefore \frac{\Delta v}{v}=\frac{\Delta s}{s}+\frac{\Delta t}{t}$

Here $\Delta s=0.4\,m$, s=40.0 m, $\Delta t=0.6\,s$, t=5.0 s

$\therefore \frac{\Delta v}{v}=\frac{0.4}{40.0}+\frac{0.6}{5.0}=0.13$

$\Rightarrow \Delta v=0.13\times8.0=1.04$

Hence, $v=(8.0\pm1.04)ms^{-1}$

$\therefore$ Percentage error $=\left(\frac{\Delta v}{v}\times100\right)=0.13\times100=13\%$

EX. 4: A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
[AIEEE 2011]

Sol. Main scale reading = 0 mm

Circular scale reading = 52 divisions

$\text{Least count}=\frac{\text{value of 1 main scale division}}{\text{Total divisions on circular scale}}=\frac{1}{100}mm$

Diameter of wire = M.S.R +(C.S.R x L.C)

$=0+52\times\frac{1}{100}mm=0.52mm$ $=0.052cm$

EX.5: The current voltage relation of diode is given by $I=\left(e^{1000V/T}-1\right)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring $\pm$ 0.01V while measuring the current of 5mA at 300K, what will be the error in the value of current in mA? (JEE MAIN-2014)

Sol. $I=\left(e^{1000V/T}-1\right)mA$

dV= $\pm$ 0.01V, T=300K, I=5mA

$I+1=e^{1000V/T}$

$\log(I+1)=\frac{1000V}{T}$

$\frac{dI}{I+1}=\frac{1000}{T}dV \Rightarrow$ dI=0.2mA

EX.6 : In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree(=0.5^0), then the least count of the instrument is (AIEEE-2009)

Sol. **Least count** $= \dfrac{\text{Values of main scale division}}{\text{No. of divisions of vernier scale}}$

$= \dfrac{1}{30} MSD = \dfrac{1}{30} \times \dfrac{1^0}{2} = \dfrac{1^0}{60} = 1$ **min**

Combination of Errors:

↳ **Error due to addition**

If $Z = A + B$;

$\Delta Z = \Delta A + \Delta B$ (Max. possible error)

$Z + \Delta Z = (A + B) \pm (\Delta A + \Delta B)$

Relative error$= \dfrac{\Delta A + \Delta B}{A + B}$ **Percentage error**$= \dfrac{\Delta A + \Delta B}{A + B} \times 100$

↳ **Error due to subtraction**

If Z=A-B

$\Delta Z = \Delta A + \Delta B$ (Max. possible error)

$Z + \Delta Z = (A - B) \pm (\Delta A + \Delta B)$

Relative error $= \dfrac{\Delta A + \Delta B}{A - B}$ **Percentage error** $= \dfrac{\Delta A + \Delta B}{A - B} \times 100$

↳ Whether it is addition or subtraction, absolute error is same.

↳ In subtraction the percentage error increases.

↳ Error due to Multiplication:

If $Z = AB$ then $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

$\dfrac{\Delta Z}{Z}$ is called fractional error or relative error.

Percentage error $= \dfrac{\Delta Z}{Z} \times 100 = \left(\dfrac{\Delta A}{A} \times 100\right) + \left(\dfrac{\Delta B}{B} \times 100\right)$

↳ Here percentage error is the sum of individual percentage errors.

↳ **Error due to division:** if $Z = \dfrac{A}{B}$

Maximum possible relative error $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

Max. percentage error in division $= \dfrac{\Delta A}{A} \times 100 + \dfrac{\Delta B}{B} \times 100$

↳ **Error due to Power:**

If Z= A^n ; $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

↳ **In more general form :** If $Z = \frac{A^p B^q}{C^r}$

then maximum fractional error in Z is $\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

As we check for maximum error a +ve sign is to be taken for the term $r\frac{\Delta C}{C}$

Maximum Percentage error in Z is

$$\frac{\Delta Z}{Z}\times 100 = p\frac{\Delta A}{A}\times 100 + q\frac{\Delta B}{B}\times 100 + r\frac{\Delta C}{C}\times 100$$

EX.7: A physical quantity is represented by x =$M^aL^bT^{-c}$. The percentage of errors in the measurements of mass,length and time are $\alpha\%, \beta\%, \gamma\%$ respectively then the maximum percentage error is

Sol. $\frac{\Delta x}{x}\times 100 = a.\frac{\Delta M}{M}\times 100 + b.\frac{\Delta L}{L}\times 100 + c.\frac{\Delta T}{T}\times 100$

$= a\alpha + b\beta + c\gamma$

EX.8:Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is [AIEEE 2012]

Sol. $R = \frac{V}{I}$ $\quad [\therefore \log R = \log V - \log I]$

$$\Rightarrow \frac{\Delta R}{R}(100) = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)(100)$$

$= 3\% + 3\% = 6\%$

EX.9: Two resistors of resistances $R_1 = (100 \pm 3)$ ohm and $R_2 = (200 \pm 4)$ ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$ and for (b)

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \text{ and } \frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

Sol. **(a) The equivalent resistance of series combination**

$R = R_1 + R_2 = (100 \pm 3)ohm + (200 \pm 4)ohm$

$= (300 \pm 7)ohm.$

(b) The equivalent resistance of parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\, ohm$$

Then, from $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

we get, $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$$\Delta R' = \left(R'^2\right)\frac{\Delta R_1}{R_1^2} + \left(R'^2\right)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 \;= 1.8$$

Then, $R' = (66.7 \pm 1.8)\, ohm$

Significant Figures :

- A significant figure is defined as the figure, which is considered reasonably, trust worthy in number.

  Ex: $\pi = 3.141592654$
  (upto 10 digits)
  =3.14 (with 3 figures)
  =3.1416 (upto 5 digits)
- The significant figures indicate the extent to which the readings are reliable.

Rules for determining the number of significant figures:

- All the non-zero digits in a given number are significant without any regard to the location of the decimal point if any.

  Ex: 18452 or 1845.2 or 184.52 all have the same number of significant digits,i.e. 5.
- All zeros occurring between two non zero digits are significant without any regard to the location of decimal point if any.

  Ex: 106008 has six significant digits.
  106.008 or 1.06008 has also got six significant digits.
- If the number is less than one, all the zeros to the right of the decimal point but to the left of first non-zero digit are not significant.

  Ex: 0.000308
  In this example all zeros before 3 are insignificant.
- a)All zeros to the right of a decimal point are significant if they are not followed by a non-zero digit.

  Ex: 30.00 has 4 significant digits
- b) All zeros to the right of the last non-zero digit after the decimal point are significant.

  Ex: 0.05600 has 4 significant digits
- c) All zeros to the right of the last non-zero digit in a number having no decimal point are not significant.

  Ex: 2030 has 3 significant digits

Rounding off numbers:

- The result of computation with approximate numbers, which contain more than one uncertain digit,should be rounded off.

Rules for rounding off numbers:

- The preceding digit is raised by 1 if the immediate insignificant digit to be dropped is more than 5.
  Ex: 4728 is rounded off to three significant figures as 4730.
- The preceding digit is to be left unchanged if the immediate insignificant digit to be dropped is less than 5.
  Ex: 4723 is rounded off to three significant figures as 4720
- If the immediate insignificant digit to be dropped is 5 then there will be two different cases
  a) If the preceding digit is even then it is to be unchanged and 5 is dropped.
  Ex: 4.7253 is to be rounded off to two decimal places. The digit to be dropped here is 5 (along with 3) and the preceding digit 2 is even and hence to be retained as two only
  4.7253=4.72
  b)If the preceding digit is odd, it is to be raised by 1
  Ex: 4.7153 is to be rounded off to two decimal places. As the preceding digit '1' is odd, it is to be raised by 1.
  4.7153=4.72

Rules for Arithmetic Operations with significant Figures:

- In multiplication or division, the final result should retain only that many significant figures as are there in the original number with the least number of significant figures.
  Ex: $1.2 \times 2.54 \times 3.26 = 9.93648$ .But the result should be limited to the least number of significant digits- that is two digits only. So final answer is 9.9.
- In addition or subtraction the final result should retain only that many decimal places as are there in the number with the least decimal places.
  Ex:2.2+4.08+3.12+6.38=15.78.Finally we should have only one decimal place and hence 15.78 is to be rounded off as 15.8.

EX.10:The respective number of significant figures for the numbers 23.023,0.0003and 21×10^{-3} are (AIEEE-2010)

Sol.(i)All non -zero numbers are significant figures

(ii) If the number is less than one,zero between the decimal and first non zero digit are not significant.

(iii) Powers of 10 is not a significant figure.

$\therefore$ 5,1,2

S.No.	Physical Quantity	Formula	Dimensional Formula	SI Unit
1.	Displacement, Wave length, Radius of gyration, Circumference, Perimeter,Light year,		$\left[M^0\ L^1\ T^0\right]$	m
2.	Mass		$\left[M^1\ L^0\ T^0\right]$	kg
3.	Period of oscillation,	$\frac{\text{total time}}{\text{no.of oscillations}}$		
	Time, Time constant	T = Capacity $\times$ Resistance	$\left[M^0\ L^0\ T^1\right]$	s
4.	Frequency	Reciprocal of time period $n = \frac{1}{T}$	$\left[M^0 L^0 T^{-1}\right]$	hertz (Hz)
5.	Area	A = length $\times$ breadth	$\left[M^0 L^2 T^0\right]$	m^2
6.	Volume	V=length $\times$ breadth $\times$ height	$\left[M^0 L^3 T^0\right]$	m^3
7.	Density	d= $\frac{\text{mass}}{\text{volume}}$	$\left[M^1 L^{-3} T^0\right]$	kgm^{-3}
8.	Linear mass density	λ= $\frac{\text{mass}}{\text{length}}$	$\left[M^1 L^{-1} T^0\right]$	kgm^{-1}
9.	Speed, Velocity	v= $\frac{\text{displacement}}{\text{time}}$	$\left[M^0 L^1 T^{-1}\right]$	ms^{-1}
10.	Acceleration	a = $\frac{\text{change in velocity}}{\text{time}}$	$\left[M^0 L^1 T^{-2}\right]$	ms^{-2}
11.	Linear momentum	P= mass $\times$ velocity	$\left[M^1 L^1 T^{-1}\right]$	$kgms^{-1}$
12.	Force	F = Mass $\times$ acceleration	$\left[M^1 L^1 T^{-2}\right]$	N
13.	Impulse	J= Force $\times$ time	$\left[M^1 L^1 T^{-1}\right]$	Ns
14.	Work,Energy,PE, KE, Strain energy, Heat energy	W = Force $\times$ displacement P.E= mgh KE = $\frac{1}{2}$ (Mass) (velocity)^2 SE= $\frac{1}{2}\times$Stress$\times$Strain$\times$volume	$\left[M^1 L^2 T^{-2}\right]$	J(or) N.m
15.	PoEX.r	P= $\frac{\text{Work}}{\text{time}}$	$\left[M^1 L^2 T^{-3}\right]$	watt
16.	Pressure , Stress,	$\frac{\text{Force}}{\text{Area}}$		
	Modulus of elasticity (Y, η, k)	Y= $\frac{\text{Stress}}{\text{Strain}}$	$\left[M^1 L^{-1} T^{-2}\right]$	pascal or Nm^{-2}

17.	Strain	$=\frac{\text{change in dimension}}{\text{original dimension}}$	$\left[M^0L^0T^0\right]$	no units
18.	Strain energy density	$E=\frac{\text{work}}{\text{volume}}$	$\left[M^1L^{-1}T^{-2}\right]$	Jm^{-3}
19.	Angular displacement	$\theta=\frac{\text{length of arc}}{\text{radius}}$	$\left[M^0L^0T^0\right]$	rad
20.	Angular velocity	$\omega=\frac{\text{angular dispacement}}{\text{time}}$	$\left[M^0L^0T^{-1}\right]$	$rads^{-1}$
21.	Angular acceleration	$\alpha=\frac{\text{change in angular velocity}}{\text{time}}$	$\left[M^0L^0T^{-2}\right]$	$rads^{-2}$
22.	Angular momentum	L=linear momentum $\times$perpendicular distance	$\left[M^1L^2T^{-1}\right]$	Js
23.	Planck's constant	$h=\frac{\text{energy}}{\text{frequency}}$	$\left[M^1L^2T^{-1}\right]$	Js
24.	Angular impulse	Torque $\times$ time	$\left[M^1L^2T^{-1}\right]$	Js
25.	Torque	τ=force$\times\perp$ distance	$\left[M^1L^2T^{-2}\right]$	Nm
26.	Acceleration due to gravity(g)	$g=\frac{\text{weight}}{\text{mass}}$	$\left[M^0LT^{-2}\right]$	ms^{-2} or Nkg^{-1}
27.	Universal gravitational Constant	$G=\frac{\text{Force}\times(\text{distence})^2}{\text{Mass}_1\times\text{Mass}_2}$	$\left[M^{-1}L^3T^{-2}\right]$	$Nm^2\ kg^{-2}$
28.	Moment of inertia	I=Mass $\times$ (radius of gyration)^2	$\left[M^1L^2T^0\right]$	kgm^2
29.	Velocity gradient	$=\frac{dv}{dx}$	$\left[M^0L^0T^{-1}\right]$	S^{-1}
30.	Surface tension, Surface energy	$S=\frac{\text{surface energy}}{\text{change in area}}=\frac{\text{force}}{\text{length}}$	$\left[M^1L^0T^{-2}\right]$	Nm^{-1} or Jm^{-2}
	Spring constant Force constant	$K=\frac{\text{force}}{\text{elongation}}$		
31.	Coefficient of viscosity	$\eta=\frac{\text{tangential stress}}{\text{velocity gradient}}$	$\left[M^1L^{-1}T^{-1}\right]$	Pa s (or) $Nm^{-2}s$
32.	Gravitational potential	Gravitational field $\times$ distance	$\left[M^0L^2T^{-2}\right]$	J/Kg
33.	Heat energy	$ms\theta$	$\left[M^1L^2T^{-2}\right]$	joule
34.	Temperature	θ	$\left[M^0L^0T^0\theta^1\right]$	kelvin(K)
35.	Specific heat capacity	S (or) $C=\frac{\text{heat energy}}{\text{mass}\times\text{temp.}}$	$\left[M^0L^2T^{-2}\theta^{-1}\right]$	$Jkg^{-1}\ K^{-1}$
36.	Thermal capacity	$\frac{dQ}{d\theta}=\text{mass}\times\text{sp.ht}$	$\left[M^1L^2T^{-2}\theta^{-1}\right]$	JK^{-1}

37. Latent heat (or) Calorific value	$L=\frac{\text{heat energy}}{\text{mass}}$	$\left[M^0L^2T^{-2}\right]$	Jkg^{-1}
38. Water equivalent	W=Mass × specific heat	$\left[M^1L^0T^0\right]$	kg
39. Coefficient of thermal expansion	$\alpha=\frac{\Delta l}{l\Delta\theta};\beta=\frac{\Delta A}{A\Delta\theta};\gamma=\frac{\Delta V}{V\Delta\theta}$	$\left[M^0L^0T^0\theta^{-1}\right]$	K^{-1}
40. Universal gas constant	$R=\frac{PV}{nT}$	$\left[M^1L^2T^{-2}\theta^{-1}mol^{-1}\right]$	$Jmol^{-1}K^{-1}$
41. Gas constant (for 1 gm)	$r=\frac{R}{\text{Mol.wt}}$	$\left[M^0L^2T^{-2}\theta^{-1}mol^{-1}\right]$	$Jkg^{-1}K^{-1}$
42. Boltzmann's constant (for 1 Molecule)	$k=\frac{R}{\text{Avagadro number}}$	$\left[M^1L^2T^{-2}\theta^{-1}\right]$	$JK^{-1}molecule^{-1}$
43. Mechanical equivalent of heat	$J=\frac{W}{H}$	$\left[M^0L^0T^0\right]$	no SI units
44. Coefficient of thermal conductivity	$K=\frac{Qd}{A\ \Delta\theta t}$	$\left[M^1L^1T^{-3}\theta^{-1}\right]$	$Js^{-1}\ m^{-1}\ K^{-1}$ (or) $Wm^{-1}\ K^{-1}$
45. Entropy	$\frac{dQ}{T}=\frac{\text{heat energy}}{\text{temperature}}$	$\left[M^1L^2T^{-2}\theta^{-1}\right]$	JK^{-1}
46. Stefan's constant	$\sigma=\frac{\Delta E}{\Delta A \Delta T\theta^4}$	$\left[M^1L^0T^{-3}\theta^{-4}\right]$	$Js^{-1}m^{-2}K^{-4}$ (or) $Wm^{-2}K^{-4}$
47. Thermal resistance	$R=\frac{d\theta}{\left(\frac{dQ}{dt}\right)}=\frac{\text{temp}\times\text{time}}{\text{Heat}}$ (or) $R=\frac{d}{KA}$	$\left[M^{-1}L^{-2}T^3\theta^1\right]$	KsJ^{-1}
48. Temperature gradient	$\frac{\text{Change in temp}}{\text{length}}=\frac{d\theta}{dl}$	$\left[M^0L^{-1}T^0\theta^1\right]$	Km^{-1}
49. Pressure gradient	$\frac{\text{Change in pressure}}{\text{length}}=\frac{dp}{dl}$	$\left[M^1L^{-2}T^{-2}\right]$	pascal m^{-1}
50. Solar constant	$\frac{\text{Energy}}{\text{area}\times\text{time}}=\frac{\Delta E}{AT}$	$\left[M^1L^0T^{-3}\right]$	$Js^{-1}m^{-2}$ (or) Wm^{-2}
51. Enthalpy	heat (ΔQ)	$\left[M^1L^2T^{-2}\right]$	joule
52. Pole strength	m=IL (or) $\frac{\text{Magnetic Momement}}{\text{Mag.Length}}$	$\left[M^0LT^0A\right]$	Am
53. Magnetic moment	M= $2l\times m$	$\left[M^0L^2T^0A\right]$	Am^2

54.	Magnetic intensity (or) Magnetising field	$H=\frac{m}{4\pi d^2}$	$\left[M^0L^{-1}T^0A\right]$	Am^{-1}
55.	Intensity of magnetisation	$I=\frac{\text{Magnetic moment}}{\text{Volume}}$	$\left[M^0L^{-1}T^0A\right]$	Am^{-1}
56.	Magnetic flux	$\varphi = \vec{B}\times\vec{A}$ =(Magnetic induction $\times$ Area)	$\left[M^1L^2T^{-2}A^{-1}\right]$	Wb
57.	Magnetic induction	$\vec{B} = \frac{\phi}{A} = \frac{\text{Magnetic flux}}{\text{Area}} = \frac{F}{il}$	$\left[M^1L^0T^{-2}A^{-1}\right]$	Tesla (or) Wbm^{-2} (or) $NA^{-1}m^{-1}$
58.	Magnetic permeability	$\mu=\frac{4\pi Fd^2}{m_1m_2}$	$\left[M^1L^1T^{-2}A^{-2}\right]$	Hm^{-1}
59.	Magnetic susceptibility	$\chi=\frac{I}{H}$	$\left[M^0L^0T^0\right]$	no units
60.	Electric current	I	$\left[M^0L^0T^0A\right]$	A
61.	Charge	Q =Current $\times$ Time	$\left[M^0L^0TA\right]$	C
62.	Electric dipole moment	P=Charge $\times$ Distance	$\left[M^0L^1AT\right]$	Cm
63.	Electric field strength (or) Electric field intensity	$E=\frac{\text{Force}}{\text{Charge}}$	$\left[M^1LT^{-3}A^{-1}\right]$	NC^{-1}
64.	Electrical flux (ϕ_E)	Electrical intensity $\times$ area	$\left[M^1L^3T^{-3}A^{-1}\right]$	$Nm^2\ C^{-1}$
65.	Electric potential (or) Potential difference	$V=\frac{\text{Work}}{\text{Charge}}$	$\left[M^1L^2T^{-3}A^{-1}\right]$	V
66.	Electrical resistance	$R=\frac{\text{Pot.diff}}{\text{Current}}$	$\left[M^1L^2T^{-3}A^{-2}\right]$	Ω
67.	Electrical conductance	$C=\frac{1}{R}=\frac{1}{\text{Resistance}}$	$\left[M^{-1}L^{-2}T^3A^2\right]$	mho (or) Siemen (S)
68.	Specific resistance (or Resistivity ρ (or) s	$\rho=\frac{RA}{l}$	$\left[M^1L^3T^{-3}A^{-2}\right]$	Ohm-m
69.	Electrical conductivity	$\sigma = \frac{1}{\text{Resistivity}}$	$\left[M^{-1}L^{-3}T^3A^2\right]$	$Ohm^{-1}\ m^{-1}$ (or) Siemen m^{-1}
70.	Current density (current per unit area of cross section)	J = Electrical intensity $\times$ Conductivity or $\left(\frac{\text{Current}}{\text{Area}}\right)$	$\left[M^0L^{-2}T^0A\right]$	Am^{-2}
71.	Capacitance	$C=\frac{Q}{V}=\frac{\text{Charge}}{\text{Potential}}$	$\left[M^{-1}L^{-2}T^4A^2\right]$	F
72.	Self (or) Mutual inductance	$L=\frac{d\varepsilon}{\left(\frac{dI}{dt}\right)}=\frac{\text{Voltage}\times\text{Time}}{\text{Current}}$	$\left[M^1L^2T^{-2}A^{-2}\right]$	H(or) Wb/amp

73.	Electrical permittivity	$\varepsilon=\frac{q_1q_2}{4\pi Fd^2}$	$\left[M^{-1}L^{-3}T^4A^2\right]$	farad/m
74.	Surface charge density	$\frac{\text{Charge}}{\text{Area}}$	$\left[M^0L^{-2}T^1A^1\right]$	Cm^{-2}
75.	Luminous flux	$\frac{\text{Light energy}}{\text{Time}}$	$\left[M^1L^2T^{-3}\right]$	lumen
76.	Intensity of illumination (or) Iluminance	$I=\frac{\Delta E}{\Delta t\Delta A}=\left(\frac{\text{Luminous flux}}{\text{Area}}\right)$	$\left[M^1L^0T^{-3}\right]$	lumen m^{-2} (or) lux.
77.	Focal poEX.r	$P=\frac{1}{\text{Focal length}}$	$\left[M^0L^{-1}T^0\right]$	dioptre
78.	Wave number (Propagation constant)	$\bar{\nu}=\frac{1}{\lambda}$	$\left[M^0L^{-1}T^0\right]$	m^{-1}
79.	Rydberg's constant	$R=\frac{Z^2e^4m}{8\varepsilon_0^2ch^3}$	$\left[M^0L^{-1}T^0\right]$	m^{-1}

- Dimensions of a physical quantity are the powers to which the fundamental quantities are to be raised to represent that quantity.

Dimensional Formula :

- An expression showing the powers to which the fundamental quantities are to be raised to represent the derived quantity is called dimensional formula of that quantity.

In general the dimensional formula of a quantity can be written as $\left[M^xL^yT^z\right]$. Here x,y,z are dimensions.

Dimensional Constants:

- The physical quantities which have dimensions and have a fixed value are called dimensional constants.

Ex: Gravitational constant (G), Planck's constant (h), Universal gas constant (R), Velocity of light in vacuum (c) etc.,

Dimensionless Quantities:

- Dimensionless quantities are those which do not have dimensions but have a fixed value.

(a):Dimensionless quantities without units.

Ex: Pure numbers,angle trigonometric functions , logarthemic functions etc.,

(b)Dimensionless quantities with units.

Ex: Angular displacement - radian, Joule's constant etc.,

Dimensional variables:

- Dimensional variables are those physical quantities which have dimensions and do not have fixed value.

Ex: velocity, acceleration, force, work, power.etc.

Dimensionless variables:

- Dimensionless variables are those physical quantities which do not have dimensions and do not have fixed value.,

Ex: Specific gravity, refractive index, Coefficient of friction, Poisson's Ratio etc.,

Limitationas of Dimensional analysis method

- Dimensionless quantities cannot be determined by this method. Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
- This method is not applicable to trigonometric, logarithmic and exponential functions.
- In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.
- In some cases, the constant of proportionality also possesses dimensions. In such cases we cannot use this system.
- If one side of equation contains addition or subtraction of physical quantities, we cannot use this method.

EX.11: Let $[\varepsilon_0]$ denote the dimensional formula of permittivity of vacuum .If M is mass ,L is length,T is time and A is electric current,then (JEE-MAIN 2013)

Sol. **From coulomb's law** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

Substituting the units

$$\varepsilon_0 = \frac{c^2}{N-m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$$

EX.12: The dimensional formula of magnetic field strength in M, L, T and C (coulomb) is given as (AIEEE 2008)

Sol. From F = Bqv

$$B = \frac{F}{qv} = \frac{[MLT^{-2}]}{C[LT^{-1}]} = [M^1L^0T^{-1}C^{-1}]$$

Physical Quantities Having Same Dimensional Formulae:

- Distance, Displacement, radius, wavelength, radius of gyration [L]
- Speed, Velocity, Velocity of light $[LT^{-1}]$
- acceleration ,acceleration due to gravity, intensity of gravitational field, centripetal acceleration $[LT^{-2}]$
- Impulse, Change in momentum $[MLT^{-1}]$
- Force, weight, Tension,energy gradient, Thrust $[MLT^{-2}]$
- Work, Energy, Moment of force or Torque, Moment of couple $[ML^2T^{-2}]$
- Force constant, Surface Tension, Spring constant, surface energy i.e. Energy per unit area $[MT^{-2}]$
- Angular momentum, Angular impulse, Planck's constant $[ML^2T^{-1}]$
- Angular velocity, Frequency, angular frequency,Velocity gradient, Decay constant, rate of disintegration $[T^{-1}]$
- Stress, Pressure, Modulus of Elasticity, Energy density $[ML^{-1}T^{-2}]$

- Latent heat, Gravitational potential $\left[L^2 T^{-2}\right]$
- Specific heat, Specific gas constant $\left[L^2 T^{-2} \theta^{-1}\right]$
- Thermal capacity, Entropy, Boltzmann constant, Molar thermal capacity, $\left[M L^2 T^{-2} \theta^{-1}\right]$
- Wave number, Power of a lens, Rydberg's constant $\left[L^{-1}\right]$
- Time, RC, $\frac{L}{R}$, $\sqrt{LC}\,(T)$
- Power, Rate of dissipation of energy, $\left[ML^2T^{-3}\right]$
- Intensity of sound, Intensity of radiation [MT^{-3}]
- Electric potential, potential difference, electromotive force [$ML^2T^{-3}I^{-1}$]
- Intensity of magnetic field, Intensity of magnetization $\left[I L^{-1}\right]$
- Electric field and potential gradient $\left[MLT^{-3}A^{-1}\right]$
- Rydberg's constant and propagation constant $\left[M^0L^{-1}T^0\right]$
- Strain , Poisson's ratio, refractive index, dielectric constant, coefficient of friction, relative permeability, magnetic susceptibility, electric susceptibility, angle, solid angle, trigonometric ratios,logarithm function, exponential constant are all dimensionless.
- If L,C and R stands for inductance, capacitance and resistance respectively then $\frac{L}{R}, \sqrt{LC}, RC$ and time $\left[M^0L^0T\right]$
- Coefficient of linear expansion, coefficient of superficial expansion and coefficient of cubical expansion,temperature coefficient of resistance $\left[M^0L^0T^0K^{-1}\right]$
- Solar constant and poynting vector $\left[ML^0T^{-3}\right]$

Principle of homogeneity:

- It states that only quantities of same dimensions can be added, subtracted and equated.

EX.13: The dimensional formula of $\frac{a}{b}$ in the equation $P = \frac{a - ct^2}{bx}$ where P = pressure, x = displacement and t = time

Sol. $[P] = \left[\frac{a}{bx}\right] - \left[\frac{ct^2}{bx}\right]$

By principle of Homogeneity, $\left[\frac{a}{bx}\right]$ should represent pressure

$\therefore \left[\frac{a}{b}\right]\frac{1}{L} = \left[ML^{-1}T^{-2}\right] \Rightarrow \left[\frac{a}{b}\right] = \left[MT^{-2}\right]$

Uses of dimensional analysis method:

- ↳ **To check the correctness of the given equation.** (This is based on the principle of homogeneity)
- ↳ **To convert one system of units into another system.**
- ↳ **To derive the equations showing the relation between different physical quantities.**

EX.14:Check whether the relation $S = ut + \frac{1}{2}at^2$ is dimensionally correct or not, where symbols have their usual meaning.

Sol. we have $S = ut + \frac{1}{2}at^2$. checking the dimensions on both sides, LHS=$[S] = [M^0 L^1 T^0]$,

RHS=$[ut] + \left[\frac{1}{2}at^2\right] = [LT^{-1}][T] + [LT^{-2}][T^2]$

$= [M^0 L^1 T^0] + [M^0 L^1 T^0] = [M^0 L^1 T^0]$

we find LHS=RHS.

Hence, the formula is dimensionally correct.

EX.15:Young's modulus of steel is $19\times10^{10}\, N/m^2$. Express it in $dyne/cm^2$. Here dyne is the CGS unit of force.

Sol. The SI unit of Young's modulus is N/m^2.

Given $Y = 19\times10^{10}\frac{N}{m^2} = 19\times10^{10}\left(\frac{10^5\, dyne}{(10^2\, cm)^2}\right) = 19\times10^{11}\left(\frac{dyne}{cm^2}\right)$

EX.16 : For a particle to move in a circular orbit uniformly, centripetal force is required, which depends upon the mass (m), velocity (v) of the particle and the radius (r) of the circle. Express centripetal force in terms of these quantities

Sol. According to the provided information,

let $F \propto m^a v^b r^c$. $\Rightarrow F = km^a v^b r^c$

$\Rightarrow [M^1 L^1 T^{-2}] = [M^a (LT^{-1})^b L^c]$

$\Rightarrow [M^1 L^1 T^{-2}] = [M^a L^{b+c} T^{-b}]$

using principle of homogeneity we have

a = 1 ,b + c = 1 ,b = 2

on solving we have a = 1, b = 2, c = -1

using these values we get F = $km^1 v^2 r^{-1}$

$\Rightarrow F = k\frac{mv^2}{r}$

Note: The value of the dimensionless constant k is to be found experimentally.

EX.17: Derive an expression for the time period of a simple pendulum of mass(m), length (l) at a place where acceleration due to gravity is (g).

Sol. Let the time period of a simple pendulum depend upon the mass of bob m, length of pendulum l, and acceleration due to gravity g, then

$$t \propto m^a l^b g^c \Rightarrow t = km^a l^b g^c$$

$$M^0L^0T^1 = M^aL^b\left[LT^{-2}\right]^c \Rightarrow M^0L^0T^1 = M^aL^{b+c}T^{-2c}$$

comparing the powers of M, L, and T on both sides, we get a = 0, b + c = 0, -2c=1

$\Rightarrow$ a = 0, b = 1/2 and c = -1/2. Putting these values, we get $T = km^0\frac{l^{1/2}}{g^{1/2}} \Rightarrow T = k\sqrt{\frac{l}{g}}$,

which is the required relation.

EX.18: If C is the velocity of light, h is Planck's constant and G is Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is.(Eamcet - 2014)

Sol. $C = \left[LT^{-1}\right] \rightarrow (1)$; $h = \left[ML^2T^{-1}\right] \rightarrow (2)$

$G = \left[M^{-1}L^3T^{-2}\right] \rightarrow (3)$

Solving (2) and (3)

$$\frac{h}{G} = \left[\frac{ML^2T^{-1}}{M^{-1}L^3T^{-2}}\right] = \left[M^2L^{-1}T^1\right]$$

Substituting (1) in above

$$\frac{h}{G} = \frac{M^2}{C} \Rightarrow [\mathrm{M}] = \left[\mathrm{h}^{\frac{1}{2}}\,\mathrm{G}^{\frac{-1}{2}}\,\mathrm{C}^{\frac{1}{2}}\right]$$

EX.19: If E, M, J and G respectively denote energy, mass, angular momentum and universal gravitational constant, the quantity, which has the same dimensions as the dimensions of $\frac{EJ^2}{M^5G^2}$ (Eamcet - 2013)

Sol. D.F. of $\frac{EJ^2}{M^5G^2}$

Substituting D.F. of E, J, M, and G in above formula

$$= \frac{ML^2T^{-2}\left[ML^2T^{-1}\right]^2}{M^5\left[M^{-1}L^3T^{-2}\right]^2} = \left[M^0L^0T^0\right]$$

EX.20: In the equation $\left(\frac{1}{p\beta}\right) = \frac{y}{k_BT}$ where p is the pressure, y is the distance, k_B is Boltzmann constant and T is the temperature. Dimensions of β are (Med- 2013)

Sol. $\frac{1}{p\beta} = \frac{y}{k_BT}$

Dimension of $[\beta]=\frac{[Dimensional\ formulae\ of\ k_B][Dimensional\ formulae\ of\ T]}{[Dimensional\ formulae\ of\ p][Dimensional\ formulae\ of\ y]}$

$$=\frac{\left[ML^2T^{-3}\right][T]}{\left[ML^{-1}T^{-2}\right][L]}=\left[M^0L^2T^0\right]$$

$\therefore$ Dimensions of M,L,T in β are 0,2,0

EX.21: The vander Waal's equation for n moles of a real gas is $\left(p+\frac{a}{V^2}\right)(V-b)=nRT$ where p is pressure, V is volume, T is absolute temperature, R is molar gas constant a, b and c are vander Waal's constants. The dimensional formula for ab is (Med- 2012)

Sol. By principle of homogenity of dimensions P can added to P only. It means $\frac{a}{V^2}$ also gives pressure.

Dimension formulae for pressure $(P)=\left[M^1L^{-1}T^{-2}\right]$ and Volume $(V)=\left[M^0L^3T^0\right]$

Since $\frac{a}{V^2}=$ pressure

$$\therefore \frac{a}{\left(M^0L^3T^0\right)}=\left[M^1L^{-1}T^{-2}\right]\Rightarrow\frac{a}{M^0L^6T^0}=\left[M^1L^{-1}T^{-2}\right]$$

$$\therefore a=\left[M^1L^5T^{-2}\right]$$

similarly, b will have same dimensions as volume $V-b=volume$

$$\therefore b=\left[M^0L^3T^0\right]$$

$$\therefore [ab]=\left[M^1L^5T^{-2}\right]\left[M^0L^3T^0\right]=\left[M^1L^8T^{-2}\right]$$

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$$\therefore [ab]=\left[M^1L^5T^{-2}\right]\left[M^0L^3T^0\right]=\left[M^1L^8T^{-2}\right]$$

EX.22: **A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47th circular division coincides with the main scale. Find the curved surface area of the wire in cm^2 to appropriate significant figures.(Use π = 22/7)**

Sol. Least Count $= \frac{1mm}{100} = 0.01mm$

Diameter = MSR + CSR(LC) = 1 mm+47 (0.01) mm = 1.47 mm

Surface area = $\pi Dl = \frac{22}{7} \times 1.47 \times 56 mm^2$

$= 2.58724\, cm^2 = 26 cm^2$

EX.23: **In Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data.**

Sol. Maximum percentage error in Y is given by

$$Y = \frac{W}{\frac{\pi D^2}{4}} \times \frac{L}{x} \Rightarrow \left(\frac{\Delta Y}{Y}\right) = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta x}{x} + \frac{\Delta L}{L}$$

$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489$$

EX.24: **The side of a cube is measured by vernier calipers (10 divisions of the vernier scale coincide with 9 divisions of the main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures.**

Sol. Least count of vernier calipers

$$= \frac{1\, division\ of\ main\ scale}{Number\ of\ divisions\ in\ vernier\ scale} = \frac{1}{10} = 0.1mm$$

The side of cube = 10 mm + $1 \times 0.1 mm = 1.01 cm$

Now, density $= \frac{Mass}{Volume} = \frac{2.736 g}{(1.01)^3\, cm^3} = 2.66 g\, cm^{-3}$

Accuracy, precision, types of errors and combination of errors

EX 25. The accuracy in the measurement of the diameter of hydrogen atom as 1.06×10^{-10} **m is**

1) 0.01 2) 106×10^{-10} 3) $\frac{1}{106}$ 4) 0.01×10^{-10}

Sol : $\frac{\Delta d}{d} = \frac{0.01 \times 10^{-10}}{1.06 \times 10^{-10}} = \frac{1}{106}$ key-3

EX 26. **The length of a rod is measured as 31.52 cm. Graduations on the scale are up to**

1) 1 mm 2) 0.01 mm 3) 0.1 mm 4) 0.02 cm

Sol : $0.01cm$ is the least count of varnier caliperse.

key-3

EX 27. **If $L=(20\pm 0.01)m$ and $B=(10\pm 0.02)m$ then L/B is**

1) $(2\pm 0.03)m$ 2) $(2\pm 0.015)m$

3) $(2\pm 0.01)m$ 4) $(2\pm 0.005)m$

Sol : $\frac{\Delta x}{x}=\frac{\Delta L}{L}+\frac{\Delta B}{B}\Rightarrow \Delta x=x\left[\frac{\Delta L}{L}+\frac{\Delta B}{B}\right]$

$=\frac{20}{10}\left[\frac{0.01}{20}+\frac{0.02}{10}\right]$

$x\pm\Delta x=(2\pm 0.005)m$

key-4

EX 28. **The radius of a sphere is measured as $(10\pm 0.02\%)cm$. The error in the measurement of its volume is**

1) 25.1cc 2)25.12cc 3)2.51cc 4)251.2cc

Sol : $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta v}{v}\times 100=3\frac{\Delta r}{r}\times 100$

$\Delta v=3\times\frac{\Delta r}{r}\times v$, key-3

EX 29. **If length and breadth of a plate are (40 ± 0.2)cm and $(30\pm 0.1)cm$, the absolute error in measurement of area is**

1) $10cm^2$ 2) $8cm^2$ 3) $9cm^2$ 4) $7cm^2$

Sol : $A=lb\Rightarrow\frac{\Delta A}{A}=\frac{\Delta l}{l}+\frac{\Delta b}{b}\Rightarrow \Delta A=A\left[\frac{\Delta l}{l}+\frac{\Delta b}{b}\right]$

$\Delta A=b\Delta l+l\Delta b=10cm^2$, key-1

EX 30. **If the length of a cylinder is measured to be 4.28 cm with an error of 0.01 cm, the percentage error in the measured length is nearly**

1) 0.4 % 2) 0.5 % 3) 0.2 % 4) 0.1 %

Sol : $\frac{\Delta l}{l}\times 100=\frac{0.01}{4.28}\times 100=0.2\%$

key-3

EX 31. **When 10 observations are taken, the random error is x. When 100 observations are taken, the random error becomes**

1) x/10 2) x^2 3) 10 x 4) $\sqrt{x}$

Sol : $X \alpha \frac{1}{N} \Rightarrow \frac{X_1}{X_2} = \frac{N_2}{N_1} = \frac{10}{100}$

key-1

EX 32. If $L_1 = (2.02 \pm 0.01)m$ **and** $L_2 = (1.02 \pm 0.01)m$ **then** $L_1 + 2L_2$ **is (in m)**

1) 4.06 ± 0.02 2) 4.06 ± 0.03

3) 4.06 ± 0.005 4) 4.06 ± 0.01

Sol : $L_1 + 2L_2 = 2.02 + 2 \times 1.02 = 4.06$

$\Delta L_1 + 2\Delta L_2 = 0.01 + 2 \times 0.01 = 0.03$

key-2

EX 33. A body travels uniformly a distance of $(20.0 \pm 0.2)m$ **in time** $(4.0 \pm 0.04)s$**. The velocity of the body is**

1) $(5.0 \pm 0.4)\ ms^{-1}$ 2) $(5.0 \pm 0.2)\ ms^{-1}$

3) $(5.0 \pm 0.6)\ ms^{-1}$ 4) $(5.0 \pm 0.1)\ ms^{-1}$

Sol : $V = \frac{S}{T} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta S}{S} + \frac{\Delta T}{T}$

key-4

Significant figures & Rounding off

EX 34. If the value of 103.5 kg is rounded off to three significant figures, then the value is

1) 103 2) 103.0 3) 104 4) 10.3

Sol : If last digit is 5, if the preceding digit is odd then it should be increased by adding 1 and last digit 5 has to be ignored.

key-3

EX 35. The number of significant figures in $6.023 \times 10^{23}\ mole^{-1}\ is$

1) 4 2) 3 3) 2 4) 23

Sol : Use limitation of significant figures

key-1

EX 36. The side of a cube is 2.5 metre. The volume of the cube to the significant figures is

1) 15 2) 16 3) 1.5 4) 1.6

Sol : $V = l^3$ and rounded off to minimum significant

key-2

EX 37. If the unit of length is doubled and that of mass and time is halved, the unit of energy will be

1) doubled 2)4 times 3)8 times 4) same

Sol : $\frac{E_2}{E_1} = \frac{M_2}{M_1}\left(\frac{L_2}{L_1}\right)^2\left(\frac{T_2}{T_1}\right)^{-2}$

key-3

EX 38. Given M is the mass suspended from a spring of force constant. k. The dimensional formula for $[M/k]^{1/2}$ is same as that for

1) frequency 2) time period 3) velocity 4) wavelength

Sol : Here [k] = force/ length = ML^0T^{-2}

Hence $\left[\frac{M}{k}\right]^{1/2} = M^0L^0T$

key-2

EX 39. The dimensional formula for the product of two physical quantities P and Q is [ML^2T^{-2}]. The dimensional formula of $\frac{P}{Q}$ is [MT^{-2}]. Then P and Q respectively are(2001 M)

1) Force and velocity
2) Momentum and displacement
3) Force and displacement
4) Work and velocity

Sol : $PQ = ML^2T^{-2}$ ----(1); $\frac{P}{Q} = MT^{-2}$ -----(2)

(1) X (2) = $P^2 = M^2L^2T^{-4}$

$\Rightarrow P = MLT^{-2} = FORCE$ (1) / (2) = $Q^2 = L^2$

key-3

EX 40. If the unit of length is doubled and that of mass and time is halved, the unit of energy will be

1) doubled 2)4 times 3)8 times 4) same

Sol : $\frac{E_2}{E_1} = \frac{M_2}{M_1}\left(\frac{L_2}{L_1}\right)^2\left(\frac{T_2}{T_1}\right)^{-2}$

key-3

EX 41. Given M is the mass suspended from a spring of force constant. k. The dimensional formula for $[M/k]^{1/2}$ is same as that for

1) frequency 2) time period
3) velocity 4) wavelength

Sol : Here [k] = force/ length = ML^0T^{-2}

Hence $\left[\frac{M}{k}\right]^{1/2} = M^0L^0T$, key-2

EX 42. The dimensional formula for the product of two physical quantities P and Q is [ML^2T^{-2}]. The dimensional formula of $\frac{P}{Q}$ is [MT^{-2}]. Then P and Q respectively are(2001 M)

1) Force and velocity
2) Momentum and displacement
3) Force and displacement
4) Work and velocity

Sol :

$PQ = ML^2T^{-2}$ ----(1); $\frac{P}{Q} = MT^{-2}$ -----(2)

(1) X (2) = $P^2 = M^2L^2T^{-4}$

$\Rightarrow P = MLT^{-2} = FORCE$ (1) / (2) = $Q^2 = L^2$

key-3

EX 43. If minute is the unit of time, 10 ms^{-2} is the unit of acceleration and 100 kg is the unit of mass, the new unit of work in joule is

1) 10^5 2) 10^6 3) 6 x 10^6 4) 36x 10^6

Sol : $W \alpha Ma^2T^2$; $\frac{W_2}{W_1} = \frac{M_2 a_2^2 T_2^2}{M_1 a_1^2 T_1^2}$

key-4

EX 44. The magnitude of force is 100 N. What will be its value if the units of mass and time are doubled and that of length is halved?

1) 25 2)100 3) 200 4) 400

Sol : $n_1[M_1L_1T_1^{-2}] = n_2[M_2L_2T_2^{-2}]$

key-1

EX 45. If force (F), work (W) and velocity (V) are taken as fundamental quantities then the dimensional formula of Time (T) is (2007 M)

1) $[W^1F^1V^1]$ 2) $[W^1F^1V^{-1}]$

3) $[W^{-1}F^{-1}V^{-1}]$ 4) $[W^1F^{-1}V^{-1}]$

Sol : $T \alpha F^x W^y V^z$; $M^0L^0T^1 \alpha [MLT^{-2}]^x [ML^2T^{-2}]^y [LT^{-1}]^z$

key-4

EX 46. **The error in the measurement of the length of the simple pendulum is 0.2 % and the error in time period 4%. The maximum possible error in measurement of $\frac{L}{T^2}$ is**

1) 4.2% 2) 3.8% 3) 7.8% 4) 8.2%

Sol : Let $x = \frac{L}{T^2}$; $\frac{\Delta x}{x} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

key-4

EX 47. **The least count of a stop watch is (1/5) s. The time of 20 oscillations of a pendulum is measured to be 25 s. The maximum percentage error in this measurement is**

1) 8 % 2) 1 % 3) 0.8 % 4) 16 %

Sol :. $\Delta T = \frac{1/5}{20}$ *and* $T = \frac{25}{20}$; % *error* $= \frac{\Delta T}{T} \times 100$

key-3

EX 48. **The diameter of a wire as measured by a screw gauge was found to be 1.002 cm, 1.004 cm and 1.006 cm. The absolute error in the third reading is**

1) 0.002 cm 2) 0.004 cm
3) 1.002 cm 4) zero

Sol :. $\Delta x_3 = |x_3 - x_{mean}|$

key-1

EX 49. **Force and area are measured as 20 N and 5m² with errors 0.05 N and 0.0125m². The maximum error in pressure is (SI unit)**

1) 4 ± 0.0625 2) 4 ± 0.05
3) 4 ± 0.125 4) 4 ± 0.02

Sol :. $p = \frac{F}{A} \Rightarrow \frac{\Delta p}{p} = \frac{\Delta F}{F} + \frac{\Delta A}{A} \Rightarrow \Delta p = p\left(\frac{\Delta F}{F} + \frac{\Delta A}{A}\right)$

key-4

EX 50. **The length and breadth of a rectangular object are 25.2cm and 16.8cm respectively and have been measured to an accuracy of 0.1cm. Relative error and percentage error in the area of the object are**

1) 0.01 & 1% 2) 0.02 & 2%
3) 0.03 & 3% 4) 0.04 & 4%

Sol :. $a = l \times b$; $\frac{\Delta a}{a} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

$$\frac{\Delta a}{a} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) \times 100$$

key-1

EX 51. Dimensional analysis of the equation $(\text{Velocity})^x = (\text{Pressure difference})^{\frac{3}{2}}.(\text{density})^{\frac{-3}{2}}$ **gives the value of x as: (1986 E)**

1) 1 2) 2 3) 3 4)-3

Sol :17. Substitute dimension formulae

key-3

EX 52. For the equation F =A^{a}v^{b}d^c where F is force, A is area, v is velocity and d is density, with the dimensional analysis gives the following values for the exponents. (1985 E)

1) a=1, b = 2, c =1 2) a =2, b =1, c= 1

3)a =1, b =1, c= 2 4) a = 0, b =1 , c = 1

Sol :18. F = $A^a v^b d^c$; $MLT^{-2} = (L^2)^a (LT^{-1})^b (ML^{-3})^c$ comparing the powers on both sides

key-1

EX 53. The length of pendulum is measured as 1.01m and time for 30 oscillations is measured as one minute 3 seconds. Error in length is 0.01 m and error in time is 3 secs. The percentage error in the measurement of acceleration due to gravity is. (Engg. - 2012)

1) 1 2) 5 3) 10 4) 15

Sol :19. $T = 2\pi\sqrt{\frac{l}{g}}$; $\frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + 2\frac{\Delta T}{T}\times 100$

key-3

EX 54. The Energy (E), angular momentum (L) and universal gravitational constant (G) are chosen as fundamental quantities. The dimensions of universal gravitational constant in the dimensional formula of Planks constant (h) is (Eng - 2008)

1) 0 2) -1 3) 5/3 4) 1

Sol : $h \propto E, L, G$

$$ML^2T^{-1} = (ML^2T^{-2})^a (ML^2T^{-1})^b (M^{-1}L^3T^{-2})^c$$

key-1

EX 55. A body weighs 22.42 g and has a measured volume of 4.7 cc the possible errors in the measurement of mass and volume are0.01g and 0.1 cc. Then the maximum percentage error in the density will be(Med- 2010)

1) 22% 2) 2.2% 3) 0.22% 4) 0.022%

Sol :. The density of $d = \frac{M}{V}$; % Error of density

$\frac{\Delta d}{d}\times 100 = \frac{\Delta M}{M}\times 100 + \frac{\Delta V}{V}\times 100$, key-2

EX 56. If energy E, velocity v and time T are taken as fundamental quantities, the dimensional formula for surface tension is (Med-2009)

1) $[Ev^{-2}T^{-2}]$ 2) $[E^2vT^{-2}]$

3) $[Ev^{-2}T^{-1}]$ 4) $[E^{-2}v^{-2}T^{-1}]$

Sol : $[S] \propto [E]^a \times [v]^b \times [T]^c$

$\left[MT^{-2}\right] = \left[ML^2T^{-2}\right]^a \times \left[LT^{-1}\right]^b \times [T]^c$

Comparing the powers on both sides we get a,b,c

key-1

EX 57. **The measured mass and volume of a body are 53.63 g and 5.8 cm^3 respectively, with possible errors of 0.01 g and 0.1 cm^3. The maximum percentage error in density is about**

1) 0.2% 2) 2% 3) 5% 4) 10%

Sol :Density $\rho = \frac{M}{V}$; $\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta m}{m} + \frac{\Delta V}{V}\right]100$

key-2

EX 58. **A vernier calipers has 1 mm marks on the main scale . It has 20 equal divisions on the vernier scale,which match with 16main scale divisions. For this vernier calipers the least count is**

1) 0.02mm 2) 0.05 mm 3) 0.1mm 4) 0.2mm

Sol :. 16 M.S.D = 20 V.S.D$\Rightarrow 1V.S.D = 4/5\,M.S.D$

L.C = 1M.S.D - 1 V.S.D

key-4

EX 59. **The resistance of metal is given by V=IR. The voltage in the resistance is $V = (8 \pm 0.5)$ V and current in the resistance is $I = (2 \pm 0.2)$ A, the value of resistance with its percentage error is**

1) $(4 \pm 16.25\%)\Omega$ 2) $(4 \pm 2.5\%)\Omega$

3) $(4 \pm 0.04\%)\Omega$ 4) $(4 \pm 1\%)\Omega$

Sol :. $R = \frac{V}{I}$; $100 \times \frac{\Delta R}{R} = \left[\frac{\Delta V}{V} + \frac{\Delta I}{I}\right] \times 100$

$\text{Resistance} = \left[R \pm \frac{\Delta R}{R} \times 100\right]$

key-1

EX 60. **In an experiment, the values of refractive indices of glass were found to be 1.54, 1.53, 1.44, 1.54, 1.56 and 1.45 in successive measurements**

i) mean value of refractive index of glass **ii) mean absolute error**

iii) relative error and **iv) percentage error are respectively,**

1)1.51,0.04,0.03,3% 2)1.51,0.4,0.03,3 %

3)15.1,0.04,0.03,3% 4)15.1,0.04,0.3,3 %

Sol: $\mu_{mean} = \frac{\sum \mu}{6}; \Delta\mu_{mean} = \frac{\sum(\mu_{mean} - \mu_i)}{6}$;

relative % error in $\mu = \frac{\Delta\mu_{mean}}{\mu_{mean}} \times 100$ key-1

EX 60. **A student performs an experiment for determination of $g\left[=\frac{4\pi^2 L}{T^2}\right]$,L $\approx$ 1m, and he commits an error of ΔL for T he tajes the time of n oscillations with the stop watch of least count ΔT .For which of the following data the measurement of g will be most accurate?**

1) $\Delta L = 0.5, \Delta T = 0.1, n = 20$

2) $\Delta L = 0.5, \Delta T = 0.1, n = 50$

3) $\Delta L = 0.5, \Delta T = 0.01, n = 20$

4) $\Delta L = 0.5, \Delta T = 0.05, n = 50$

Sol : $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$ (Δl *and* ΔT are least, and the number of readings are maximum)

key- 4

EX 61. **A rectangular metal slab of mass 33.333 has its length 8.0 cm, breadth 5.0 cm and thickness 1mm. The mass is measured with accuracy up to 1 mg with a sensitive balance. The length and breadth are measured with vernier calipers having a least count of 0.01 cm. The thickness is measured with a screw gauge of least count 0.01 mm. The percentage accuracy in density calculated from the above measurements is**

1) 13 % 2)130 % 3)1.6 % 4)16 %

Sol : Percentage error gives percentage accuracy $d = \frac{m}{lbh}$

relative error, $\frac{\Delta d}{d} = \frac{\Delta m}{m} + \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$

and calculate $\left(\frac{\Delta d}{d}\right) \times 100$

key- 3

EX 62. In the relation $p = \frac{\alpha}{\beta} e^{-\alpha z / K\theta}$; P is pressure, K is Boltzmann's constant, Z is distance and θ is temperature. The dimensional formula of β will be

1) $\left[M^0 L^2 T^0\right]$ 2) $\left[M^1 L^2 T^1\right]$

3) $\left[M L^0 T^{-1}\right]$ 4) $\left[M^0 L^2 T^{-1}\right]$

Sol : $\left[\frac{\alpha z}{k\theta}\right] = 1$; 20)Here$[A] = IT^{-2}$ *and* $[B] = KT$

key-1

EX 63. **The heat generated in a circuit is given by Q = i² Rt joule , where 'i' is current, R is resistance and t is time. If the percentage errors in measuring i, R and t are 2%, 1% and 1% respectively, the maximum error in measuring heat will be**

1) 2 % 2) 3 % 3) 4 % 4) 6 %

Sol : $Q = i^2Rt$; $\frac{\Delta Q}{Q}\times100 = \frac{2\Delta i}{i}\times100+\frac{\Delta R}{R}\times100+\frac{\Delta t}{t}\times100$

key-4

EX 64. **You measure two quantities as A=1.0m $\pm$ 0.2m, B=2.0m $\pm$ 0.2m. We should report correct value for $\sqrt{AB}$ as**

1)1.4m $\pm$ 0.4m 2) 1.41m $\pm$ 0.15m

3) 1.4m $\pm$ 0.3m 4) 1.4m $\pm$ 0.2m

Sol : $Y = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414m$

$$\frac{\Delta y}{y} = \frac{1}{2}\left[\frac{\Delta A}{A}+\frac{\Delta B}{B}\right] = \frac{1}{2}\left[\frac{0.2}{10}+\frac{0.2}{2.0}\right] = \frac{0.6}{2\times2.0}$$

Rounding off to one significant digit Δy =0.2 cm

key-4

EX 65. **Which of the following measurement is most precise ?**

1) 5.00 mm 2) 5.00 cm 3) 5.00 m 4) 5.00 km

Sol : All given measurement are correct upto two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence, 5.00 mm is mpst precise.

key-1

EX 66. The mean length of an object is 5 cm. Which of the following measurement is most accurate ?

1) 4.9 cm 2) 4.805 cm3) 5.25 cm 4) 5.4 cm

Sol : Given length

Now, checking the errors with each options one by one, we get

$\Delta l_1 = 5 - 4.9 = 0.1$ cm

$\Delta l_2 = 5 - 4.805 = 0.195$ cm

$\Delta l_3 = 5.25 - 5 = 0.25$ cm

$\Delta l_4 = 5.4 - 5 = 0.4$ cm

Error Δl_1 is least

Hence, 4.9 cm is most precise.

key-1

JEE MAIN PREVIOUS YEAR QUESTIONS

TOPIC-1.....Unit of Physical Quantities

1. The density of a material in SI unit is 128 kg m^{-3} In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is:

[10 Jan. 2019]

(a) 40 (b) 16 (c) 640 (d) 410

sol. (a) Density of material in SI unit,$= \frac{128\text{kg}}{\text{m}^3}$

Density of material in new system

$= \frac{128(50\text{g})(20)}{(25\text{cm})^3(4)^3} = \frac{128}{64}(20) = 40\text{units}$

2. A metal sample carrying a currentalong X-axis with density J_x is subjected to a magnetic field B_z(along z-axis). The electric field E_y developed along Y-axis is directly proportional to J_x as well as B_z. The constant of proportionality has SI unit

[Online April 25, 2013]

(a) $\frac{m^2}{A}$ (b) $\frac{m^3}{As}$ (c) $\frac{m^2}{As}$ (d) $\frac{As}{m^3}$

sol. (b) According to question $E_y \propto J_x B_Z$

Constant of proportionality $K = \frac{E_y}{B_Z J_x} = \frac{C}{J_x} = \frac{m^3}{As}$

[As $\frac{E}{B} = C$ (speed of light) and $J = \frac{I}{\text{Area}}$]

TOPIC-2.....Dimensions of Physical Quantities

3. The quantities $= \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{1}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ε_0 , μ_0 , - free space permittivity and permeability respectively. Then : [Sep. 05, 2020 (II)]

(a) x, y and z have the same dimension.

(b) Only x and z have the same dimension.

(c) Only x and y have the same dimension.

(d) Only y and z have the same dimension.

sol. (a) We know that Speed of light, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = x$

Also, $c = \frac{E}{B} = y$

Time constant, $\Gamma = Rc = t$

$z = \frac{l}{Rc} = \frac{l}{t} =$ Speed

Thus, x, y, z will have the same dimension of speed.

4. Dimensional formula for thermal conductivity is (here K denotes the temperature): [Sep. 04, 2020 (I)]

(a) $\mathrm{MLT^{-2}K}$ (b) $\mathrm{MLT^{-2}K^{-2}}$ (c) $\mathrm{MLT^{-3}K}$ (d) $\mathrm{MLT^{-3}K^{-1}}$

sol. (d) From formula, $\frac{dQ}{dt} = kA\frac{dT}{dx} \Rightarrow k = \frac{\left(\frac{dQ}{dt}\right)}{A\left(\frac{dT}{dx}\right)}$

$$[k] = \frac{[\mathrm{ML^2T^{-3}}]}{[\mathrm{L^2}][\mathrm{KL^{-1}}]} = [\mathrm{MLT^{-3}K^{-1}}]$$

5. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of: [Sep. 04, 2020 (II)]

(a) planck's constant (b) force constant

(c) energy density (d) coefficient of viscosity

sol. (c) Dimension ofΓorce $F = \mathrm{M^1L^1T^{-2}}$

Dimension of velocity $V = \mathrm{L^1T^{-1}}$

Dimension of work $= \mathrm{M^1L^2T^{-2}}$

Dimension of length $= \mathrm{L}$

Moment of inertia $= \mathrm{ML^2}$

$x = \frac{IFv^2}{WL^4} \qquad = \frac{(M^1L^2)(M^1L^1T^{-2})(L^1T^{-2})^2}{(M^1L^2T^{-2})(L^4)}$

$= \frac{M^1L^{-2}T^{-2}}{L^3} = M^1L^{-1}T^{-2} =$ Energy density

6. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is: [Sep. 03, 2020 (II)]

(a) ML^2T^{-2} (b) ML^0T^{-3} (c) $M^2L^0T^{-1}$ (d) MLT^{-2}

sol. (b) Solar constant $= \frac{\text{Energy}}{\text{TimeArea}}$

Dimension of Energy, $E = ML^2T^{-2}$

Dimension of Time $= T$

Dimension of Area $= L^2$

Dimension of Solar constant $= \frac{M^1L^2T^{-2}}{TL^2} = M^1L^0T^{-3}$

7. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be: [Sep. 02, 2020 (I)]

(a) FA^2V^{-1} (b) FA^2V^{-3} (c) FA^2V^{-2} (d) $FA^{-1}V^0$

sol. (d) Young's modulus, $Y = \frac{\text{stress}}{\text{strain}}$

$$\Rightarrow Y = \frac{F}{A} / \frac{\Delta\ell}{\ell_0} = FA^{-1}V^0$$

8. If momentum(P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is : [Sep. 02, 2020 (II)]

(a) $[P^2AT^{-2}]$ (b) $[PA^{-1}T^{-2}]$ (c) $[PA^{1/2}T^{-1}]$ (d) $[P^{1/2}AT^{-1}]$

sol. (c) Energy $E \propto A^aT^bP^c$

or, $E = kA^aT^bP^c$ ---(i)

where k is a dimensionless constant and a, b and c are the exponents.

Dimension of momentum, $P = M^1L^1T^{-1}$

Dimension of area, $A = L^2$

Dimension of time, $T = T^1$

Putting these values in equation (i), we get $M^1L^2T^{-2} = M^cL^{2a+c}T^{b-c}$

by comparison

$c = 1$

$2a + c = 2$

$b - c = -2$

$c = 1\ , a = 1/2, b = -1$

$E = A^{1/2}T^{-1}P^1$

9. Which of the following combinations has the dimension of electrical resistance ($\in_0$ is the permittivity of vacuum and μ_0 is the permeability of vacuum)? [12 April 2019 I]

(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

sol. (a) $\sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0^2}{\varepsilon_0\mu_0}} = \mu_0 c$

$\mu_0 c \rightarrow \mathrm{M}L\mathrm{T}^{-2}\mathrm{A}^{-2} \times \mathrm{LT}^1$ $=\mathrm{ML^2T^3A^2}$

Dimensions of resistance

10. In the formula $\mathrm{X = 5YZ^2}$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? [10 Apri12019 II]

(a) $[\mathrm{M^3L^2T^8A^4}]$ (b) $[\mathrm{M^1L^2T^4A^2}]$ (c) $[\mathrm{M^2L^0T^{\triangleleft}A^2}]$ (d) $[\mathrm{M^2L^2T^6A^3}]$

sol. (a) $\mathrm{X = 5YZ^2}$

$\Rightarrow Y \propto \frac{X}{Z^2}$ --(i)

$X = \text{Capacitance} = \frac{Q}{\mathrm{V}} = \frac{Q^2}{W} = \frac{[A^2T^2]}{[ML^2T^{-2}]}$

$\mathrm{X = [M^1L^2T^4A^2]}$

$Z = B = \frac{F}{IL}\,[\because F = ILB]$

$\mathrm{Z = [MT^2A^1]}$

$$Y = \frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$$

$Y = [M^3L^2T^8A^4]$ (Using (i))

11. In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is: [8 April 2019 I]

(a) $A^{-1}T^1M^1L^3$ (b) $AT^2M^{-1}L^{-1}$ (c) $AT^{-3}ML^{3/2}$ (d) $A^2T^3M^{-1}L^{-2}$

sol. (d) $\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \sqrt{\frac{\varepsilon_0^2}{\mu_0\varepsilon_0}} = \left[\frac{\varepsilon_0}{\sqrt{\mu_0\varepsilon_0}}\right] = \varepsilon_0 C[LT^{-1}] \times [\varepsilon_0]$

$$\left[\because \frac{1}{\sqrt{\mu_0\varepsilon_0}} = C\right]$$

$$F = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow [\varepsilon_0] = \frac{[AT]^2}{[MLT^{-2}] \times [L^2]} = [A^2M^{-1}L^{-3}T^4]$$

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = [LT^{-1}] \times [A^2M^{-1}L^{-3}T^4]$$

$$= [M^{-1}L^{-2}T^3A^2]$$

12. Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be: [12 Jan. 2019 II]

(a) $[LA^{-2}]$ (b) $[A^{-1}]$ (c) $[LTA]$ (d) $[LT^2]$

sol. (b) As we know,

$\left[\frac{\ell}{r}\right] = [T]$ and $[cv] = [AT]$

$\left[\frac{\ell}{rcv}\right] = \left[\frac{T}{AT}\right] = [A^{-1}]$

13. The force of interaction between two atoms is given by $= \alpha\beta \exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two

constants. The dimensions of β is: [11 Jan. 2019 I]

(a) $M^0L^2T^{-4}$ (b) M^2LT^{-4} (c) MLT^{-2} (d) $M^2L^2T^{-2}$

sol. (b) Force of interaction between two atoms,

$$\Gamma = \alpha\beta e\left(\frac{-x^2}{\alpha kT}\right)$$

Since exponential terms are dimensionless

$$\left[\frac{x^2}{\alpha kT}\right] = M^0L^0T^0$$

$$\Rightarrow \frac{L^2}{[\alpha]ML^2T^{-2}} = M^0L^0T^0$$

$$\Rightarrow [\alpha] = M^{-1}T^2$$

$$[\Gamma] = [\alpha][\beta]$$

$$MLT^{-2} = M^{-1}T^2[\beta]$$

$$\Rightarrow [\beta] = M^2LT^{-4}$$

14. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be: [11 Jan. 2019 II]

(a) $V^{-2}A^2F^{-2}$ (b) $V^{-2}A^2F^2$ (c) $V^{-4}A^{-2}F$ (d) $V^{-4}A^2F$

sol. (d) Let $[Y] = [V]^a[\Gamma]^b[A]^c$

$$[ML^{-1}T^{-2}] = [LT^{-1}]^a[MLT^{-2}]^b[LT^{-2}]^c$$

$$[ML^{-1}T^{-2}] = \left[M^bL^{a+b+c}T^{-a-2b-2c}\right]$$

Comparing power both side of similar terms we get,

$b = 1$, $a + b + c = -1, -a - 2b - 2c = -2$

solving above equations we get:

$a = -4, b = 1, c = 2$

so $[Y] = [V^{-4}\Gamma A^2] = [V^{-4}A^2\Gamma]$

15. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational

constant and h is the Planck's constant. Dimension of f is that of: [9 Jan. 2019 I]

(a) area (b) energy (c) momentum (d) volume

sol. (b) Dimension of $[h] = [ML^2T^1]$

$[C] = [LT^1]$

$[G] = [M^1L^3T^2]$

Hence dimension of

$$\left[\sqrt{\frac{hC^5}{G}}\right] = \frac{[ML^2T^{-1}]\cdot[L^5T^{-5}]}{[M^{-1}L^3T^{-2}]}$$

$= [ML^2T^2] =$ energy

16. Expression for time in terms of G (universal gravitational constant), h (Planck's constant) and c (speed of light) is proportional to: [9 Jan. 2019 II]

(a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$ (c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

sol. (c) Let $t \propto G^x h^y C^z$

Dimensions of $G = [M^{-1}L^3T^{-2}]$,

$h = [ML^2T^{-1}]$ and $C = [LT^{-1}]$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

By comparing the powers of M, L, T both the sides

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ (i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ (ii) Solving eqns. (i) and (ii),

$x = y = \frac{1}{2}, z = -\frac{5}{2}\ t \propto \sqrt{\frac{Gh}{C^5}}$

17. The dimensions of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is:

[8 Jan. 2019 I]

(a) $h^{U3}G^{2/3}c^{U3}A^{-1}$ (b) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$ (c) $h^{2/3}C^{1/3}G^{4/3}\ A^{-1}$ (d) $h^{2}G^{3/2}C^{1/3}A^{-1}$

sol. (None)

Stopping potential $(V_0) \propto h^x I^y G^Z C^r$

Here, $h =$ Planck's constant $= [ML^2T^{-1}]$

$I =$ current $= [A]$

$G =$ Gravitational constant $= [M^1L^3T^2]$

and $c =$ speed oflight $= [LT^1]$

$$V_0 = \text{potentia1} = [ML^2T^3A^1]$$

$[ML^2T^3A^1] = [ML^2T^1]^x[A]^y[M^1L^3T^2]^z[LT^1]r$

$$M^{x-z}; L^{2x+3z+r}; I^{\leftarrow x-2z-r}; A^y$$

Comparing dimension ofM, L, T, A, we get

$y = -1, x = 0, z = -1\ , r = 5$

$V_0 \propto h^0 I^{-1} G^{-1} C^5$

18. The dimensions of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is: [8 Jan. 2019 II]

(a) MLT^{-2} (b) ML^2T^{-1} (c) ML^2T^{-2} (d) $ML^{-1}T^{-2}$

sol. (d) The quantity $\frac{B^2}{2\mu_0}$ is the energy density ofmagnetic field.

$\Rightarrow \left[\frac{B^2}{2\mu_0}\right] = \frac{\text{Energy}}{Volume} = \frac{\Gamma\text{orce}\times\text{disp1acement}}{(\text{disp1acement})^3}$

$= \left[\frac{ML^2T^{-2}}{L^3}\right] = ML^{-1}T^{-2}$

19. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, h and c. Which of the following correctly gives the Planck length? [Online April 15, 2018]

(a) G^2hc (b) $\left(\frac{Gh}{c^3}\right)^{\frac{1}{2}}$ (c) $G^{\frac{1}{2}}h^2c$ (d) Gh^2c^3

sol. (b) Plank length is a unit of length, $l_p = 1.616229 \times 10^{-35}$m

$$l_p = \sqrt{\frac{hG}{c^3}}$$

20. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be: [Online April 8, 2017]

(a) $[\mathrm{M}] = [\mathrm{T}^{-1}\mathrm{C}^{-2}\mathrm{h}]$ (b) $[\mathrm{M}] = [\mathrm{T}^{-1}\mathrm{C}^{2}\mathrm{h}]$

(c) $[\mathrm{M}] = [\mathrm{T}^{-1}\mathrm{C}^{-2}\mathrm{h}^{-1}]$ (d) $[\mathrm{M}] = [\mathrm{T}\mathrm{C}^{-2}\mathrm{h}]$

sol. (a) Let mass, related as $\mathrm{M} \propto \mathrm{T}^{x}\mathrm{C}^{y}\mathrm{h}^{z}$

$$\mathrm{M}^1\mathrm{L}^0\mathrm{T}^0 = (\mathrm{T}')^x(\mathrm{L}^1\mathrm{T}^1)^y(\mathrm{M}^1\mathrm{L}^2\mathrm{T}^1)^z$$

$$\mathrm{M}^1\mathrm{L}^0\mathrm{T}^0 = \mathrm{M}^z\mathrm{L}^{y+2z} + \mathrm{T}^{xyz}$$

$z = 1$

$y + 2z = 0 \; x - y - z = 0$

$y = -2 \; x + 2 - 1 = 0$

$x = -1$

$$\mathrm{M} = [\mathrm{T}^1\mathrm{C}^2\mathrm{h}^1]$$

21. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation AD = C ln (BD) holds true. Then which of the combination is not a meaningful quantity? [Online Apri110, 2016]

(a) $\frac{\mathrm{C}}{\mathrm{BD}} - \frac{\mathrm{AD}^2}{\mathrm{C}}$ (b) $\mathrm{A}^2 - \mathrm{B}^2\mathrm{C}^2$ (c) $\frac{\mathrm{A}}{\mathrm{B}} - \mathrm{C}$ (d) $\frac{(\mathrm{A}-\mathrm{C})}{\mathrm{D}}$

sol. (d) Dimension ofA ≠ dimension of(C)

Hence A-C is not possible.

22. In the following I refers to current and other sүnbols have their usual meaning, Choose the option that corresponds to the dimensions of electrical conductivity:

[Online April 9, 2016]

(a) $M^{-1}L^{-3}T^3I$ (b) $M^{-1}L^{-3}T^3I^2$ (c) $M^{-1}L^3T^3I$ (d) $ML^{-3}T^{-3}I^2$

sol. (b) We know that resistivity

$$p = \frac{RA}{\ell}$$

$$\text{Conductivity} = \frac{1}{\text{resistivity}} = \frac{\ell}{RA}$$

$$= \frac{\ell I}{VA} (\because V = RI)$$

$$= \frac{[L][I]}{|\frac{[ML^2T^{-2}}{[I][T]}| \times [L^2]} \,.\,.\, V = \frac{W}{q} = \frac{W}{it}$$

$$= [M^{-1}L^{-3}T^3][I^2] = [M^{-1}L^{-3}T^3I^2]$$

23. If electronic charge e, electron mass m, speed of light in vacuum c and Planck's constant h are taken as fundamental quantities, the permeability of vacuum μ_0 can be expressed in units of: [Online April 11, 2015]

(a) $\left(\frac{h}{me^2}\right)$ (b) $\left(\frac{hc}{me^2}\right)$ (c) $\left(\frac{h}{ce^2}\right)$ (d) $\left(\frac{mc^2}{he^2}\right)$

sol. (c) Let μ_0 related with e, m, c and h as follows.

$$\mu_0 = ke^a m^b c^c h^d$$

$[ML\Gamma^2 A^{-2}] = [AT]^a [M]^b [L\Gamma^1]^c [ML^2\Gamma^1]^d$

$= [M^{b+d} L^{c+2d} T^{l-c-d} A^a]$

On comparing both sides we get

$a = -2$ (i)

$b + d = 1$ (ii)

$c + 2d = 1$ (iii)

$a - c - d = -2$ (iv)

By equation (i), (ii), (iii) & (iv) we get,

$a = -2, b = 0, c = -1 , d = 1$

$$[\mu_0] = \left[\frac{h}{ce^2}\right]$$

24. If the capacitance of a nanocapacitor is measured in terms of a unit '*u*' made by combining the electric charge '*e*', Bohr radius 'a0', Planck's constant'h' and speed of light' *c*' then: [Online Apri110, 2015]

(a) $u = \frac{e^2h}{a_0}$ (b) $u = \frac{hc}{e^2a_0}$ (c) $u = \frac{e^2c}{ha_0}$ (d) $u = \frac{e^2a_0}{hc}$

sol. (d) Let unit '*u*' related with e, a_0, h and c as follows.

$$[u] = [e]^a[a_0]^b[h]^c[C]^d$$

Using dimensional method,

$$[M^{-1}L^{-2}T^{\vdash 4}A^{+2}] = [A^1T^1]^a[L]^b[ML2\Gamma^1]^c[L\Gamma^1]^d$$

$$[M^{-1}L^{-2}T^{\vdash 4}A^{+2}] = [M^cL^{b+2c+d}\Gamma^{r-c-d}A^a]$$

$$a = 2, b = 1 , c = -1 , d = -1$$

$$u = \frac{e^2a_0}{hc}$$

25. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is: [Online Apri112, 2014]

(a) $\frac{ch}{2\pi\varepsilon_0^2}$ (b) $\frac{e^2}{2\pi\varepsilon_0 Gm_e^2}$($m_e$ = mass of electron)

(c) $\frac{\mu_0\varepsilon_0 G}{c^2he^2}$ (d) $\frac{2\pi\sqrt{\mu_0\varepsilon_0}h}{ce^2G}$

sol. (b) The dimensional formulae of

$$e = [M^0L^0T^1A^1]$$

$\varepsilon_0 = [M^{-1}L^3T^4A^2]$

$G = [M^{-1}L^3T^{-2}]$ and $m_e = [M^1L^0T^0]$

Now, $\frac{e^2}{2\pi\varepsilon_0 Gm_e^2}$

$$= \frac{[M^0L^0T^1A^1]^2}{2\pi[M^{-1}L^{-3}T^4A^2][M^{-1}L^3T^{-2}][M^1L^0T^0]^2}$$

$$= \frac{[T^2A^2]}{2\pi[M^{-1-1+2}L^{-3+3}T^{4-2}A^2]}$$

$$= \frac{[T^2A^2]}{2\pi[M^0L^0T^2A^2]} = \frac{1}{2\pi}$$

$\frac{1}{2\pi}$ is dimensionless.

Thus the combination $\frac{e^2}{2\pi\varepsilon_0 G m_e^2}$ would have the same value in different systems of units.

26. In terms of resistance R and time T, the dimensions ofratio $\frac{\mu}{\varepsilon}$ of the permeability μ and permittivity ε is: [Online April 11, 2014]

(a) $[RT^{-2}]$ (b) $[R^2T^{-1}]$ (c) $[R^2]$ (d) $[R^2T^2]$

sol. (c) Dimensions of $\mu = [MLT^{-2}A^{-2}]$

Dimensions of$_\in = [M^{-1}L^{-3}T^4A^2]$

Dimensions of$R = [ML^2T^{-3}A^{-2}]$

$$\frac{\text{Dimensionsof}\mu}{\text{Dimensionsof} \in} = \frac{[MLT^{-2}A^{-2}]}{[M^{-1}L^{-3}T^4A^2]}$$

$$= [M^2L^4\Gamma^{-6}A^{-4}] = [R^2]$$

27. Let $[\in_0]$ denote the dimensional formula ofthe permittivity of vacuum. If M =mass, L =length, T = time and A = electric current, then: [2013]

(a) $\in_0 = [M^{-1}L^{-3}T^2A]$ (b) $\in_0 = [M^{-1}L^{-3}T^4A^2]$

(c) $\in_0 = [M^1L^2T^1A^2]$ (d) $\in_0 = [M^1L^2T^1A]$

sol. (b) As we know, $\Gamma = \frac{1 q_1 q_2}{4\pi\varepsilon_0 R^2}$

$$\Rightarrow \varepsilon_0 = \frac{q_1 q_2}{4\pi\Gamma R^2}$$

Hence, $\varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$

$$= [M^{-1}L^{-3}T^4A^2]$$

28. If the time period t ofthe oscillation of a drop ofliquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b}s^c d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$. The value of b should therefore be: [Online April 23, 2013]

(a) $\frac{3}{4}$ (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

sol. (c)

29. The dimensions of angular momentum, latent heat and capacitance are, respectively. [Online April 22, 2013]

(a) $ML^2T^1A^2$, L^2T^{-2} , $M^{-1}L^{-2}T^2$ (b) ML^2T^{-2}, L^2T^2, $M^{-1}L^{-2}T^4A^2$

(c) ML^2T^{-1}, L^2T^{-2}, ML^2TA^2 (d) ML^2T^{-1} , L^2T^{-2} , $M^{-1}L^{-2}T^4A^2$

sol. (d) Angular momentum $= m \times v \times r = ML^2T^{-1}$ Latent heat $L = \frac{Q}{m} = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$

Capacitance $C = \frac{\text{Charge}}{\text{P.d}} = M^{-1}L^{-2}T^4A^2$

30. Given that K =energy, V =velocity, T =time. Ifthey are chosen as the fundamental units, then what is dimensional formula for surface tension? [Online May 7, 2012]

(a) $[KV^{-2}T^{-2}]$ (b) $[K^2V^2\Gamma^{-2}]$ (c) $[K^2V^{-2}\Gamma^{-2}]$ (d) $[KV^2T^2]$

sol. (a) Surface tension, $T = \frac{F}{\ell} = \frac{F}{\ell}.\frac{\ell}{\ell}.\frac{T^2}{T^2}$

(As, $F.\ell = K$(energy); $\frac{T^2}{\ell^2} = V^{-2}$)

Therefore, surface tension $= [Kr^2\Gamma^{-2}]$

31. The dimensions of magnetic field in M, L, T and C (coulomb) is given as [2008]

(a) [MLT-1 C^{-1}] (b) [MT2 C^{-2}] (c) $[MT^{-1}C^{-1}]$ (d) $[MT^{-2}C^{-1}]$

sol. (c) Magnitude of Lorentz formula $F = qvB \sin\theta$

$$B = \frac{F}{qv} = \frac{MLT^{-2}}{C \times LT^{-1}} = [MT^{-1}C^{-1}]$$

32. Which of the following units denotes the dimension ML^2Q^2 , where Q denotes the electric charge? [2006]

(a) Wb/m^2 (b) Henry (H) (c) H/m^2 (d) Weber (Wb)

sol. (b) Mutual inductance $= \frac{\varphi}{I} = \frac{BA}{I}$ [Henry] $= \frac{[MT^{-1}Q^{-1}L^2]}{[QT^{-1}]} = ML^2Q^{-2}$

33. Out of the following pair, which one does NOT have identical dimensions? [2005]

(a) Impulse and momentum

(b) Angular momentum and planck's constant

(c) Work and torque

(d) Moment of inertia and moment of a force

sol. (d) Moment of Inertia, $I = MR^2$

$$[I] = [ML^2]$$

Moment of force, $\vec{\Gamma} = \vec{r} \times \vec{F}$

$$\rightarrow {}'\mathrm{r} = [L][MLT^{-2}] = [ML^2T^{-2}]$$

34. Which one of the following represents the correct dimensions of the coefficient of viscosity? [2004]

(a) $[ML^{-1}T^{-1}]$ (b) $[MLT^{-1}]$ (c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$

sol. (a) According to, Stokes law,

$$F = 6\pi\eta rv \Rightarrow \eta = \frac{F}{6\pi rv}$$

$$\eta = \frac{[MLT^{-2}]}{[L][LT^{-1}]} \Rightarrow \eta = [ML^{-1}T^{-1}]$$

35. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are [2003]

(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

sol. (c) As we know, the velocity of light in free space is given by

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_o}} \frac{1}{\mu_0 8_0} = e^2 = Z_1^2T^2$$

$$\frac{1}{\mu_0\varepsilon_o} = C^2[\mathrm{m/s}]^2$$

$$= [L\Gamma^1]^2$$

$$= [M^0L^2\Gamma^2]$$

36. The physical quantities not having same dimensions are [2003]

(a) torque and work (b) momentum and planck's constant

(c) stress and young's modulus (d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

sol. (b) Momentum, $= mv = [ML\Gamma^1]$

Planck's constant,

$$h = \frac{E}{v} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2\Gamma^1]$$

37. Identify the pair whose dimensions are equal [2002]

(a) torque and work (b) stress and energy

(c) force and stress (d) force and work

sol. (a) Work $W = \vec{F} \cdot \vec{s} = Fs \cos\theta$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$= [MLT^{-2}][L] = [ML^2T^{-2}];$$

Torque, $\vec{\Gamma} = \vec{r} \times \vec{F} \Rightarrow /\tau = rF \sin\theta$

$$\vec{A} \times \vec{B} = AB \sin\theta$$

$$= [L][MLT^{-2}] = [ML^2T^{-2}]$$

TOPIC-3…..Errors in Measurements

38. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively:

[Sep. 06, 2020 (I)]

(a) Negative, 2 μm (b) Positive, 10 μm (c) Positive, 0.1 mm (d) Positive, 0.1 μm

sol. (b) Given: No. of division on circular scale of screw gauge = 50 Pitch = 0.5 mm

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. ofdivisiononcircularscale}}$$

$= \frac{0.5}{50}$ mm $= 1 \times 10^5$m $= 10\mu$m

And nature of zero error is positive.

39. The density of a solid metal sphere is determined by measuring its mass and its diameter.

The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)$ %. Ifthe relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .

[NA Sep. 06, 2020 (I)]

sol. (1050)

Density, $\mathrm{p} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} \Rightarrow \mathrm{p} = \frac{6}{\pi} M D^{-3}$

$$\% \left(\frac{\Delta \mathrm{p}}{\mathrm{p}}\right) = \frac{\Delta m}{m} + 3\left(\frac{\Delta D}{D}\right) = 6 + 3 \times 1.5 = 10.5\%$$

$\% \left(\frac{\Delta \mathrm{p}}{\mathrm{p}}\right) = \frac{1050}{100}\% = \left(\frac{x}{100}\right)\%$

$$x = 1050.00$$

40. A student measuring the diameter of a pencil of circular cross-section with the help of a Vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm, 5.65 mm, The average ofthese four reading is 5.5375 mm and the standard deviation ofthe data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as:

[Sep. 06, 2020 (II)]

(a) (5.5375 ± 0.0739) mm (b) (5.5375 ± 0.0740) mm

(c) (5.538 ± 0.074) mm (d) (5.54 ± 0.07) mm

sol. (d) Average diameter, $d_{\mathrm{av}} = 5.5375$ mm

Deviation of data, $\Delta d = 0.07395$ mm

As the measured data are up to two digits after decimal, therefore answer should be in two digits after decimal. $d = (5.54 \pm 0.07)$ mm

41. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b}{\sqrt{c} d^3}\frac{2}{3}$. The percentages of error in the measurement ofa, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage oferror in z is:

[Sep. 05, 2020 (I)]

(a) 12.25% (b) 16.5% (c) 13.5% (d) 14.5%

sol. (d) Given: $Z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$

Percentage error in Z,

$$= \frac{\Delta Z}{Z} = \frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{3\Delta d}{d}$$

$= 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5 = 14.5\%$.

42. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as : [Sep. 03, 2020 (I)]

(a) 2.12lcm (b) 2.124cm (c) 2.125cm (d) 2.123cm

sol. (a) Thickness = M. S.Reading + Circular Scale Reading (L.C.)

Here LC $= \frac{\text{Pitch}}{\text{Circularsca1e}}$division $= \frac{0.1}{50} = 0.002$ cm per division

So, correct measurement is measurement of integral multiple of L.C.

43. The least count of the main scale of a Vernier calipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7^{th} division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side ofthe zero ofmain scale. When this vernier is used to measure length ofa cylinder the zero of the Vernier scale between 3.1 cm and 3.2 cm and 4^{th} VSD coincides with a main scale division. The length ofthe cylinder is: (VSD is vernier scale division)

[Sep. 02, 2020 (I)]

(a) 3.2 cm (b) 3.21 cm (c) 3.07 cm (d) 2.99 cm

sol. (c) L.C. of Vernier calipers = 1 MSD-I VSD

$= \left(1 - \frac{9}{10}\right) \times 1 = 0.1\text{mm} = 0.01$ cm

Here 7^{th} division ofvernier scale coincides with a division of main scale and the zero of Vernier scale is lying right side of the zero of main scale.

Zero error $= 7 \times 0.1 = 0.7\text{mm} = 0.07$ cm.

Length of the cylinder = measured value- zero error

$= (3.1 + 4 \times 0.01) - 0.07 = 3.07$ cm.

44. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is:

[9 Jan. 2020 I]

(a) 0.00lcm (b) 0.02mm (c) 0.01 cm (d) 0.00l mm

sol. (d) When screw on a screw-gauge is given six rotations, it moves by 3mm on the main scale

$\text{Pitch} = \frac{3}{6} = 0.5 \text{ mm}$

$\text{Least count L.C.} = \frac{\text{Pitch}}{CSD} = \frac{0.5\text{mm}}{50}$

$$= \frac{1}{100}\text{mm} = 0.01mm = 0.00\text{lcm}$$

45. For the four sets of three measured physical quantities as given below. Which of the following options is correct?

[9 Jan. 2020 II]

(A) $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$

(B) $A_2 = 24.44, B_2 = 16.082, C_2 = 240.2$

(C) $A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183$

(D) $A_4 = 25, B_4 = 236.191, C_4 = 19.5$

(a) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

(b) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

(c) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$

(d) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

sol. (None)

$$D_1 = A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2 = 280.6 D_2 = A_2 + B_2 + C_2$$

$$= 24.44 + 16.082 + 240.2 = 280.7 D_3 = A_3 + B_3 + C_3$$

$$= 25.2 + 19.2812 + 236.183 = 280.7$$

$$D_4 = A_4 + B_4 + C_4 = 25 + 236.191 + 19.5 = 281$$

None of the option matches.

46. A simple pendulum is being used to determine the value ofgravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1 s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is:

[8 Jan. 2020 II]

(a) 5.40% (b) 3.40% (c) 4.40% (d) 2.40%

sol. (c) Given, Length of simple pendulum, $l = 25.0$ cm

Time of 40 oscillations, $T = 50s$

Time period of pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = \frac{4\pi^2\ell}{g} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$$

$\Rightarrow$Fractional error in g $= \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

$$\Rightarrow \frac{\Delta g}{g} = \left(\frac{0.1}{25.0}\right) + 2\left(\frac{1}{50}\right) = 0.044$$

Percentage error in $g = \frac{\Delta g}{g} \times 100 = 4.4\%$

47. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01)m, respectively. The error in the measurement of density is: [9 April 2019 I]

(a) $0.01\text{k}g/\text{m}^3$ (b) $0.10\text{k}g/\text{m}^3$ (c) $0.013\text{k}g/\text{m}^3$ (d) $0.07\text{k}g/\text{m}^3$

sol. (Bonus) $6 = \frac{M}{V} = \frac{M}{l^3} = Ml^{-3}$

$$\frac{\Delta\delta}{6} = \frac{\Delta M}{M} + 3\frac{\Delta l}{l} = \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31\text{k}y\text{m}^3$$

48. The area of a square is $5.29\ \text{cm}^2$. The area of 7 such squares taking into account the significant figures is: [9 April 2019 II]

(a) 37cm^2 (b) 37.030cm^2 (c) 37.03cm^2 (d) 37.0cm^2

sol. (d) $A = 7 \times 5.29 = 37.03\text{cm}^2$

The result should have three significant figures, so

$$A = 37.0\text{cm}^2$$

49. In a simple pendulum experiment for determination of acceleration due to gravity (g) ,

time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale ofleast count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to: [8 April 2019 II]

(a) 0.7% (b) 0.2% (c) 3.5% (d) 6.8%

sol. (d) We have

$T = 2\pi\sqrt{\frac{\ell}{g}}$ or $g = 4\pi^2 \frac{\ell}{T^2}$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta R}{Q} \times 100 + 2\frac{\Delta T}{T} \times 100$$

$$= \frac{0.1}{55} \times 100 + 2\left(\frac{1}{30}\right) \times 100$$

$= 0.18 + 6.67 = 6.8\%$

50. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter ofa wire is: [12 Jan. 2019 I]

(a) 50 (b) 200 (c) 100 (d) 500

sol. (b) Least count of main scale of screw gauge = lmm

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Numberofdivisiononcircularscale}}$$

$$5 \times 10^{-6} = \frac{10^{-3}}{\text{N}}$$

$$\Rightarrow \text{N} = 200$$

51. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures? [10 Jan. 2019 II]

(a) $4264 \pm 81\text{cm}^3$ (b) $4264.4 \pm 81.0\text{cm}^3$

(c) $4260 \pm 80\text{cm}^3$ (d) $4300 \pm 80\text{cm}^3$

sol. (c)

52. The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 division below the mean line. The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of the sheet is: [9 Jan. 2019 II]

(a) 5.755mm (b) 5.950mm (c) 5.725mm (d) 5.740mm

sol. (c) Least count of screw gauge,

$$\mathrm{LC} = \frac{\mathrm{Pitch}}{\mathrm{No.\,ofdivision}}$$

$= 0.5 \times 10^{-3} = 0.5 \times 10^{-2}\mathrm{mm} + \mathrm{ve\ error} = 3 \times 0.5 \times 10^{-2}\ \mathrm{mm}$

$$= 1.5 \times 10^{-2}\mathrm{mm} = 0.015\mathrm{mm}$$

Reading $= \mathrm{MSR} + \mathrm{CSR} - (+\mathrm{ve\ error})$

$$= 5.5\mathrm{mm} + (48 \times 0.5 \times 10^{-2}) - 0.015$$

$= 5.5 + 0.24 - 0.015 = 5.725\ \mathrm{mm}$

53. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is: [2018]

(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

sol. (c) $= 1.5\ \% + 3\ (1\%) = 4.5\%$

54. The percentage errors in quantities P, Q, Rand S are 0.5%, 1%, 3% and 1.5% respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{RS}}$. The maximum percentage error in the value ofA will be

[Online Apri116, 2018]

(a) 8.5% (b) 6.0% (c) 7.5% (d) 6.5%

sol. (d) Maximum percentage error in A

$= 3(\%\ \text{error in P}) + 2(\%\ \text{error in Q})$

$+ \frac{1}{2}(\%\ \text{error in R}) + 1(\%\ \text{error in S})$

$$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1 \times 1.5$$

$= 1.5 + 2 + 1.5 + 1.5 = 6.5\%$

55. The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius ofthe orbit is negligible, the relative uncertainty in the mass of the earth is [Online Apri116, 2018]

(a) $3 \times 1\sigma - 2$ (b) 10^{-2} (c) 2×10^{-2} (d) 6×10^{-2}

sol. (c) From Kepler's law, time period of a satellite,

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} T^2 = \frac{4\pi^2}{GM} r^3$$

Relative uncertainty in the mass of the earth

$|\frac{\Delta M}{M}| = 2\frac{\Delta T}{T} = 2 \times 10^{-2}$ (4π & G constant and relative uncertainty in radius $\frac{\Delta r}{r}$ negligible)

56. The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination Of its volume is [Online April 15, 2018]

(a) $\frac{2}{3}\alpha$ (b) $\frac{2}{3}\alpha$ (c) $\frac{3}{2}\alpha$ (d) α

sol. (c) Relative error in Surface area, $\frac{\Delta s}{s} = 2 \times \frac{\Delta r}{r} = \alpha$ and

relative error in volume, $\frac{\Delta v}{v} = 3 \times \frac{\Delta r}{r}$

Relative error in volume w.r. t. relative error in area,

$$\frac{\Delta v}{v} = \frac{3}{2}\alpha$$

57. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness ofa wire measured by this screw gauge gives a reading of4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is: [Online Apri115, 2018]

(a) 0.0430 cm (b) 0.3150 cm (c) 0.4300 cm (d) 0.2150 cm

sol. (d) Least count $= \frac{\text{Va1ueof1partonmainsca1e}}{\text{Numberofpartsonverniersca1e}}$

$= \frac{0.25}{5\times100}$ cm $= 5 \times 10^{-4}$ cm

Reading $= 4 \times 0.05$cm $+ 30 \times 5 \times 10^{-4}$ cm

$= (0.2 + 0.0150)$cm $= 0.2150$ cm(Thickness of wire)

58. The following observations were taken for determining surface tension T of water by capillary method: Diameter of capillary, D $= 1.25 \times 1T^2$m , rise of water, h $= 1.45 \times 10^2$m. Using g $= 9.80\text{m/s}^2$ and the simplified relation, $T = \frac{rhg}{2} \times 10^3\text{N/m}$, the possible error in surface tension is closest to : [2017]

(a) 2. 4% (b) 10% (c) 0.15% (d) 1.5%

sol. (d) Surface tension, $T = \frac{rhg}{2} \times 10^3$

Relative error in surface tension,

$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$ (g, 2 & 10^3 are constant) Percentage error

$$100 \times \frac{\Delta T}{T} = (\quad)^{100}(.)$$

$$= (0.8 + 0.689)$$

=(1.489)= 1.489%≅ 1.5%

59. A physical quantity P is described by the relation P $= a^{1/2}b^2c^3d^{-4}$. If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be: [Online April 9, 2017]

(a) 8% (b) 12% (c) 32% (d) 25%

sol. (c) Given, P $= \text{a}^{1/2}\text{b}^2\text{c}^2\text{d}^{\rightarrow\downarrow}$,

Maximum relative error,

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$

$$= \frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5 = 32\%$$

60. A screw gauge with a pitch of0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet ofAluminium. Before starting the measurement, it is found that wen the two jaws of the screw gauge are brought in contact, the 45$^{\text{th}}$ division

coincides with the main scale line and the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? [2016]

(a) 0.70 mm (b) 0.50 mm (c) 0.75mm (d) 0.80mm

sol. (d) L.C. $= \frac{0.5}{50} = 0.01\text{mm}$

Zero error $= 5 \times 0.01 = 0.05$ mm

(Negative) Reading $= (0.5 + 25 \times 0.01) + 0.05 = 0.80$ mm

61. A student measures the time period of100 oscillations of a simple pendulum four times. The data set is90 s, 91s, 95s, and 92 s. Ifthe minimum division in the measuring clock is 1 s, then the reported mean time should be: [2016]

(a) 92 ± 1.8s (b) 92 ± 3s (c) 92 ± 1.5s (d) 92 ± 5.0s

sol. (c) $\Delta T = \frac{|\Delta T_1|+|\Delta T_2|+|\Delta T_3|+|\Delta T_4|}{4}$

$$= \frac{2+1+3+0}{4} = 1.5$$

As the resolution of measuring clock is 1.5 therefore the mean time should be 92 ± 1.5

62. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value ofL is 20.0 cm known to 1 mm accuracy and time for 100 oscillations ofthe pendulum is found to be 90 s using a wrist watch ofls resolution. The accuracy in the determination of g is: [2015]

(a) 1% (b) 5% (c) 2% (d) 3%

sol. (d) As, $g = 4\pi^2 \frac{L}{T^2}$

So, $\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2\frac{\Delta T}{T} \times 100$

$$= \frac{0.1}{20} \times 100 + 2 \times \frac{1}{90} \times 100 = 2.72 = 3\%$$

63. Diameter of a steel ball is measured using a Vernier calipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions ofits vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are given as: [Online Apri110, 2015]

S.No. MS(cm) VS divisions

1.	0.5	8
2.	0.5	4
3.	0.5	6

If the zero error is-0.03 cm, then mean corrected diameter is:

(a) 0.52 cm (b) 0.59 cm (c) 0.56 cm (d) 0.53 cm

sol. (b) Least count $= \frac{0.1}{10} = 0.01$ cm

$d_1 = 0.5 + 8 \times 0.01 + 0.03 = 0.61$ cm

$d_2 = 0.5 + 4 \times 0.01 + 0.03 = 0.57$ cm

$d_3 = 0.5 + 6 \times 0.01 + 0.03 = 0.59$ cm

Mean diameter $= \frac{0.61+0.57+0.59}{3}$

$= 0.59$ cm

64. The current voltage relation of a diode is given by $\mathrm{I} = \left(\mathrm{e}^{1000\mathrm{V}/\mathrm{T}} - 1\right)$mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. Ifa student makes an error measuring ± 0.01V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? [2014]

(a) 0.2mA (b)0.02mA (c) 0.5mA (d) 0.05mA

sol. (a) The current voltage relation of diode is $I = \left(e^{1000V/T} - 1\right)$mA (given)

When, $I = 5mA$, e1000 $Vl\tau_{=6mA}$

Also, $dI = (e^{1000}V/\tau_{)\times\frac{1000}{T}}$

Error $= \pm 0.01$ (By exponential function)

$$= (6mA) \times \frac{1000}{300} \times (0.01) = 0.2\text{mA}$$

65. A student measured the length ofa rod and wrote it as 3.50 cm. Which instrument did he use to measure it? [2014]

(a) A meter scale.

(b) A vernier caliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(d) A screwgauge having 50 divisions in the circular scale and pitch as 1 mm.

sol. (b) Measured length of rod = 3.50 cm

For Vernier Scale with 1 Main Scale Division = 1 mm

9 Main Scale Division = 10 Vernier Scale Division,

Least count = 1 MSD-I VSD = 0.1 mm

66. Match List-I(Event) with List-II(Order of the time interval for happening of the event) and select the correct option from the options given below the lists:

[Online Apri119, 2014]

List-I	List- II
(1) Rotation period of earth	(i) 10^5s
(2) Revolution period of earth	(ii) 10^7s
(3) Period of light wave	(iii) 10^{-15}s
(4) Period of sound wave	(iv) 10^{-3}s

(a) (1) -(i), (2) – (ii), (3) -(iii), (4) – (iv)

(b) (1) -(ii), (2) – (i), (3) -(iv), (4) -(iii)

(c) (1) -(i), (2) -(ii), (3) -(iv), (4) -(iii)

(d) (1) -(ii), (2) – (i), (3) -(iii), (4) – (iv)

sol. (a) Rotation period of earth is about 24 hrs = 10^5s

Revolution period of earth is about 365 days = 10^7s

Speed of light wave $C = 3 \times 10^8 m/s$

Wavelength of visible light of spectrum

$$\lambda = 4000 - 7800A$$

$C = f\lambda \quad (and\, T = \frac{1}{f})$

Therefore period of light wave is 10^{-15}s (approx)

67. In the experiment of calibration of voltmeter, a standard cell of e. m. f. 1.1 volt is balanced against 440 cm ofpotential wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of voltmeter will be:

[Online Apri112, 2014]

(a) -0.15 volt (b) 0.15 volt (c) 0.5 volt (d) -0.05 volt

sol. (d) In a voltmeter

$$V \propto l$$

$$V = kl$$

Now, it is given $E = 1.1$ volt for $l_1 = 440$ cm

and $V = 0.5$ volt for $l_2 = 220$ cm

Let the error in reading of voltmeter be ΔV then,

$1.1 = 400K$ and $(0.5 - \Delta V) = 220$ K.

$$\Rightarrow \frac{1.1}{440} = \frac{0.5 - \Delta V}{220}$$

$\Delta V = -0.05$ volt

68. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured byusing a watch of 1 second least count and the value is 90.0 seconds. The length L is measured byusing a meter scale ofleast count 1 mm and the value is 20.0 cm. The error in the determination ofg would be:

[Online April 9, 2014]

(a) 1.7% (b) 2.7% (c) 4.4% (d) 2.27%

sol. (b) According to the question.

$t = (90 \pm 1)$ or, $\frac{\Delta t}{t} = \frac{1}{90}$

$l = (20 \pm 0.1)$ or, $\frac{\Delta l}{l} = \frac{0.1}{20}$

$\frac{\Delta g}{g}$ %=?

As we know,

$$t = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{t^2}$$

or, $\frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l} + 2\frac{\Delta t}{t}\right) = \left(\frac{0.1}{20} + 2 \times \frac{1}{90}\right) = 0.027$

$\frac{\Delta g}{g}\% = 2.7\%$

69. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is [2012]

(a) 6% (b) zero (c) 1% (d) 3%

sol. (a) According to ohm's law, V = IR

$R = \frac{V}{I}$

Percentage error $= \frac{\text{Abso1uteerror}}{\text{Measurement}} \times 10^2$

where, $\frac{\Delta V}{V} \times 100 = \frac{\Delta I}{I} \times 100 = 3\%$

then, $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 10^2 + \frac{\Delta I}{I} \times 10^2$

$= 3\% + 3\% = 6\%$

70. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading: 58.5 degree

Vernier scale reading: 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the Vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is [2012]

(a) 58.59 degree (b) 58.77 degree (c) 58.65 degree (d) 59 degree

sol. (c) Reading of Vernier = Main scale reading+ Vernier scale reading × least count.

Main scale reading = 58.5

Vernier scale reading = 09 division

least count of Vernier $= 0.5^\circ/30$

Thus, $R = 58.5^\circ + 9 \times \frac{0.5^\circ}{30}$

$$R = 58.65^\circ$$

71. N divisions on the main scale of a Vernier calipers coincide with $(N+1)$ divisions of the Vernier scale. Ifeach division of main scale is 'a' units, then the least count of the

instrument is [Online May 19, 2012]

(a) a (b) $\frac{a}{N}$ (c) $\frac{N}{N+1} \times a$ (d) $\frac{a}{N+1}$

sol. (d) No. of divisions on main scale $= N$

No. of divisions on Vernier scale $= N + 1$

size of main scale division $= a$

Let size of Vernier scale division be b

then we have

$$aN = b(N+1) \Rightarrow b = \frac{aN}{N+1}$$

Least count is a—b = a $- \frac{aN}{N+1}$

$$= a\left[\frac{N+1-N}{N+1}\right] = \frac{a}{N+1}$$

72. A student measured the diameter of a wire using a screw gauge with the least count 0.001 cm and listed the measurements. The measured value should be recorded as [Online May 12, 2012]

(a) 5.3200 cm (b) 5.3 cm (c) 5.32 cm (d) 5.320cm

sol. (d) The least count(L.C.) of a screw guage is the smallest length which can be measured accurately with it.

As least count is 0.001 cm $= \frac{1}{1000}$ cm

Hence measured value should be recorded upto 3 decimal places i.e., 5.320 cm

73. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading: 0 mm

Circular scale reading : 52 divisions

Given that l mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is [2011]

(a) 0.052 cm (b) 0.026 cm (c) 0.005 cm (d) 0.52cm

sol. (a) Least count, L.C. $= \frac{1}{100}$ mm $= \frac{1}{30}$ MSD

Diameter ofwire $= MSR + CSR \times L.C. = \frac{1}{30} \times 0.5° = 1$ minute.

$1 \text{ mm} = 0.1 \text{ cm}$

$= 0 + \times 52 = 0.52\text{mm} = 0.052 \text{ cm}\underline{1}$

74. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are [2010]

(a) 5, 1, 2 (b) 5, 1, 5 (c) 5, 5, 2 (d) 4, 4, 2

sol. (a) Number of significant figures in 23.023 = 5

Number of significant figures in 0.0003 = 1

Number of significant figures in $2.1 \times 10^{-3} = 2$

75. In an experiment the angles are required to be measured using an instrument, 29 divisions of the main scale exactly coincide with the 30 divisions of the Vernier scale. If the smallest division of the main scale is half- a degree $(= 0.5^\circ)$, then the least count of the instrument is: [2009]

(a) half minute (b) one degree (c) half degree (d) one minute

75. (d) 30 Divisions of V.S. coincide with 29 divisions of M.S.

I V.S. D $= \frac{29}{30}$ MSD

L.C. = 1 MSD- IVSD

$= 1$ MSD — $\frac{29}{30}$ MSD

Reading $= [\text{M.S.R.} + \text{C.S.R.} \times \text{L.C.}] -$(zero error)

$= [3 + 35 \times 0.01] - (-0.03) = 3.38 \text{ mm}$

$$= \frac{1}{30} MSD$$

$$= \frac{1}{30} \times 0.5^0 = 1 \min ute$$

76. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00ms^{-1}. The magnitude ofits momentum is recorded as [2008]

(a) 17.6kgms^{-1} (b) $17.565 \text{ kg ms}^{-1}$ (c) 17.56kgms^{-1} (d) 17.57kgms^{-1}

sol. (a) Momentum, $p = m \times v$

Given, mass of a body $= 3.513\text{kg}$

speed of body $= (3.513) \times (5.00) = 17.565\text{kgm/s}$

$= 17.6$ (Rounding offto get three significant figures)

77. Two full turns of the circular scale of a screw gauge cover a distance of l mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of-0.03 mm. While measuring the diameter ofa thin wire, a student notes the main scale reading of3 mm and the number ofcircular scale divisions in line with the main scale as 35. The diameter of the wire is [2008]

(a) 3.32mm (b) 3.73 mm (c) 3.67mm (d) 3.38mm

sol. (d) Least count of screw gauge $= 0.01$ mm

$\frac{0.5}{50} mm$

Reading = [M.S.R. + C.S.R. $\times$ L.C.] $-$(zero error)

= [3 + 35 $\times$ 0.01] $-$ ($-$ 0.03)=3.38 mm

Chapter 3 - MOTION IN A STRAIGHT LINE

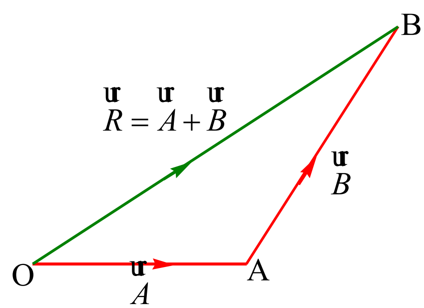
- Motion in a straight line deals with the motion of an object which changes its position with time along a straight line.
- The study of the motion of objects without considering the cause of motion is called kinematics.

Rest and Motion

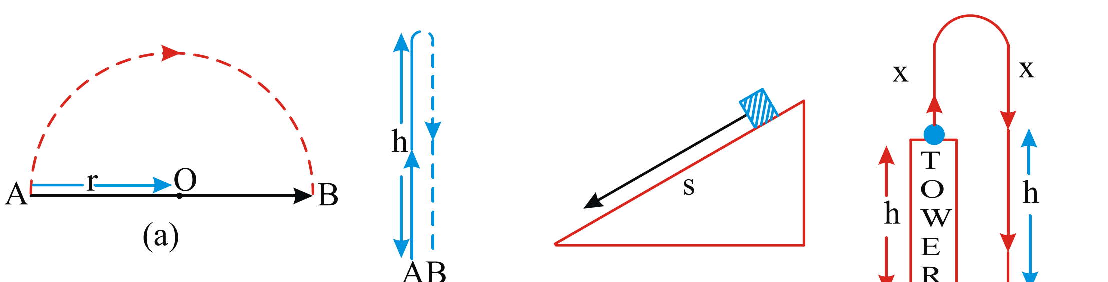
If the position of a body does not change with time with respect to the surroundings then it is said to be at rest, if not it is said to be in motion.

Distance and Displacement

- Distance is the actual path covered by a moving particle in a given interval of time while displacement is the change in position vector,i.e., a vector joining initial to final position. If a particle moves from A to B as shown in Fig. the distance travelled is Δs while displacement is $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$



- Distance is a scalar while displacement is a vector, both having same dimensions[L] and SI unit is metre.



(b) (c) (d)

Distance = πr
Displacement = 2r

Distance = 2h
Displacement = 0

Distance = s
Displacement = s

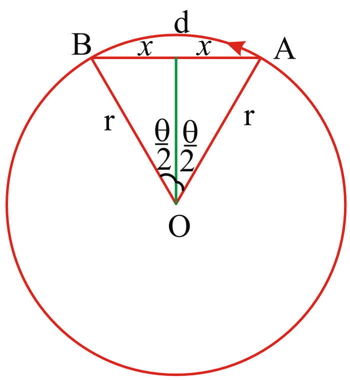
distance = h + 2x
displacement = h

- The magnitude of displacement is equal to minimum possible distance between two positions; so Distance $\geq |Displacement|$
- For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
- For a moving particle distance can never decrease with time while displacement can.
- Decrease in displacement with time means body is moving towards the initial position.
- For a moving particle distance can never be negative or zero while displacement can be negative.
- In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.
- Magnitude of displacement is less than the distance travelled in case of curvilinear motion.

  Ex : If an object turns through an angle θ along a circular path of radius r from point A to point B then

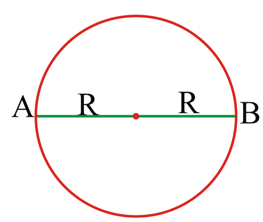
  i) distance $d = r\theta$

  ii)displacement $2x = 2r\sin(\theta/2)\left[\because \sin\frac{\theta}{2} = \frac{x}{r}\right]$



EX.1:An athlete completes one round of a circular track of radius R in 40 s. What will be his displacement at the end of 2 min 20 s? (2010E)

Sol. The time = 2 min 20s = 140s



In 40 seconds athlete completes = 1 round

In 140 seconds athlete will completes

$$=\frac{140}{40}\text{round} = 3.5 \text{ rounds}$$

The displacement in 3 rounds = 0

So net displacement =2R

EX.2 : If the position of a particle along Y axis is represented as a function of time t by the equation y(t)=t^3 then find displacement of the particle during the period t to $t+\Delta t$

Sol. Position at time t is $y(t)=t^3$

Position at time $t+\Delta t$ is $y(t+\Delta t)=(t+\Delta t)^3$

$\therefore$ displacement of the particle from t to $t+\Delta t$ is $y(t+\Delta t)-y(t)=(t+\Delta t)^3-t^3$

$=t^3+3t^2\Delta t+3t(\Delta t)^2+(\Delta t)^3-t^3$

$=3t^2\Delta t+3t(\Delta t)^2+(\Delta t)^3$

Average Speed and Average Velocity:

- Average speed or velocity is a measure of overall 'fastness' of motion during a specified interval of time.
- The average speed of a particle for a given 'interval of time' is defined as the ratio of distance travelled to the time taken while average velocity is defined as the ratio of displacement to time taken.
- Thus, if a particle in time interval Δt after travelling distance Δr is displaced by $\Delta \vec{r}$

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}, \qquad \text{i.e., } V_{avg}=\frac{\Delta r}{\Delta t} \text{ ....(i)}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}, \qquad \text{i.e, } \vec{V}_{avg}=\frac{\Delta \vec{r}}{\Delta t} \text{ ....(ii)}$$

- Average speed is a scalar while average velocity is a vector both having same units (m/s) and dimensions $\left[LT^{-1}\right]$.
- For a given time interval average velocity is single valued while average speed can have many values depending on path followed.
- During the motion if the body comes back to its initial position.

  $\vec{V}_{avg}=0 \quad (\because \Delta \vec{r}=0)$

  but $V_{avg}>0$ and finite $(\because \Delta r>0)$
- For a moving body average speed can never be negative or zero while average velocity can be negative.
- If a graph is plotted between distance (or displacement) and time, the slope of chord during a given interval of time gives average speed (or) average velocity

  $$V_{avg}=\frac{\Delta r}{\Delta t}=\tan\phi=\text{slope of chord}$$

Instantaneous speed and Instantaneous velocity

- Instantaneous velocity is defined as rate of change of position of the particle with time. If the position $\vec{r}$ of a particle at an instant t changes by $\Delta \vec{r}$ in a small time interval Δt

  $$\vec{V}=\lim_{\Delta t\to o}\frac{\Delta \vec{r}}{\Delta t}=\frac{d\vec{r}}{dt}$$
- The magnitude of velocity is called speed,i.e

  Speed=$|velocity|$ i.e, $V=|\vec{V}|$
- Velocity is a vector while speed is a scalar , both having same units (m/s) and dimensions $\left(LT^{-1}\right)$.

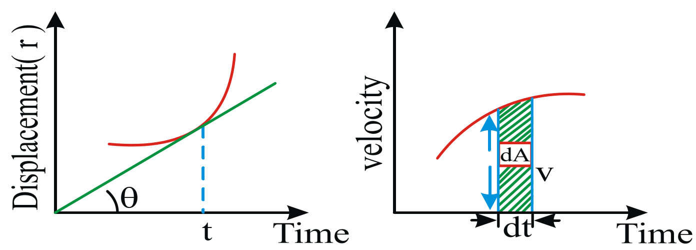
↳ If during motion velocity remains constant throughout a given interval of time, then the motion is said to be uniform and for uniform motion, $\vec{V}$=constant=$\vec{V}_{avg}$

↳ If velocity is constant, speed will also be constant. However , the converse may or may not be true,i.e, if speed=constant,velocity may or may not be constant as velocity has a direction in addition to magnitude which may or may not change,e.g, in case of uniform rectilinear motion $V = \text{constant}$ and $\vec{V} = \text{constant}$
While in case of uniform circular motion
V=constant but $\vec{V}$ is not constant, due to change in direction.

↳ velocity can be positive or negative as it is a vector but speed can never be negative as it is magnitude of velocity, i.e, $V = |\vec{V}|$

↳ As by definition $V = \frac{dr}{dt}$, the slope of displacement-versus time graph gives velocity , i.e,

$$V = \frac{dr}{dt} = \tan\theta = slope\ of\ r - t\ curve$$



(a) (b)

↳ As by definition $V = \frac{dr}{dt}$; *i.e*, $dr = Vdt$

and from fig. $Vdt = dA$

So, $dA = dr$ i.e, $r = \int dA = \int Vdt$

i.e, area under velocity-time graph gives displacement while without sign gives distance.

↳ Average speed is the total distance divided by total time

$$v_{avg} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

↳ If a body travels a distance s_1 in time t_1 , s_2 in time t_2 and s_3 in time t_3 then the average speed is $v_{avg} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$

↳ If an object travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 respectively in the same direction. Then

$$\text{Average speed} = \frac{s_1 + s_2 + s_3}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}}$$

↪ If an object travels first half of the total journey with a speed v_1 and next half with a speed v_2

then its average speed is $$v_{avg} = \frac{s+s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2}{\frac{1}{v_1}+\frac{1}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$

↪ If a body travels first 1/3 rd of the distance with a speed v_1 and second 1/3rd of the distance with a speed v_2 and last 1/3rd of the distance with a speed v_3, then the average speed

$$v_{avg} = \frac{\frac{s}{3}+\frac{s}{3}+\frac{s}{3}}{\frac{s}{3v_1}+\frac{s}{3v_2}+\frac{s}{3v_3}}$$

$$v_{avg} = \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$$

↪ If an object travels with speeds v_1, v_2, v_3 etc., during time intervals t_1, t_2, t_3 etc.,

then its average speed $$= \frac{v_1t_1+v_2t_2+v_3t_3+....}{t_1+t_2+t_3+....}$$

If $t_1 = t_2 = t_3 = ... = t$, then

If $t_1 = t_2 = t_3 = ... = t$, then

$$v_{avg} = \frac{v_1t+v_2t+v_3t+.....}{nt} = \frac{v_1+v_2+...}{n}$$

i.e.The average speed is equal to the arithmetic mean of individual speeds.

↪ The actual path length traversed by a body is called distance.

Note : If $\vec{v} = v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$ varies with time t then

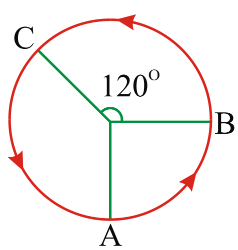
$$\vec{dr} = \vec{v}dt \left(\because \vec{v} = \frac{\vec{dr}}{dt}\right)$$

Integrating on both sides then $$\int_{\vec{r}_i}^{\vec{r}_f} \vec{dr} = \int_{t_1}^{t_2} \vec{v}dt$$

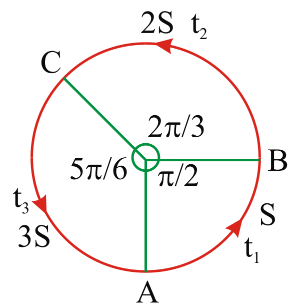
$\Rightarrow$ displacement of the particle from time t_1 to t_2 is given by

$$\vec{s} = \vec{r}_f - \vec{r}_i = \int_{t_1}^{t_2} v_x\hat{i}dt + \int_{t_1}^{t_2} v_y\hat{j}dt + \int_{t_1}^{t_2} v_z\hat{k}dt$$

EX.3: A particle starting from point A, travelling upto B with a speed S, then upto point ‘C’ with a speed 2S and finally upto ‘A’ with a speed 3S, then find its average speed.



Sol. Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$



Total distance travelled = AB + BC + CA = $2\pi r$

Total time taken is

$$T = t_1 + t_2 + t_3 = \frac{AB}{V_1} + \frac{BC}{V_2} + \frac{CA}{V_3} = \frac{\pi r}{2S} + \frac{2\pi r}{6S} + \frac{5\pi r}{18S}$$ (arc length = radius × angle)

$$V_{avg} = \frac{2\pi r}{\frac{\pi r}{2S} + \frac{2\pi r}{6S} + \frac{5\pi r}{18S}} = 1.8S$$

EX.4: For a man who walks 720 m at a uniform speed of 2 m/s, then runs at a uniform speed of 4 m/s for 5 minute and then again walks at a speed of 1 m/s for 3 minutes. His average speed is

Sol. Where s_1 = 720 m and $t_1 = \frac{s_1}{v_1} = 360s = 6\,\text{min}$.

s_2 = (4)(5)(60) = 1200m, t_2 = 300 s s_3 = (1)(3)(60) = 180 m, t_3 = 180 s

$$v_{avg} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3} = \frac{720 + 1200 + 180}{360 + 300 + 180}$$

$$\Rightarrow v_{avg} = 2.5m/s$$

EX.5: A particle is at x = +5 m at t = 0, x = -7m at t = 6s and x = +2m at t = 10s. Find the average velocity of the particle during the interval (a) t = 0 to t = 6s; (b) t = 6s to t = 10s, (c) t = 0 to t = 10s.

Sol. x_1 = +5m, t_1 = 0, x_2 = -7m; t_2 = 6s, x_3 =+2m, t_3= 10s

a) The average velocity between the times t = 0 to t = 6s is

$$|\vec{v}_1| = \frac{|x_2 - x_1|}{t_2 - t_1} = \frac{|-7-5|}{6-0} = 2m/s$$

b) The average velocity between the times t_2 = 6s to t_3 = 10s is

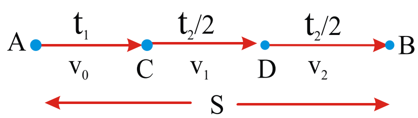
$$|\vec{v}_2| = \frac{x_3 - x_2}{t_3 - t_2} = \frac{2-(-7)}{10-6} = \frac{9}{4} = 2.25m/s$$

c) The average velocity between times $t_1 = 0$ to $t_3 = 10s$ is

$$\left|\vec{v}_3\right| = \frac{x_3 - x_1}{t_3 - t_1} = \frac{2-5}{10-0} = 0.3 m/s$$

EX.6: A particle traversed one third of the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with a velocity v_2 for the remaining half of time. Assuming motion to be rectilinear, find the mean velocity of the particle averaged over the whole time of motion.

Sol.



For AC; $\frac{S}{3} = v_0 t_1 \Rightarrow t_1 = \frac{S}{3v_0}$ ---(1)

For CB; $\frac{2S}{3} = CD + DB$

$$\Rightarrow \frac{2S}{3} = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right) \Rightarrow t_2 = \frac{4S}{3(v_1 + v_2)}$$

Since, average velocity is defined as

$$v_{avg} = \frac{\text{Total Displacement}}{\text{Total Time}} = \frac{\frac{S}{3} + \frac{2S}{3}}{t_1 + \frac{t_2}{2} + \frac{t_2}{2}} = \frac{S}{t_1 + t_2}$$

$$\Rightarrow v_{avg} = \frac{3v_0(v_1 + v_2)}{4v_0 + v_1 + v_2}$$

Acceleration

- The rate of change of velocity is equal to acceleration.

Average and Instantaneous acceleration

- If the velocity of a particle changes (either in magnitude or direction or both) with time the motion is said to be accelerated or non-uniform. In case of non-uniform motion if change in velocity is $\Delta\vec{V}$ in time interval Δt, then

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \quad ....(1)$$

Instantaneous acceleration or simply acceleration is defined as rate of change of velocity with time at a given instant. So if the velocity of a particle $\vec{v}$ at time t changes by $\Delta\vec{V}$ in a small time interval Δt then

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt} \quad .... (2)$$

Regarding acceleration it is worth noting that:

↳ It is a vector with dimensions $\left[LT^{-2}\right]$ and SI units $\left[m/s^2\right]$.

↳ If acceleration is zero, velocity will be constant and the motion will be uniform. However, if acceleration is constant(uniform), motion is non-uniform and if acceleration is not constant then both motion and acceleration are non-uniform.

↳ As by definition $\vec{V} = \left(d\vec{s}/dt\right)$

So, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right) = \frac{d^2\vec{s}}{dt^2}$ ....(3)

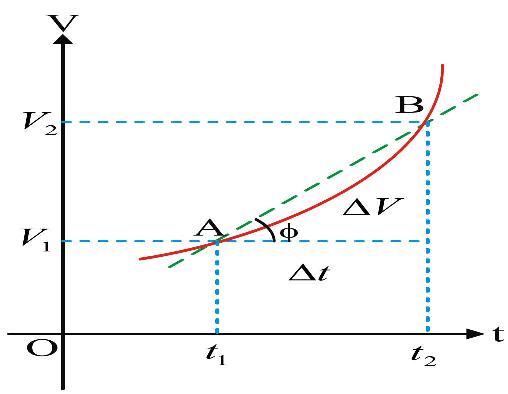
i.e., if $\vec{s}$ is given as a function of time, second derivative of displacement w.r.t. time gives acceleration.

↳ If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} \text{ or } a = v\frac{dv}{dx} \quad \left[\text{as} \frac{dx}{dt} = v\right]$$

↳ As acceleration $\vec{a} = \left(d\vec{v}/dt\right)$, the slope of velocity-time graph gives acceleration.

Velocity - Time Graph



$$\vec{a}_{avg} = \frac{\Delta\vec{V}}{\Delta t} = \tan\phi$$

$\vec{a}_{avg}$ = slope of the line joining two points in v-t graph.

↳ The slope of a versus t curve, i.e, $d\vec{a}/dt$ is a measure of rate of non-uniformity of acceleration(usually it is known as JERK).

↳ Acceleration can be positive or negative. Positive acceleration means velocity is increasing with time while negative acceleration called retardation means velocity is decreasing with time.

Equations of motion :

↳ If a particle starts with an initial velocity u, acceleration a and it gains velocity v in time t then

$$a = \frac{dv}{dt} \text{ or } dv = a\,dt$$

or $\int_u^v dv = a\int_0^t dt$ or $\left[V\right]_u^V = a\left[t\right]_0^t$

or $v = u + at$ ....(1A)

In vector form $\vec{v} = \vec{u} + \vec{a}t$ ....(1B)

↪ Now again by definition of velocity, Eqn. (1A) reduces to $\frac{ds}{dt} = u + at$

or $\int_0^s ds = \int_0^t (u + at)\,dt$ or

$$s = ut + \frac{1}{2}at^2 \qquad \text{........(2A)}$$

In vector form $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ ....(2B)

From eqns. (1A) and (2A), we get

$$s = u\frac{(v - u)}{a} + \frac{1}{2}a\left[\frac{(v - u)}{a}\right]^2$$

or $2as = 2uv - 2u^2 + v^2 + u^2 - 2uv$

i.e., $v^2 = u^2 + 2as$ ....(3)

↪ In scalar form

$\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$ or $v^2 = u^2 + 2\vec{a}.\vec{s}$

↪ Distance travelled = average speed x time

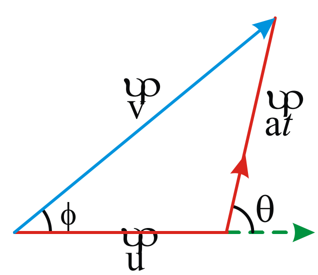
$$s = \left(\frac{u + v}{2}\right)t \qquad \text{.............(4)}$$

↪ Distance travelled in n^{th} second $\qquad s_n = u + a\left(n - \frac{1}{2}\right)$..... (5)

↪ If acceleration and velocity are not collinear, v can be calculated using

$$v = \left[u^2 + (at)^2 + 2uat\cos\theta\right]^{1/2}$$

with $\tan\phi = \frac{at\sin\theta}{u + at\cos\theta}$ ....(6)



Graphs

Characteristics of s-t and v-t graphs

↪ Slope of displacement time graph gives velocity.

↪ Slope of velocity-time graph gives acceleration.

↪ Area under velocity-time graph gives displacement

Let us plot v-t and s-t graphs of some standard results. To draw the following graphs assume that the particle has got either a one-dimensional motion with uniform velocity or with constant acceleration.

↳ If a particle starts from rest and moves with uniform acceleration 'a' such that it travels distances s_m and s_n in the m^{th} and n^{th} seconds

then $$S_n = 0 + \frac{a}{2}(2n - 1) \rightarrow (1) \quad (u=0)$$

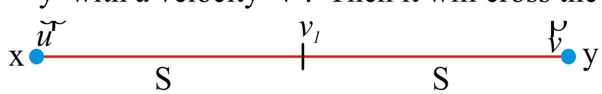
$$S_m = 0 + \frac{a}{2}(2m - 1) \rightarrow (2)$$

subtracting eq (2) from eq (1) $a = \dfrac{s_n - s_m}{(n-m)}$.

↳ A particle starts from rest and moves along a straight line with uniform acceleration. If 's' is the distance travelled by it in n seconds and s_n is the distance travelled in the n^{th} second, then

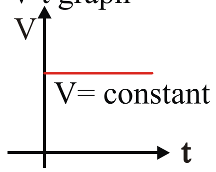
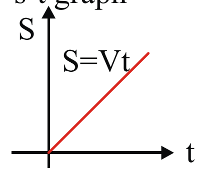
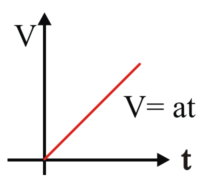
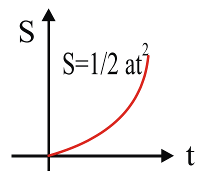
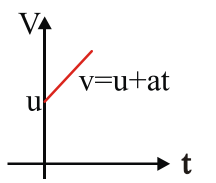
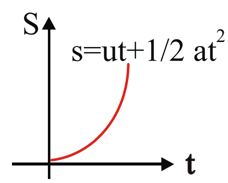
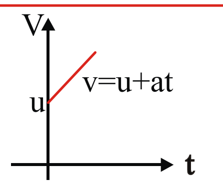
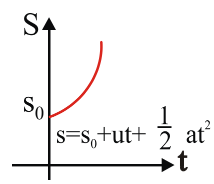
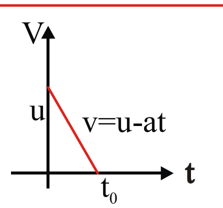
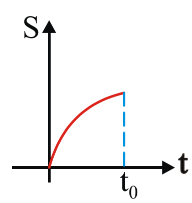
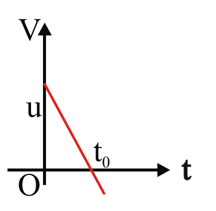
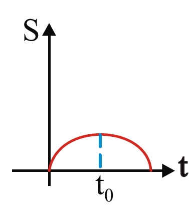
$\dfrac{s_n}{s} = \dfrac{(2n-1)}{n^2}$ (fraction of distance fallen in n^{th} second during free fall)

↳ Moving with uniform acceleration, a body crosses a point 'x' with a velocity 'u' and another point 'y' with a velocity 'v'. Then it will cross the mid point of 'x' and 'y' with velocity

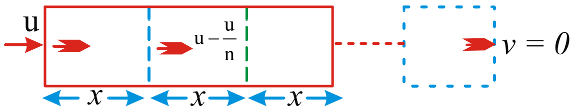


In this case acceleration $a = \dfrac{v_1^2 - u^2}{2S} = \dfrac{v^2 - v_1^2}{2S}$

$$\therefore \text{ on solving, } v_1 = \sqrt{\frac{v^2 + u^2}{2}}.$$

S.No	Situation	v-t graph	s-t graph	Interpretation
1	Uniform motion	V-t graph 	s-t graph 	i)Slope of s-t graph = v = constant. ii) In s-t- graph s = 0 at t = 0
2	Uniformly accelerated motion with u=0 and s=0 at t = 0			I) u=0, i.e.,v=0 and t=0 ii) u=0, i.e., slope of s-t graph at t=0, should be zero iii) a or slope of v-t graph is constant
3	Uniformly accelerated motion with $u \neq 0$ *and* $s = s_0$ *at* $t = 0$			I) $u \neq 0$, i.e., v or slope of s-t graph at t=0 is not zero ii) v or slope of s-t graph gradually goes on increasing
4	Uniformly accelerated motion with $u \neq 0$ *and* $s = s_0$ *at* $t = 0$			I) s = s_0 at t=0
5	Uniformly retarded motion till velocity be comes zero			I)Slope of s-t graph at t=0 gives u. ii) Slope of s-t graph at $t=t_0$ becomes zero iii) In this case u can't be zero.
6	Uniformly retarded then accelerated in opposite direction			I)At time $t=t_0$, v=0 or slope of s-t graph is zero. ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

↳ If a bullet looses $(1/n)^{th}$ of its velocity while passing through a plank, then the minimum no. of such planks required to just stop the bullet is .



Let m be the no of planks required to stop the bullet

$$\therefore \quad \left(u - \frac{u}{n}\right)^2 - u^2 = 2ax \quad \rightarrow (1) \qquad 0^2 - u^2 = 2amx \quad \rightarrow (2)$$

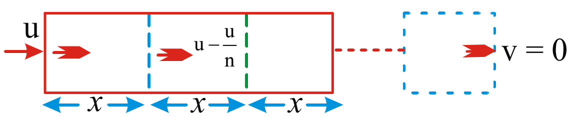
Dividing eq (2) with eq (1)

$$\frac{0^2 - u^2}{\left(u - \frac{u}{n}\right)^2 - u^2} = \frac{2amx}{2ax}$$

$$\rightarrow \quad m = \frac{u^2}{u^2 - \left(u - \frac{u}{n}\right)^2} = \frac{1}{1 - \left(1 - \frac{1}{n}\right)^2} \quad \rightarrow \quad m = \frac{1}{1 - \left(\frac{n-1}{n}\right)^2} = \frac{n^2}{n^2 - n^2 - 1 + 2n}$$

$$m = \frac{n^2}{2n-1}$$

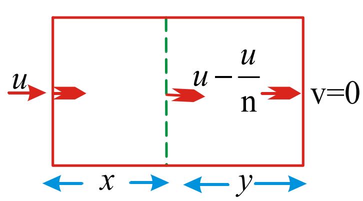
↳ The velocity of a bullet becomes $\frac{1}{n}$ of the initial velocity while penetrating a plank. The number of such planks required to stop the bullet.



$$\therefore \quad \left(\frac{u}{n}\right)^2 - u^2 = 2ax \quad \rightarrow (1) \qquad 0^2 - u^2 = 2amx \quad \rightarrow (2)$$

From eq (1) and eq (2) ; $\quad m = \dfrac{n^2}{n^2 - 1}$

↳ A bullet loose $\frac{1}{n}$ of its velocity while penetrating a distance x into the target. The further distance travelled before coming to rest.



Let x is the distance covered by the bullet to loose the $\frac{1}{n}th$ of initial velocity

$\therefore \quad v^2 - u^2 = 2as$

$$\left(u - \frac{u}{n}\right)^2 - u^2 = 2ax \quad \rightarrow (1)$$

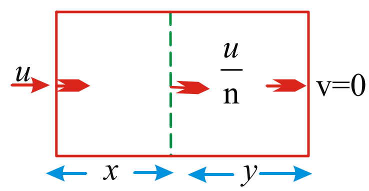
Let y is the further distance covered by the bullet to come to rest

$\therefore \quad 0^2 - u^2 = 2a(x+y) \quad \rightarrow (2)$

$\therefore$ From eq (1) and (2)

$$y = x\left(\frac{(n+1)^2}{2n}\right)$$

↳ If the velocity of a body becomes $\left(\frac{1}{n}\right)^{th}$ of its initial velocity after a displacement of 'x' then it will come to rest after a further displacement of



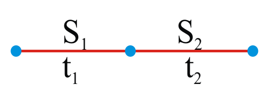
$$\Rightarrow \quad \left(\frac{u}{n}\right)^2 - u^2 = 2ax \quad \rightarrow (1) \qquad \Rightarrow \quad 0^2 - \left(\frac{u}{n}\right)^2 = 2ay \quad \rightarrow (2)$$

From eq. (1) and eq (2) ; $y = \frac{x}{n^2 - 1}$

EX.7. **A body covers 100cm in first 2 seconds and 128cm in the next four seconds moving with constant acceleration. Find the velocity of the body at the end of 8sec?**

Sol. distance in first two seconds is

$$s_1 = ut_1 + \frac{1}{2}at_1^2$$



$$100 = 2u + \frac{1}{2}a(4) \quad ......(1)$$

distance in (2+4)sec from starting point is

$$s_1 + s_2 = u(t_1 + t_2) + \frac{1}{2}a(t_1 + t_2)^2 \qquad 228 = 6u + \frac{1}{2}a(36) \; ......(2)$$

from eq (1) and (2)

We get a = -6 cm/s^2, sub a=-6 in eq - (1)

$$\Rightarrow 100 = 2u - \frac{1}{2} \times 6 \times 4$$

2u = 112 u = 56 cm/s

v = u + at = 56 - 6 x 8 $\Rightarrow v = 8cm/s$

EX.8. A car starts from rest and moves with uniform acceleration 'a'. At the same instant from the same point a bike crosses with a uniform velocity 'u'. When and where will they meet ? What is the velocity of car with respect to the bike at the time of meeting ?

Sol. $s_{car} = \frac{1}{2}at^2 \rightarrow (1)$, $s_{bike} = ut \rightarrow (2)$

if they meet at the same point

$s_{car} = s_{bike}$

$\frac{1}{2}at^2 = ut \Rightarrow t = \frac{2u}{a}$

$s_{bike} = ut = u\frac{2u}{a} = \frac{2u^2}{a}$

$v_{car} = at = 2u$

v_{car} w.r.t. bike at the time of meeting, $|\vec{v}_{cb}| = |\vec{v}_{car} - \vec{v}_{bike}| = 2u - u = u$

EX.9: What does $d|\vec{v}|/dt$ **and** $|d\vec{v}/dt|$ **represent? can these be equal ? can:**

(a) $d|\vec{v}|/dt = 0$ **while** $|d\vec{v}/dt| \neq 0$

(b) $d|\vec{v}|/dt \neq 0$ **while** $|d\vec{v}/dt| = 0$ **?**

Sol. $d|\vec{v}|/dt$ represents time rate of change of speed as $|\vec{v}| = u$, while $|d\vec{v}/dt|$ represents magnitude of acceleration.

If the motion of a particle has translational acceleration (without change in direction)

as $\vec{v} = |\vec{v}|\hat{n}, \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[|\vec{v}|\hat{n}\right]$

or $\frac{d\vec{v}}{dt} = \hat{n}\frac{d}{dt}|\vec{v}|$ [as $\hat{n}$ is constant]

or $\left|\frac{d\vec{v}}{dt}\right| = \frac{d}{dt}|\vec{v}|(\neq 0)$

In this case both these will be equal and not equal to zero.

(a) The given condition implies that:

$|d\vec{v}/dt| \neq 0$, i.e., $|acc.| \neq 0$ while $d|\vec{v}|/dt = 0$, i.e., speed =constant.

This actually is the case of uniform circular motion. In case of uniform circular motion

$\left|\frac{d\vec{v}}{dt}\right| = |\vec{a}| = \frac{u^2}{r} = \text{constant} \neq 0$

while $|\vec{v}| = \text{constant}$. i.e., $\frac{d}{dt}|\vec{v}| = 0$

(b) $\left|d\vec{v}/dt\right| = 0$ means $|\vec{a}| = 0$, i.e., $\vec{a} = \vec{0}$

or $\left(d\vec{v}/dt\right) = \vec{0}$ or $\vec{v} = \text{constant}$

And when velocity $\vec{v}$ is constant speed will be constant,

i.e., speed $= |\vec{v}|$=constant or $\frac{d}{dt}|\vec{v}| = 0$

So, it is not possible to have $\left|\frac{d\vec{v}}{dt}\right| = 0$ while $\frac{d}{dt}|\vec{v}| \neq 0$.

EX.10:In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point . Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2 respectively , the value of $\frac{V}{t}$ is

Sol. The distance covered by both cars is same

Thus, $s_1=s_2=s$

If the cars take time t_1 and t_2 for the race and their velocities at the end of race be v_1 and v_2 , then it is given that

$v_1 - v_2 = v$ and $t_2 - t_1 = t$

Now, $\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{\frac{1}{a_2}} - \sqrt{\frac{1}{a_1}}}$ $\therefore \frac{v}{t} = \sqrt{a_1 a_2}$ \

EX.11 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1sec. How long the drunkard takes to fall in a pit 13m away from the start?

Sol. Distance of the pit from the start =13-5=8m

Time taken to move first 5m=5sec

5 steps (i.e., 5m) forward and 3steps(i.e.,3m) backward means that net distance moved =5-3=2m

and time taken during this process =5+3=8sec

$\therefore$ Time taken in moving $8\text{m} = \frac{8 \times 8}{2} = 32\text{sec}$

$\therefore$ Total time taken to fall in the pit =32+5=37sec

EX.12 : An α particle travels inside a straight hollow tube 2m long of a particle accelerator under uniform acceleration. How long is the particle in the tube if it enters at a speed of 1000 m/s and leaves at 9000 m/s. What is its acceleration during this interval ?

Sol. Let 'a' be the uniform acceleration of α-particle. According to given problem s = 2m, v = 9000 m/s and u = 1000 m/s. Since $v^2 - u^2 = 2aS$,

$(9000)^2 - (1000)^2 = 2a(2)$ $\Rightarrow a = 2 \times 10^7 \text{ m/s}^2$

Let the particle remains in the tube for time 't', then $v = u + at$

$$\Rightarrow t = \frac{v-u}{a} = \frac{9000-1000}{2\times 10^7} = 4\times 10^{-4} s$$

EX.13:A car starts from rest and moves with uniform acceleration of 5 m/s^2 for 8 sec. If the acceleration ceases after 8 seconds then find the distance covered in 12s starting from rest.

Sol. The velocity after 8 sec v = 0 + 5 x 8 = 40 m/s

Distance covered in 8 sec

$$s_0 = 0 + \frac{1}{2}\times 5\times 64 = 160m$$

After 8s the body moves with uniform velocity and distance covered in next 4s with uniform velocity.

s = vt = 40 x 4 = 160 m

The distance covered in12 s =160 +160 =320m.

EX.14 : A car is moving with a velocity of 40 m/s. The driver sees a stationary truck ahead at a distance of 200 m. After some reaction time Δt the breaks are applied producing a (reaction) retardation of 8m/s^2. What is the maximum reaction time to avoid collision ?

Sol. The car before coming to rest (v = 0)

$$v^2 = u^2 + 2as \rightarrow 0 = (40)^2 - 2\times 8\times s$$

$$\Rightarrow s = 100m$$

The distance travelled by the car is 100m

To avoid the clash, the remaining distance 200 - 100 = 100m must be covered by the car with uniform velocity 40 m/s during the reaction time Δt.

$$\frac{100}{\Delta t} = 40 \Rightarrow \Delta t = 2.5s$$

The maximum reaction time $\Delta t = 2.5s$

EX.15 : Two trains one travelling at 54 kmph and the other at 72 kmph are headed towards one another along a straight track. When they are 1/2 km apart, both drivers simultaneously see the other train and apply their brakes. If each train is decelerated at the rate of 1 m/s^2, will there be a collision ?

Sol. Velocity of the first train is 54 kmph = 15 m/s

Distance travelled by the first train before coming to rest

$$s_1 = \frac{u^2}{2a} = \frac{225}{2} = 112.5m$$

Velocity of the second train is 72 kmph= 20 m/s

Distance travelled by the second train before coming to rest

$$s_2 = \frac{u^2}{2a} = \frac{400}{2} = 200m$$

Total distance travelled by the two trains before coming to rest = $s_1 + s_2$ = 112.5 + 200 = 312.5m

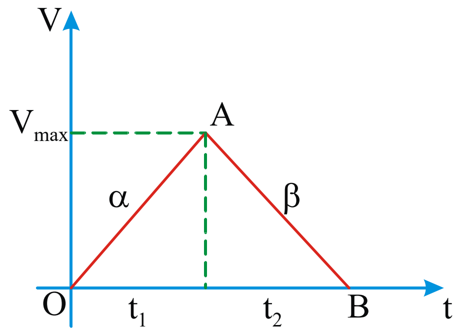
Because the initial distance of separation is 500m which is greater than 312.5m, there will be no collision between the two trains.

EX.16. **A bus accelerates from rest at a constant rate 'α' for some time, after which it decelerates at a constant rate 'β' to come to rest. If the total time elapsed is t seconds. Then evaluate following parameters from the given graph**

a) the maximum velocity achieved

b) the total distance travelled graphically and

c) Average velocity



Sol. **a)** α = Slope of line OA = $\frac{v_{max}}{t_1} \Rightarrow t_1 = \frac{v_{max}}{\alpha}$

β = Slope of line AB = $\frac{v_{max}}{t_2} \Rightarrow t_2 = \frac{v_{max}}{\beta}$

$$t = t_1 + t_2 = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

b) Displacement = area under the v-t graph

= area of ΔOAB

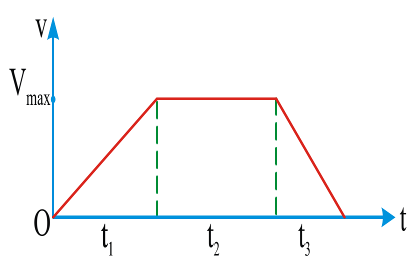
$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(t_1 + t_2)v_{max} = \frac{1}{2}tv_{max}$$

$$= \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

$$v_{avg} = \frac{\text{total displacement}}{\text{total time}} = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t = \frac{v_{max}}{2}$$

EX.17: **A body starts from rest and travels a distance S with uniform acceleration, then moves uniformly a distance 2S and finally comes to rest after moving further 5S under uniform retardation. Find the ratio of average velocity to maximum velocity**

Sol.



area of $\Delta OAC \quad S = \frac{1}{2} V_{max} t_1 \text{ (or) } t_1 = \frac{2S}{V_{max}}$;

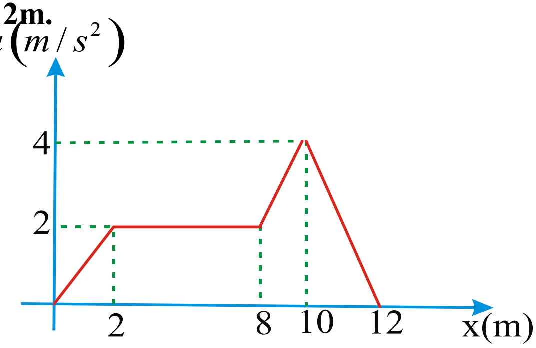
area of ABCD $\quad 2S = V_{max} t_2 \text{ (or) } t_2 = \frac{2S}{V_{max}}$

area of $\Delta BDE \quad 5S = \frac{1}{2} V_{max} t_3 \text{ (or) } t_3 = \frac{10S}{V_{max}}$

$$V_{avg} = \frac{\text{Total displacement}}{\text{Total time}}; \quad V_{avg} = \frac{S + 2S + 5S}{\frac{2S}{V_{max}} + \frac{2S}{V_{max}} + \frac{10S}{V_{max}}} = \frac{8S}{\frac{14S}{V_{max}}}$$

$$\frac{V_{avg}}{V_{max}} = \frac{8S}{14S} = \frac{4}{7}$$

EX.18: The acceleration-displacement (a - x) graph of a particle moving in a straight line is as shown. If the particle starts from rest,then find the velocity of the particle when displacement of the particle is 12m.



Sol. $v^2 - u^2 = 2ax$

$$\Rightarrow ax = \frac{v^2 - u^2}{2} = \frac{v^2}{2} \qquad (\because u=0)$$

$$\Rightarrow v = \sqrt{2(\text{area under a} - \text{x graph})}$$

Area under a-x graph =

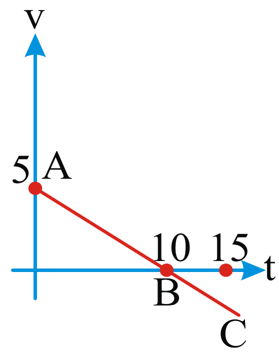
Area of ΔOAE + Area of rectangle ABEF

+ Area of trapezium BFGC + Area of ΔCGD

$$\text{Area} = \frac{1}{2}(2)(2) + 6\times 2 + \frac{1}{2}(2+4)(2) + \frac{1}{2}\times 2\times 4 = 24$$

$$\Rightarrow v = \sqrt{2\times 24} = 4\sqrt{3} \text{m/s}$$

EX.19: Velocity-time graph for the motion of a certain body is shown in Fig. Explain the nature of this motion. Find the initial velocity and acceleration and write the equation for the variation of displacement with time. What happens to the moving body at point B ? How will the body move after this moment ?



Sol. The velocity -time graph is a straight line with -ve slope, the motion is uniformly retarding one upto point B and uniformly accelerated after with-ve side of velocity axis .
At point B the body stops and then its direction of velocity reversed.

The initial velocity at point A is $v_0 = 7ms^{-1}$

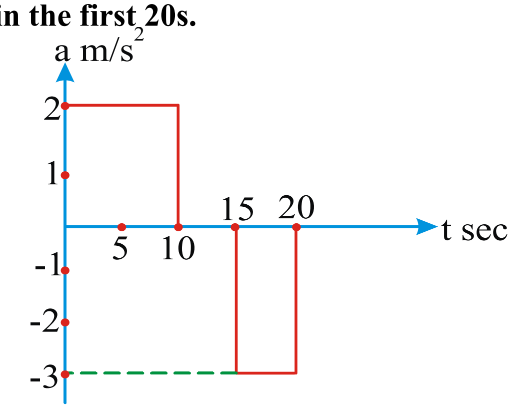
The acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v_t - v_0}{\Delta t} = \frac{0-7}{11} = -0.6364ms^{-2} \approx -0.64ms^{-2}$$

The equation of motion for the variation of displacement with time is

$$s = 7t - \frac{1}{2}0.64t^2 = 7t - 0.32t^2$$

EX.20: **A particle starts from rest and accelerates as shown in the graph. Determine**
a) the particle's speed at t = 10s and at t = 20s
b) the distance travelled in the first 20s.



Sol. a) Upto 10 sec the particle moves with uniform acceleration, hence the velocity at t = 10s,
$v_1 = u + at = 0 + 2 \times 10 = 20m/s$
From t = 10s to t = 15s the acceleration is zero,so The velocity of the particle at t=15s is 20m/s
Velocity at t = 20sec, $v_2 = v_1 + at$
$= 20 + (-3)5 = 5$ m/s
b) Distance travelled in first 10 sec

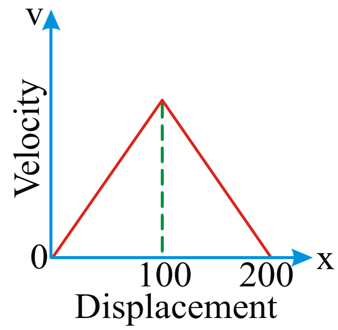
$$s_1 = ut + \frac{1}{2}a_1t^2 = 0\times 10 + \frac{1}{2}\times 2\times (10)^2 = 100m$$ Distance travelled when t = 10 sec to t = 15 sec

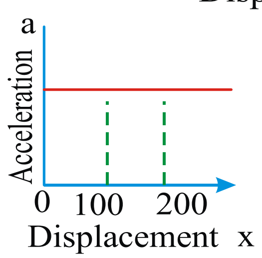
$$s_2 = vt = 20\times 5 = 100m$$ Distance travelled when t = 15 sec to t = 20 sec

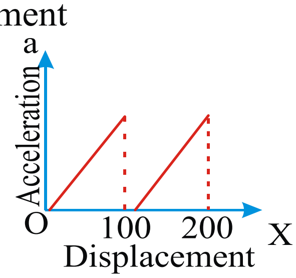
$$s_3 = 20 \times 5 + \frac{1}{2} \times (-3) \times 5^2 = 62.5\text{m}$$

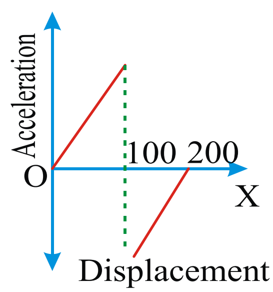
Total distance travelled in 20 sec = $s_1 + s_2 + s_3$ = 100 + 100 + 62.5 = 262.5 m

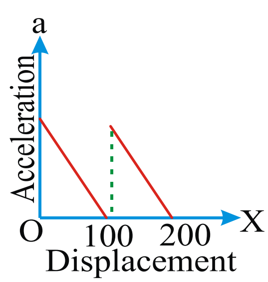
EX.21: Velocity (v) versus displacement (x) plot of a body moving along a straight line is as shown in the graph. The corresponding plot of acceleration (a) as a function of displacement (x) is (EAM-2014)



1)


2)


3)


4)


Sol. From the given graph equation for velocity v = kx

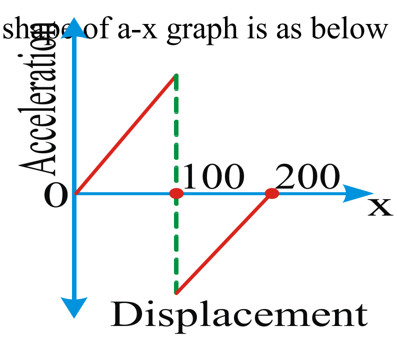
on differentiation

$$\frac{dv}{dt} = kv \qquad \text{------ (i)}$$

$$\frac{dv}{dt} = k(kx) = k^2x; \qquad a = k^2x \qquad \text{and } v = -kx + v_0$$

on differentiation $\frac{dv}{dt} = -kv = -k(-kx + v_0)$ $\qquad a = k^2x - kv_0 \qquad$ -------(ii)

So, according to the eq. (ii) the shape of a-x graph is as below



Equations of Motion for Variable Acceleration :

↪ When acceleration 'a' of the particle is a function of time i.e., a = f(t)

$$\Rightarrow \frac{dv}{dt} = f(t) \Rightarrow dv = f(t)dt$$

Integrating both sides within suitable limits, we have

$$\int_u^v dv = \int_0^t f(t)dt \Rightarrow v = u + \int_0^t f(t)dt$$

↪ When acceleration 'a' of the particle is a function of distance a = f(x)

$$\Rightarrow \frac{dv}{dt} = f(x) \Rightarrow \frac{dv}{dx}\frac{dx}{dt} = f(x)$$

$$\int_u^v vdv = \int_{x_0}^x f(x)dx \Rightarrow v^2 = u^2 + 2\int_{x_0}^x f(x)dx$$

EX.22: A particle located at x = 0, at time t = 0, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of particle varies with time as

Sol. $\frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt$

On integrating, we get $\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha dt$

[at t = 0, x = 0 and let at any time t, particle be at x]

$$\Rightarrow \left[\frac{x^{1/2}}{1/2}\right]_0^x = \alpha t \text{ or } x^{1/2} = \frac{\alpha}{2}t \Rightarrow x \;\alpha\; t^2$$

EX.23 : The acceleration (a) of a particle moving in a straight line varies with its displacement (S) as a = 2S. The velocity of the particle is zero at zero displacement. Find the corresponding velocity-displacement equation.

Sol. $a = 2S \Rightarrow v\frac{dv}{ds} = 2S \Rightarrow vdv = 2SdS$

$$\Rightarrow \int_0^v vdv = \int_0^s 2SdS \qquad \Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{S^2}{2}\right)_0^S$$

$$\Rightarrow \frac{v^2}{2} = S^2 \Rightarrow v = \pm\sqrt{2}S$$

EX.24: An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is instantaneous speed. The time taken by the object, to come to rest, would be (AIEEE-2011)

Sol. $\frac{dv}{dt} = -2.5\sqrt{v}$ (or) $\frac{1}{\sqrt{v}}dv = -2.5dt$

On integrating, within limits

($v_1 = 6.25$ m/s to $v_2 = 0$)

$$\int_{v_1=6.25m/s}^{v_2=0} v^{-1/2}dv = -2.5\int_0^t dt$$

$$2\times\left[v^{1/2}\right]_{6.25}^{0} = -(2.5)t \text{ (or)} \qquad t = \frac{-2\times(6.25)^{1/2}}{-2.5} = 2s$$

EX.25:The velocity of a particle moving in the positive direction of the X-axis varies as $V = K\sqrt{S}$ where K is a positive constant. Draw V-t graph.

Sol. $V = K\sqrt{s}$

$$\frac{dS}{dt} = K\sqrt{S} \Rightarrow \int_0^S \frac{dS}{\sqrt{S}} = \int_0^t K\ dt$$

$$\Rightarrow 2\sqrt{S} = Kt \text{ and } S = \frac{1}{4}K^2t^2$$

$$\Rightarrow V = \frac{dS}{dt} = \frac{1}{4}K^2 2t = \frac{1}{2}K^2t \qquad \Rightarrow V \propto t$$

$\therefore V \propto t$

$\therefore$ The V-t graph is a straight line passing through the origin.

Acceleration due to gravity

- The uniform acceleration of a freely falling body towards the centre of earth due to earth's gravitational force is called acceleration due to gravity
- It is denoted by 'g'
- Its value is constant for all bodies at a given place. It is independent of size, shape, material, nature of the body.
- Its value changes from place to place on the surface of the earth.
- It has maximum value at the poles of the earth. The value is nearly 9.83 m/s^2.
- It has minimum value at equator of the earth. The value is nearly 9.78 m/s^2.
- On the surface of the moon, $g_{moon} = \frac{g_{earth}}{6}$
- The acceleration due to gravity of a body always directed downwards towards the centre of the earth, whether a body is projected upwards or downwards.
- When a body is falling towards the earth, its velocity increases, g is positive.
- The acceleration due to gravity at the centre of earth is zero.

Motion under gravity

Equation of motion for a body projected vertically downwards :

- When a body is projected vertically downwards with an initial velocity u from a height h then a = g, s = h

a) $v = u + gt$ b) $h = ut + \frac{1}{2}gt^2$ c) $v^2 - u^2 = 2gh$ d) $S_n = u + \frac{g}{2}(2n-1)$

↪ In case of freely falling body $u = 0$, $a = +g$

a) $v = gt$ b) $S = \frac{1}{2}gt^2$ c) $v^2 = 2gS$ d) $S_n = g\left(n - \frac{1}{2}\right)$

↪ For a freely falling body, the ratio of distances travelled in 1 second, 2seconds, 3 seconds, ... = 1 : 4 : 9 : 16....

↪ For a freely falling body, the ratio of distances travelled in successive seconds = 1 : 3 : 5 : 9 ....

↪ A freely falling body passes through two points A and B in time intervals of t_1 and t_2 from the start, then the distance between the two points A and B is $= \frac{g}{2}\left(t_2^2 - t_1^2\right)$

↪ A freely falling body passes through two points A and B at distances h_1 and h_2 from the start, then the time taken by it to move from A to B is

$$T = \sqrt{\frac{2h_2}{g}} - \sqrt{\frac{2h_1}{g}} = \sqrt{\frac{2}{g}}\left(\sqrt{h_2} - \sqrt{h_1}\right)$$

↪ Two bodies are dropped from heights h_1 and h_2 simultaneously. Then after any time the distance between them is equal to $(h_2 \sim h_1)$.

↪ A stone is dropped into a well of depth 'h', then the sound of splash is heard after a time of 't'.

Time taken by the body to reach water, $t_1 = \sqrt{\frac{2h}{g}}$

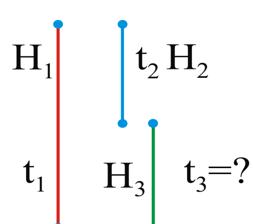
Time taken by sound to travel a distance 'h', $t_2 = \frac{h}{V_{sound}}$

$\therefore$ Time to hear splash of sound is $t = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{V_{sound}}$

↪ A stone is dropped into a river from the bridge and after 'x' seconds another stone is projected down into the river from the same point with a velocity of 'u'. If both the stones reach the water simultaneously, then $S_{1(t)} = S_{2(t-x)}$

$$\frac{1}{2}gt^2 = u(t - x) + \frac{1}{2}g(t - x)^2$$

↪ A body dropped freely from a multistoried building can reach the ground in t_1 sec. It is stopped in its path after t_2 sec and again dropped freely from the point. The further time taken by it to reach the ground is $t_3 = \sqrt{t_1^2 - t_2^2}$.



We know that $H_1 = H_2 + H_3$

$$\Rightarrow \frac{1}{2}gt_1^2 = \frac{1}{2}gt_2^2 + \frac{1}{2}gt_3^2$$

$$\Rightarrow t_1^2 = t_2^2 + t_3^2 \qquad \therefore t_3 = \sqrt{t_1^2 - t_2^2}$$

Equations of motion of a body Projected Vertically up :

↳ Acceleration (a) = -g

a) $v = u - gt$ b) $s = ut - \frac{1}{2}gt^2$ c) $v^2 - u^2 = -2gh$ d) $s_n = u - \frac{g}{2}(2n-1)$

↳ Angle between velocity vector and acceleration vector is 180^0 until the body reaches the highest point.

↳ At maximum height, v = 0 and a = g

↳ $H_{max} = \frac{u^2}{2g} \Rightarrow H_{max} \propto u^2$ (independent of mass of the body)

↳ A body is projected vertically up with a velocity ‘u’ from ground in the absence of air resistance ‘R’. then. $(t_a = t_d)$

i) $t_a = t_d = \frac{u}{g}$

ii) Time of flight $T = t_a + t_d = \frac{2u}{g}$

↳ A body is projected vertically up with a velocity ‘u’ from ground in the presence of constant air resistance ‘R’. If it reaches the ground with a velocity ‘v’, then

a) Height of ascent = Height of descent

b) Time of ascent $t_a = \frac{mu}{mg + R}$

c) Time of descent $t_d = \frac{mv}{mg - R}$

d) $t_a < t_d$

e) $\frac{v}{u} = \sqrt{\frac{mg - R}{mg + R}}\,(v < u)$

f) For a body projected vertically up under air resistance, retardation during its motion is > g

↳ At any point of the journey, a body possess the same speed while moving up and while moving down.

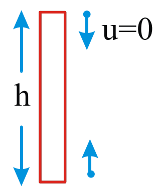
↳ Irrespective of velocity of projection, all the bodies pass through a height $\frac{g}{2}$ in the last second of ascent. Distance traveled in the last second of its journey $u - \frac{g}{2}$.

↳ The change in velocity over the complete journey is ‘2u’ (downwards)

↳ If a vertically projected body rises through a height ‘h’ in n^{th} second, then in $(n-1)^{th}$ second it will rise through a height (h+g) and in $(n+1)^{th}$second it will rise through height (h-g).

↳ If velocity of body in n^{th} second is 'v' then in $(n-1)^{th}$ second it is $(v+g)$ and that in $(n+1)^{th}$ second is (v - g) while ascending

↳ A body is dropped from the top edge of a tower of height 'h' and at the same time another body is projected vertically up from the foot of the tower with a velocity 'u'.



a) The separation between them after 't' seconds is = $(h-ut)$

b) The time after which they meet $t=\frac{h}{u}$

c) The height at which they meet above the ground = $\left(h-\frac{gh^2}{2u^2}\right)$

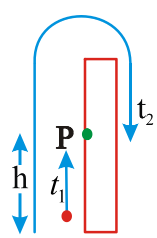
d) The time after which their velocities are equal in magnitudes is $t=\frac{u}{2g}$

e) If they meet at mid point, then the velocity of thrown body is $u=\sqrt{gh}$ and its velocity of meeting is zero

f) Ratio of the distances covered when the magnitudes of their velocities are equal is 1 : 3.

↳ A body projected vertically up crosses a point P at a height 'h' above the ground at time 't_1' seconds and at time t_2 seconds (t_1 *and* t_2 are measured from the instant of projection) to same point while coming down.

$h=ut-\frac{1}{2}gt^2$; $\qquad gt^2-2ut+2h=0$



This is quadratic equation in t

Sum of the roots, $t_1+t_2=\frac{2u}{g}$ (time of flight)

Velocity of projection , $u=\frac{1}{2}g(t_1+t_2)$

Product of the roots, $t_1 t_2 = \frac{2h}{g}$

Height of P is $h = \frac{1}{2} g t_1 t_2$

Maximum height reached above the ground $H = \frac{1}{8} g (t_1 + t_2)^2$

Magnitude of velocity while crossing P is $\frac{g(t_2 - t_1)}{2}$

↳ A body is projected vertically up with velocity u_1 and after 't' seconds another body is projected vertically up with a velocity u_2.

a) If $u_2 > u_1$, the time after which both the bodies will meet with each other is $h_1 = h_2$

$$u_1 x - \frac{1}{2} g x^2 = u_2 (x - t) - \frac{1}{2} g (x - t)^2$$

$$x = \frac{u_2 t + \frac{1}{2} g t^2}{(u_2 - u_1) + gt}$$ for the first body.

b) If $u_1 = u_2 = u$, the time after which they meet is $\left(\frac{u}{g} + \frac{t}{2}\right)$ for the first body and $\left(\frac{u}{g} - \frac{t}{2}\right)$ for the second body.

Height at which they meet $= \frac{u^2}{2g} - \frac{gt^2}{8}$

↳ A rocket moves up with a resultant acceleration a. If its fuel exhausts completely after time 't' seconds, the maximum height reached by the rocket above the ground is

$$h = h_1 + h_2$$

$$= \frac{1}{2} a t^2 + \frac{1}{2g} a^2 t^2 \left[h_2 = \frac{v^2}{2g} = \frac{a^2 t^2}{2g}\right], [v = at]$$

$$h = \frac{1}{2} a t^2 \left(1 + \frac{a}{g}\right)$$

↳ An elevator is accelerating upwards with an acceleration a. If a person inside the elevator throws a particle vertically up with a velocity u relative to the elevator, time of flight is $t = \frac{2u}{g + a}$

↳ In the above case if elevator accelerates down, time of flight is $t = \frac{2u}{g - a}$

↳ The zero velocity of a particle at any instant does not necessarily imply zero acceleration at that instant. A particle may be momentarily at rest and yet have non-zero acceleration.
For example, a particle thrown up has zero velocity at its uppermost point but the acceleration at that instant continues to be the acceleration due to gravity.

EX.26: Drops of water fall at regular intervals from the roof of a building of height h = 16m. The first drop striking the ground at the same moment as the fifth drop is ready to leave from the roof. Find the distance between the successive drops.

Sol. **Step-I** : Time taken by the first drop to touch the ground $= t = \sqrt{\frac{2h}{g}}$

For h = 16m, $t = \sqrt{\frac{16(2)}{g}} = 4\sqrt{\frac{2}{g}}$

Time interval between two successive drops is $\Delta t = \left(\frac{1}{n-1}\right)t = \left(\frac{1}{4}\right)t = \sqrt{\frac{2}{g}}$

Where n = number of drops

Step-II :

Distance travelled by 1st drop $S_1 = \frac{1}{2}g(4\Delta t)^2 = \frac{1}{2}g(16)\left(\frac{2}{g}\right) = 16m$

Distance travelled by 2nd drop $S_2 = \frac{1}{2}g(3\Delta t)^2 = \frac{1}{2}g(9)\left(\frac{2}{g}\right) = 9m$

Distance travelled by 3rd drop $S_3 = \frac{1}{2}g(2\Delta t)^2 = \frac{1}{2}g(4)\left(\frac{2}{g}\right) = 4m$

Distance travelled by 4th drop $S_4 = \frac{1}{2}g(\Delta t)^2 = \frac{1}{2}g\left(\frac{2}{g}\right) = 1m$

Distance between 1st and 2nd drops$= S_1 - S_2 = 16 - 9 = 7m$

Distance between 2nd and 3rd drops$= S_2 - S_3 = 9 - 4 = 5m$

Distance between 3rd and 4th drops$= S_3 - S_4 = 4 - 1 = 3m$

Distance between 4th and 5th drops$= S_4 - S_5 = 1 - 0 = 1m$

EX.27: A body falls freely from a height of 125m ($g = 10\ m/s^2$). After 2 sec gravity ceases to act Find time taken by it to reach the ground ?

Sol. 1) Distance covered in 2s under gravity $S_1 = \frac{1}{2}gt^2 = \frac{1}{2}(10)(2)^2 = 20m$

Velocity at the end of 2s $V = gt = 10 \times 2 = 20$ m/s

Now at this instant gravity ceases to act, there after velocity becomes constant.The remaining distance which is 125 - 20 = 105m is covered by body with constant velocity20 m/s.Time taken to cover 105 m with constant velocity is given by

$t_1 = \frac{S}{V} \Rightarrow t_1 = \frac{105}{20} = 5.25s$

Total time taken = 2 + 5.25 = 7.25 s

EX.28: A parachutist drops freely from an aeroplane for 10 seconds before the parachute opens out. Then he descends with a net retardation of 2 m/s². His velocity when he reaches the ground is 8 m/s. Find the height at which he gets out of the aeroplane ?

Sol. Distance he falls before the parachute opens is

$$S_1 = \frac{1}{2}g \times 100 = 490m$$

Then his velocity , u=gt = 98.0 m/s

Velocity on reaching ground = v=8 m/s

retardation = 2 m/s^2

$v^2 - u^2 = 2aS_2 \qquad \Rightarrow S_2 = 2385m$

Total distance $S = S_1 + S_2$ = 2385 + 490

=2875 m(height of aeroplane)

EX.29: If a freely falling body covers half of its total distance in the last second of its journey. Find its time of fall

Sol. Suppose t is the time of free fall

$$S_n = \frac{g}{2}(2n-1) \text{ and } S = \frac{g}{2}(n^2)$$

$$\frac{S_n}{S} = \frac{(2n-1)\times\frac{g}{2}}{n^2\times\left(\frac{g}{2}\right)} = \frac{1}{2}$$

$n^2 - 4n + 2 = 0$ and $n = \left(2+\sqrt{2}\right)$ sec

EX.30: A body is projected vertically up with a velocity u. Its velocity at half of its maximum height and at 3/4th of its maximum height are

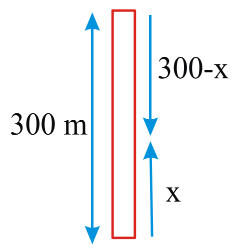
Sol. From $v^2 - u^2 = 2aS$, here a = –g; when S = H/2, then

$$v^2 - u^2 = 2(-g)\frac{u^2}{4g} \quad \Rightarrow v = \frac{u}{\sqrt{2}}$$

When S = 3H/4, then $v^2 - u^2 = 2(-g)\frac{3u^2}{4(2g)} \quad \Rightarrow v = \frac{u}{2}$

EX.31: A stone is allowed to fall from the top of a tower 300m height and at the same time another stone is projected vertically up from the ground with a velocity 100 m/s. Find when and where the two stones meet ?

Sol.



height of the tower, h= 300m

Suppose the two stones meet at a height x from ground after t seconds

$$t = \frac{S_r}{u_r},\ u_r = u + 0 = u,\ S_r = h$$

$$t = \frac{h}{u} = \frac{300}{100} = 3\,\text{sec}$$

height of the stone from the point of projection is

$$h_1 = ut - \frac{1}{2}gt^2 = 100 \times 3 - \frac{1}{2} \times 9.8 \times 9 = 255.9m$$

EX.32: A stone is dropped from certain height above the ground. After 5s a ball passes through a pane of glass held horizontally and instantaneously loses 20% of its velocity. If the ball takes 2 more seconds to reach the ground, the height of the glass above the ground is

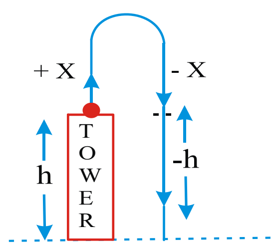
Sol. In 5s velocity gained v = gt = 50 m/s.

Velocity after passing through the glass pane $\frac{80}{100} \times 50 = 40\text{m/s}$

Height of the glass pane above the ground is $h = ut + \frac{1}{2}gt^2 = 40 \times 2 + \frac{1}{2} \times 10 \times (2)^2 = 100\text{m}$

Body Projected Vertically up from a Tower

A body projected vertically up from a tower of height 'h' with a velocity 'u' (or) a body dropped from a rising balloon (or) a body dropped from an helicopter rising up vertically with constant velocity 'u' reaches the ground exactly below the point of projection after a time 't'. Then



(a) Height of the tower is $h = -ut + \frac{1}{2}gt^2$

(b) Time taken by the body to reach the ground

$$t = \frac{u + \sqrt{u^2 + 2gh}}{g}$$

(c) The velocity of the body at the foot of the tower $v = \sqrt{u^2 + 2gh}$

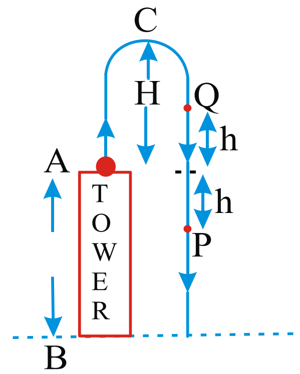
(d) Velocity of the body after 't' sec. is

$$v = u - gt$$

(e) Distance between the body and balloon after this time = $\frac{1}{2}gt^2$

EX.33: A ball is thrown vertically upwards from the top of a tower. Velocity at a point 'h' m vertically below the point of projection is twice the downward velocity at a point 'h' m vertically above the point of projection. The maximum height reached by the ball above the top of the tower is (MED-2012)

Sol. If AB is the tower then according to the problem, velocity at 'P' is given as twice the velocity at 'Q'



$$V_P = \sqrt{(u^2 + 2gh)} \; ; \; V_Q = \sqrt{(u^2 - 2gh)} \; ; \; V_P = 2(V_Q)$$

$$\sqrt{(u^2 + 2gh)} = 2\sqrt{(u^2 - 2gh)} \Rightarrow u^2 = \frac{10gh}{3}$$

From the top of the tower maximum height reached $H = \frac{u^2}{2g} \Rightarrow H = \frac{\left(\frac{10gh}{3}\right)}{2g} = \frac{5h}{3}$

EX.34: From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground is n times that taken by it to reach the highest point of its path. The relation between H, u and n is (jee main- 2014)

Sol. Time taken to reach the maximum height $t_1 = \frac{u}{g}$

If t_2 is the time taken to hit ground

i.e., $-H = ut_2 - \frac{1}{2}gt^2$

But $t_2 = nt_1$; So, $-H = u\left(\frac{nu}{g}\right) - \frac{1}{2}g\left(\frac{n^2u^2}{g^2}\right)$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\left(\frac{n^2u^2}{g}\right) \Rightarrow 2gH = nu^2(n-2)$$

EX.35 : A balloon starts from rest, moves vertically upwards with an acceleration g/8 ms^{-2}. A stone falls from the balloon after 8 s from the start. Further time taken by the stone to reach the ground (g = 9.8 ms^{-2}) is

Sol. The distance of the stone above the ground about which it begins to fall from the balloon is

$$h = \frac{1}{2}\left(\frac{g}{8}\right)8^2 = 4g$$

The velocity of the balloon at this height can be obtained from $v = u + at$; $v = 0 + \left(\frac{g}{8}\right)8 = g$

This becomes the initial velocity (u) of the stone as the stone falls from the balloon at the height h.

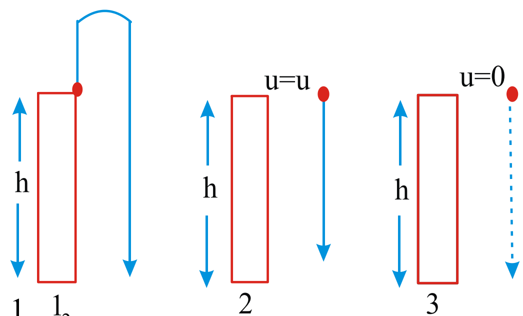
$\therefore u' = g$

For the total motion of the stone $h = -u^{1}t + \frac{1}{2}gt^2$

$\therefore -4g = gt - \frac{1}{2}gt^2$ and $t^2 - 2t - 8 = 0$

Solving for 't' we get t = 4 and -2s. Ignoring negative value of time, t = 4s.

- Three bodies are projected from towers of same height as shown. 1st one is projected vertically up with a velocity 'u'. The second one is thrown down vertically with the same velocity and the third one is dropped as a freely falling body. If t_1, t_2, t_3 are the times taken by them to reach ground, then,



For 1st body, $h = -ut_1 + \frac{1}{2}gt_1^2 \quad \rightarrow (1)$

For 2nd body, $h = ut_2 + \frac{1}{2}gt_2^2 \quad \rightarrow (2)$

For 3rd body, $h = \frac{1}{2}gt_3^2 \quad \rightarrow (3)$

from $(1)\times t_2 + (2)\times t_1$

$\Rightarrow h(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1 + t_2)$

i) Height of the tower $h = \frac{1}{2}gt_1t_2 \quad \rightarrow (4)$

ii) From eq (3) & (4), $t_3 = \sqrt{t_1t_2} \quad \rightarrow (5)$

iii) Equating R.H.S of (1) & (2),

velocity of projection $u = \frac{1}{2}g(t_1 - t_2) \rightarrow (6)$

iv) Time difference between first two bodies to reach the ground $\Delta t = \frac{2u}{g} \quad \rightarrow (7)$

Relative Motion in one dimension

- Velocity of one moving body with respect to other moving body is called Relative velocity.
- A and B are two objects moving uniformly with average velocities v_A and v_B in one dimension, say along x-axis having the positions $x_A(0)$ and $x_B(0)$ at t = 0.
- If $x_A(t)$ and $x_B(t)$ are positions of objects A and B at time t then

$x_A(t) = x_A(0) + v_At\ ;\ x_B(t) = x_B(0) + v_Bt$

- The displacement from object A to B is given by

$$x_{BA}(t) = x_B(t) - x_A(t)$$

$$= [x_B(0) - x_A(0)] + (v_B - v_A)t$$

$$= x_{BA}(0) + (v_B - v_A)t$$

Velocity of A w.r.t B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

- Two bodies are moving in a straight line in same direction then, $|\vec{v}_{AB}| = |\vec{v}_A| - |\vec{v}_B|$ $(\because \theta = 0^0)$
- Two bodies are moving in a straight line in opposite direction then, $|\vec{v}_{AB}| = |\vec{v}_A| + |\vec{v}_B|$, $(\because \theta = 180^0)$
- If two bodies move with same velocity and in same direction, then position between them does not vary with time.
- If two bodies move with unequal velocity and in same direction, then position between them first decreases to minimum and then increases.
- If the particles are located at the sides of n sided symmetrical polygon with each side a and each particle moves towards the other, then time after which they meet is

$$T = \frac{\text{Initial separation}}{\text{Relative velocity of approach}}$$

$$T = \frac{a}{v - v\cos\left(\frac{2\pi}{n}\right)} \Rightarrow T = \frac{a}{v\left(1 - \cos\left(\frac{2\pi}{n}\right)\right)}$$

and $T = \dfrac{a}{2v\sin^2\left(\frac{\pi}{n}\right)}$

Shortcut to solve the problems

For Triangle $n = 3 \Rightarrow T = \frac{2a}{3v}$; For Square $n = 4 \Rightarrow T = \frac{a}{v}$

For hexagon, $n = 6 \Rightarrow T = \frac{2a}{v}$

EX.36: A passenger is at a distance 'd' from a bus, when the bus begins to move with a constant acceleration a. Then find the minimum constant velocity with which the passenger should run towards the bus so as to catch is

Sol. $S_{passenger} = d + S_{bus}$; $vt = d + \frac{1}{2}at^2$

$$at^2 - 2vt + 2d = 0 \Rightarrow t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

for minimum velocity $v^2 - 2ad = 0 \Rightarrow v = \sqrt{2ad}$

EX.37. Two trains, each travelling with a speed of 37.5kmh^{-1}, are approaching each other on the same straight track. A bird that can fly at 60kmph flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird before trains collide is

Sol. Relative speed of trains $= 37.5 + 37.5 = 75\text{kmh}^{-1}$

Time taken by them to meet $t = \frac{S_r}{u_r} = \frac{90}{75} = \frac{6}{5}\text{h}$

Distance travelled by the bird, $x = V_{bird} \times t$ $= 60 \times \frac{6}{5} = 72\text{km}$ ($\because$ $V_{bird} = 60\text{kmh}^{-1}$) ,

EX.38: On a two-lane road, car A is travelling with a speed of 36 kmph. Two cars B and C approach car A in opposite directions with speed of 54 kmph each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Sol. Velocity of a car A, $V_A = 36\text{km/h} = 10\text{m/s}$

Velocity of car B, $V_B = 54\text{km/h} = 15\text{m/s}$

Velocity of car C, $V_C = 54\text{km/h} = 15\text{m/s}$

Relative velocity of car B with respect to car A, $V_{BA} = V_B - V_A = 15 - 10 = 5\text{m/s}$

Relative velocity of car C with respect to car A, $V_{CA} = V_C + V_A = 15 + 10 = 25m/s$

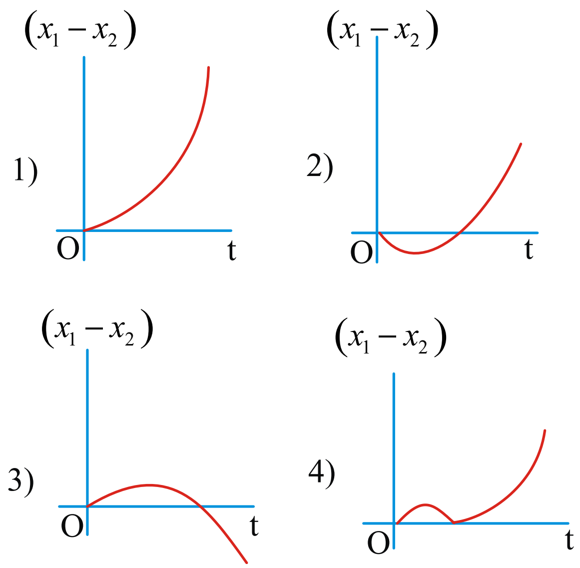
At a certain instance, both cars B and C are at the same distance from car A i.e., $s = 1\text{km} = 1000\text{m}$

Time taken (t) by car C to cover 1000m is $t = \frac{1000}{25} = 40\text{s}$

The acceleration produced by car B is

$1000 = 5 \times 40 + \frac{1}{2}\text{at}^2 \Rightarrow \text{a} = \frac{1600}{1600} = 1\text{m/s}^2$

EX.39: A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't' and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't' ? [AIEEE - 2008]



Sol. As, $x_1(t) = \frac{1}{2}at^2$ and $x_2(t) = vt$ $\therefore$ $x_1 - x_2 = \frac{1}{2}at^2 - vt$ (parabola)

Clearly, graph (2) represents it correctly.

EX.40. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. It speed after 10s is (AIEEE-2009)

Sol. $\vec{v} = \vec{u} + \vec{a}t$

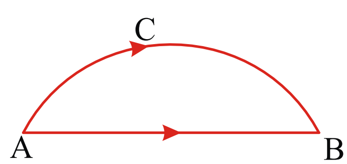
$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})10 = 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

$|\vec{v}| = \sqrt{49 + 49} = 7\sqrt{2}$

EX.41. A body is moving along the circumference of a circle of radius 'R' and completes half of the revolution. Then, the ratio of its displacement to distance is

1) $\pi : 2$ 2) 2:1 3) $2 : \pi$ 4) 1:2

Sol. .



Displacement : Distance $= \pi R : 2R$ **key-3**

EX.42. A body completes one round of a circle of radius 'R' in 20 second. The displacement of the body after 45 second is

1) $\frac{R}{\sqrt{2}}$ 2) $\sqrt{2}$ R 3) $2\sqrt{R}$ 4) 2R

Sol. In 40sec body completes two revolutions.

In 5 sec it covers 1/4 th of the circle and angle traced is $\frac{\pi}{2}$. So displacement $s = 2R\sin\frac{\theta}{2}$

key-2

EX.43. **If a body covers first half of its journey with uniform speed v_1 and the second half of the journey with uniform speed v_2, then the average speed is**

1) $v_1 + v_2$ 2) $\frac{2v_1v_2}{v_1 + v_2}$ 3) $\frac{v_1v_2}{v_1 + v_2}$ 4) v_1v_2

Sol. . Average speed $= \frac{s_1 + s_2}{t_1 + t_2} \Rightarrow v = \frac{2v_1v_2}{v_1 + v_2}$ key-2

EX.44. **A car is moving along a straight line, say OP in figure. It moves from O to P in 18 s and return from P to Q in 6 s. What are the average velocity and average speed of the car in going from O to P and back to Q?**

1) $10ms^{-1}, 20ms^{-1}$ 2) $20ms^{-1}, 10ms^{-1}$ 3) $10ms^{-1}, 10ms^{-1}$ 4) $20ms^{-1}, 20ms^{-1}$

Sol. . $v_{avg} = \frac{\text{total displacement}}{\text{total time}} = \frac{s_1 + s_2}{t_1 + t_2}$ key-1

EX.45. **For a body moving with uniform acceleration 'a', initial and final velocities in a time interval 't' are 'u' and 'v' respectively. Then, its average velocity in the time interval 't' is**

1) $(v + at)$ 2) $\left(v - \frac{at}{2}\right)$ 3) $(v - at)$ 4) $\left(u - \frac{at}{2}\right)$

Sol. $v_{avg} = \frac{v + u}{2} = \frac{v + v - at}{2}$ key-2

EX.46. **Two cars 1 & 2 starting from rest are moving with speeds v_1 and v_2 m/s $(v_1 > v_2)$. Car 2 is ahead of car '1' by s meter when the driver of the car '1' sees car '2'. What minimum retardation should be given to car '1' to avoid collision.** **(2002 A)**

1) $\frac{v_1 - v_2}{s}$ 2) $\frac{v_1 + v_2}{s}$ 3) $\frac{(v_1 + v_2)^2}{2s}$ 4) $\frac{(v_1 - v_2)^2}{2s}$

Sol. $u_{rel} = v_1 - v_2 ; v_{rel} = 0$; $v_{rel}^2 = u_{rel}^2 = 2as$

Key-4

EX.47. **If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is**

1) $\frac{1}{2}\sqrt{v_1v_2}$ 2) $\frac{v_1 + v_2}{2}$ 3) $\frac{2v_1v_2}{v_1 + v_2}$ 4) $\frac{5v_1v_2}{3v_1 + 2v_2}$ \

Sol. avg speed $= \frac{\text{total distance}}{\text{total time}} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$

key-4

EX.48. The coordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by

1) $\sqrt{\alpha^2 + \beta^2}$ 2) $3t\sqrt{\alpha^2 + \beta^2}$ 3) $3t^2\sqrt{\alpha^2 + \beta^2}$ 4) $t^2\sqrt{\alpha^2 + \beta^2}$

Sol. $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$; $v = \sqrt{v_x^2 + v_y^2}$

key-3

EX.49. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerate at the rate (f/2) to come to rest. If the total distance travelled is 15S, then

1) $S = ft$ 2) $S = \frac{1}{6}ft^2$ 3) $S = \frac{1}{72}ft^2$ 4) $S = \frac{1}{4}ft^2$

Sol.

s

$\left.\begin{array}{l} v^2 - o^2 = 2fs \\ o^2 - v^2 = -2\frac{f}{2}.s_3 \end{array}\right\}$ ® $s_3 = 2s$

$\therefore s_2 = 12s$; $12s = vt$ also $v = \sqrt{2fs}$

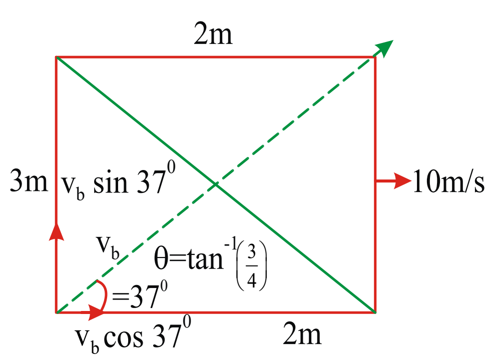
$\therefore 12s = \sqrt{2fs}.t$; $144s^2 = 2fs.t$; $s = \frac{1}{72}ft^2$

key-3

EX.50. An armored car 2m long and 3 m wide is moving at $10ms^{-1}$ when a bullet hits it in a direction making an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the length of the car as seen by a stationary observer. The bullet enters one edge of the car at the corner and passes out at the diagonally opposite corner. Neglecting any interaction between the car and the bullet and effect of gravity, the time for the bullet to cross the car is

1) 0.20 s 2) 0.15 s 3) 0.10 s 4) 0.50 s

Sol.



Relative velocity along x-axis is $\left(v_b \cos 37^0 - 10\right)$

Distance travelled by bullet along x- axis is

$\left(v_b \cos 37^0 - 10\right) t = 2$ ....(1)

Distance travelled by bullet along y-axis is

$\left(v_b \sin 37^0\right) t = 3$ ....(2)

Solving equation (1) and (2) we get t=0.20s key-1

EX.51. Two particles start simultaneously from the same point and move along two straight lines. One with uniform velocity v and other with a uniform acceleration a. If α is the angle between the lines of motion of two particles then the least value of relative velocity will be at time given by

1) $\frac{v}{a}\sin\alpha$ 2) $\frac{v}{a}\cos\alpha$ 3) $\frac{v}{a}\tan\alpha$ 4) $\frac{v}{a}\cot\alpha$

Sol. At any time velocity of first car is V and that of second car is $v = v + at = 0 + at$

$$v_{rel} = \sqrt{v^2 + (at)^2 - 2vat\cos\alpha}$$

v_{rel} is minimum if $\frac{d}{dt}\left(v_r^2\right) = 0$; $t = \frac{v\cos\alpha}{a}$

key-2

EX.52. A jet airplane travelling at the speed of 500 km/h ejects its products of combustion at the speed of 1500 km/h relative to the jet plane. What is the speed of the later with respect to an observer on ground?

1) -100kmph 2) -1000kmph 3) -10kmph 4) -11kmph

Sol. Velocity of jet aeroplane $= 500\hat{j}$

velocity of fuel w.r.to plane $= -1500\hat{j}$

$\therefore \vec{v}_f - \vec{v}_p = -1500\hat{j}$; $\vec{v}_f = \vec{v}_p - 1500\hat{j}$

$= 500\hat{j} - 1500\hat{j} = -1000\hat{j}$

$\therefore$ speed of fuel w.r.to ground is -1000km/hr.

key-2

EX.53. **A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?**

1) $x < 0, v < 0, a > 0$ 2) $x > 0, v < 0, a < 0$ 3) $x > 0, v < 0, a > 0$ 4) $x > 0, v > 0, a < 0$

Sol. As the lift is coming in downward direction dis-placement will be negative. We have to see whether the moton is accelerating or retarding.

We know that due to downward motion displace-ment will be negative. When the lift reaches 4th floor is about to stop hence, motion is retarding in nature hence, $x < 0, a > 0$. As displacement is in negative direction, velocity will also be negative i.e., $V < 0$.

key-1

EX.54. **A vehicle travels half the distance 'l' with speed v_1 and the other half with speed v_2, then its average speed is**

1) $\frac{v_1 + v_2}{2}$ 2) $\frac{2v_1 + v_2}{v_1 + v_2}$ 3) $\frac{2v_1 v_2}{v_1 + v_2}$ 4) $\frac{L(v_1 + v_2)}{v_1 v_2}$

Sol. Total time = $t_1 + t_2$

$$= \frac{1}{2v_1} + \frac{1}{2v_2} = \frac{1}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$$

key-3

EX.55. **The displacement of a particle is given by x = $(t - 2)^2$ if where x is in metre and t in second. The distance coverred by the particle in first 4 seconds is**

1) 4 m 2) 8 m 3) 12 m 4) 16m

Sol. Given x = $(t - 2)^2$

velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t-2)^2 = 2(t-2)\,m/s$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left[2(t-2)\right]$

$= 2\,[1 - 0] = 2\ m/s^2$

key-2

EX.56. **At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be**

l) $(t_1+t_2)/2$ 2) $t_1t_2/(t_2-t_1)$ 3) $t_1t_2/(t_2+t_1)$ 4) $t_1 - t_2$

Sol. Velocity of girl $v_g = \frac{L}{t_1}$

Velocity of escalator $v_e = \frac{L}{t_2}$

Net velocity of the girl $v_g + v_e = \frac{L}{t_1} + \frac{L}{t_2}$

$$\frac{L}{t} + \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$$

key-3

EX.57. **A body moving along a straight line traversed one third of the total distance with a velocity 4 m/sec in the first stretch. In the second stretch the remaining distance is covered with a velocity 2 m/sec for some time t_0 and with 4m/s for the remaining time. if the average velocity is 3 m/sec, find the time for which body moves with velocity 4 m/sec in second stretch:**

A) $\frac{3}{2}t_0$ B) t_0 C) $2t_0$ D) $\frac{t_0}{2}$

Sol. $t_1 = \frac{s/3}{4} = \frac{s}{12}$

$\frac{2s}{3} = 2(t_0) + 4(kt_0) = t_0(2+4k)$ or $t_0 = \frac{2s}{3(2+4k)}$

Average velocity $= \frac{s}{t_1 + t_0 + t_0 k}$

$= \frac{s}{\frac{s}{12} + \frac{2s(1+k)}{3(2+4k)}} = \frac{6(2+4k)}{(5+6k)}$

$v_{av}(5+6k) = 12 + 24k$ gives $k = \frac{1}{2}$

Required time $= kt_0 = \frac{t_0}{2}$. key-D

EX.58. **For motion of an object along the x-axis, the velocity v depends on the displacement x as $v = 3x^2 - 2x$, then what is the acceleration at x = 2 m?**

A) $48\ ms^{-2}$ B) $80\ ms^{-2}$ C) $18\ ms^{-2}$ D) $10\ ms^{-2}$

Sol. Given $v = 3x^2 - 2x$; differentiating v, we get

$\frac{dv}{dt} = (6x-2)\frac{dx}{dt} = (6x-2)v$

$\Rightarrow a = (6x-2)(3x^2 - 2x)$ Now put x= 2 m

$\Rightarrow a = (6\times 2 - 2)(3(2)^2 - 2\times 2) = 80\ ms^{-2}$ key-B

EX.59. **A police party is chasing a dacoit in a jeep which is moving at a constant speed v. The dacoit is on a motorcycle. When he is at a distance x from the jeep, he accelerates from rest at a constant rate α ? Which of the following relations is true if the police is able to catch the dacoit ?**

A) $v^2 \le \alpha x$ B) $v^2 \le 2\alpha x$ C) $v^2 \ge 2\alpha x$ D) $v^2 \ge \alpha x$

Sol. If police is able to catch the dacoit after time t, then

$vt = x + \frac{1}{2}\alpha t^2$. This gives $\frac{\alpha}{2}t^2 - vt + x = 0$

or $t = \frac{v \pm \sqrt{v^2 - 2ax}}{\alpha}$ For t be real , $v^2 \ge 2ax$ key-C

EX.60. A point moves in a straight line so that its displacement x metre time t second is given by $x^2=1+t^2$. Its acceleration in ms^{-2} at time t second is

A) $\frac{1}{x^{3/2}}$ B) $\frac{-t}{x^3}$ C) $\frac{1}{x}-\frac{t^2}{x^3}$ D) $\frac{1}{x}-\frac{1}{x^2}$

Sol. $x^2=1+t^2$ or $x=\left(1+t^2\right)^{1/2}$

$$\frac{dx}{dt}=\frac{1}{2}\left(1+t^2\right)^{-1/2}2t=t\left(1+t^2\right)^{-1/2}$$

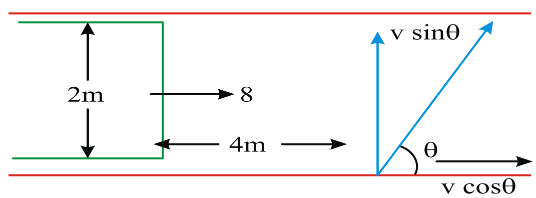
$$a=\frac{d^2x}{dt^2}=\left(1+t^2\right)^{-1/2}+t\left(-\frac{1}{2}\right)\left(1+t^2\right)^{-3/2}2t$$

$$=\frac{1}{x}-\frac{t^2}{x^3}$$ key-C

EX.61. A 2m wide truck is moving with a uniform speed $v_0=8\,ms^{-1}$ along a straight horizontal road. A pedestrain strarts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v so that he can cross the road safely is

A) $2.62\,ms^{-1}$ B) $4.6\,ms^{-1}$ C) $3.57\,ms^{-1}$ D) $1.414\,ms^{-1}$

Sol.



Time of crosing $=\frac{2}{\mathrm{v}\sin\theta}$

Time in which truck just able to catch the man $=\frac{4}{8-\mathrm{v}\cos\theta}$

For safe crosing $\frac{2}{\mathrm{v}\sin\theta}=\frac{4}{8-\mathrm{v}\cos\theta}$

or $16-2\mathrm{v}\cos\theta=4\mathrm{v}\sin\theta$

or $\mathrm{v}=\frac{16}{\cos\theta+2\sin\theta}$

For v minimum $\cos\theta+2\sin\theta$ is maximum

so, $\frac{d}{d\theta}\left(\cos\theta+2\sin\theta\right)=0$

$\Rightarrow -\sin\theta+2\cos\theta=0$

$\Rightarrow \tan\theta=2$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{5}},\sin\theta=\frac{2}{\sqrt{5}}$

Now $\mathrm{v}_{min}=\frac{16\sqrt{5}}{5}=\frac{16}{\sqrt{5}}$ key-C

$=3.57\,\mathrm{m/s}$

EX.62. The velocity of a particle along a straight line increases according to the linear law $v = v_0 + kx$, where k is a constant. Then

A) the acceleration of the particle is $k(v_0 + kx)$

B) the particle takes a time $\frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$ to attain a velocity v_1

C) velocity varies linearly with displacement with slope of velocity displacement curve equal to k.

D) data is insufficient to arrive at a conclusion.

Sol. Acceleration $= \frac{dv}{dt} = \dot{v} = 0 + kx$

$$\left\{\because \dot{x} = \frac{dx}{dt} = v\right\} \Rightarrow \dot{v} = a = kv = k(v_0 + kx)$$

Further, $a = \frac{dv}{dt} = kv \quad \Rightarrow \frac{dv}{dt} = kv$

$$\Rightarrow \frac{dv}{v} = kdt$$

$$\Rightarrow \int_{v_0}^{v_1} \frac{dv}{v} = k\int_0^t dt \quad \Rightarrow t = \frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$$

Since, $v = v_0 + kx$. Hence slope of velocity displacement curve is $\frac{dv}{dx} = k$ key-ABC

EX.63. Two particles P and Q move in a straight line AB towards each other. P starts from A with velocity u_1 and an acceleration a_1. Q starts from B with velocity u_2 and acceleration a_2. They pass each other at the midpoint of AB and arrive at the other ends of AB with equal velocities.

A) They meet at midpoint at time $t = \frac{2(u_2 - u_1)}{(a_1 - a_2)}$

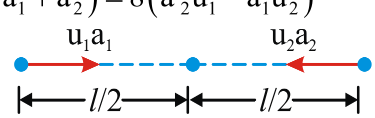
B) The length of path specified i.e., AB is $l = \frac{4(u_2 - u_1)(a_1u_2 - a_2u_1)}{(a_1 - a_2)^2}$

C) They reach the other ends of AB with equal velocities if $(u_2 + u_1)(a_1 - a_2) = 8(a_1u_2 - a_2u_1)$

D) They reach the other ends of AB with equal velocities if

$(u_2 - u_1)(a_1 + a_2) = 8(a_2u_1 - a_1u_2)$

Sol. .



$\frac{l}{2} = u_1t + \frac{1}{2}a_1t^2$ ...(1) and $-\frac{l}{2} = -u_1t + \frac{1}{2}(-a_2)t^2$

$$\Rightarrow \frac{l}{2} = u_2t + \frac{1}{2}a_2t^2 \text{ ....(2)}$$

subtracting (1) and (2) , we get $t = 2\left(\frac{u_2 - u_1}{a_1 - a_2}\right)$ ....(3)

Substituting (3) in (1) or(2) and rearranging, we get. $l = \frac{4(u_2 - u_1)}{(a_1 - a_2)^2}(a_1u_2 - a_2u_1)$ ...(4)

Since the particle P & Q reach the other ends of A and B with equal velocities say v

For particle P $\quad v^2 - u_1^2 = 2a_2 l \ldots (5)$

For particle Q $\quad v^2 - u_2^2 = 2a_2 l \ldots (6)$

Substracting and then substituting value of l and rearranging, we get

$(u_2 + u_1)(a_1 - a_2) = 8(a_1 u_2 - a_2 u_1)$

key-ABC

EX.64. A particle moves along a straight line so that its velocity depends on time as $v = 4t - t^2$. Then for first 5s.

A) Average velocity is $25/3\ ms^{-1}$

B) Average speed is $10\ ms^{-1}$

C) Average velocity is $5/3\ ms^{-1}$

D) Acceleration is $4\ ms^{-2}\ at\ t = 0$

Sol. Average velocity $\vec{v} = \dfrac{\int_0^5 v\,dt}{\int_0^5 dt} = \dfrac{\int_0^5 (4t - t^2)\,dt}{\int_0^5 dt} = \dfrac{\left[2t^2 - \frac{t^3}{3}\right]_0^5}{5} = \dfrac{50 - \frac{125}{3}}{5} = \dfrac{25}{3\times 5} = \dfrac{5}{3}$

For average speed, let us put v= 0, which gives $t = 0$ amd $t = 4s$

$\therefore$ average speed =

$$\frac{\left|\int_0^4 v\,dt\right| + \left|\int_4^5 v\,dt\right|}{\int_0^5 dt} = \frac{\left|\int_0^4 (4t - t^2)\,dt\right| + \left|\int_4^5 v\,dt\right|}{5} \qquad = \frac{\left[2t^2 - \frac{t^3}{3}\right]_0^4 + \left[2t^2 - \frac{t^3}{3}\right]_4^5}{5}$$

$$= \frac{\left|\left[2t^2 - \frac{t^3}{3}\right]_0^4\right| + \left|\left[2t^2 - \frac{t^3}{3}\right]_4^5\right|}{5} = \frac{13}{5}\ ms^{-1}$$

For acceleration :

$a = \dfrac{dv}{dt} = \dfrac{d}{dt}(4t - t^2) = 4 - 2t \qquad$ At $t = 0,\ a = 4\ ms^{-2}$ key-C,D

EX.65. A particle moves with an initial velocity v_0 and retardation αv, where v is velocity at any time t.

A) The particle will cover a total distance $\dfrac{v_0}{\alpha}$

B) The particle will come to rest after time $\dfrac{1}{\alpha}$

C) The particle will continue to move for a long time.

D) The velocity of particle will become $\dfrac{v_0}{e}$ after time $\dfrac{1}{\alpha}$

Sol. $v.\frac{dv}{dx} = -\alpha v \Rightarrow or \int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$

$v_0 = \alpha x_0 \Rightarrow x_0 = \frac{v_0}{\alpha};$

$\frac{dv}{dt} = -\alpha v (or) \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$

$v = v_0 e^{-\alpha t} (or) v = 0 \; for \; t = \infty$

$\Rightarrow v = \frac{v_0}{e} \; when \; t = \frac{1}{\alpha}$ .key-A,C,D

EX.66. **A particle is moving along X–axis whose position is given by $x = 4 - 9t + \frac{t^3}{3}$. Mark the correct statement(s) in relation to its motion.**

A) direction of motion is not changing at any of the instants
B) direction of motion is changing at t = 3 s
C) for 0 < t < 3 s, the particle is slowing down
D) for 0 < t < 3 s, the particle is speeding up.

Sol. The particle's velocity is getting zero at t = 3 s, where it changes its direction of motion.

For 0 < t < 3 s, V is negative, a is positive, so particle is slowing down.

For t < 3, both V and a are positive, so the particle is speeding up.

key-B,C

PASSAGE TYPE QUESTIONS

Passage-1

A train starts from rest with constant acceleration, $a = 1 m/s^2$. A passenger at a distance 'S' from the train runs at his maximum velocity of 10 m/s to catch the train at the same moment at which the train starts.

67. If S=25.5 m and passenger keeps running, find the time in which he will catch the train:

A) 5 sec B) 4 sec C) 3 sec D) $2\sqrt{2}$ sec.

68. Find the critical distance 'S_c' for whcih passenger will take the ten seconds time to catch the train:

A) 50m B) 35m C) 30m D) 25m

69. Find the speed of the train when the passenger catches it for the critical distacne:

A) 8 m/s B) 10 m/s C) 12 m/s D) 15m/s

Sol. 67. At time t, X_t and X_p are coordinates of train and passenger respectively.

$X_t = \frac{1}{2} a_1 t^2$ and $X_p = v_p t - S$

If passenger catches the train,

$X_t = X_p$

or $\frac{1}{2} a_1 t^2 = v_P t - S$ or $t = \frac{v_P - \sqrt{v_P^2 - 2a_1 S}}{a_1}$

$= \frac{10 - \sqrt{(10)^2 - 2(1)(25.5)}}{1} = 3 \text{seconds}$

68. The critical distacne 'S_c' for which passenger will take the ten seconds time to catch the train is given by $S_c = \frac{v_P^2}{2a_1}$

The time is 10 seconds, if $v_P^2 - 2a_1S = 0$

$$S_c = \frac{v_P^2}{2a_t} = \frac{(10)^2}{2(1)} = 50m$$

69. For critical distance, passenger catches the train in time, $t = \frac{v_P}{a_t}$ So, required velocity of train = $a_t.t$

$$= a_t\left(\frac{v_P}{a_t}\right) = V_P/2 = 10m/sec$$

key-67-.C 68- A 69- B

Passage-2

A body is moving with uniform velocity of $8\,ms^{-1}$. When the body just crossed another body, the second one starts and moves with uniform acceleration of $4\,ms^{-2}$.

70. The time after which two bodies meet will be

A) 2 s B) 4 s C) 6 s D) 8 s

71. The distance covered by the second body when they meet is

A) 8 m B) 16 m C) 24 m D) 32 m

Sol. 70. Let they meet after time t, then the distance travelled by both in time t should be same

$$s = 8t = \frac{1}{2}4t^2 \Rightarrow t = 4\,s$$

71. $s = 8t = 8 \times 4 = 32\,m$

key- 70-. B 71-. D

EX.72. An elevator in which a man is standing is moving upwards with a speed of $10\,ms^{-1}$. If the man drops a coin from a height of 2.45 m from the floor of elavator, it reaches the floor of the elavator after time ($g = 9.8\,ms^{-2}$)

A) $\sqrt{2}\ s$ B) $1/\sqrt{2}\ s$ C) 2 s D) $1/2\,s$

Sol. Let the initial relative velocity, rerlative acceleration and relative displacement of the with respect to the floor of the lift be u_r, a_r and s_r, then $s_r = u_r t + (1/2)a_r t^2$

and $u_r = u_c - u_l = 10 - 10 = 0$

$$a_r = a_c - a_l = (-9.8) - 0 = -9.8\,ms^{-2}$$

$$s_r = s_c - s_l = -2.45m$$

$$-2.45 = 0(t) + (1/2)(-9.8)t^2$$

or $t^2 = 1/2$ $or\ t = 1\sqrt{2}s$

key-B

EX.73. A body is thrown vertically upwards from A, the top of a tower. It reaches the ground in time t_1. If it is thrown vertically downward from A with the same speed, it reaches the ground in time t_2. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by

A) $t = \frac{t_1 + t_2}{2}$ B) $t = \frac{t_1 - t_2}{2}$ C) $t = \sqrt{t_1 t_2}$ D) $t = \sqrt{\frac{t_1}{t_2}}$

Sol. Suppose the body be projected vertically upwards from A with a speed u_0.

Using equation $s = ut + \left(\frac{1}{2}\right)at^2$, we get

For first case: $-h = u_0 t_1 - \left(\frac{1}{2}\right)gt_1^2$ (i)

For second case: $-h = -u_0 t_2 - \left(\frac{1}{2}\right)gt_2^2$ (ii)

(i)- (ii) $\Rightarrow 0 = u_0\left(t_2 + t_1\right) + \left(\frac{1}{2}\right)g\left(t_2^2 - t_1^2\right)$

$\Rightarrow u_0 = \left(\frac{1}{2}\right)g\left(t_1 - t_2\right)$ (iii)

Putting the value of u_0 in (ii), we get

$-h = -\left(\frac{1}{2}\right)g\left(t_1 - t_2\right)t_2 - \left(\frac{1}{2}\right)gt_2^2$

$\Rightarrow h = \frac{1}{2}gt_1 t_2$ (iv)

For third case: $u = 0, t = ?$

$-h = 0 \times t - \left(\frac{1}{2}\right)gt^2$ or $h = \left(\frac{1}{2}\right)gt^2$ (v)

Combining Eq. (iv) and (v), we get

$\frac{1}{2}gt^2 = \frac{1}{2}gt_1 t_2$ or $t = \sqrt{t_1 t_2}$

key-C

EX.74. The decelaration experienced by a moving motor boat, after its engine is cut-off is given by $\frac{dv}{dt} = -kv^3$, where k is constant.If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is

A) $v_0 / 2$ B) v C) $v_0 e^{-kt}$ D) $\frac{v_0}{\sqrt{2v_0^2 kt + 1}}$

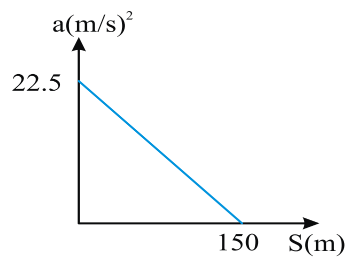
Sol. Here $\frac{dv}{dt}=-kv^3$

or $\frac{dv}{v^3}=-kdt \; or \int_{v_0}^{v}\frac{dv}{v^3}=\int_0^t -kdt$

or $\left[-\frac{1}{2v^2}\right]_{V_0}^{v}=-kt \; or -\frac{1}{2v^2}+\frac{2}{2v_0^2}=-kt$, or $v^2=\frac{v_0^2}{1+2v_0^2kt} \; or \; v=\frac{v_0}{\sqrt{2v_0^2kt+1}}$

EX.75. A jet plane starts from rest at S = 0 and is subjected to the acceleration shown. Determine the speed of the plane when it has travelled 60 m.

A) 46.47 m/s B) 36.47 m/s C) 26.47 m/s D) 16.47 m/s



Sol. $a=\frac{dv}{ds}v \Rightarrow \int_0^s a.ds=\int_u^v v.dv=v^2-u^2$

$\Rightarrow$ Area under a : s curve $=v^2-u^2$

$\Rightarrow \left(\frac{1}{2}\times 150\times 22.5\right)-\frac{1}{2}(90\times 13.5)=v^2-0$

$\Rightarrow v^2=75\times 22.5-45\times 13.5$

$\Rightarrow v=\sqrt{75\times 22.5-45\times 13.5} \Rightarrow 46.47$

EX.76. The relation between time t and distance x is $t=ax^2+bx$. Where a and b are constants. The retardation is

A) $2av^3$ B) $2bv^3$ C) $2abv^3$ D) $2b^3v^3$

Sol. $t=\alpha x^2+\beta n \quad 1=2\alpha xv+\beta v\left[\frac{1}{v}=2\alpha x+\beta\right]$

$0=2\alpha[x.a+v.v]+\beta a \quad 0=(2\alpha x+\beta)a+2\alpha v^2$

$\Rightarrow a=\frac{2\alpha v^2}{2\alpha x+\beta}=\frac{2\alpha v^2}{\frac{1}{v}}=2\alpha v^3$

EX.77. **The motion of a body falling from rest in a resisting medium is described by the equation $\frac{dv}{dt}=a-bv$ where a and b are constants. The velocity at any time t is given by**

A) $v=\frac{a}{b}\left(1-e^{-bt}\right)$ B) $v=\frac{b}{a}\left(e^{-bt}\right)$

C) $v=\frac{a}{b}\left(1+e^{-bt}\right)$ D) $v=\frac{b}{a}e^{bt}$

Sol. $\frac{dv}{dt}=a-bv\ \int_0^v \frac{dv}{a-bv}=\int_0^t dt$

$\left(\frac{-2}{b}\right)\left[\ln(a-bv)\right]_0^v=t \Rightarrow \ln\frac{a-bv}{a}=-bt$

$\Rightarrow a-bv=ae^{-bt} \Rightarrow v=\frac{a}{b}\left(1-e^{-bt}\right)$

EX.78 **A train stops at two stations s distance apart and takes time t on the journey from one station to the other. Its motion is first of uniform acceleration a and then immediately of uniform retardation b, then**

A) $\frac{1}{a}-\frac{1}{b}=\frac{t^2}{s}$ B) $\frac{1}{a}+\frac{1}{b}=\frac{t^2}{s}$ C) $\frac{1}{a}+\frac{1}{b}=\frac{t^2}{2s}$ D) $\frac{1}{a}-\frac{1}{b}=\frac{t^2}{2s}$

Sol. $s=\frac{v^2}{2a}+\frac{v^2}{2b}=\frac{v^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

Again $t=\frac{\upsilon}{a}+\frac{\upsilon}{b}\Rightarrow v=t\left(\frac{ab}{a+b}\right)$

$\Rightarrow s=\frac{t^2}{2\left(\frac{1}{a}+\frac{1}{b}\right)^2}\times\left(\frac{1}{a}+\frac{1}{b}\right) \Rightarrow \frac{t^2}{2s}=\frac{1}{a}+\frac{1}{b}$

EX.79. **A ball is thrown from the top of a tower in vertically upward direction. Velocity at apoint h metre below the point of projection is twice of the velocity at a point h metre above the point of projection. find the maximum height reached by the ball above the top of tower.**

A) $2h$ B) $3h$ C) $(5/3)h$ D) $(4/3)h$

Sol. $H=\frac{u^2}{2g}$; given $v_2=2v_1$

(i) A to B: $v_1^2=u^2-2gh$

(ii) A to C $v_2^2=u^2-2g(-h)$

(iii) solving (i), (ii),(iii) we get the value of u^2 as 10g/h/3 and then we get the value of H by using $H=\frac{u^2}{2g}$

key-C

EX.80. A parachutist drops first freely from an aeroplane for 10 s and then his parachute opens out. Now he descends with a net retardation of $2.5\ ms^{-2}$. If he bails out of the plane at a height of 2495 m and $g=10\ ms^{-2}$, his velocity on reaching the ground will be

A) $5\ ms^{-1}$ B) $10\ ms^{-1}$ C) $15\ ms^{-1}$ D) $20\ ms^{-1}$

Sol. The velocity v acquired by the parachutist after 10 s.

$v=u+gt=0+10\times10=100ms^{-1}$

Then, $s_1=ut+\frac{1}{2}gt^2=0+\frac{1}{2}\times10\times10^2=500m$

The distance travelled by the parachutist under retadation is

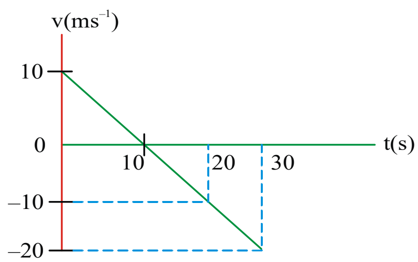
$s_2=2495-500=1995\ m$

Let v_g be the velocity on reaching the ground. Then $v_g^2-v^2=2as_2$

or $v_g^2-(100)^2=2\times(-2.5)\times1995\ \ or\ v_g=5\ ms^{-1}$

key-A

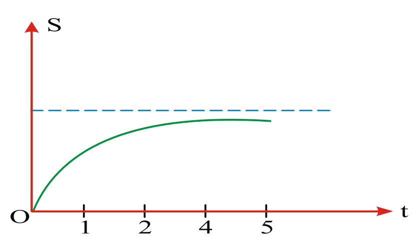
EX.81. The velocity-time plot a particle moving on a straight line is shown in figure.



A) The particle has a constant acceletration
B) The particle has never tuned around
C) The particle has zero displacement
D) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s

Sol. Since the graph is astraight line, its slope is constant, it means acceleration of the particle is constant.
Velocity of the particle changes from positive to negative at $t=10\ s$, so particle changes direction at this time.
The particle has zero displacement up to 20 s, but not for the entire motion.
The average speed in the interval of 0 to 10 s is the same as the average speed in the internal of 10 s to 20 s because distance covered in both time interval is same. key-AD

EX.82. The displacement of a particle as a function of time is shown in figure. It indicates



A) The particle starts with a certain velocity, but the motion is retarded and finally the particle stops

B) The velocity of the particle decreases

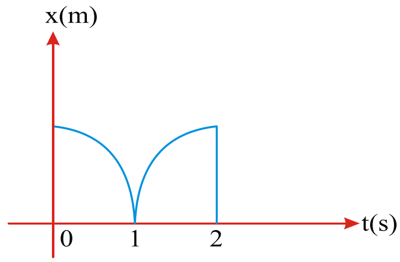
C) The acceleration of the particle is in opposite direction to the velocity

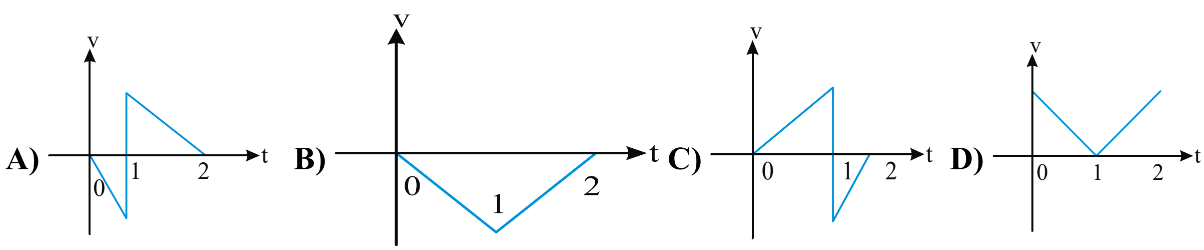
D) The particle starts with a constant velocity, the motion is accelerated and finally the particle moves with another constant velocity.

Sol. Initially at orogin, slope is not zero, so the particle has some initial velocity but with time we see that slope is decreasing and finally the slope necomes zero, so the particle stops finally.

key-ABC

EX.83. The displacement-time graph of a moving particle with constant acceleration is shown in the figure. The velocity time graph is given by





Sol. At $t=0$, slope of the x-t graph is zero; hence, velocity is zero at $t=0$. As time increases, slope increases in negative direction; hence, velocity increases in negative deirection. At point'I', slope changes suddenly from negative to positive value: hence, velocity changes suddenly from negative to positive and then velocity starts decreasing and becomes zero at'2', option (A) represents all these clearly.

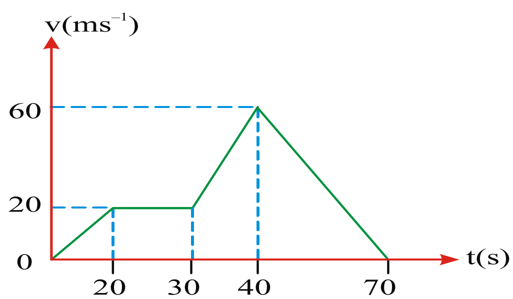
EX.84 The velocity-time graph of abody is given in figure. The maximum accceleration in ms^{-2} is

A) 4

B) 3

C) 2

D) 1



Sol. Maximum acceleration will befrom 30 to 40 s, because slope in this interval maximum

$$a=\frac{v_2-v_1}{t_2-t_1}=\frac{60-20}{40-30}=4\ ms^{-2}$$

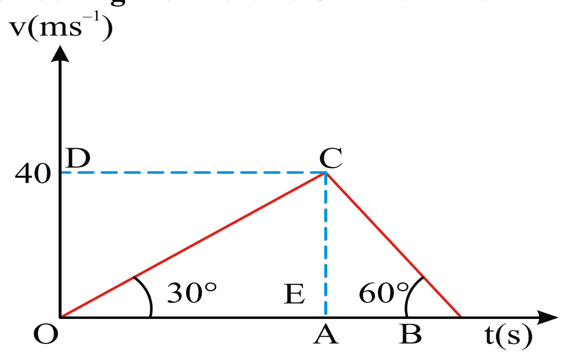
EX.85. The velocity-time graph of a body is shown in figure. The ratio of magnitude of average acceleration during the intervals OA and AB is

A) 1

B) 1/2

C) 1/3

D) 3

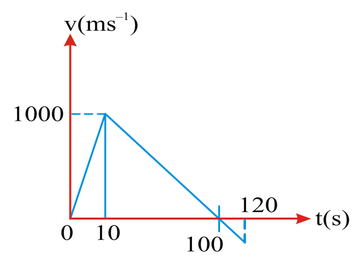


Sol. During OA, acceleration $= \tan 30^0 = \frac{1}{\sqrt{3}} ms^{-2}$

During AB, acceleration $= -\tan 60^0 = -\sqrt{3}\ ms^{-2}$.

required ratio $= \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

EX.86. The following graph shows the variation of velocity of a rocket with time. Then the maximum height attained by the rocket is

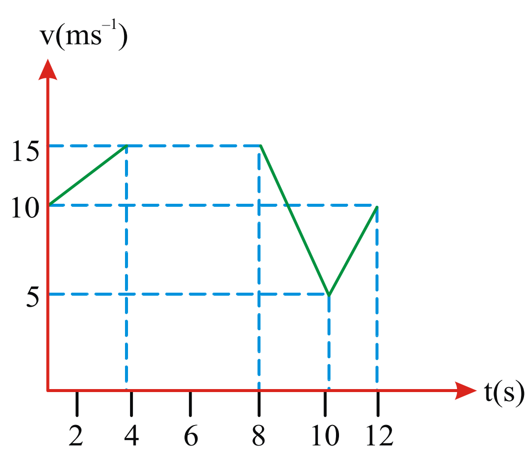


A) 1.1 km B) 5 km C) 55 km D) None

Sol. Maximum height will be attained at 110 s. Because after 110 s, velocity becomes negative and rocket will start coming down. Area from 0 to 110 s is $\frac{1}{2} \times 110 \times 1000 = 55,000\ m = 55\ km$

key-C

EX.87. The velocity-time graph of a particle moving in a straight line is shown in figure. The acceleration of the particle at $t = 9$ is

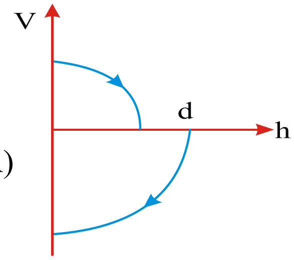


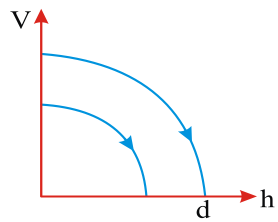
A) Zero B) $5\ ms^{-2}$ C) $-5\ ms^{-2}$ D) $-2\ ms^{-2}$

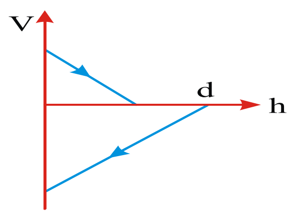
Sol. Acceleration between 8 to 10 s(or at t= 9 s) . $a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5-15}{10-8} = -5m/s^2$

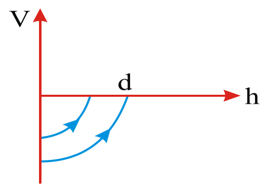
key-C

EX.88. A ball is dropped vertically from a height *d* above the ground. It hits the ground and bounces up vertically to a height $\frac{d}{2}$. Neglecting subsequent motion and air resistance, its velocity *v* varies with height *h* above the ground as: [2004]

A) 

B) 

C) 

D) 

Sol. (i) For uniformly accelerated/decelerated motion

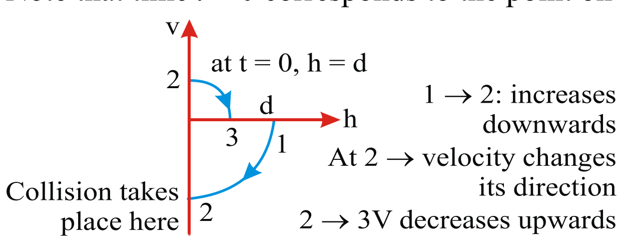
$v^2 = u^2 \pm 2gh$

i.e., v – h graph will be a parabola Because equation is quadratiC).

(ii) Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude. *I.e.*, velocity is upwards (+ve). Graph A) satisfies both these conditions.

Therefore, correct answer is A).

Note that time $t = 0$ corresponds to the point on the graph where $h = d$



JEE MAIN PREVIOUS YEAR QUESTIONS

MOTION IN A STRAIGHT LINE

TOPIC-1 ….Distance, Displacement & Uniform Motion

1. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$(assume that the particle is at origin at $t = 0$).
[12 Apr. 2019 II]

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$ (c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

sol. (b) $Given, v = b\sqrt{x}$

$$or\ dx/dt = bx^{(1/2)}$$

$$or \int_0^x x^{-1/2}dx = \int_0^t bdt$$

$$or\ x^{(1/2)}/(1/2) = 6t$$

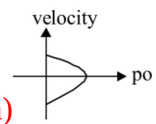
$$or\ x = \frac{b^2t^2}{4}$$

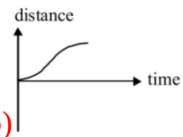
Differentiating w. r.t. time, we get

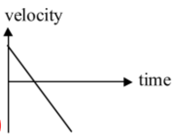
$$\frac{dx}{dt} = \frac{b^2 \times 2t}{4}\ (t = \tau)$$

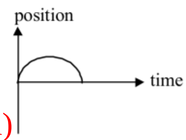
or $v = \frac{b^2\tau}{2}$

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.
[2018]

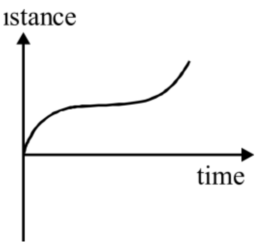
(a) 

(b) 

(c) 

(d) 

sol. (b) Graphs in option (c) position-time and option (a) velocity-position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.



3. A car covers the first half of the distance between two places at 40 km/h and other half at 60 km/h. The average speed of the car is [Online May 7, 2012]

(a) 40 km/h (b) 45 km/h (c) 48km/h (d) 60km/h

sol. (c) Average speed $= \frac{\text{Tota1distancetrave11ed}}{\text{Tota1timetaken}} = \frac{x}{T}$

$$= \frac{x}{\frac{x}{2\times 40}+\frac{x}{2\times 60}} = 48\text{km/h}$$

4. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is [2007]

(a) $v_0 + gl2 + f$ (b) $v_0 + 2g + 3f$ (c) $v_0 + gl2 + fl3$ (d) $v_0 + g + f$

sol. (c) We know that, $v = \frac{dx}{dt}$

$\Rightarrow dx = vdi$

Integrating, $\int_0^x dx = \int_0^t v\, dt$

or $x = \int_0^t (v_0 + gt + ft^2)\, dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$

or, $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$, $x = v_0 + \frac{g}{2} + \frac{f}{3}$

5. A particle located at $x = 0$ at time $t = 0$, starts moving along with the positive x-direction with a velocity'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as [2006]

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

sol. (a) $v = \alpha\sqrt{x}$,

$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt$

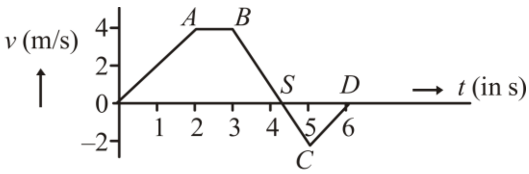
Integrating both sides,

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t d\,t; \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$$

$\Rightarrow 2\sqrt{x} = \alpha t \Rightarrow x = \frac{\alpha^2}{4} t^2$

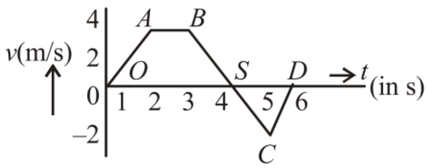
TOPIC-2 ….Non-Uniform Motion

6. The velocity(v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is:
[05 Sep. 2020 (II)]



(a) $\frac{37}{3}$ m (b) 12 m (c) 11 m (d) $\frac{49}{4}$ m

sol. (a)



$$OS = 4 + \frac{1}{3} = \frac{13}{3}$$

$$SD = 2 - \frac{1}{3} = \frac{5}{3}$$

Distance covered by the body = area of v-t graph = ar$(OABS)$ + ar (SCD)

$$= \frac{1}{2}\left(\frac{13}{3} + 1\right) \times 4 + \frac{1}{2} \times \frac{5}{3} \times 2 = \frac{32}{3} + \frac{5}{3} = \frac{37}{3} \text{ m}$$

8. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as xn, where n is an integer, the value of n is .
[NA 9 Jan 2020 I]

sol. (3) Distance X varies with time t as $x^2 = at^2 + 2bt + c$

$$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b$$

$$\Rightarrow x\frac{dx}{dt} = at + b \Rightarrow \frac{dx}{dt} = \frac{(at+b)}{x}$$

$$\Rightarrow x\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2=a$$

$$\Rightarrow \frac{d^2x}{dt^2}=\frac{a-\left(\frac{dx}{dt}\right)^2}{x}=\frac{a-\left(\frac{at+b}{x}\right)^2}{x}$$

$$=\frac{ax^2-(at+b)^2}{x^3}=\frac{ac-b^2}{x^3}$$

$\Rightarrow$ $a \propto x^3$ Hence, $n=3$

9. A bullet of mass 20g has an initial speed of 1 m/s, just before it starts penetrating a mud wall ofthickness 20 cm. Ifthe wall offers a mean resistance of 2.5×10^{-2}N, the speed ofthe bullet after emerging fi$_i$om the other side of the wall is close to: [10Apr. 2019 II]

(a) 0.1 m/s (b) 0.7 m/s (c) 0.3 m/s (d) 0.4 m/s

sol. (b) From the third equation of motion

$$v^2-u^2=2aS$$

But, $a=F/m$

$$v^2=u^2-2\left(\frac{F}{m}\right)S$$

$$\Rightarrow v^2=(1)^2-(2)\left[\frac{2.5\times10^{-2}}{20\times10^{-3}}\right]\frac{20}{100}$$

$$\Rightarrow v^2=1-\frac{1}{2}$$

$$\Rightarrow v=\frac{1}{\sqrt{2}}\text{m/s}=0.7\text{m/s}$$

10. The position of a particle as a function of time t, is given by $x(t)=at+bt^2-ct^3$ where, a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

(a) $a+\frac{b^2}{4c}$ (b) $a+\frac{b^2}{3c}$ (c) $a+\frac{b^2}{c}$ (d) $a+\frac{b^2}{2c}$

sol. (b) $x=at+bt^2-ct^3$

Velocity, $v=\frac{dx}{dt}=\frac{d}{dt}(at+bt^2+ct^3)$

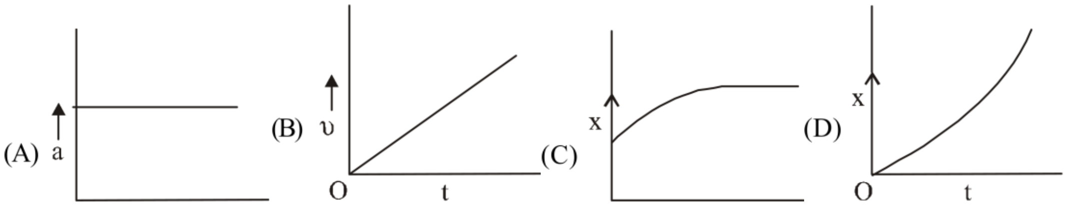
$$=a+2bt-3ct^2$$

Acceleration, $\frac{dv}{dt}=\frac{d}{dt}(a+2bt-3ct^2)$

or $0=2b-3c\times2t$ $\therefore$ $t=\left(\frac{b}{3c}\right)$

and $v=a+2b\left(\frac{b}{3c}\right)-3c\left(\frac{b}{3c}\right)^2$

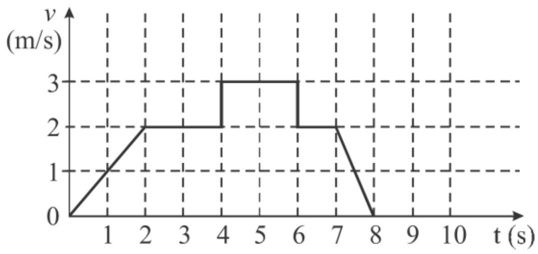
11. A particle starts fi$_{j}$om origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represents the motion qualitatively (a =acceleration, v =velocity,x =displacement, t =time)
[8 Apr. 2019 II]



(a) (B), (C) (b) (A) (c) (A), (B), (C) (d) (A), (B), (D)

sol. (d) For constant acceleration, there is straight line parallel to t-axis on $a-t$.
Inclined straight line on $v-t$, and parabola on $x-t$.

12. A particle starts from the origin at time $t=0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t=5s$?
[10 Jan. 2019 II]



(a) 10 m (b) 6 m (c) 3m (d) 9m

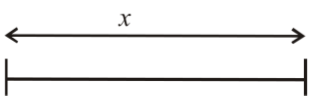
sol. (d) Position of the particle,
S = area under graph (time $t=0$ to 5s)

$$=\frac{1}{2}\times 2\times 2+2\times 2+3\times 1=9\text{m}$$

13. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed v more than of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then v is equal to:
[9 Jan. 2019 II]

(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}t$ (c) $\sqrt{a_1a_2}t$ (d) $\frac{a_1+a_2}{2}t$

sol. (c) Let time taken by A to reach finishing point is t_0 Time taken by B to reach finishing point $=t_0+t$



$$u = 0$$

$$v_A = a_1 t_0$$

$$v_B = a_2(t_0 + t)$$

$$v_A - v_B = v$$

$$\Rightarrow v = a_1 t_0 - a_2(t_0 + t) = (a_1 - a_2)t_0 - a_2 t \ldots \text{(i)}$$

$$x_B = x_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

$$\Rightarrow \sqrt{a_1}t0 = \sqrt{a_2}(t_0 + t)$$

$$\Rightarrow (\sqrt{a_1} - \sqrt{a_2})t_0 = \sqrt{a_2}t$$

$$\Rightarrow t_o = \frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}}$$

Putting this value of t_0 in equation (i)

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= (\sqrt{a_1} + \sqrt{a_2})\sqrt{a_2}t - a_2 t = \sqrt{a_1 a_2}t + a_2 t - a_2 t$$

or, $v = \sqrt{a_1 a_2}t$

14. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 kmph, the minimum stopping distance, in metres, is (assume no skidding)

[Online Apri115, 2018]

(a) 75m (b) 160m (c) 100m (d) 150m

sol. (b) According to question, $u_1 = 40$ kmni, $v_1 = 0$ and $s_1 = 40\text{m}$

using $v^2 - u^2 = 2as; 0^2 - 40^2 = 2a \times 40$ (i)

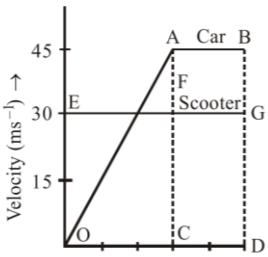
Again, $0^2 - 80^2 = 2as$ (ii)

From eqn. (i) and(ii)

Stopping distance, $s = 160\text{m}$

15. The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively

[Online Apri115, 2018]



0 5 10 15 20 25

Time in (s) →

(a) 337.5m and 25 s (b) 225.5m and 10 s

(b) 112.5m and 22.5 s (d) 112.5m and 15 s

sol. (c) Using equation, $a=\frac{v-u}{t}$ and $S=ut+\frac{1}{2}at^2$

Distance travelled by car in 15 sec $=\frac{1}{2}\frac{(45)}{15}$ (15)2

$=\frac{675}{2}$m

Distance travelled by scooter in 15 seconds $=30\times 15=450$ (distance =speed ×time)

Difference between distance travelled by car and scooter in 15 sec, $450-337.5=112.5$m

Let car catches scooter in time t;

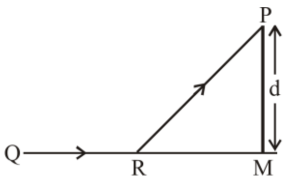
$$\frac{675}{2}+45(t-15)=30t$$

$$337.5+45t-675=30t \Rightarrow 15t=337.5$$

$$\Rightarrow t=22.5 \text{ sec}$$

16. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?

[Online Apri115, 2018]



(a) $\frac{d}{\sqrt{3}}$ (b) $\frac{d}{2}$ (c) $\frac{d}{\sqrt{2}}$ (d) d

sol. (a) Let the car turn of the highway at a distance x from the point M. So, RM $=x$

And if speed of car in field is v, then time taken bythe car to cover the distance $QR=QM-x$ on the highway,

$t_1=\frac{QM-x}{2v}$ (i)

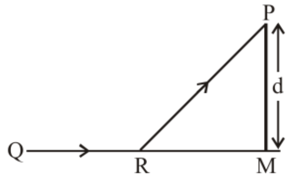
Time taken to travel the distance RP' in the field

$t_2=\frac{\sqrt{d^2+x^2}}{v}$ ..... (ii)

Total time elapsed to move the car from Q to $Pt=t_1+t_2=\frac{QM-x}{2v}+\frac{\sqrt{d^2+x^2}}{v}$

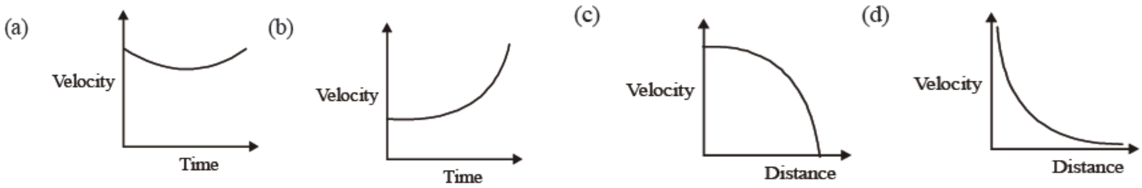
For't' to be minimum $\frac{dt}{dx} = 0$

$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$$



or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

17. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity?
[Online April 8, 2017]



sol. (c) According to question, object is moving with constant negative acceleration
i.e., a = − constant (C)

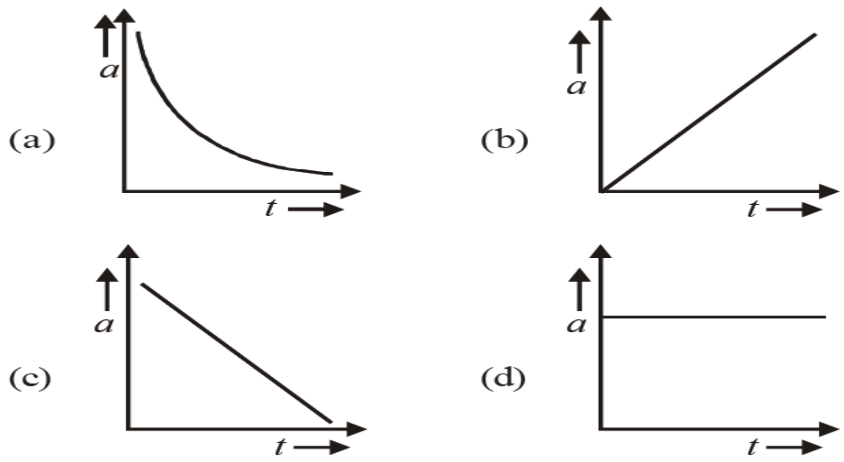
$$\frac{vdv}{dx} = -C$$

vdv =-Cdx

$$\frac{v^2}{2} = -Cx + k \quad x = -\frac{v^2}{2C} + \frac{k}{C}$$

Hence, graph (3) represents correctly.

18. The distance travelled by a body moving along a line in time t is proportional to t^3. The acceleration-time (a, t) graph for the motion of the body will be
[Online May 12, 2012]



sol. (b) Distance along a line i.e., displacement $(s) = t^3$ $(s \propto t^3$ given)

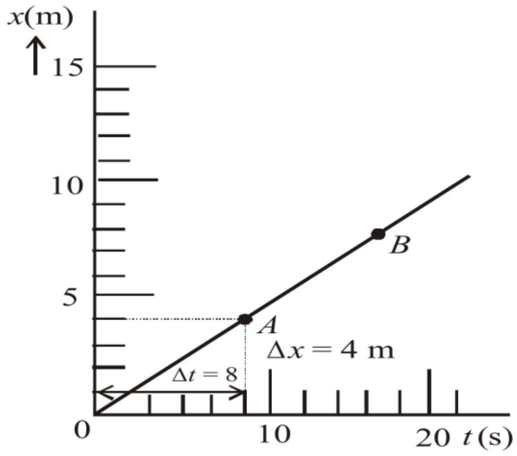
By double differentiation of displacement, we get acceleration.

$V = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2$ and $a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$

$a = 6t$ or $a \propto t$

Hence graph (b) is correct.

19. The graph of an object's motion (along the x-axis) is shown in the figure. The instantaneous velocity of the object at points A and B are v_A and v_B respectively. Then [Online May 7, 2012]



(a) $v_A = v_B = 0.5\text{m/s}$

(b) $v_A = 0.5\text{m/s} < v_B$

(c) $v_A = 0.5\text{m/s} > v_B$

(d) $v_A = v_B = 2\text{m/s}$

sol. (a) Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$

From graph, $v_A = \frac{\Delta x_A}{\Delta t_A} = \frac{4m}{8s} = 0.5\text{m/s}$

and $v_B = \frac{\Delta x_B}{\Delta t_B} = \frac{8m}{16s} = 0.5\text{m/s}$

i.e., $v_A = v_B = 0.5\text{m/s}$

20. An object, moving with a speed of 6.25m/s, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be: [2011]

(a) 2 s (b) 4 s (c) 8 s (d) 1 s

sol. (a) Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \frac{dv}{\sqrt{v}} = -2.5dt$$

Integrating, $\int_{6.25}^{0} v^{-1/2} dv = -2.5\int_0^t dt$

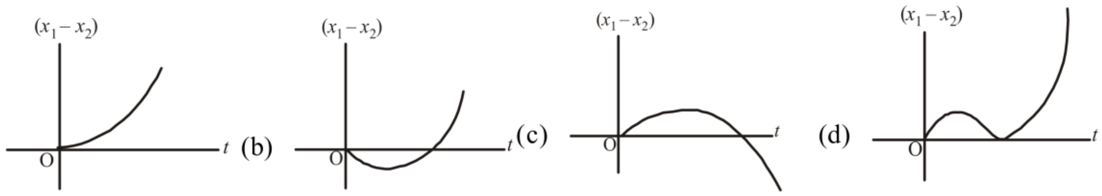
$$\Rightarrow \left[\frac{v^{+1/2}}{(/2)}\right]_{625}^{0} = -2.5[t]_0^t$$

$$\Rightarrow -2(6.25)^1/_2 = -2.5t$$

$$\Rightarrow -2 \times 2.5 = -2.5t$$

$$\Rightarrow t = 25$$

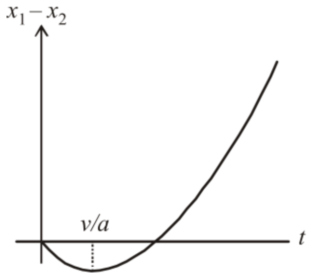
21. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't'; and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'? [2008]

(a)


sol. (b) For the body starting from rest, distance travelled (x_1) is given by

$$x_1 = 0 + \frac{1}{2}at^2$$

$$\Rightarrow x_1 = \frac{1}{2}at^2$$



For the body moving with constant speed

$$x_2 = vt$$

$$x_1 - x_2 = \frac{1}{2}at^2 - vt$$

at $t = 0,\ x_1 - x_2 = 0$

This equation is of parabola.

For $< \frac{v}{a}$; the slope is negative

For $= \frac{v}{a}$; the slope is zero

For $> \frac{v}{a}$; the slope is positive

These characteristics are represented by graph (b).

22. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S , then

[2005]

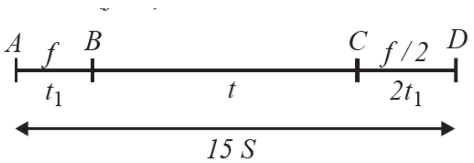
(a) $S = \frac{1}{6}ft^2$ (b) $S = ft$ (c) $S = \frac{1}{4}ft^2$ (d) $S = \frac{1}{72}ft^2$

22. (d) Let car starts from A from rest and moves up to point B with acceleration f.

Distance, $AB = S = \frac{1}{2}f{t_1}^2$

Distance, $BC = (ft_1)t$

Distance, $CD = \frac{u^2}{2a} = \frac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



Total distance, $AD = AB + BC + CD = 15S$

$$AD = S + BC + 2S$$

$$\Rightarrow S + ft_1t + 2S = 15S$$

$\Rightarrow ft_1t = 12S$ (i)

$\frac{1}{2}ft_1^2 = S$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \frac{t}{6}$

$$\Rightarrow S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$$

23. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is
[2005]

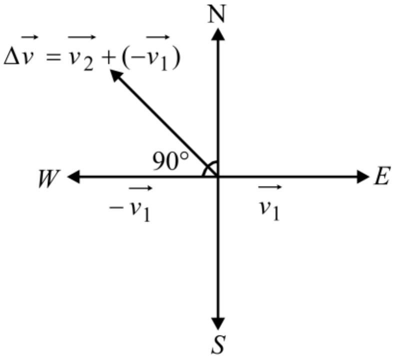
(a) $\frac{1}{2}ms^{-2}$ towards north

(b) $\frac{1}{\sqrt{2}}ms^{-2}$ towards north- east

(c) $\frac{1}{\sqrt{2}}ms^{-2}$ towards north- west

(d) zero

sol. (c) v_2



Initial velocity, $\overline{v_1} = 5\hat{\imath}$,
Final velocity, $\overline{v_2} = 5\hat{\jmath}$,
Change in velocity $\Delta\overline{v} = (\overline{v}_2 - \overline{v}_1)$

$$= \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos 90}$$

$$= \sqrt{5^2 + 5^2 + 0} = 5\sqrt{2}\text{m/s}$$

[As $|v_1| = |v_2| = 5\text{m/s}$]

Avg. acceleration $= \frac{\Delta\overline{v}}{t}$

$$= \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\text{m/s}^2$$

$$\tan\theta = \frac{5}{-5} = -1$$

which means θ is in the second quadrant. (towards north-west)

24. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is
[2005]
(a) $2bv^3$ (b) $-2abv^2$ (c) $2av^2$ (d) $-2av^3$

sol. (d) Given, $t = ax^2 + bx$;
Diff. with respect to time(t)

$$\frac{d}{dt}(t) = a\frac{d}{dt}(x^2) + b\frac{dx}{dt} = a.2x\frac{dx}{dt} + \text{b.v.}$$

$\Rightarrow 1 = 2axv + bv = v(2ax + b)(v = \text{velocity})$

$$2ax + b = \frac{1}{v}.$$

Again differentiating, we get

$$2a\frac{dx}{dt} + 0 = -\frac{1dv}{v^2dt}$$

$\Rightarrow a = \frac{dv}{dt} = -2av^3 \left(\because \frac{dx}{dt} = v\right)$

25. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20m. If the car is going twice as fast i.e., 120 km/h, the stopping distance will be
[2004]
(a) 60m (b) 40m (c) 20m (d) 80m

sol. (d) In first case speed,

$$u = 60 \times \frac{5}{18}\text{m/s} = \frac{50}{3}\text{m/s}$$

$$d = 20\text{m},$$

Let retardation be a then

$0 - u^2 = -2ad$
or $u^2 = 2ad$ ... (i)

In second case speed, $u' = 120 \times \frac{5}{18}$

$$= \frac{100}{3}\text{m/s}$$

and $0 - u'^2 = -2ad'$
or $u'^2 = 2ad'$ ... (ii)
(ii) divided by (i) gives,

$$4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80\text{m}$$

26. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same

car is moving at a speed of 100 $\mathrm{km/hr}$, the minimum stopping distance is
[2003]

(a) 12m (b) 18m (c) 24m (d) 6m

sol. (c) Fir first case : Initial velocity,

$$u = 50 \times \frac{5}{18}\mathrm{m/s},$$

$$v = 0, s = 6\mathrm{m}, a = a$$

Using, $v^2 - u^2 = 2as$

$$\Rightarrow 0^2 - \left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$$

$$\Rightarrow -\left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$$

$$a = -\frac{250 \times 250}{324 \times 2 \times 6} \approx = -16\mathrm{ms}^{-2}.$$

Case-2 : Initial velocity, $u = 100\mathrm{km/hr}$

$$= 100 \times \frac{5}{18}\mathrm{m/\sec}$$

$$v = 0,\ s = s,\ a = a$$

As $v^2 - u^2 = 2as$

$$\Rightarrow 0^2 - \left(100 \times \frac{5}{18}\right)^2 = 2as$$

$$\Rightarrow -\left(100 \times \frac{5}{18}\right)^2 = 2 \times (-16) \times 5$$

$$s = \frac{500 \times 500}{324 \times 32} = 24\mathrm{m}$$

27. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?
[2002]

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4cm.

sol. (a) In first case

$u_1 = u; v_1 = \frac{u}{2},\ s_1 = 3\ \mathrm{cm},\ a_1 = ?$

Using, $v_1^2 - u_1^2 = 2a_1s_1$ (i)

$$\left(\frac{u}{2}\right)^2 - u^{2=2\times a\times 3}$$

$$-u^2$$

$$\Rightarrow a = \overline{8}$$

In second case: Assuming the same retardation

$$u_2 = ul2; v_2 = 0; =? a_2 = \frac{-u^2}{8}$$

$v_2^2 - u_2^2 = 2a_2 \times s_2$ (ii)

$$0 - \frac{u^2}{4} = 2\left(\frac{-u^2}{8}\right) \times s_2$$

$\Rightarrow s_2 = 1$ cm

28. Speeds of two identical cars are u and $4u$ at the specific instant. The ratio ofthe respective distances in which the two cars are stopped from that instant is
[2002]
(a) 1 : 1 (b) 1 :4 (c) 1 :8 (d) 1 : 16

sol. (d) For first car

$$u_1 = u,\ v_1 = 0,\ a_1 = -a,\ s_1 = s_1$$

As $v_1^2 - u_1^2 = 2a_1 s_1$

$$\Rightarrow -u^2 = -2as_1$$
$$\Rightarrow u^2 = 2as_1$$

$\Rightarrow s_1 = \mathrm{u/2a}$ (i)

For second car

$$u_2 = 4u,\ v_1 = 0,\ a_2 = -a,\ s_2 = s_2$$
$$v_2^2 - u_2^2 = 2a_2 s_2$$
$$\Rightarrow -(4u)^2 = 2(-a)s_2$$
$$\Rightarrow 16u^2 = 2as_2$$

$\Rightarrow s_2 = \frac{8u^2}{a}$ (ii)

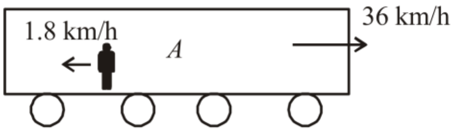
Dividing(i) and(ii),

$$\frac{s_1}{s_2} = \frac{u^2}{2a} \cdot \frac{a}{8u^2} = \frac{1}{16}$$

TOPIC-3 …..Relative Velocity

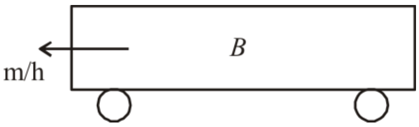
29. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) ofthis person as observed from train B will be close to : (take the distance between the tracks as negligible)
[2 Sep. 2020 (I)]
(a) $29.5\ \mathrm{ms}^{-1}$ (b) $28.5\ \mathrm{ms}^{-1}$ (c) $31.5\ \mathrm{ms}^{-1q}$ (d) $30.5\ \mathrm{ms}^{-1}$

sol. (a) According to question, train A and B are running on parallel tracks in the opposite direction.



$$V_A = 36\text{km/h} = 10\text{m/s}$$

72k



$V_B = -72\ \text{kimh}\ = -20\text{m/s}$

$V_{MA} = -1.8\text{knyh}\ = -0.5\text{m/s}$

$$V_{\text{man},B} = V_{\text{man},A} + V_{A,B}$$

$$= V\ + V - V = -0.5 + 10 - (-20)$$

$$\text{man}, A\ A\ B$$

$$= -0.5 + 30 = 29.5\text{m/s}.$$

30. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train oflength 120 m travels at a speed of 30 km/h. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in same direction, and (ii) in the opposite directions is:
[12 Jan. 2019 II]

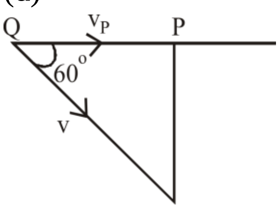
(a) $\frac{11}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{25}{11}$

sol. (a)

31. A person standing on an open ground hears the sound of a jet aero plane, coming from north at an angle 60° with ground level. But he finds the aero plane right vertically above his position. If v is the speed of sound, speed of the plane is:
[12 Jan. 2019 II]

(a) $\frac{\sqrt{3}}{2}v$ (b) $\frac{2v}{\sqrt{3}}$ (c) v (d) $\frac{v}{2}$

sol. (d)



R (Observer)

Distance, PQ $= v_p \times t$ (Distance =speed ×time) Distance, $QR = V.t$

$$\cos 60^{\circ} = \frac{PQ}{QR}$$

$$\frac{1}{2}=\frac{v_p \times t}{V.t} \Rightarrow v_p=\frac{v}{2}$$

32. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration $2\ m/s^2$ and the car has acceleration $4\ m/s^2$. The car will catch up with the bus after a time of:
[Online April 9, 2017]

(a) $\sqrt{110}s$ (b) $\sqrt{120}s$ (c) $10\sqrt{2}s$ (d) 15 s

sol. (c) $\rightarrow 4m/sec^2$ $\rightarrow 2m/sec^2$

200 m

Given, $u_C=u_B=0,\ a_C=4m/s^2,\ a_B=2m/s^2$

hence relative acceleration, $a_{CB}=2m/sec^2$

Now, we know, $s=ut+\frac{1}{2}at^2$

$$200=\frac{1}{2}\times 2t^2 \quad u=0$$

Hence, the car will catch up with the bus after time $t=10\sqrt{2}$ second

33. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator?
[Online Apri112, 2014]

(a) 37 s (b) 27 s (c) 24 s (d) 45 s

sol. (c) Person's speed walking only is $\frac{1}{60}$

Standing the escalator without walking the speed is $\frac{1}{40}$

Walking with the escalator going, the speed add.

So, the person's speed is $\frac{1}{60}+\frac{1}{40}=\frac{15}{120}$

So, the time to go up the escalator $t=\frac{120}{5}=24$ second.

34. A goods train accelerating uniformly on a straight railway track, approaches an electric pole standing on the side of track. Its engine passes the pole with velocity u and the guard's room passes with velocity v. The middle wagon of the train passes the pole with a velocity.
[Online May 19, 2012]

(a) $\frac{u+v}{2}$ (b) $\frac{1}{2}\sqrt{u^2+v^2}$ (c) $\sqrt{uv}$ (d) $\sqrt{\left(\frac{u^2+v^2}{2}\right)}$

Sol. (d) Let S be the distance between two ends a be the constant acceleration

As we know $v^2 - u^2 = 2aS$

Or $aS = \frac{v^2 - u^2}{2}$

Let v be velocity at mid point.

Therefore, $v_c^2 - u^2 = 2a\frac{S}{2}$

$$v_c^2 = u^2 + aS$$

$$v_c^2 = u^2 + \frac{v^2 - u^2}{2}$$

$$v_c = \sqrt{\frac{u^2 + v^2}{2}}$$

TOPIC-4 ….Motion Under Gravity

35. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the accelertion due to gravity] :
[5 Sep. 2020 (I)]

(a) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$ (c) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (d) $t = \sqrt{\frac{2h}{3g}}$

sol. (c) For upward motion of helicopter,

$$v^2 = u^2 + 2gh \Rightarrow v^2 = 0 + 2gh \Rightarrow v = \sqrt{2gh}$$

Now, packet will start moving under gravity.

Let 't' be the time taken by the food packet to reach the ground.

$$s = ut + \frac{1}{2}at^2$$

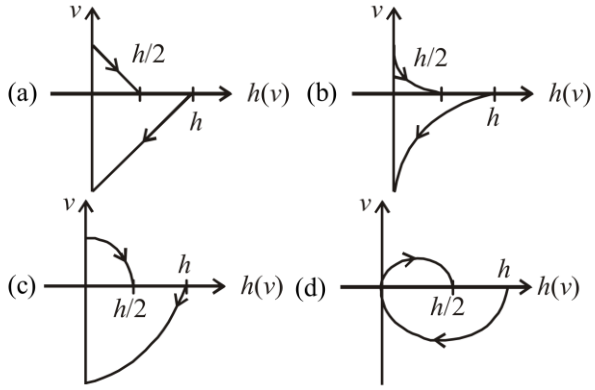
$$\Rightarrow -h = \sqrt{2gh}t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - \sqrt{2gh}t - h = 0$$

or, $t = \frac{\sqrt{2gh} \pm \sqrt{2gh + 4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}}$

or, $t = \sqrt{\frac{2gh}{g}}(1+\sqrt{2}) \Rightarrow t = \sqrt{\frac{2h}{g}}(1+\sqrt{2})$

or, $t = 3.4\sqrt{\frac{h}{g}}$

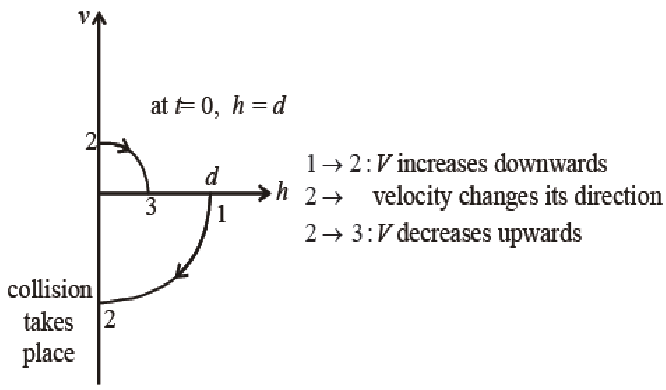
36. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by: (graph are drawn schematically and on not to scale)
[4 Sep. 2020 (I)]



sol. (c) For uniformly accelerated/ deaccelerated motion :

$$v^2 = u^2 \pm 2gh$$

As equation is quadratic, so, v-h graph will be a parabola



Initially velocity is downwards (-ve) and then after collision it reverses its direction with lesser magnitude, i. e. velocity is upwards (+ve) .
Note that time $t = 0$ corresponds to the point on the graph where $h = d$.
Next time collision takes place at 3.

37. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in m/s 2) near the surface on that planet is .
[NA 8 Jan. 2020 II]

sol. (08.00) Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$$\Rightarrow S = 0 \times t + \frac{1}{2}gt^2$$

$\Rightarrow\ 200 = gt^2\ \ [2S = 100m] \Rightarrow t = \sqrt{\frac{200}{g}}$ ...(i)

In last $\frac{1}{2}s$, body travels a distance of19 m, so in $\left(t - \frac{1}{2}\right)$ distance travelled $= 81$

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

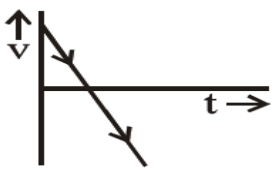
$$g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$$

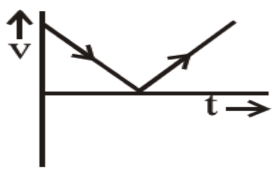
$$\Rightarrow \left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$$

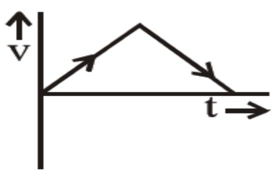
$\frac{1}{2} = \frac{1}{\sqrt{g}}\left(\sqrt{200} - \sqrt{81 \times 2}\right)$ using (i)

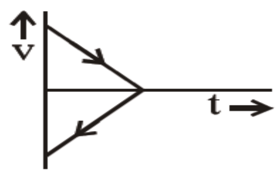
$$\Rightarrow \sqrt{g} = 2\left(10\sqrt{2} - 9\sqrt{2}\right)$$
$$\Rightarrow \sqrt{g} = 2\sqrt{2}$$
$$g = 8mls^2$$

38. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?
[2017]

(a)


(b)


(c)


(d)


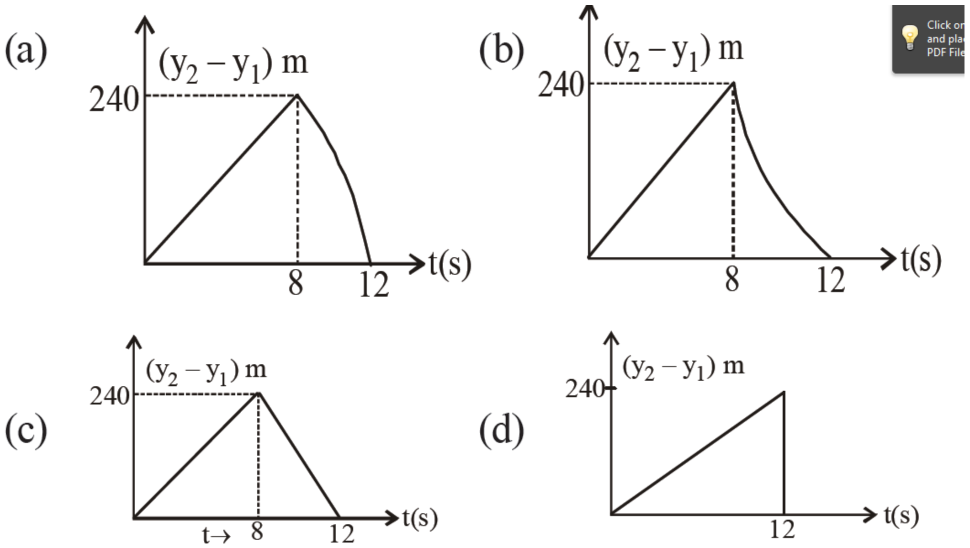
sol. (a) For a body thrown vertically upwards acceleration remains constant $(a = -g)$ and velocity at anytime t is given by $V = u - gt$
During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.
Hence graph (a) correctly depicts velocity versus time.

39. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 m/s^2$)

[2015]

(The figures are schematic and not drawn to scale)



sol. (b) $y_1 = 10t - 5t^2; y_2 = 40t - 5t^2$ for $y_1 = -240m$, $t = 8s$

$y_2 - y_1 = 30t$ for $t \leq 8s$.

for $t > 8s$,

$$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$$

40. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:

[2014]

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2d$ (c) $2gH = nu^2$ (n-2) (d) $gH = (n-2)u^2$

sol. (c) Speed on reaching ground

$$v = \sqrt{u^2 + 2gh}$$

Now, $v = u + at$

$$\Rightarrow \sqrt{u^2 + 2gh} = -u + gt$$

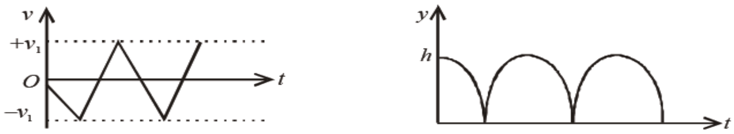
Time taken to reach highest point is $t = \frac{u}{g}$,

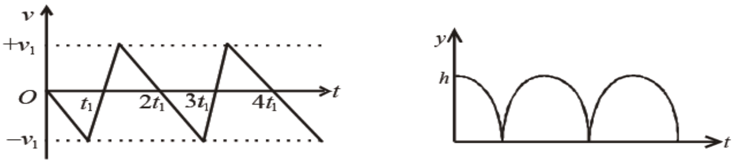
$$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$$

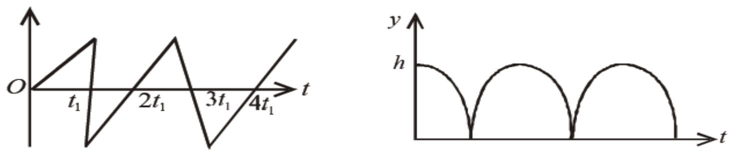
(from question)

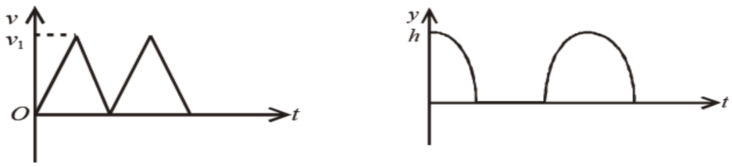
$$\Rightarrow \quad 2gH = n(n-2)u^2$$

41. Consider a rubber ball freely falling from a height $h = 4.9\text{m}$ onto a horizontal elastic plate. Assume that the duration ofcollision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be :
[2009]

(a) 

(b) 

(c) 

(d) 

sol. (b) For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2}at^2$

$$y - h = -\frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$$

The graph between y and t is a parabola with $y = h$ at $t = 0$.

As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocityjust after collision.

As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

42. A parachutist after bailing out falls 50m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?
[2005]

(a) 182 m (b) 91 m (c) 111 m (d) 293m

sol. (d) Initial velocity of parachute after bailing out,

$$u = \sqrt{2gh}$$

$$u = \sqrt{2 \times 98 \times 50} = 14\sqrt{5}$$

The velocity atground v=3 m/s

$$S = \frac{v^2 - u^2}{2 \times 2} = \frac{3^2 - 980}{4} \approx 243\text{m}$$

Initially he has fallen 50 m.

Total height from where he bailed out $= 243 + 50 = 293\text{m}$

43. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball at $\frac{T}{3}$ second

[2004]

(a) $\frac{8h}{9}$ meters from the ground (b) $\frac{7h}{9}$ meters from the ground

(c) $\frac{h}{9}$ meters from the ground (d) $\frac{17h}{18}$ meters from the ground

Sol. (a) We have $s = ut + \frac{1}{2}gt^2$

$$\Rightarrow h = 0 \times T + \frac{1}{2}gT^2$$

$$\Rightarrow h = \frac{1}{2}gT^2$$

Vertical distance moved in time $\frac{T}{3}$ is

$$h^{\uparrow} = \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h^{\uparrow} = \frac{1}{2} \times \frac{gT^2}{9} = \frac{h}{9}$$

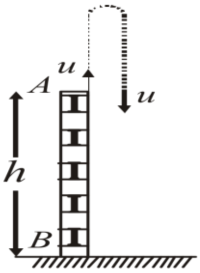
Position of ball from ground $= h - \frac{h}{9} = \frac{8h}{9}$

44. From abuilding two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then [2002]

(a) $v_B > v_A$ (b) $v_A = v_B$ (c) $v_A > v_B$

(d) their velocities depend on their masses.

sol. (b)



Ball A is thrown upwards with velocity u from the building. During its downward journey when it comes back to the point of throw, its speed is equal to the speed of throw (u) . So, for the journey of both the balls from point A to B.

We can apply $v^2 - u^2 = 2gh.$

As $u,\ g,\ h$ are same for both the balls, $v_A = v_B$

Chapter 4 -- MOTION I N A PLANE

Relative Velocity

↳ If body A is moving with a velocity $\vec{V}_A$ w.r.t. ground and body B is moving with velocity $\vec{V}_B$ w.r.t. ground then

1) The relative velocity of body 'A' w.r.t. 'B' is given by $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

2) The relative velocity of body 'B' w.r.t. 'A' is given by $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

3) Both $\vec{V}_A - \vec{V}_B$ and $\vec{V}_B - \vec{V}_A$ are equal in magnitude but opposite in direction.

$\vec{V}_{AB} = -\vec{V}_{BA}$ and $|\vec{V}_{AB}| = |\vec{V}_{BA}| = \sqrt{V_A{}^2 + V_B{}^2 - 2V_A V_B \cos\theta}$

4) For two bodies moving in same direction, magnitude of relative velocity is equal to the difference of magnitudes of their velocities. $(\theta = 0^0, \cos\ 0 = 1)$ $\therefore |\vec{V}_{AB}| = V_A - V_B, |\vec{V}_{BA}| = V_B - V_A$

5) For two bodies moving in opposite directions, magnitude of relative velocity is equal to the sum of the magnitudes of their velocities. $(\theta = 180^0;\ \cos\ 180^0 = -1)$

$\therefore |\vec{V}_{AB}| = |\vec{V}_{BA}| = V_B + V_A$

6) Relative displacement of A w.r.t. B is

$\vec{X}_{AB} = \vec{X}_{AG} - \vec{X}_{BG}$ Where $\vec{X}_{AG}$ = displacement of 'A' w.r.t ground

and $\vec{X}_{BG}$ = displacement of 'B' w.r.t ground

7) Relative velocity of A w.r.t. B is $\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$

8) Relative acceleration of A w.r.t. B is $\vec{a}_{AB} = \vec{a}_{AG} - \vec{a}_{BG}$

9) Two trains of lengths l_1 and l_2 are moving on parallel tracks with speeds v_1 and v_2 $(v_1 > v_2)$ w.r.t ground. The time taken to cross each other

when they move in same direction is $t_1 = \frac{S_{rel}}{V_{rel}} = \frac{l_1 + l_2}{v_1 - v_2}$

when they move in opposite direction is $t_2 = \frac{S_{rel}}{V_{rel}} = \frac{l_1 + l_2}{v_1 + v_2}$

Application:

Relative Motion on a moving train

If a boy in a train is running with velocity $\vec{V}_{BT}$ relative to train and train is moving with velocity $\vec{V}_{TG}$ relative to ground, then the velocity of the boy relative to ground $\vec{V}_{BG}$ will be given by

$\vec{V}_{BG} = \vec{V}_{BT} + \vec{V}_{TG}$

So, if boy in a train is running along the direction of train. $\vec{V}_{BG} = \vec{V}_{BT} + \vec{V}_{TG}$

If the boy in train is running in a direction opposite to the motion of train, then
$\vec{V}_{BG} = \vec{V}_{BT} - \vec{V}_{TG}$

EX.1: When two objects move uniformly towards each other, they get 4 metres closer each second and when they move uniformly in the same direction with original speed, they get 4 metres closer each 5s. Find their individual speeds.

Sol. Let their speeds be v_1 and v_2 and let $v_1 > v_2$.

In First case :

Relative velocity, $v_1 + v_2 = \frac{4}{1} = 4\,m/s$ ....(1)

In Second case:

Relative velocity $= v_1 - v_2 = \frac{4}{5} = 0.8\,m/s$...(2)

solving eqns.(1) and (2), weget $v_1 = 2.4ms^{-1}, v_2 = 1.6ms^{-1}$

EX.2: A person walks up a stationary escalator in time t_1. If he remains stationary on the escalator, then it can take him up in time t_2. How much time would it take for him to walk up the moving escalator?

Sol. Let L be the length of escalator .

Speed of man w.r.t. escalator is $v_{ME} = \frac{L}{t_1}$

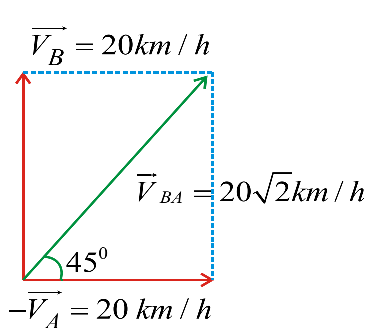
Speed of escalator $v_E = \frac{L}{t_2}$

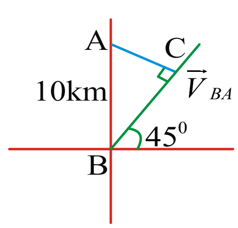
Speed of man with respect to ground would be $v_M = v_{ME} + v_E = L\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$

$\therefore$ The desired time is $t = \frac{L}{v_M} = \frac{t_1 t_2}{t_1 + t_2}$.

EX.3: Two ships A and B are 10km apart on a line running south to north. Ship A farther north is streaming west at 20km/h and ship B is streaming north at 20km/h. What is their distance of closest approach and how long do they take to reach it?

Sol.





Velocity of B relative to A is $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$|\vec{V}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2} km/h$

i.e., $\vec{V}_{BA}$ is $20\sqrt{2}$ km/h at an angle of 45^0 from east towards north.

A is at rest and B is moving with $\vec{V}_{BA}$ in the direction shown in Fig.

Therefore the minimum distance between ships

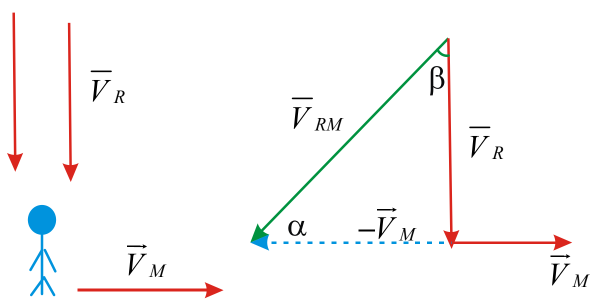
$$S_{\min} = AC = AB \sin 45^0 = 10\left(\frac{1}{\sqrt{2}}\right) km = 5\sqrt{2} km$$

and time taken is $t = \frac{BC}{|\vec{V}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} = \frac{1}{4} h = 15 \min$

Rain umbrella Concept

↳ If rain is falling with a velocity $\vec{V}_R$ and man moves with a velocity $\vec{V}_M$ relative to ground, he will observe the rain falling with a velocity $\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$.

Case - I : If rain is falling vertically with a velocity $\vec{V}_R$ and an observer is moving horizontally with velocity $\vec{V}_M$, then the velocity of rain relative to observer will be :



$\vec{V}_{RM} = \overline{V}_R - \overline{V}_M$

The magnitude of velocity of rain relative to man is $V_{RM} = \sqrt{V_R^2 + V_M^2}$

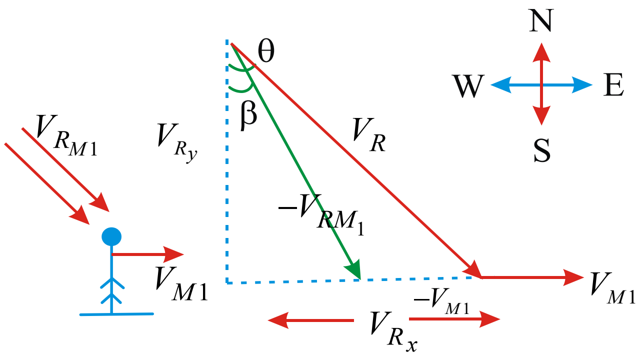
If α is the angle made by the umbrella with horizontal, then, $\tan\alpha = \left|\frac{V_R}{V_M}\right|$

If β is the angle made by the umbrella with vertical, then, $\tan \beta = \left|\frac{V_M}{V_R}\right|$

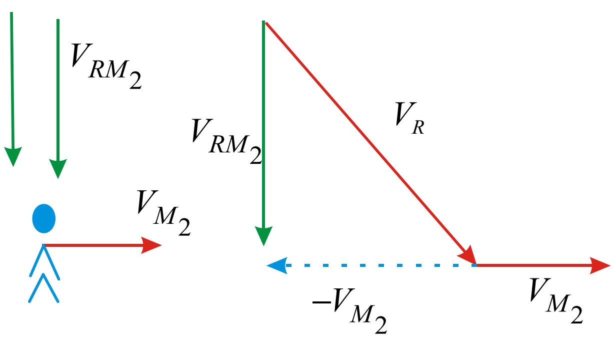
Case - II : When the man is moving with a velocity V_{M_1} relative to ground towards east(positive x-axis), and the rain is falling with a velocity $\vec{V}_R$ relative to ground by making an angle θ with vertical(negative z-axis). Then the velocity of rain relative to man $\vec{V}_{RM_1}$ is as shown in figure.

$\vec{V}_R = V_{R_x}\hat{i} - V_{R_y}\hat{k}$; $\quad \vec{V}_{M1} = V_{M1}\hat{i}$

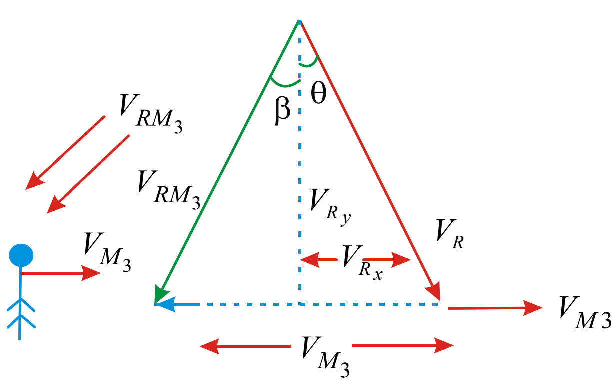
and $\tan\beta = \dfrac{V_{R_x} - V_{M_1}}{V_{R_y}}$ ...... (2)



Case - III : If the man speeds up, at a particular velocity $\vec{V}_{M_2}$, the rain will appear to fall vertically with $\vec{V}_{RM_2}$, then $\vec{V}_{RM_2} = \vec{V}_R - \vec{V}_{M_2}$ as shown in figure.



Case - IV : If the man increases his speed further, he will see the rain falling with a velocity as shown in figure.



$\vec{V}_{RM_3} = \vec{V}_R - \vec{V}_{M_3}$; $\tan\beta = \dfrac{V_{M_3} - V_{R_x}}{V_{R_y}}$

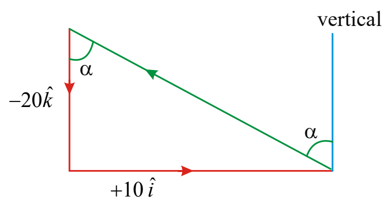
EX.4: **Rain is falling vertically with a speed of $20ms^{-1}$., A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both from the west) The angle with the vertical at which the person should hold his umbrella so that he may not get drenched is :**

Sol. $\vec{V}_{Rain} = \vec{V}_R = 20(-\hat{k})$

$\vec{V}_{Man} = \vec{V}_M = 5\hat{i}$, $\vec{V}_{Wind} = \vec{V}_W = 15\hat{i}$

Resultant velocity of rain and wind is $\vec{V}_{RW} = -20\hat{k} + 15\,\hat{i}$

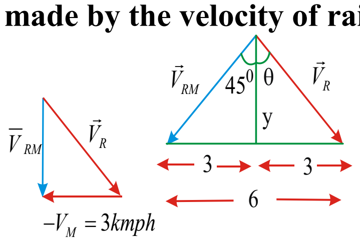
Now, velocity of rain **relative** to man is $\vec{V}_{RW} - \vec{V}_M = (-20\hat{k} + 15\hat{i}) - (5\hat{i}) = -20\hat{k} + 10\hat{i}$



$Tan\alpha = \frac{1}{2} \Rightarrow \alpha = Tan^{-1}\frac{1}{2}$

EX.5: **To a man walking at the rate of 3km/h the rain appears to fall vertically.When he increases his speed to 6km/h it appears to meet him at an angle of 45^0 with vertical. Find the angle made by the velocity of rain with the vertical and its speed.**

Sol :

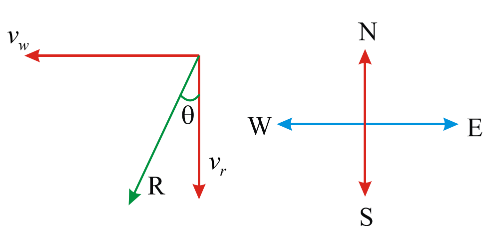


From the diagram $Tan45^0 = \frac{3}{y}$.........(1) and $Tan\theta = \frac{3}{y}$ ....(2)

From (1) and (2) $\theta = 45^0 \Rightarrow \therefore \sin 45^0 = \frac{3}{V_R}, \frac{1}{\sqrt{2}} = \frac{3}{V_R}$

$V_R = 3\sqrt{2}kmph$

EX.6: **Rain is falling, vertically with a speed of 1m/s .Wind starts blowing after sometime with a speed of 1.732 m/s in east to west direction.In which direction should a boy waiting at a bus stop hold his umbrella.?**



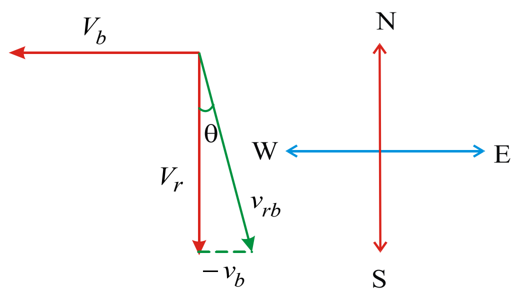
Sol. If R is the resultant of velocity of rain (V_R) and velocity of wind (V_W) then

$$R = \sqrt{v_r^2 + v_w^2} = \sqrt{1^2 + (1.732)^2}\,ms^{-1} = 2ms^{-1}$$

The direction θ that R makes with the vertical is given by $\tan\theta = \frac{v_w}{v_r} = \frac{\sqrt{3}}{1} = 60^0$ Therefore, the boy should hold his umbrella in the vertical plane at an angle of about 60^0 with the vertical towards the east.

EX.7: Rain is falling vertically with a speed of 1m/s . A woman rides a bicycle with a speed of 1.732 m/s in east to west direction. What is the direction in which she should hold her umbrella ?

Sol. In Fig. v_r represents the velocity of rain and v_b, the velocity of the bicycle, the woman is riding. Both these velocities are with respect to the ground.Since the woman is riding a bicycle, the velocity of rain as experienced by



here is the velocity of rain relative to the velocity of the bicycle she is riding. That is $v_{rb} = v_r - v_b$

This relative velocity vector as shown in Fig. makes an angle θ with the vertical.It is given by

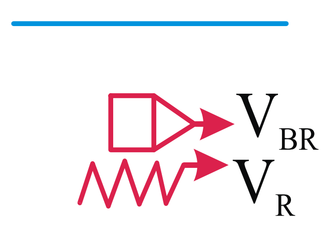
$$\text{Tan}\,\theta = \frac{v_b}{v_r} = \frac{\sqrt{3}}{1} \Rightarrow \theta = 60^o$$

Therefore,the woman should hold her umbrella at an angle of about 60^0 with the vertical towards the west.

Motion of a Boat in the River

Boat motion is classified into three categories based on angle between V_{BR} and V_R they are

1) Down stream $(\theta = 0^0)$:



Resultant velocity of the boat = $V_{BR} + V_R$

The time taken for the boat to move a distance `d' along the direction of flow of water is.

$$t_1 = \frac{d}{V_{BR} + V_R} \text{ ....(1)}$$

2) Up stream $(\theta = 180^0)$:

Resultant velocity of the boat = $V_{BR} - V_R$

The time taken for the boat to move a distance `d' opposite to the direction of flow of water is.

$$t_2 = \frac{d}{V_{BR} - V_R} \text{ ....(2)}$$

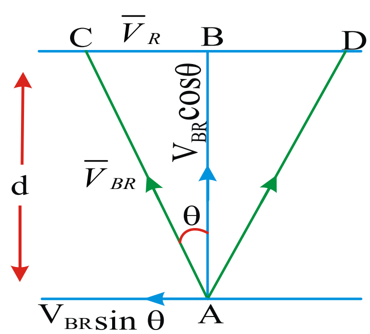
From equation (1) and (2) $\frac{t_1}{t_2} = \frac{V_{BR} - V_R}{V_{BR} + V_R}$

time taken by person to go down stream a distance `d' and come back is

$$T = t_1 + t_2 = \frac{d}{V_{BR} + V_R} + \frac{d}{V_{BR} - V_R}$$

3) General approach :

Suppose the boat starts at point A on one bank with velocity V_{BR} and reaches the other bank at point D



The component of velocity of boat anti parallel to the flow of water is $V_{BR} \sin \theta$

The component of velocity of boat perpendicular to the flow of water is $V_{BR} \cos \theta$

↳ The time taken by the boat to cross the river is, $t = \frac{d}{V_{BR} \cos \theta}$

↳ Along the flow of water, distance travelled by the boat (or) drift is $x = (V_R - V_{BR} \, Sin\theta)t$

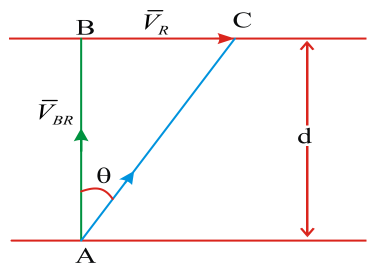
$$x = (V_R - V_{BR} sin\theta)\left[\frac{d}{V_{BR} cos\theta}\right]$$

↳ (a) The boat reaches the other end of the river to the right of B if $V_R > V_{BR} \sin \theta$

(b) The boat reaches the other end of the river to the left of B if $V_R < V_{BR} \sin \theta$

(c) The boat reaches the exactly opposite point on the bank if $V_R = V_{BR} \sin \theta$

Motion of a Boat Crossing the River in Shortest Time



If $\vec{V}_{BR}, \vec{V}_R$ are the velocities of a boat and river flow respectively then to cross the river in shortest time, the boat is to be rowed across the river i.e., along normal to the banks of the river.

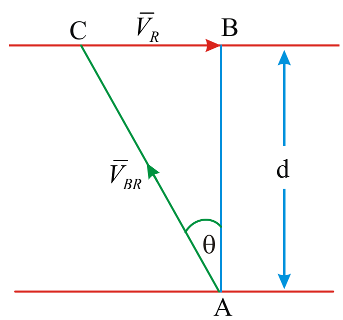
i) Time taken to cross the river, $t=\dfrac{d}{V_{BR}}$ where $d=$ width of the river. This time is independent of velocity of the river flow

ii) Velocity of boat w.r.t. ground has a magnitude of $V_B=\sqrt{V_{BR}^2+V_R^2}$

iii) The direction of the resultant velocity is $\theta=\tan^{-1}\left(\dfrac{V_R}{V_{BR}}\right)$ with the normal.

iv)The distance (BC) travelled downstream $=V_R\left(\dfrac{d}{V_{BR}}\right)$ is called drift

Motion of a Boat Crossing the River in Shortest Distance



i) The boat is to be rowed upstream making some angle 'θ with normal to the bank of the river which is given by $\theta=\sin^{-1}\left(\dfrac{V_R}{V_{BR}}\right)$

ii) The angle made by boat with the river flow (or) bank is $=90^0+\theta$

iii) Velocity of boat w.r.t. ground has a magnitude of $V_B=\sqrt{V_{BR}^2-V_R^2}$

iv) The time taken to cross the river is $t=\dfrac{d}{\sqrt{V_{BR}^2-V_R^2}}$

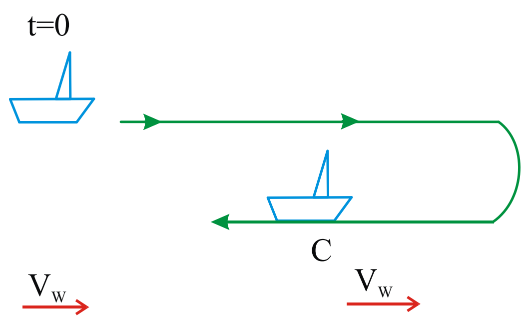
Note : V_{BR} = Relative velocity of the boat w.r.t river (or) velocity of boat in still water.

EX.8: **A boat is moving with a velocity $v_{bw} = 5$ km/hr relative to water. At time $t = 0$. the boat passes through a piece of cork floating in water while moving down stream. If it turns back at time $t_1 = 30$ min.**

a) when the boat meet the cork again ?

b) The distance travelled by the boat during this time.

Sol.



Let AB =d is the distance travelled by boat along down stream in `$t_1$' sec and it returns back and it meets the cork at point C after `t_2' sec.

$\therefore$ Let AC=x is the distance travelled by the cork during $(t_1 + t_2)$ sec.

$$d = (V_B + V_W)t_1 ....(1)$$
$$d - x = (V_B - V_w)t_2 ....(2)$$
$$and\ \ x = V_w(t_1 + t_2) ....(3)$$

Substitute (1) and (3) in (2) weget $t_1 = t_2$

$\therefore$ The boat meets the cork again after $T = 2t_1 = 60$ min and the distance (AB+BC) travelled by the boat before meets the cork is

$$D = 2d - x$$
$$D = 2(V_B + V_w)\mathrm{t}_1 - V_w 2t_1$$
$$D = 2V_B t_1 + 2V_w t_1 - 2V_w t_1$$
$$D = 2V_B t_1 = 2 \times 5 \times \frac{30}{60} = 5km$$

EX.9: **A swimmer crosses a flowing stream of width `d' to and fro normal to the flow of the river in time t_1. The time taken to cover the same distance up and down the stream is t_2. If t_3 is the time the swimmer would take to swim a distance 2d in still water, then relation between t_1, t_2 & t_3.**

Sol : Let v be the river velocity and u be the velocity of swimmer in still water. Then

$$t_1 = 2\left(\frac{d}{\sqrt{u^2 - v^2}}\right).....(i)$$

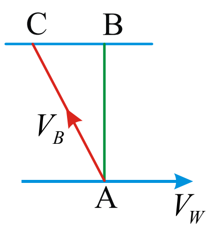
$$t_2 = \frac{d}{u+v} + \frac{d}{u-v} = \frac{2ud}{u^2 - v^2}........(ii)$$

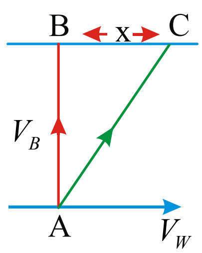
and $\mathrm{t}_3 = \dfrac{2d}{u}.............(iii)$

from equation (i) ,(ii) and (iii) $t_1^2 = t_2 t_3 \Rightarrow t_1 = \sqrt{t_2 t_3}$

EX.10: Two persons P and Q crosses the river starting from point A on one side to exactly opposite point B on the other bank of the river.The person P crosses the river in the shortest path. The person Q crosses the river in shortest time and walks back to point B. Velocity of river is 3 kmph and speed of each person is 5kmph w.r.t river.If the two persons reach the point B in the same time, then the speed of walk of Q is.

Sol : For person (P) : For person(Q) :





$$t_p = \frac{d}{\sqrt{V_B^{\ 2} - V_w^{\ 2}}} = \frac{d}{\sqrt{5^2 - 3^2}} = \frac{d}{4} \qquad t_Q = \frac{d}{V_B} = \frac{d}{5}, t_p = t_Q + \Delta t$$

$$\frac{d}{4} = \frac{d}{5} + \frac{x}{V_{man}}, \quad \text{But } \ x = V_W \frac{d}{V_B}$$

$$\frac{d}{4} = \frac{d}{5} + \frac{V_W d}{V_B V_{man}}, \quad \frac{d}{4} = \frac{d}{5} + \frac{3d}{(5)V_{man}}$$

$$\frac{1}{4} - \frac{1}{5} = \frac{3}{5V_{man}}, \quad \frac{1}{20} = \frac{3}{5V_{man}}$$

$$V_{man} = \frac{(3)(20)}{5} = 12 kmph$$

↳ **When a body is moving in a plane**

a) A body can have any angle between velocity and acceleration
b) If the angle between velocity and acceleration is acute, velocity increases.
c) If the angle between velocity and acceleration is obtuse, velocity decreases.
d) If the angle between velocity and acceleration is a right angle, velocity remains constant.
e) A body can have constant speed and changing velocity
f) A body cannot have constant velocity and changing speed.

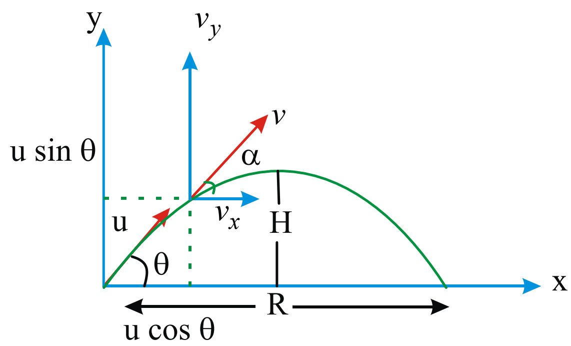
Projectiles :

Oblique Projectile :

↳ Any body projected into air with some velocity at an angle 'θ' [$\theta \neq (90^0$ and $0^0)$] with the horizontal is called an oblique projectile.

↳ Horizontal component of velocity $u_x = u\cos\theta$, remains constant throughout the journey.

Vertical component of velocity $u_y = u\sin\theta$, gradually decreases to zero and then gradually increases to $u\sin\theta$. It varies at the rate 'g'.



Vertical component of velocity $u_y = u\sin\theta$, gradually decreases to zero and then gradually increases to $u\sin\theta$. It varies at the rate 'g'.

horizontal component of acceleration, $a_x = 0$

vertical component of acceleration , $a_y = -g$

At the Point of Projection

(a) Horizontal component of velocity $u_x = u\cos\theta$

(b)Vertical component of velocity $u_y = u\sin\theta$

(c) velocity vector $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

(d) Angle between velocity and acceleration is $(90+\theta)$

At any instant 't'

Velocity vector ($\vec{v}$) is $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Here $v_x = u\cos\theta$ and $v_y = u_y + a_y t = u\sin\theta - gt$

Hence $\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

magnitude of velocity is given by $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

direction of velocity is given by $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$

Displacement vector ($\vec{s}$)

displacement $\vec{s} = x\hat{i} + y\hat{j}$ here

horizontal displacement during a time t $x = u_x t = (u\cos\theta)t$

vertical displacement during a time t $y = u_y t - \frac{1}{2}gt^2 = (u\sin\theta)t - \frac{1}{2}gt^2$

Equation of projectile

$$y=(\tan\theta)x-\left(\frac{g}{2u^2\cos^2\theta}\right)x^2=Ax-Bx^2$$

Where A and B are constants $A=\tan\theta,\ B=\dfrac{g}{2u^2\cos^2\theta}$

Time of flight (T)

Time of ascent (t_a) = Time of descent $(t_d)=\dfrac{u_y}{g}=\dfrac{u\sin\theta}{g}$

Time of flight $T=t_a+t_d=\dfrac{2u_y}{g}=\dfrac{2u\sin\theta}{g}$

↳ During time of flight

1) angle between velocity and acceleration vectors changes from $(90^0+\theta)$ to $(90^0-\theta)$.

2) change in momentum is $2mu\sin\theta$.(In general, change in momentum $\Delta P=mgT\downarrow$)

3) vertical displacement is 0.

4) The angle between velocity and acceleration during the rise of projectile is $180^0>\theta>90^0$

5) The angle between velocity and acceleration during the fall of projectile is $0^0<\theta<90^0$

Maximum height (H)

$$H=\frac{u_y^2}{2g}=\frac{u^2\sin^2\theta}{2g}$$

↳ At maximum height

1) The vertical component of velocity becomes zero.

2) The velocity of the projectile is minimum at the highest point and is equal to $u\cos\theta$ and is horizontal.

3) Acceleration is equal to acceleration due to gravity 'g', and it always acts vertically downwards.

4) The angle between velocity and acceleration is 90^0.

Range (R):

↳ $R=u_xT=\dfrac{2u_xu_y}{g}$ (or)

$$R=(u\cos\theta)T=u\cos\theta\times\frac{2u\sin\theta}{g}=\frac{u^2\sin2\theta}{g}$$

1) Range is maximum when $\theta=45^0$

2) Maximum range, $R_{max}=\dfrac{u^2}{g}$

3) When 'R' is maximum, $H_{Max}=\dfrac{R_{Max}}{4}=\dfrac{u^2}{4g}$

4) For given velocity of projection range is same for complimentary angles of projection i.e $(\theta_1+\theta_2=90^0)$

↪ Relation between H, T and R

1) $\frac{H}{T^2} = \frac{g}{8}$ (b) $\frac{H}{R} = \frac{\tan\theta}{4}$ (c) $\frac{R}{T^2} = \frac{g}{2\tan\theta}$

2) $R = \frac{gT^2}{2\tan\theta}$ and if $\theta = 45^0$ then $R = \frac{gT^2}{2} \Rightarrow T = \sqrt{\frac{2R}{g}}$

↪ If $y = Ax - Bx^2$ represents equation of a projectile then

1) Angle of projection $\theta = \tan^{-1}(A)$

2) Initial velocity $|\vec{u}| = \sqrt{\frac{g\left(1+A^2\right)}{2B}}$

3) Range of the projectile R = $\frac{A}{B}$

4) Maximum height H = $\frac{A^2}{4B}$

5) Time of flight $(T) = \sqrt{\frac{2A^2}{Bg}}$

↪ If horizontal and vertical displacement of projectile are respectively $x = at$ and $y = bt - ct^2$ then

1) angle of projection $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

2) velocity of projection $u = \sqrt{a^2 + b^2}$

3) acceleration of projectile = 2c

4) maximum height reached = $\frac{b^2}{4c}$

5) horizontal range = $\frac{ab}{c}$

↪ In case of complimentary angles of projection

1) If T_1 and T_2 are the times of flight then

i) $\frac{T_1}{T_2} = \tan\theta$ ii) $T_1T_2 = \frac{2R}{g} \Rightarrow T_1 T_2 \alpha R$

2)If H_1 and H_2 are maximum heights then

i) $\frac{H_1}{H_2} = \tan^2\theta$ ii) $H_1 + H_2 = \frac{u^2}{2g}$ iii) $R = 4\sqrt{H_1H_2}$ iv) $R_{\max} = 2\left(H_1 + H_2\right)$

↪ If a man throws a body to a maximum distance 'R' then he can project the body to maximum vertical height R/2.

↪ If a man throws a body to a maximum distance 'R' then maximum height attained by it in its path is R/4.

At the point of striking the ground

1) Horizontal component of velocity = $u\cos\theta$

2) Vertical component of velocity = $-u\sin\theta$

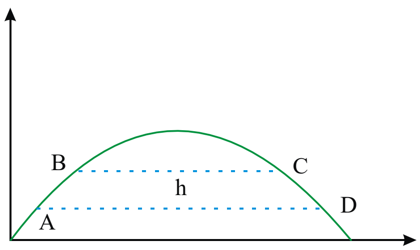
3) Speed of projection is equal to striking speed of projectile.

4) Angle of projection is equal to the striking angle of projectile

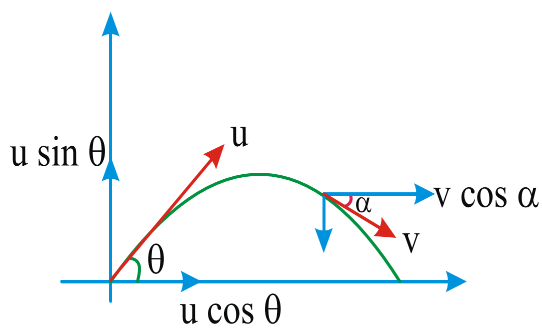
5) If the angle of projection with the horizontal is θ then angle of deviation is 2θ

↳ The projectile crosses the points A, D in time interval t_1 seconds and B,C in time interval t_2 seconds

then $t_1^2 - t_2^2 = \frac{8h}{g}$ (h is the distance between BC and AD)



↳ A projectile is fired with a speed u at an angle θ with the horizontal. Its speed when its direction of motion makes an angle α with the horizontal. $v = u\cos\theta\sec\alpha$



$\because v\cos\alpha = u\cos\theta \qquad v = u\cos\theta\sec\alpha$

↳ If a body is projected with a velocity u making an angle θ with the horizontal, the time after which direction of velocity is perpendicular to the initial velocity is $t = \frac{u\ \text{cosec}\theta}{g} = \frac{u}{g\sin\theta}$

and its velocity at that instant is $v = u\cot\theta$

↳ **The path of projectile as seen from another projectile**

Suppose two bodies A and B are projected simultaneously from the same point with initial velocities u_1 and u_2 at angles θ_1 and θ_2 with horizontal.

The instantaneous positions of the two bodies are given by

Body A : $x_1 = u_1\cos\theta_1 t$, $y_1 = u_1\sin\theta_1 t - \frac{1}{2}gt^2$

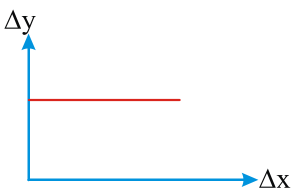
Body B : $x_2 = u_2\cos\theta_2 t$, $y_2 = u_2\sin\theta_2 t - \frac{1}{2}gt^2$

$$\Delta x = \left(u_1\cos\theta_1 - u_2\cos\theta_2\right)t$$

$$\Delta y = \left(u_1\sin\theta_1 - u_2\sin\theta_2\right)t$$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}$$

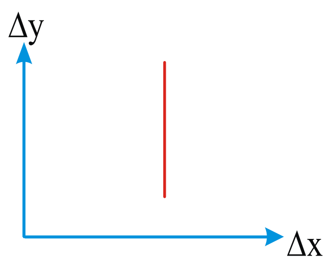
i) If $u_1\sin\theta_1 = u_2\sin\theta_2$ (initial vertical components) then slope $\frac{\Delta y}{\Delta x} = 0$



The path is a horizontal straight line

ii) If $u_1\cos\theta_1 = u_2\cos\theta_2$ (initial horizontal components)

Then slope $\frac{\Delta y}{\Delta x} = \infty$



The path is a vertical straight line

- For a projectile, 'y' component of velocity at $\frac{1}{n^{th}}$ of the maximum height is $\frac{u\sin\theta}{\sqrt{n}}$
- Resultant velocity at a height of $\frac{1}{n^{th}}$ of maximum height

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(u\cos\theta\right)^2 + \left(\frac{u\sin\theta}{\sqrt{n}}\right)^2} \qquad = u\sqrt{\frac{(n-1)\cos^2\theta + 1}{n}}$$

If n=2, velocity of a projectile at half of maximum height $= u\sqrt{\frac{1+\cos^2\theta}{2}}$

- For a projectile, w.r.t stationary frame path (or) trajectory is a parabola.
- Path of projectile w.r.t frame of another projectile is a straight line
- Acceleration of a projectile relative to another projectile is zero
- A body is projected vertically up from a topless car relative to the car which is moving horizontally relative to earth

a) If car velocity is constant, ball will be caught by the thrower.

b) If car velocity is constant, path of ball relative to the ground is a parabola and relative to this car is straight up and then straight down

c) If the car accelerates, ball falls back relative to the car

d) If the car retards ball falls forward relative to the car

↪ If a gun is aimed towards a target and the bullet is fired, the moment when the target falls, the bullet will always hit the target irrespective of the velocity of the bullet if it is with in the range.

Note : If air resistance is taken into consideration then

a) trajectory departs from parabola.

b) time of flight may increase or decrease.

c) the velocity with which the body strikes the ground decreases

d) maximum height may decrease.

e) striking angle increases

f) range decreases.

↪ A particle is projected with a velocity $\vec{u} = a\hat{i} + b\hat{j}$ then the radius of curvature of the trajectory of the particle at the

(i) point of projection is $r = \frac{(a^2+b^2)^{3/2}}{ga}$ (ii) Highest point is $r = \frac{a^2}{g}$

↪ Expression for radius of curvature is $r = \frac{(\text{velocity})^2}{\text{normal acceleration}}$ $r = \frac{u^2\cos^2\theta}{g\cos^3\alpha}$

α is angle made by $\vec{v}$ with horizontal

EX.11: A bullet fired at an angle of 30^0 with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to be fixed, and neglect air resistance.

Sol . weare given that angle of projection with the horizontal, $\theta = 30^0$, horizontal range R = 3km.

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2 \sin 60^0}{g} = \frac{u_0^2}{g} \times \frac{\sqrt{3}}{2} \quad \text{or} \quad \frac{u_0^2}{g} = 2\sqrt{3}km$$

Since the muzzle speed (u_0) is fixed

$$R_{\max} = \frac{u_0^2}{g} = 2\sqrt{3} = 2\times 1.732 = 3.464km$$

so, it is not possible to hit the target 5km away.

EX.12: A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag ?

Sol . Here, u_0=240 ms^{-1}, R =5.10 km =5100m,

$$g = 9.8ms^{-2}, \alpha = ?$$

$$R = \frac{u_0^2 \sin 2\alpha}{g}$$

$$\sin 2\alpha = \frac{Rg}{u_0^2} \qquad \Rightarrow \alpha = 30^0 \ or \ 60^0$$

using, $T = \dfrac{2u_0 \sin\alpha}{g}$

$$When \quad \alpha = 30^0, T_1 = \frac{2\times 240\times 0.5}{9.8} = 24.5s$$

$$When \quad \alpha = 60^0, T_2 = \frac{2\times 240\times 0.867}{9.8} = 42.46s$$

EX.13: The ceiling of a long hall is 20 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40ms^{-1} can go without hitting the ceiling of the hall $\left(g = 10ms^{-2}\right)$?

Sol. : Here, H =20 m, $u = 40ms^{-1}$.

Suppose the ball is thrown at an angle θ with the horizontal.

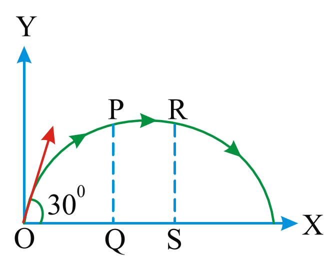
$$Now \quad H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow 20 = \frac{(40)^2 \sin^2\theta}{2\times 10}$$

$$or, \quad \sin\theta = 0.5 \Rightarrow \theta = 30^0$$

$$Now \quad R = \frac{u^2 \sin 2\theta}{g} = \frac{(40)^2 \times \sin 60^0}{10}$$

$$= \frac{(40)^2 \times 0.866}{10} = 138.56m$$

EX.14: A ball projected with a velocity of 10m/s at angle of 30^0 with horizontal just clears two vertical poles each of height 1m. Find separation between the poles.



Sol. $h = u_y t + \frac{1}{2} g t^2 = \left(10\sin 30^0\right)t + \frac{1}{2}(-10)t^2$

$1 = 5t - 5t^2 \Rightarrow t = 0.72s,\ 2.76s$

are the instants at which projectile crosses the poles.

$\therefore$ separation between poles = OS - OQ

$= u\cos\theta\left(t_2 - t_1\right)$

$= 10\cos 30^0\left(2.76 - 0.72\right) = 17.7m$

EX.15: A body is projected with velocity u at an angle of projection θ with the horizontal.The body makes 30^0 with horizontal at t = 2 second and then after 1 second it reaches the maximum height. Then find

a) angle of projection b) speed of projection.

Sol. During the projectile motion, angle at any instant t is such that $\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$

For $t = 2$ seconds, $\alpha = 30^0$

$\frac{1}{\sqrt{3}} = \frac{u\sin\theta - 2g}{u\cos\theta}$ ---------1

For t = 3 seconds, at the highest point $\alpha = 0^o$

$0 = \frac{u\sin\theta - 3g}{u\cos\theta}$

$u\sin\theta = 3g$ ------------(2)

using eq. (1) and eq. (2)

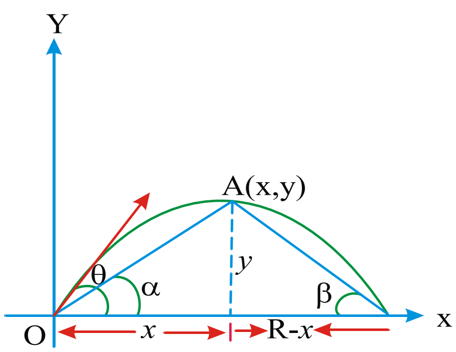
$u\cos\theta = \sqrt{3}g$.....................(3)

Eq. (2) $\div$ eq.(3) give $\theta = 60^0$ squaring and adding equation (2) and (3)

$u = 20\sqrt{3}\ m/s.$

EX.16: A particle is thrown over a triangle from one end of horizontal base and grazing the vertex falls on the other end of the base.If α and β are the base angles and θ be the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

Sol.: The situation is shown in figure.From figure,we have



$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$

$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)}$ _______(1)

But equation of trajectory is $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$\tan\theta = \left[\frac{yR}{x(R-x)}\right]$------$(ii)$

From Eqs. (i) and (ii), $\tan\theta = \tan\alpha + \tan\beta$

EX.17: The velocity of a projectile at its greatest height is $\sqrt{\frac{2}{5}}$ times its velocity, at half of its greatest height, find the angle of projection.

Sol.: $u\cos\theta = \sqrt{\frac{2}{5}} \times u\sqrt{\frac{1+\cos^2\theta}{2}}$

Squaring on both sides $u^2\cos^2\theta = \frac{2}{5}u^2\left(\frac{1+\cos^2\theta}{2}\right)$

$10\cos^2\theta = 2 + 2\cos^2\theta \Rightarrow 8\cos^2\theta = 2 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \theta = 60^0$

EX.18: A foot ball is kicked off with an initial speed of 19.6 m/s to have maximum range. Goal keeper standing on the goal line 67.4 m away in the direction of the kick starts running opposite to the direction of kick to meet the ball at that instant. What must his speed be if he is to catch the ball before it hits the ground?

Sol.: $R = \frac{u^2\sin 2\theta}{g} = \frac{(19.6)^2 \times \sin 90}{9.8}$

or R= 39.2 metre.

Man must run 67.4 m -39.2m=28.2m

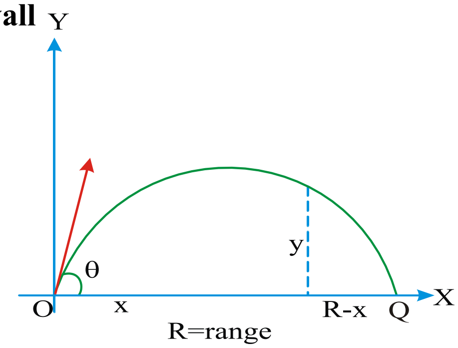
in the time taken by the ball to come to ground Time taken by the ball.

$t = \frac{2u\sin\theta}{g} = \frac{2\times 19.6\times \sin 45^0}{9.8} = \frac{4}{\sqrt{2}}$

$t = 2\sqrt{2} = 2\times 1.41 = 2.82\,\text{sec}.$

Velocity of man $= \frac{28.2m}{2.82\,\text{sec}} = 10m/\text{sec}.$

EX.19: A body projected from a point `0' at an angle $\theta$, just crosses a wall `y' m high at a distance `x' m from the point of projection and strikes the ground at `Q' beyond the wall as shown, then find height of the wall



Sol . we know that the equation of the trajectory is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ can be written as

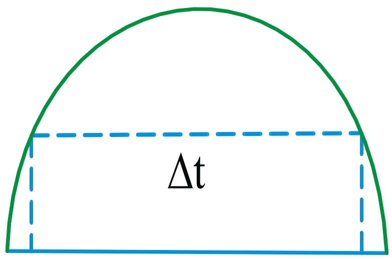
$y = x\tan\theta - \left(\frac{gx^2}{2u^2\cos^2\theta}\right)\frac{\sin\theta}{\sin\theta}$

$$y = x\tan\theta - \frac{gx^2\tan\theta}{u^2(2\sin\theta\cos\theta)} \qquad \Rightarrow y = x\tan\theta - \frac{x^2\tan\theta}{\frac{u^2\sin 2\theta}{g}}$$

$$\Rightarrow y = x\tan\theta\left[1-\frac{x}{R}\right] \quad [\because R = \frac{u^2\sin 2\theta}{g}]$$

EX.20: A particle is projected with a velocity of $10\sqrt{2}$ m/s at an angle of 45^0 with the horizontal . Find the interval between the moments when speed is $\sqrt{125}$ m/s $\left(g = 10m/s^2\right)$

Sol.



$v = \sqrt{125}\ m/s$

$u_x = 10\sqrt{2}\cos 45^0 = 10m/s, \quad u_y = 10\sqrt{2}\sin 45^0 = 10m/s$

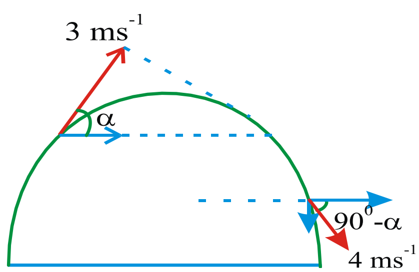
$v^2 = v_x^2 + v_y^2$

$125 = 100 + v_y^2 \Rightarrow v_y = 5m/s \quad (\because v_x = u_x)$

The required time interval is $\Delta t = \frac{2v_y}{g} = \frac{2\times 5}{10} = 1\,s$

EX.21: A projectile of 2 kg has velocities 3 m/s and 4 m/s at two points during its flight in the uniform gravitational field of the earth. If these two velocities are $\perp$ to each other then the minimum KE of the particle during its flight is

Sol. $V_1\cos\alpha = V_2\cos(90-\alpha)$



$3\cos\alpha = 4\sin\alpha$

$\tan\alpha = \frac{3}{4}$

$KE_{\min} = \frac{1}{2}mv_1^2\cos^2\alpha$

$= \frac{1}{2}\times 2\times 3^2\times\left(\frac{4}{5}\right)^2 = \frac{9\times 16}{25} = 5.76J$

EX.22: In the absence of wind the range and maximum height of a projectile were R and H. If wind imparts a horizontal acceleration a =g/4 to the projectile then find the maximum range and maximum height.

Sol . $H^1 = H$ ($\because$ u sinθ remains same)

$T^1 = T$

$R^1 = u_x T + \frac{1}{2} a T^2 = R + \frac{1}{2}\frac{g}{4}T^2$

$= R + \frac{1}{8} g T^2 \quad = R + H$ $\boxed{R^1 = R + H}$ $\boxed{H^1 = H}$

↳ **If a body is projected with a velocity**

$\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}$

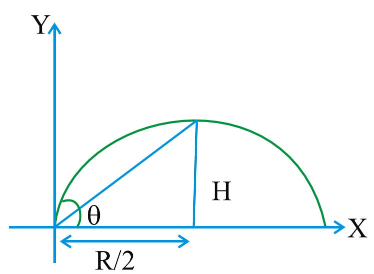
($\vec{i} - east \quad \vec{j} - north \quad \vec{k} - \quad vertical$) then

$u_x = \sqrt{a^2 + b^2}$; $u_y = c$

$T = \frac{2c}{g}; H = \frac{c^2}{2g}, R = \frac{2\left(\sqrt{a^2+b^2}\right)c}{g}$

EX.23: A particle is projected from the ground with an initial speed v at an angle θ with horizontal.The average velocity of the particle between its point of projection and highest point of trajectory is [EAM 2013]

Sol.

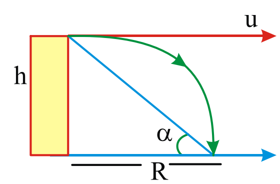


$\vec{v}_{avg} = \frac{\vec{v} + \vec{u}}{2} = \frac{u\cos\theta\hat{i} + (u\cos\theta\hat{i} + u\sin\theta\hat{j})}{2}$, $\quad v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$

↳ **Horizontal projectile**

When a body is projected horizontally with a velocity from a point above the ground level, it is called a Horizontal Projectile.

↳ Path of the Horizontal Projectile is parabola



a) Time of descent $t = \sqrt{\frac{2h}{g}}$ (is independent of u)

b) The horizontal displacement (or) range

$$R = u\sqrt{\frac{2h}{g}}$$

c) The velocity of projectile at any instant of time is $v = \sqrt{u^2 + g^2t^2}$

The direction of velocity $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

d) The velocity with which it hits the ground $v = \sqrt{u^2 + 2gh}$

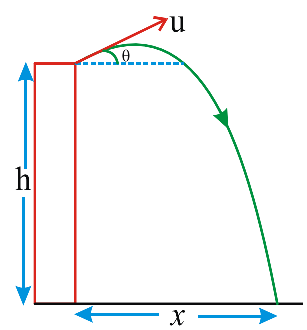
e) The angle at which it strikes the ground $\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{u}\right)$

f) If α is angle of elevation of point of projection from the point where body hits the ground then

$$\tan\alpha = \frac{h}{R} = \frac{gt^2/2}{ut} = \frac{gt}{2u} \qquad \Rightarrow \tan\alpha = \frac{\tan\theta}{2}$$

θ is the angle with which body reaches the ground

Case (i) : If the body is projected at an angle θ in upward direction from the top of the tower,then



a) The time taken by projectile to reach same level as point of projection is $T = \frac{2u\sin\theta}{g}$

b) The time taken by projectile to reach ground is calculated from $h = (-u\sin\theta)t + \frac{1}{2}gt^2$

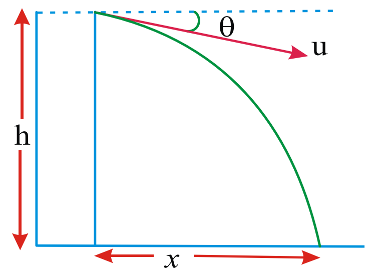
c) The horizontal distance from foot of the tower where the projectile lands is given by $x = u\cos\theta \times t$

d) The velocity with which it strikes the ground $v = \sqrt{u^2 + 2gh}$

e) The angle at which it strikes the ground

$$\alpha = \tan^{-1}\left[\frac{-u\sin\theta + gt}{u\cos\theta}\right] \qquad \text{(or)} \quad \alpha = \tan^{-1}\left[\frac{\sqrt{u^2\sin^2\theta + 2gh}}{u\cos\theta}\right]$$

Case (ii) : If the body is projected at angle θ from top of the tower in the downward direction, then



a) The time taken by projectile to reach ground is calculated from $h=(u\sin\theta)t+\frac{1}{2}gt^2$

b) The horizontal distance from foot of the tower where the projectile lands is given by $x=u\cos\theta\times t$

c) The velocity with which it strikes the ground $v=\sqrt{u^2+2gh}$

d) The angle at which it strikes the ground $\alpha=\tan^{-1}\left[\frac{\sqrt{u^2\sin^2\theta+2gh}}{u\cos\theta}\right]$

↳ When an object is dropped from an aeroplane moving horizontally with constant velocity
a) Path of the object relative to the earth is parabola
b) Path of the object relative to pilot is a straight line vertically down.

↳ Two bodies are projected horizontally from top of the tower of height h in opposite directions with velocities u_1 and u_2 then

a) The time after which their velocity vectors are making an angle θ with each other

$$t=\frac{\sqrt{u_1u_2}}{g}\cot\frac{\theta}{2}$$

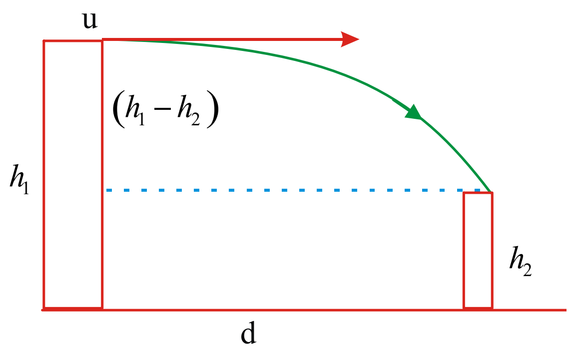
b) The distance between them when their velocity vectors are making an angle θ with each other $x=(u_1+u_2)\frac{\sqrt{u_1u_2}}{g}\cot\frac{\theta}{2}$

c) The time after which their position vectors are making an angle θ with each other $=\frac{2\sqrt{u_1u_2}}{g}\cot\frac{\theta}{2}$

d) The distance between them when their displacement vectors are making an angle θ with each other is

$$x=(u_1+u_2)\frac{2\sqrt{u_1u_2}}{g}\cot\frac{\theta}{2}$$

↳ Two tall towers having heights h_1 and h_2 are separated by a distance d. A person throws a ball horizontally with velocity u from the top of the first tower to reach the top of the second tower then

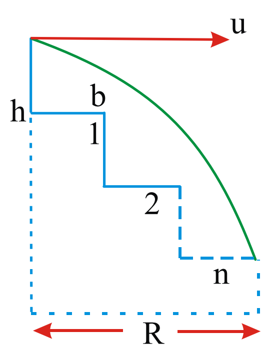


a) Time taken $t = \sqrt{\frac{2(h_1 - h_2)}{g}}$

b) Horizontal distance travelled $d = ut$

↳ A ball rolls off from the top of a stair case with a horizontal velocity u. If each step has a height 'h' and width "b" then the ball will just hit the n^{th} step, directly if n equal to

nb = ut and nh = $\frac{1}{2}gt^2$



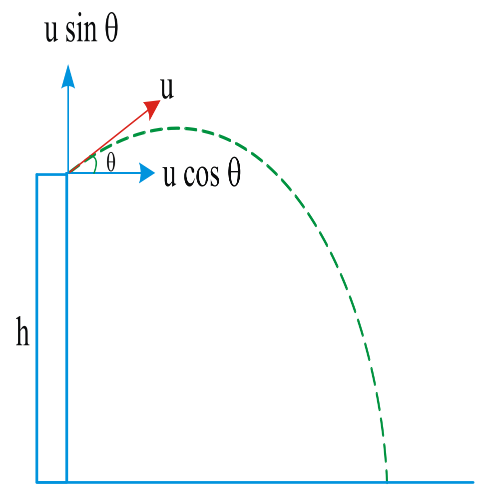
$$n = \frac{2hu^2}{gb^2}$$

↳ From the top of the tower of height h , one stone is thrown towards east with velocity u_1 and another is thrown towards north with velocity u_2. The distance between them after striking the ground,

$$d = t\sqrt{u_1^2 + u_2^2}\ ,\ t = \sqrt{\frac{2h}{g}}$$

EX.24: **A ball is thrown from the top of a tower of 61 m high with a velocity 24.4 ms^{-1} at an elevation of 30^0 above the horizontal . What is the distance from the foot of the tower to the point where the ball hits the ground?**

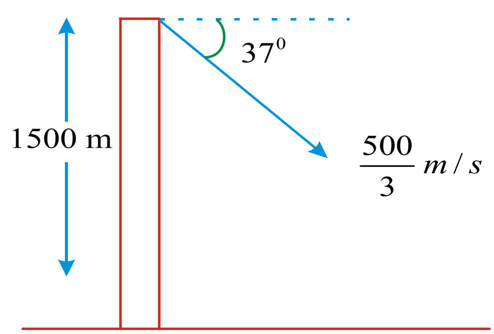
Sol. :



$$h = \frac{1}{2}gt^2 - (u\sin\theta)t \qquad \Rightarrow t = 5\sec onds$$

Also, $d = (u\cos\theta)t = 105.65m$

EX.25: **A particle is projected from a tower as shown in figure, then find the distance from the foot of the tower where it will strike the ground.** $(g = 10m/s^2)$



Sol.:

$$u_y = u\sin\theta = \frac{500}{3}\sin 37^0 \qquad s = ut + \frac{1}{2}at^2$$

$$1500 = \left(\frac{500}{3}\sin 37\right)t + \frac{1}{2}10t^2$$

$$1500 = \frac{500}{3}\left(\frac{3}{5}\right)t + 5t^2$$

$300 = 20t + t^2 \Rightarrow t = 20s$ $\therefore$ horizontal distance = (u cos θ) t

$= \frac{500}{3}\left(\frac{4}{5}\right)10 = \frac{4000}{3}m$

EX.26: A golfer standing on the ground hits a ball with a velocity of 52 m/s at an angle θ above the horizontal if $\tan\theta = \frac{5}{12}$ find the time for which the ball is at least 15m above the ground? $\left(g = 10m/s^2\right)$

Sol. $v_y = \sqrt{u_y^{\ 2} - 2gy}$, $u_y = u\sin\theta$

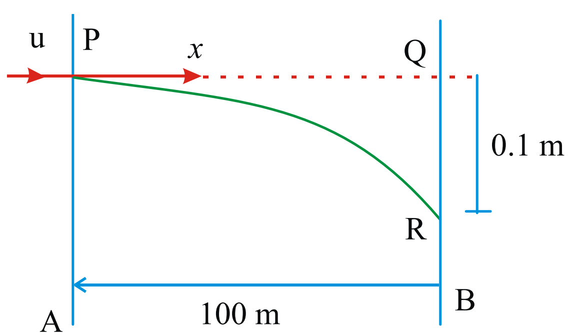
$v_y = \sqrt{52\times 52\times \frac{5\times 5}{13\times 13} - 2\times 10\times 15}$

$= \sqrt{16\times 25 - 300} = 10$

$\Delta t = \frac{2v_y}{10} = \frac{2\times 10}{10} = 2s$

EX.27: Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect resistance of paper and air.

Sol. **:** The situation is shown in Fig.



$d = u\sqrt{\frac{2(h_1 - h_2)}{g}} \Rightarrow 100 = u\sqrt{\frac{2\times 0.1}{9.8}} \Rightarrow u = 700m/s.$

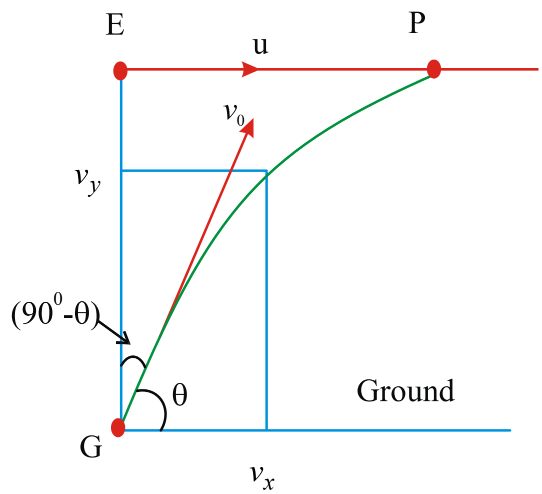
EX.28: A boy aims a gun at a bird from a point, at a horizontal distance of 100m. If the gun can impart a velocity of 500m/sec to the bullet, at what height above the bird must he aim his gun in order to hit it?

Sol : x = vt or $100 = 500\times t$; $t = 0.2\,\text{sec}$.

Now h $= 0 + \frac{1}{2}\times 10\times (0.2)^2 = 0.20m = 20cm.$

EX.29: An enemy plane is flying horizontally at an altitude of 2 km with a speed of 300 ms^{-1}. An army man with an anti - aircraft gun on the ground sights enemy plane when it is directly overhead and fires a shell with a muzzle speed of 600ms^{-1}. At what angle with the vertical should the gun be fired so as to hit the plane?

Sol. Let G be the position of the gun and E that of the enemy plane flying horizontally with speed.



$u = 300\,\mathbf{ms^{-1}}$, when the shell is fired with a speed v_0 , $v_x = v_0\cos\theta$

The shell will hit the plane, if the horizontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizontal direction in the same time, i.e.

$u \times t = v_x \times t$ or $u = v_x \Rightarrow u = v_0\cos\theta$

or $\cos\theta = \dfrac{u}{v_0} = \dfrac{300}{600} = 0.5$ or $\theta = 60^0$

Therefore, angle with the vertical $= 90^0 - \theta = 30^0$.

EX.30: From the top of a tower, two balls are thrown horizontally with velocities u_1 and u_2 in opposite directins. If their velocities are perpendicular to each other just before they strike the ground, find the height of tower.

Sol. Time taken to reach ground $t = \sqrt{\dfrac{2h}{g}}$

at time of reaching ground respective velocities are $\vec{v}_1 = u_1\hat{i} + gt\hat{j}$, $\vec{v}_2 = -u_2\hat{i} + gt\hat{j}$

Given $\vec{v}_1 \cdot \vec{v}_2 = 0$, $t = \dfrac{\sqrt{u_1u_2}}{g}$

$\therefore \sqrt{\dfrac{2h}{g}} = \dfrac{\sqrt{u_1u_2}}{g} \Rightarrow h = \dfrac{u_1u_2}{2g}$

is the height of the tower.

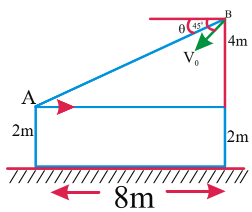
EX.31: From points A and B, at the respective heights of 2m and 6m, two bodies are thrown simultaneously towards each other, one is thrown horizontally with a velocity of 8m/s and the other, downward at an angle 45^0 to the horizontal at an initial velocity v_0 such that the bodies collide in flight. The horizontal distance between points A and B equal to 8m . Then find

a) The initial velocity V_0 of the body thrown at an angle 45^0

b) The time of flight t of the bodies before colliding

c) The coordinate (x,y) of the point of collision (consider the bottom of the tower A as origin) is

Sol :



a) From diagram $\tan\theta = \frac{4}{8} \Rightarrow \tan\theta = \frac{1}{2}$ ....(1)

$\vec{v}_A = 8\hat{i}, \vec{v}_B = -v_0\cos45^0\,\hat{i} - v_0\sin45^0\,\hat{j}$

$$\vec{v}_{BA} = \left(-\frac{v_0}{\sqrt{2}} - 8\right)i - \frac{v_0}{\sqrt{2}}\,j$$

Direction of $\vec{v}_{BA}$

$$\tan\theta = \frac{v_0}{\sqrt{2}} \frac{\sqrt{2}}{\left(v_0 + 8\sqrt{2}\right)} \text{ ....(2)}$$

From eq (1) and eq (2) $\quad 2v_0 = v_0 + 8\sqrt{2} \quad , \quad v_0 = 11.28\,m/s$

b) $\vec{v}_{BA} = \left(-\frac{v_0}{\sqrt{2}} - 8\right)\hat{i} - \frac{v_0}{\sqrt{2}}\hat{j}$

$\because v_0 = 8\sqrt{2} \Rightarrow \vec{v}_{BA} = -16\hat{i} - 8\hat{j}$

$|\vec{v}_{BA}|\,t = S_{BA} \qquad \left(\sqrt{(-16)^2 + (-8)^2}\right)t = \sqrt{8^2 + 4^2}$

$$t = \sqrt{\frac{80}{320}} = \sqrt{\frac{1}{4}} \Rightarrow t = 0.5s$$

c)

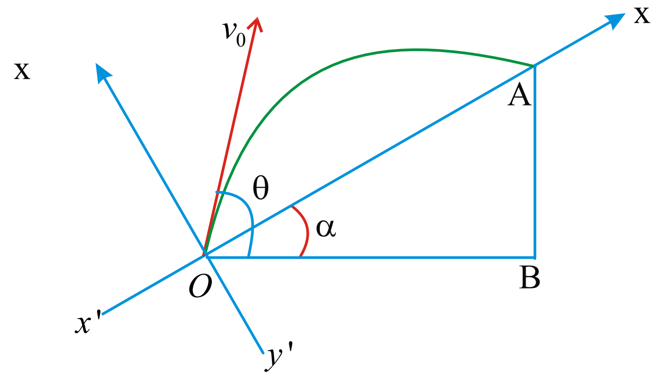
$x = v_x t = (8)(0.5) = 4$

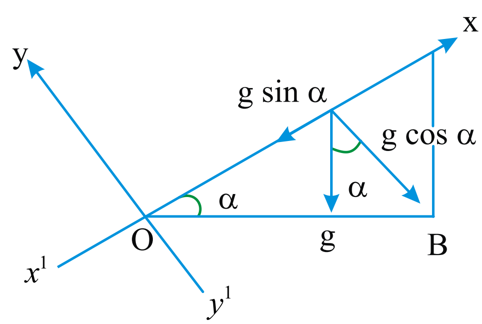
$y' = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \frac{1}{4} = 1.25$

$y = 2 - y' = 0.75$

Motion of a Projected Body on an inclined plane :

↳ A body is projected up the inclined plane from the point O with an initial velocity v_0 at an angle θ with horizontal.





a) Acceleration along $x-axis$, $a_x = -g\sin\alpha$

b) Acceleration along $y-axis$, $a_y = -g\cos\alpha$

c) Component of velocity along $x-axis$ $\quad u_x = v_0\cos(\theta-\alpha)$

d) Component of velocity along $y-axis$ $\quad u_y = v_0\sin(\theta-\alpha)$

e) Time of flight $T = \dfrac{2v_0\sin(\theta-\alpha)}{g\cos\alpha}$

f) Range of projectile (OA)

$$R = \frac{v_0^2}{g\cos^2\alpha}\left[\sin(2\theta-\alpha)-\sin\alpha\right]. \quad \textbf{(or)} \qquad R = \frac{2v_0^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

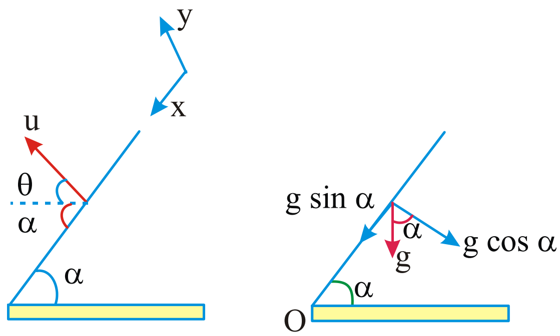
For maximum range $(2\theta-\alpha) = \dfrac{\pi}{2}$

$$\therefore R_{max} = \frac{v_0^2(1-\sin\alpha)}{g\cos^2\alpha}$$

g) $T^2 g = 2R_{max}$

horizontal range (OB) $x = R\cos\alpha$

Down the plane : Here, x and y-directions are down the plane and perpendicular to plane respectively



$u_x = u\cos(\alpha+\theta),\ a_x = g\sin\alpha \quad u_y = u\sin(\alpha+\theta),\ a_y = -g\cos\alpha$

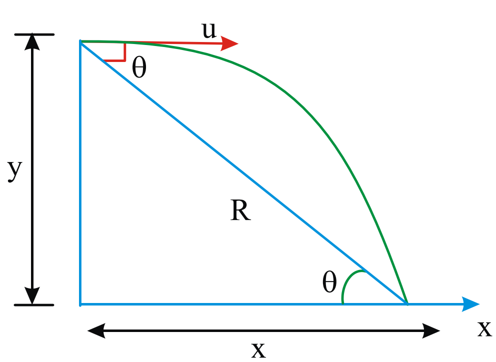
Proceeding in the similar manner , weget the following results

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\alpha},$$

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin\left(2\theta+\alpha\right)+\sin\alpha\right]$$

EX.32: A particle is projected horizontally with a speed "u" from the top of plane inclined at an angle "θ" with the horizontal. How far from the point of projection will the particle strike the plane ?

Sol:



$$R = \sqrt{x^2+y^2} \qquad \left(\frac{y}{x} = \tan\theta\right)$$

$$= \sqrt{x^2+\left(x\tan\theta\right)^2} = x\sqrt{1+\tan^2\theta} = x\sec\theta$$

$x = ut; \quad y = \frac{1}{2}gt^2; \qquad \frac{y}{x} = \frac{1}{2}\frac{gt^2}{ut}$

$\tan\theta = \frac{gt}{2u}; \quad t = \frac{2u}{g}\tan\theta$

$x = ut = \frac{2u^2}{g}\tan\theta;$

$\therefore R = \frac{2u^2}{g}\tan\theta\sec\theta$

EX.33: A projectile has the maximum range of 500m. If the projectile is now thrown up on an inclined plane of 30^0 with the same speed , what is the distance covered by it along the inclined plane?

Sol:

$$R_{max} = \frac{u^2}{g}$$

$$\therefore 500 = \frac{u^2}{g} \quad \text{or} \quad u = \sqrt{500g}$$

$$v^2 - u^2 = 2gs$$

$$0 - 500g = 2 \times \left(-g\sin 30^0\right) \times x$$

$$x = 500m.$$

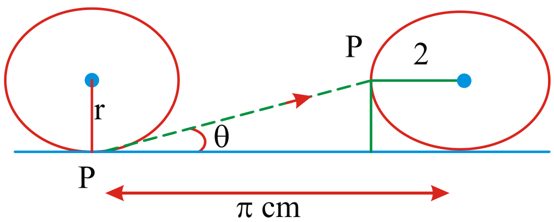
EX. 34: The displacement of the point of a wheel initially in contact with the ground when the wheel rolls forward quarter revolution where perimeter of the wheel is 4π m, is (Assume the forward direction as x-axis)

1) $\sqrt{(\pi+2)^2+4}$ along $Tan^{-1}\frac{2}{\pi}$ with x - axis

2) $\sqrt{(\pi-2)^2+4}$ along $Tan^{-1}\frac{2}{\pi-2}$ with x - axis

3) $\sqrt{(\pi-2)^2+4}$ along $Tan^{-1}\frac{2}{\pi}$ with x - axis

4) $\sqrt{(\pi+2)^2+4}$ along $Tan^{-1}\frac{2}{\pi-2}$ with x - axis

Sol. . 

$S=\sqrt{y^2+x^2}$ and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$ key-2

EX.35: A particle starts from the origin at $t=0s$ with a velocity of $10.0\hat{j}$ m/s and moves in the $xy-$plane with a constant acceleration of $\left(8\hat{i}+2\hat{j}\right)ms^{-2}$. Then $y-$coordinate of the particle in 2 sec is

1) 24 m 2) 16 m 3) 8 m 4) 12 m

Sol. . $\vec{r}=\vec{v}_0 t+\frac{1}{2}\vec{a}t^2$, $y\left(t\right)=t^2+10t$ key-1

EX.36: A car moving at a constant speed of 36 kmph moves north wards for 20 minutes then due to west with the same speed for $8\frac{1}{3}$ minutes. what is the average velocity of the car during this run in kmph

1) 27.5 2) 40.5 3)20.8 4) 32.7

Sol. . $v_{avg}=\frac{\vec{v}_1 t_1+\vec{v}_2 t_2}{t_1+t_2}$ key-1

EX 37: Velocity of a particle at time t = 0 is $2ms^{-1}$. A constant acceleration of $2\,ms^{-2}$ acts on the particle for 1 second at an angle of 60^0 with its initial velocity . Find the magnitude of velocity at the end of 1 second.

1) $\sqrt{3}\ m/s$ 2) $2\sqrt{3}\ m/s$ 3) $4\ m/s$ 4) $8\ m/s$

Sol. . $\overline{v}=v_x\hat{i}+v_y\hat{j}$: $v_x=u_x+a_x t$, $v_y=u_y+a_y t$

$a_x=a\cos\theta$, $a_y=a\sin\theta$ key-2

EX.38: An aeroplane moving in a circular path with a speed 250 km/h. The change in velocity in half of the revolution is.

1) 500km/h 2) 250km/h 3) 120 km/h 4) zero

Sol. . $\Delta V=2V\sin\frac{\theta}{2}$ 6. $\Delta v=\sqrt{v_1^2+v_2^2}$ key-1

EX.39: A ship is moving due east with a velocity of 12 m/sec, a truck is moving across on the ship with velocity 4m/sec. A monkey is climbing the vertical pole mounted on the truck with a velocity of 3m/sec. Find the velocity of the monkey as observed by the man on the shore (m/sec)

1) 10 2) 15 3) 13 4)20

Sol. . $V=\sqrt{V_x^{\ 2}+V_y^{\ 2}+V_z^{\ 2}}$ key-3

EX.40: A man is walking due east at the rate of 2Kmph. The rain appears to him to come down vertically at the rate of 2kmph. The actual velocity and direction of rainfall with the vertical respectively are (2008 M)

1) $2\sqrt{2}kmph, 45^0$ 2) $\frac{1}{\sqrt{2}}kmph, 30^0$ 3) 2 kmph, 0^0 4) 1kmph, 90^0

Sol. . $V_R=\sqrt{V^2+V_m^2}$; $Tan\theta=\frac{V_m}{V}$ key-1

EX.41: A boat takes 2 hours to travel 8km and back in still water lake.With water velocity of 4 kmph, the time taken for going upstream of 8km and coming back is

1) 160 minutes 2) 80 minutes 3) 320 minutes 4) 180 minutes

Sol. . $V_B=\frac{8+8}{2}=8kmph$

$t=t_1+t_2=\frac{d}{v_B+v_r}+\frac{d}{v_B-v_r}$ key-1

EX.42: A ball is thrown with a velocity of u making an angle θ with the horizontal. Its velocity vector normal to initial vector (u) after a time interval of

1) $\frac{u\sin\theta}{g}$ 2) $\frac{u}{g\cos\theta}$ 3) $\frac{u}{g\sin\theta}$ 4) $\frac{u\cos\theta}{g}$

Sol. $\overline{u}=(u\cos\theta)\hat{i}+(u\sin\theta)\hat{j}$

$\overline{v}=(u\cos\theta)\hat{i}+(u\sin\theta-gt)\hat{j}$; $\overline{u}.\overline{v}=0$ key-3

EX.43: A number of bullets are fired in all possible directions with the same initial velocity u. The maximum area of ground covered by bullets is

1) $\pi\left(\frac{u^2}{g}\right)^2$ 2) $\pi\left(\frac{u^2}{2g}\right)$ 3) $\pi\left(\frac{u}{g}\right)^2$ 4) $\pi\left(\frac{u}{2g}\right)^2$

Sol. . Max area $\pi\left(R_{\max}\right)^2$ key-1

EX.44: An aeroplane flies along a straight line from A to B with a speed v_0 and back again with the same speed v_0. A steady wind v is blowing. If AB = l then

a) total time for the trip is $\frac{2v_0l}{v_0^2-v^2}$ if wind blows along the line AB

b) total time for the trip is $\frac{2l}{\sqrt{v_0^2-v^2}}$, if wind blows perpendicular to the line AB

c) total time for the trip decrease because of the presence of wind

d) total time for the trip increase because of the presence of wind

1) a , b , d are correct 2) a , b , c are correct
3) only a , d are correct 4) only b , d are correct

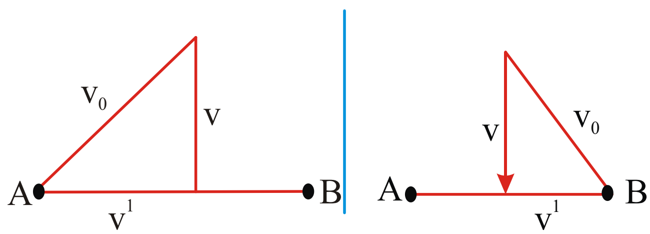
Sol. . a,b,d are correct

When wind blows along the line AB,

$$t = t_{A\to B} + t_{B\to A}$$

$$\Rightarrow t = \frac{l}{v+v_0} + \frac{l}{v_0 - v} \quad \Rightarrow t = \frac{2lv_0}{v_0^2 - v^2}$$

If wind blows perpendicular to AB



$$t = t_{A\to B} + t_{B\to A}$$

$$v' = \sqrt{v_0^2 - v^2} \qquad v' = \sqrt{v_0^2 - v^2}$$

$$t_{A\to B} = \frac{l}{\sqrt{v_0^2 - v^2}} \qquad t_{B\to A} = \frac{l}{\sqrt{v_0^2 - v^2}}$$

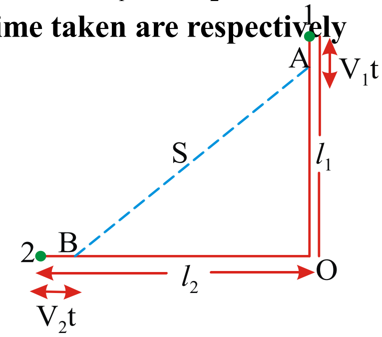
Hence $t = \dfrac{2l}{\sqrt{v_0^2 - v^2}}$

If the wind were not present then total time

taken for the trip would have been $t = \dfrac{2l}{v_0}$

i.e. the total time for the trip increases because of the presence of wind. key-2

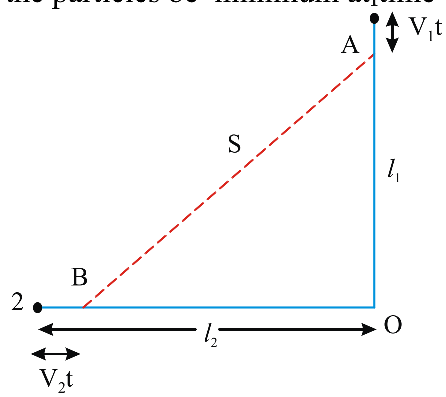
EX.45: Two particles A and B move with constant velocity $\vec{v}_1$ and $\vec{v}_2$ along two mutually perpendicular straight lines towards intersection point O as shown in figure. At moment t = 0 particles were located at distance l_1 and l_2 respectively from O. Then minimum distance between the particles and time taken are respectively



1) $\dfrac{|l_1v_2 - l_2v_1|}{\sqrt{v_1^2+v_2^2}}, \dfrac{l_1v_1 + l_2v_2}{v_1^2+v_2^2}$ 2) $\dfrac{|l_1v_1 - l_2v_2|}{\sqrt{v_1^2+v_2^2}}, \dfrac{l_1v_2 + l_2v_1}{v_1^2+v_2^2}$

3) $\dfrac{|l_1v_2 - l_2v_1|}{\sqrt{v_1^2+v_2^2}}\sqrt{\dfrac{l_1}{l_2}}, \dfrac{(l_1v_1 + l_2v_2)l_1}{(v_1^2+v_2^2)l_2}$ 4) $\dfrac{|l_1v_2 - l_2v_1|}{\sqrt{v_1^2+v_2^2}}\sqrt{\dfrac{l_2}{l_1}}, \dfrac{(l_1v_1 + l_2v_2)l_2}{(v_1^2+v_2^2)l_1}$

Sol. . Let the separation between the particles be minimum at time t, Then



Since $OB = l_2 - v_2 t$ and $OA = l_1 - v_1 t$ and

$$AB^2 = OB^2 + OA^2 \Rightarrow s^2 = (l_1 - v_1 t)^2 + (l_2 - v_2 t)^2$$

For s to be minimum $\dfrac{ds}{dt} = 0$ or $\dfrac{d}{dt}(s^2) = 0$

$$\Rightarrow 2s\frac{ds}{dt} = 2(l_1 - v_1 t) - v_1 + 2(l_2 - v_2 t) - v_2 = 0$$

$$-l_1v_1 + v_1^2 t - l_2v_2 + v_2^2 t = 0 \Rightarrow t = \frac{l_1v_1 + l_2v_2}{v_1^2 + v_2^2}$$

$$S^2_{\min} = \left[l - \left(\frac{l_1v_1 + l_2v_2}{v_1^2+v_2^2}\right)v_1\right]^2 + \left[l_2 - v_2\left(\frac{l_1v_1 + l_2v_2}{v_1^2+v_2^2}\right)\right]^2$$

$$\Rightarrow S^2_{\min} = \frac{(l_1v_2 - l_2v_1)^2}{v_1^2+v_2^2} \Rightarrow S_{\min} = \frac{|l_1v_2 - l_2v_1|}{\sqrt{v_1^2+v_2^2}}$$ key-3

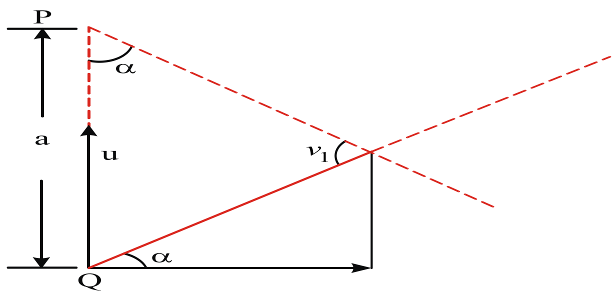
EX.46: The distance between two moving particles P and Q at any time is a. If v_r be their relative velocity and if u and v be the components of v_r, along and perpendicular to PQ.The closest distance between P and Q and time that elapses before they arrive at their nearest distance is

1) $\dfrac{a(v+v_r)}{v}, a\left(1+\dfrac{v_r}{u}\right)^2$ 2) $\dfrac{av}{(v+v_r)}, a\left(1+\dfrac{u}{v_r}\right)^2$

3) $\frac{av_r}{v}, \frac{av_r}{u^2}$ 4) $\frac{av}{v_r}, \frac{au}{v_r^2}$

Sol. . Assuming P to be at rest, particle Q is moving with velocity v_r, in the direction shown in figure. components of v_r along and perpendicular to PQ are u and v respectively, In the figure

$\sin\alpha = \frac{u}{v_r}, \cos\alpha = \frac{v}{v_r}$



The closest distance between the particles is PR.

$$S_{\min} = PR = PQ\cos\alpha = (a)\left(\frac{v}{v_r}\right) \Rightarrow S_{\min} = \frac{av}{v_r}$$

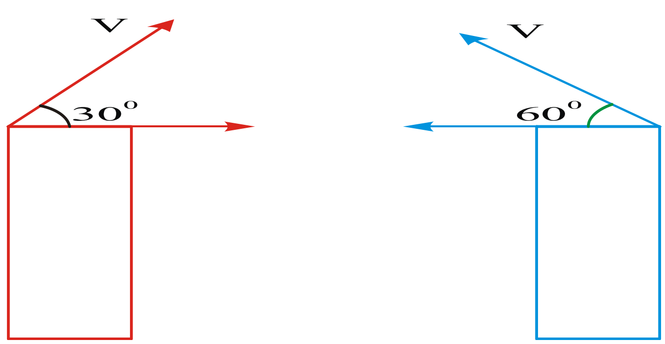
Time after which they arrive at their nearest distance is

$$t = \frac{QR}{v_r} = \frac{(PQ)\sin\alpha}{v_r} = \frac{(a)\left(\frac{v}{v_r}\right)}{v_r} = \frac{au}{v_r^2}$$ key-2

EX.47: Two stones are projected from the top of a tower in opposite direction, with the same velocity V but at 30^0 & 60^0 with horizontal respectively.The relative velocity of first stone relative to second stone is

1) 2v 2) $\sqrt{2}v$ 3) $\frac{2V}{\sqrt{3}}$ 4) $\frac{V}{\sqrt{2}}$

Sol.



$\vec{V}_1 = V\cos 30\hat{i} + V\sin 30\hat{j}$

$\vec{V}_2 = -V\cos 60\hat{i} + V\sin 60\hat{j}$ key-2

$\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

EX.48: A motor boat going down stream comes over a floating body at a point A. 60 minutes later it turned back and after some time passed the floating body at a distance of 12 km from the point A. Find the velocity of the stream assuming constant velocity for the motor boat in still water.

1) 2 Km/hr 2) 3 Km/hr 3) 4 Km/hr 4) 6 Km/hr

Sol. . $d = (v_B + v_W)t_1 -- (1)$;

$d - 12 = \left(v_B - v_w\right)t_2 --(2)$;

$12 = v_w\left(t_1 + t_2\right) ---(3)$

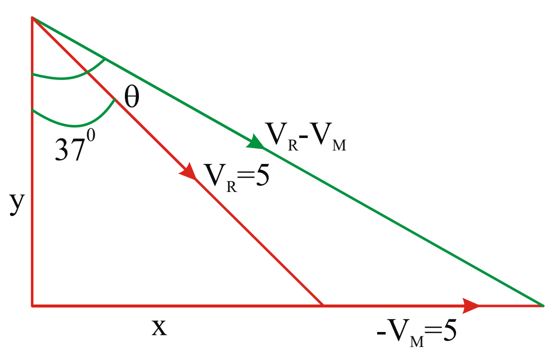
solve above equtions key-2

EX.49: It is raining at a speed of 5 m/s^{-1} at an angle 37^0 to vertical, towards east.A man is moving to west with a velocity of 5 m/s^{-1}. The angle with the vertical at which he has to hold the umbrella to protect himself from rain is.

1) $Tan^{-1}(2)$ to west 2) $Tan^{-1}(2)$ to east

3) $Tan^{-1}\left(\frac{1}{2}\right)$ to south 4) $Tan^{-1}\left(\frac{1}{2}\right)$ to east

Sol. .

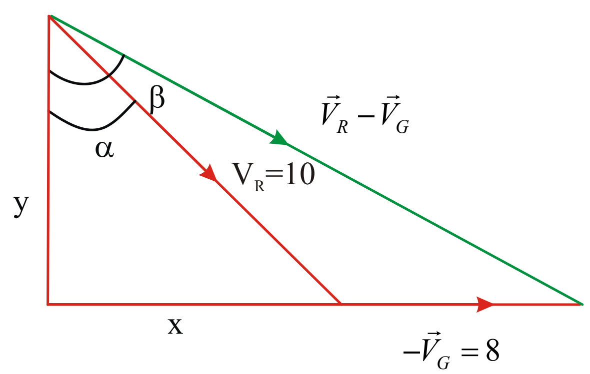


$\sin 37 = \frac{x}{V_R} = \frac{3}{5}; \cos 37 = \frac{y}{V_R}; Tan\theta = \frac{x + V_M}{y}$ key-1

EX.50: Rain, pouring down at an angle α with the vertical has a speed of $10ms^{-1}$. A girl runs against the rain with a speed of $8ms^{-1}$ and sees that the rain makes an angle β with the vertical, then relation between α and β is

1) $\tan\alpha = \frac{8 + 10\sin\beta}{10\cos\beta}$ 2) $\tan\beta = \frac{8 + 10\sin\alpha}{10\cos\alpha}$ 3) $\tan\alpha = \tan\beta$ 4) $\tan\alpha = \cot\beta$

Sol.

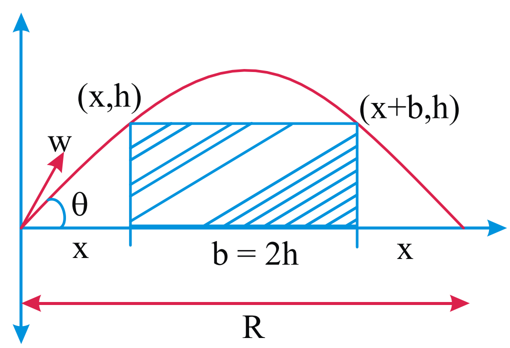


$\sin\alpha = \dfrac{x}{V_R}$; $x = V_R \sin\alpha$ $\cos\alpha = \dfrac{y}{V_R}$; $y = V_R\cos\alpha; Tan\beta = \dfrac{x+V_G}{y}$ key-2

EX.51: A particle when fired at an angle $\theta = 60^0$ along the direction of the breadth of a rectangular building of dimension $9m\times 8m\times 4m$ so as to sweep the edges. Find the range of the projectile.

1) $8\sqrt{3}$ 2) $4\sqrt{3}$ 3) $\dfrac{8}{\sqrt{3}}$ 4) $\dfrac{4}{\sqrt{3}}$

Sol.



$y = h = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$,

$R = x + x + 2h \Rightarrow x = \dfrac{R}{2} - h \ \ and \ \ R = \dfrac{u^2\sin 2\theta}{g}$

using equations 1 and 2 $h = \left(\dfrac{R}{2} - h\right)\tan\theta - \dfrac{1}{R}\left(\dfrac{R}{2} - h\right)^2 \tan\theta$

$\Rightarrow R = 2h\cot\left(\dfrac{\theta}{2}\right)$ key-1

$putting\ \theta = 60^0, h = 4m \ \ then\ R = 8\sqrt{3}m$

EX.52: **.The direction of projectile at certain instant is inclined at angle α to the horizontal after t sec.If it is inclined at an angle β then the horizontal component of velocity is**

1) $\dfrac{g}{\tan\alpha-\tan\beta}$ 2) $\dfrac{gt}{\tan\alpha-\tan\beta}$ 3) $\dfrac{t}{g(\tan\alpha-\tan\beta)}$ 4) $\dfrac{gt}{(\tan\alpha+\tan\beta)}$

Sol. . $Tan\alpha=\dfrac{V_y}{u\cos\alpha}$; $Tan\beta=\dfrac{V_y-gt}{u\cos\beta}$; $Tan\alpha-Tan\beta=\dfrac{gt}{u\cos\beta}$

$u\cos\beta=u\cos\alpha\Rightarrow u\cos\beta=\dfrac{gt}{Tan\alpha-Tan\beta}$ key-2

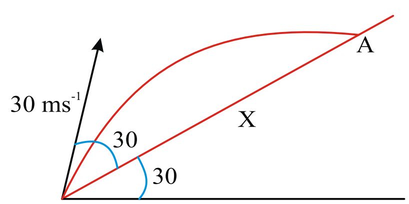
EX.53: **Two bodies are projected from the same point with same speed in the directions making an angle α_1 and α_2 with horizontal and strike at the same point in the horizontal plane through a point of projection. If t_1 and t_2 are their time of flights. Then $\dfrac{t_1^2-t_2^2}{t_1^2+t_2^2}$**

1) $\dfrac{\tan(\alpha_1-\alpha_2)}{\tan(\alpha_1+\alpha_2)}$ 2) $\dfrac{\sin(\alpha_1+\alpha_2)}{\sin(\alpha_1-\alpha_2)}$ 3) $\dfrac{\sin(\alpha_1-\alpha_2)}{\sin(\alpha_1+\alpha_2)}$ 4) $\dfrac{\sin^2(\alpha_1-\alpha_2)}{\sin^2(\alpha_1+\alpha_2)}$

Sol. . $\alpha_1+\alpha_2=90^0\Rightarrow\sin(\alpha_1+\alpha_2)=1$

$t_1=\dfrac{2u\sin\alpha_1}{g},t_2=\dfrac{2u\sin\alpha_2}{g}$ key-2

EX.54: **An object in projected up the inclined at the angle shown in the figure with an initial velocity of $30ms^{-1}$. The distance x up the incline at which the object lands is**



1) 600 m 2) 104m 3) 60 m 4) 208 m

Sol. . From figure $\alpha=30^0,\theta=60^0$ key-2

$$R=\frac{2u^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}$$

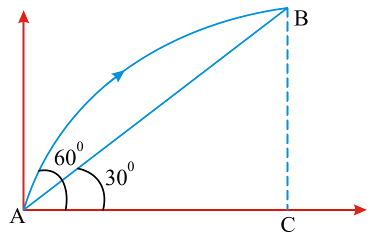
EX.55: **A projectile fired with velocity u at right angle to the slope which is inclined at an angle θ with horizontal. The expression for R is**

1. $\dfrac{2u^2}{g}\tan\theta$ 2. $\dfrac{2u^2}{g}\sec\theta$ 3. $\dfrac{u^2}{g}\tan^2\theta$ 4. $\dfrac{2u^2}{g}\tan\theta\sec\theta$

Sol. . $\alpha = \theta$ $\quad \theta^1 = 90^0 + \theta, \theta^1 - \theta = 90^0$ key-4

$$R = \frac{2u^2 \cos\theta^1 \sin\left(\theta^1 - \theta\right)}{g\cos^2\theta}$$

EX.56: In figure shown below, the time taken by the projectile to reach from A to B is t then, the distance AB is equal to



1) $\frac{ut}{\sqrt{3}}$ 2) $\frac{\sqrt{3}ut}{2}$ 3) $\sqrt{3}ut$ 4) $2ut$

Sol. . $u_x = u\cos\theta = u\cos 60^0$

$$x = AC = u_x.t;\ from\quad figure\quad \cos 30^0 = \frac{AC}{AB}$$

$$AB = \frac{AC}{\cos 30^0}$$ key-1

EX.57: Two particles are projected in air with speed υ_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then thick the right choices

1) angle of projection : $\theta_1 > \theta_2$

2) time of flight: $T_2 > T_1$

3) horizontal range: $R_1 > R_2$

4) total energy: $U_1 > U_2$

Sol. . $H_1 > H_2$

$\sin\theta_1 - \sin\theta_2$ or $\theta_1 > \theta_2$

$T_1 > T_2$

$$\frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} > 1$$

$$U_1 = KE + PE = \frac{1}{2}m_1 v_0^2$$

$$U_2 = KE + PE = \frac{1}{2}m_2 v_0^2$$

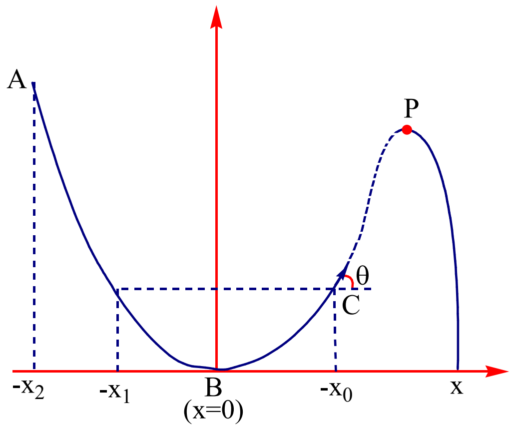
If $m_1 = m_2$ then $U_1 = U_2$

$m_1 > m_2$ then $U_1 > U_2$

$m_1 < m_2$ then $U_1 < U_2$

key-1,2,3

EX.58: A particle slides down friction less parabolic ($y = x^2$) track (A - B - C) starting from rest at point A. Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then A .



1) KE at P = KE at B
2) height at P = height at A
3) total energy at P = total energy at A
4) time of travel from A to B = time of travel from B to P

Sol. . In this type of question, nature of track is very important of consider, as friction is not in this track, total energy of the particle will remain constant throughout the journey.

$(KE)_B > (KE)_P$
$(PE)_P < (PE)_A$
As, height of p < Height of A
Hence, path length AB > path length of BP
Hence, time of travel from A to B $\neq$ Time of travel from B to P.

key-3

EX.59: Two particles having position vectors $\vec{r}_1 = (3\hat{i} + 5\hat{j})m$ and $\vec{r}_2 = (-5\hat{i} + 3\hat{j})m$ are moving with velocities $\vec{V}_1 = (4\hat{i} - 4\hat{j})ms^{-1}$ and $\vec{V}_2 = (a\hat{i} - 3\hat{j})ms^{-1}$. If they collide after 2 seconds , the value of ' a' is

1) 2　　2) 4　　3) 6　　4) 8

Sol. . $\bar{r}_1 + \bar{v}_1 t = \bar{r}_2 + \bar{v}_2 t$; 2. $4t - 5t^2 = 0$

key-3

EX.60: A particle starts from origin at t = 0 with a constant velocity $5\hat{i}\, ms^{-1}$ and moves in xy plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})ms^{-2}$. The y - coordinate of the particle at the instant its x co-ordinate is 84 m in m is

1) 6　　2) 36　　3) 18　　4) 9

Sol. . $\vec{r} = ut + \frac{1}{2}at^2$

equate x coordinate to 84 to find time t

key-2

JEE MAIN PREVIOUS YEAR QUESTIONS

MOTION IN A PLANE

1. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$ .The angle between $\vec{A}$ and $\vec{B}$ is: [10 Jan. 2019 II]

(a) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (c) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

Sol. (a) Let magnitude of two vectors $\vec{A}^{\text{and}}\vec{B} = a$

$|\vec{A}+\vec{B}| = \sqrt{a^2+a^2+2a^2\cos\theta}$ and $|\vec{A}-\vec{B}| = \sqrt{a^2+a^2-2a^2[\cos(180^\circ-\theta)]}$

$$= \sqrt{a^2+a^2-2a^2\cos\theta}$$

and according to question,

$$|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$$

or, $\frac{a^2+a^2+2a^2\cos\theta}{a^2+a^2-2a^2\cos\theta} = n^2$

$$\Rightarrow \frac{\mathcal{H}'(1+1+2\cos\theta)}{\swarrow(1+1-2\cos\theta)}n^2 \Rightarrow \frac{(1+\cos\theta)}{(1-\cos\theta)} = n^2$$

using componendo and dividendo theorem, we get

$$\theta = \cos^1\left(\frac{n^2-1}{n^2+1}\right)$$

2 Let $\vec{A} = (\hat{\imath}+\hat{\jmath})$ and $\vec{B} = (\hat{\imath}-\hat{\jmath})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by [Online April 16, 2018]

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$ (c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

Sol. (a) If $\vec{C} = a\hat{\imath}+b\hat{\jmath}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a+b=1$ (i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a-b=1$ (ii)

Solving equation (i) and(ii) we get

$a = \frac{1}{3}, \ b = \frac{2}{3}$

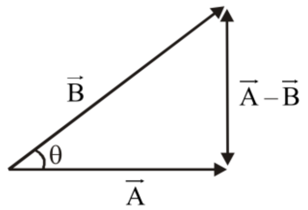
Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

3. A vector $\overline{A}$ is rotated by a small angle $\Delta\theta$ radian $(\Delta\theta << 1)$ to get a new vector $\overline{B}$. In that case $|\overline{B} - \overline{A}|$ is : [Online April 11, 2015]

(a) $|\overline{A}|\Delta\theta$ (b) $|\overline{B}|\Delta\theta - |\overline{A}|$ (c) $|\overline{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$ (d) 0

Sol (a) Arc length $=$ radius $\times$ angle

So, $|\vec{B} - \vec{A}| = |\overline{A}|\Delta\theta$



4. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is [2004]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

Sol. (c) $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

$\vec{A} \times \vec{B} = 0$

Angle between them is $0, \ \pi$, or $2 \ \pi$

from the given options, $\theta = \pi$

TOPIC-1 ….Vectors

5. Two forces P and Q, ofmagnitude 2Γ and 3Γ, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: [10 Jan. 2019 II]

(a) 120° (b) 60° (c) 90° (d) 30°

sol. (a) Using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$4\ \Gamma^2 + 9\Gamma^2 + 12\Gamma^2\cos\theta = R^2$$

When forces Q is doubled,

$$4\ \Gamma^2 + 36\Gamma^2 + 24\Gamma^2\cos\theta = 4R^2$$

$$4\ \Gamma^2 + 36\Gamma^2 + 24\Gamma^2\cos\theta$$

$$= 4(13\Gamma^2 + 12\Gamma^2\cos\theta) = 52\Gamma^2 + 48\Gamma^2\cos\theta$$

$$\cos\theta = -\frac{12\Gamma^2}{24\Gamma^2} = -\frac{1}{2} \Rightarrow \theta = 120^o$$

6. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The ma$\mathcal{X}$i$^{\text{i}}$mde of $(\overline{A} + \overline{B})$ is 'n' times the *magnitude* of $(\overline{A} - \overline{B})$. The angle between $\vec{A}$ and $\vec{B}$ is:

[10 Jan. 2019 II]

(a) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (c) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

sol. (a) Let magnitude of two vectors $\vec{A}^{\text{and}}\vec{B} = a$

$|\vec{A} + \vec{B}| = \sqrt{a^2 + a^2 + 2a^2\cos\theta}$ and

$$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2[\cos(180^\circ - \theta)]}$$

$$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$$

and according to question,

$$|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$$

or, $\frac{a^2+a^2+2a^2\cos\theta}{a^2+a^2-2a^2\cos\theta} = n^2$

$$\Rightarrow \frac{(1 + 1 + 2\cos\theta)}{(1 + 1 - 2\cos\theta)}n^2 \Rightarrow \frac{(1 + \cos\theta)}{(1 - \cos\theta)} = n^2$$

using componendo and dividendo theorem, we get

$$\theta = \cos^1\left(\frac{n^2 - 1}{n^2 + 1}\right)$$

7. Let $\vec{A} = (\hat{\imath} + \hat{\jmath})$ and $\vec{B} = (\hat{\imath} - \hat{\jmath})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} =$

$\vec{A}.\vec{B}$ is given by

[Online April 16, 2018]

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$ (c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

sol. (a) If $\vec{C} = a\hat{i}+b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ (i)

$$\vec{B}.\vec{C} = \vec{A}.\vec{B}$$

$2a - b = 1$ (ii)

Solving equation (i) and(ii) we get

$$a = \frac{1}{3}, \ b = \frac{2}{3}$$

Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$

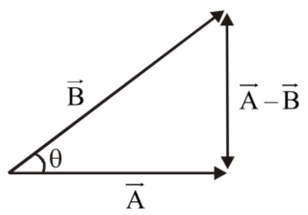
8. A vector $\overline{A}$ is rotated by a small angle $\Delta\theta$ radian $(\Delta\theta << 1)$ to get a new vector $\overline{B}$. In that case $|\overline{B} - \overline{A}|$ is :

[Online April 11, 2015]

(a) $|\overline{A}|\Delta\theta$ (b) $|\overline{B}|\Delta\theta - |\overline{A}|$ (c) $|\overline{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$ (d) 0

sol. (a) Arc length $=$ radius $\times$ angle

So, $|\vec{B} - \vec{A}| = |\overline{A}|\Delta\theta$



9. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is

[2004]

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

sol. (c) $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

$$\vec{A} \times \vec{B} = 0$$

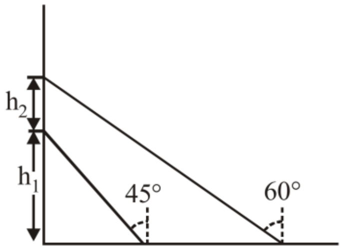
Angle between them is $0, \ \pi$, or $2 \ \pi$

from the given options, $\theta = \pi$

TOPIC-2 ....Motion in a plane with constant acceleration

10. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45^o with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60^o with respect to the vertical ifthe girl moves further bya distance $2.464d$∞intC). Then the height h_2 is (given $\tan 30^o = 0.5774$):

[Sep. 05, 2020 (I)]

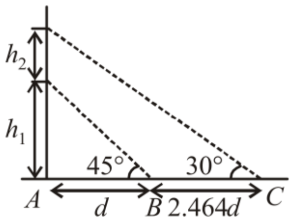


A ← d → B + 2.464d ∗ C

(a) $1.464d$ (b) $0.732d$ (c) $0.464d$ (d) d

sol. (d) From figure/ trigonometry

$$\frac{h_1}{d} = \tan 45^\circ \quad h_1 = d$$



And, $\frac{h_1 + h_2}{d + 2.464d} = \tan 30^\circ$

$$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46d$$

$$\Rightarrow (h_1 + h_2) = \frac{3.46d}{\sqrt{3}} \Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$$

$$h_2 = d$$

11. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}\text{ms}^{-1}$, a particle moves in the $x - y$ plane with a constant acceleration of (10î+4j) ms^{-2}. At time t, its coordiantes are $(20\text{m}, y_0\text{m})$. The values oft and y_0 are, respectively:

[Sep. 04, 2020 (I)]

(a) 2 s and 18 m (b) 4 s and52 m (c) 2 sand24m (d) 5 s and25 m

sol. (a) Given: $\vec{u} = 5\hat{j}nvs$

Acceleration, $\vec{a} = 10\hat{\imath} + 4\hat{j}$ and

final coordinate $(20,\ y_0)$ in time t.

$S_x = u_x t + \frac{1}{2} a_x t^2$ $[\because u_x = 0]$

$$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2\text{s}$$

$$S_y = u_y \times t + \frac{1}{2} a_y t^2$$

$$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18\text{m}$$

12. The position vector of a particle changes with time according to the relation $\vec{r}(\text{t}) = 15\text{t}^2\hat{\imath} + (4 - 20\text{t}^2)\hat{\jmath}$. What is the magnitude ofthe acceleration at $t = 1$?

[9 April 2019 II]

(a) 40 (b) 25 (c) 10 (d) 50

sol. (d) $\vec{r} = 15t^2\hat{\imath} + (4 - 20t^2)\hat{\jmath}$

$v = \frac{dr\rightarrow}{dt}$ =30tî-40tj

Acceleration, $\rightarrow a = \frac{dv\rightarrow}{dt}$ =30î-40j

$$a = \sqrt{30^2 + 40^2} = 50\text{m/s}^2$$

13. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$ m, at $t = 0$, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})ms^{-2}$. What is the distance of the particle from the origin at time 2s?

[11 Jan. 2019 II]

(a) 15m (b) $20\sqrt{2}$m (c) 5m (d) $10\sqrt{2}$m

sol. (b) As $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i}+4\hat{j})4$

$= 10\hat{i}+8\hat{j}+8\hat{i}+8\hat{j}$

$$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$$

[as $\vec{s} =$ change in position $= \vec{r}_f - \vec{r}_i$]

$$\vec{r}_r = 20\hat{i} + 20\hat{j}$$

$$|\vec{r}_r| = 20\sqrt{2}$$

14. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

[9 Jan. 2019 I]

(a) $y = x^2 +$ constant (b) $y^2 = x +$ constant

(c) $y^2 = x^2 +$ constant (d) $xy =$ constant

sol. (c) From given equation,

$\vec{V} = K(y\hat{i}+x\hat{j})$

$\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

Now $\frac{dy}{dt} / \frac{dx}{dt} = \frac{x}{y} = \frac{dy}{dx}, \Rightarrow ydy = xdx$

Integrating both side

$$y^2 = x^2 + c$$

TOPIC-3 …Projectile Motion

15. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the *x*-*y* plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s^2. The *x* coordinate of the particle at the instant when its *y* coordinate is 32 m is D meters. The value of D is:

[9 Jan. 2020 II]

(a) 32 (b) 50 (c) 60 (d) 40

sol. (c) Using $S = ut + \frac{1}{2}at^2$

$y = u_y t + \frac{1}{2}a_y t^2$ (along y Axis)

$$\Rightarrow 32 = 0 \times t + \frac{1}{2}(4)t^2$$

$$\Rightarrow \frac{1}{2} \times 4 \times t^2 = 32$$

$$\Rightarrow t = 4s$$

$S_x = u_x t + \frac{1}{2}a_x t^2$ (Along x Axis)

$$\Rightarrow x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$$

16. A particle is moving along the x-*ax*is with its coordinate with time t' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v(in m/s) is_

[NA 8 Jan. 2020 I]

sol. (580)

For pariticle '*A*'

$X_A = -3t^2 + 8t + 10$

For particle '*B*'

$Y_B = 5 - 8t^3$

$\vec{V}_A = (8 - 6t)\hat{\imath}$ $\vec{V}_B = -24t^2 j$

$\vec{a}_A$ =- 6î $\vec{a}_B = -48t\hat{\jmath}$

At $t = 1$ sec

$$\vec{V}_A = (8 - 6t)\hat{\imath} = 2\hat{\imath} \text{ and } \vec{v}_B = -24j$$

$$\vec{V}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{\imath}-24j$$

Speed of B w.r.t. A, $\sqrt{v} = \sqrt{2^2 + 24^2}$

$$= \sqrt{4 + 576} = \sqrt{580}$$

$$v = 580(\text{m/s})$$

17. A particle moves such that its position vector $\vec{r}(t) = \cos \omega t\, \hat{\imath} + \sin \omega t\, j$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}$ (t) and acceleration $\vec{a}(t)$ ofthe particle:

[8 Jan. 2020 II]

(a) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

(b) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(c) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

sol. (d) Given, Position vector,

$$\vec{r} = \cos wt\hat{\imath} + \sin wtj$$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \text{w}\ (-\sin \text{w}t\hat{\imath} + \cos \text{w}t\hat{\jmath})$

Acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = -\text{w}^2(\cos \text{w}t\hat{\imath} + \sin \text{w}t\hat{\jmath})$$

$$\vec{a} = -\text{w}^2\vec{r}$$

$\vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0$ $\vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

18. A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is

[2010]

(a) $y = x^2 +$ constant (b) $y^2 = x +$ constant

(c) $xy =$ constant (d) $f = x^2 +$ constant

sol. (d) $v = k(yi + xj)$

$$v = kyi + kxj$$

$$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{kx}{ky}$$

$ydy = xdx$ (i) Integrating equation (i)

$$\int y\,dy = \int x \cdot dx$$

$y^2 = x^2 + c$

19. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is:

[2009]

(a) $7\sqrt{2}$ units (b) 7 units (c) 8.5 units (d) 10units

sol. (a) Given $\vec{u} = 3\hat{\imath} + 4j, \vec{a} = 0.4\hat{\imath} + 0.3j, t = 10$s

From 1st equation of motion.

$$a = \frac{v - u}{t}$$

$v = at + u$

$\Rightarrow v = (0.4\hat{\imath} + 0.3j) \times 10 + (3\hat{\imath} + 4j) \Rightarrow 4\hat{\imath} + 3j + 3j + 4j$

$\Rightarrow v = 7\hat{\imath} + 7j$

$\Rightarrow |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2}$ unit.

20. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed ofthe particle at time t' is given by

[2003]

(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$ (c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

sol. (b) Coordinates of moving particle at time t' are

$x = \alpha t^3$ and $y = \beta t^3$

$v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

21. A particle of mass m is projected with a speed $u$fi¡om the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely in elastically with another particle of the same mass and velocity $u\ \hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

[9 Jan. 2020 II]

(a) $\frac{3\sqrt{3}u^2}{8g}$ (b) $\frac{3\sqrt{2}u^2}{4g}$ (c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

sol. (a) Using principal of conservation of linear momentum for horizontal motion, we have

$$2mv_x = mu + mu\cos 60^\circ$$

$$v_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Let R is the horizontal distance travelled by the body.

$R = v_x T + \frac{1}{2}(0)(T)^2$ (For horizontalmotion)

$$R = v_x T = \frac{3u}{4} \times \sqrt{\frac{2h}{g}}$$

$$\Rightarrow R = \frac{3\sqrt{3}u^2}{8g}$$

22. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. Ifit were launched at an angle θ_0 with speed v_0 then ($g = 10$ ms 2):

[12 April 2019 I]

(a) $\theta_0 = \sin^{1\frac{1}{\sqrt{5}}}$ and $v_0 = \frac{5}{3}$ m/s

(b) $\theta_0 = \cos^1\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m/s

(c) $\theta_0 = \cos^1\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{9}{3}$ m/s

(d) $\theta_0 = \sin^1\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m/s

sol. (c) Given, $y = 2x - 9x^2$

On comparing with,

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have,

$\tan\theta = 2$ or $\cos\theta = \frac{1}{\sqrt{5}}$

and $\frac{g}{2u^2\cos^2\theta} = 9$ or $\frac{10}{2u^2(1/\sqrt{5})^2} = 9$

$u = 5/3\text{m/s}$

23. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is :

[12 Apri12019 I]

(a) R/4g (b) R/g (c) R/2g (d) 2R/g

sol. (d) R will be same for θ and $90° - \theta$.

Time of flights:

$t_1 = \frac{2u \sin\theta}{g}$ and

$$t_2 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u \cos\theta}{g}$$

Now, $t_1 t_2 = \left(\frac{2u \sin\theta}{g}\right)\left(\frac{2u \cos\theta}{g}\right)$

$$= \frac{2}{g}\frac{u^2 \sin 2\theta}{g} = \frac{2R}{g}$$

24. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

[12 April 2019 II]

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$ (c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

sol. (b) For same range, the angle of projections are:

θ and $90^\circ - \theta$. So,

$h_1 = \frac{u^2 \sin^2\theta}{2g}$ and $h_2 = \frac{u^2 \sin^2(90^o - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$

Also, $R = \frac{u^2 \sin 2\theta}{g}$

$$h_1 h_2 = \frac{u^2 \sin^2\theta}{2g} \times \frac{u^2 \cos^2\theta}{2g}$$

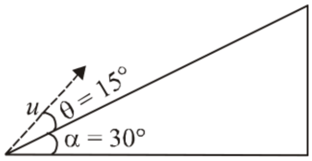
$$= \frac{u^2 u^2 (2\sin\theta\cos\theta)^2}{16g^2}$$

$$= \frac{R^2}{16}$$

or $R^2 = 16h_1h_2$

25. A plane is inclined at an angle $\alpha = 30^o$ with respect to the horizontal. A particle is projected with a speed $u = 2$ ms 1, fi$_{\text{i}}$om the base ofthe plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10$ m/s 2)

[10 April 2019 II]



(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

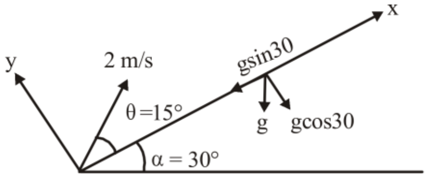
sol. (a) On an inclined plane, time of flight (T) is given by

$$T = \frac{2u \sin\theta}{g \cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2 \sin 15^\circ)}{g \cos 30} \circ = \frac{4 \sin 15^\circ}{10 \cos 30^\circ}$$

Distance, $S = (2 \cos 15^\circ)T - \frac{1}{2} g \sin 30^\circ (T)^2$



$$= (2 \cos 15^\circ)\frac{4 \sin 15^o}{1010 \cos 30^o} - \left(\frac{1}{2} \times 10 \sin 30^\circ\right) \frac{16 \sin^2 15^o}{100 \cos^2 30^o}$$

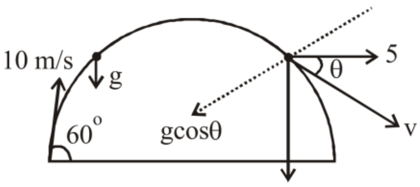
$$= \frac{16\sqrt{3} - 16}{60} = 0.1952\text{m} = 20\text{cm}$$

26. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius ofcurvature of its trajectory at $t = 1s$ is R. Neglecting air resistance and taking acceleration due to gravity $g = 10 ms^{-2}$, the value of R is:

[11 Jan. 2019 I]

(a) 10.3 m (b) 2.8 m (c) 2.5m (d) 5.1m

sol. (b)



Horizontal component of velocity $v_x = 10 \cos 60^\circ = 5 m/s$

vertical component of velocity $v_y = 10 \cos 30^\circ = 5\sqrt{3}\text{m/s}$

After $t = 1$ sec.

Horizontal component of velocity $v_x = 5\text{m/s}$

Vertical component of velocity

$$v_y = |(5\sqrt{3} - 10)|\text{m/s} = 10 - 5\sqrt{3}$$

Centripetal, acceleration a $= \frac{V^2}{R}$

$$\Rightarrow R = \frac{v_x^2 + v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10 \cos \theta} \text{ (i)}$$

From figure (using (i))

$$\tan \theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} \Rightarrow \theta = 15^\circ$$

$$R = \frac{100(2 - \sqrt{3})}{10 \cos 15} = 2.8\text{m}$$

27. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is:

[10 Jan. 2019 I]

(a) 1:16 (b) 1:2 (c) 1:4 (d) 1:8

sol. (a) As we know, range $R = \frac{u^2 \sin 2\theta}{g}$

and, area $A = \pi R^2$

$A \propto R^2$ or, $A \propto u^4$

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

28. The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the center of a target 700 m away at the same level as the target. How far above the centre ofthe target?

[Online April, 2014]

(a) 1.0m (b) 4.2m (c) 6.1m (d) 9.8m

sol. (c) Let t' be the time taken by the bullet to hit the target.

$$700 \text{ m} = 630\text{ms}^{-1}t$$

$$\Rightarrow t = \frac{700\text{m}}{630\text{ms}^{-1}} = \frac{10}{9} \text{ sec}$$

For vertical motion,

Here, $u = 0$

$$h = \frac{1}{2}gt^2$$

$$= \frac{1}{2} \times 10 \times \left(\frac{10}{9}\right)^2$$

$$= \frac{500}{81}\text{m} = 6.1\text{m}$$

Therefore, the rifle must be aimed 6.1m above the centre of the target to hit the target.

29. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = (40\hat{\imath} + 50\hat{\jmath})\text{m}$ at $t = 2\text{s}$. If the projectile was launched at an angle θ from the horizontal, then θ is (take $g = 10\text{ms}^{-2}$) [Online April 9, 2014]

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$ (c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

sol. (c) From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$$\Rightarrow u_y \times 2 + \frac{1}{2}(-10) \times 4$$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35\text{m/s}$

$$\tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

$\Rightarrow$ Angle $\theta = \tan^{-1}\frac{7}{4}$

30. A projectile is given an initial velocity of $(\hat{\imath}+2\hat{j})$ m/s, where $\hat{\imath}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10 m/s^2$, the equation of its trajectory is:

[2013]

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$ (c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

sol. (b) From equation, $\vec{v} = \hat{\imath} + 2\hat{j}$

$$\Rightarrow x = t \quad \text{(i)}$$

$y = 2t - \frac{1}{2}(10t^2)$ ... (ii)

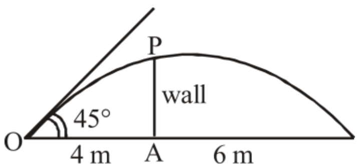
From(i) and(ii), $y = 2x - 5x^2$

31. The maximum range of a bullet fired fi;om a toy pistol mounted on a car at rest is $R_0 = 40m$. What will be the acute angle ofinclination ofthe pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20 m/s$, on a horizontal surface?

$(g = 10 m/s^2)$ [Online April 25, 2013]

(a) 30° (b) 60° (c) 75° (d) 45°

sol. (b)



32. A ball projected from ground at an angle of 45° just clears a wall in fiiont. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is:

[Online April 22, 2013]

(a) 4.4m (b) 2.4m (c) 3.6m (d) 1.6m

sol. (b)

As ball is projected at an angle 45° to the horizontal

therefore Range $= 4\mathrm{H}$

or $10 = 4\mathrm{H} \Rightarrow \mathrm{H} = \frac{10}{4} = 2.5\mathrm{m}$

(Range $= 4\mathrm{m} + 6\mathrm{m} = 10\mathrm{m}$)

Maximum height, $\mathrm{H} = \frac{\mathrm{u}^2\sin^2\theta}{2\mathrm{g}}$

$$\mathrm{u}^2 = \frac{\mathrm{H} \times 2\mathrm{g}}{\sin^2\theta} = \frac{2.5 \times 2 \times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

or, $\mathrm{u} = \sqrt{100} = 10\mathrm{ms}^{-1}$

Height of wall PA

$$= \mathrm{OA}\tan\theta - \frac{1}{2}\frac{\mathrm{g}(\mathrm{OA})^2}{\mathrm{u}^2\cos^2\theta}$$

$$= 4 - \frac{1}{2} \times \frac{10 \times 16}{10 \times 10 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 2.4\mathrm{m}$$

33. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be

[2012]

(a) $20\sqrt{2}$m (b) 10 m (c) $10\sqrt{2}$m (d) 20m

sol. (d) $R = \frac{u^2\sin^2\theta}{g}$, $H = \frac{u^2\sin^2\theta}{2g}$

$H_{\max}$ at $2\theta = 90^\circ$

$$H_{\max} = \frac{u^2}{2g}$$

$$\frac{u^2}{2g} = 10 \Rightarrow u^2 = 10g \times 2$$

$$R = \frac{u^2\sin 2\theta}{g} \Rightarrow R_{\max} = \frac{u^2}{g}$$

$R_{\max} = 20$ metre

34. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is:

[2011]

(a) $\pi \frac{v^4}{g^2}$ (b) $\frac{\pi}{2} \frac{v^4}{g^2}$ (c) $\pi \frac{v^2}{g^2}$ (d) $\pi \frac{v^2}{g}$

sol. (a) Let, total area around fountain

$A = \pi R^2_{\max}$ (i)

Where $R_{\max} = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}$ (ii)

From equation (i) and(ii)

$$A = \pi \frac{v^4}{g^2}$$

35. A projectile can have the same range 'R' for two angles of projection. If T_1 ' and T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to.

[2004]

(a) R (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) R^2

sol. (a) A projectile have same range for two angle

Let one angle be θ, then other is $90° - \theta$

$$T_1 = \frac{2u \sin \theta}{g}, T_2 = \frac{2u \cos \theta}{g}$$

then, $T_1 T_2 = \frac{4u^2 \sin \theta \cos \theta}{g} = 2R$

$$\left(R = \frac{u^2 \sin^2 \theta}{g}\right)$$

Thus, it is proportional to R. (Range)

36. A ball is thrown from a point with a speed v_0 at an elevation angle of θ. From the same point

and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ?

[2004]

(a) No (b) Yes, 30° (c) Yes, 60° (d) Yes, 45°

sol. (c) Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$$\frac{v_o}{2} = v_o \cos \theta$$

or $\theta = 60^{\circ}$

37. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30^{o} with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

$\left[g = 10 \text{m/s}^2,\ \sin 30^{o} = \frac{1}{2},\ \cos 30^{o} = \frac{\sqrt{3}}{2}\right]$ [2003]

(a) 5.20m (b) 4.33m (c) 2.60m (d) 8.66m

sol. (d) Horizontal range is required

$$R = \frac{(10)^2 \sin(2 \times 30^{o})}{10} = 5\sqrt{3} = 8.66 \text{m}$$

TOPIC-4 .Relative Velocity in Two Dimensions & Uniform Circular Motion

38. A clock has a continuously moving second's hand of 0.1m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:

[Sep. 06, 2020 (I)]

(a) 10^{-3} (b) 10^{-4} (c) 10^{-2} (d) 10^{-1}

sol. (a) Here, $R = 0.1 \text{m}$

$w = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \, rad/s$

Accelaration of the tip of the clock seconds hand,

$a = w^2 R = (0.105)^2 (0.1) = 0.0011 = 1.1 \times 10^{-3}$

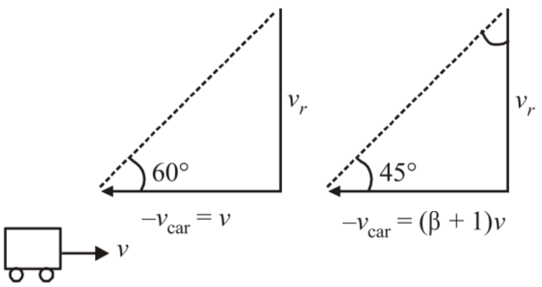
Hence, average accelaration is of the order of 10^{-3}

39. When a car is at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60° from the horizontal. On furter increasing the speed of the car to $(1+\beta)$v, this angle changes to 45°. The value of β is close to:

[Sep. 06, 2020 (II)]

(a) 0.50 (b) 0.41 (c) 0.37 (d) 0.73

sol. (d) The given situation is shown in the diagram. Here v_r be the velocity of rain drop.



When car is moving with speed v,

$\tan 60^\circ = \frac{v_r}{v}$ (i)

When car is moving with speed $(1+\beta)v$, $\tan 45^\circ = \frac{v_r}{(\beta+1)v}$ (ii)

Dividing (i) by(ii) we get,

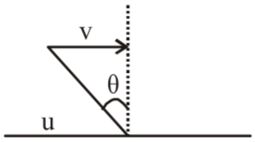
$$\sqrt{3}v = (\beta+1)v \Rightarrow \beta = \sqrt{3} - 1 = 0.732.$$

40. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

[9 April 2019 I]

(a) 90° (b) 150° (c) 120° (d) 60°

sol. (c) $\sin\theta = \frac{u}{v} = \frac{2}{4} = \frac{1}{2}$ or $\theta = 30°$



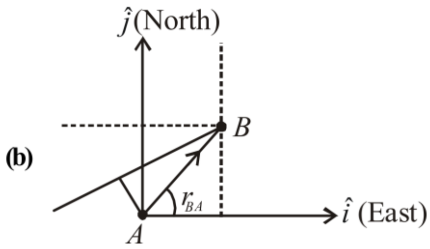
with respect to flow, $= 90° + 30° = 120°$

41. Ship A is sailing towards north-east with velocity km/hr where points east and , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

[8 April 2019 I]

(a) 4.2hrs. (b) 2.6hrs. (c) 3.2hrs. (d) 2.2hrs.

sol.



$\vec{v}_A = 30\hat{\imath} + 50\hat{j}$ km/ hr

$\vec{v}_B = (-10\hat{\imath})$km/hr

r_{BA} =(80î+150j) km

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A =$ −10î-30î-50î=40î-50j

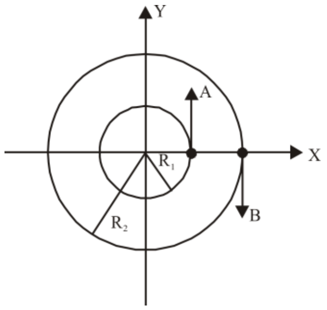
$$t_{\text{mmimum}} = \frac{|(\vec{r}_{BA})\cdot(\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2}$$

$$= \frac{|(80\hat{\imath}+150\hat{\jmath})(-40\hat{\imath}-50\hat{\jmath})|}{\left(10\sqrt{41}\right)^2}$$

$$t = \frac{10700}{10\sqrt{41}\times 10\sqrt{41}} = \frac{107}{41} = 2.6\text{hrs.}$$

42. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed w. At $t=0$, their positions and direction of motion are shown in the figure:

[12 Jan. 2019 II]

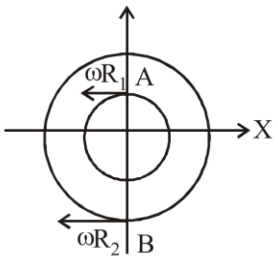


The relative velocity $v_{\vec{A}}-v_{\vec{B}}$ and $t=\frac{\pi}{2w}$ is given by:

(a) $w(R_1+R_2)\ \hat{\imath}$ (b) $-w(R_1+R_2)\ \hat{\imath}$ (c) $w(R_2-R_1)\ \hat{\imath}$ (d) $w(R_1-R_2)\ \hat{\imath}$

sol. (c) From, $\theta=\mathrm{wt}=\frac{\pi}{2}$

So, both have completed quarter circle



Relative velocity,

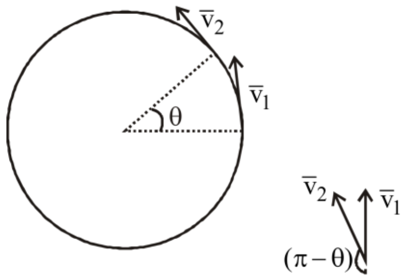
$v_A-v_B=wR_1(-\hat{\imath})-wR_2(-i)=w(R_2-R_1)i$

43. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity ofthe particle, when it moves through an angle of 60° around the centre of the circle?

[Online Apri110, 2015]

(a) $10\sqrt{3}m/s$ (b) zero (c) $10\sqrt{2}m/s$ (d) 10 m/s

sol. (d)



Change in velocity, $|\Delta\overline{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos(\pi - \theta)}$

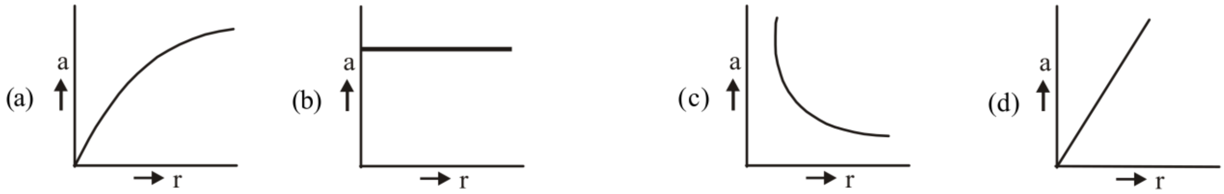
$$= 2v \sin\frac{\theta}{2} \; (\because |\vec{v}_1| = |\vec{v}_2|) = v$$

$$= (2 \times 10) \times \sin(30^\circ) = 2 \times 10 \times \frac{1}{2}$$

$$= 10 \text{m/s}$$

44. If a body moving in circular path maintains constant speed of 10 ms^{-1} , then which of the following correctly describes relation between acceleration and radius?

[Online Apri110, 2015]



sol. (c) Speed, $V =$ constant (from question)

Centripetal acceleration, $a = \frac{v^2}{r}$

$ra =$ constant

Hence graph (c) correctly describes relation between acceleration and radius.

NEWTON'S LAWS OF MOTION

Inertia :

- It is the inability of a body to change its state of rest or of uniform motion or its direction by itself.
- Mass is a measure of inertia in translatory motion
- Heavier the mass, larger the inertia & vice–versa.

Types of inertia: There are three types of inertia. (i) Inertia of rest (ii) Inertia of motion and (iii)Inertia of direction.

- **Inertia of rest:** It is the inability of a body to change its state of rest by itself .

Ex: When a bus is at rest and starts suddenly moving forward the passengers inside it will fall back.

Inertia of motion: It is the inability of a body to change its state of uniform motion by itself.

Ex: Passengers in a moving bus fall forward, when brakes are applied suddenly.

Inertia of direction: It is the inability of a body to change its direction of motion by itself .

Ex: When a bus takes a turn, passengers in it experience an outward force.

- A person sitting in a moving train, throws a coin vertically upwards, then

i) it falls behind him, if the train is accelerating
ii) it falls infront of him, if the train is retarding
iii) it falls into the hand of the person, if the train is moving with uniform velocity.
iv) It falls into the hand of the person if the train is at rest

Newton's First Law (law of Inertia)

- Every body continues to be in its state of rest (or) uniform motion in a straight line unless it is acted upon by a net external force to change its state
- It defines inertia,force and mechanical equilibrium.
- If the net external force on an object is zero , then acceleration of object is zero.

Linear momentum :

- Linear momentum is the product of the mass of a body and its velocity. $\vec{p} = m\vec{v}$
- Linear momentum is a vector.It has the same direction as the direction of velocity of the body. SI unit: kg m s^{-1} , CGS unit: g cm s^{-1}
- D.F: MLT^{-1}

Change in momentum of a body in different cases

- Consider a body of mass m moving with velocity $\vec{v_i}$ and momentum $\vec{P_i}$. Due to a collision (or) due to the action of a force on it suppose its velocity changes to $\vec{v_f}$ and momentum changes to $\vec{P_f}$ in a small time interval Δt.

Change in momentum of body = $\Delta\vec{P} = \vec{P_f} - \vec{P_i}$

Where P_i = initial momentum

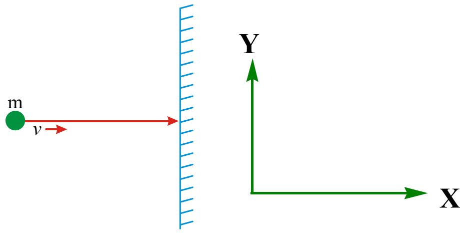
P_f = final momentum

$$\Delta\vec{P} = m\vec{v_f} - m\vec{v_i}$$

$$\left|\Delta\vec{P}\right| = \left|\vec{P_f} - \vec{P_i}\right| = \sqrt{P_f^2 + P_i^2 - 2P_fP_i\cos\theta}$$ where θ = angle between $\vec{P_f}$ and $\vec{P_i}$

Consider a body of mass ‘m’ moving with velocity ‘$\vec{v}$’along a straight line

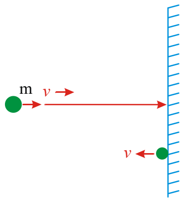
↳ **Case (i) :** If it hits a wall and comes to rest, Change in momentum of the body



$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0 - (m\vec{v})\hat{i}$

$= -m\vec{v}\hat{i}$; $|\Delta\vec{P}| = mv$, along the normal and away from the wall.

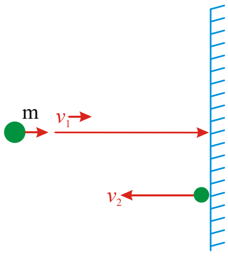
↳ **Case(ii) :** If the body rebounds with same speed ‘v’ then $\theta = 180^0$



$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \left[-(m\vec{v})\hat{i}\right] - \left[(m\vec{v})\hat{i}\right] = -(2m\vec{v})\hat{i}$

$\therefore$ $|\Delta\vec{P}| = 2mv$, along the normal and away from the wall.

↳ **Case (iii) :** If the body hits a rigid wall normally with speed v_1 and rebounds with speed v_2 then $\theta = 180^0$,



$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \left[-(m\vec{v}_2)\hat{i}\right] - \left[(m\vec{v}_1)\hat{i}\right]$,

$|\Delta\vec{P}| = m(v_2 + v_1)$, along the normal and away from the wall.

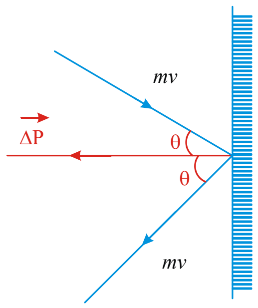
↳ **Case (iv) :** A body of mass ‘m’ moving with speed ‘v’ hits a rigid wall at an angle of incidence θ and rebounds with same speed ‘v’

$\Delta\vec{P}$ is along the normal,away from the wall

$\Delta\vec{P}_x = -mv\cos\theta\ i - mv\cos\theta\ i$ $\quad \Delta\vec{P}_y = mv\sin\theta\ j - mv\sin\theta\ j$

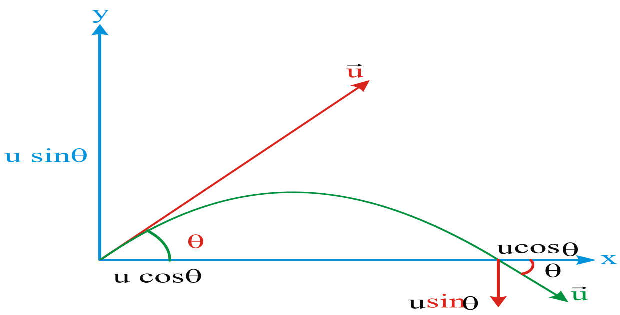
$\Delta\vec{P} = \Delta\vec{P}_x + \Delta\vec{P}_y = 2mv\cos\theta\ (-i)$

$|\Delta\vec{P}| = 2mv\cos\theta$



↳ **Case(v) :** In the above case if θ is the angle made with wall then $|\Delta \vec{P}| = 2mv \sin\theta$, along the normal and away from the wall.

↳ **Case(vi) : Projectile motion :**



a) In case of projectile motion the change in momentum of a body between highest point and point of projection is

$\vec{P}_i = (mu\cos\theta)\hat{i} + (mu\sin\theta)\hat{j}$

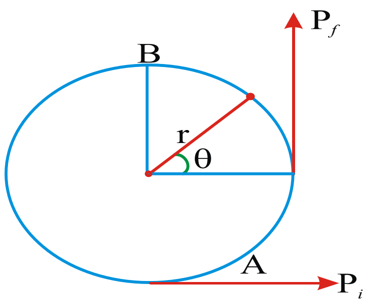
$\vec{P}_f = (mu\cos\theta)\hat{i} + 0$, $\quad \overrightarrow{\Delta P} = -(mu\sin\theta)\hat{j}$

b) The change in momentum of the projectile between the striking point and point of projection is

$\vec{P}_i = (mu\cos\theta)\hat{i} + (mu\sin\theta)\hat{j}$

$\vec{P}_f = (mu\cos\theta)\hat{i} - (mu\sin\theta)\hat{j} \quad \overrightarrow{\Delta P} = -(2mu\sin\theta)\hat{j}$

↳ A particle of mass 'm' is moving uniformly with a speed 'v' along a circular path of radius 'r'. As it moves from a point A to another point B, such that the arc AB subtends an angle θ at the centre, then the magnitude of change in momentum is $2mv\sin(\theta/2)$ and is directed towards the centre of the circle.



Newton's second law:

- The rate of change of momentum of a body is directly proportional to the resultant (or) net external force acting on the body and takes place along the direction of force.
- $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ (or) $\vec{F}_{net} = \frac{d(m\vec{v})}{dt}$
- In a system if only velocity changes and mass remain constant , $\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$
- In a system, if only mass changes and velocity remians constant $\vec{F}_{net} = \vec{v}\frac{dm}{dt}$
- Force is a vector and the acceleration produced in the body is in the direction of net force,
- SI unit : newton (N). CGS unit :dyne.
- One newton = 10^5 dyne.
- D.F=MLT^{-2}

  Gravitational units of force: Kilogram weight (kg wt) and gram weight (g wt); 1 kg.wt = 9.8 N, 1 gm.wt= 980 dyne.
- A metallic plate of mass 'M' is kept held in mid air by firing 'n' bullets in 't' seconds each of mass 'm' with a velocity 'v' from below.

  (a) If the bullet falls dead after hitting the plate then $\frac{mnv}{t} = Mg$

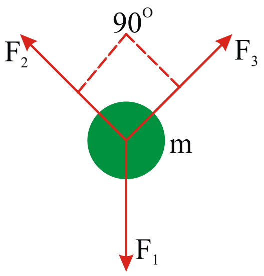
  (b) If the bullet rebounds after hitting the plate with same velocity then $\frac{2mnv}{t} = Mg$

EX.1: A force produces an acceleration16 ms^{-2} in a mass 0.5 kg and an acceleration 4 ms^{-2} in an unknown mass when applied separately. If both the masses are tied together, what will be the acceleration under same force?

Sol. Force F=ma=0.5×16=8N when both masses are joined and same force acts, acceleration is given by $a^1 = \frac{F}{m+m^1} = \frac{8}{0.5+(8/4)} = 3.2ms^{-2}$

EX.2: When the forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remain stationary. If the force F_1 is now removed, then find the acceleration of the particle .

Sol. If mass 'm' is stationary under three forces,



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$
$$\vec{F}_1 = -(\vec{F}_2 + \vec{F}_3)$$
$$\sqrt{F_2^2 + F_3^2} = F_1$$

Obviously if F_1 is removed then the mass will have $a = \frac{\sqrt{F_2^2+F_3^2}}{m} (or)\ a = \frac{F_1}{m}$

EX.3: **A body of mass m=3.513 kg is moving along the x-axis with a speed of 5ms^{-1}.The magnitude of its momentum is recorded as**

Sol. m=3.513kg,v=5ms^{-1} momentum,

p = mv=3.513×5 =17.565kgms^{-1}

EX.4: **A very flexible chain of length L and mass M is vertically suspended with its lower end just touching the table. If it is released so that each link strikes the table and comes to rest. What force the chain will exert on the table at the moment 'y' part of length falls on the table ?**

Sol. Since chain is uniform , the mass of 'y' part of the chain will be $\left(\frac{M}{L}y\right)$. When this part reaches the table, its total force exerted must be equal to the weight of y part resting on table + Force due to the momentum imparted

$$F = \frac{M}{L}yg + \frac{\left(\frac{M}{L}dy\right)\sqrt{2gy}}{dt} = \frac{Mg}{L}y + \frac{M}{L}v.\sqrt{2gy}$$

$$\left(\because \frac{dy}{dt} = v\right) = \frac{Mg}{L}y + \frac{M}{L}\sqrt{2gy}.\sqrt{2gy} = 3\frac{My}{L}g$$

EX.5: **A body of mass 8kg is moved by a force F =(3x)N, where x is the distance covered. Initial position is $x = 2$ m and final position is x=10m. If initially the body is at rest, find the final speed. [2014E]**

Sol: F=ma $\Rightarrow$ F=m$\frac{dv}{dt}$ $\Rightarrow$ 3x=m$\frac{dv}{dx}\frac{dx}{dt}$

$$3x = 8\frac{dv}{dx}v \Rightarrow 3xdx = 8vdv$$

$$3\int_2^{10} xdx = 8\int_0^v vdv \Rightarrow 3\left[\frac{x^2}{2}\right]_2^{10} = 8\left[\frac{v^2}{2}\right]_0^v$$

3[100-4]=8v^2 $\Rightarrow v^2 = \frac{3\times 96}{8} = 36 \Rightarrow v$=6ms^{-1}

EX. 6: **Sum of magnitudes of the two forces acting at a point is 16 N. If their resultant is normal to the smaller force, and has a magnitude 8 N, then the forces are (2012E)**

Sol. $F_1 + F_2 = 16$——(1) Resultant force is perpendicular to F_1, then $F_2^2 - F_1^2 = F^2$

$$F_2{}^2 - F_1{}^2 = 8^2 \Rightarrow (F_2 - F_1)(F_2 + F_1) = 64$$

$$(F_2 - F_1)\times 16 = 64 \text{———}(2)$$

Solving(1) &(2),we get $F_1 = 6N, F_2 = 10N$

EX.7: **A particle is at rest at x=a. A force $\vec{F} = -\frac{b}{x^2}\vec{i}$ begins to act on the particle. The particle starts its motion, towards the origin, along X–axis. Find the velocity of the particle, when it reaches a distance x from the origin.**

Sol. $F = -\frac{b}{x^2} \Rightarrow \frac{d}{dt}(p) = -\frac{b}{x^2}$

$$\frac{d}{dt}(mv) = -\frac{b}{x^2} \Rightarrow m\frac{dv}{dx}\frac{dx}{dt} = -\frac{b}{x^2}$$

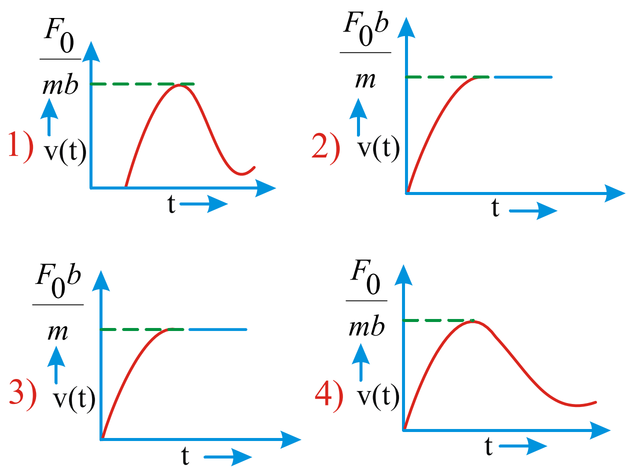
$$mvdv = -\frac{b}{x^2}dx \Rightarrow vdv = -\frac{b}{mx^2}dx$$

$$\int_0^v v\,dv = \int_a^x -\frac{b}{mx^2}dx \Rightarrow \frac{v^2}{2} = \frac{b}{m}\left[\frac{1}{x}\right]_a^x$$

$$\frac{v^2}{2} = \frac{b}{m}\left[\frac{1}{x} - \frac{1}{a}\right] \quad \therefore v = \sqrt{\frac{2b}{m}\left(\frac{a-x}{xa}\right)}$$

EX.8: A particle of mass m is at rest at the origin at time t=0. It is subjected to a force F(t)= $F_0 e^{-bt}$ in the X-direction. Its speed V(t) is depicted by which of the following curves.

(AIEEE-2012)



Sol: As the force is exponentially decreasing, its acceleration, i.e,rate of increase of velocity will decrease with time.Thus, the graph of velocity will be an increasing curve with decreasing slope with time.

$$a = \frac{F}{m} = \frac{F_0}{m}e^{-bt} \Rightarrow \frac{dv}{dt} = \frac{F_0}{m}e^{-bt} \Rightarrow \int_0^v dv = \int_0^t \frac{F_0}{m}e^{-bt}dt$$

$$\Rightarrow v = \left[\frac{F_0}{m}\left(\frac{1}{-b}\right)e^{-bt}\right]_0^t = \left[\frac{F_0}{m}\left(\frac{1}{b}\right)e^{-bt}\right]_t^0$$

$$= \frac{F_0}{mb}\left(e^0 - e^{-bt}\right) = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

So, velocity increases continuously and attains a maximum value, $v_{\max} = \frac{F_0}{mb}$ **Ans: 3**

EX.9: A bus moving on a level road with a velocity v can be stopped at a distance of x, by the application of a retarding force F. The load on the bus is increased by 25% by boarding the passengers. Now, if the bus is moving with the same speed and if the same retarding force is applied, the distance travelled by the bus before it stops is

[2014E]

Sol : By using equations of motion $v^2 - u^2 = 2as$

$$v^2 - u^2 = -2\left(\frac{F}{m}\right)s \quad -u^2 = -2\left(\frac{F}{m}\right)s$$

$$u^2 = 2\frac{Fs}{m} \Rightarrow m = \frac{2Fs}{u^2} \Rightarrow m \propto s \Rightarrow \frac{m_1}{m_2} = \frac{s_1}{s_2}$$

Given $s_1 = x$, $m_1 = m$, and

$$m_2 = m + \frac{25}{100}(m) = m + \frac{m}{4} = \frac{5m}{4} \quad \Rightarrow \frac{m}{5m/4} = \frac{x}{s_2} \Rightarrow s_2 = \frac{5x}{4} = (1.25x)\,\text{m}$$

Applications of variable mass :

- When a machine gun fires 'n' bullets each of mass 'm' with a velocity v in a time interval 't' then force needed to hold the gun steadily is $F = \frac{nmv}{t}$
- When a jet of liquid coming out of a pipe strikes a wall normally and falls dead , then force exerted by the jet of liquid on the wall is $F = Adv^2$ A = Area of cross section of the pipe v = Velocity of jet d = density of the liquid
- If the liquid bounces back with the same velocity then the force exerted by the liquid on the wall is $F = 2Adv^2$
- If the liquid bounces back with velocity v' then the force exerted on the wall is $F = Adv(v + v')$
- When a jet of liquid strikes a wall by making an angle $'\theta'$ with the wall with a velocity 'v' and rebounds with same velocity then force exerted by the water jet on wall is $F = 2Adv^2 \sin\theta$
- If gravel is dropped on a conveyor belt at the rate of $\frac{dm}{dt}$, extra force required to keep the belt moving with constant velocity $'u'$ is $F = u\left(\frac{dm}{dt}\right)$

EX.10: A gardener is watering plants at the rate 0.1litre/sec using a pipe of cross- sectional area 1 cm². What additional force he has to exert if he desires to increase the rate of watering two times?

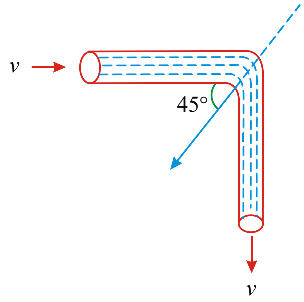
Sol : $F = Adv^2 = \frac{(Av)^2 d}{A}$. If rate of watering of plant (Av) is doubled, it means that the amount of water poured/sec is doubled which is possible only if velocity is doubled. Hence, force is to be made 4 times.

$\therefore$ Additional force = 3 times initial force $\quad = 3Adv^2 = 3\frac{(Av)^2}{A}d$

$$= \frac{3 \times 0.1 \times 0.1 \times 10^3}{10^{-4}} = 3 \times 10^5 N$$

EX.11: A liquid of density ρ flows along a horizontal pipe of uniform cross – section A with a velocity v through a right angled bend as shown in Fig. What force has to be exerted at the bend to hold the pipe in equilibrium?

Sol : Change in momentum of mass Δm of liquid as it passes through the bend



$$\Delta P = P_f - P_i = \sqrt{2}\Delta mv$$

$$F = \frac{\Delta P}{\Delta t} = \left(\sqrt{2}\right)v\frac{\Delta m}{\Delta t};\left[as\ \ \Delta m = \rho A\Delta L\right]$$

$$F = \sqrt{2}v\frac{\left(\rho.A\Delta L\right)}{\Delta t};\left[as\ \ \Delta L/\Delta t = v\right]$$

$$F = \sqrt{2}\rho Av^2$$

EX.12:A flat plate moves normally with a speed v_1 towards a horizontal jet of water of uniform area of cross section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is r. Assume that water splashes along the surface of the plate at right angles to the original motion. The magnitude of the force acting on the plate due to the jet is

Sol. Force acting on the plate $F = \frac{dp}{dt} = u_r\frac{dm}{dt}$ Since $Av_2 = V \Rightarrow \frac{dm}{dt} = A(v_1+v_2)\rho = \frac{V}{v_2}(v_1+v_2)\rho$

($u_r = v_1 + v_2$ = velocity of water coming out of jet w.r.t plate)

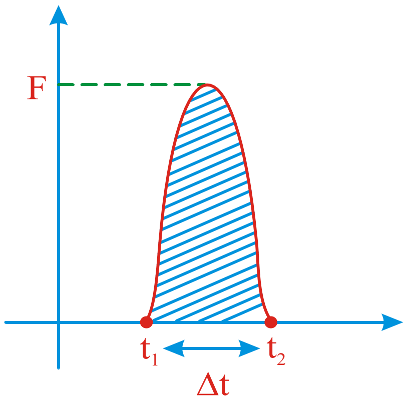
$$F = (v_1+v_2).\frac{V}{v_2}(v_1+v_2)\rho = \frac{V}{v_2}(v_1+v_2)^2\rho \text{ N}$$

Impulse ($\vec{J}$) :

- It is the product of impulsive force and time of act ion that produces a finite change in momentum of body.
- J=Ft = m(v-u) = change in momentum. SI unit: Ns (or) Kg - ms^{-1}; DF: MLT^{-1}
- It is a vector directed along the force
- change in momentum and Impulse are always in the same direction.
- For constant force, J=Ft,
- Impulsive force is a variable, then

$$\vec{F} = \frac{d\vec{p}}{dt}, \qquad J = \int_{t_1}^{t_2} Fdt$$

- The area bounded by the force-time graph measures Impulse.



Application of Impulse :

a) shock absorbers are used in vehicles to reduce the magnitude of impulsive force.

b) A cricketer lowers his hands, while catching the ball to reduce the impulsive force.

EX.13: Find the impulse due to the force $\vec{F} = a\hat{i} + bt\hat{j}$, where a=2 N and b=4 Ns^{-1} if this force acts from t_i=0 to t_f=0.3s

Sol: $J = \int_{t_i}^{t_f} \vec{F} dt = \int_0^{0.3} (a\hat{i} + bt\hat{j}) dt$

$$J = a\int_0^{0.3} dt\,\hat{i} + b\int_0^{0.3} t\,dt\,\hat{j} = a[t]_0^{0.3}\hat{i} + b\left[\frac{t^2}{2}\right]_0^{0.3}\hat{j}$$

$$= 2\times 0.3\times\hat{i} + 4\times\frac{(0.3)^2}{2}\times\hat{j} = 0.6\hat{i} + 0.18\hat{j} \text{ NSec}$$

EX.14: A ball falling with velocity $\vec{v}_i = (-0.65\hat{i} - 0.35\hat{j})$ ms^{-1} is subjected to a net impulse $\vec{I} = \left(0.6\hat{i} + 0.18\hat{j}\right)$ Ns. If the ball has a mass of 0.275kg, calculate its velocity immediately following the impulse

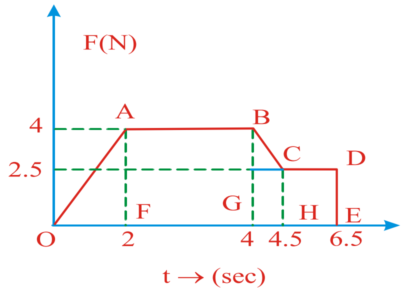
Sol: $m\vec{v}_f - m\vec{v}_i = \vec{I}$; $\vec{v}_f = \vec{v}_i + \frac{\vec{I}}{m}$

$$\vec{v}_f = -0.65\hat{i} - 0.35\hat{j} + \frac{0.6\hat{i} + 0.18\hat{j}}{0.275}$$

$$\vec{v}_f = -0.65\hat{i} - 0.35\hat{j} + 2.18\hat{i} + 0.655\hat{j}$$

$$\vec{v}_f = \left(1.53\hat{i} + 0.305\hat{j}\right) ms^{-1}$$

EX.15: A body of mass 2kg has an initial speed 5 ms^{-1}. A force acts on it for some time in the direction of motion. The force–time graph is shown in figure. Find the final speed of the body



Sol. Area of $OAF = \frac{1}{2}\times 2\times 4 = 4$

Area of ABG F = 2 $\times$ 4 = 8

Area of $BGHC = \frac{1}{2}(4+2.5)\times 0.5 = 1.625$

Area of CDEH = 2 $\times$ 2.5 = 5

Total area under F-t graph = Change in momentum

$\Rightarrow$ m(v – u) = 18.625

$\Rightarrow v = \frac{18.625}{2} + 5 = 14.25 ms^{-1}$

EX.16:A bullet is fired from a gun. The force on a bullet is, $F = 600 - 2\times 10^5 t$ newton. The force reduces to zero just when the bullet leaves barrel.Find the impulse imparted to the bullet.

Sol. F = 600 – 2 × 10^5 t , F becomes zero as soon as the bullet leaves the barrel.

0 = 600 – 2 × 10^5 t $\Rightarrow 600 = 2\times 10^5 t$

t = 3 × 10^{-3} s $\Rightarrow$ Impulse = $\int_0^t F dt$

$= \int_0^t \left(600 - 2\times 10^5 t\right) dt = \left[600t - 2\times 10^5 \frac{t^2}{2}\right]_0^{3\times 10^{-3}}$

= 600 × 3 × 10^{-3} – 10^5 × 9 × 10^{-6} = $0.9 Ns$

Equilibrium: The necessary and sufficient conditions for the translational equilibrium of the rigid body.

$\sum F = 0$; $\sum F_x = 0,\ \sum F_y = 0,\ \sum F_z = 0$ For rotational equilibrium

$\sum \tau = 0$: $\sum \tau_x = 0,\ \sum \tau_y = 0,\ \sum \tau_z = 0$

↳ As for a body , $\vec{F} = 0, m\vec{a} = 0$ $\left(\text{as } \vec{F} = m\vec{a}\right)$ $\frac{d\vec{v}}{dt} = 0$ (as m is finite and $\vec{a} = \frac{d\vec{v}}{dt}$) $\vec{v}$ =constant or zero

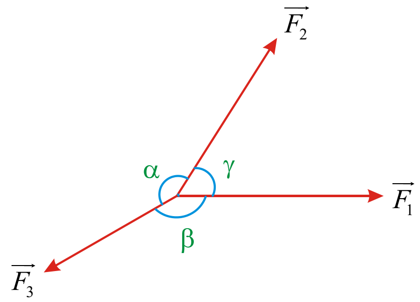
↳ If a body is in translatory equilibrium it will be either at rest or in uniform motion.If it is at rest, the equilibrium is called static,otherwise dynamic.

↳ If 'n' coplanar forces of equal magnitudes acting simultaneously on a particle at a point, with the angle between any two adjacent forces is 'θ' and keep it in equilibrium, then $\theta = \frac{360}{n}$

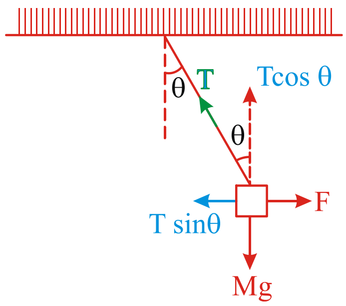
Lami's Theorem :

- If an object O is in equilibrium under three concurrent forces $\vec{F_1}, \vec{F_2}$ and $\vec{F_3}$ as shown in figure.

  Then, $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



- If the bob of simple pendulum is held at rest by applying a horizontal force 'F' as shown in fig



If body is in equilibrium

$T\sin\theta = F$, $\quad T\cos\theta = mg$,

$F = mg\tan\theta$, $\quad \sqrt{F^2 + (mg)^2} = T$

$$\frac{x}{F} = \frac{l}{T} = \frac{\sqrt{l^2 - x^2}}{mg}$$

EX.17:A mass of 3kg is suspended by a rope of length 2m from the ceiling. A force of 40N in the horizontal direction is applied at midpoint P of the rope as shown. What is the angle the rope makes with the vertical in equilibrium and the tension in part of string attached to the ceiling? (Neglect the mass of the rope, g= 10m/s^2)

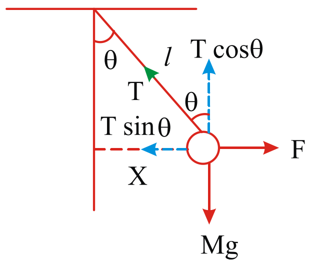
Sol : Resolving the tension T_1 into two mutually perpendicular components, we have

$T_1\cos\theta = W = 30N \qquad T_1\sin\theta = 40N$

$\therefore \tan\theta = \frac{4}{3}$ (or) $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^0$

EX.18:A mass M is suspended by a weightless string. The horizontal force required to hold the mass at 60^0 with the vertical is (2013E)

Sol :



$F = T\sin\theta$ ----- (1)

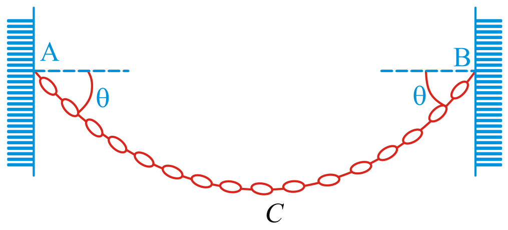
$Mg = T\cos\theta$ ----- (2)

Dividing Eq.(1) and Eq.(2)

$\frac{F}{Mg} = \frac{T\sin\theta}{T\cos\theta}$; $F = Mg\tan\theta$

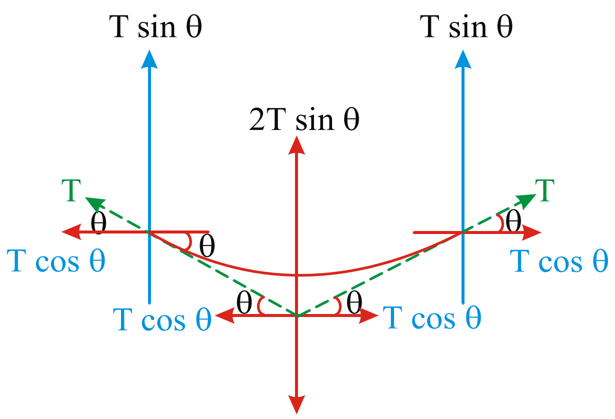
$F = Mg\tan 60^0$; $F = \sqrt{3}Mg$

EX.19:A chain of mass 'm' is attached at two points A and B of two fixed walls as shown in the figure. Find the tension in the chain near the walls at point A and at the mid point C.



Sol.

i)



$2T\sin\theta = mg \Rightarrow T = \frac{1}{2}mg\,\text{cosec}\theta$

ii) Tension along horizontal direction is same everywhere

∵ (no external force is acting on it in horizontal direction.)

$T^1 = T\cos\theta = \frac{mg\cos\theta}{2\sin\theta} = \frac{mg\cot\theta}{2}$

Newton's third law:

- For every action there is always an equal and opposite reaction
- Action and reaction do not act on the same body and they act on different bodies at same instant of time
- Action and reaction, known as pair of forces, are equal in magnitude and opposite in directions acting on different bodies in interaction. So they never cancel each other
- Newton’s third law is not applicable to pseudo forces.
- Newton’s third law defines nature of force and gives the law of conservation of linear momentum.

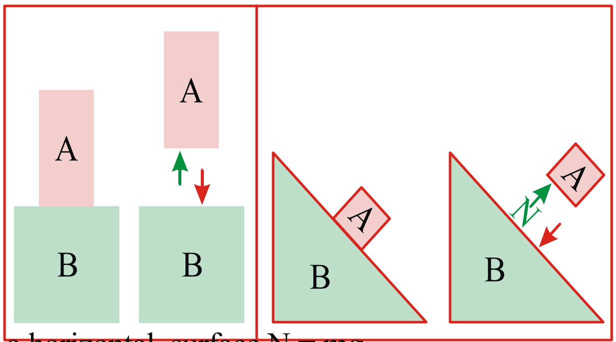
Examples:

- When we walk on a road we push the road backwards and road applies equal (in magnitude) and opposite force on us, so that we can move forward.
- When we swim on water we push water backward and water applies equal (in magnitude) and opposite force on us,so that we can move forward.
- A bird is in a wire cage hanging from a spring balance. When the bird starts flying in the cage, the reading of the balance decreases.
- If the bird is in a closed cage (or) air tight cage and it hovers in the cage the reading of the spring balance does not change.
- In the closed cage if the bird accelerates upward the reading of the balance is $R = W_{bird} + ma$

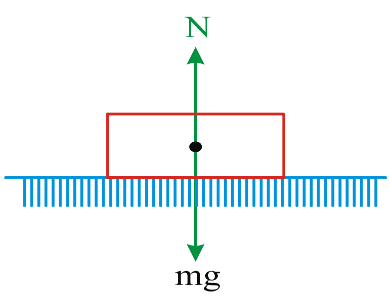
Limitations of newton’s third law:-

- Newton’s third law is not strictly applicable for the interaction between two bodies separated by large distances, of the order of astronomical units.
- It does not apply strictly when the objects move with velocity nearer to that of light
- It does not apply where the gravitational field is strong.

Normal reaction/force : Normal force acts perpendicular to the surfaces in contact when one body tries to press on the surface of the second body.In this way second body tries to push away the first body.

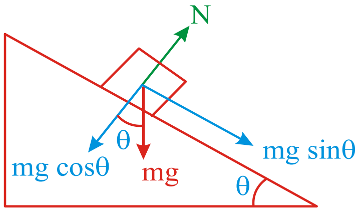


- When the body lies on a horizontal surface N = mg



↳ When the body lies on an inclined surface

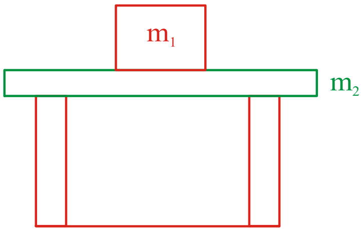
N = mg cos θ .



Free Body Diagram:- When several bodies are connected by strings, springs, surfaces of contact, then all the forces acting on a body are considered and sketched on the body under consideration by just isolating it . Then the diagram so formed is called Free Body Diagram (FBD).

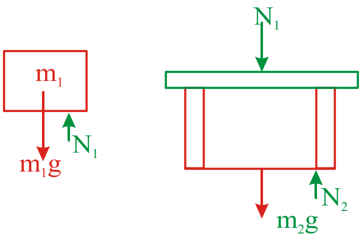
Some examples:

i) A block is placed on a table and the table is kept on earth.Assuming no other body in the universe exerts any force on the system,make the FBD of block and table.



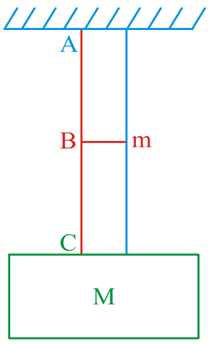
FBD of block, $N_1 = m_1 g$

FBD of table $N_2 = N_1 + m_2 g = m_1 g + m_2 g = (m_1 + m_2) g$



ii) A block of mass M is suspended from the ceiling by means of a uniform string of mass m.Find the tension in the string at points A,B and C. B is the mid point of string. Also find the tensions A,B and C if the mass of string is negligible or it is massless.

ii) A block of mass M is suspended from the ceiling by means of a uniform string of mass m.Find the tension in the string at points A,B and C. B is the mid point of string. Also find the tensions A,B and C if the mass of string is negligible or it is massless.

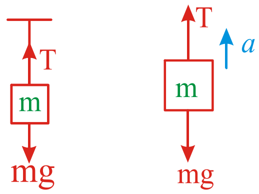


Tension at any point will be weight of the part below it.

So, $T_A = (M+m)g, T_B = \left(\frac{m}{2}+M\right)g, T_C = Mg$.

Now if the string is massless: m=0 then $T_A = T_B = T_C = Mg$. So in a massless string, tension is the same at every point.

(iii) Find the tension in the massless string connected to the block accelerating upward.



Net force :

$F_{net} = T - mg$ Now apply $F_{net} = ma$

$\Rightarrow T - mg = ma \Rightarrow T = mg + ma = m(g+a)$

Note: If ‘a’ is downward, then replace a with -a; we get $T = m(g-a)$

In free fall a=g then T=0.

Frames of Reference:

↳ A system of coordinate axes which defines the position of a particle or an event in two or three dimensional space is called a frame of reference.

There are two types of frames of reference

a) inertial or unaccelerated frames of reference

b) non-inertial or accelerated frames of reference

Inertial frames of reference :

a) Frames of reference in which Newton’s Laws of motion are applicable are called inertial frame.

b) Inertial frames of reference are either at rest or move with uniform velocity with respect to a fixed imaginary axis.

c) In inertial frame, acceleration of a body is caused by real forces.

d) Equation of motion of mass ‘m’ moving with acceleration ‘a’ relative to an observer in an inertial frame is $\sum \vec{F}_{real} = ma$

Examples:

1) A lift at rest,

2) Lift moving up(or)down with constant velocity,

3) Car moving with constant velocity on a straight road.

Real Force : Force acting on an object due to its interaction with another object is called a real force.

Ex: Normal force, Tension, weight, spring force, muscular force etc.

a) All fundamental forces of nature are real.

b) Real forces form action, reaction pair.

Non-Inertial frames :

a) Frames of reference in which Newton Laws are not applicable are called non-inertial frames.
b) Accelerated frames move with either uniform acceleration or non uniform acceleration.
c) All the accelerated and rotating frames are non-inertial frames of reference.

d) Examples:

1) Accelerating car on a road.
2) Merry go round.
3) Artificial satellite around the earth.

Pseudo force :

a) In non-inertial frame Newton's second law is not applicable. In order to make Newton's second law applicable in non-inertial frame a pseudo force is introduced.

b) If $\vec{a}$ is the acceleration of a non-inertial frame, the pseudo force acting on an object of mass m, as measured by an observer in the given non-inertial frame is $\vec{F}_{Pseudo} = -m\vec{a}$
i.e. Pseudo force acts on an object opposite to the direction of acceleration of the non-inertial frame.

c) Pseudo forces exist for observers only in non-inertial frames, such forces have no existence relative to an inertial frame.

d) Equation of motion relative to non-inertial frame is $\sum\left(\vec{F}_{real} + \vec{F}_{Pseudo}\right) = m\vec{a}'$

Where a' is the acceleration of body as measured in non-inertial frame.

e) Earth is an inertial frame for an observer on the earth but it is an accelerated frame for an observer at centre of earth (or) in a satellite.

Examples : (i) Centrifugal force and deflection of pendulum relative to accelerating car.(ii) Gain or loss of weight experienced in an accelerating elevator.

Apparent weight of a body in a moving elevator

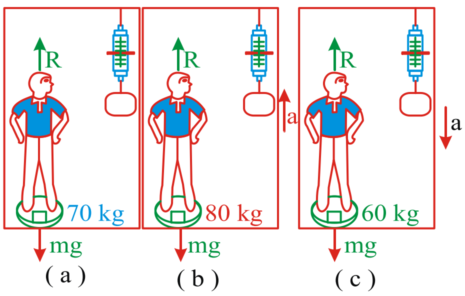
Weight of a body on a surface comes due to the reaction of a supporting surface, i.e.,Apparent weight of a body in a lift

W_{app} = Reaction of supporting surface. Consider a person standing on a spring balance , or in a lift. The following situations are possible:

Case(i) :If lift is at rest or moving with constant velocity then the person will be in translatory equilibrium. So, $R = mg$

$\therefore\ W_{app} = mg \qquad$ [as $W_{app} = R$]

or $W_{app} = W_0$ [as $W_0 = mg =$ true weight]



i.e., apparent weight (reading of balance) will be equal to true weight.

Case(ii) : If lift is accelerated up or retarding down with acceleration a from Newton's II law we have

$R - mg = ma$ or $R = m(g + a)$

or $W_{app} = m(g+a) \quad = mg\left[1+\frac{a}{g}\right] = W_0\left[1+\frac{a}{g}\right]$ or $W_{app} > W_0$

i.e., apparent weight (reading of balance) will be more than true weight.

Case (iii) : If lift is accelerated down: or retarding up with acceleration 'a' $mg - R = ma$ i.e., $R = m(g-a)$

or $W_{app} = m(g-a)$ [as $W_{app} = R$] $= mg\left[1-\frac{a}{g}\right]$

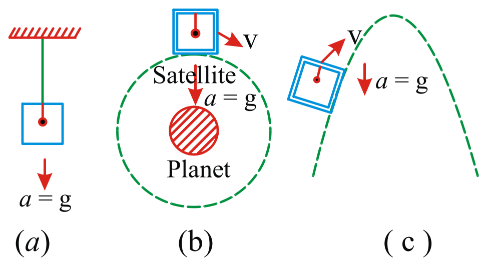
i.e., $W_{app} = W_0\left[1-\frac{a}{g}\right] \quad W_{app} < W_0$

i.e., apparent weight (reading of balance) will be lesser than true weight.

Note: If $a > g, W_{app}$ will be negative; negative weight will mean that the body is pressed against the roof of the lift instead of floor (as lift falls more faster than the body) and so the reaction will be downwards, the direction of apparent weight will be upwards.

Case (iv) : If lift is in freely falling, Then a=g ,

So $mg - R = mg$ i.e., $R = 0$. So, $W_{app} = 0$



(a) Freely falling lift
(b) Satellite motion
(c) Projectile motion

i.e., apparent weight of a freely falling body is zero.

↳ This is why the apparent weight of a body is zero, or body is weightless if it is in a (i) lift whose cable has broken, (ii) orbiting satellite.

EX.20: A mass of 1kg attached to one end of a string is first lifted up with an acceleration 4.9m/s^2 and then lowered with same acceleration. What is the ratio of tension in string in two cases.

Sol : When mass is lifted up with acceleration 4.9m/s² $T_1 = m(g+a) = 1\,(9.8+4.9) = 14.7N$

When mass is lowered with same acceleration $T_2 = m(g-a) = 1(9.8-4.9) = 4.9\ N$

$$\therefore \frac{T_1}{T_2} = \frac{14.7}{4.9} = 3:1$$

EX.21: The apparent weight of a man in a lift is W_1 when lift moves upwards with some acceleration and is W_2 When it is accelerating down with same acceleration. Find the true weight of the man and acceleration of lift.

Sol : (a) $W_1 = m(g+a), W_2 = m(g-a)$

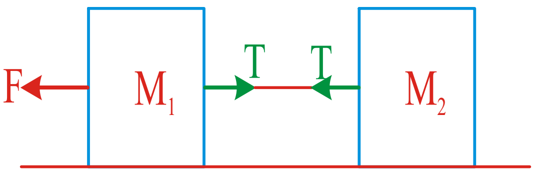
$$W_1 + W_2 = 2mg \Rightarrow W_1 + W_2 = 2W (\because W = mg) \qquad \Rightarrow \frac{W_1 + W_2}{2} = W$$

(b) $$\frac{W_1}{W_2} = \frac{m(g+a)}{m(g-a)} = \frac{g+a}{g-a}$$

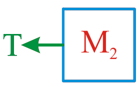
$$\frac{g}{a} = \frac{W_1 + W_2}{W_1 - W_2} \Rightarrow a = g\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Connecting Bodies:

↪ If masses are connected by strings then acceleration of system and tension in the strings on smooth horizontal surface are

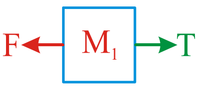
a) 

Free body diagram for M_2



$T = M_2 a$ .....(1)

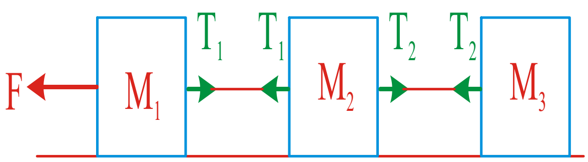
Free body diagram for M_1



$F - T = M_1 a$ ....(2)

from (1) and (2)

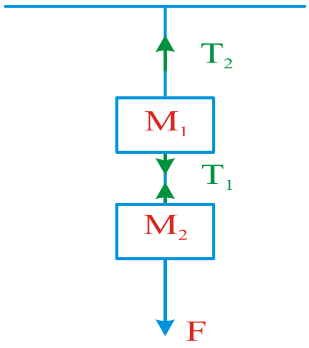
$$a = \frac{F}{(M_1 + M_2)} \text{ and } T = \frac{M_2 F}{(M_1 + M_2)}$$

b) 

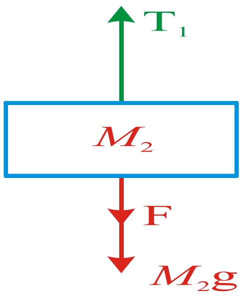
$$a = \frac{F}{M_1 + M_2 + M_3}; \; T_1 = \frac{(M_2 + M_3)F}{(M_1 + M_2 + M_3)}$$

$$T_2 = \frac{M_3 F}{(M_1 + M_2 + M_3)}$$

↳ If masses are connected by a string and suspended from a support then tension in the string when force F is applied downwards as shown in the figure

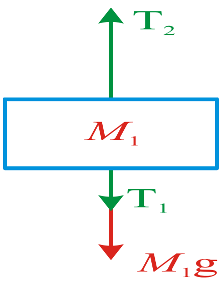


Free body diagram for M_2



$$T_1 = F + M_2 g \quad \text{............(1)}$$

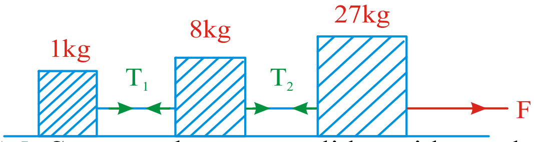
Free body diagram for M_1



$$T_2 = T_1 + M_1 g \quad \text{............(2)}$$

From (1) and (2), $T_2 = F + (M_1 + M_2)g$

EX.22: Three blocks connected together by strings are pulled along a horizontal surface by applying a force F. If F= 36N, What is the tension T_2?



Sol : Suppose the system slides with acceleration 'a'.

$m_1 = 1kg, m_2 = 8kg, m_3 = 27kg$

$F - T_2 = m_3 a\ , T_2 - T_1 = m_2 a\ , T_1 = m_1 a$

Solving the above equations ,we get

$$a = \frac{F}{m_1 + m_2 + m_3} = \frac{36}{1+8+27} = \frac{36}{36} = 1\ ms^{-2}$$

From the above equation, $T_2 = F - m_3 a$

$T_2 = 36 - 27 \times 1 = 9$ N

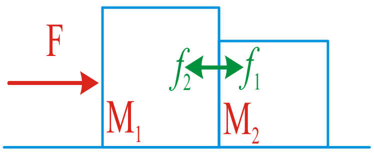
Contact Forces : When two objects are in contact with each other, the molecules at the interface interact with each other. This interaction results in a net force called contact force. The contact force can be resolved into two components.

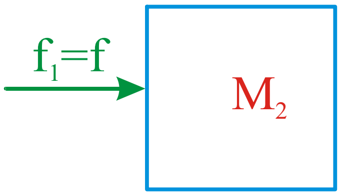
(a) **Normal force (N):** Component of the contact force along the normal to the interface. Normal force is independent of nature of the surfaces in contact.

(b) **Friction (f):** Component of the contact force along the tangent at the interface. Friction depends on the roughness of the surfaces in contact. This component can be minimised by polishing the surfaces.

↳ The tension and contact forces are self adjustable forces. Their magnitude and direction change when other forces involved in a physical arrangement change.

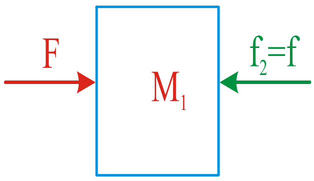
↳ Masses are in contact on a smooth horizontal surface:





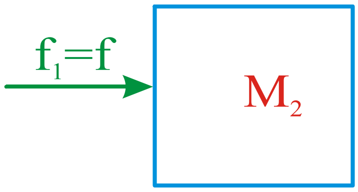
contact force $f_1 = f_2 = f = M_2 a$

free body diagram for M_1



$F - f = M_1 a$ ....(1)

free body diagram for M_2

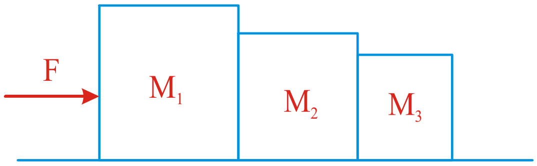


$f = M_2 a$ ....(2)

From (1) and (2)

$a = \frac{F}{(M_1 + M_2)}$, contact force, $f = \frac{M_2 F}{M_1 + M_2}$

↳ Contact forces are as shown in the figure



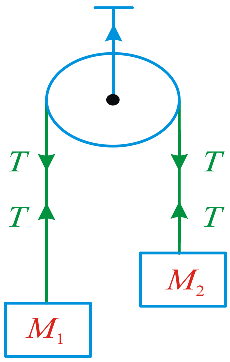
a) Acceleration of system, $a = \frac{F}{(M_1 + M_2 + M_3)}$

b) Contact force between M_1 and M_2 $f = (M_2 + M_3)a$

c) Contact force between M_2 and M_3, $f^1 = M_3 a$

Atwood's Machine :

↳ Masses M_1 and M_2 ($M_1 > M_2$)are tied to a string , which passes over a frictionless light pulley The string is light and inextensible.



Acceleration of the system, $a = \frac{(M_1 - M_2)g}{M_1 + M_2}$

Tension in the string , $T = \left(\frac{2M_1 M_2}{M_1 + M_2}\right)g$

Thrust on the pulley , $2T = \left(\frac{4M_1 M_2}{M_1 + M_2}\right)g$

↳ If the pulley begins to move with acceleration $\vec{a}$ then

i) If the pulley accelerates upward , then $a_{net} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(g+a)$ and $T_{net} = \left(\frac{2M_1M_2}{M_1 + M_2}\right)(g+a)$

ii) If the pulley accelerates downward, then $a_{net} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(g-a)$ and

$$T_{net} = \left(\frac{2M_1M_2}{M_1 + M_2}\right)(g-a)$$

↳ Thrust on the pulley when it comes downward with acceleration 'a' is $T = \frac{4M_1M_2}{(M_1 + M_2)}(g-a)$

EX.23:The maximum tension a rope can withstand is 60 kg-wt.The ratio of maximum acceleration with which two boys of masses 20kg and 30kg can climb up the rope at the same time is (2011E)

Sol. $m_1 = 20kg, m_2 = 30kg, T = 60kgwt = 600N$

For 'm_1'; $T - m_1 g = m_1 a_1$

$600 - 20 \times 10 = 20 \times a_1 \Rightarrow a_1 = 20ms^{-2}$

For 'm_2'; $T - m_2 g = m_2 a_2$

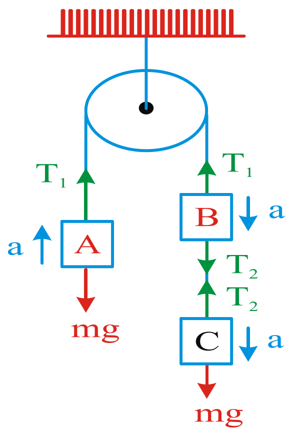
$600 - 30 \times 10 = 30 \times a_2 \Rightarrow a_2 = 10ms^{-2}$

$a_1 : a_2 = 20 : 10 = 2 : 1$

EX.24:Figure shows three blocks of mass 'm' each hanging on a string passing over a pulley. Calculate the tension in the string connecting A to B and B to C?

Sol. Net pulling force = 2 mg – mg = mg

Total mass = m+ m + m = 3m



Acceleration, $a = \frac{mg}{3m} = \frac{g}{3}$

Considering block A,

$T_1 - mg = ma$; $T_1 = mg + ma$

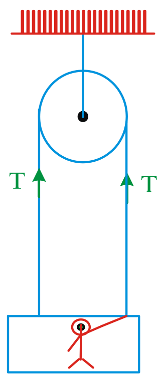
$T_1 = mg + m\left(\frac{g}{3}\right) \Rightarrow T_1 = \frac{4}{3}mg$

Considering block C,

$mg - T_2 = ma \Rightarrow T_2 = mg - ma$

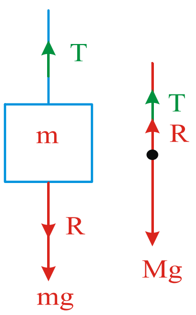
$\Rightarrow T_2 = mg - \frac{mg}{3} = \frac{2}{3}mg$.

EX.25:A man of mass 60 kg is standing on a weighing machine kept in a box of mass 30 kg as shown in the diagram. If the man manages to keep the box stationary, find the reading of the weighing machine.



Sol. we know that Normal reaction = scale reading

For man , $T = Mg - R$

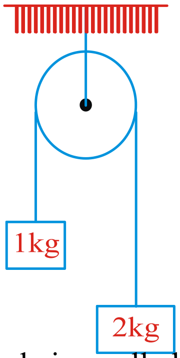


For box : $T = mg + R$

$Mg - R = mg + R$; 2 R =(M –m)g

$R = \frac{(60 - 30) \times 10}{2} = 150N$

EX.26: **Two unequal masses are connected on twosides of a light string passing over a light and smooth pulley as shown in figure. The system is released from rest. The larger mass is stopped for a moment, 1sec after the system is set into motion. Find the time elapsed before the string is tight again . ($g = 10\ m/s^2$)**



Sol. Net pulling force = 2g – 1g = 10N Mass being pulled = 2 + 1 = 3 kg

Acceleration of the system is $a = \frac{10}{3} m/s^2$ Velocity of both the blocks at t = 1 s will be

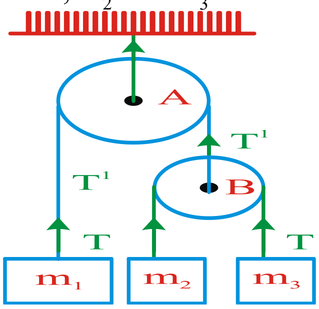
$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3}$ m/s. Now, at this moment velocity of 2kg block becomes zero, while that of 1kg block is $\frac{10}{3}$ m/s upwards. Hence, string becomes tight again when displacement of 1 kg block = displacement of 2 kg block.

$$v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \Rightarrow gt^2 = v_0 t$$

$$t = \frac{v_0}{g} = \frac{(10/3)}{10} = \frac{1}{3}s$$

EX.27: **In the figure, if m_1 is at rest, find the relation among m_1 , m_2 and m_3 ?**

Sol. m_1 is at rest $\Rightarrow$ point B does not move, m_2 and m_3 move with acceleration

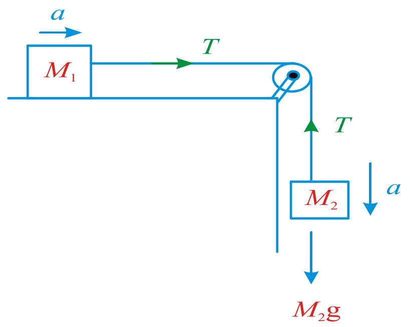


$$a = \left(\frac{m_3 - m_2}{m_2 + m_3}\right)g\ ;\ m_3 > m_2$$

$$T = \frac{2m_2 m_3 g}{m_2 + m_3}\ ; \qquad T^1 = 2T = \frac{4m_2 m_3 g}{m_2 + m_3}$$

$$m_1 \not{g} = \frac{4m_2 m_2 \not{g}}{m_2 + m_3} \qquad \boxed{\frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}}$$

↳ Two blocks are connected by a string passing over a pulley fixed at the edge of a horizontal table then the acceleration of system and tension in the string $(M_2 > M_1)$

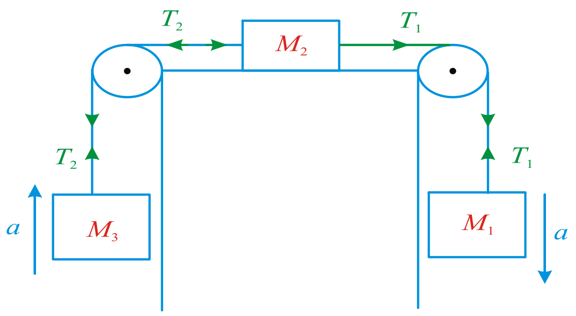


$M_2g - T = M_2a$ and $T = M_1a$

$\Rightarrow a = \frac{M_2g}{(M_1 + M_2)}$

$T = M_1a = \frac{M_1M_2g}{(M_1 + M_2)}$

↳ Acceleration and Tension in the string when bodies are connected as shown in the figure if $M_1 > M_3$.



$M_1g - T_1 = M_1a \quad ; T_1 - T_2 = M_2a$

$T_2 - M_3g = M_3a$

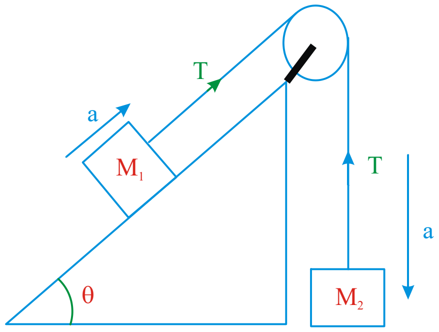
$\Rightarrow a = \frac{(M_1 - M_3)g}{(M_1 + M_2 + M_3)}$

$T_2 = \frac{M_3g(2M_1 + M_2)}{M_1 + M_2 + M_3}; \quad T_1 = \frac{M_1g(2M_3 + M_2)}{M_1 + M_2 + M_3}$

↳ Masses are attached to a string passing through the pulley attached to the edge of an inclined plane, acceleration of system and tension in the string if M_2 moves down

$a = \left(\frac{M_2 - M_1 \sin\theta}{M_1 + M_2}\right)g;$

$T = \left[\frac{M_1M_2(1 + \sin\theta)}{(M_1 + M_2)}\right]g$

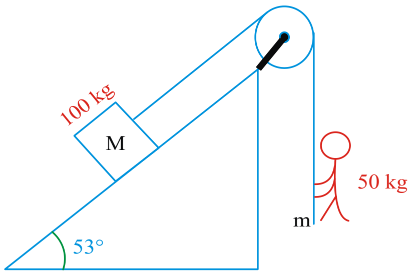


Thrust on the pulley : Resultant Tension =

$$T_g = \sqrt{T^2 + T^2 + 2T^2\cos(90-\theta)}$$

$$T_g = \sqrt{2T^2(1+\sin\theta)} = T\sqrt{2(1+\sin\theta)}$$

EX.28:By what acceleration the boy must go up so that 100 kg block remains stationary on the wedge. The wedge is fixed and is smooth.(g = 10m/s²)



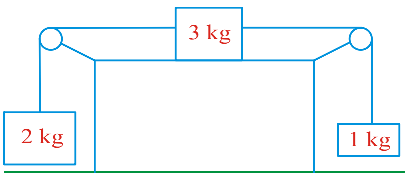
Sol : For the block to remain stationary,

$$T = Mg\sin\theta = 100 \times 10 \times \sin 53 = 100 \times 10 \times \frac{4}{5} = 800\text{N}$$

For man ; T – mg = ma

$T = m(g+a) \Rightarrow 800 = 50(10+a)$ $a = 6\text{m/s}^2$

EX.29:The system as shown in fig is released from rest. Calculate the tension in the strings and force exerted by the strings on the pulley. Assuming pulleys and strings are massless



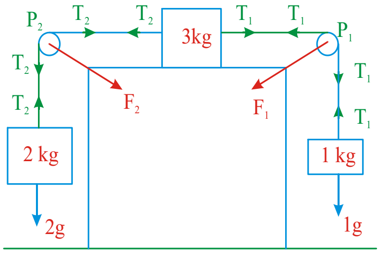
Sol: T_1-1g=1a — (1)

T_2-T_1=3a — (2)

2g- T_2=2a —(3)

Solving the above equations,

we get , $a = \frac{g}{6} m/s^2$

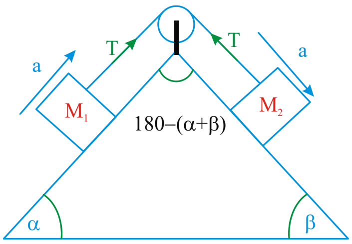


$T_1 = \frac{7g}{6}\ N, T_2 = \frac{5g}{3} N$

Force on pulley P_1 is $F_1 = \sqrt{T_1^2 + T_1^2} \quad = \sqrt{2}T_1 = \frac{7g}{3\sqrt{2}}$ N

Force on pulley P_2 is $F_2 = \sqrt{T_2^2 + T_2^2} \quad = \sqrt{2}T_2 = \frac{5\sqrt{2}g}{3}$ N

↪ If position of masses is interchanged,then the tension in the string and acceleration remains unchanged.



↪ If M_2 slides down then M_1 moves up on double smooth inclined plane then the acceleration of system and tension in the string are given by, acceleration , a = $\left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right)g$

Tension , T = $\frac{M_1 M_2 g}{(M_1 + M_2)}(\sin\alpha + \sin\beta)$

Resultant Tension

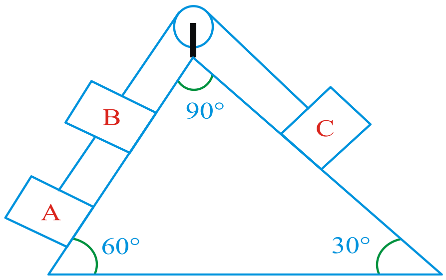
$T_R = \sqrt{T^2 + T^2 + 2T^2 \cos[180 - (\alpha + \beta)]}$

$= \sqrt{2T^2[1 - \cos(\alpha + \beta)]}$

Note:- If $M_2 \sin\beta = M_1 \sin\alpha \Rightarrow a = 0$

$\Rightarrow$ System does not accelerate

EX.30:In the adjacent fig, masses of A, B and C are 1kg, 3kg and 2kg respectively. Find a) the acceleration of the system b) tension in the string (g = 10m/s²)



Sol . a) In this case net pulling force

$= m_A g \sin 60^0 + m_B g \sin 60^0 - m_C g \sin 30^0$

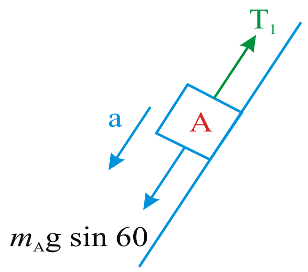
$=(1)(10)\frac{\sqrt{3}}{2}+(3)(10)\left(\frac{\sqrt{3}}{2}\right)-(2)(10)\left(\frac{1}{2}\right) = 24.64\text{N}$

Total mass = 1 + 3 + 2 = 6kg

$\therefore$ Acceleration of the system $a = \frac{24.64}{6} = 4.1\, m/s^2$

b) For the tension in the string between A and B.

FBD of Body A



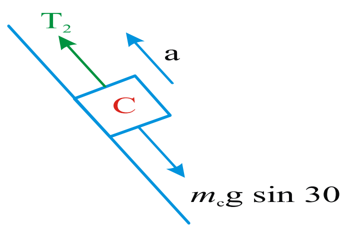
$m_A g \sin 60 - T_1 = m_A a$

$T_1 = m_A g \sin 60 - m_A a = m_A \left(g \sin 60 - a\right)$

$T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56N$

For the tension in the string between B and C

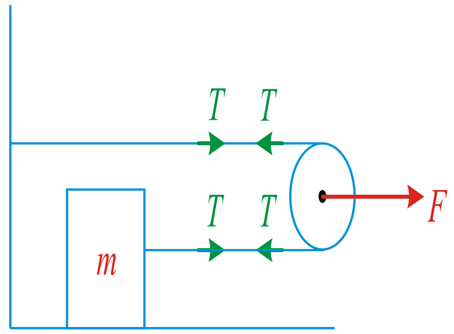
FBD of body C



$T_2 - m_C g \sin 30^0 = m_C a$; $T_2 = m_C \left(g \sin 30^0 + a\right)$

$T_2 = 2\left(10\left(\frac{1}{2}\right) + 4.1\right) = 18.2N$

- A force F is applied on the massless pulley as shown in the figure and string is connected to the block on smooth horizontal surface. Then



F= 2T and $T = ma_{block}$

- If the block moves a distance 'x' the pulley moves x/2 (Total length of the string remains constant)

Therefore acceleration of the pulley $= \frac{a_{block}}{2}$

$= \frac{T}{2m} = \frac{F/2}{2m} = \frac{F}{4m}$.

Constrained Motion:

- **(a) Constraint :** Restriction to the free motion of body in any direction is called constraint.

(b) Constrained Body : A body, whose displacement in space is restricted by other bodies, either connected to or in contact with it, is called a constrained body.

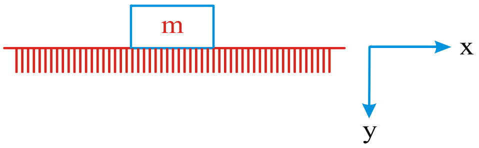
(c) Kinematic Constraints : These are equations that relate the motion of two or more particles.

(d) Types of Constraints :

i) General constraints ii) Pulley constraints
iii) Wedge constraints iv) Mixed constraints

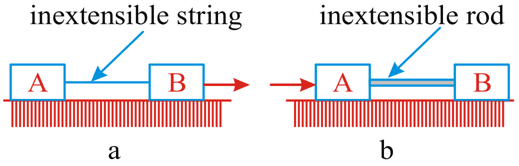
General Constraints:

i) A body placed on floor : The floor acting as a constraint restricts the kinematical quantities in the downward direction such that



y = 0 ; $v_y = 0$ and $a_y = 0$ for the body placed on the floor.

ii) Two bodies connected with a string or rod.



The string / rod is inextensible.

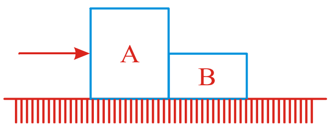
$\therefore$ Displacements of A and B are equal in horizontal direction $\Rightarrow s_A = s_B$

Differentiating w.r.t time, $\frac{ds_A}{dt} = \frac{ds_B}{dt} \Rightarrow v_A = v_B$

Again differentiating $\frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$

iii) Two bodies in contact with each other

↳ Displacement of A and B are equal in horizontal direction.

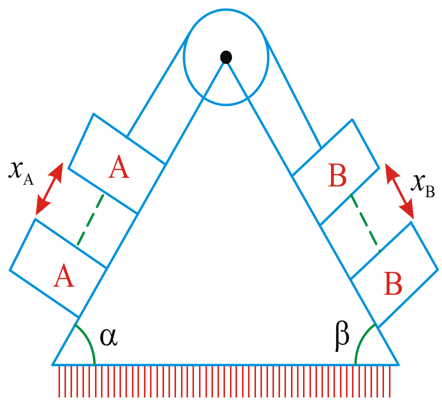


$\Rightarrow s_A = s_B$

By differentiating, we will get $v_A = v_B$ and $a_A = a_B$ in horizontal direction

String Constraints:

↳ For example, the motion of block A is downwards along the inclined plane in fig. will cause a corresponding motion of block B up the other inclined plane.Assuming string AB length is inextensible, i.e., length of AB is constant.



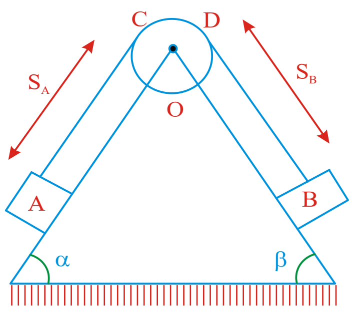
$\therefore$ The displacements of $A(x_A)$ and $B(x_B)$ are equal $\therefore$ $x_A = x_B$

Differentiating w.r.t. time, $\Rightarrow v_A = v_B$

Once again differentiating w.r.t. time, $\Rightarrow a_A = a_B$

i.e., if one body (A) moves down the inclined plane with certain acceleration, then the other body will move up inclined plane with an equal acceleration (magnitude).

Alternate Method : First specify the location of the blocks using position co-ordinates S_A and S_B.



From the fig. the position co-ordinates are related by the equation $s_A + l_{CD} + s_B = L$

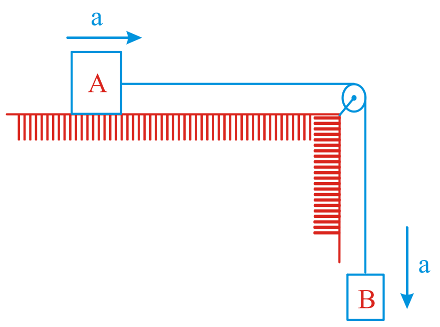
where l_{CD} = the length of the string over arc

CD = constant L = total length of the string = constant Differentiating w.r.t. time, we get

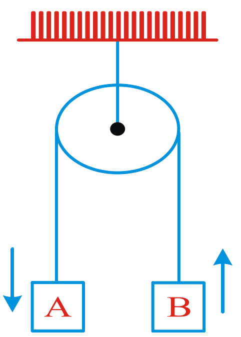
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \quad \Rightarrow v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive s_A, it causes a corresponding upward velocity of block B, i.e., B moves in the negative s_B direction.

Again differentiating w.r.t. time, $\frac{dv_B}{dt} = -\frac{dv_A}{dt} \Rightarrow a_B = -a_A$ Similarly

1) 

$$x_A = x_B \Rightarrow v_A = v_B \Rightarrow a_A = a_B$$

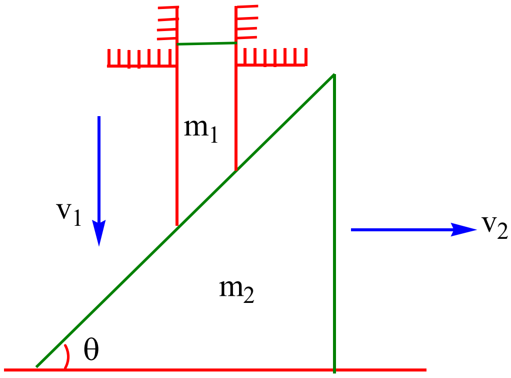
2) 

$x_A = x_B \Rightarrow v_A = v_B \Rightarrow a_A = a_B$

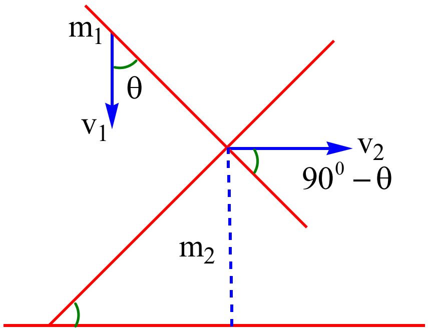
Wedge constraints :

For wedges in contact the constriant is that velocity and acceleration along common normal is same for both bodies

Ex. 30a : Find the relation between velocity of rod and that of wedge at any instant



Sol. Component of velocityalong perpendicular to the contact plane must be equal.

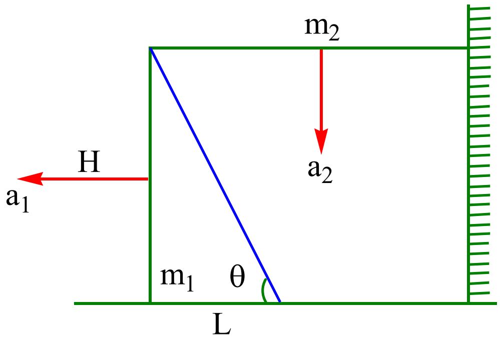


$v_1 \cos\theta = v_2 \cos\theta\left(90^0 - \theta\right)$

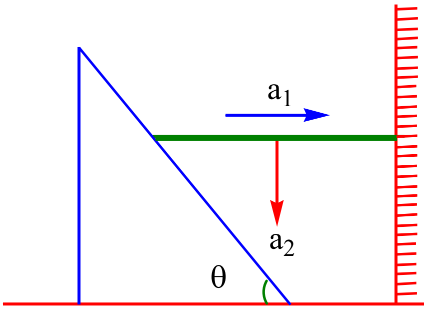
$v_1 \cos\theta = v_2 \sin\theta$

$\tan\theta = \frac{v_1}{v_2}$

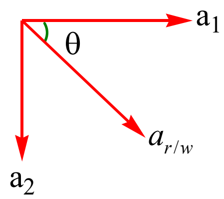
Ex. 30b. Find the relation between a_1 and a_2



Sol. Accelaration of rod w.r.t wedge



$$a_{r/w} = a_1\hat{i} - a_2\hat{j}$$



$\vec{a}_{r/w}$ must be along the incline

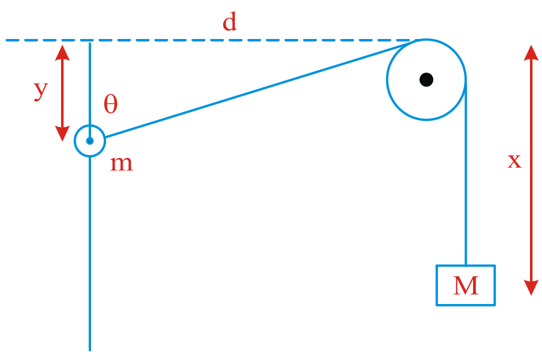
$$\tan\theta = \frac{a_2}{a_1}$$

Mixed constraints :

Ring sliding on a smooth rod :

↳ Consider a ring of mass m connected through a string of length L with a block of mass M. If the ring is moving up with acceleration a_m and a_M is the acceleration of block. As the length of the string is constant,

$$L = \sqrt{d^2 + y^2} + x$$



Since, L is constant, differentiating with respect to time t, we get

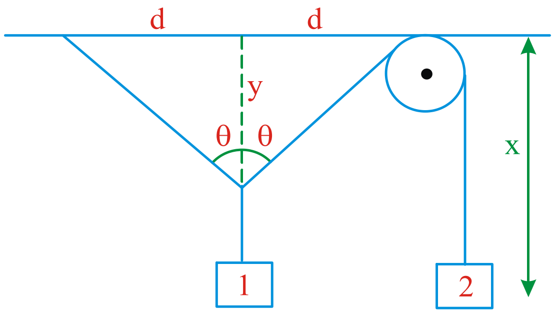
$$\frac{dL}{dt} = \frac{1}{2}\frac{2y}{\left(d^2 + y^2\right)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

Since $\dfrac{dy}{dt} = v_m$ *and* $\dfrac{dx}{dt} = v_M$ and

$\cos\theta = \frac{y}{\sqrt{d^2+y^2}}$ so $v_M = -v_m \cos\theta$.

By differentiating, relation between a_m and a_M can be obtained, however, while doing so remember that $\cos\theta$ is not constant, but it is variable.

Two blocks connected with pulley : If the blocks are connected as shown in fig, then the length of the string is



$$L = 2\sqrt{d^2+y^2} + x$$

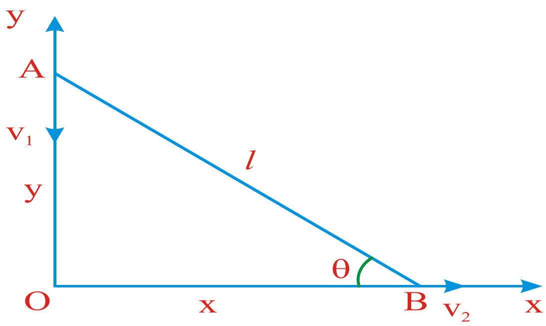
Since, L is constant, differentiating with respect to time t, we get

$$\frac{dL}{dt} = \frac{2\times 2y}{2\left(d^2+y^2\right)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

$$\Rightarrow 2v_1\cos\theta + v_2 = 0;\ v_2 = -2v_1\cos\theta$$

EX.31: A rod of length 'l' is inclined at an angle 'θ' with the floor against a smooth vertical wall. If the end A moves instantaneously with velocity v_1, what is the velocity of end B at the instant when rod makes 'θ' angle with the horizontal.

Sol: Let at any instant, end B and A are at a distance x and y respectively from the point 'O'.



Thus we have, $x^2 + y^2 = l^2$ ....(1)

Here l is the length of the rod, which is constant. Differentiating eq (1) with respect to time, we get

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(l^2);\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

If $\dfrac{dx}{dt} = v_2$ and $\dfrac{dy}{dt} = -v_1$

$x(v_2) + y(-v_1) = 0 \qquad \Rightarrow v_2 = \dfrac{y}{x} v_1 \ = v_1 \tan\theta$

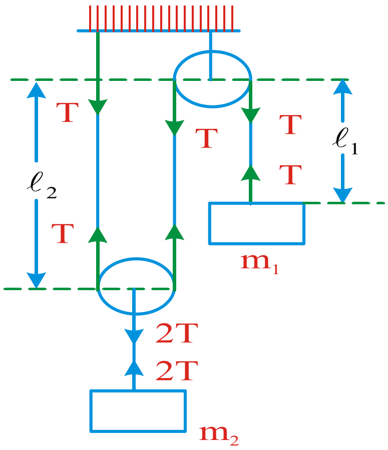
EX.32:In the fig, find the acceleration of mass m_2

Sol: $\ell_1 + 2\ell_2 = constant$

on differentiating $\quad v_1 + 2v_2 = 0$

Again differentiating

$a_1 + 2a_2 = 0 \Rightarrow a_1 = -2a_2$

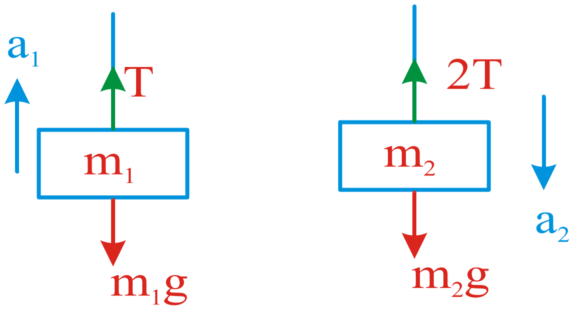


'- ve' sign indicates that the accelerations are in opposite direction.Suppose acceleration of m_2 is a_2 downward and then acceleration of m_1 will be a_1 upwards.

$T - m_1 g = m_1 a_1$

$T = m_1 g + m_1 a_1$

$m_2 g - 2T = m_2 a_2$



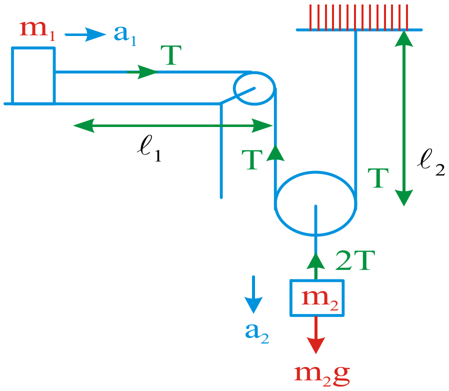
$m_2 g - 2(m_1 g + m_1 a_1) = m_2 a_2$

$m_2 g - 2m_1 g = m_2 a_2 + 4m_1 a_2 \quad (\because a_1 = 2a_2)$

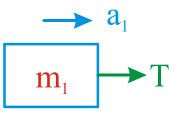
'–' sign should not be substituted

$a_2 = \dfrac{(m_2 - 2m_1)g}{4m_1 + m_2} \quad ms^{-2}$

EX.33: In the fig, find the acceleration of m_1 and m_2



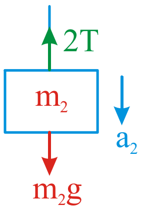
Sol. $\ell_1 + 2\ell_2 = constant$



$a_1 = 2a_2$; $T = m_1a_1$

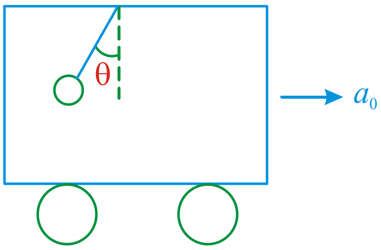
$m_2g - 2T = m_2a_2$; $m_2g = 2m_1a_1 + m_2a_2$

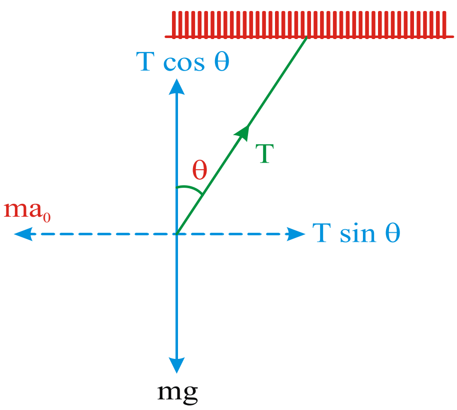
$m_2g = 2m_1(2a_2) + m_2a_2$



$a_2 = \dfrac{m_2g}{4m_1 + m_2}$, $a_1 = \dfrac{2m_2g}{4m_1 + m_2}$

EX.34: A pendulum is hanging from the ceiling of a car having an acceleration a_0 with respect to the road. Find the angle made by the string with vertical at equilibrium. Also find the tension in the string in this position.





Sol : $T\sin\theta = ma_0 ......(i)$; $T\cos\theta = mg......(ii)$

dividing (i)&(ii) $\tan\theta = \frac{a_0}{g}$

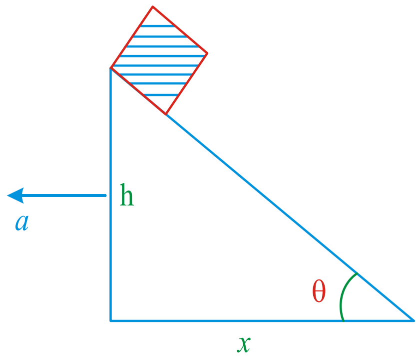
$\therefore$ The string is making an angle $\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$ with the vertical at equilibrium

Squaring and adding (i) and (ii)

$$T^2\sin^2\theta + T^2\cos^2\theta = m^2\left(a_0^2 + g^2\right)$$

$$T = m\sqrt{a_0^2 + g^2}$$

EX.35: For what value of 'a' the block falls freely?

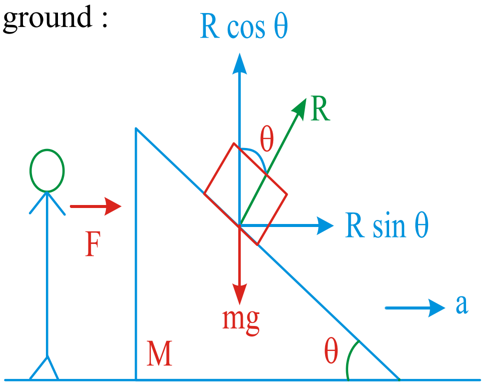


Sol : In the time the wedge moves a distance 'x' towards left with an acceleration a the block falls from a height 'h' with acceleration 'g'

$$x = \frac{1}{2}at^2, h = \frac{1}{2}gt^2 \Rightarrow \frac{x}{h} = \frac{a}{g}, \Rightarrow \cot\theta = \frac{a}{g} \Rightarrow a = g\cot\theta$$

EX.36: A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. Find the i) Acceleration of the wedge ii) Force to be applied on the wedge iii) Force exerted by the wedge on the block.

Sol. (i). For an observer on the ground :



$R\sin\theta = ma, R\cos\theta = mg$

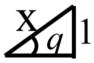
$\Rightarrow a = g\tan\theta$

ii) F = (M + m)a = (M + m) g tan θ

iii) Force exerted by the wedge on the block

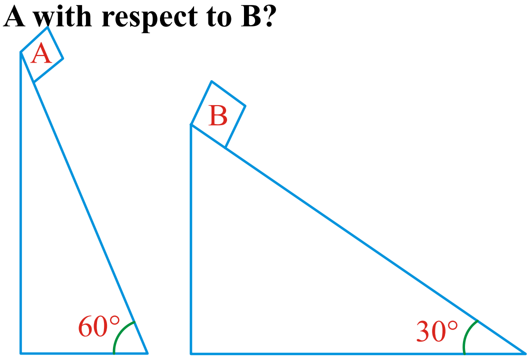
$$\Rightarrow R = \frac{mg}{\cos\theta} \text{ or } R = mg\sec\theta$$

Note : If inclination is given as 1 in x, $\sin\theta = \frac{1}{x}$

$\tan\theta = \frac{1}{\sqrt{x^2-1}}$ 

$\Rightarrow$ Acceleration $\quad a = g\tan\theta = \frac{g}{\sqrt{x^2-1}}$

EX.37: Two fixed frictionless inclined planes making an angles 30^0 and 60^0 with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? (AIEEE-2010)



Sol: $mg\sin\theta = ma \Rightarrow a = g\sin\theta$

where a is along the inclined plane

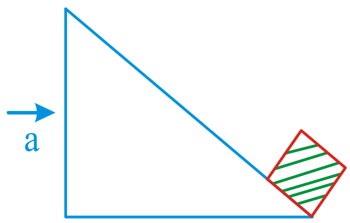
$\therefore$ vertical component of acceleration is $g\sin^2\theta$

$a_r = a_{AB} = a_A - a_B$

$\therefore$ relative vertical acceleration of A with respect to B is $g\left(\sin^2 60^0 - \sin^2 30^0\right) = \frac{g}{2} = 4.9ms^{-2}$

(in vertical direction)

EX.38: For what value of 'a' block slides up the plane with an acceleration 'g' relative to the inclined plane.

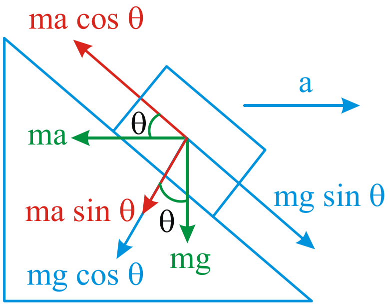


Sol.

$F_{net} = ma\cos\theta - mg\sin\theta$

$ma' = ma\cos\theta - mg\sin\theta$

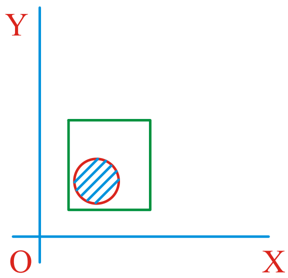
If $a' = g$, $mg = ma\cos\theta - mg\sin\theta$



$$a\cos\theta = g + g\sin\theta \Rightarrow a = g\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

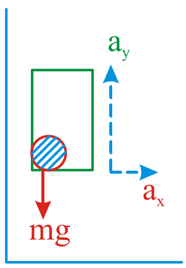
$$\Rightarrow a = g(\sec\theta + \tan\theta)$$

EX.39: **A solid sphere of mass 2kg rests inside a cube as shown. The cube is moving with velocity $\vec{\upsilon} = (5t\hat{i} + 2t\hat{j})ms^{-1}$ where 't' is in sec and 'υ' is in m/s. What force does sphere exert on cube?**



Sol. As given, $\vec{\upsilon} = 5t\hat{i} + 2t\hat{j}$;

$$\therefore a_x = \frac{d\upsilon_x}{dt} = 5,\ a_y = \frac{d\upsilon_y}{dt} = 2$$



When cube is moving with above accelerations along x and y-axes, the forces that exert on cube are

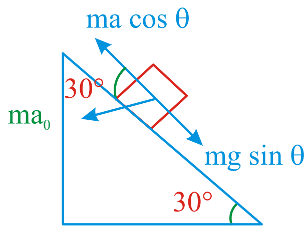
$$F_x = -ma_x = -2\times 5 = -10N$$

$$F_y = -\left(mg + ma_y\right) = -\left(20 + 2\times 2\right) = -24N$$

Net force, $F = \sqrt{(F_x)^2 + (F_y)^2}$

$$= \sqrt{(10)^2 + (24)^2} = 26N$$

EX.40: **A block is placed on an inclined plane moving towards right with an acceleration $a_0 = g$. The length of the inclined plane is l_0. All the surfaces are smooth. Find the time taken by the block to reach from bottom to top.**



Sol. $ma = ma_0 \cos 30^0 - mg \sin 30^0$

$$a = \frac{ma_0 \cos 30^0 - mg \sin 30^0}{m}$$

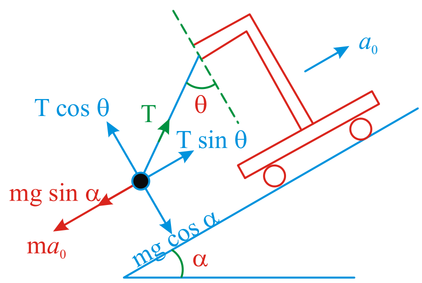
$$a = \frac{ma_0 \frac{\sqrt{3}}{2} - mg \frac{1}{2}}{m} \qquad = g\left(\frac{\sqrt{3}-1}{2}\right)$$

from $s = ut + \frac{1}{2}at^2$; $\qquad \ell_0 = \frac{1}{2}at^2$

$$\ell_0 = \frac{1}{2} g \left(\frac{\sqrt{3}-1}{2}\right) t^2 \Rightarrow t = \sqrt{\frac{4\ell_0}{g\left(\sqrt{3}-1\right)}} \text{ sec}$$

EX.41: **A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is**

Sol.



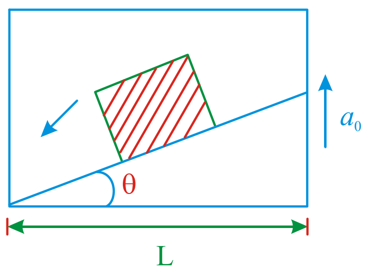
$T \sin\theta = ma_0 + mg \sin\alpha$ -------------(1)

$T \cos\theta = mg \cos\alpha$ -------------(2)

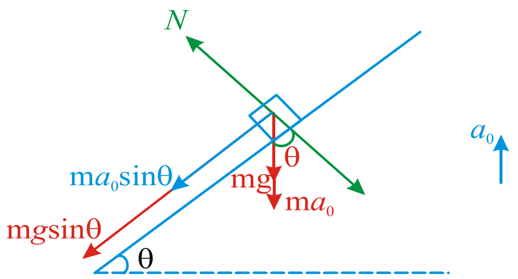
$$\frac{①}{②} \Rightarrow \tan\theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right]}$$

EX.42:A block slides down from top of a smooth inclined plane of elevation θ fixed in an elevator going up with an acceleration a_0.The base of incline has length L. Find the time taken by the block to reach the bottom.



Sol. Let us solve the problem in the elevator frame. The free body diagram is shown. The forces are



(i) N normal reaction to the plane,
(ii) mg acting vertically downwards,
(iii) ma_0 (pseudo force).acting vertically down

If a is acceleration of the body with respect to inclined plane, taking components of forces parallel to the inclined plane.

$mg\sin\theta + ma_0\sin\theta = ma \quad \therefore \quad a = (g + a_0)\sin\theta$

This is the acceleration with respect to the elevator

The distance travelled is $\frac{L}{\cos\theta}$. If 't' is the time for reaching the bottom of inclined plane

$$\frac{L}{\cos\theta} = 0 + \frac{1}{2}(g + a_0)\sin\theta . t^2$$

$$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{\frac{1}{2}} = \left[\frac{4L}{(g + a_0)\sin 2\theta}\right]^{\frac{1}{2}}$$

Law of conservation of momentum:

- When the resultant external force acting on a system is zero, the total momentum (vector sum) of the system remains constant. This is called "law of conservation of linear momentum".
- Newton's third law of motion leads to the law of conservation of linear momentum.
- Walking, running, swimming, jet propulsion, motion of rockets, rowing of a boat, recoil of a gun etc., can be explained by Newton's third law of motion.
- Explosions, disintegration of nuclei, recoil of gun, collisions etc., can be explained on the basis of the law of conservation of linear momentum.

Applications:

- When a shot is fired from a gun, while the shot moves forwards, the gun moves backwards. This motion of gun is called **recoil of the gun**. When a gun of mass 'M' fires a bullet of mass 'm' with a muzzle velocity 'v', the gun recoils with a velocity 'V' given by V = mv/M.
- When a bullet of mass 'm' moving with a velocity 'v' gets embedded into a block of mass M at rest and free to move on a smooth horizontal surface, then their common velocity ` V = mv/ (M + m).
- A boy of mass 'm' walks a distance 's' on a boat of mass 'M' that is floating on water and initially at rest. If the boat is free to move, it moves back a distance d = ms / (M + m).

Explosion of Bomb

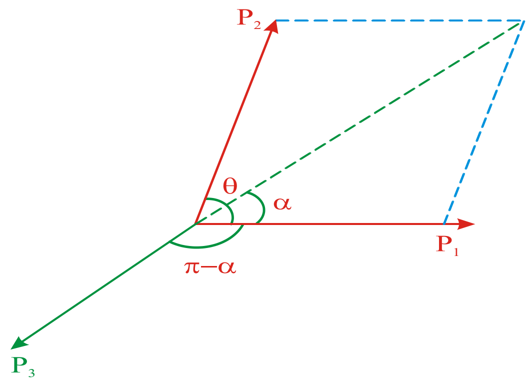
- A shell of mass 'M'at rest explodes into two fragments and one of masses 'm' moves out with a velocity 'v' the other piece of mass (M– m) moves in opposite direction with a velocity of V = mv/ (M – m).
- Suppose a shell of mass m at rest explodes into three pieces of masses m_1, m_2 and m_3, moving with velocities $\overrightarrow{v_1}, \overrightarrow{v_2}$ and $\overrightarrow{v_3}$ respectively.

$m_1\vec{v}_1 = \vec{p}_1; m_2\vec{v}_2 = \vec{p}_2; m_3\vec{v}_3 = \vec{p}_3$

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + m_3\overrightarrow{v_3} = 0$

(as shell is at rest initially) $\therefore \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

So the third piece moves with the same magnitude of the resultant momentum of the other two pieces but in opposite direction.



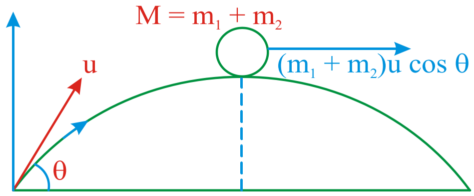
$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}$ $\theta =$ angle between $\vec{P}_1, \vec{P}_2$

$(\pi - \alpha) =$ angle between $\vec{P}_3, \vec{P}_1$

$$\tan\alpha = \frac{P_2\sin\theta}{P_1 + P_2\cos\theta}$$

Explosion of a shell travelling in a parabolic path at its highest point: (into two fragments)

- Consider a shell of mass M as a projectile with velocity u and angle of projection θ. Suppose the shell breaks into two fragments at maximum height and their initial velocities are $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$



Total momentum of the two parts is constant just before and just after the explosion.

$[m_1 + m_2]u\cos\theta\vec{i} = m_1\vec{v}_1 + m_2\vec{v}_2$

Case : (i) If the fragments travel in opposite direction after explosion then

$(m_1 + m_2)u\cos\theta\vec{i} = m_1 v_1\vec{i} - m_2 v_2\vec{i}$

Case : (ii) If one fragment retraces its path and falls at the point of projection

$(m_1 + m_2)u\cos\theta\vec{i} = -m_1 u\cos\theta\vec{i} + m_2\vec{v}_2$

Case:(iii) If one fragment falls freely after explosion

$(m_1 + m_2)u\cos\theta\vec{i} = m_1 0 + m_2\vec{v}_2$

$(m_1 + m_2)u\cos\theta\vec{i} = m_2\vec{v}_2$

EX.43:A bomb moving with velocity(40i+50j-25k)m/s explodes into two pieces of mass ratio1:4.After explosion the smaller piece moves away with velocity(200i+70j+15k)m/s.The velocity of larger piece after explosion is (EAM-2010)

Sol: From Law of conservation of linear momentum

$MU = m_1 v_1 + m_2 v_2; M = 5x, m_1 = x, m_2 = 4x$

$U = 40\hat{i} + 50\hat{j} - 25\hat{k}ms^{-1};$

$v_1 = 200\hat{i} + 70\hat{j} + 15\hat{k}ms^{-1}$

here v_2 is the velocity of the larger piece

$5x(40\hat{i} + 50\hat{j} - 25\hat{k}) =$

$x(200\hat{i} + 70\hat{j} + 15\hat{k}) + 4x(v_2)$

On simplification, we get $v_2 = 45\hat{j} - 35\hat{k}$

EX.44:A particle of mass 4 m explodes into three pieces of masses m,m and 2m. The equal masses move along X-axis and Y- axis with velocities 4ms^{-1} and 6 ms^{-1} respectively . The magnitude of the velocity of the heavier mass is (E - 2009)

Sol: M=4m ,U=0, $m_1 = m, \quad m_2 = m, \quad m_3 = 2m$

$v_1 = 4ms^{-1}, \quad v_2 = 6ms^{-1}, v_3 = ?$

According to law of conservation of momentum,

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$

$\vec{P}_3 = -(\vec{P}_1 + \vec{P}_2), |\vec{P}_3| = |\vec{P}_1 + \vec{P}_2|$

$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2Cos\theta}$

P_1 and P_2 are perpendicular to each other

$$P_3 = \sqrt{P_1^2 + P_2^2}\ , m_3v_3 = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$$

$$2mv_3 = \sqrt{(m\times 4)^2 + (m\times 6)^2}$$

$$2v_3 = \sqrt{16+36} \Rightarrow v_3 = \sqrt{13}ms^{-1}$$

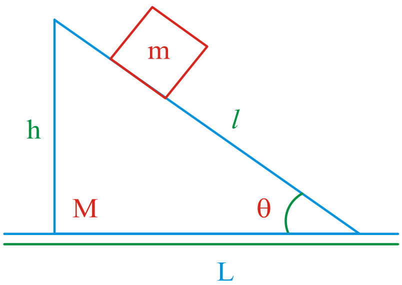
EX.45:A rifle of 20kg mass can fire 4 bullets/s. The mass of each bullet is 35×10^{-3} kg and its final velocity is $400ms^{-1}$. Then, what force must be applied on the rifle so that it does not move backwards while firing the bullets?(2007E)

Sol : Law of conservation of momentum MV + 4mv = 0

$$\Rightarrow V = -\frac{4mv}{M} = -\frac{4\times35\times10^{-3}\times400}{20} = -2.8\,ms^{-1}$$

Force applied on the rifle $F = \frac{MV}{t} = -\frac{20\times2.8}{1} =$ -56 N

EX.46:All surfaces are smooth.Find the horizontal displacements of the block and the wedge when the block slides down from top to bottom.



Sol : When the block slides down on the smooth wedge, the wedge moves backwards. In the horizontal direction there is no external force ; $\vec{F}_x = 0$

$\therefore\ \vec{P}_x$ =constant

$\vec{P}_f = \vec{P}_i$ (along x-axis) ; $\quad m\vec{u} + M\vec{V} = \vec{0}$

x_1 = forward distance moved by the block along X-axis.

x_2 = backward distance moved by the wedge along X-axis.

$m\vec{u} = -M\vec{V}$;

$$m\frac{x_1}{t} = M\frac{x_2}{t}$$

$$mx_1 = Mx_2\ ,\ x_1 = \frac{ML}{M+m} = \frac{M\ell\cos\theta}{M+m}$$

$$x_2 = \frac{mL}{M+m} = \frac{m\ell\cos\theta}{M+m}$$

EX.47: A bomb of 1 kg is thrown vertically up with speed 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400gm goes down with speed 25m/s. What will happen to the other part just after explosion

Sol : After 5 sec, velocity of the bomb,

$v = u + at$

$\vec{v} = u\hat{j} - gt\hat{j} = (100 - 10 \times 5)\hat{j} = 50\hat{j}$ m/s

$m = 1kg, m_1 = 0.4kg, m_2 = 0.6kg, v_1 = 25ms^{-1}$

According to law of conservation of momentum $m\overline{v} = m_1\overline{v}_1 + m_2\overline{v}_2$

$1 \times 50\hat{j} = -0.4 \times 25\hat{j} + 0.6\vec{v}_2$

$\Rightarrow v_2 = 100\hat{j}$

$\therefore v_2 = 100ms^{-1}$, vertically upwards

EX.48: A particle of mass 2m is projected at an angle 45^0 with horizontal with a velocity of $20\sqrt{2}$ m/s. After 1sec, explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. The maximum height attained by the other part from the ground is ($g = 10m/s^2$)

Sol : $M = 2m, \theta = 45^0, u = 20\sqrt{2}ms^{-1}$

$u_x = u\cos\theta = 20\sqrt{2} \times \frac{1}{\sqrt{2}} = 20ms^{-1}$

$u_y = u\sin\theta = 20\sqrt{2} \times \frac{1}{\sqrt{2}} = 20ms^{-1}$

But height attained before explosion , $H_1 = ut - \frac{1}{2}gt^2 = 20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15m$

After 1sec, $v_x = 20ms^{-1}$

$v_y = u_y - gt = 20 - 10 = 10ms^{-1}$

Due to explosion one part comes to rest,

$m_1 = m_2 = m, v_1 = 0$

$M(v_x i + v_y j) = m_1\overline{v}_1 + m_2\overline{v}_2$

$2m(20i + 10j) = m(0) + m\overline{v}_2$

$v_2 = 40i + 20j$

$v_y{}^1 = 20ms^{-1}$

Height attained after explosion $= H_2 = \frac{(v_y{}^1)^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20m$

$H_{TOT} = H_1 + H_2 = 15 + 20 = 35m$

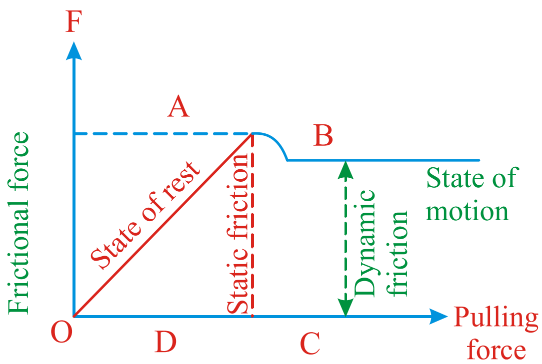
Friction:

If we slide or try to slide a body over another surface, the motion of the body is resisted by bonding between the body and the surface.This resistance is called friction.

- The force of friction is parallel to the contact surfaces and opposite to the direction of intended or relative motion.
- There are three types of frictional forces

  i. Static friction ii. Dynamic friction

  iii. Rolling friction
- If a body is at rest and no pulling force is acting on it,force of friction on it is zero.
- If a force is applied to move the body and it does not move,the friction developed is called **static friction**, which is equal in magnitude and opposite in direction to the applied force (static friction friction is self adjusting force).
- If a force is applied to move the body and it moves,the friction developed is called **dynamic or kinetic friction**.
- When a body rolls or rotates on the surface of another body friction developed is called as **rolling friction**.
- It is due to the deformation at the point of contact and depends on area of contact.



Note-i: If you are walking due east the feet slides relatively due west so the frictional force is due east.

Note-ii: Engine is connected to rear wheels of a car. When the car is accelerated, direction of frictional force on the rear wheels will be in the direction of motion and on the front wheels in the opposite direction of motion

Note-iii: In cycling ,the force exerted by rear wheel on the ground makes the force of friction to act on it in the forward direction. Front wheel moving by itself experience force of friction in backward direction.

Note-iv: When pedaling is stopped,the frictional force is in backward direction for both the wheels.

Laws of Friction:

- Friction is directly proportional to the normal reaction acting on the body.
- The law of static friction may thus be written as

  $f_s \le \mu_s N$. Where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force.

  $(f_s)_{max} = f_l = \mu_s N;\ f_l =$ Limiting friction
- Coefficient of static friction (μ_s) depends on the nature of the two surfaces in contact and is independent of the area of contact.

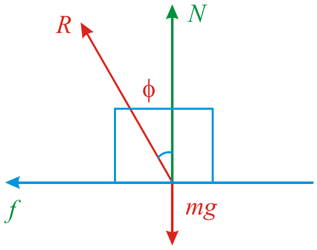
- Static friction is independent of the area of contact between the two surfaces
- Coefficient of kinetic friction $(\mu_k) = \frac{f_k}{N}$. It is independent of velocity of the body.
- Coefficient of rolling friction $(\mu_R) = \frac{f_R}{N}$
- Rolling friction depends on the area of the surfaces in contact.

Note : $\mu_S > \mu_K > \mu_R$

- Friction depends on the nature of the two surfaces in contact i.e., nature of materials, surface finish, temperature of the two surfaces etc.

Angle of Friction:

- Angle made by the resultant of f and N with the normal reaction N is called angle of friction.
- Friction is parallel component of contact force to the surfaces.
- Normal force is perpendicular component of contact force to the surfaces.



$$R = \sqrt{f^2 + N^2}$$

When the block is static $\tan\phi = \frac{f}{N}; \ \phi \le \phi_s$

When the block is in impending state, $\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$

Where $\phi_s \rightarrow$ maximum angle of friction.

When block is sliding, $\tan\phi_k = \frac{\mu_k N}{N} = \mu_k$

Since $\mu_s > \mu_k$, it follows that $\phi_s > \phi_k$.

$$F_R = \sqrt{f_l^2 + N^2} = \sqrt{(\mu_s N)^2 + N^2} = N\sqrt{\mu_s^2 + 1}$$

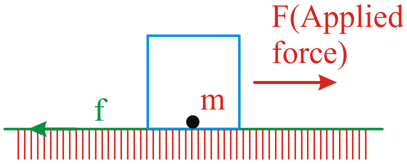
$$F_R = mg\sqrt{\tan^2\phi + 1} \qquad (\therefore \mu_s = \tan\phi)$$

$$F_R = mg\sec\phi$$

Motion on a horizontal rough surface:

Consider a block of mass m place on a horizontal surface with normal reaction N.

Case (i) : If applied force F = 0, the force of friction is zero.



Case (ii) : If applied force $F < (f_s)_{max}$, the block does not move and the force of friction is $\boxed{f_s = F}$

Case (iii) : If applied force F = $(f_s)_{max}$ block just ready to slide and frictional force $\boxed{(f_s)_{max} = f_l = \mu_s N}$

$$F = \mu_s mg \qquad (\because N = mg); \quad \text{(at time t=0)}$$

Case(iv) : If the above applied force continues to act $(t \neq 0)$ the body gets motion, static friction converts as kinetic friction and body possesses acceleration

$$a = \frac{F_{ext} - f_k}{m} = \frac{f_l - f_k}{m} = (\mu_s - \mu_k) g$$

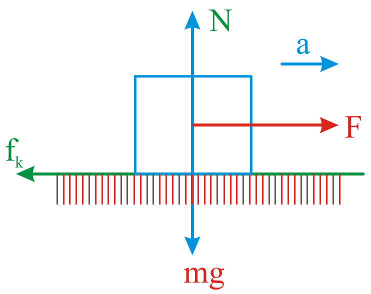
Case (v) : If the applied force is greater than limiting friction the body starts moving and gets acceleration

$$a = \frac{F^1{}_{ext} - f_k}{m} \qquad Here\ F^1{}_{ext} > F_{ext}$$

↳ If the block slides with an acceleration 'a' under the influence of applied force 'F',

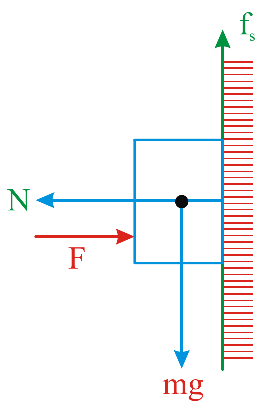
$$F_R = F - f_k \quad ; \quad ma = F - f_k$$

$$\therefore a = \frac{F - f_k}{m} = \frac{F - \mu_k mg}{m} \quad (f_k = \mu_K N = \mu_K mg)$$



Bodies in contact with vertical surfaces:

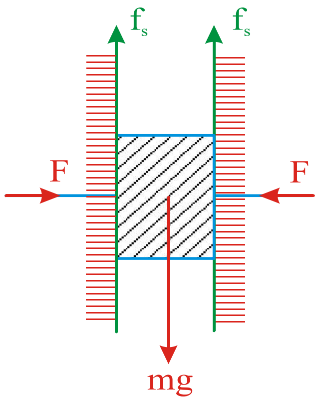
↳ A block of mass m is pressed against a wall without falling, by applying minimum horizontal force F. Then $F = \frac{mg}{\mu_s}$



$$f_s = mg \; ; \quad \mu_s N = mg$$

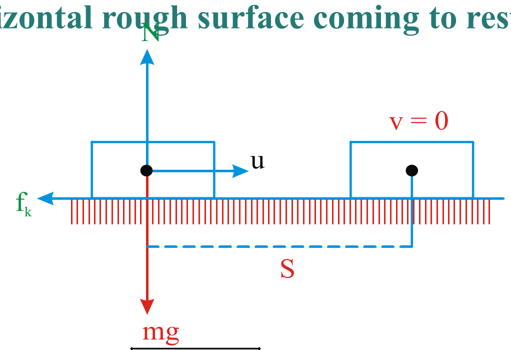
$$\mu_s F = mg (\because N = F) \quad \Rightarrow F = \frac{mg}{\mu_s}$$

↳ A block is pressed between two hands without falling, by applying minimum horizontal force 'F' by each hand. Then $F = \frac{mg}{2\mu_s}$



$W = 2 f_s$; $mg = 2\mu_s F \Rightarrow F = \frac{mg}{2\mu_s}$

Sliding block on a horizontal rough surface coming to rest :

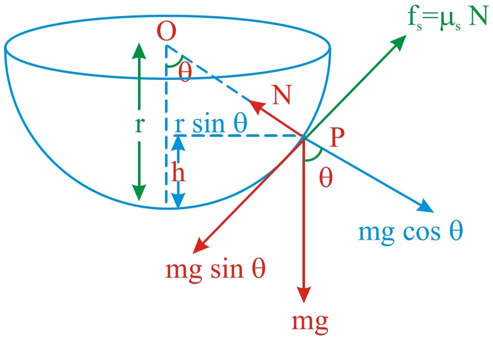


a) The acceleration of the block is $\boxed{a = -\mu_k g}$

b) Distance travelled by the block before coming to rest is $\boxed{S = \frac{u^2}{2\mu_k g}}$

c) Time taken by the block to come to rest is $t = \frac{u}{\mu_k g}$

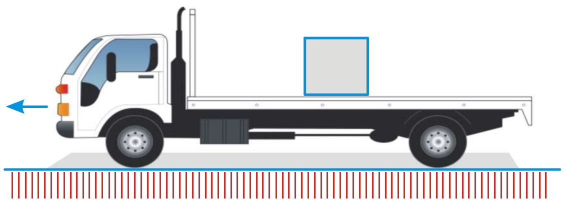
↳ An insect is crawling in a hemispherical bowl of radius 'r'. Maximum height upto which it can crawl is



$$h = r\left(1-\cos\theta\right) = r\left(1-\frac{1}{\sqrt{\mu_s^2+1}}\right)$$

Maximum angular displacement upto which it can crawl is 'θ'. Then $\mu_s = \tan\theta$

↳ A block is placed on rear horizontal surface of a truck moving along the horizontal with an acceleration 'a'. Then



1) The maximum acceleration of the truck for which block does not slide on the floor of the truck is $a = \mu_s g$

2) If $a < \mu_s g$ block does not slide and frictional force on the block is f=ma.

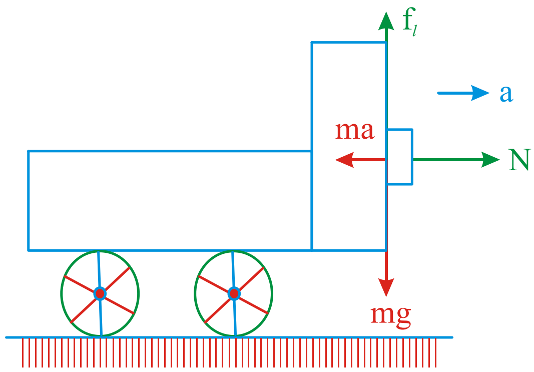
3) If $a > \mu_s g$ block slips or slides on the floor the acceleration of the block relative to the truck is $a' = a - \mu_k g$

4) If ℓ is the distance of the block from rear side of the truck, time taken by the block to cover a distance ℓ. $\quad t = \sqrt{\frac{2\ell}{a - \mu_k g}}$

4) Acceleration of the block relative to ground is $a'' = \mu_k g$

Body placed in contact with the front surface of accelerated truck:

↳ When a block of mass 'm' is placed in contact with the front face of the vehicle moving with acceleration a then a pseudo force 'F_{pf}' acts on the block in a direction opposite to the direction of motion of the vehicle

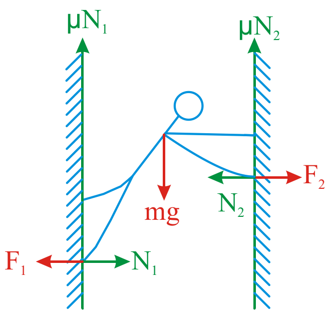


Under equilibrium, $\quad f_l = mg;\ N = ma$

$$\mu_S N = mg \Rightarrow \mu_S .ma = mg \Rightarrow a_{min} = \frac{g}{\mu_s}$$

EX.49: **A man of mass 40 kg is at rest between the walls as shown in the figure. If 'm' between the man and the walls is 0.8, find the normal reactions exerted by the walls on the man.**

Sol. Since man is at rest,



$N_1 - N_2 = 0$ (horizontal equilibrium)

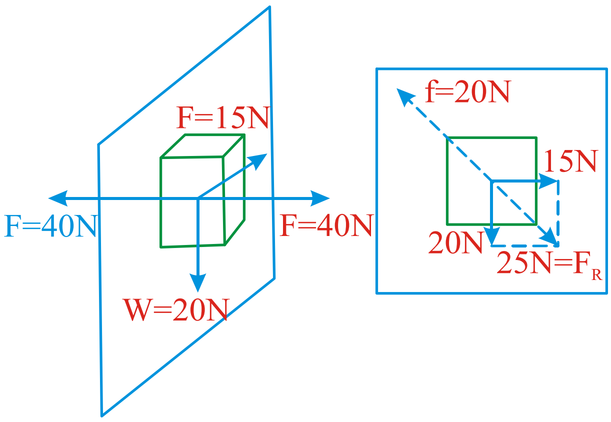
$\therefore N_1 = N_2 = N$, $F_1 = F_2 = \text{F (say)}$

$\therefore 2\mu N = mg$ (vertical equilibrium)

$= 2 \times 0.8 \times N = 400 \quad \therefore N = 250N$

EX.50: **A 2 kg block is in contact with a vertical wall having coefficient of friction 0.5 between the surfaces. A horizontal force of 40N is applied on the block at right angles to the wall. Another force of 15N is applied, on the plane of the wall and at right angles to 40N force. Find the acceleration of the block.**

Sol.



Resultant of W=20N and 15N

$F_R = \sqrt{20^2 + 15^2} = 25\text{N}$

frictional force $f = \mu N = 0.5 \times 40 = 20\text{N}$

This acts in a direction, opposite to 25N force.

$\therefore$ Net force acting on the block, $F_{net} = 25 - 20 = 5$

$\therefore$ acceleration of the block $a = \frac{5}{2} = 2.5\text{ms}^{-2}$

EX.51:A block of mass 4 kg is placed on a rough horizontal plane. A time dependent horizontal force F = kt acts on the block (k = 2 N/s). Find the frictional force between the block and the plane at t = 2 seconds and t = 5 seconds (μ = 0.2)

Sol. Given F = kt

When t = 2sec ; F = 2(2)=4N ..... case (i)

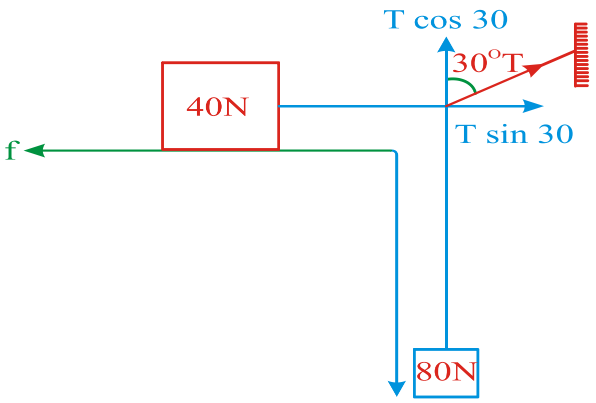
$f_{ms} = \mu_s mg = 0.2 \times 4 \times 10 = 8N$

Here F < f_{ms} $\therefore$ friction = applied force=4N

When t = 5 sec ; F = 2 (5) = 10N......case(ii)

F > f $\therefore$ frictional force < 8N

EX.52:A block on table shown in figure is just on the edge of slipping. Find the coefficient of static friction between the block and table



Sol.

$f_l = T\sin\theta$

$\mu mg = T\sin\theta$ ....(1)

$80 = T\cos\theta$ ....(2)

$\frac{T\sin 30^0}{T\cos 30^0} = \frac{\mu mg}{80}$;

$\text{Tan}\,30^0 = \frac{\mu 40}{80}$; $\frac{1}{\sqrt{3}} = \frac{\mu}{2}$ $\Rightarrow \mu = \frac{2}{\sqrt{3}} = 1.15$

EX.53:When a car of mass 1000 kg is moving with a velocity of 20ms^{-1} on a rough horizontal road, its engine is switched off. How far does the car move before it comes to rest if the coefficient of kinetic friction between the road and tyres of the car is 0.75 ?

Sol. Here v = $20ms^{-1}$, $\mu_k = 0.75$, g = $10ms^{-2}$

Stopping distance $S = \frac{v^2}{2\mu_k g} = 26.67m$

EX.54:**A horizontal conveyor belt moves with a constant velocity V. A small block is projected with a velocity of 6 m/s on it in a direction opposite to the direction of motion of the belt. The block comes to rest relative to the belt in a time 4s.** $\mu = 0.3$**, g = 10 m/s^2. Find V**

Sol. $\left|\vec{V}_{b,c}\right| = V_b + V_c = 6 + V$

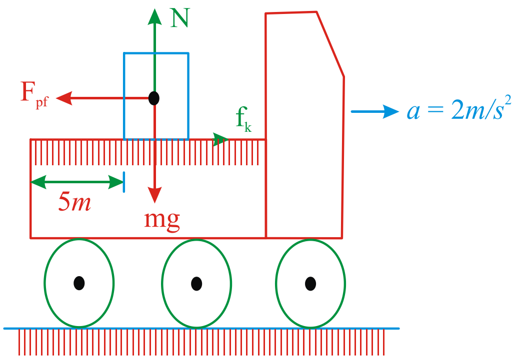
$f = \mu mg = 0.3 \times m \times 10 = 3m$

Retardation $a = \dfrac{F}{m} = \dfrac{3m}{m} = 3m/s^2$

$u_r = 6 + V, V_r = 0, t = 4\sec$, $a_r = -3ms^{-2}$

$V_r = u_r + a_r t$, $0 = (6+V) - 3 \times 4, V = 6$ m/s

EX.55:The rear side of a truck is open. A box of 40 kg mass is placed 5m away from the open end as shown in figure. The coefficient of friction between the box and the surface is 0.15. On a straight road, the truck starts from rest and accelerating with 2 m/s^2. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).



Sol : Because of the acceleration of the truck the pseudo force on the box = m x a = 40 x 2 = 80N. This force acts opposite to the acceleration of the truck. The frictional force on the truck which acts in the forward direction $f_k = \mu N = 0.15 \times 40g$ = 58.8N Since pseudo force is greater than frictional force, the block will accelerate in backward direction relative to truck with a magnitude

$a = \dfrac{80 - 58.8}{40} = 0.53\, m/s^2$

The time taken by box to cover the distance 5m is given by

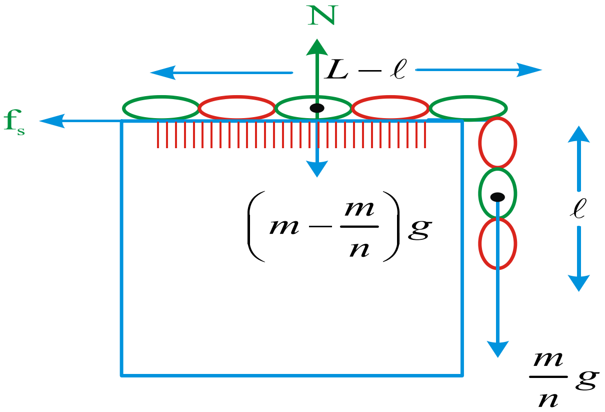
$s = 0 + \dfrac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{2s}{a}} = 4.34\sec$

The distance travelled by truck in this time is , $a' = 2ms^{-2}$

$s' = \dfrac{1}{2}a't^2 = \dfrac{1}{2} \times 2 \times (4.34)^2 = 18.87m$

Sliding of a chain on a horizontal table:

↳ Consider a uniform chain of mass "m" and length "L" lying on a horizontal table of coefficient of friction "μ_s". When 1/nth of its length is hanging from the edge of the table, the chain is found to be about to slide from the table. Weight of the hanging part of the chain = $\dfrac{mg}{n}$



Weight of the chain lying on the table = $mg-\frac{mg}{n}=mg\left(1-\frac{1}{n}\right)$

When the chain is about to slide from edge of the table,
The weight of the hanging part of the chain = frictional force between the chain and the table surface.

$$\frac{mg}{n}=\mu_s mg\left(1-\frac{1}{n}\right)$$

$$\Rightarrow \frac{mg}{n}=\mu_s mg\left(\frac{n-1}{n}\right) \qquad \therefore \mu_s=\frac{1}{(n-1)}$$

If l is the length of the hanging part, then $n=\frac{L}{l}$ Substituting this in the above expression we get,

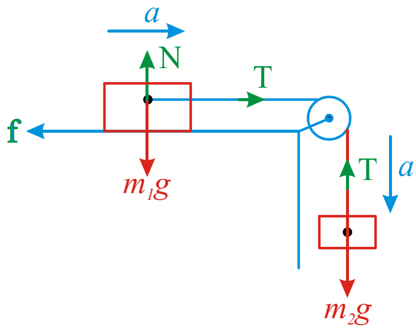
$$\mu_s=\frac{l}{L-l}(or)\,n=\frac{L}{l}=\frac{\mu_s+1}{\mu_s}$$

$\therefore$ The maximum fractional length of chain hanging from the edge of the table in equilibrium is $\frac{\ell}{L}=\frac{\mu_s}{\mu_s+1}$

↳ Fractional length of chain on the table $\frac{L-\ell}{L}=\frac{1}{\mu_s+1}$

Connected Bodies :

↳ A block of mass m_1 placed on a rough horizontal surface, is connected to block of mass m_2 by a string which passes over a smooth pulley. The coefficient of frictionbetween m_1 and the table is μ.



For body of mass m_2

$m_2g - T = m_2a$ ———— (i)

For body of mass m_1

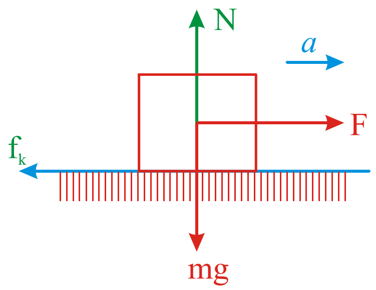
$T - f_k = m_1a \Rightarrow T - \mu_k N = m_1a$ ——— (ii)

Solving Eqs (i) and (ii), we get

$$a = \left(\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right)g \; ; \; T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \mu)$$

EX.56:A block of mass 10kg is pushed by a force F on a horizontal rough plane is moving with acceleration $5ms^{-2}$. When force is doubled, its acceleration becomes $18ms^{-2}$.Find the coefficient of friction between the block and rough horizontal plane. $\left(g = 10ms^{-2}\right)$.

Sol :



On a rough horizontal plane, acceleration of a block of mass 'm' is given by $a = \frac{F}{m} - \mu_k g$ ....(i)

Initially, a = $5ms^{-2}$

$5 = \frac{F}{10} - \mu_k(10)$............$(ii)$ $(\because m = 10kg)$

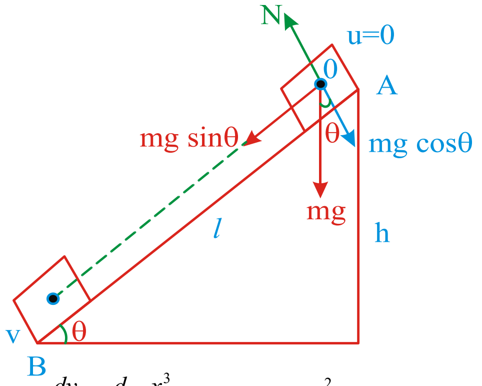
When force is doubled a =$18ms^{-2}$.

$18 = \frac{2F}{10} - \mu_k(10)$..........$(iii)$

Multiplying Eq(ii) with 2 and subtracting from Eq.(iii) $8 = \mu_k(10) \Rightarrow \mu_k = \frac{8}{10} = 0.8$

EX.57:A block of mass 'm' is placed on a rough surface with a vertical cross section of $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is(JEE MAIN -2014)

Sol:



$Tan\theta = \frac{dy}{dx} = \frac{d}{dx}(\frac{x^3}{6}) \Rightarrow Tan\theta = \frac{x^2}{2}$

At limiting equilibrium,we get $\mu = Tan\theta \Rightarrow 0.5 = \frac{x^2}{2}$

$x^2 = 1 \Rightarrow x = \pm 1$

Now putting the values of 'x'in $y = \frac{x^3}{6}$, we get

When $x = 1 \Rightarrow y = \frac{1}{6}; x = -1 \Rightarrow y = -\frac{1}{6}$

So the maximum height above the ground at which the block can be placed without slipping is

$y = \frac{1}{6}m$

Motion of a body on an inclined plane :

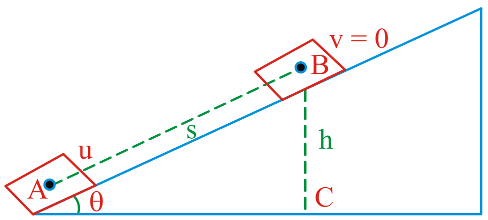
Case (i) :Body sliding down on a smooth inclined plane :

Let us consider a body of mass 'm' kept on the plane as shown in fig.

- Normal reaction $N = mg\cos\theta$
- Acceleration of sliding block a = g sin θ
- If l is the length of the inclined plane and h is the height. The time taken to slide down starting from rest from the top is $t = \sqrt{\frac{2l}{g\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$
- Sliding block takes more time to reach the bottom than to fall freely in air from the top of the inclined plane to the ground.
- Velocity of the block at the bottom of the inclined plane is same as the speed attained if block falls freely from the top of the inclined plane.

$V = \sqrt{2gl\sin\theta} = \sqrt{2gh}$

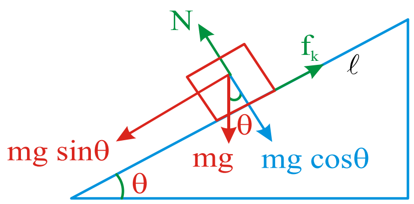
Case(ii) :Body projected up on a smooth inclined plane :



- If a block is projected up the plane with a velocity u, the acceleration of the block is $a = -g\sin\theta$
- Distance travelled by the block up the plane before its velocity becomes zero is $S = \dfrac{u^2}{2g\sin\theta}$
- Time of ascent $t = \dfrac{u}{g\sin\theta}$

Case (iii) Motion of a body down the rough inclined plane:

- Let a body of mass m be sliding down a rough inclined plane of angle of inclination θ and coefficient of kinetic friction μ_k.

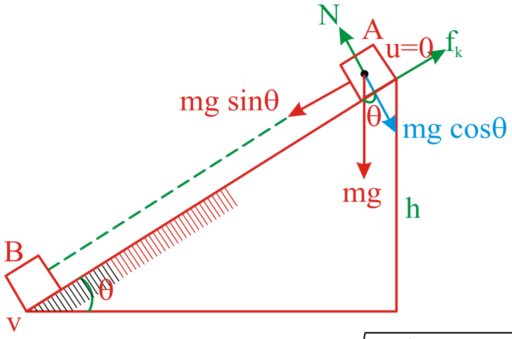


Angle of Repose (α) : Angle of repose is the minimum angle of the rough inclined plane for which body placed on it may just start sliding down. It is numerically equal to the angle of friction.

- Let θ be the angle of inclination of a rough inclined plane, α be the angle of repose, m be the mass of the body and μ be the coefficient of friction.
  At limiting equilibrium (about to slide)
  $mg\sin\alpha = \mu_s mg\cos\alpha \Rightarrow \tan\alpha = \mu_s \Rightarrow \alpha = \tan^{-1}(\mu_s)$

1. When $\theta_1 < \alpha$; the block remains at rest on the inclined plane. Frictional force $mg\sin\theta_1$ (self adjusting) acceleration a=0
2. When $\theta_2 = \alpha$; the block remains at rest on inclined plane or impending state of motion is achieved.
   $mg\sin\theta_2 = \mu_s mg\cos\theta_2$ (at time t=0)
   Here $\theta_2 > \theta_1$ and $f_s = f_l$ acceleration a=0
3. When $\theta_2 > \alpha$; and the same inclination is continued the block moves downwards with accleration a.
   $mg\sin\theta_2 > \mu_k mg\cos\theta_2$ acceleration
   $$a = \frac{mg\sin\theta_2 - \mu_k mg\cos\theta_2}{m}$$
   $$a = \frac{\mu_s mg\cos\theta_2 - \mu_k mg\cos\theta_2}{m} = g\cos\theta_2(\mu_s - \mu_k)$$
4. When $\theta > \alpha$, the body slides $f_k = \mu_k mg\cos\theta$
   The resultant force acting on the body down the plane is $F_R = mg\sin\theta - f_K$,
   $F_R = mg(\sin\theta - \mu_k\cos\theta)$

The acceleration of the body $a = g(\sin\theta - \mu_k\cos\theta)$



↳ Velocity of the body at the bottom of the plane $V = \sqrt{2g(\sin\theta - \mu_k \cos\theta)l}$

↳ If 't'is the time taken to travel the distance 'l' with initial velocity $u = 0$, at the top of the plane,

$$t = \sqrt{\frac{2l}{g(\sin\theta - \mu_k \cos\theta)}}$$

↳ The time taken by a body to slide down on a rough inclined plane is 'n' times the time taken by it to slide down on a smooth inclined plane of same inclination and length then coefficient of friction is....

$$n = \frac{t_{rough}}{t_{smooth}} = \frac{\sqrt{\frac{2l}{g(\sin\theta - \mu_k \cos\theta)}}}{\sqrt{\frac{2l}{g\sin\theta}}} \qquad n^2 = \frac{\sin\theta}{\sin\theta - \mu_k \cos\theta}$$

$$\Rightarrow n^2 \sin\theta - n^2 \mu_k \cos\theta = \sin\theta \qquad \Rightarrow \mu_k = Tan\theta\left[1 - \frac{1}{n^2}\right]$$

Body projected up a rough inclined plane:

If a body is projected with an initial velocity 'u'to slide up the plane, the kinetic frictional force acts down the plane and the body suffers retardation due to a resultant force

$F_R = (mg \sin\theta + f_k)$

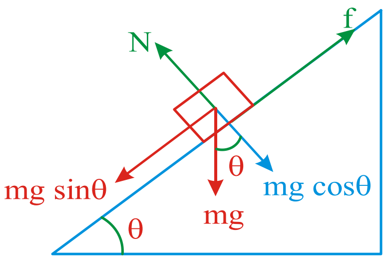
↳ acceleration $a = -g(\sin\theta + \mu_k \cos\theta)$

↳ Time taken to stop after travelling a distance l along the plane, $t = \sqrt{\frac{2l}{g(\sin\theta + \mu_k \cos\theta)}}$

↳ Force required to drag with an acceleration 'a' is $F = (\mu_k mg\cos\theta + mg\sin\theta + ma)$

EX.58:A body is moving down a long inclined plane of angle of inclination 'θ'for which the coefficient of friction varies with distance x as $\mu(x) = kx$, where k is a constant. Here x is the distance moved by the body down the plane. The net force on the body will be zero at a distance x_0 is given by

Sol :



$F = mg\sin\theta - f$

$N = mg\cos\theta$; $f = \mu N = \mu mg\cos\theta$

$F = mg\sin\theta - \mu mg\cos\theta$

$F = mg(\sin\theta - kx\cos\theta)$

If F = 0 ; $\sin\theta - kx_0\cos\theta = 0 \Rightarrow x_0 = \dfrac{\tan\theta}{k}$

EX.59:A body of mass 'm' slides down a smooth inclined plane having an inclination of 45^0 with the horizontal. It takes 2s to reach the bottom. If the body is placed on a similar plane having coefficient of friction 0.5 Then what is the time taken for it to reach the bottom?

Sol : Mass = m, $\theta=45^0, \mu=0.5$ Time taken by the body to reach the bottom without friction is

$$T_1 = \sqrt{\frac{2l}{g\sin\theta}} = 2\sec$$

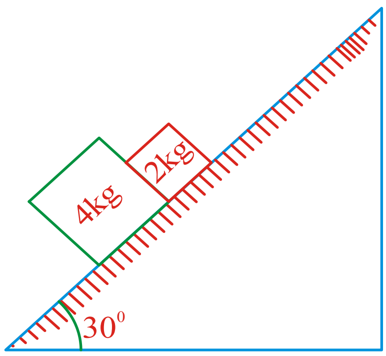
Time taken with friction is

$$T_2 = \sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{\sin\theta - \mu\cos\theta}{\sin\theta}}$$

$$T_2 = T_1\sqrt{\frac{\sin\theta}{\sin\theta - \mu\cos\theta}} = 2\sqrt{\frac{\sin 45^0}{\sin 45^0 - (0.5)\cos 45^0}}$$

$$= 2\sqrt{\frac{(1/\sqrt{2})}{(1/\sqrt{2}) - (0.5)(1/\sqrt{2})}} = 2\times\sqrt{2} = 2.828s$$

EX.60:Two blocks of masses 4 kg and 2 kg are in contact with each other on an inclined plane of inclination 30^0 as shown in the figure. The coefficient of friction between 4 kg mass and the inclined plane is 0.3, where as between 2 kg mass and the plane is 0.2. Find the contact force between the blocks.



Sol : The acceleration of 4 kg mass,

If $\theta = 30^0, \mu_k = 0.3$

$$a_4 = g(\sin\theta - \mu_k\cos\theta) = 10\left[\frac{1}{2} - 0.3\times\frac{\sqrt{3}}{2}\right] = 2.6ms^{-2}$$

The acceleration of 2 kg mas

$$a_2 = 10\left[\frac{1}{2} - 0.2\times\frac{\sqrt{3}}{2}\right] = 3.27 ms^{-2}$$

$\therefore a_2 > a_4$

Thus, there will be contact force between the blocks and they move together. If 'a' is the common acceleration,

$$(m_1 + m_2)a = (m_1 + m_2) g\sin\theta - (\mu_1 m_1 + \mu_2 m_2) g\cos\theta$$

$$6a = 6\times10\times\frac{1}{2} - (0.3\times4 + 0.2\times2)\times10\times\frac{\sqrt{3}}{2}$$

$$6a = 30 - 13.856 \Rightarrow a = 2.7 ms^{-2}$$

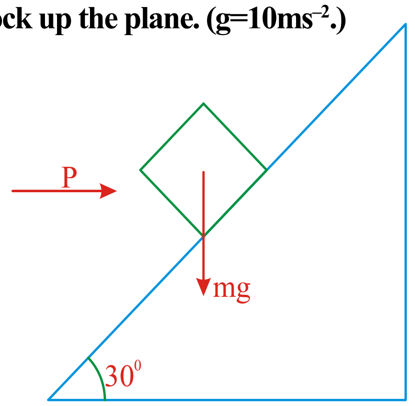
For, 4 kg mass; $mg\sin\theta + f_{contact} - f_{friction} = ma$

$$4\times10\times\frac{1}{2} + f_c - 0.3\times10\times10\times\frac{\sqrt{3}}{2} = 4 \times 2.7$$

$f_c = 10.8 + 10.4 - 20 \Rightarrow f_c = 1.2N$

EX.61: A 30kg block is to be moved up an inclined plane at an angle 30^0 to the horizontal with a velocity of $5ms^{-1}$. If the frictional force retarding the motion is 150N, find the horizontal force required to move the block up the plane. ($g=10ms^{-2}$.)

Sol.



The force required to move a body up an inclined plane is $F = mg\sin\theta + f_k$

$= 30(10)\sin 30^0 + 150 = 300N.$

If P is the horizontal force, $F = P\cos\theta$

$$P = \frac{F}{\cos\theta} = \frac{300}{\cos 30^0} = \frac{300\times2}{\sqrt{3}} = 200\sqrt{3} = 346N$$

EX.62: A body is sliding down an inclined plane having coefficient of friction 0.5. If the normal reaction is twice that of resultant downward force along the inclined plane, then find the angle between the inclined plane and the horizontal

Sol : $\mu = 0.5$, $N = mg\cos\theta$

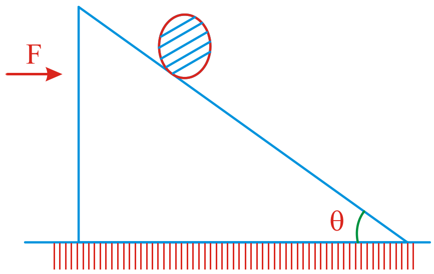
$N = 2F, F = mg(\sin\theta - \mu\cos\theta)$ $\quad$ $N = 2mg(\sin\theta - \mu\cos\theta)$

$$mg\cos\theta = 2mg(\sin\theta - \mu\cos\theta)$$

$$c\text{os}\,\theta = 2\cos\theta(\tan\theta - \mu)$$

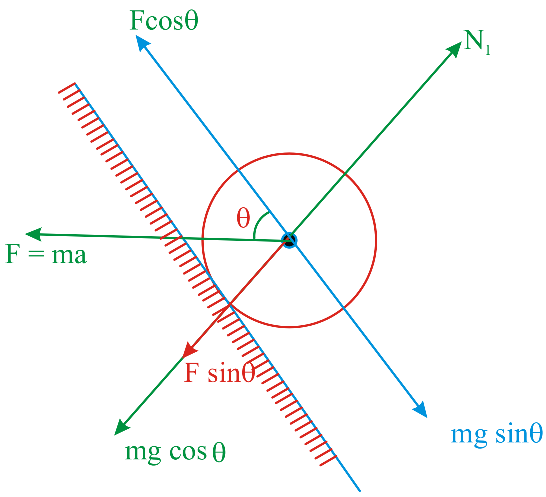
$$\frac{1}{2} = \tan\theta - \frac{1}{2} \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^0$$

EX.63:In the given figure, the wedge is acted upon by a constant horizontal force 'F'. The wedge is moving on a smooth horizontal surface. A ball of mass 'm' is at rest relative to the wedge. The ratio of forces exerted on 'm' by the wedge when 'F' is acting and 'F' is withdrawn assuming no friction between the edge and the ball,is equal to



Sol.

When Force F is applied



$N_1 = \text{mgcos}\,\theta + \text{Fsin}\,\theta$

$(F\cos\theta = mg\sin\theta \Rightarrow \;\; F = mg\tan\theta)$

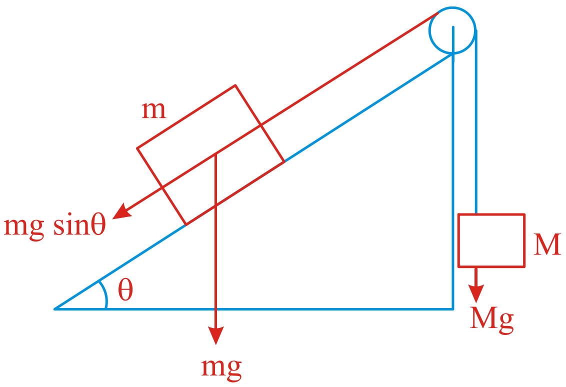
If F=0 ; $N_2 = \text{mgcos}\,\theta$, $\frac{N_1}{N_2} = 1 + \frac{F\sin\theta}{mg\cos\theta}$

$$\frac{N_1}{N_2} = 1 + \frac{mg\tan\theta\sin\theta}{mg\cos\theta} = 1 + \tan^2\theta = \sec^2\theta$$

↳ Two blocks of mass m and M are placed on a rough inclined plane as shown, when $(\theta > \alpha)$

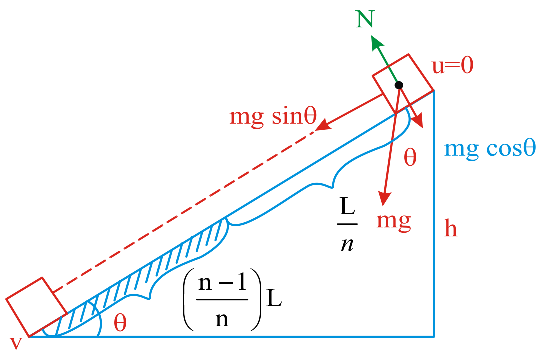
i)Minimum value of M for which m slides upwards is

$M = m(\sin\theta + \mu_s\cos\theta)$



ii) Maximum value of M for which m slides downwards: $M = m(\sin\theta - \mu_s \cos\theta)$

↳ A body is released from rest from the top of an inclined plane of length 'L' and angle of inclination 'θ'. The top of plane of length $\frac{L}{n}(n > 1)$ is smooth and the remaining part is rough. If the body comes to rest on reaching the bottom of the plane then find the value of coefficient of friction of rough



For smooth part :

Using $v^2 - u^2 = 2as$; $V^2 = 2a_1 \frac{L}{n}$,

$a_1 = g\sin\theta$, $a_2 = g(\sin\theta - \mu\cos\theta)$

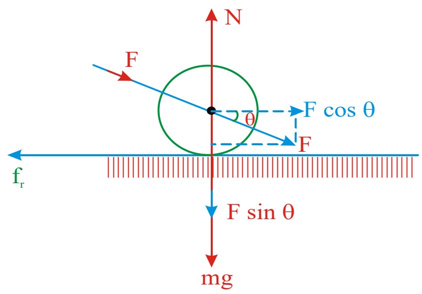
For rough part $\quad 0 - V^2 = 2a_2\left(\frac{n-1}{n}\right)L \qquad 2a_1\frac{L}{n} = -2a_2\left(\frac{n-1}{n}\right)L$

$g\sin\theta = -g[\sin\theta - \mu\cos\theta](n-1) \qquad \mu = Tan\theta\left[\frac{n}{n-1}\right]$

↳ A body is pushed down with velocity 'u' from the top of an inclined plane of length 'L' and angle of inclination 'θ'. The top of plane of length $\frac{L}{n}(n > 1)$ is rough and the remaining part is smooth. If the body reaches the bottom of the plane with a velocity equal to the initial velocity 'u', then the value of coefficient of friction of rough plane is $\boxed{\mu_K = n(\tan\theta)}$

Pushing & Pulling of a Lawn Roller :

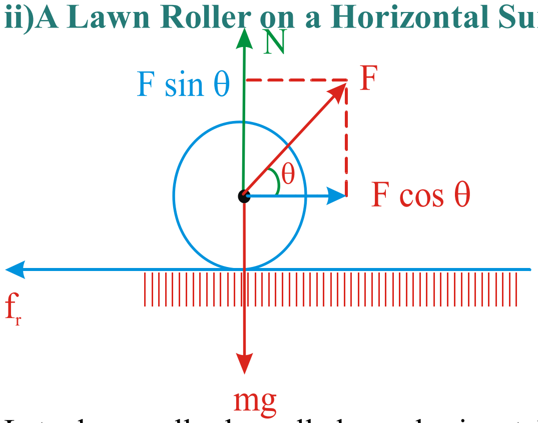
i) A Roller on Horizontal Surface Pushed by an Inclined Force :



- When a lawn roller is pushed by a force 'F',which makes an angle θ with the horizontal, then
- Normal reaction N = mg + F sin θ.
- Frictional force $f_r = \mu_r N = \mu_r\left(mg + F\sin\theta\right)$

$\therefore$ The net horizontal pushing force is given by $F_1 = F(\cos\theta - \mu_r \sin\theta) - \mu_r mg$

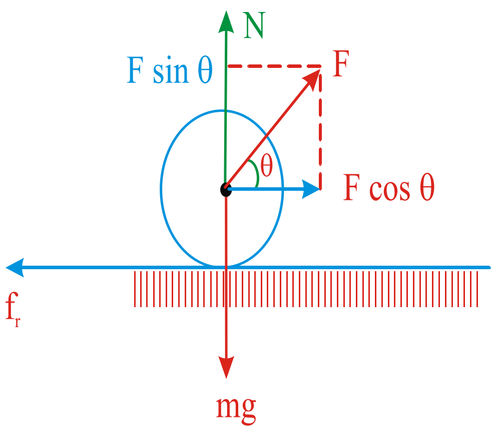
ii)A Lawn Roller on a Horizontal Surface Pulled by an Inclined Force



- Let a lawn roller be pulled on a horizontal road by a force 'F', which makes an angle θ with the horizontal.
- Normal reaction N =mg – Fsin θ
- Frictional force $f_r = \mu_r N = \mu_r(mg - F\sin\theta)$
- The net horizontal pulling force is $F_2 = F(\cos\theta + \mu_r \sin\theta) - \mu_r mg$ Pulling is easier thanPushing.

Applying an Inclined Pulling Force :

Let an inclined force F be applied on the body so as to pull it on the horizontal surface as shown in the figure.



The body is in contact with the surface,and just ready to move

$N + F\sin\theta = mg \Rightarrow N = mg - F\sin\theta$

frictional force $f_r = F\cos\theta$

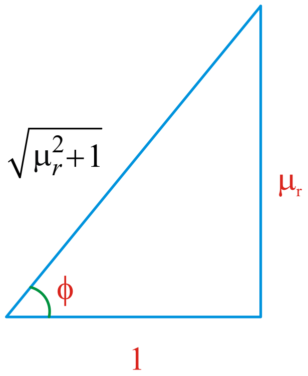
$F\cos\theta = \mu_r N$,$F\cos\theta = \mu_r(mg - F\sin\theta)$

$$F = \frac{\mu_r mg}{(\cos\theta + \mu_r \sin\theta)} \qquad \Rightarrow F = \frac{mg\sin\phi}{\cos(\theta-\phi)} \ (\because Tan\phi = \mu_r)$$

For F to be minimum $\cos(\theta-\phi)$ should be

maximum $\Rightarrow \cos(\theta-\phi) = 1 \Rightarrow \theta-\phi = 0, \theta = \phi$

$\phi =$ angle of friction.



$\therefore F_{\min} = mg\sin\theta = mg\sin\phi$ From the figure, $\sin\phi = \frac{\mu_r}{\sqrt{\mu_r^2+1}}$, $F_{\min} = \frac{\mu_r mg}{\sqrt{\mu_r^2+1}}$

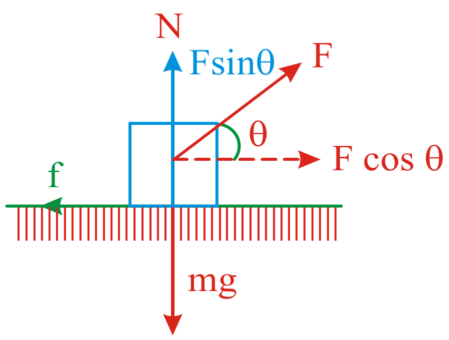
Minimum horizontal pulling force, when $\theta = 0$

$\cos(0-\phi) = \cos\phi$

$$F = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi$$

Applying an Inclined Pushing Force :

Let an inclined force F is applied on the body so as to push it on the horizontal surface as shown in the figure.



The body is in contact with the surface, and just ready to move, N=mg+F$\sin\theta$

frictional force $f_l = F\cos\theta$

$$F\cos\theta = \mu_s N \Rightarrow F\cos\theta = \mu_s\left(mg + F\sin\theta\right)$$

$$F = \frac{\mu_s mg}{\left(\cos\theta - \mu_s \sin\theta\right)}$$

$$F = \frac{mg\sin\phi}{\cos\left(\theta + \phi\right)} \qquad \left(\because Tan\phi = \mu_s\right)$$

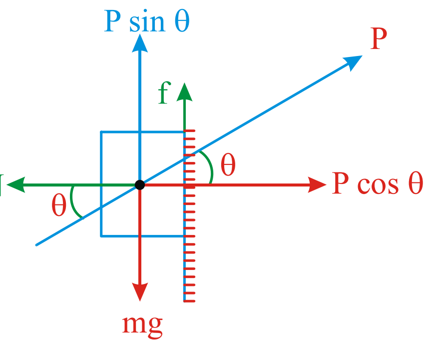
For F to be minimum $\theta = 0$

$$\therefore F_{\min} = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi = \mu_s mg$$

(since $\mu_s = \tan\phi$)

EX.64: A block of mass m kg is pushed up against a wall by a force P that makes an angle 'θ' with the horizontal as shown in figure. The coefficient of static friction between the block and the wall is μ. The minimum value of P that allows the block to remain stationary is

Sol :



At equilibrium, $P\sin\theta + f = mg$, $N = P\cos\theta$

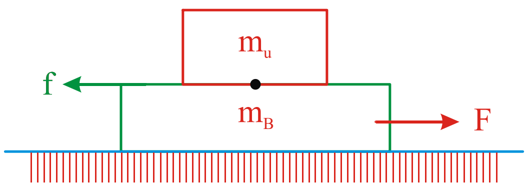
$f = \left(mg - P\sin\theta\right)$; $\mu N = \left(mg - P\sin\theta\right)$

$\mu P\cos\theta = mg - P\sin\theta$; $P\left[\sin\theta + \mu\cos\theta\right] = mg$

$$\boxed{P = \frac{mg}{\left(\sin\theta + \mu\cos\theta\right)}}$$

Block on Block:

Case I: Bottom block is pulled and there is no friction between bottom block and the horizontal surface.



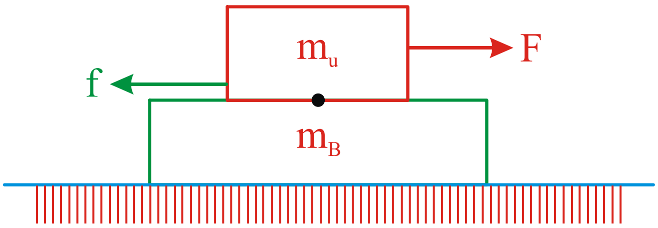
↳ When the bottom block is pulled upper block is accelerated by the force of friction acting upon it.

↳ The maximum acceleration of the system of two blocks to move together without slipping is $a_{max} = \mu_s g$, where μ_s is the coefficient of static friction between the two blocks. The maximum applied force for which both the blocks move together is $F_{max} = \mu_s g(m_u + m_B)$

↳ If $a < \mu_s g$ blocks move together and applied force is $F = (m_B + m_u)a$ In this case frictional force between the two block $f = m_u a$.

↳ If $a > \mu_s g$, blocks slip relative to each other and have different accelerations. The acceleration of the upper block is $a_u = \mu_k g$ and that for the bottom block is $a_B = \dfrac{F - \mu_k m_u g}{m_B}$

↳ **Case - II**:Upper block pulled and there is no friction between bottom block and the horizontal surface.



↳ When the upper block is pulled, bottom block is accelerated by the force of friction acting on it.

↳ The maximum acceleration of the system of two blocks to move together without slipping is $a_{max} = \mu_s \dfrac{m_u}{m_B} g$ where , μ_s=coefficient of static friction between the two blocks The maximum force for which both blocks move together is $F_{max} = \mu_s \dfrac{m_u}{m_B} g(m_u + m_B)$

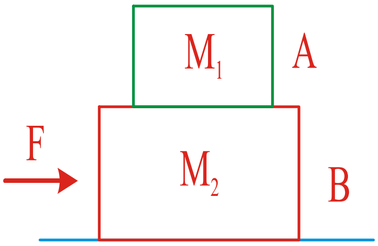
↳ If $a < a_{max}$, blocks move together and frictional force between the two blocks is $f = m_B a$ The applied force on the upper block is $F = (m_B + m_u)a$

↳ If $a > a_{max}$ blocks slide relative to each other and hence they have different accelerations.The acceleration of the bottom block is $a_B = \mu_K \dfrac{m_u}{m_B} g$ and the acceleration of the upper block is

$$a_u = \frac{F - \mu_k m_u g}{m_u}$$

↳ A number of blocks of identical masses m each are placed one above the other. Force required to pull out Nth block from the top is
F = (2N–1) μ mg

EX.65: **A block of mass 4kg is placed on another block of mass 5kg, and the block B rests on a smooth horizontal table, for sliding the block A on B, a horizontal force 12N is required to be applied on it .How much maximum horizontal force can be applied on 'B' so that both A and B move together? Also find out the acceleration produced by this force.**



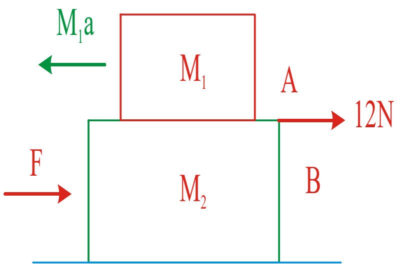
Sol: Here M_1 =4kg and M_2 =5kg

Limiting friction between the blocks is f_{lim} Acceleration of system is

$$a = \frac{F}{M_1 + M_2} = \frac{F}{4+5} = \frac{F}{9} m/s^2$$

Because of this acceleration the block A experiences a pseudo force of magnitude

$$F_{pseudo} = M_1 a = 4 \times \frac{F}{9}$$

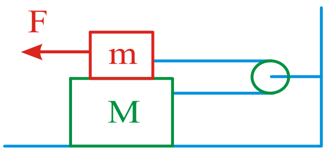


As block A move together with B, $F_{pseudo} \le f_{\lim}$ For maximum value of applied force

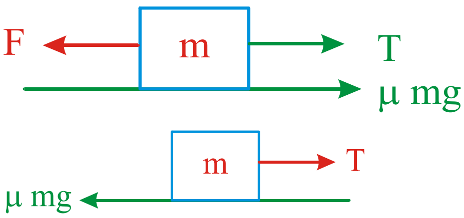
$$F_{pseudo} = f_{\lim}\,; \quad \frac{4F}{9} = 12 \Rightarrow F = 27N$$

$$\text{The acceleration of blocks} = \frac{27}{9} = 3m/s^2$$

EX.66: **Two blocks of masses 'm' and 'M' are arranged as shown in the figure. The coefficient of friction between the two blocks is 'μ', where as between the lower block and the horizontal surface is zero. Find the force 'F' to be applied on the upper block, for the system to be under equilibrium?**



Sol :



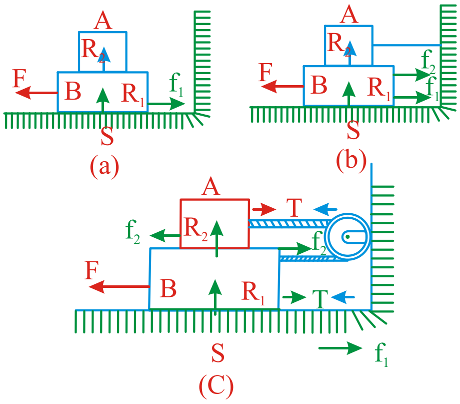
On the upper block,

F = T + f = T + μ N ;   F = T + μ mg ......(1)

On the lower block   T = μ mg .... (2)

from (1) and (2), we get, F = 2 μ mg

EX.67 : Block A weighs 4N and block B weighs 8N.The coefficient of kinetic friction is 0.25 for all surfaces.Find the force F to slide B at a constant speed when (a) A rests on B and moves with it (b) A is held at rest and (c) A and B are connected by a light cord passing over a smooth pulley as shown in fig (a),(b) and (c) respectively



Sol :(a) When A moves with B the force opposing the motion is the only force of friction between B and S the horizontal and as velocity of system is constant,

$F = f_1 = \mu R_1 = 0.2(4+8) = 3N$

(b)When A is held stationary, the friction opposing the motion is between A and B. So

$F = \mu R_1 + \mu R_2 = 3 + 0.25(4) = 4N$

(c)In this situation for dynamic equilibrium of B

$F = \mu R_1 + \mu R_2 + T$ ....(i)

While for the uniform motion of A,

$T = \mu R_2$ ....(ii)

Substituting T from eqn (ii) in (i) we get $F = \mu R_1 + 2\mu R_2 = 3 + 2\times 1 = 5N$

EX.68 : The apparent weight of a person inside a lift is W_1 when lift moves up with certain acceleration and is W_2 when lift moves down with same acceleration. The weight of person when lift moves up with constant speed is

1) $\frac{W_1 + W_2}{2}$ 2) $\frac{W_1 - W_2}{2}$ 3) $2W_1$ 4) $2W_2$

Sol : key-1

$w_1 = m(g+a), w_2 = m(g-a)$,

$w_3 = mg$

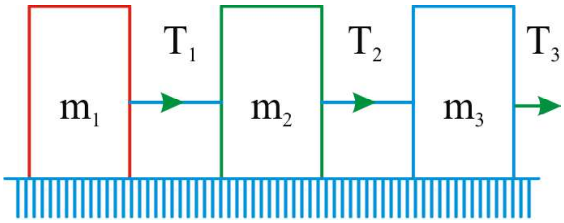
EX.69 : A rope of length 10m and linear density 0.5kg/m is lying length wise on a smooth horizontal floor. It is pulled by a force of 25 N. The tension in the rope at a point 6m away from the point of application is

1) 20 N 2) 15 N 3) 10 N 4) 5 N

Sol : key-3. $F = ma$,For one unit $\rightarrow \frac{F}{L}$

For l units$\rightarrow \frac{F}{L}l$,$T = F\left(1-\frac{l}{L}\right)$

EX.70 : Three blocks of masses m_1 , m_2 and m_3 are connected by a massless string as shown in figure on a frictionless table. They are pulled with a force T_3 = 40 N. If $m_1 = 10kg$, $m_2 = 6kg$ and $m_3 = 4kg$, then tension T_2 will be

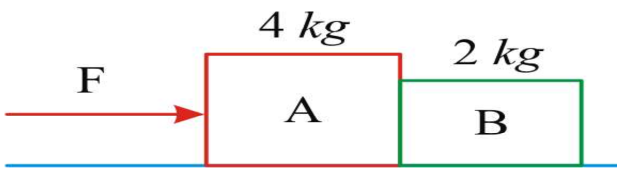


1) 10 N 2) 20 N 3) 32 N 4) 40 N

Sol : key-3.. $F_{net} = ma$, $T_3 - T_2 = m_3 a$, $T_2 - T_1 = m_2 a$, $T_1 = m_1 a$

solving the above equations,we get

EX.71 : A horizontal force F pushes a 4 kg block (A) which pushes against a 2 kg block (B) as shown. The blocks have an acceleration of $3m/s^2$ to the right. There is no friction between the blocks and the surfaces on which they slide. What is the net force B exerts on A?



1) 6 N to the right 2) 12 N to the right 3) 6 N to the left 4) 12 N to the left

Sol : key-3. $F = ma$, $F_{net} = F - f$, $f = ma$

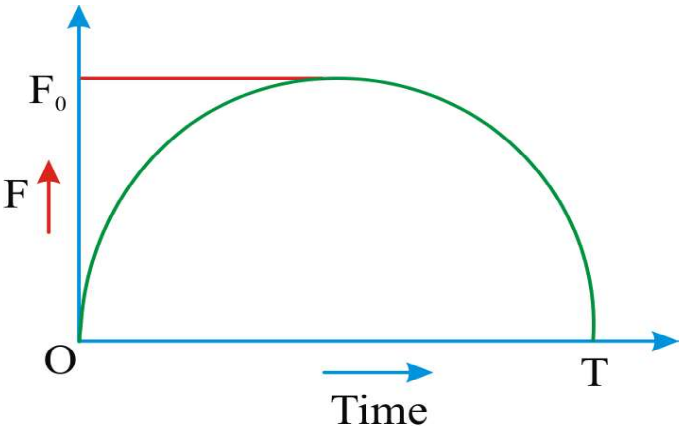
$a = \frac{T_3}{m_1 + m_2 + m_3}$, $T_2 = (m_1 + m_2)a$

EX.72 : The momentum of a body in two perpendicular directions at any time't' are given by $P_X = 2t^2 + 6$ and $P_Y = \frac{3t^2}{2} + 3$.The force acting on the body at t = 2 sec is

1) 5 units 2) 2 units 3) 10 units 4) 15 units

Sol : key-3. $F_x = \frac{dP_x}{dt}, F_y = \frac{dP_y}{dt}, F = \sqrt{F_x^2 + F_y^2}$

EX.73 : A particle of mass m, initially at rest is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle.



1) $u=\dfrac{\pi F_0^2}{2m}$ 2) $u=\dfrac{\pi T^2}{8m}$ 3) $u=\dfrac{\pi F_0 T}{4m}$ 4) $u=\dfrac{\pi F_0 T}{2m}$

Sol : key-3. Impulse = Area of semi circle

EX.74 : A balloon of mass M is descending at a constant acceleration α. When a mass m is released from the balloon it starts rising with the same acceleration α. Assuming that its volume does not change, what is the value of m?

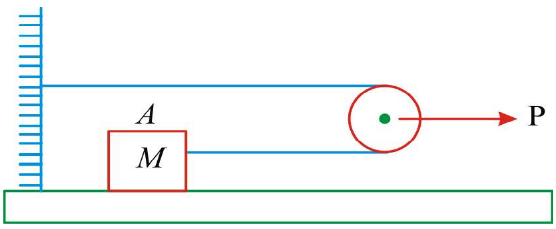
1) $\dfrac{\alpha}{\alpha+g}M$ 2) $\dfrac{2\alpha}{\alpha+g}M$ 3) $\dfrac{\alpha+g}{\alpha}M$ 4) $\dfrac{\alpha+g}{2\alpha}M$

Sol : key-2. While descending, $Mg-F_B=M\alpha$

While ascending $F_B-(M-m)g=(M-m)\alpha$

Where 'F_B' is the buoyancy force

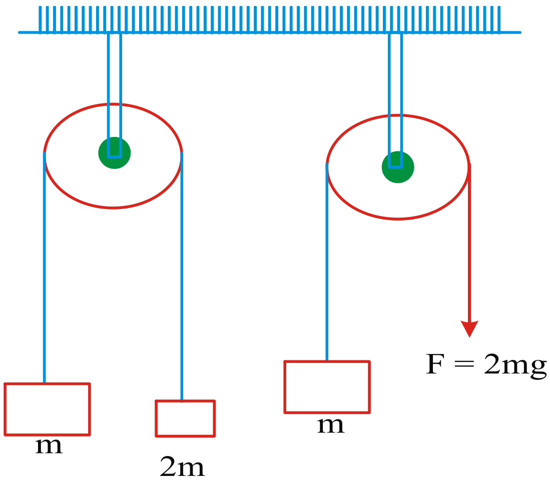
EX.75 : In the following figure, the pulley is massless and frictionless. There is no friction between the body and the floor. The acceleration produced in the body when it is displaced through a certain distance with force 'P' will be



1) $\dfrac{P}{M}$ 2) $\dfrac{P}{2M}$ 3) $\dfrac{P}{3M}$ 4) $\dfrac{P}{4M}$

Sol : key-2. $F=ma$, $2T=p$, $a=\dfrac{T}{m}$

EX.76 :The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case I,the mass m is lifted by attaching a mass 2m to the other end of rope with a constant downward force F = 2mg, where g is acceleration due to gravity. The acceleration of mass m in case I is



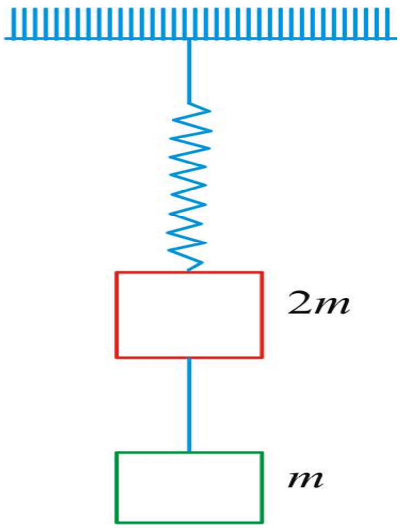
1) zero
2) more than that in case II
3)less than that in case II
4)equal to that in case II

Sol : key-3. $F = ma$, $a = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$,F –T = 0 and

T = 2mg also T–mg = ma[1]

Finally a < a[1]

EX.77 :The string between blocks of masses 'm' and '2m' is massless and inextensible.The system is suspended by a massless spring as shown. If the string is cut, the magnitudes of accelerations of masses 2m and m (immediately after cutting)



1) g, g
2) $g, \frac{g}{2}$
3) $\frac{g}{2}, g$
4) $\frac{g}{2}, \frac{g}{2}$

Sol : key-3. For $m_1, F = ma = m_1 g$;For $m_2, T - m_2 g = m_2 a^1$

EX.78 :Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t=0$. They collide at time t_0. Their velocities become $\vec{v}_1^1$ and $\vec{v}_2^1$ at time $2t_0$ while still moving in air. The value of $\left|\left(m_1\vec{v}_1^1+m_2\vec{v}_2^1\right)-\left(m_1\vec{v}_1+m_2\vec{v}_2\right)\right|$ is

1) zero 2) $(m_1+m_2)gt_0$

3) $2(m_1+m_2)gt_0$ 4) $\frac{1}{2}(m_1+m_2)gt_0$

Sol : key-3. $mv^1 = m(u+at)$

$m_1v_1^1+m_2v_2^1 = m[v_1+2gt_0]+m[v_2+2gt_0]$

EX.79 :In order to raise a block of mass 100kg a man of mass 60kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with an acceleration $\frac{5g}{4}$ relative to rope. The tension in the rope is $\left(g=10ms^{-2}\right)$

1) 1432N 2) 928 N 3) 1218N 4) 642N

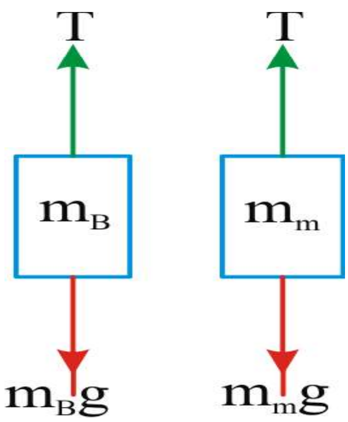
Sol : key-3. $m_m = 60kg, m_B = 100kg$

'a' be acceleration of rope.

$a_{rel} = \frac{5g}{4}, a_{rel} = a_m + a \,; a_m = a_{rel} - a$

$a_m = \frac{5g}{4} - a$

$T - m_B g = m_B a \rightarrow (1)$



$T - m_m g = m_2 a_m \rightarrow (2)$

solving (1) & (2), we get

T - 100g = 100a -------(3)

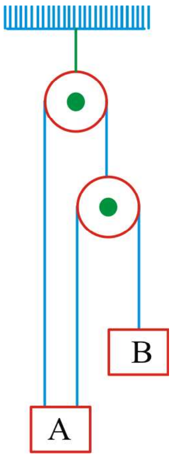
T - 60g = $60\left(\frac{5g}{4}-a\right)$ -------(4)

(3) - (4) , $-40g = 100a - 75g + 60a$

$160a = 350 \Rightarrow a = \frac{35}{16} ms^{-2}$

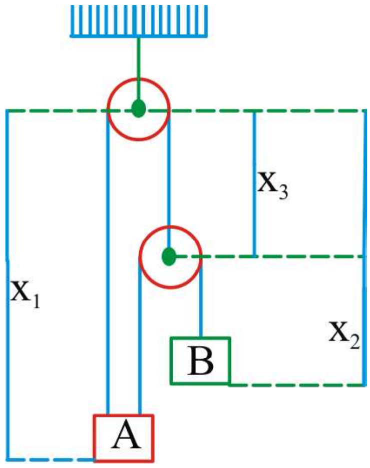
T = 100g + 100a = 1000 + $100 \times \frac{35}{16} = 1218N$

EX.80 :In the pulley-block arrangement shown in figure.Find the relation between acceleration of block A and B.



1) a_B=-3a_A 2) a_B=-a_A 3) a_B=-2a_A 4) a_B=-4a_A

Sol : key-1



$x_1 + x_3 = \ell_1$

differentiating with respect to time,

we get $v_1 + v_3 = 0$

Again differentiating w.r.to to time,

$a_1 + a_3 = 0 \Rightarrow a_1 = -a_3, a_3 = -a_1$

$(x_1 - x_3) + (x_2 - x_3) = \ell_2$ $\qquad x_1 + x_2 - 2x_3 = \ell_2$

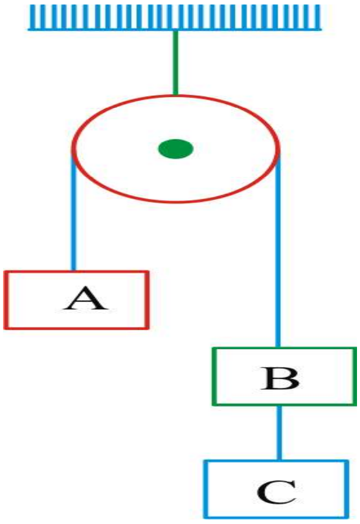
differentiating w.r.to time, $v_1 + v_2 - 2v_3 = 0$

Again differentiating w.r.to time, $a_1 + a_2 - 2a_3 = 0$

$a_1 + a_2 + 2a_1 = 0$; $\qquad 3a_1 + a_2 = 0$

$a_2 = -3a_1$; $\quad a_B = -3a_A$

EX.81 :Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the fig. The tension in the string connecting weights B and C is



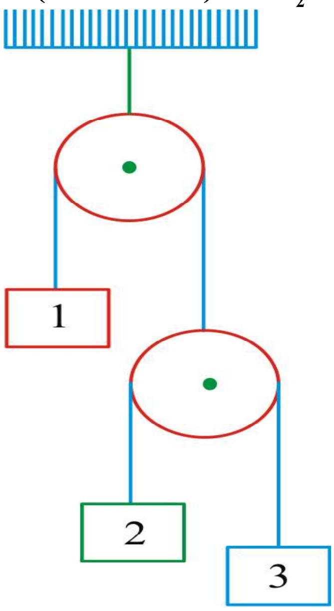
1) zero 2) 13 N 3) 3.3 N 4) 19.6 N

Sol : key-2. For A, $T_1 - m_1 a = m_1 g$

For B, $(m_2 + m_3) g - T_2 = (m_2 + m_3) a$

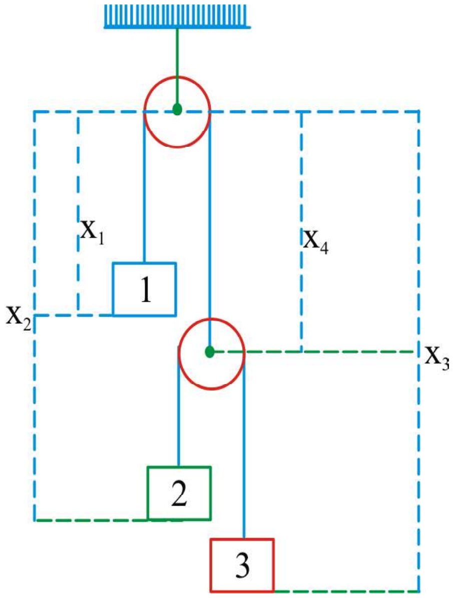
$a = \dfrac{F_{net}}{m_1 + m_2 + m_3}$;For C , $m_3 g - T_2 = m_3 a$

EX.82 :In the figure shown $a_3 = 6m/s^2$ (downwards) and $a_2 = 4m/s^2$ (upwards). Find acceleration of 1.



1) $1m/sec^2$ upwards 2) $2m/sec^2$ upwards
3) $1m/sec^2$ downwards 4 $2m/sec^2$ downwards

Sol : key-1



Since the points 1,2,3 and are movable, so let their displacements are x_1, x_2, x_3 and x_4 We observe that the length of the strings between 1 and 4 and 2 and 3 are constants.

$x_1 + x_4 = \ell_1$; $(x_2 - x_4) + (x_3 - x_4) = \ell_2$

Differentiating twice w.r.t time, we get

$a_1 + a_4 = 0 \Rightarrow a_1 = -a_4 \rightarrow (1)$

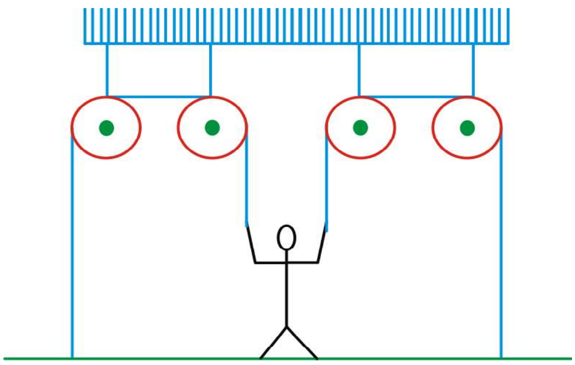
$a_2 - a_4 + a_3 - a_4 = 0 \Rightarrow a_2 + a_3 - 2a_4 = 0 \rightarrow (2)$

$a_2 = -4ms^{-2}, a_3 = 6ms^{-2}$; $\quad -4 + 6 - 2a_4 = 0$

$2a_4 = 2 \Rightarrow a_4 = 1ms^{-2}$; From (1),

$a_1 = -a_4$; $\quad a_1 = -1ms^{-2}$; $\quad a_1 = 1ms^{-2}$ upwards

EX.83 :A man of mass m stands on a platform of equal mass m and pulls himself by two ropes passing over pulleys as shown in figure. If he pulls each rope with a force equal to half his weight, his upward acceleration would be



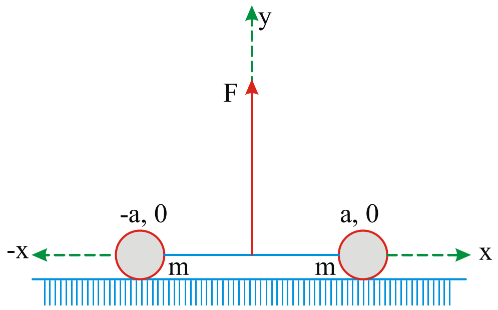
1) $\frac{g}{2}$ 2) $\frac{g}{4}$ 3) g 4) zero

Sol : key-4. $F = 4\left(\frac{mg}{2}\right) \to upwards$

$W = 2mg \to downwards$

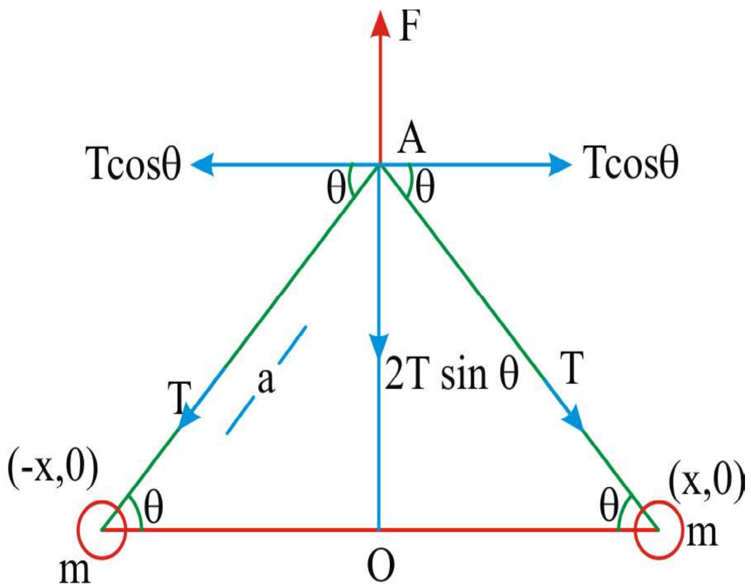
F = W $, \therefore a = 0$

EX.84 :Two masses each equal to m are lying on X-axis at (-a, 0) and (+a, 0), respectively, as shown in fig. They are connected by a light string. A force F is applied at the origin along vertical direction. As a result, the masses move towards each other without loosing contact with ground. What is the acceleration of each mass? Assume the instantaneous position of the masses as (-x, 0) and (x, 0), respectively

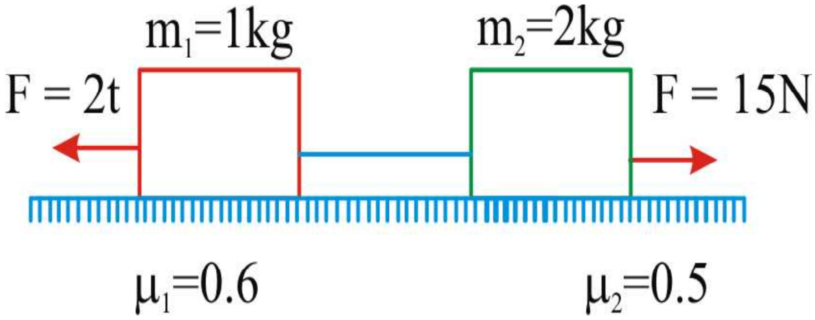


1) $\frac{2F}{m}\frac{\sqrt{(a^2 - x^2)}}{x}$ 2) $\frac{2F}{m}\frac{x}{\sqrt{(a^2 - x^2)}}$ 3) $\frac{F}{2m}\frac{x}{\sqrt{(a^2 - x^2)}}$ 4) $\frac{F}{m}\frac{x}{\sqrt{(a^2 - x^2)}}$

Sol : key-3. $F = 2T\sin\theta, ma^1 = T\cos\theta$

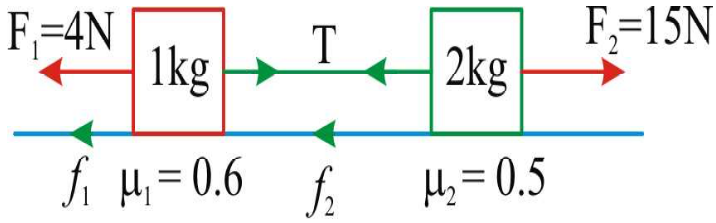


EX.85 :Two blocks A and B are separated by some distance and tied by a string as shown in the figure. The force of friction in both the blocks at t = 2s is.



1) 4N(→), 5N(←) 2) 2N(→), 5N(←)
3) 0N(→), 10N(←) 4) 1N(←), 10N(←)

Sol : key-4. At t = 2s, $F_1 = 4N$



$f_1 = \mu_1 m_1 g = 0.6 \times 1 \times 10 = 6N$

$f_2 = \mu_2 m_2 g = 0.5 \times 2 \times 10 = 10N$

$F_{net} = F_2 - F_1 = 15 - 4 = 11N$

As, $F_{net} > f_1 + f_2$.

The system will remain at rest and the values of frictional forces on the blocks will be,

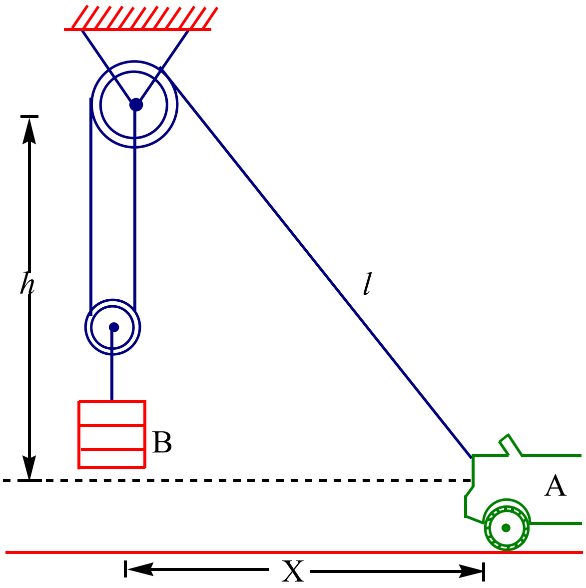
$T = 4 + f_1$ and $T = 15 - f_2 ; 4 + f_1 = 15 - f_2$

$f_1 + f_2 = 11N \rightarrow (1)$

4th option,

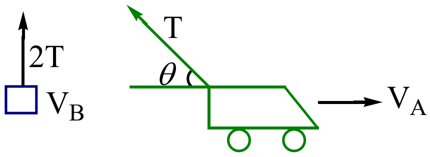
$f_1 = +1N,\ f_2 = +10N;\ f_1 + f_2 = 11N$

EX.86 :The car A is used to pull a load B with the pulley arrangement shown. If A has a forward velocity v_A **determine an expression for the upward velocity** v_B**, of the load in terms of** V_A **and** θ. θ **is angle between string and horizontal**



1) $\frac{1}{2}V_A\cos\theta$ 2) $V_A\sin\theta$ 3) $V_A\cos\theta$ 4) $\frac{1}{2}V_A\tan\theta$

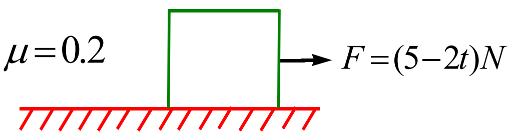
Sol : key-1. Let θ angle between string and horizontal T is tension in string



$\sum T.V = 0$; $-T\cos\theta V_A + 2TV_B = 0$

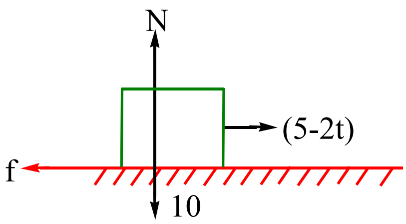
$V_B = \frac{1}{2}V_A\cos\theta$

EX.87 :The force acting on the block of mass 1kg is given by F = 5 -2t. The frictional force acting on the block after time t = 2 seconds will be $(\mu = 0.2)$



1) 2N 2) 3N 3) 1N 4) Zero

Sol : key-2.



$f_{max} = 10 \times 0.2 = 2N$
Initial force = 5N > 2N
$\therefore$ block will move with acceleration

$$a = \frac{5 - 2t - f}{1} = 5 - 2t - 2$$

$$\frac{dv}{dt} = 3 - 2t \qquad v = 3t - t^2$$

$(\because at\ t = 0, v = 0)\ v = 0 \Rightarrow t = 0, 3\sec$
$\therefore$ at t = 2sec block is moving
$\therefore f_{max}$ will act i.e., frictional force acting = 2N

EX.88 :**A body of mass 2kg travels according to the law x(t) = pt + qt^2 + rt^3 where, q = 4ms^{-2}, p = 3ms^{-1} and r = 5ms^{-3}. The force acting on the body at t = 2 seconds is**

a) 136 N b) 134 N c) 158 N d) 68 N

Sol: (a) Given, mass = 2kg
x(t) =pt + qt^2 + rt^3

$$v = \frac{dx}{dt} = p + 2qt + 3rt^2$$

$$a = \frac{dv}{dt} = 0 + 2qt + 6rt$$

at t = 2s; a = 2q + 6 x 2 xr
= 2q +12r
= 2 x 4 + 12 x 5
= 8 + 60 = 68 m/s
Force = F = ma = 2 x 68 = 136N

EX.89 :**A body with mass 5 kg is acted upon by a force $F = \left(-3\hat{i} + 4\hat{j}\right)N$. If its initial velocity at t = 0 is $v = \left(6\hat{i} - 12\hat{j}\right)ms^{-1}$, the time at which it will just have a velocity along they- axis is**

a) never b) 10 s c) 2 s d) 15 s

Sol: (b) Given, mass = m = 5kg

Acting force = $F = \left(-3\hat{i} + 4\hat{j}\right)N$

Initial velocity at t = 0, $v = \left(6\hat{i} - 12\hat{j}\right)ms^{-1}$

Retardation, $a = \frac{F}{m} = \left(-\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}\right)m/s^2$

As final velocity is along Y-axis only, its x- component must be zero.
From v = u + at, for X-component only,

EX.90 :**The motion of a particle of mass m is given by x = 0 for t < 0s, x(t) = Asin 4πt for 0< t<(1/4) s (A > 0), and x = 0 for t >(1/4) s. Which of the following statements is true?**

a) The force at t = (1/8) s on the particle is -m16π^2 A.

b) The particle is acted upon by an impulse of magnitude m47π^2 A at t = 0 s and t = (1/4) s.

c) The particle is not acted upon by any force.

d) The particle is not acted upon by a constant force.

e)There is no impulse acting on the particle.

Sol: (a,b,d)

Given, x = 0 for t < 0 s

$x(t) = A \sin 4\pi t$; for $0 < t < \frac{1}{4}$ s

x =0; for $t > \frac{1}{4}$ s

For, $0 < t < \frac{1}{4}$ s

$$v(t) = \frac{dx}{dt} = 4\frac{dv(t)}{dt} \text{Acos } 4\pi t$$

$$a(t) = \frac{dv(t)}{dt} = -16\pi^2 \text{ a sin}4\pi t$$

At = $\frac{1}{8}$ s, $a(t) = -16\pi^2 \text{Asin}4\pi \text{x } \frac{1}{8} = -16\pi^2 A$

$F = ma(t) = -16\pi^2 A \text{ x m} = -16\pi^2 mA$

Impulse = Change in linear momentum

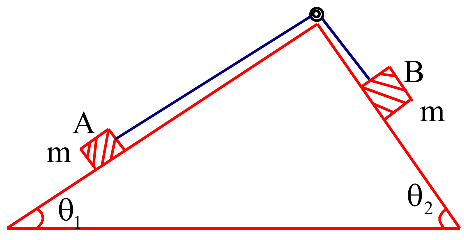
$I = Fxt = (-16\pi^2 Am) \text{ x } \frac{1}{4} = -4\pi^2 Am$

the impulse (Change in linear momentum)

at t = 0 is same as, t = $\frac{1}{4}$ s

Clearly, force depends upon A which is not constant. Hence, force is also not constant.

EX.91 :**In figure a body A of mass m slides on plane inclined at angle θ_1 ,to the horizontal and μ is the coefficent of friction between Aand the plane. A is connected by a light string passing over a frictioness pulley to another body B, also of mass m, sliding on a frictionless plane inclined at angle θ_2 to the horizontal. Which of the following statements are true?**



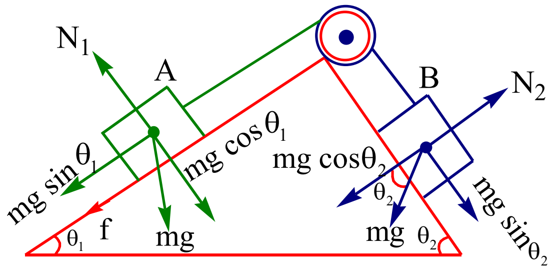
a) A will never move up the plane.

b) A will just start moving up the plane when $\mu = \dfrac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}$

c) For A to move up the plane, θ_2 must always be greater than θ_1.

d) B will always slide down with constant speed.

Sol: (b,c) Let A moves up the plane frictional force on A will be downward as shown.



When A just starts moving up

$mg\sin\theta_1 + f = mg\sin\theta_2$

$\Rightarrow mg\sin\theta_1 + \mu mg\cos\theta_1 = mg\sin\theta_2$

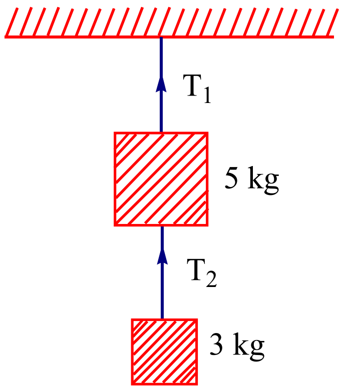
$\mu = \dfrac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}$

When A moves upwards

$f = mg\sin\theta_2 - mg\sin\theta_1 > 0$

$\Rightarrow \sin\theta_2 > \sin\theta_1 \Rightarrow \theta_2 > \theta_1$

EX.92 :**Two masses of 5 kg and 3 kg are suspended with help of massless inextensible strings as shown in figure. Calculate T_1 and T_2 when whole system is going upwards with acceleration = $2 m/s^2$ (use $g = 9.8\ ms^{-2}$).**

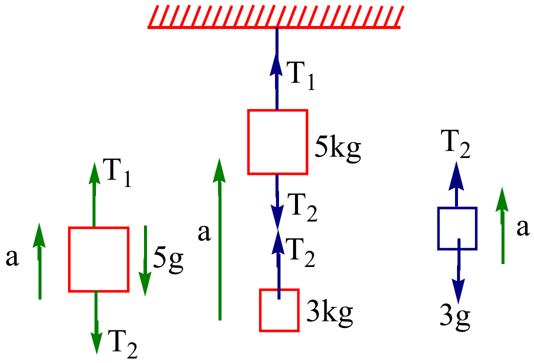


a) $T_1 = 5N, T_2 = 38N$ b) $T_1 = 35.4N, T_2 = 94.4N$

c) $T_1 = 94.4N, T_2 = 35.4N$ d) $T_1 = 0N, T_2 = 35.4N$

Sol:Given, $m_1 = 5kg$, $m_2 = 3kg$

$g = 9.8 m/s^2$ and $a = 2 m/s^2$



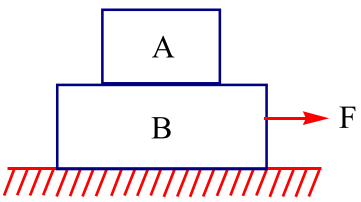
For the upper block T_1 - T_2 - 5g = 5a
$\Rightarrow$ T_1 - T_2 = 5 (g + a)
For the lower block T_2 - 3g = 3a
$\Rightarrow$ T_2 = 3(g + a) = 3(9.8+2) = 35.4N
From Eq. (i) T_1 = T_2 + 5 (g + a)
= 35.4 + 5 (9.8 + 2) = 94.4N

EX.93 :**A cricket ball of mass 150 g has an initial velocity** $u = \left(-3\hat{i} + 4\hat{j}\right) ms^{-1}$ **and a final velocity** $v = -\left(3\hat{i} + 4\hat{j}\right) ms^{-1}$ **after being hit. The change in momentum (final momentum- initial momentum) is (in kg ms^{-1})**

a) zero b) $-\left(0.45\hat{i} + 0.6\hat{j}\right)$ c) $-\left(0.9\hat{i} + 1.2\hat{j}\right)$ d) $-5\left(\hat{i} + \hat{j}\right)\hat{i}$

Sol: (c) Given, $u = \left(3\hat{i} + 4\hat{j}\right)$ m/s and
$v = -\left(3\hat{i} + 4\hat{j}\right)$ m/s
mass ofthe ball = 150 g = 0.15 kg
ΔP= mv-mu
$\Delta P = m (v - w) = -(0.15)\left[\left(3\hat{i} + 4\hat{j}\right) - \left(3\hat{i} + 4\hat{j}\right)\right]$
$= (0.15)\left[-6\hat{i} - 8\hat{j}\right]$
Hence, $\Delta p = -\left[0.9\hat{i} + 1.2\hat{j}\right]$

EX.94 :**In figure the co-efficient of friction between the floor and the body B is 0.1. The co-efficient of friction between the bodies B and A is 0.2. A force F is applied as shown B. The mass of A is m/2 and of B is m. Which of the following statements are true?**



a) The bodies will move together if F= 0.25 mg.
b) The body A will slip with respect to B if F=0.5 mg.
c) The bodies will move together if F= 0.5 mg.
d) The bodies will be at rest if F=0.1 mg.

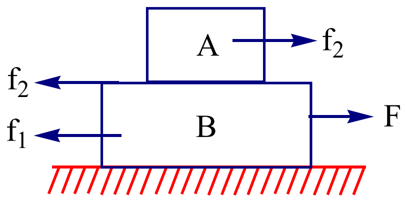
Sol: (a,b,d)

Consider the adjacent diagram Frictonal force on B(f_1) and frictional force on A(t_2) will be as shown.
Let A and B are moving together

$$a_{common} = \frac{F - f_1}{m_A + m_B} = \frac{F - f_1}{(m/2) + m} = \frac{2(F - f_1)}{3m}$$

Pseudo force on A = (m_A) x a_{common}

$$= m_A \times \frac{2(F - f_1)}{3m} = \frac{m}{2} \times \frac{2(F - f_1)}{3m} = \frac{(F - f_1)}{3m}$$



The force (F) will be maximum when
Pseudo force on A = Frictional force on A

$$\Rightarrow \frac{F_{max} - f_1}{3} = \mu m_A g$$

$$\Rightarrow \frac{F_{max} - f_1}{3} = 0.2 \times \frac{m}{2} \times g = 0.1mg$$

$$\Rightarrow F_{max} = 0.3mg + f_1$$

$$= 0.3mg + (0.1)\frac{3}{2}mg = 0.45mg$$

$\Rightarrow$ Hence, maximum force upto which bodies will move together is F_{max} = 0.45mg
a) Hence, for F = 0.25 mg < F_{max} bodies will move together
b) For F = 0.5mg > F_{max}, body A will slip with respect to B
c) For F = 0.5mg > F_{max}, bodies slip

$$(f_1)_{max} = \mu m_B g = (0.1) \times \frac{3}{2} m \times g = 0.15mg$$

$$(f_2)_{max} = \mu m_A g = (0.2) \times \frac{m}{2} \times g = 0.1mg$$

Hence, minimum force required for movement of the system (A + B)
f_{min} = $(f_1)_{max}$ + $(f_2)_{max}$
= 0.15mg + 0.1mg = 0.25mg
d) Ginve, force F = 0. lmg < Fmin
Hence, the bodies will be at rest

EX.95 :**A body of mass 10kg is acted upon by two perpendicular forces, 6N and 8N. The resultant acceleration of the body is**

a) 1 ms^{-2} at an angle of $\tan^{-1}(4/3)$ 6N force.

b) 0.2 ms^{-2} at an angle of $\tan^{-1}(4/3)$ w.r.t. 6N force.

c) 1 ms^{-2} at an angle of $\tan^{-1}(4/3)$ w.r.t.8N force.

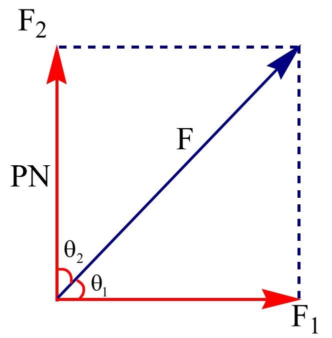
d) 0.2 ms^{-2} at an angle of $\tan^{-1}(4/3)$ w.r.t.8N force.

Sol: (a) Consider the adjacent diagram

Given, mass = m = 10 kg

$F_1 = 6N, F_2 = 8N$

Resultant force= $F = \sqrt{F_1^2 + F_2^2} = \sqrt{36+64} = 10N$



$$a = \frac{F}{m} = \frac{10}{10} = 1m/s^2 \text{ ; along R}$$

Let θ_1, be angle between R and F_1

$$\tan\theta_1 = \frac{8}{6} = \frac{4}{3}$$

$$\theta_1 = \tan^{-1}\left(\frac{4}{3}\right) \text{ w.r.t. } F_1 = 6N$$

Let θ_2 be angle between F and F_2

$$\tan\theta_2 = \frac{6}{8} = \frac{3}{4}$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) \text{ w.r.t } F_2 = 8N$$

EX.96 :**Mass m_1 moves of a slope making an angle q with the horizontal and is attached to mass m_2 by a string passing over a frictionless pulley as shown in figure. The coefficient of friction between m_1 and the sloping surface is μ. Which of the following statements are true ?**

a) If $m_2 > m_1$ sin , the body will move up the plane

b)If $m_2 > m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane

c) If $m_2 < m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane

d) If $m_2 < m_1 (\sin\theta - \mu\cos\theta)$,the body will move down the plane

Sol: (b,d)

Let m_1, moves up the plane.

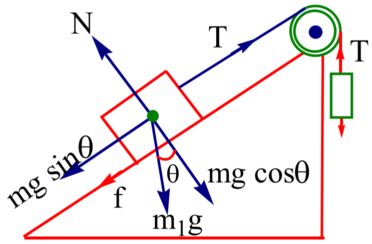
Different forces involved are shown in the diagram

N = Normal reaction

f = Frictional force

T = Tension in the string

$f = \mu N = \mu m_1 g \cos\theta$



For the system $(m_1 + m_2)$ to move up

$m_2g - (m_1g\sin\theta + f) > 0$

$\Rightarrow m_2g - (m_1g\sin\theta + \mu m_1g\cos\theta) > 0$

$\Rightarrow m_2 > m_1(\sin\theta + \mu\cos\theta)$

Hence, option (b) is corrected

Let the body moves down the plane, in this case facts up the plane.

Hence, $m_1g\sin\theta - f > m_2g$

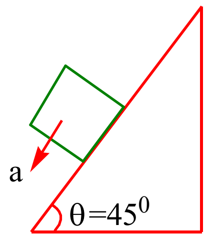
$\Rightarrow m_1g\sin - \mu m_1g\cos\theta > m_2g$

$\Rightarrow m_1(\sin\theta - \mu\cos\theta) > m_2$

$\Rightarrow m_2 < m_1(\sin\theta - \mu\cos\theta)$

Hence, option (d) is correct.

EX.97 :**When a body slides down from rest along a smooth inclined plane making an angle of 45^0 with the horizontal, it takes timeT. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time pT, where p is some number greater than 1. Calculate the coefficient of friction between the body and the rough plane.**



a) $\left(1-\frac{1}{P^2}\right)$ b) $\left(1+\frac{1}{P^2}\right)$ c) $\frac{1}{P^2}$ d) $-\frac{1}{P^2}$

Sol: (a) consider the diagram where a body slides down from along an inclined plane of inclination $\theta(=45^0)$

On smooth inclined plane Acceleration of a body sliding down a smooth inclined plane

Here $a = g\sin\theta$, $\theta = 45^0$

$\therefore a = g\sin45^0 = \frac{g}{\sqrt{2}}$

Let the travelled distance be s.

Using equation of motion, $s = ut + \frac{1}{2} at^2$, weget

$s = 0.t + \frac{1}{2} \frac{g}{\sqrt{2}} T^2$ or $s = \frac{gT^2}{2\sqrt{2}}$

On rough inclined plane Acceleration fo the body

$a = g(\sin\theta - \mu \cos\theta)$

$$= \frac{g(1-\mu)}{\sqrt{2}}$$

$\left(\text{As, } \sin 45^0 = \cos 45^0 = \frac{1}{\sqrt{2}}\right)$

Again using equation of motion,

$s = ut + \frac{1}{2} at^2$, we get

$$s = 0(pT) + \frac{1}{2} \frac{g(1-\mu)}{\sqrt{2}} (pT)^2$$

or $s = \frac{g(1-\mu)p^2T^2}{2\sqrt{2}}$

From Eqs. (i) and (ii), we get

$$\frac{gT^2}{2\sqrt{2}} = \frac{g(1-\mu)p^2T^2}{2\sqrt{2}}$$

or $(1-\mu)p^2 = 1$

or $(1-\mu) = \frac{1}{p^2}$

or $\mu = \left(1 - \frac{1}{p^2}\right)$

JEE MAIN PREVIOUS YEAR QUESTIONS

TOPIC-1 $1^{ST}, 2^{ND}$ & 3^{RD} Laws of Motion

1. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{\imath} + v_x\hat{\jmath})$, where v_x and v_y are x and y components of its velocity $\vec{v}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle? [Sep. 06, 2020 (II)]

(a) Quantity $\vec{v} \times \vec{a}$ is constant in time

(b) $\vec{F}$ Arises due to a magnetic field

(c) Kinetic energy of particle is constant in time

(d) Quantity $\vec{v} \cdot \vec{a}$ is constant in time

sol. (a) Given

$$\vec{F} = k(v_y\hat{\imath} + v_x j)$$
$$F_x = kv_y\hat{\imath}, F_y = kv_x j$$
$$\frac{mdv_x}{dt} = kv_y \Rightarrow \frac{dv_x}{dt} = \frac{k}{m}v_y$$

Similarly, $\frac{dv_y}{dt} = \frac{k}{m}v_x$

$$\frac{dv_y}{dv_x} = \frac{v_x}{v_y} \Rightarrow \int v_y dv_y = \int v_x dv_x$$

$$v_y^2 = v_X^2 + C$$

$v_y^2 - v_X^2 =$ constant

$$\vec{v} \times \vec{a} = (v_x\hat{\imath} + v_y\hat{\jmath}) \times \frac{k}{m}(v_y\hat{\imath} + v_x\hat{\jmath})$$

$= (v_x^2\hat{k} - v_y^2\hat{k})\frac{k}{m} = (v_x^2 - v_y^2)\frac{k}{m}\hat{k} =$ constant

2. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is : [Sep. 05, 2020 (II)]

(a) $-bv^3(t)$ (b) $-\frac{bv^3}{M(t)}$ (c) $-\frac{2bv^3}{M(t)}$ (d) $-\frac{bv^3}{2M(t)}$

sol. (b) From the Newton's second law,

$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\left(\frac{dm}{dt}\right)$ (i)

We have given, $\frac{dM(t)}{dt} = bv^2(t)$ (ii)

Thrust on the satellite,

$F=-v\left(\frac{dm}{dt}\right)=-v(bv^2)=-bv^3$ [Using (i) and (ii)]

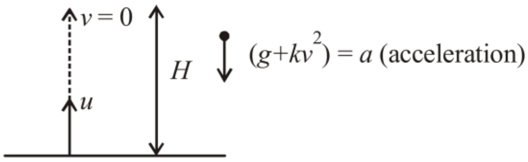
$$\Rightarrow F=M(t)a=-bv^3 \Rightarrow a=\frac{-bv^3}{M(t)}$$

3. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mky^2 where v is its speed. The maximum height attained by the ball is:
[Sep. 04, 2020 (II)]

(a) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ (b) $\frac{1}{k}\ln\left(\left(+\frac{ku^2}{2g}\right)\right)$ (c) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (d) $\frac{1}{2k}\ln\left(\left(+\frac{ku^2}{g}\right)\right)$

sol.

(d)



$\vec{F}=mkv^2-mg$ ($\because mg$ and mky^2 act opposite to each other)

$\vec{a}=-[kv^2+g]$

$\Rightarrow v\cdot\frac{dv}{dh}=-[ky^2+g]$ $\left(\because a=v\frac{dv}{dh}\right)$

$$\Rightarrow \int_u^0 \frac{v\cdot dv}{ky^2+g}=\int_0^h d\,h$$

$$\Rightarrow \frac{1}{2k}\ln\left[kv^2+g\right]_u^0=-h$$

$$\Rightarrow \frac{1}{2k}\ln\left[\frac{ku^2+g}{g}\right]=h$$

4. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass ofthe ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is: [10 April 2019 I]

(a) $\frac{1}{\sqrt{\gamma}g}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$ (b) $\frac{1}{\sqrt{\gamma}g}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(c) $\frac{1}{\sqrt{\gamma}g}ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$ (d) $\frac{1}{\sqrt{2\gamma}g}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

sol. (a) Net acceleration

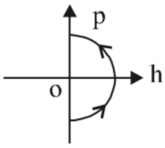
$$\frac{dv}{dt}=a=-(g+\gamma v^2)$$

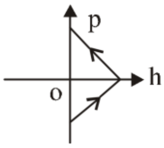
Let time t required to rise to its zenith $(v=0)$ so,

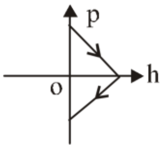
$\int_{v0}^{0}\frac{-dv}{g+\gamma v2}=\int_0^t d\,t$ [for H_{max}, $v=0$]

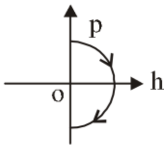
$$t = \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(\frac{\sqrt{\gamma} v_0}{\sqrt{g}}\right)$$

5. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is: [9 April 2019 I]

(a) 

(b) 

(c) 

(d) 

sol. (d) $v^2 = u^2 - 2gh$

or $v = \sqrt{u^2 - 2gh}$

Momentum, $P = mv = m\sqrt{u^2 - 2gh}$

At $h = 0,\ P = mu$ and at $h = \frac{u^2}{f}, P = 0$

upward direction is positive and downward direction is negative.

6. A particle of mass m is moving in a straight line with momentump. Starting at timet $= 0$, a force $F = k$t acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is: [11 Jan. 2019 II]

(a) $2\sqrt{\frac{k}{p}}$ (b) $2\sqrt{\frac{p}{k}}$ (c) $\sqrt{\frac{2k}{p}}$ (d) $\sqrt{\frac{2p}{k}}$

sol. (b) From Newton's second law

$$\frac{dp}{dt} = F = kt$$

Integrating both sides we get,

$$\int_p^{3p} dp = \int_0^T k\, t dt \Rightarrow [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$

$$\Rightarrow 2p = \frac{kT^2}{2} \Rightarrow T = 2\sqrt{\frac{p}{k}}$$

7. A particle of mass m is acted upon by a force F given by the empirical law $= \frac{R}{t^2} v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot: [Online Apri110, 2016]

(a) $\log v(t)$ against $\frac{1}{t}$ (b) $v(t)$ against t^2

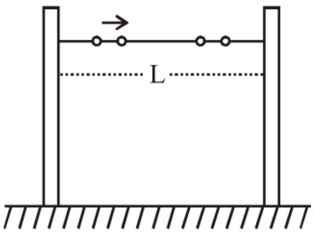
(c) $\log v(t)$ against $\frac{1}{t^2}$ (d) $\log v(t)$ against t sol. (a) From $F = \frac{R}{t^2} v(t) \Rightarrow m\frac{dv}{dt} = \frac{R}{t^2} v(t)$

Integrating both sides $\int \frac{dv}{dt} = \int \frac{Rdt}{mt^2}$

In $v = -\frac{R}{mt}$

.. $\ln v \propto \frac{1}{t}$

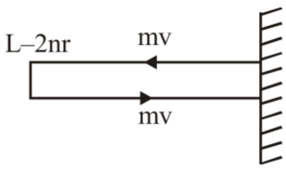
8. A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length $L(L >> r)$ and are at rest at random positions. The rod is mounted between two rigid supports (see figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic):
[Online April 11, 2015]



(a) $\frac{mv^2}{2(L-nr)}$ (b) $\frac{mv^2}{L-2nr}$ (c) $\frac{mv^2}{L-nr}$ (d) zero

sol. (b) Space between the supports for motion of beads is $L - 2nr$

Average force experienced by each support, $F = \frac{2mV}{\frac{2(L-2nr)}{V}} = \frac{mV^2}{L-2nr}$



9. A body of mass 5 kg under the action of constant force $\vec{F} = F_x\hat{i} + F_y\hat{j}$ has velocity at $t = 0s$ as $\vec{v} = (6\hat{i} - 2\hat{j})m/s$ and at $t = 10s$ as $\vec{v} = +6\hat{j}m/s$. The force $\vec{F}$ is: [Online April 11, 2014]

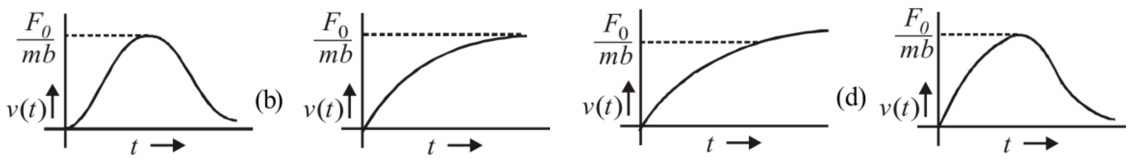
(a) $(-3\hat{j} + 4j)N$ (b) $\left(-\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)N$ (c) $(3\hat{i}\text{-}4j)N$ (d) $\left(\frac{3}{5}\hat{i}\text{-}\frac{4}{5}\hat{j}\right)N$

sol. (a) From question,
Mass of body, $m = 5$ kg
Velocity at $t = 0$, $u = (6\hat{i}\text{-}2j)$ m/s
Velocity at $t = 10s$, $v = +6j m/s$
Force, $F = ?$

Acceleration, $a=\frac{v-u}{t}=\frac{6\hat{j}-(6\hat{i}-2\hat{j})}{10}=\frac{-3\hat{i}+4\hat{j}}{5}\mathrm{m/s^2}$

Force, $F=ma \quad =5\times\frac{(-3\hat{i}+4\hat{j})}{5}=(-3\hat{i}+4\hat{j})N$

10. A particle of mass m is at rest at the origin at time $t=0$. It is subjected to a force $F(t)=F_0\mathrm{e}^{-bt}$ in the x direction. Its speed $\mathrm{v}(t)$ is depicted by which of the following curves? [2012]



sol. (c) Given that $F(t)=F_0e^{-bt}$

$$\Rightarrow m\frac{dv}{dt}=F_0e^{-bt}$$

$$\frac{dv}{dt}=\frac{F_0}{m}e^{-bt}$$

$$\int_0^v d\,v=\frac{F_0}{m}\int_0^t e^{-bt}\,dt$$

$$v=\frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t=\frac{F_0}{mb}[-(e^{-bt}-e^{-0})]$$

$$\Rightarrow v=\frac{F_0}{mb}[1-e^{-bt}]$$

11. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: If you push on a cart being pulled by a horse so that it does not move, the cart pushes you back with an equal and opposite force.

Statement 2: The cart does not move because the force described in statement 1 cancel each other. [Online May 26, 2012]

(a) Statement 1 is true, Statement2 is true, Statement 2 is the correct explanation of Statement 1.

(b) Statement 1 is false, Statement 2 is true.

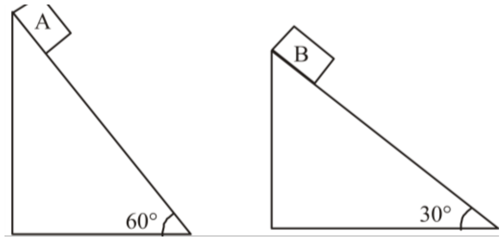
(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is true, Statement2 is true, Statement 2 is not the correct explanation of Statement 1.

sol. (a) According to newton third law of motion i.e. every action is associated with equal and opposite reaction.

12. Two fixed frictionless inclined planes making an angle 30^{O} and $60°$ with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical

acceleration of A with respect to B? [2010]



(a) 4.9ms^{-2} in horizontal direction
(b) 9.8ms^{-2} in vertical direction
(c) Zero
(d) 4.9ms^{-2} in vertical direction

sol. (d) $mg \ \sin\theta = ma$

$a = g\sin\theta$

Vertical component of acceleration $= g\sin^2\theta$

Relative vertical acceleration ofA with respect to B is

$$g(\sin^2 60 - \sin^2 30]$$

$$= g\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{g}{2} = 4.9\text{m/s}^2 \text{ in vertical direction}$$

13. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. (Consider $g = 10\text{m/s}^2$). [2006]

(a) 4N
(b) 16N
(c) 20N
(d) 22N

sol. (d) For the motion of ball, just after the throwing

$$v = 0,\ s = 2\text{m},\ a = -g = -10\text{ms}^{-2}$$

$v^2 - u^2 = 2as$ for upwardjourney

$$\Rightarrow -u^2 = 2(-10) \times 2 \Rightarrow u^2 = 40$$

When the ball is in the hands of the thrower

$$u = 0,\ v = \sqrt{40}\text{ms}^{-1}$$

$$s = 0.2\text{m}$$

$$v^2 - u^2 = 2as$$

$$\Rightarrow 40 - 0 = 2(a)0.2 \Rightarrow a = 100\text{m/s}^2$$

$$F = ma = 0.2 \times 100 = 20N$$

$$\Rightarrow N - mg = 20 \Rightarrow N = 20 + 2 = 22N$$

Note :

$$W_{hand} + W_{gravity} = \Delta K$$

$$\Rightarrow F(0.2) + (0.2)(10)(2.2) = 0 \Rightarrow F = 22N$$

14. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force ofthe blow exerted by the ball on the hand of the player is equal to [2006]

(a) 150N
(b) 3 N
(c) 30N
(d) 300N

sol. (c) Given, mass of cricket ball, $m=150g=0.15$ kg

Initial velocity, $u=20\mathrm{m/s}$

Force, $F=\frac{m(v-u)}{t}=\frac{0.15(0-20)}{0.1}=30N$

15. A particle of mass 0.3 kg subject to a force $F=-kx$ with $k=15\mathrm{N/m}$. What will be its initial acceleration if it is released from a point 20 cm away from the origin? [2005]

(a) $15\mathrm{m/s^2}$(b) $3\mathrm{m/s^2}$ (c) $10\mathrm{m/s^2}$(d) $5\mathrm{m/s^2}$

sol. (c) Mass $(m)=0.3$ kg

Force, $F=m.a=-kx$

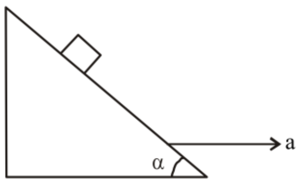
$$\Rightarrow ma=-15x$$

$$\Rightarrow 0.3a=-15x$$

$$\Rightarrow a=-\frac{15}{0.3}x=\frac{-150}{3}x=-50x$$

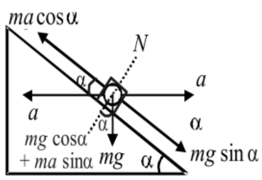
$$a=-50\times 0.2=10\mathrm{m/s^2}$$

16. A block is kept on a frictionless inclined surface with angle of inclinationα'. The incline is given an acceleration 'a' to keep the block stationary. Then a is equal to [2005]



(a) g cosec α (b) $g/\tan\alpha$ (c) $g\tan\alpha$ (d) g

sol. (c) When the incline is given an acceleration a towards the right, the block receives a reaction ma towards left.



For block to remain stationary, Net force along the incline should be zero.

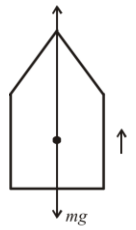
$mg\ \sin\alpha=ma\cos\alpha\Rightarrow a=g\tan\alpha$

17. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial accelerationof $10\mathrm{m/s^2}$. Then the initial thrust of the blast is [2003]

(a) $3.5\times10^5\mathrm{N}$ (b) $7.0\times10^5\mathrm{N}$ (c) $14.0\times10^5\mathrm{N}$ (d) $1.75\times10^5\mathrm{N}$

sol. (b) In the absence of air resistance, if Thrust (F) the rocket moves up with an acceleration a, then thrust

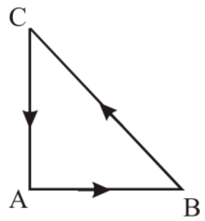
$F=mg+ma$



$F = m(g + a) = 3.5 \times 10^4$ (10+10)

$= 7 \times 10^5$N

18. Three forces start acting simultaneously on a particle moving with velocity, $\vec{v}$. These forces are represented in maguitude and direction by the three sides of a triangle ABC. The particle will now move with velocity [2003]



(a) less than $\vec{v}$ (b) greater than $\vec{v}$

(c) $|\vec{v}|$ in the direction of the largest force BC (d) $\vec{v}$, remaining unchanged

sol. (d) Resultant force is zero, as three forces are represented by the sides of a triangle taken in the same order. From Newton's second law, $\vec{F}_{net} = m\vec{a}$.

Therefore, acceleration is also zero *i.e.*, velocity remains unchanged.

19. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) [2002]

(a) solid sphere (b) hollow sphere (c) ring (d) all same

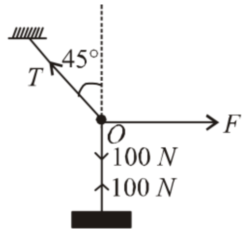
sol. (d) This is a case of sliding (if plane is friction less) and therefore the acceleration of all the bodies is same.

TOPIC 2, **Motion of Connected Bodies, Pulley& Equilibrium of Forces**

20. A mass of10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the midpoint ofthe rope such that the top half of the rope makes an angle of $45°$ with the vertical. Then F equals: (Take $g = 10$ m/s2 and the rope to be massless) [7 Jan. 2020 II]

(a) 100 N (b) 90 N (c) 70N (d) 75N

sol. (a) From the free body diagram



10 kg

$T \cos 45^{\circ}=100 N$ (i)

$T \sin 45^{\circ}=F$ (ii)

On dividing (i) by(ii) we get

$$\frac{T \cos 45^{\circ}}{T \sin 45^{\circ}}=\frac{100}{F}$$

$$\Rightarrow F=100 \mathrm{~N}$$

21. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass ofthe elevator itself is 920 kg and it moves with a constant speed of 3 $\mathrm{m/s}$. The frictional force opposing the motion is 6000 N. Ifthe elevator is moving up with its full capacity, the power delivered by the motor to the elevator $\left(g=10 \mathrm{m/s}^{2}\right)$ must be at least: [7 Jan. 2020 II]

(a) 56300 W (b) 62360 W (c) 48000 W (d) 66000 W

sol. (d) Net force on the elevator $=$ force on elevator $+$ frictional force

$$\Rightarrow F=(10 m+M) g+f$$

where, $m=$ mass ofperson, $M=$ mass ofelevator, $f=$ fiictional force

$$\Rightarrow F=(10 \times 68+920) \times 9.8+600$$

$$\Rightarrow F=22000 N$$

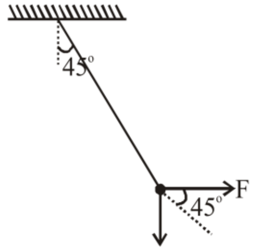
$\Rightarrow P=F V=22000 \times 3=66000 W$

22. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45^{o} at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is $\left(\mathrm{g}=10 \mathrm{ms}^{-2}\right)$ [9 Jan. 2019 II]

(a) 200 N (b) 140 N (c) 70 N (d) 100 N

sol.

(d)

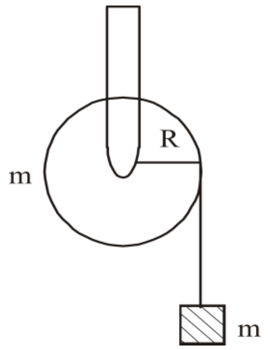


At equilibrium,

$$\tan 45^{\circ}=\frac{\mathrm{mg}}{F}=\frac{100}{F}$$

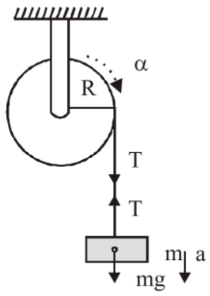
$$F = 100\text{N}$$

23. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. Ifthe string does not slip on the cylinder, with what acceleration will the mass fall or release? [2014]



(a) $\frac{2g}{3}$ b) $\frac{g}{2}$ (c) $\frac{5g}{6}$ (d) g

sol. (b) From figure,



Acceleration $a = R\alpha$ ... (i)

And $mg - T = ma$ ...(ii)

From equation (i) and (ii)

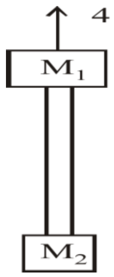
$$T \times R = mR^2\alpha = mR^2\left(\frac{a}{R}\right)$$

or $T = ma$

$\Rightarrow mg - ma = ma$

$\Rightarrow a = \frac{g}{2}$

24. Two blocks of mass $M_1 = 20$ kg and $M_2 = 12$ kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is: [Online April 22, 2013]



(a) 144 N (b) 96N (c) 240 N (d) 192N

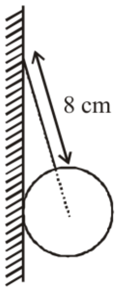
sol. (d) Acceleration produced in upward direction $a = \frac{F}{M_1 + M_2 + \text{Mass of metal rod}}$

$= \frac{480}{20+12+8} = 12 \text{ ms } -2$

Tension at the mid point

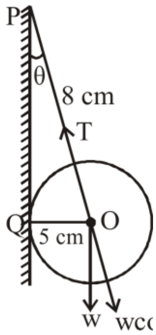
$T = \left(M_2 + \frac{\text{Mass of rod}}{2}\right) a = (12 + 4) \times 12 = 192\text{N}$

25. A uniform sphere of weight W and radius 5 cm is being held by a string as shown in the figure. The tension in the string will be: [Online April 9, 2013]



(a) $12\frac{W}{5}$ (b) $5\frac{W}{12}$ (c) $13\frac{W}{5}$ (d) $13\frac{W}{12}$

sol. (d)



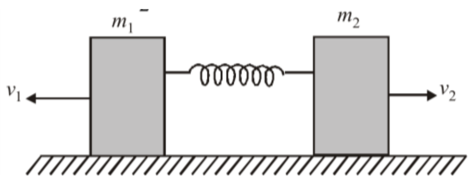
$$PQ = \sqrt{oP^2 + OQ^2}$$
$$= \sqrt{13^2 + 5^2} = 12$$

Tension in the string $T = w\cos\theta = \frac{13}{12}W$

26. A spring is compressed between two blocks of masses m_1 and m_2 placed on a horizontal

fiictionless surface as shown in the figure. When the blocks are released, they have initial velocity of v_1 and v_2 as shown. The blocks travel distances x_1 and x_2 respectively before coming to rest.

The ratio $\left(\frac{x_1}{x_2}\right)$ is [Online May 12, 2012]



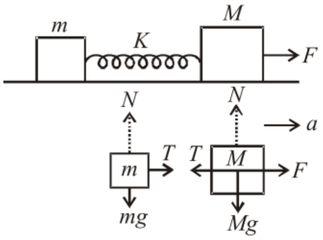
(a) $\frac{m_2}{m_1}$ (b) $\frac{m_1}{m_2}$ (c) $\sqrt{\frac{m_2}{m_1}}$ (d) $\sqrt{\frac{m_1}{m_2}}$

sol. (a)

27. A block of mass m is connected to another block ofmass M by a spring (massless) of spring constant$k$. The block is kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force of the block of mass m. [2007]

(a) $\frac{MF}{(m+M)}$ (b) $\frac{mF}{M}$ (c) $\frac{(M+m)F}{m}$ (d) $\frac{mF}{(m+M)}$

sol. (d) Writing free body-diagrams for m &M,



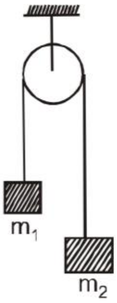
we get $T = ma$ and $F - T = Ma$ where T is force due to spring

$\Rightarrow F\text{-}ma = Ma$ or, $F = Ma + ma$

Acceleration of the system $a = \frac{F}{M+m}$.

Now, force acting on the block of mass m is $ma = m\left(\frac{F}{M+m}\right) = \frac{mF}{m+M}$

28. Two masses $m_1 = 5\text{g}$ and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when left free to move? $(\text{g} = 10\text{ms}^{-2})$ [2004]



(a) 5 m/s^2 (b) $9.8 m/s^2$ (c) $0.2 m/s^2$ (d) $4.8 m/s^2$

sol. (c)Here,$m_1 = 5$kgand$m_2 = 48$kg

If a is the acceleration of the masses,

$m_1 a = m_1 g - T$ (i)

$m_2 a = T - m_2 g$ (ii)

Solving (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$\Rightarrow a = \frac{(5 - 4.8) \times 9.8}{(5 + 4.8)} \text{m/s}^2 = 0.2 \text{m/s}^2$$

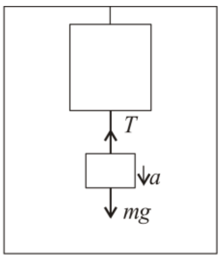
29. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance Will be [2003]

(a) 24N (b) 74N (c) 15N (d) 49N

sol. (a) When lift is stationary, $W_1 = mg$ (i)

When the lift descends with acceleration a, $W_2 = m(g - a)$

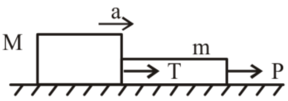
$$W_2 = \frac{49}{10}(10 - 5) = 24.5N$$



30. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end ofthe rope, the force exerted by the rope on the block is [2003]

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$ (c) P (d) $\frac{PM}{M+m}$

sol. (d) Taking the rope and the block as a system



we get $P=(m+M)a$

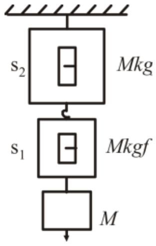
Acceleration produced, $a=\frac{P}{m+M}$

Taking the block as a system,

Force on the block, $F=Ma \quad F=\frac{MP}{m+M}$

31. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs liom the former one. Then the true statement about the scale reading is [2003]
(a) both the scales read M kg each
(b) the scale of the lower one reads M kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the reading will be M kg
(d) both the scales read *M/2* kg each

sol. (a) The Earth exerts a pulling force *Mg*. The block in turn exerts a reaction force *Mg* on the spring of spring balance S_1 which therefore shows a reading of *M* kgf.
As both the springs are massless. Therefore, it exerts a force of *Mg* on the spring of spring balance S_2 which shows the reading of $Mkgf$.



32. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively [2002]
(a) g, g (b) $g-a,\ g-a$ (c) $g-a,\ g$ (d) $a,\ g$

sol. (c) Case - I: For the man standing in the lift, the acceleration of the ball

$$\vec{a}_{bm}=\vec{a}_b-\vec{a}_m \Rightarrow a_{bm}=g-a$$

Case- II: The man standing on the ground, the acceleration of the ball

$$\vec{a}_{bm}=\vec{a}_b-\vec{a}_m \Rightarrow a_{bm}=g-0=g$$

33. When forces $F_1,\ F_2,\ F_3$ are acting on a particle ofmass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. Ifthe force F_1 is nowremoved then the acceleration of the particle is [2002]
(a) F_1/m (b) F_2F_3/mF_1 (c) $(F_2-F_3)/m$ (d) F_2/m.

sol. (a) When forces F_1 , F_2 and F_3 are acting on the particle, it remains in equilibrium. Force F_2 and F_3 are perpendicular to each other,

$$F_1 = F_2 + F_3$$

$$F_1 = \sqrt{F_2^2 + F_3^2}$$

The force F_1 is now removed, so, resultant of F_2 and F_3 will now make the particle move with force equal to F_1. Then, acceleration, $a = \frac{F_1}{m}$

34. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are [2002]

(a) $12\text{N}, 6\text{N}$ (b) $13\text{N}, 5\text{N}$ (c) 10 N, 8N (d) $16\text{N}, 2\text{N}$.

sol. (b) Let the two forces be F_1 and F_2 and let $F_2 < F_1$. R is the resultant force.

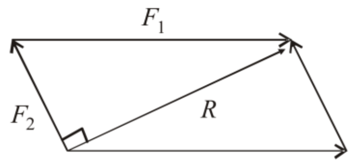
Given $F_1 + F_2 = 18$ (i)

From the figure $F_2^2 + R^2 = F_1^2$

$F_1^2 - F_2^2 = R^2$

$\therefore \quad F_1^2 - F_2^2 = 144$

Only option (b) follows equation (i) and (ii).

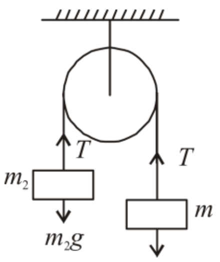


35. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is [2002]

(a) 8: 1 (b) 9: 7 (c) 4: 3 (d) 5: 3

sol. (b) For mass m_1 $m_1 g - T = m_1 a$ (i)

For mass m_2 $T - m_2 g = m_2 a$ (ii)

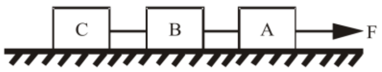


Adding the equations we get $a = \frac{(m_1 - m_2)g}{m_1 + m_2}$

Here $a = \frac{g}{8}$

$$\frac{1}{8}=\frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1}\Rightarrow\frac{m_1}{m_2}+1=8\frac{m_1}{m_2}-8\Rightarrow\frac{m_1}{m_2}=\frac{9}{7}$$

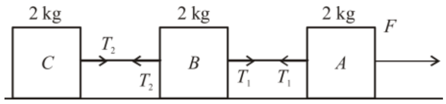
36. Three identical blocks of masses $m=2$ kg are drawn by a force $F=10.2\mathrm{N}$ with an acceleration of $0.6\mathrm{ms}^{-2}$ on a fiictionless surface, then what is the tension (in N) in the string between the blocks B and C? [2002]



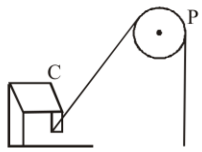
(a) 9.2 (b) 3.4 (c) 4 (d) 9.8

sol. (b) Γorce $=$mass $\times$ acceleration

$$F=(m+m+m)\times a$$
$$F=3\mathrm{m}\times\mathrm{a}$$
$$a=\frac{F}{3m}$$
$$a=\frac{10.2}{6}\mathrm{m/s^2}$$
$$T_2=ma=2\times\frac{10.2}{6}=3.4\mathrm{N}$$



37. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? [2002]



(a) 16 (b) 6 (c) 4 (d) 8

sol. (c) Tension, $T=360\mathrm{N}$

Mass of a man $\mathrm{m}=60$ kg

$$mg-T=ma$$
$$a=g-\frac{T}{m}$$
$$=10-\frac{360}{60}=4m/s^2$$

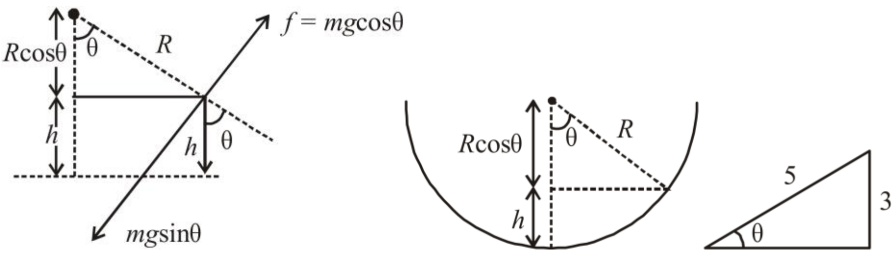
TOPIC-3 **Friction**

38. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is: $(\mathrm{g}=10 \mathrm{~ms}^{-2})$ [Sep. 06, 2020 (I)]

(a) 0.20 m (b) 0.45 m (c) $0.60 \mathrm{m}$ (d) $0.80 \mathrm{m}$

sol. (a) For balancing, $mg \sin \theta=f=\mu m g \cos \theta$

$$\Rightarrow \tan \theta=\mu=\frac{3}{4}=0.75$$



$$h=R-R \cos \theta=R-R\left(\frac{4}{5}\right)=\frac{R}{5}$$

$h=\frac{R}{5}=0.2 \mathrm{m}$ [radius, $R=1 \mathrm{m}$]

39. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_{0}. It comes back to its initial position with velocity $\frac{v_{0}}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is

[NA Sep. 03, 2020 (II)]

sol. (346)

Acceleration of block while moving up an inclined plane,

$$a_{1}=g \sin \theta+\mu g \cos \theta$$

$$\Rightarrow a_{1}=g \sin 30^{\circ}+\mu g \cos 30^{\circ}$$

$=\frac{g}{2}+\frac{\mu g \sqrt{3}}{2}$ (i) $(\theta=30^{\mathrm{o}})$

Using $v^{2}-u^{2}=2 a(s)$

$$\Rightarrow v_{0}^{2}-0^{2}=2 a_{1}(s) \quad(\mathrm{u}=0)$$

$$\Rightarrow v_{0}^{2}-2 a_{1}(s)=0$$

$\Rightarrow s=\frac{v_{0}^{2}}{a_{1}}$ (ii)

Acceleration while moving down an inclined plane

$$a_{2}=g \sin \theta-\mu g \cos \theta$$

$$\Rightarrow a_2 = g \sin 30° - \mu g \cos 30°$$

$\Rightarrow a_2 = \frac{g}{2} - \frac{\mu\sqrt{3}}{2} g$ (iii)

Using again $v^2 - u^2 = 2as$ for downward motion

$\Rightarrow \left(\frac{v_0}{2}\right)^2 = 2a_2(s) \Rightarrow s = \frac{v_0^2}{4a_2}$ (iv)

Equating equation (ii) and (iv) $\frac{v_0^2}{a_1} = \frac{v_0^2}{4a_2} \Rightarrow a_1 = 4a_2$

$$\Rightarrow \frac{g}{2} + \frac{\mu g\sqrt{3}}{2} = 4\left(\frac{g}{2} - \frac{\mu\sqrt{3}}{2}\right)$$

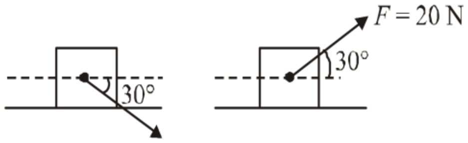
$\Rightarrow 5 + 5\sqrt{3}\mu = 4(5 - 5\sqrt{3}\mu)$ (Substituting, $g = 10\text{m/s}^2$)

$$\Rightarrow 5 + 5\sqrt{3}\mu = 20 - 20\sqrt{3}\mu \Rightarrow 25\sqrt{3}\mu = 15$$

$$\Rightarrow \mu = \frac{\sqrt{3}}{5} = 0.346 = \frac{346}{1000}$$

So, $\frac{l}{1000} = \frac{346}{1000}$

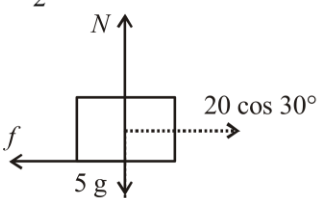
40. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20\text{N}$, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be: ($g = 10 \ \text{ms}^{-2}$) [12 April 2019 II]



(a) $0.4 \ \text{ms}^{-2}$ (b) $3.2 \ \text{ms}^{-2}$ (c) $0.8 \ \text{ms}^{-2}$ (d) $0 \ \text{ms}^{-2}$

sol. (c) A: $N = 5\text{g} + 20 \sin 30°$

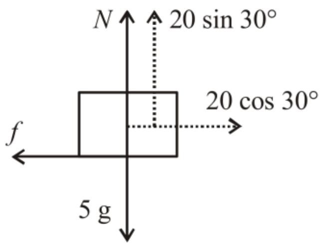
$= 50 + 20 \times \frac{1}{2} = 60$ N



20 sin 30°

Accelaration, $a_1 = \frac{F-f}{m} = \frac{20 \cos 30° - \mu N}{5}$

$$= \left[\frac{20 \times \frac{\sqrt{3}}{2} - 0.2 \times 60}{5}\right] = 1.06\text{m/s}^2$$



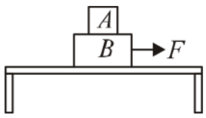
B: $N = 5g - 20\sin 30^\circ$

$$= 50 - 20 \times \frac{1}{2} = 40\text{N}$$

$$a_2 = \frac{F - f}{m} = \left[\frac{20\cos 30^\circ - 0.2 \times 40}{5}\right] = 1.86\text{m/s}^2$$

Now $a_2 - a_1 = 1.86 - 1.06 = 0.8\text{m/s}^2$

41. Two blocks A and B masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient offiiction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is: [Take $g = 10\text{m/s}^2$] [10 April 2019 II]



(a) 8N (b) 16N (c) 40N (d) 12N

sol. (b) Taking $(A + B)$ as system

$$F - \mu(M + m)g = (M + m)a$$

$$\Rightarrow a = \frac{F - \mu(M + m)g}{(M + m)}$$

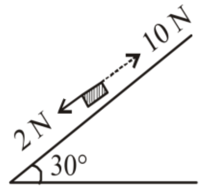
$a = \frac{F - (0.2)4 \times 10}{4} = \left(\frac{F - 8}{4}\right)$ (i)

But, $a_{\max} = \mu g = 0.2 \times 10 = 2$

$$\frac{F - 8}{4} = 2$$

$$\Rightarrow F = 16\text{N}$$

42. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is: [Take $g = 10\text{m/s}^2$] [12 Jan. 2019 II]



(a) $\frac{\sqrt{3}}{2}$ ` (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

sol. (a) From figure,

$2+\text{mg}\sin 30° = \mu\text{mg}\cos 30°$ and $10 = \text{mg}\sin 30° + \mu$ mg $\cos 30°$

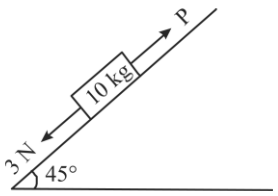
$= 2\mu\text{mg}\cos 30° - 2$

$\Rightarrow 6 = \mu\text{mg}\cos 30°$ and $4 = \text{mg}\cos 30°$

By dividing above two $\Rightarrow \frac{3}{2} = \mu \times \sqrt{3}$

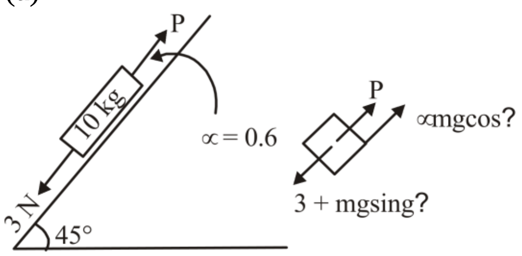
Coefficient of friction, $\mu = \frac{\sqrt{3}}{2}$

43. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (take $g = 10\text{ms}^2$) [9 Jan. 2019 I]



(a) 32N (b) 18N (c) 23 N (d) 25 N

sol. (a)



$$\text{mg}\sin 45° = \frac{100}{\sqrt{2}} = 50\sqrt{2}$$

$$[\because \text{m} = 10\text{kg}, \text{g} = 9.8\text{ms}^{-2}]$$

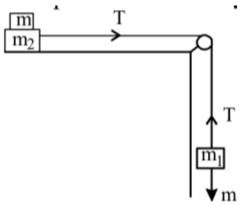
$$\mu\text{mg}\cos\theta = 0.6 \times \text{mg}^{\times}\frac{1}{\sqrt{2}} = 0.6'50\sqrt{2}$$

$$3 + \text{mg}\sin\theta = \text{P} + \mu\text{mg}\cos\theta$$

$$3 + 50\sqrt{2} = \text{P} + 30\sqrt{2}$$

$$\text{P} = 31.28 = 32\text{N}$$

44. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is[2018]



(a) 18.3 kg (b) 27.3 kg (c) 43.3 kg (d) 10.3 kg

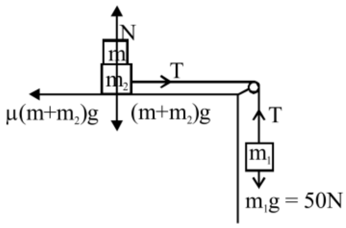
sol. (b) Given: $m_1 = 5kg; m_2 = 10kg; \mu = 0.15$

FBD for m_1 , $m_1 g - T = m_1 a \quad \Rightarrow 50 - T = 5 \times a$

and, $T - 0.15(m + 10)g = (10 + m)a$

For rest $a = 0$

or, $50 = 0.15(m + 10)10$



$\Rightarrow 5 = \frac{3}{20}(m + 10)$

$\frac{100}{3} = m + 10 \;\; m = 23.3kg;$ close to option (b)

45. A given object takes n times more time to slide down a $45°$ rough inclined plane as it takes to slide down a perfectly smooth 45^{O} incline. The coefficient of kinetic friction between the object and the incline is: [Online Apri115, 2018]

(a) $\sqrt{1 - \frac{1}{n^2}}$ (b) $1 - \frac{1}{n^2}$ (c) $\frac{1}{2-n^2}$ (d) $\sqrt{\frac{1}{1-n^2}}$

sol. (b) The coefficients of kinetic friction between the object and the incline

$$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) \Rightarrow \mu = 1 - \frac{1}{n^2} \; (\theta = 45°)$$

46. A body of mass 2kg slides down with an acceleration of $3m/s^2$ on a rough inclined plane having a slope of $30°$. The external force required to take the same body up the plane with the same acceleration will be: $(g = 10m/s^2)$ [Online April 15, 2018]

(a) 4N (b) 14N (c) 6N (d) 20N

sol. (d) Equation of motion when the mass slides down $Mg \;\; \sin\theta - f = Ma$

$\Rightarrow 10 - f = 6(M = 2kg,\; a = 3m/s^2,\; \theta = 30°$ given)

$$f = 4N$$

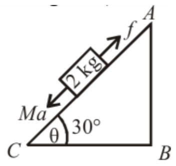
Equation of motion when the block is pushed up

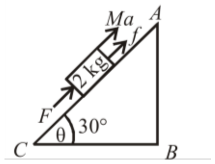
Let the external force required to take the block up the plane acceleration be F

F-$Mg \quad \sin\theta - f = Ma$

$$\Rightarrow F - 10 - 4 = 6$$

$$F = 20\text{N}$$

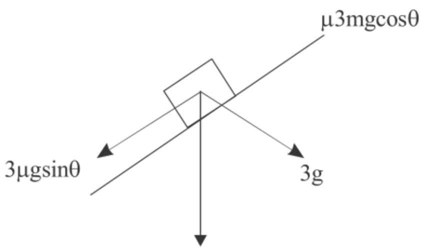




47. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient offiiction $\mu_{\text{m ln}}$ between the mass and the inclined surface such that the mass does not move is: [Online April 9, 2016]

(a) $\tan 2\theta$ (b) $\tan\theta$ (c) $3 \quad \tan\theta$ (d) $2 \quad \tan\theta$

sol. (b) Let μ be the minimum coefficient of friction

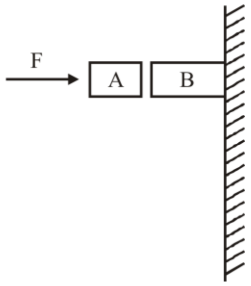


At equilibrium, mass does not move so,

$$3\text{mg} \sin\theta = \mu 3\text{mg} \cos\theta$$

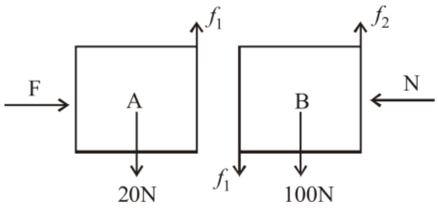
$.. \ \mu_{\text{min}} = \tan\theta$

48. Given in the figure are two blocks A and B ofweight $20 \ \text{N}$ and $100 \ \text{N}$, respectively. These are being pressed against a wall by a force Γ as shown. If the coefficient of fiiction between the blocks is 0.1 and between block B and the wall is 0.15, the fiictional force applied by the wall on block B is: [2015]



(a) 120N (b) 150N (c) 100N (d) 80N

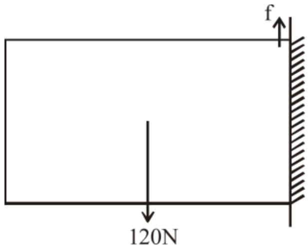
sol. (a)



Assuming both the blocks are stationary $N = F$

$f_1 = 20N$

$f_2 = 100 + 20 = 120N$



Considering the two blocks as one system and due to equilibrium f$=120N$

49. A block of mass $m = 10$ kg rests on a horizontal table. The coefficient of fiiction between the block and the table is 0.05. When hit by a bullet ofmass 50 g moving with speed v, that gets embedded in it, the block moves and comes to stop after moving a distance of2 m on the table. If a freely falling object were to acquire speed $\frac{v}{10}$ after being dropped from height H, then neglecting energy losses and taking $g = 10ms^{-2}$, the value of H is close to: [Online April 10, 2015]

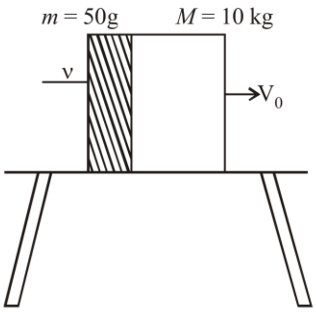
(a) 0.05 km (b) 0.02 km (c) 0.03km (d) 0.04km

sol. (d) $f = \mu(M+m)g$

$$a = \frac{f}{M+m} = \frac{\mu(M+m)g}{(M+m)} = \mu g$$

$$= 0.05 \times 10 = 0.5ms^{-2}$$

$$V_0 = \frac{\text{Initia1momenmm}}{(M+m)} = \frac{0.0.5V}{1005}$$



$v^2 - u^2 = 2as$

$0 - u^2 = 2as$

$u^2 = 2as$

$$\left(\frac{0.05v}{10.05}\right)^2 = 2 \times 0.5 \times 2$$

Solving we get $v = 201\sqrt{2}$

Object falling from height H.

$$\frac{V}{10} = \sqrt{2gH}$$

$$\frac{201\sqrt{2}}{10} = \sqrt{2 \times 10 \times H}$$

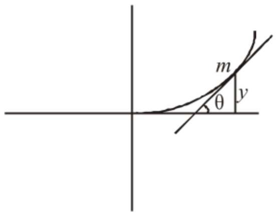
$H = 40\text{m} = 0.04$ km

50. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: [2014]

(a) $\frac{1}{6}$m (b) $\frac{2}{3}$m (c) $\frac{1}{3}$m (d) $\frac{1}{2}$m

sol. (a) At limiting equilibrium,

$\mu = \tan\theta$



$\tan\theta = \mu = \frac{dy}{dx} = \frac{x^2}{2}$ (from question)

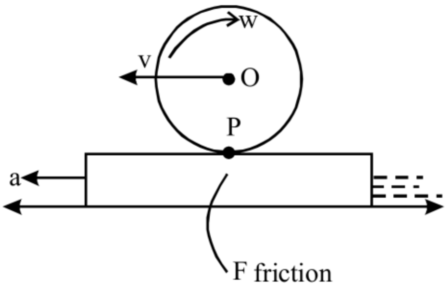
Coefficient of friction $\mu = 0.5$

$$0.5 = \frac{x^2}{2}$$

$\Rightarrow x = \pm 1$

Now, $y = \frac{x^3}{6} = \frac{1}{6}m$

51. Consider a cylinder of mass M resting on a rough horizontal rug that is pulled out from under it with acceleration 'a' perpendicular to the axis of the cylinder. What is F_{fiiction} at point P? It is assumed that the cylinder does not slip. [Online Apri119, 2014]



(a) Mg (b) Ma (c) $\frac{\text{Ma}}{2}$ (d) $\frac{\text{Ma}}{3}$

sol. (d) Force of friction at point P,

$\Gamma_{\text{fiiction}} = \frac{1}{3}$ Ma $\sin\theta$

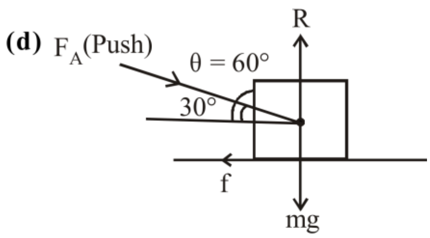
$= \frac{1}{3}$ Ma $\sin 90°$ [here $\theta = 90°$]

$= \frac{\text{Ma}}{3}$

52. A heavy box is to dragged along a rough horizontal floor. To do so, person A pushes it at an angle $30°$ from the horizontal and requires a minimum force F_A, while person B pulls the box at an angle 60^o from the horizontal and needs minimum force F_B. If the coefficient of friction between the box and the floor is $\frac{\sqrt{3}}{5}$, the ratio $\frac{F_A}{F_B}$ is [Online Apri119, 2014]

(a) $\sqrt{3}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{2}{\sqrt{3}}$

sol.



$$F_A = \frac{\mu mg}{\sin\theta - \mu\cos\theta}$$

Similarly,

$$F_B = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$$

$$\frac{F_A}{F_B} = \frac{\frac{\mu mg}{\sin\theta - \mu\cos\theta}}{\frac{\mu mg}{\sin\theta + \mu\cos\theta}} \qquad [\mu = \frac{\sqrt{3}}{5} \text{ given}]$$

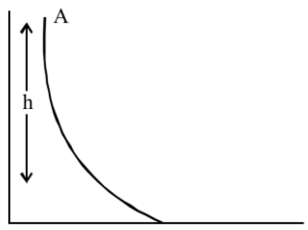
$$= \frac{\sin 30° + \frac{\sqrt{3}}{5} \cos 30°}{\sin 60° - \frac{\sqrt{3}}{5} \cos 60°}$$

$$= \frac{\frac{1}{2} + \frac{\sqrt{3}}{5} \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{5} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{2}\left(1+\frac{3}{5}\right)}{\frac{\sqrt{3}}{5}\left(1-\frac{1}{5}\right)} = \frac{\frac{1}{2} \times \frac{8}{5}}{\frac{\sqrt{3}\times 4}{10}}$$

$$= \frac{8}{\sqrt{3} \times 4} = \frac{2}{\sqrt{3}}$$

53. A small ball of mass m starts at a point A with speed v_o and moves along a fiictionless track AB as shown. The track BC has coefficient of friction μ. The ball comes to stop at C after travelling a distance L which is: [Online April 11, 2014]



B ← L ← C

(a) $\frac{2h}{\mu} + \frac{v_o^2}{2\mu g}$ (b) $\frac{h}{\mu} + \frac{v_o^2}{2\mu g}$ (c) $\frac{h}{2\mu} + \frac{v_o^2}{\mu g}$ (d) $\frac{h}{2\mu} + \frac{v_o^2}{2\mu g}$

sol. b) Initial speed at point A, $u = v_0$

Speed at point B, $v = ?$

$$v^2 - u^2 = 2gh$$
$$v^2 = v_0^2 + 2gh$$

Let ball travels distance 'S' before coming to rest

$$S = \frac{v^2}{2\mu g} = \frac{v_0^2 + 2gh}{2\mu g}$$
$$= \frac{v_0^2}{2\mu g} + \frac{2gh}{2\mu g} = \frac{h}{\mu} + \frac{v_0^2}{2\mu g}$$

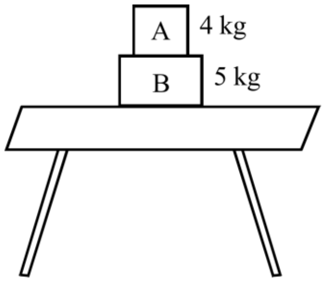
54. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied to B for the blocks to move together will

be: [Online April 9, 2014]

(a) 30N (b) 25 N (c) 27N (d) 48N

sol. (c) Minimum force on A

$=$ frictional force between the surfaces $=12N$

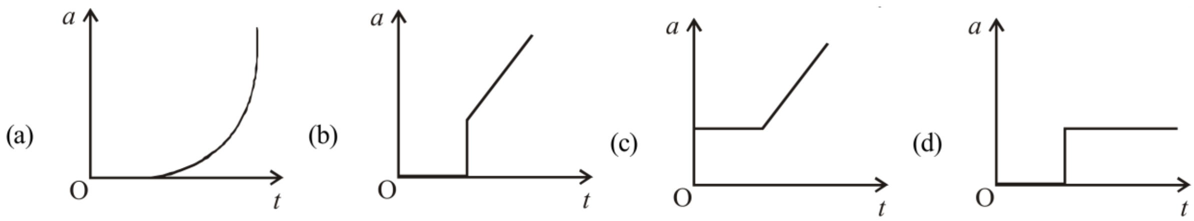


Therefore maximum acceleration $a_{\max}=\frac{12N}{4kg}=3m/s^2$

Hence maximum force, $F_{\max}=$ total mass $\times a_{\max}$

$=9\times 3=27N$

55. A block is placed on a rough horizontal plane. A time dependent horizontal force $F=kt$ acts on the block, where k is a positive constant. The acceleration- time graph of the block is :
[Online April 25, 2013]



sol. (b) Graph (b) correctly depicts the acceleration-time graph of the block.

56. A body starts from rest on a long inclined plane of slope $45°$. The coefficient of fiiction between the body and the plane varies as $\mu=0.3x$, where x is distance travelled down the plane. The body will have maximum speed (for $g=10m/s^2$) when $x=$ [Online April 22, 2013]

(a) 9.8m (b) 27m (c) 12m (d) 3.33m

sol. (d) When the body has maximum speed then

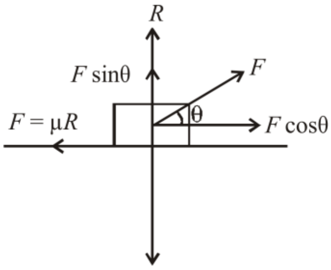
$$\mu=0.3x=\tan 45°$$
$$x=3.33m$$

57. A block of weight W rests on a horizontal floor with coefficient of static friction μ. It is desired to make the block move by applying minimum amount of force. The angle θ from the horizontal at which the force should be applied and magnitude of the force F are respectively.
[Online May 19, 2012]

(a) $\theta=\tan^{-1}(\mu), F=\frac{\mu W}{\sqrt{1+\mu^2}}$ (b) $\theta=\tan^{-1}\left(\frac{1}{\mu}\right), F=\frac{\mu W}{\sqrt{1+\mu^2}}$

(c) $\theta=0, F=\mu W$ (d) $\theta=\tan^{-1}\left(\frac{\mu}{1+\mu}\right), F=\frac{\mu W}{1+\mu}$

sol. (a) Let the force F is applied at an angle θ with the horizontal.



For horizontal equilibrium,

$F\cos\theta=\mu R$ (i)

For vertical equilibrium,

$$R+F\sin\theta=\text{mg}$$

or, $R=mg-F\sin\theta$ (ii)

Substituting this value of R in eq. (i), we get $F\cos\theta=\mu(\text{mg}-F\ \text{sine})$

$$=\mu\text{mg}-\mu F\sin\theta$$

or, $F(\cos\theta+\mu\sin\theta)=\mu\text{mg}$

or, $\Gamma=\frac{\mu\text{mg}}{\cos\theta+\mu\sin\theta}$ (iii)

For F to be minimum, the denominator ($\cos\theta+\mu$ sine) should be maximum.

$\frac{d}{d\theta}(\cos\theta+\mu\sin\theta)=0$ or, —sine $+\mu\cos\theta=0$ or, $\tan\theta=\mu$

or, $\theta=\tan^{-1}(\mu)$

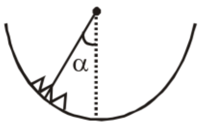
Then, $\sin\theta=\frac{\mu}{\sqrt{1+\mu^2}}$ and

$$\cos\theta=\frac{1}{\sqrt{1+\mu^2}}$$

Hence, F_{min}

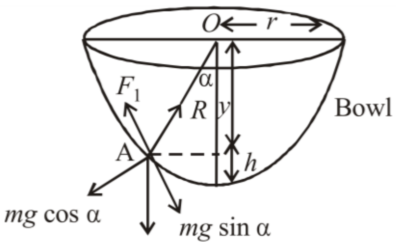
$$=\frac{\mu w}{\frac{1}{\sqrt{1+\mu^2}}+\frac{\mu^2}{\sqrt{1+\mu^2}}}=\frac{\mu w}{\sqrt{1+\mu^2}}$$

58. An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α so that the insect does not slip is given by [Online May 12, 2012]



(a) $\cot \alpha = 3$ (b) $\sec \alpha = 3$ (c) $\operatorname{cosec} \alpha = 3$ (d) $\cos \alpha = 3$

sol. (a)



The insect crawls up the bowl upto a certain height h only till the component ofits weight along the bowl is balanced by limiting frictional force.

For limiting condition at point A

$R = mg \cos \alpha$ (i)

$F_1 = mg \sin \alpha$ (ii)

Dividing eq. (ii) by(i)

$$\tan \alpha = \frac{1}{\cot \alpha} = \frac{F_1}{R} = \mu [As\ F_1 = \mu R]$$

$\Rightarrow \tan \alpha = \mu = \frac{1}{3} [\because \mu = \frac{1}{3} \text{(Given)}]$

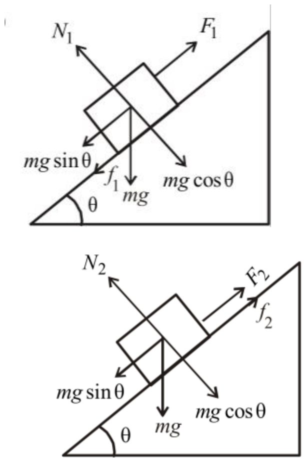
$\cot \alpha = 3$

59. The minimum force required to start pushing a body up rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan \theta = 2\mu$ then the ratio $\frac{F_1}{F_2}$ is

[2011 RS]

(a) 1 (b) 2 (c) 3 (d) 4

sol. (c)



When the body slides up the inclined plane, then $\text{mg} \sin \theta + f_1 = F_1$

or, $\Gamma_1 = \text{mg} \sin \theta + \mu \text{mg} \cos \theta$

When the body slides down the inclined plane, then $\text{mg} \sin \theta - f_2 = F_2$

or $F_2 = \text{mg} \sin\theta - \mu \text{mg} \cos\theta$

$$\frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3$$

60. If a spring of stiffness 'k' is cut into parts ' and 'B' of length $\ell_A : \ell_B = 2:3$, then the stiffness of spring A ' is given by [2011 RS]

(a) $\frac{3k}{5}$ (b) $\frac{2k}{5}$ (c) k (d) $\frac{5k}{2}$

sol. (d) It is given $\ell_A : \ell_B = 2:3$

$$\ell_A = \frac{2\ell}{5},\ p_B = \left(\frac{3l}{5}\right)$$

We know that $k \propto \frac{1}{\ell}$

If initial spring constant is k, then

$$k\ell = k_A \ell_A = k_B \ell_B$$

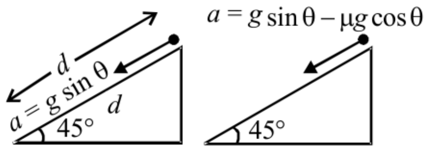
$k\ell = k_A\left(\frac{2\ell}{5}\right)$ $k_A = \frac{5k}{2}$

61. A smooth block is released at rest on a 45^{o} incline and then slides a distance d'. The time taken to slide is n' times as much to slide on rough incline than on a smooth incline. The coefficient of friction is [2005]

(a) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$ (b) $\mu_k = 1 - \frac{1}{n^2}$ (c) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$ (d) $\mu_s = 1 - \frac{1}{n^2}$

sol.

(b)



Smooth rough

On smooth inclined plane, acceleration of the body= $g\sin\theta$. Let d be the distance travelled

$$d = \frac{1}{2}(g\sin\theta)t_1^2,$$

$$t_1 = \sqrt{\frac{2d}{g\sin\theta}},$$

On rough inclined plane,

$$a = \frac{mg\sin\theta - \mu R}{m}$$

$$\Rightarrow a = \frac{mg \sin \theta - \mu mg \cos \theta}{m}$$

$$\Rightarrow a = g \sin \theta - \mu_k g \cos \theta$$

$$d = \frac{1}{2}\left(g \sin \theta - \mu \hat{k} g \cos \theta\right) t_2^2$$

$$t_2 = \sqrt{\frac{2d}{g \sin \theta - \mu \hat{k} g \cos \theta}}$$

According to question, $t_2 = n t_1$

$$n\sqrt{\frac{2d}{g \sin \theta}} = \sqrt{\frac{2d}{g \sin \theta - \mu \hat{k} g \cos \theta}}$$

Here, μ is coefficient of kinetic friction as the block moves over the inclined plane.

$$\sin \theta = \left(\sin \theta - \mu \hat{k} \cos \theta\right) n^2$$

$$\Rightarrow n = \frac{1}{\sqrt{1-\mu_k}} \Rightarrow n^2 = \frac{1}{1-\mu_k}$$

$$\Rightarrow \mu_k = 1 - \frac{1}{n^2}$$

62. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of fiiction for the lower half is given by [2005]

(a) 2 $\cos \phi$ (b) 2 $\sin \phi$ (c) $\tan \phi$ (d) 2 $\tan \phi$

sol. (d) For first half

acceleration $= g \sin \varphi$;

For second half

acceleration $= -(g \sin \varphi - \mu g \cos \varphi)$

For the block to come to rest at the bottom, acceleration in I half $=$ retardation in II half.

$$g \sin \varphi = -(g \sin \varphi - \mu g \cos \varphi)$$

$$\Rightarrow \mu = 2 \tan \varphi$$

NOTE

According to work-energy theorem, $W = \Delta K = 0$

(Since initial and final speeds are zero) Work done by $\mathrm{fi_i}$iction $+$ Work done by gravity $= 0 i.e.,$

$-\left(\mu mg \ \cos (|)\right)\frac{\ell}{2} + mg\ell \sin \varphi = 0$

or $\frac{\mu}{2} \cos \varphi =$ $\sin \varphi$ or $\mu = 2 \tan \varphi$

63. Consider a car moving on a straight road with a speed of 100 $\mathrm{m/s}$. The distance at which car can be

stopped is $[\mu_k = 0.5]$ [2005]

(a) 1000 m (b) 800 m (c) 400 m (d) 100 m

sol. (a) Given, initial velocity, $u = 100\text{m/s}$. Final velocity, $v = 0$.

Acceleration, $a = \mu_k g = 0.5 \times 10$

$v^2 - u^2 = 2as$ or

$$\Rightarrow 0^2 - u^2 = 2(-\mu_k g)s$$

$$\Rightarrow -100^2 = 2 \times -\frac{1}{2} \times 10 \times s$$

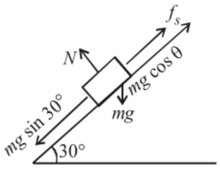
$$\Rightarrow s = 1000\text{m}$$

64. A block rests on a rough inclined plane making an angle of $30°$ with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass ofthe block (in kg) is (take $g = 10\text{m/s}^2$) [2004]

(a) 1.6 (b) 4.0 (c) 2.0 (d) 2.5

sol.

(c)

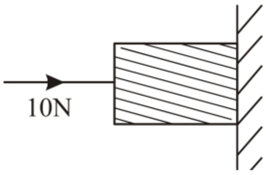


Since the body is at rest on the inclined plane, $mg \sin 30° = \Gamma$orce of fiiction

$$\Rightarrow m \times 10 \times \sin 30° = 10$$

$\Rightarrow m \times 5 = 10 \Rightarrow m = 2.0$ kg

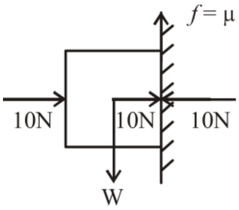
65. A horizontal force of 10 N is necessarytojust hold a block stationary against a wall. The coefficient offiiction between the block and the wall is 0.2. The weight of the block is [2003]



(a) 20N (b) 50N (c) 100N (d) 2N

sol. (d) Horizontal force, $N = 10$ N.

Coefficient of friction $\mu = 0.2$.



The block will be stationary so long as Force of friction = weight of block

$$\mu N = W$$

$$\Rightarrow 0.2 \times 10 = W$$
$$\Rightarrow W = 2N$$

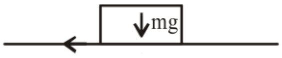
66. A marble block of mass 2 kg lying on ice when given a velocity of 6 $\mathrm{m/s}$ is stopped by friction in 10 s. Then the coefficient of friction is [2003]

(a) 0.02 (b) 0.03 (c) 0.04 (d) 0.06

sol. (d) $u = 6\mathrm{m/s},\ v = 0,\ t = 10s,\ a = ?$ Acceleration $a = \frac{v-u}{t}$

$$\Rightarrow a = \frac{0-6}{10}$$

$$\Rightarrow a = \frac{-6}{10} = -0.6\mathrm{m/s^2}$$



$$\mathrm{f} = \mu \mathrm{N} \uparrow \mathrm{N}$$

The retardation is due to the frictional force

$$f = ma \Rightarrow \mu N = ma$$

$$\Rightarrow \mu mg = ma \Rightarrow \mu = \frac{ma}{mg}$$

$$\Rightarrow \mu = \frac{a}{g} = \frac{0.6}{10} = 0.06$$

TOPIC-4, **Circular Motion, Banking of Road**

67. A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is $(g = 10\mathrm{m/s^2})$ [Online Apri115, 2018]

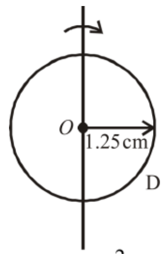
(a) 0.5 (b) 0.7 (c) 0.3 (d) 0.6

sol. (d) Using, $\mu mg = \frac{mv^2}{r} = mrw^2$

$\mathrm{w} = 2\pi n = 2\pi \times 3.5 = 7\pi \mathrm{rad/\,sec}$

Radius, $r = 1.25$ cm $= 1.25 \times 10^{-2}\mathrm{m}$ Coefficient of friction, $\mu = ?$

$$\mu mg = \frac{mrw^2}{r}\ (v = r\mathrm{w}))$$

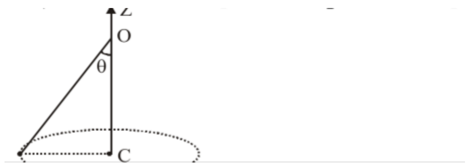


$mg = mr\omega^2$

$$\Rightarrow \ \mu = \frac{r(j)^2}{g} = \frac{1.25\times10^{-2}\times\left(7\times\frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25\times10^{-2}\times22^2}{10} = 0.6$$

68. A conical pendulum of length 1 m makes an angle $\theta = 45°$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4m and its centre is vertically below O. The speed of the pendulum, in its circular path, will be: $(g = 10\text{m/s}^2)$ [Online April 9, 2017]



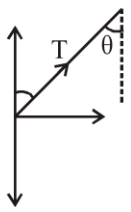
(a) 0.4 m/s (b) 4 m/s (c) 0.2 m/s (d) 2 m/s

sol. (d) Given, $\theta = 45°$, $r = 0.4\text{m}$, $g = 10\text{m/s}^2$

$T \sin\theta = \frac{mv^2}{r}$ (i)

$T \cos\theta = mg$ (ii)

From equation (i) & (ii) we have, $\tan\theta = \frac{v^2}{rg}$

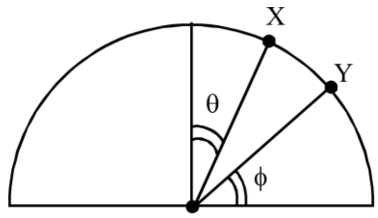


$$v^2 = rg \ \ \theta = 45°$$

Hence, speed of the pendulum in its circular path,

$$v = \sqrt{rg} = \sqrt{04\times10} = 2\text{m/s}$$

69. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle φ with the horizontal. Then: [Online April 19, 2014]



(a) $\sin\varphi = \cos\varphi$ (b) $\sin\varphi = \frac{1}{2}\cos\theta$ (c) $\sin\varphi = \frac{2}{3}\cos\theta$ (d) $\sin\varphi = \frac{3}{4}\cos\theta$

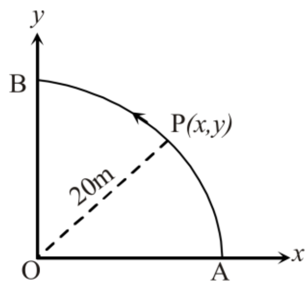
sol. (c)

70. A body of mass '*m*' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is 1 cm. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is: [Online April 23, 2013]

(a) 15 cm (b) 12 cm (c) 16 cm (d) 10 cm

sol. (a)

71. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of'P' when $t = 2s$ is nearly. [2010]



(a) 13m/s^2 (b) 12 m/s^2 (c) 7.2ms^2 (d) 14m/s^2

sol. (d) $s = t^3 + 5$

$\Rightarrow$ velocity, $v = \frac{ds}{dt} = 3t^2$

Tangential acceleration $a_t = \frac{dv}{dt} = 6t$

Radial acceleration $a_c = \frac{v^2}{R} = \frac{9t^4}{R}$

At $t = 2s$, $a_t = 6 \times 2 = 12\text{m/s}^2$

$$a_c = \frac{9 \times 16}{20} = 7.2\text{m/s}^2$$

Resultant acceleration

$$= \sqrt{a_t^2 + a_c^2}$$

$$= \sqrt{(12)^2 + (72)^2} = \sqrt{144 + 51.84}$$

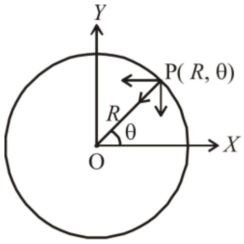
$$= \sqrt{19584} = 14 m/s^2$$

72. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle ofradius R is (Here θ is measured fi¡om the x-axis) [2010]

(a) $-\frac{v^2}{R}\cos\theta\ \hat{\imath} + \frac{v^2}{R}\sin\theta j$ (b) $-\frac{v^2}{R}\sin\theta\ \hat{\imath} + \frac{v^2}{R}\cos\theta j$

(c) $-\frac{v^2}{R}\ cos\ \theta\ \hat{\imath} - \frac{v^2}{R}\sin\theta j$ (d) $\frac{v^2}{R}\hat{\imath} + \frac{v^2}{R} j$

sol. (c) Clearly $\vec{a} = a_c\, cos\theta(-\hat{\imath}) + a_c \sin\theta(-\hat{j})$ $= \frac{-v^2}{R}\cos\ \theta\hat{\imath} - \frac{v^2}{R}\sin\theta\hat{\jmath}$

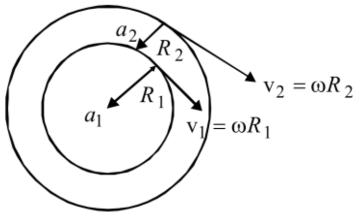


73. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring , $\frac{F_1}{F_2}$ is [2005]

(a) $\left(\frac{R_1}{R_2}\right)^2$ (b) $\frac{R_2}{R_1}$ (c) $\frac{R_1}{R_2}$ (d) 1

sol.

(c)



Let m is the mass of each particle and w is the angular speed of the annular ring.

$$a_1 = \frac{v_1^2}{R_1} = \frac{w^2 R_1^2}{R_1} = R_1 w^2$$

$$a_2 = \frac{v_2^2}{R_2} = w^2 R_2$$

Taking particle masses equal

$$\frac{F_1}{F_2} = \frac{ma_1}{ma_2} = \frac{mR_{1^{0)}}^2}{mR_{2^{0)^2}}} = \frac{R_1}{R_2}$$

NOTE:

The force experienced by any particle is only along radial direction.

Force experienced by the particle, $F = m w^2 R$

$$\frac{F_1}{F_2} = \frac{R_1}{R_2}$$

74. Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed? [2004]

(a) The acceleration vector points to the centre of the circle

(b) The acceleration vector is tangent to the circle

(c) The velocity vector is tangent to the circle

(d) The velocity and acceleration vectors are perpendicular to each other.

sol. (b) Only option (b) is false.

since acceleration vector is always radial (i.e. towards the center) for uniform circular motion.

75. The minimum velocity (in ms^{-1}) withwhich acardriver must traverse a flat curve ofradius 150 m and coefficient offriction 0.6 to avoid skidding is [2002]

(a) 60 (b) 30 (c) 15 (d) 25

sol. (b) The maximum velocity of the car is

$$v_{\max} = \sqrt{\mu r g}$$

Here $\mu = 0.6,\ r = 150\mathrm{m}, g = 9.8$

$$v_{\max} = \sqrt{06 \times 150 \times 9.8} \simeq 30\mathrm{m/s}$$

WORK ENERGY AND POWER

Work :

Work is said to be done on a body, only when energy of the body changes (Mechanical energy or Thermal energy)

A force or a Torque is responsible for work done on a body.

Workdone by a force :

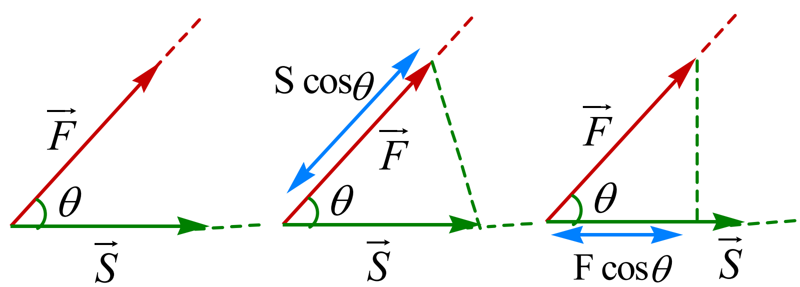
Work is said to be done on a body by a force, if it displaces the point of application of force in its direaction or opposite to its direction.

Here force may be constant or variable i.e function of time or function of position

Work done by constant force:

♦ When a constant force $\vec{F}$ acts on a particle and the particle moves through a displacement $\vec{S}$, then the force is said to do work W on the particle.

$$W = \vec{F}.\vec{S}$$



The scalar (dot) product of $\vec{F}$ and $\vec{S}$, can be evaluated as

$$W = \vec{F}.\vec{S} = FS\cos\theta$$

Where

F is the magnitude of $\vec{F}$,

S is the magnitude of $\vec{S}$

θ is the angle between $\vec{F}$ *and* $\vec{S}$.

♦ W=magnitude of the force x component of displacement in the direction of force

$$W = FS\cos\theta = F(S\cos\theta)$$

♦ W =component of the force in the direction of displacement x magnitude of the displacement

$$W = (F\cos\theta)S$$

♦ Work is a scalar quantity.

♦ SI Unit is Nm or joule (J).

♦ CGS unit is erg.

♦ $1J = IN \times 1m; \quad 1erg = 1\,dyne \times 1cm$

♦ Dimensional formula of work is $\left[ML^2T^{-2}\right]$.

♦ Relation between joule and erg: 1 joule = 10^7 *erg*

Other units of work:

Electron Volt $(eV) = 1.6\times10^{-19} J$

Kilowatt hour $= 3.6\times10^{6} J$

Work done by multiple forces:

If a number of forces act on a body or particle then:

$$W = W_1 + W_2 + W_3 + ...$$

$$W = \int \vec{F_1}.\vec{ds} + \int \vec{F_2}.\vec{ds} + ...$$

$$W = \int \left(\vec{F_1} + \vec{F_2} + ...\right)\vec{ds}$$

$$W = \int \vec{F_R}.\vec{ds}\left[as\ \vec{F_R} = \Sigma\vec{F}\right]$$

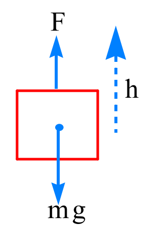
Work done in displacing a particle under the action of a number of forces is equal to the work done by the resultant force.

Nature of Work:

Work done by a force may be positive or negative or zero.

Ex:

(a) If we lift a body from rest to a height h



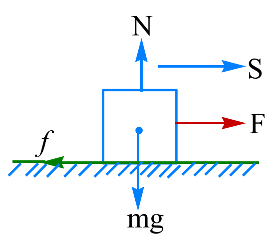
Work done by lifting force F , $W_1 = Fh\cos 0^0 = Fh \quad (+ve)$

Work done by gravitational force , $W_2 = mgh\cos 180^0 = -mgh \quad (-ve)$

So, net work $W = W_1 + W_2 = Fh - mgh = (F - mg)h$

Now, if the body is in equilibrium F=mg, W=0

(b) If a body is pulled on a rough horizontal road through a displacement S



Work done by normal reaction and gravity $W_1 = 0$ as force is $\perp$ to S

Work done by pulling force F, $W_2 = FS\cos 0^0 = FS \quad (+ve)$

Work done by frictional force f, $W_3 = fs\cos 180^0 = -\mu mg \quad (-ve)$

Net work $W = W_1 + W_2 + W_3 = 0 + FS - fS = (F - f)S$

Now, if the body is in dynamic equilibrium $f = F$; So, W=0

Zero Work:

♦ Work done is zero if

1. Force and displacement are perpendicular.
2. Displacement of point of application of forced is zero.
3. Net force acting on the body is zero.

♦ As $W = \int \vec{F}.d\vec{s}$

so, if $d\vec{s} = 0, W = 0$

i.e., if the displacement of a particle or body is zero whatever be the force, work done is zero (except non-conservative force)

(a) When a person tries to displace a wall or stone by applying a force and it (actually its centre of mass) does not move, the work done is zero.

(b) A weight lifter does work in lifting the weight from the ground but does not work in holding it up.

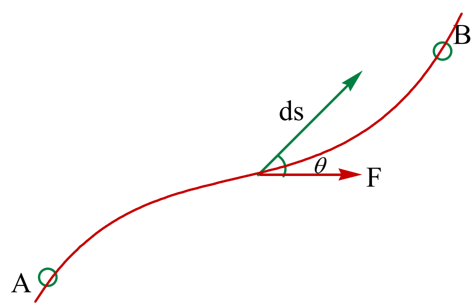
♦ As $W = \int Fds \cos\theta,\ so\ W = 0,\ if\ \theta = 90^0,\ i.e.,$

if force is always perpendicular to motion, work done by the force will be zero though neither forced nor displacement is zero. This is why:

(a) When a porter moves with a suitcase on his head on a horizontal level road, the work done by the lifting force or force of gravity is zero.

(b) When the bob of a simple pendulum swings, the work done by tension in the string is zero.

Work done by Variable Force:



When the magnitude and direction of a force varies with position, then the work done by such a force for an infinitesimal displacement ds is given by

$$dW = \vec{F}.\overleftarrow{ds}$$

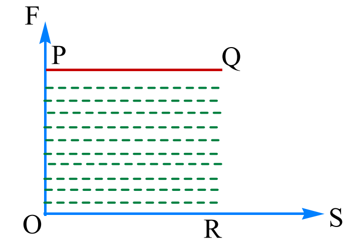
The total work done in going from A to B is $W_{AB} = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\ \cos\theta)\ ds$

In terms of rectangular components $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}; \vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$$W = \int_A^B F_x dx + \int_A^B F_Y dy + \int_A^B F_z dz$$

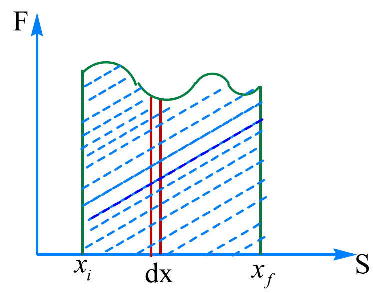
Graphical representation of work done:

- The area enclosed by the F-S graph and displacement axis gives the amount of work done by the force.



Work = FS = Area of OPQR

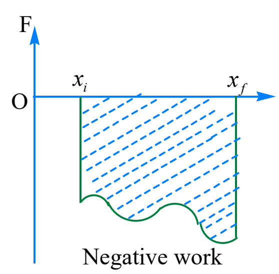
- Work done by variable force



For a small displacement dx the work done will be the area of the strip of width dx

$$W = \int_{X_i}^{X_f} dw = \int_{X_i}^{X_f} F\,dx$$

- If area lies above X-axis work done is +ve
  if the area enclosed below X-axis work done is –ve



Negative work

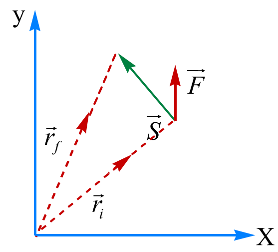
Applications on work:

♦ If force is changing linearly from F_1 to F_2 over a displacement S then work done is

$$W = \left(\frac{F_1 + F_2}{2}\right)s$$

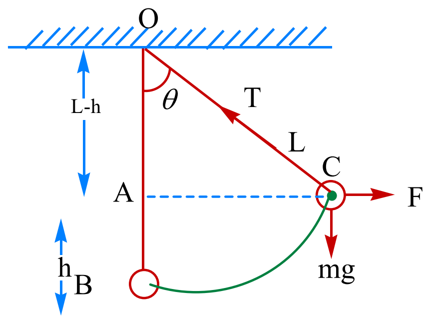
♦ If force displaces the particle from its initial position $\vec{r_i}$ to final position $\vec{r_f}$ then displacement vector

$$\vec{S} = \vec{r_f} - \vec{r_i}$$



$$W = \vec{F}.\vec{S} = \vec{F}.\left(\vec{r_f} - \vec{r_i}\right)$$

♦ Work done in pulling the bob of mass m of a simple pendulum of length L through an angle θ to vertical by means of a horizontal force F.



$$\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L}; \frac{h}{L} = 1 - \cos\theta$$

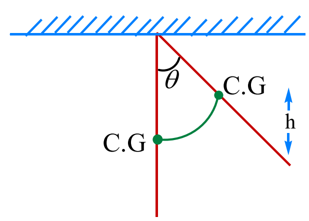
$$h = L\left(1-\cos\theta\right)$$

•Work done by gravitational force $W = -mgh = -mgL\left(1-\cos\theta\right)$

•Work done by horizontal force F is W=FL sin θ

•Work done by tension T in the string is zero.

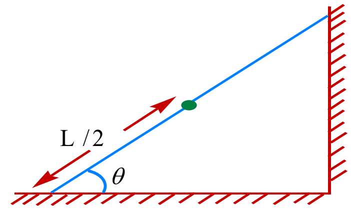
♦ Work done by gravitational force in pulling a uniform rod of m ass m and length l through an angle θ is given by



$$W = -mg\frac{l}{2}\left(1-\cos\theta\right),$$

Where $\frac{l}{2}$ is the distance of centre of mass from the support.

♦ A ladder of mass ‘m’ and length ‘L’ resting on a level floor is lifted and held against a wall at an angle θ with the floor



Work done by gravitational force is $W_g = -mgh = -mg\left(\frac{L}{2}\right)\sin\theta$

♦ A bucket full of water of total mass M is pulled by using a uniform rope of mass m and length l.

Work done by pulling force. $W = mgl + mg\frac{l}{2}$

♦ A block of mass m is suspended vertically using a rope of negligible mass.

If the rope is used to lift the block vertically up with uniform acceleration 'a', work done by tension in the rope is

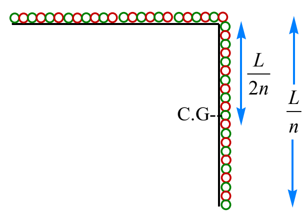
$$W = m(g+a)h \quad (h = height)$$

If block is lowered with acceleration 'a', then

$$W = -m(g-a)h$$

♦ A uniform chain of mass M and length L is kept on smooth horizontal table such that $\frac{1}{n^{th}}$ of its length is hanging over the edge of the table.

The work done by the pulling force to bring the hanging part onto the table is



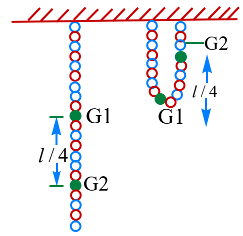
$$W = \left(\frac{M}{n}\right)gh = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$$

Mass of hanging part is $\frac{M}{n}$

♦ A uniform chain of mass M and Length L rests on a smooth horizontal table with $\frac{1}{n_1^{th}}$ part of its length is hanging from the edge of the table.

Work done in pulling the chain practically such that $\frac{1}{n_2^{th}}$ part is hanging from the edge of the table is given by $W = \frac{MgL}{2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

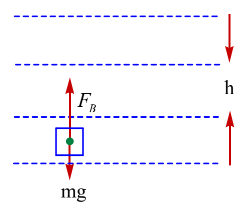
♦ A uniform chain of mass ‘M’ and length L is suspended vertically. The lower end of the chain is lifted upto point of suspension



$h = \frac{L}{4} + \frac{L}{4} = \frac{L}{2}$ = raise in centre of mass of lower half of the chain.

Work done by gravitational force is $W_g = -\frac{M}{2} g \frac{L}{2} = -\frac{MgL}{4}$

♦ The work done in lifting a body of mass ‘m’ having density 'd_1' inside a liquid of density 'd_2' through a height ‘h’ is



$$W = mg'h = mgh\left[1 - \frac{d_2}{d_1}\right]$$

♦ A body of mass 'm' is placed on a frictionless horizontal surface.

A force F acts on the body parallel to the surface such that it moves with an acceleration ‘a’, through a displacement ‘S’.

The work done by the force is $W = FS = maS\left(\because \theta = 0^0\right)$

♦ A body of mass 'm' is placed on a rough horizontal surface of coefficient of friction μ.

A force F acts on the body parallel to the surface such that it moves with an acceleration ‘a’, through a displacement ‘S’.

The work done by the friction of the force is $f = umg \cos\theta; but \theta = 0^0$

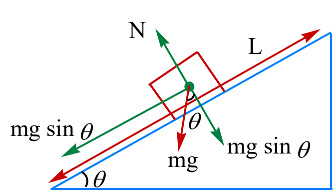
$$\therefore f = umg \cos 0^0 = umg$$

$$W = \left(f + ma\right)S = \left(umg + ma\right)S = m\left(ug + a\right)S$$

If the body moves with uniform velocity then $W = f\ S = umg\ S$

♦ A body of mass m is sliding down on a smooth inclined plane of inclination θ.

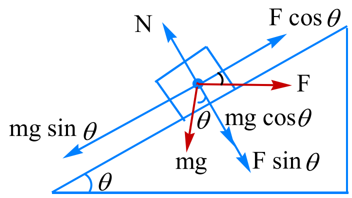
If L is length of inclined plane then work done by gravitational force is



$$W_g = FS = mg \sin\theta L$$

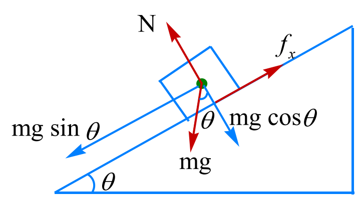
♦ A body of mass 'm' is moved up the smooth inclined plane of inclination θ and length L by a constant horizontal force F then work done by the resultant force is

$$W = (F\cos\theta - mg\sin\theta)L$$



♦ A body of mass 'm' is sliding down on rough inclined plane of inclination θ.

If L is the length of incline and μ_K is the coefficient of kinetic friction then work done by the resultant force on the body is

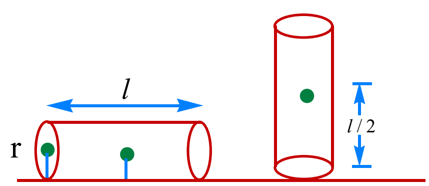


$$W = (mg\sin\theta - f_k)L = (mg\sin\theta - \mu_k mg\cos\theta)L$$

$$= mgL(\sin\theta - \mu_k\cos\theta)$$

♦ A uniform solid cylinder of mass m, length l and radius r is lying on ground with curved surface in contact with ground.

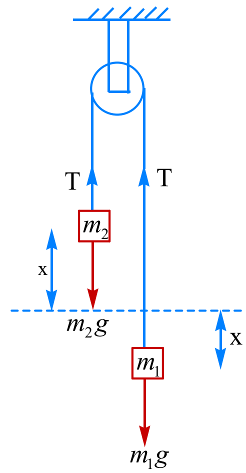
If it is turned such that its circular face is in contact with ground then work done be applied force is



$$W = mgh = mg\left(\frac{l}{2} - r\right)\left(\therefore h = \frac{l}{2} - r\right)$$

♦ Two blocks of masses m_1 *and* m_2 $(m_1 > m_2)$ connected by an inextensible string are passing over a smooth, massless pulley.

The two blocks are released from the same level. At any instant 't', if 'x' is the displacement of each block then



Work done by gravity on block $m_1, W_1 = +m_1gx$

Work done by gravity on block $m_2, W_2 = -m_2gx$

Work done by gravitational force on the system, $W_g = m_1gx - m_2gx$

$$W_g = (m_1 - m_2)gx = (m_1 - m_2)g\left(\frac{1}{2}at^2\right) \quad \left[\because v^2 - u^2 = 2as\right]$$

$$W_g = \frac{(m_1 - m_2)^2 g^2t^2}{2(m_1 + m_2)}\left[\because a = \frac{(m_1 - m_2)g}{m_1 + m_2}\right]$$

Note:

In this work done on the **two blocks** by **tension is zero.**

$$W = T(x) + T(-x) = 0$$

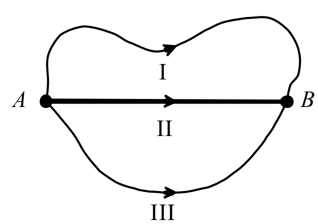
Work Done in Conservative and Non-conservative Field:

♦ In conservative field, work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) is independent of the path followed between any two points.

$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

or

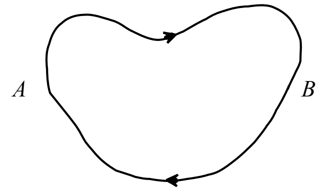
$$\underset{\text{Path I}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path II}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path III}}{\int \vec{F}.d\vec{l}}$$



♦ In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) over a closed path/loop is zero.

$$W_{A\to B} + W_{B\to A} = 0$$

$$\text{or } \oint \vec{F}.\vec{dl} = 0$$

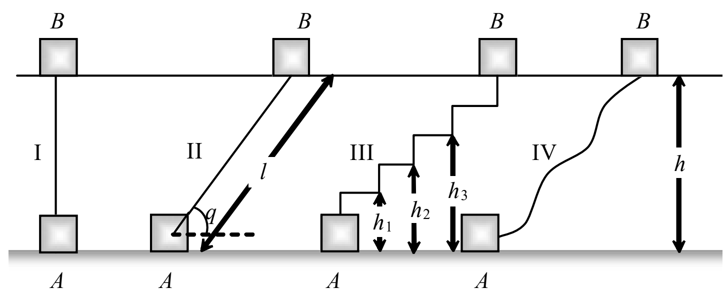


Conservative force :

The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of mass *m* lifted to height *h* from the ground level by different path as shown in the figure



Work done through different paths

$$W_I = F.s = mg \times h = mgh$$

$$W_{II} = F.s = mg\ \sin\theta \times l = mg\ \sin\theta \times \frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$$

$$= mg(h_1 + h_2 + h_3 + h_4) = mgh$$

$$W_{IV} = \int \vec{F}.\vec{ds} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position A, similar amount of work (energy) is released from the system, it means $W_{AB} = mgh$ and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round trip is zero.

$$W_{Net} = W_{AB} + W_{BA} = mgh + (-mgh) = 0$$

i.e. the gravitational force is conservative in nature.

NOTE :

Under conservative force $F = \dfrac{-dU}{dr}$ where U is Potential Energy. $U = \int dU = -\int \vec{F}.\vec{dr}$

$(\vec{F} = F_x \hat{i} + F_y \hat{k} + F_z \hat{k}$

$$\overrightarrow{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$\vec{F}\left(\frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}\right)$$

Non-conservative forces :

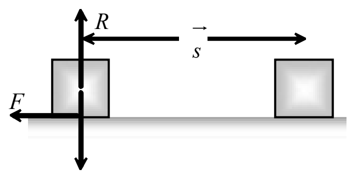
A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be zero.

Example: Frictional force, Viscous force, Airdrag *etc*.

If a body is moved from position A to another position B on a rough table, work done against frictional force shall depend on the length of the path between A and B and not only on the position A and B.

$$W_{AB} = \mu mgs$$

Further if the body is brought back to its initial position A, work has to be done against the frictional force, which opposes the motion. Hence the net work done against the friction over a round trip is not zero.



$$W_{BA} = \mu mgs.$$

$$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$$

i.e. the friction is a non-conservative force.

‘

‘

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::PROBLEMS::

1. A body is displaced from $\vec{r_A} = \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)$ to $\vec{r_B} = \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right)$ under a constant force $\vec{F} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$. Find the work done.

SOLUTION :

$$\text{Work done } W = \vec{F}.\vec{S}; W = \vec{F}.\left(\vec{r_B} - \vec{r_A}\right)$$

$$W = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)\left[\left(6\hat{i} - 4\hat{j} + 2\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)\right]$$

$$W = \left(2\hat{i} + 3\hat{j} - \hat{k}\right).\left(4\hat{i} - 8\hat{j} + 8\hat{K}\right)$$

$$W = 8 - 24 - 8 = -24\,units$$

2. A force $\vec{F} = 2x\hat{i} + 2\hat{j} + 3z^2\hat{k}$ N is acting on a particle. Find the work done by the force in displacing the body from (1,2,3) m to (3,6,1) m..

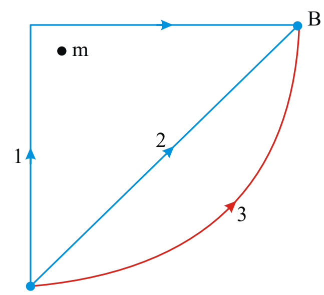
SOLUTION :

$$\text{Work done } W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

$$W = \int_1^3 2x dx + \int_2^6 2dy + \int_3^1 3z^2 dz$$

$$W = 2\left[\frac{x^2}{2}\right]_1^3 + 2\left[y\right]_2^6 + 3\left[\frac{z^3}{3}\right]_3^1 = -10J$$

3. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 respectively As shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3 [IIT-2003]



A) $W_1 > W_2 > W_3$ **B)** $W_1 = W_2 = W_3$ **C)** $W_1 < W_2 < W_3$ **D)** $W_2 > W_1 > W_3$

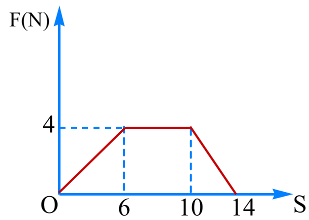
SOLUTION :

Work done will be same in all the cases because gravitational field is a conservative field.

Thus work done is independent of the point B,

therefore $W_1 = W_2 = W_3$

4. The force acting on an object varies with the distance travelled by the object as shown in the figure. Find the work done by the force in moving the object from x=0m to x=14m.



SOLUTION :

.

Work done=Area under F- S curve.

$$W = \left(\frac{1}{2}\times 6\times 4\right) + (4\times 4) + \left(\frac{1}{2}\times 4\times 4\right) = 36J$$

5. A body is displaced from $(0,0)$ to $(1m, 1m)$ along the path $x = y$ by a force $\vec{F} = \left(x^2\hat{j} + y\hat{i}\right)N$. The work done by this force will be

A) $\frac{4}{3}J$ **B)** $\frac{5}{6}J$ **C)** $\frac{3}{2}J$ **D)** $\frac{7}{5}J$

SOLUTION :

$$W = \int_{(0,0)}^{(1,1)} \vec{F}.\vec{ds}$$

Here $\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int_{(0,0)}^{(1,1)} \left(x^2dy + ydx\right) \quad (as\ x = y)$$

$$W = \left[\frac{y^3}{3} + \frac{x^2}{2}\right]_{(0,0)}^{(1,1)} = \frac{5}{6}J$$

6. The displacment x (in m), of a particle of mass m (in kg) is related to the time t (in second) by $t = \sqrt{x} + 3$. Find the work done in first six second. (in mJ)

SOLUTION :

.

$$x = (t-3)^2 = t^2 - 6t + 9$$

$$v = \frac{dx}{dt} = 2t - 6$$

at $t = 0, v = -6$;

at $t = 6, v = +6$

initial $KE = \frac{1}{2}m(-6)^2 = 18m$

final $KE = \frac{1}{2}m(6)^2 = 18m$

7. When a rubber band is stretched by a distance 'x', it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. Find the work done in stretching the unstretched rubber band by 'L'. (JEE MAIN 2014)

SOLUTION :

The restoring force exerted by the rubber band when it is stretched by a distance 'x' is $F = ax + bx^2$.

The small amount of work done on the rubber band in stretching through a small distance 'dx' is

$$dW = Fdx = \left(ax + bx^2\right)dx$$

The total work done in stretching the unstretched rubber band by 'L' is

$$W = \int_0^L Fdx = \int_0^L \left(ax + bx^2\right)dx = \int_0^L axdx + \int_0^L bx^2dx$$

$$W = a\left[\frac{x^2}{2}\right]_0^L + b\left[\frac{x^3}{3}\right]_0^L = \frac{aL^2}{2} + \frac{bL^3}{3}$$

8. The work done on a particle of mass m by a force $k\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i} + \frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (k being a constant of appropriate dimensions) when the particle is taken from the point $(a,0)$ along a circular path of radius a bout the origin in the x-y plane is [IIT-2013]

A) $\frac{2k\pi}{a}$ **B)** $\frac{k\pi}{a}$ **C)** $\frac{k\pi}{2a}$ **D) zero**

SOLUTION :

$$W = \int \vec{F}.d\vec{r}$$

$$= k\int_{r_A}^{r_B}\left[\frac{x\hat{i}}{\left(x^2+y^2\right)^{3/2}} + \frac{y\hat{j}}{\left(x^2+y^2\right)^{3/2}}\right]\left(dx\hat{i} + dy\hat{j}\right) = k\int_{r_A}^{r_B}\frac{xdx}{\left(x^2+y^2\right)^{3/2}} + \frac{ydy}{\left(x^2+y^2\right)^{3/2}}$$

$$= k\int_{r_A}^{r_B}\frac{1}{\left(x^2+y^2\right)^{3/2}}\left[d\left(\frac{x^2}{2}\right) + d\left(\frac{y^2}{2}\right)\right] = k\int_{r_A}^{r_B}\frac{1}{\left(x^2+y^2\right)^{3/2}}\left(x^2+y^2\right)$$

$$= k\int_{r_A}^{r_B}\frac{1}{2r^3}d\left(r^2\right) = k\int_{r_A}^{r_B}\frac{2rdr}{2r^3} = k\int_{r_A}^{r_B}\frac{dr}{r^2} = k\left[-\frac{1}{r}\right]_{r_A}^{r_B} = k\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

But $r_A = a$ and $r_B = a$; $\therefore W = 0$

9. Forces acting on a particle moving in a straight line varies with the velocity of the particle as $F = \frac{\alpha}{\upsilon}$ where α is constant. The work done by this force in time interval Δt is:

A) $\alpha\Delta t$ **B)** $\frac{1}{2}\alpha\Delta t$ **C)** $2\alpha\Delta t$ **D)** $\alpha^2\Delta t$

SOLUTION :

$$F = \frac{\alpha}{\upsilon}$$

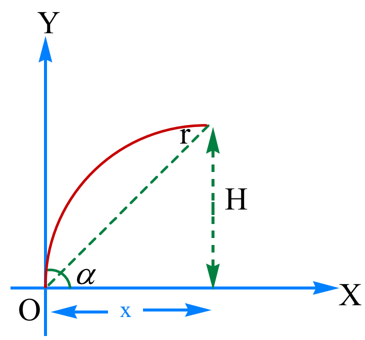
$$\Rightarrow m\frac{d\upsilon}{dt} = \frac{\alpha}{\upsilon} \Rightarrow \int m\upsilon d\upsilon = \int \alpha dt$$

$$\left(\frac{m\upsilon^2}{2}\right) = \alpha t$$

$$; \quad \Delta KE = \alpha \Delta t = \text{work done}$$

10. A particle of mass 'm' is projected at an angle α to the horizontal with an initial velocity u. Find the workdone by gravity during the time it reaches the highest point.

SOLUTION :



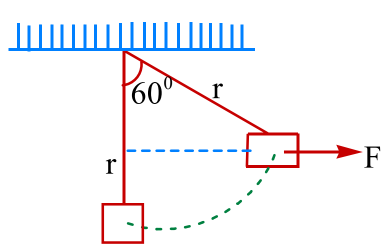
$$\overline{F_y} = mg\hat{j}; \vec{r_y} = \overline{H}_{\max} = \left(\frac{u^2 \sin^2 \alpha}{2g}\right)\hat{j}$$

$$W = \overline{F}_y \vec{r}_y = \left(-mg\hat{j}\right).\left(\frac{u^2 \sin^2 \alpha}{2g}\right)\hat{j}$$

$$W = -\frac{1}{2}mu^2 \sin^2(\alpha)$$

11. A 10 kg block is pulled along a frictionless surface in the form of an arc of a circle of radius 10 m. The applied force is 200 N. Find the work done by (a) applied force and (b) gravitational force in displacing through an angle 60^0

SOLUTION :



Work done by applied force $W = Fr\sin\theta$

$$W = 200\times10\times\sin 60^0 = 200\times10\times\frac{\sqrt{3}}{2}1732J$$

$$\text{work done by gravitational force } W = -mgr(1-\cos\theta)$$

$$W = -10\times9.8\times10(1-\cos 60^0)$$

$$W = -98\times10\left(1-\frac{1}{2}\right) = -490J$$

12. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain back onto the table?

SOLUTION :

$$M=4\text{ kg}, L=2\text{m}, \ l = 0.6m, \ g = 10m/s^2$$

$$\text{Work done } W = mg\frac{l}{2} = \left(\frac{M}{L}\right)l\,g\frac{l}{2}$$

$$W = \left(\frac{4}{2}\right)\times0.6\times10\times\frac{0.6}{2} = 3.6J$$

13. Find the work done in lifting a body of mass 20 kg and specific gravity 3.2 to a height of 8 m in water? (g=10m/s^2)

SOLUTION :

$$\text{Given specific gravity } \frac{\rho_b}{\rho_w} = 3.2$$

$$\rho_b = 3.2\times\rho_w = 3.2\times1000 = 3200$$

$$\text{Workdone W= mgh}\left(1-\frac{\rho_w}{\rho_b}\right) = 20\times10\times8\left(1-\frac{1000}{3200}\right)$$

$$W = 20\times10\times8\left(\frac{2200}{3200}\right) = 1100J$$

14. A block of mass 'm' is lowered with the help of a rope of negligible mass through a distance 'd' with an acceleration of g/3. Find the work done by the rope on he block?

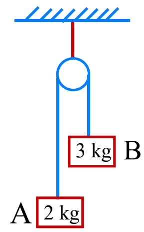
SOLUTION :

$$\text{During lowering a block, tension in rope is } T = m(g-a) \ and S = d$$

$$\text{work done} \qquad W = -m(g-a)d$$

$$W = -m\left(g-\frac{g}{3}\right)d = -\frac{2mgd}{3}$$

15. If the system shown is released from rest. Find the net workdone by tension in first one second (g=10m/s^2)



SOLUTION :

$$a=\left(\frac{m_2-m_1}{m_1+m_2}\right)g=\left(\frac{3-2}{2+3}\right)10=2m/s^2$$

$$T=\frac{2m_1m_2g}{m_1+m_2}=\frac{2\times2\times3\times10}{2+3}24N$$

for each blocks $S=\frac{1}{2}at^2=\frac{1}{2}\times2\times1=1m$

$$\therefore W_{net}=W_1+W_2=TS-TS=0$$

Energy:

♦ Energy is the ability or capacity to do work. Greater the amount of energy possessed by the body, greater the work it will be able to do.

♦ Energy is cause for doing work and work is effect of energy

♦ Energy is a scalar.

Energy and work have same units and dimensions.

♦ The different forms of energy are

- Mechanical energy,
- Light energy,
- Heat energy,
- Sound energy,
- Electrical energy,
- Nuclear energy .....etc.

♦ Mechanical energy is of two types

1) Potential Energy
2) Kinetic Energy

Potential energy (U):

♦ Potential energy of body is the energy possessed by a body by virtue of its position or configuration in the field.

♦ Potential energy is defined only for conservative forces. It does not exist for non-conservative forces.

In case of conservative forces.

$$F = -\left(\frac{dU}{dr}\right) \therefore dU = -\vec{F}.\vec{dr} \Rightarrow \int_{u_1}^{u_2} dU = -\int_{r_1}^{r_2} \vec{F}.\vec{dr}$$

$$U_2 - u_1 = -\int_{r_1}^{r_2} \vec{F}.\vec{dr} = -W$$

If $r_1 = \infty, U_1 = 0 \quad \therefore U = \int_{\infty}^{r} \vec{F}.\vec{dr} = -W$

♦ P.E can be +ve or -ve or can be zero.

♦ P.E depends on frame of reference.

EX-

1. Water stored in a dam,
2. A stretched bow,
3. A loaded spring etc., possesses P.E

♦ In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from some reference position to given position.

♦ Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.

♦ A moving body may or may not have potential energy.

♦ Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

Kinetic energy:

♦ Kinetic energy is the energy possessed by a body by virtue of its motion.

♦ Kinetic energy of a body of mass 'm' moving with a velocity 'v', $KE = \frac{1}{2}mv^2$

♦ Kinetic energy is a scalar quantity.
♦ The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion.

Examples for bodies having K.E

1) A vehicle in motion
2) Water flowing along a river
3) A bullet fired from a gun

♦ Kinetic energy depends on frame of reference.

EX-kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of earth.

Relation between KE and Linear Momentum:

♦ $KE = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}Pv \left(\because P = mv\right)$

♦ If two bodies of different masses have same momentum then lighter body will have greater KE

$\left(\because KE \alpha \frac{1}{m}\right)$

♦ When a bullet is fired from a gun the momentum of the bullet and gun are equal and opposite.

$$i.e \frac{KE_{bullet}}{KE_{gun}} = \frac{M_{gun}}{M_{bullet}}$$

Hence, the KE of the bullet is greater than that of the gun

♦ A body can have energy without momentum. But it can not have momentum without energy.

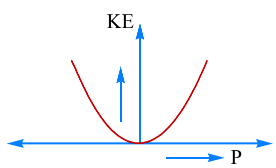
♦ A bullet of mass 'm' moving with velocity 'v' stops in wooden block after penetrating through a distance 'x'. If F is resistance offered by the block to the bullet

(Assuming F is constant inside the block)

$$\frac{1}{2}mv^2 = Fx; \ F = \frac{mv^2}{2x} \qquad \therefore v^2 ax$$

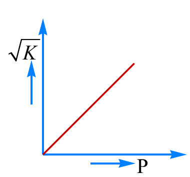
For a given body

♦ The graph between KE and P is a parabola.

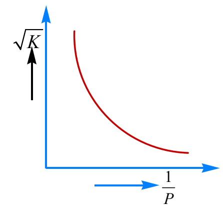


♦ The graph between $\sqrt{KE}$ and P is a straight line passing through the origin.

Its slope= $\frac{1}{\sqrt{2m}}$



♦ The graph between $\sqrt{KE}$ and $\frac{1}{P}$ is a rectangular hyperbola.

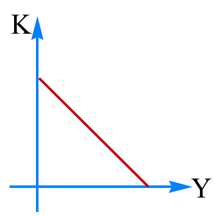


♦ A particle is projected up from a point at an angle $'\theta'$ with the horizontal. At any time 't' if 'P' is linear momentum, 'y' is vertical displacement and 'x' is horizontal displacement, then nature of the curves drawn for KE of the particle (K) against these parameters are

i) <u>K-y graph:</u>

$$K = K_i - mgy;$$

It is a straight line

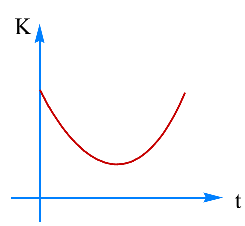


ii) <u>K-t graph:</u>

$$K = K_i - mg\left(u_y t - \frac{1}{2}gt^2\right)$$

$$\because\ y = u_y t - \frac{1}{2}gt^2;$$

It is a parabola

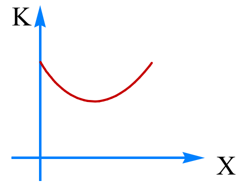


iii) <u>K - x graph:</u>

$$K = K_i - mg\left(x\tan\theta - \frac{gx^2}{2u_x^2}\right)$$

$$\because\ y = (\tan\theta)x - \left(\frac{g}{2u_x^2}\right)x^2;$$

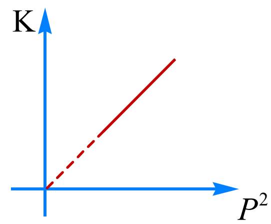
It is also parabola



iv) K -P² graph:

It is a straight line passing through origin and slope= $\dfrac{1}{2m}$

$$P^2 = 2mK$$



$$P^2 \alpha K$$

Restoring force and spring constant : When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.

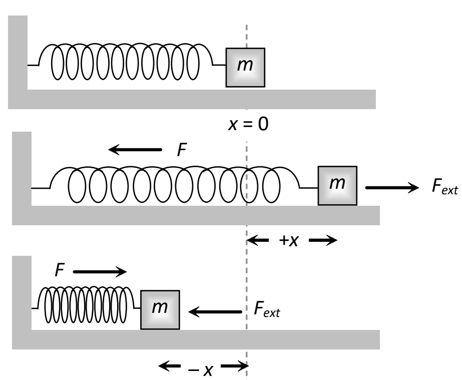


Fig. 6.20

$$i.e. \quad \vec{F} \propto -\vec{x}$$

$$\vec{F} = -k\vec{x} \ldots \text{(i)}$$

where k is called spring constant.

If $x = 1$, $F = k$ (Numerically)

$$k = F$$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa. Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$

$$\backslash\ [k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$$

Units : S.I. unit *Newton/metre*, C.G.S unit *Dyne/cm*.

- **Spring force is an example of a variable force which is conservative.**
- **In an ideal spring, the spring force F_s is directly proportional to 'x'.**
  Where x is the displacement of the block from equilibrium position.
  i.e., $F_s = Kx$.
  The constant K is called spring constant.
- **The work done on the block by the spring force as the block moves from undeformed position x=0 to x=x_1**

$$dW = \vec{F}.\vec{dx} = -Kxdx$$

$$W = \int dW = \int_0^{x_1} -Kxdx = -\frac{1}{2}K\left(x^2\right)_0^{x_1} = -\frac{1}{2}Kx_1^2$$

- **If the block moves from x=x_1 to x=x_2 the work done by spring force is** $W = \int_{x_1}^{x_2} -Kxdx$

$$W = -\frac{1}{2}K\left(x_1^2 - x_2^2\right) = \frac{1}{2}Kx_1^2 - \frac{1}{2}Kx_2^2$$

Potential energy stored in a spring:

The change in potential energy of a system corresponding to a conservative internal force is

$$du = -\int_0^x \vec{F}.\vec{dx},$$

dU = - (work done by the spring force)

$$dU = -\left(\frac{-Kx^2}{2}\right); U_f - U_i = \frac{1}{2}Kx^2$$

since U_i is zero when spring is at its natural length

$$\therefore U_f = \frac{1}{2}Kx^2$$

Work - energy theorem:

Work done by all forces acting on a body is equal to change in its kinetic energy.

$$i.e., W = K_f - K_i \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Where K_f and K_i are the final and initial kinetic energies of the body.

- **Work energy theorem is applicable not only for a single particle but also for a system of particles.**
- **When it is applied to a system of two or more particles change in kinetic energy of the system is equal to work done on the system by the external as well as internal forces.**
- **Work - energy theorem can also be applied to a system under the action of variable forces, pseudo forces, conservative as well as non-conservative forces.**

Application of work-energy theorem:

- **A body of mass *m* starting from rest acquire a velocity '*v*' due to constant force F. Neglecting air resistance.**

  Work done = change in Kinetic energy $\frac{1}{2}mv^2$

- **A particle of mass '*m*' *is* thrown vertically up with a speed '*u*'. Neglecting the air friction, the work done by gravitational force, as particle reaches maximum height is**

$$W_g = \Delta K = K_f - K_i$$

$$W_g = -\frac{1}{2}mu^2 - \frac{1}{2}m \times 0 = -\frac{1}{2}mu^2$$

- **A particles of mass 'm' falls freely from a height 'h' in air medium onto the ground. If 'v' is the velocity with which it reaches the ground, the work done by air friction is W_f and work done by gravitational force W_g then,**

$$W_g + W_f = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

- **A block of mass 'm' slides down a frictionless incline of inclination 'θ' to the horizontal. If h is the height of incline, the velocity with which body reaches the bottom of incline is**

$$W_g = \Delta K; \ mgh = \frac{1}{2}mv^2 - 0$$

$$mgh = \frac{1}{2}mv^2; \quad v = \sqrt{2gh}$$

- **A body of mass 'm' starts from rest from the top of a rough inclined plane of inclination 'θ' and length 'l'.**

The velocity 'v' with which it reaches the bottom of incline if μ_k is the coefficient of kinetic friction is

$$W_g + W_f = \Delta k$$

$$(mg\sin\theta)l + (-\mu_k mg\cos\theta)l = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{2gl(\sin\theta - \mu_k \cos\theta)}$$

♦ **A bob of mass m suspended from a string of length l is given a speed u at its lowest position then the speed of the bob v when it makes an angle θ with the vertical is**

$$W_g + W_T = \Delta K \Rightarrow mgl\left(1-\cos\theta\right)+0=\frac{1}{2}m\left(v^2-u^2\right)$$

$$v=\sqrt{u^2-2gl\left(1-\cos\theta\right)}$$

♦ **A bullet of mass 'm' moving with velocity 'v' stops in a wooden block after penetrating through a distance x.**

If 'f' is the resistance offered by the block to the bullet.

$$W_f = K_f - K_i; \quad -fx = 0 - KE_i$$

i.e., stopping distance $x=\frac{KE_i}{f}=\frac{mv^2}{2f}=\frac{P^2}{2mf}$

♦ **A block of mass 'm' attached to a spring of spring constant 'K' oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. It has a speed 'v' when the spring is at natural length. The distance it moves on a table before it comes to rest is calculated as below**

$$W_{S.F}+W_g+W_N=\Delta K \qquad \left(S.F = spring\ force\right)$$

Let the mass be oscillating with amplitude 'x',

On compressing the spring $W_{S.F}=-\frac{1}{2}Kx^2$

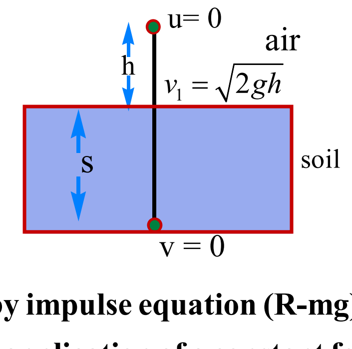
W_g =FS cos 90^0 = 0; W_N=NS cos 90^0 =0

$$W_{S.F}=K_f-K_i \Rightarrow -\frac{1}{2}Kx^2=0-\frac{1}{2}mv^2 \Rightarrow x=v\sqrt{\frac{m}{K}}$$

♦ **A pile driver of mass 'm' is dropped from a height 'h' above the ground. On reaching the ground it pierces through a distance 's' and then stops finally. If R is the average resistance offered by ground then**

$$W_g+W_R=K_f-K_i=\frac{1}{2}mu^2-\frac{1}{2}mv^2$$

$$mg\left(h+s\right)+\left(-Rs\right)=0;\ R=mg\left(1+\frac{h}{s}\right)$$



Here time of penetration is given by impulse equation (R-mg) t = $0-m\sqrt{2gh}$

♦ **A body of mass 'm' is initially at rest. By the application of a constant force, its velocity changes to v_0 in time t_0 the kinetic energy of the body at time 't' is**

$$W=\Delta K=K_f-K_i--=K_i-0$$

$$K_f = W = mas = ma\left(\frac{1}{2}at^2\right) = \frac{1}{2}ma^2t^2$$

$$\textbf{Since } a = \frac{v_o}{t_o};\ K_f = \frac{1}{2}m\left(\frac{v_0}{t_0}\right)^2 t^2$$

Types of Equilibrium:

A body is said to be in translatory equilibrium, if net force acting on the body is zero, $i.e., \vec{F}_{net} = 0$

If the forces are conservative $F = -\frac{dU}{dr}$

and for equilibrium F = 0,

so $-\frac{dU}{dr} = 0$ or $\frac{dU}{dr} = 0, \therefore$ At equilibrium position

slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant)

There are three types of equilibrium

(i) Stable equilibrium (ii) Unstable equilibrium

(iii) Neutral equilibrium

Stable equilibrium:

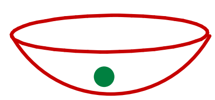
1.Net forced is Zero

2.$\frac{dU}{dr} = 0$ or slope of U-r graph is zero

3.When displaced from its equilibrium position, a net retarding forces starts acting on the body, which has a tendency to bring the body back to its equilibrium position

4.PE in equilibrium position is minimum as compared to its neighboring points as $\frac{d^2U}{dr^2}$ is positive

5.When displaced from equilibrium position the centre of gravity of the body comes down



Unstable equilibrium:

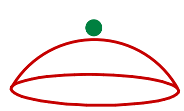
1.Net force is zero

2.$\frac{dU}{dr} = 0$ (or) slope of U-r graph is zero

3.When displaced from its equilibrium position, a net force starts acting on the body which moves the body in the direction of displacement or away from the equilibrium position

4.PE in equilibrium position is maximum as compared to other positions as $\frac{d^2U}{dr^2}$ is negative

5.When displaced from equilibrium position the centre of gravity of the body goes up



Neutral equilibrium:

1.Net force is zero

2.$\frac{dU}{dr} = 0$ (or) slope of U-r graph is zero

3.When displaced from its equilibrium position the body has neither the tendency to come back nor move away from the original position.

4.PE remains constant even if the body is moving to neighbouring points $\frac{d^2U}{dr^2} = 0$

5.When displaced from equilibrium position the centre of gravity of the body remains constant

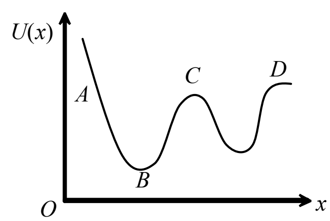
Potential energy curve :

A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.

As we know that negative gradient of the potential energy gives force.

$$-\frac{dU}{dx} = F$$



Nature of force :

(i) Attractive force :

On increasing x, if U increases,

$$\frac{dU}{dx} = \text{positive},$$

then F is in negative direction

i.e. force is attractive in nature.

In graph this is represented in region BC.

(ii) Repulsive force :

On increasing x, if U decreases,

$$\frac{dU}{dx} = \text{negative},$$

then F is in positive direction

i.e. force is repulsive in nature.

In graph this is represented in region AB.

(iii) Zero force :

On increasing x, if U does not change,

$$\frac{dU}{dx} = 0$$

then F is zero

i.e. no force works on the particle.

Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

Law of conservation of Mechanical energy:

Total mechanical energy of a system remains constant, if only conservative forces are acting on a system of particles and the work done by all other forces is zero.

$$\therefore U_f - U_i = -W$$

From work energy theorem $W = k_f - k_i$

$$\therefore U_f - U_i = -\left(k_f - k_i\right)$$

$$\therefore U_f + k_f = U_i + k_i \Rightarrow U + K = cons\tan t$$

The sum of potential energy and kinetic energy remains constant in any state.

A body is projected vertically up from the ground. When it is at height 'h' above the ground, its potential and kinetic energies are in the ration x : y .

If H is the maximum height reached by the body, then

$$\frac{x}{y} = \frac{h}{H-h} \; or \; \frac{h}{H} = \frac{x}{x+y}$$

:: PROBLEMS : :

1. Two spheres whose radii are in the ratio 1 : 2 are moving with velocities in the ratio 3 : 4. If their densities are in the ratio 3 : 2, then find the ratio of their kinetic energies.

$$\frac{r_1}{r_2}=\frac{1}{2},\frac{v_1}{v_2}=\frac{3}{4},\frac{\rho_1}{\rho_2}=\frac{3}{2}$$

$$K.E=\frac{1}{2}mv^2=\frac{1}{2}(V\rho)v^2=\frac{1}{2}\left(\frac{4}{3}\pi r^3\rho\right)v^2$$

$$\frac{KE_1}{KE_2}=\frac{\rho_1}{\rho_2}\times\left(\frac{r_1}{r_2}\right)^3\times\left(\frac{v_1}{v_2}\right)^2=\frac{3}{2}\times\left(\frac{1}{2}\right)^3\times\left(\frac{3}{4}\right)^2$$

$$\frac{KE_1}{KE_2}=\frac{3}{2}\times\frac{1}{8}\times\frac{9}{16}=\frac{27}{256}$$

2. An engine is pumping water continuously. The water passes through a nozzle with a velocity v. As water leaves the nozzle, the mass per unit length of the water jet is m_0. Find the rate at which kinetic energy is imparted to the water.

A) $\frac{1}{2}m_0v^3$ B) $\frac{1}{2}m_0v^2$ C) $\frac{1}{2}m_0v^{3/2}$ D) $\frac{1}{2}m_0v^{1/2}$

SOLUTION :

$$m_0=\frac{dm}{dx}$$

$$\frac{d}{dn}(\text{KE})=\frac{d}{dt}\left(\frac{1}{2}mv^2\right)=\frac{1}{2}\left(\frac{dm}{dt}\right)v^2$$

$$=\frac{1}{2}\left(\frac{dm}{dx}\right)\left(\frac{dx}{dt}\right)v^2=\frac{1}{2}m_0v^3$$

3. A particle is projected at 60^0 to the horizontal with a kinetic energy 'K'. Find the kinetic energy at the highest point? (JEE MAIN 2007)

SOLUTION :

Initial kinetic energy is $K=\frac{1}{2}mu^2$

The velocity at highest point $v_x=u\cos\theta$.

Kinetic energy of a particle at highest point

$$K_H=\frac{1}{2}mv_x^2=\frac{1}{2}mu^2\cos^2\theta=K\cos^2 60^0=\frac{K}{4}$$

4. An athlete in the Olympic games covers a distance of 100 m in 10s. His kinetic energy can be estimated to be in the range. (JEE MAIN 2008)

1) 200J-500J 2) $2\times10^5J-3\times10^5J$ 3) 20,000J-50,000J 4) 2,000J - 5,000J

SOLUTION :

Approximate mass of the athlete = 60kg

Average velocity = 10m/s

Approximate $K.E.\frac{1}{2}mv^2 = \frac{1}{2}\times 60\times 10^2 = 3000J$

Range of KE = 2000 J to 5000 J

5. Kinetic energy of a particle moving along a circle of radius 'r' depends on the distance as KE = cs², (c is constant, s is displacement). Find the force acting on the particle

SOLUTION :

$$KE = \frac{1}{2}mv^2 = cs^2 \Rightarrow v = \left(\sqrt{\frac{2c}{m}}\right)s$$

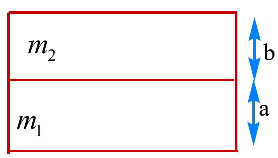
$$\alpha_t = \frac{dv}{dt} = \sqrt{\frac{2c}{m}}\times\frac{ds}{dt} = v\sqrt{\frac{2c}{m}}$$

$$F_t = m\alpha_t = mv\sqrt{\frac{2c}{m}} = \left[m\sqrt{\frac{2c}{m}}s\right]\sqrt{\frac{2c}{m}} = 2cs$$

Total force $F = \sqrt{F_t^2 + F_c^2} = \sqrt{(2cs)^2 + \left(\frac{mv^2}{r}\right)^2}$

$$F = 2cs\sqrt{1+\frac{s^2}{r^2}}$$

6. A rectangular plank of mass m_1 and height 'a' is on a horizontal surface. On the top of it another rectangular plank of mass m_2 and height 'b' is placed. Find the potential energy of the system?

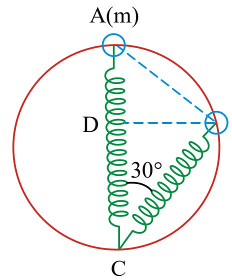


SOLUTION :

Total potential energy of system U = $U_1 + U_2$

$$= m_1 g\frac{a}{2} + m_2 g\left(a\frac{b}{2}\right) = \left[\left(\frac{m_1}{2}+m_2\right)a + m_2\left(\frac{b}{2}\right)\right]g$$

7. A ring 'A' of mass 'm' is attached to a stretched spring of force constant K, which is fixed at C on a smooth vertical circular track of radius R. Points A and C are diametrically opposite. When the ring slips from rest on the track to point B, making an angle of $30°$ with AC. ($\angle ACB = 30°$) spring becomes unstretched. Find the velocity of the ring at B



A) $\left[\frac{KR^2}{2m}\left(2-\sqrt{3}\right)^2 + gR\sqrt{3}\right]^{\frac{1}{2}}$ **B)** $\left[\frac{KR^2}{m}\left(2-\sqrt{3}\right)^2 + gR\right]^{\frac{1}{2}}$

C) $\left[\frac{2KR^2}{m}\left(2-\sqrt{3}\right)^2 + gR\sqrt{3}\right]^{\frac{1}{2}}$ **D)** $\left[\frac{KR^2}{2m}\left(\sqrt{2}-1\right)^2 + gR\right]^{\frac{1}{2}}$

SOLUTION :

Decrease in elastic PE + Decrease in PE = Increase in KE

$$\frac{1}{2}Kx^2 + mg(AD) = \frac{1}{2}mv^2$$

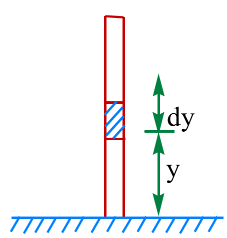
$$x = AC - CB = 2R - 2R\cos 30^\circ = R\left(2-\sqrt{3}\right) \quad \left(As\ \angle CBA = 90^\circ\right)$$

$$AD = AB\cos 60^\circ = \left(AC\sin 30^\circ\right)\cos 60^\circ = \frac{R}{2}$$

$$\text{So, } \frac{1}{2}KR^2\left(2-\sqrt{3}\right)^2 + mg\frac{R}{2} = \frac{1}{2}mv^2$$

$$v = \left[\frac{KR^2}{m}\left(2-\sqrt{3}\right)^2 + gR\right]^{\frac{1}{2}}$$

8. A rod of mass m and length L is held vertical. Find its gravitational potential energy with respect to zero potential energy at the lower end?



SOLUTION :

Choose a small element of length dy, then

mass of the element $dm = \left(\frac{m}{L}\right)dy$.

The potential energy of the element dU = (dm) g (y) Potential energy of the entire rod

$$U=\int_0^L (dm)\,gy\int_0^L\left(\frac{m}{L}\right)(dy).gy=\frac{m}{L}g\int_0^L ydy$$

$$U=\frac{m}{L}g\left(\frac{y^2}{2}\right)_0^L=\frac{mgL}{2}$$

9. The total mechanical energy of a particle is E. The speed of the particle at $x=\left(\frac{2E}{K}\right)^{1/2}$ is $\left(\frac{2E}{m}\right)^{1/2}$. Find the potential energy of the particle at x:

A) zero **B)** $\frac{1}{2}Kx^2$ **C)** $\frac{1}{4}Kx^2$ **D)** $\frac{2}{5}Kx^2$

SOLUTION :

At $x=\left(\frac{2E}{K}\right)^{1/2}$, $v=\left(\frac{2E}{m}\right)^{1/2}$

or $v^2=\frac{2E}{m}$ or $\frac{1}{2}mv^2=E$

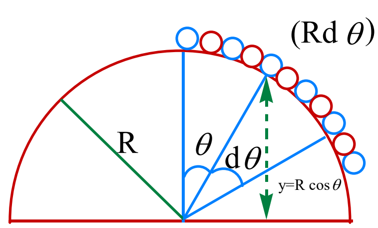
So, at $x=\left(\frac{2E}{K}\right)^{1/2}$, kinetic energy is equal to total mechanical energy.

Hence, PE at $x=\left(\frac{2E}{K}\right)^{1/2}$ is zero

So, $U(x)=0$

10. A chain of length ℓ and mass 'm' lies on the surface of a smooth hemisphere of radius R > ℓ with one end tied to top of the hemisphere. Find the gravitational potential energy of the chain?

SOLUTION :



Consider a small element of chain of width dθ at angle θ from the vertical

The mass of the element $dm=\left(\frac{m}{\ell}\right)Rd\theta$

The gravitational potential energy of the element du = (dm)gy

The gravitational potential energy of total chain

$$U=\int_0^{\frac{l}{R}}(dm)\,gy=\int_0^{\frac{l}{R}}\left(\frac{m}{l}Rd\theta\right)g\left(R\cos\theta\right)$$

$$U = \frac{mgR^2}{l}\left[\sin\theta\right]_0^{\frac{l}{R}} = \frac{mgR^2}{l}\sin\left(\frac{l}{R}\right)$$

11. A spring of force constant 'k' is stretched by a small length 'x'. Find work done in stretching it further by a small length 'y'?

SOLUTION :

Initial potential energy $U_i \frac{1}{2}kx^2$

Final potential energy $U_f = \frac{1}{2}k(x+y)^2$

Work done $W = U_f - U_i = \frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2$

$$W = \frac{1}{2}ky(2x+y)$$

12. Under the action of force 2kg body moves such that its position 'x' varies as a function of time t given by $x = \frac{t^3}{3}$, x is in meter and t in second. Calculate the workdone by the force in first two seconds.

SOLUTION :

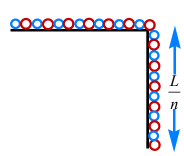
From work-energy theorem $W = \Delta KE$

$$x = \frac{t^3}{3}, Velocity\ v = \frac{dx}{dt} = t^2$$

At t = 0, v_1=0, At t= 2s, v_2= 4 m/s

$$W = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}\times 2\left(4^2 - 0\right) = 16J$$

13. A uniform chain of length 'l' and ,as 'M' is on a smooth horizontal table, with (1/n)th part of its length hanging from the edge of the table. Find the kinetic energy of the chain as it completely slips off the table.



SOLUTION :

Work done $\Delta W = U_i - U_f = K_f - K_i$

$$\frac{Mgl}{2} - \frac{Mgl}{2n^2} = \frac{1}{2}Mv^2; \quad v = \sqrt{gl\left[1 - \frac{1}{n^2}\right]}$$

14. A particle moves move on the rough horizontal ground with some initial velocity V_0. If $\frac{3}{4}$ of its kinetic energy lost due to friction in time t_0. The coefficient of friction between the particle and the ground is

A) $\frac{V_0}{2gt_0}$ B) $\frac{V_0}{4gt_0}$ C) $\frac{3V_0}{4gt_0}$ D) $\frac{V_0}{gt_0}$

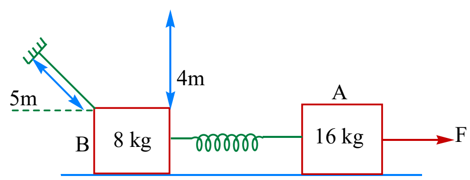
SOLUTION :

$$K.E_i = \frac{1}{2}mV_0^2;$$

$$K.E_f = \frac{1}{4}\frac{1}{2}mV_0^2$$

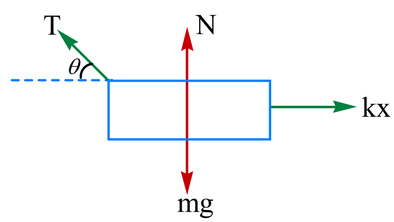
$$V = V_0 - \mu g t_0$$

15. **Two blocks having masses 8 kg and 16 kg are connected to the two ends of a light spring. The system is placed on a smooth horizontal floor. An inextensible string also connects B with ceiling as shown in figure at the initial moment. Initially the spring has its natural length. A constant horizontal force F is applied to the heavier block as shown. What is the maximum possible value of F so that lighter block doesn't loose contact with ground.**



SOLUTION :

Draw FBD of B to get extension in spring. When block B just looses contact with ground resultant force on it is zero.



$$Kx - T\cos\theta = 0 \Rightarrow T = \frac{Kx}{\cos\theta}; T\sin\theta + N - mg = 0$$

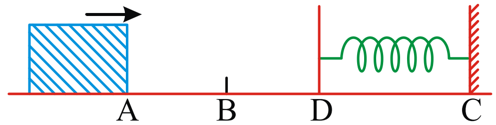
When N = 0 then $T\sin\theta = mg \Rightarrow \frac{Kx}{\cos\theta}\sin\theta = mg$

$$x = \frac{mg}{K\tan\theta} = \frac{80}{K\times(4/3)} = \frac{60}{K}$$

If spring has to just extend till this value then from work energy theorem we get

$$Fx = \frac{1}{2}Kx^2 \Rightarrow F = 30N$$

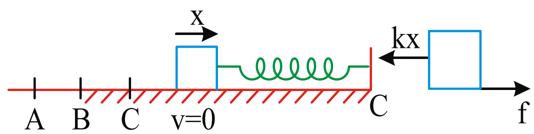
16. **A 0.5 kg block slides from the point A (see figure) on a horizontal track with an initial speed of 3m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m.The part AB of the track is frictionlessand the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distances AB and BD are 2m and 2.14 m respectively find the total distance through which the block moves before it comes to rest completely (Take g = 10 m/s²).**



A) 4.20 m **B) 4.14 m** **C) 4.24 m** **D) 4.26 m**

SOLUTION :

From A to B, there will be no loss of energy.
Now let block compresses the spring by an amount x and comes momentarily to rest.
Then, loss of energy will be equal to the work done against friction.
Therefore,



$$\mu_k mg(BD + x) = \frac{1}{2}mv^2 - \frac{1}{2}kx^2$$

Substituting the values $(0.2)(0.5)(10)(2.14 + x) = \frac{1}{2}(0.5)(3)^2 - \frac{1}{2}(2)(x)^2$

Solving this equation, we get $x = 0.1\text{m}$

17. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring and compresses it till the block is motionless. The kinetic frictional force is 15 N and spring constant is 10,000Nm^{-1}. Find the compression in the spring? (JEE MAIN 2007)

SOLUTION :

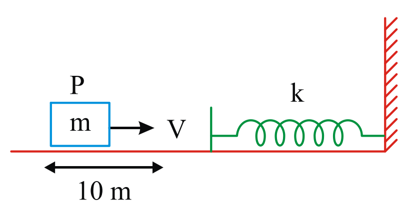
$$KE = \frac{1}{2}mv^2 = W_{friction} + \frac{1}{2}Kx^2$$

$$\Rightarrow \frac{1}{2}\times 2\times 4^2 = 15x + \frac{1}{2}\times 10000\times x^2$$

$$\Rightarrow 5000x^2 + 15x - 16 = 0$$

$$\Rightarrow x = 0.055m \quad or \quad x = 5.5\ cm$$

18. A block of mass 1 kg kept over a smooth surface is given velocity 2 m/s towards a spring of spring constant 1 N/m at a distance of 10m. Find after what time block will be passing through P again



A) $(20 + 2\pi)$sec **B)** 10sec **C)** $(10 + 2\pi)$sec **D)** $(10 + \pi)$sec

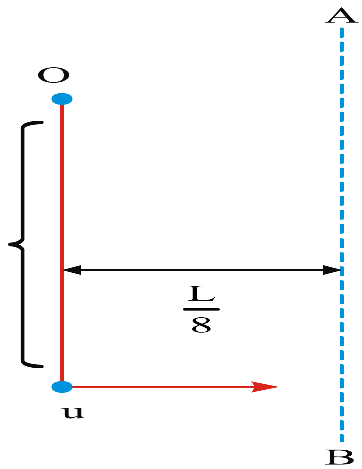
SOLUTION :

$$t = \frac{s}{v} + \pi\sqrt{\frac{m}{k}} + \frac{s}{v}$$

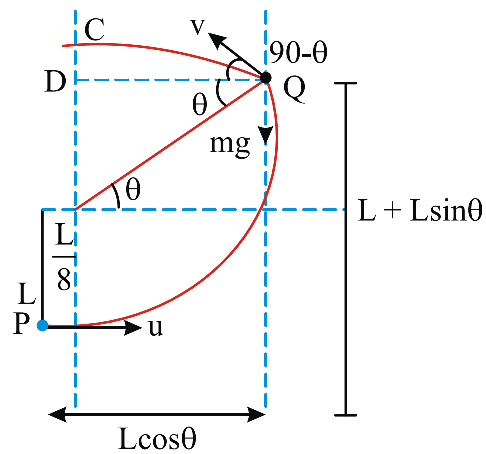
19. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is is at a distance of L/8 from O as shown. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passed through the line

AB. At the instant of crossing AB, its velocity is horizontal. Find u.

[1999]



SOLUTION :



Now, we have following equations

1) $T_Q = 0$ Therefore, $mg\sin\theta = \frac{mv^2}{L}$ ...... (1)

2) $v^2 = u^2 - 2gh = u^2 - 2gL(1+\sin\theta)$ ...... (2)

3) $QD = \frac{1}{2}(range)$ .... (3)

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

Comprehension-I

The potential energy U (in J) of a particle is given by $(ax+by)$, where *a* and *b* are constants. The mass of the particle is 1 kg and x and y are the coordinates of the particle in metre. The particle is at rest at $(4a, 2b)$ at time $t=0$.

20. Find the speed of the particle when it crosses x-axis

A) $2\sqrt{a^2+b^2}$ B) $\sqrt{a^2+b^2}$ C) $\frac{1}{2}\sqrt{a^2+b^2}$ D) $\sqrt{\frac{(a^2+b^2)}{2}}$

SOLUTION :

$$\vec{a} = \vec{F}/m = \frac{1}{m}\left(\frac{\partial U}{\partial X}\hat{i} + \frac{\partial U}{\partial Y}\hat{j}\right) = -\left(a\hat{i} + b\hat{j}\right) \text{ since, } m = 1 \text{ kg}$$

21. Find the speed of the particle when it crosses y-axis

A) $4\sqrt{a^2+b^2}$ B) $2\sqrt{2(a^2+b^2)}$ C) $\sqrt{2(a^2+b^2)}$ D) $\sqrt{(a^2+b^2)}$

SOLUTION :

$$\text{acceleration } \left|\vec{a_x}\right| = -a, \left|\vec{a_y}\right| = -b$$

$$\text{acceleration } \left|\vec{a}\right| = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{a^2+b^2}$$

$$u_x = -at, v_y = -bt$$

$$X = 4a + \frac{1}{2}a_x t^2 = 4a - \frac{a}{2}t^2$$

$$Y = 2b + \frac{1}{2}a_y t^2 = 2b - \frac{b}{2}t^2$$

particle crosses x-axis, when y = 0

22. Find the acceleration of the particle

A) $4\sqrt{(a^2+b^2)}$ B) $2\sqrt{2(a^2+b^2)}$ C) $\sqrt{2(a^2+b^2)}$ D) $\sqrt{(a^2+b^2)}$

SOLUTION :

Particle crosses y-axis, when x = 0

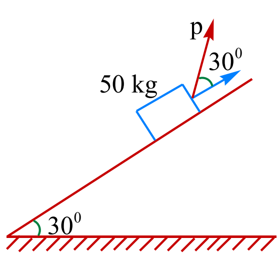
23. Find the coordinates of the particle at t = 1 second

A) $(3.5a, 1.5b)$ B) $(3a, 2b)$ C) $(3a, 3b)$ D) $(3a, 4b)$

SOLUTION :

Coordinate at $t = 1$ sec will be $(3.5a, 1.5b)$

24. In the below figure, what constant force 'P'is required to bring the 50 kg body, which starts from rest to a velocity of 10m/s in 7m? (Neglect friction)



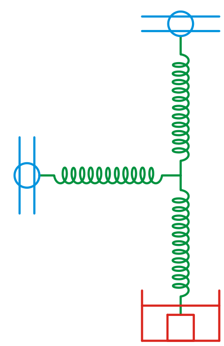
SOLUTION :

Work done by force P in displacing the block by 7 m, $W_1 (F\cos\theta)(S)$

$$W_1 (P\cos 30^0) 7 = \frac{7\sqrt{3}}{2} PJ$$

$$\mathbf{W_2 = mgh = 50 \times 9.8 \times 7 \sin 30^0 = 17}$$

25. Three springs A,B and C each of force constant K, are connected at O. The other ends of B and C can slide on smooth sliders. A pan is hanging from other end of the spring A. When a block of mass m is placed int he pan, find the amount of workdone by the gravity on block system after it stops vibrating. The spring C does not sag:



A) $\frac{3m^2g^2}{2K}$ **B)** $\frac{m^2g^2}{K}$ **C)** $\frac{2m^2g^2}{K}$ **D)** $\frac{m^2g^2}{2K}$

SOLUTION :

The system will adjust in such a way by sliding the spring C remains unstretched and spring A and B remains vertical.

Thus, effective force constant is giveny by

$$\frac{1}{K'} = \frac{1}{K} + \frac{1}{K} \quad ;$$

$$K' = \frac{K}{2}$$

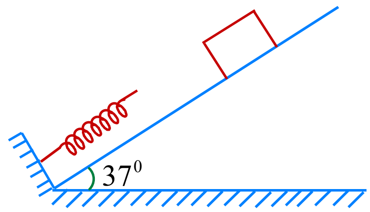
there is no effect of spring C.

$$K'x = mg \quad ;$$

$$x = \frac{2mg}{K} \quad ,$$

$$W = mg.x = mg.\frac{2mg}{k}$$

26. Figure shows a spring fixed at the bottom end of an incline of inclination 37^0. A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance 1m up and incline. Find (i) the friction coefficient between the plane and the block and (ii) the spring constant of the spring . (g= 10 ms^{-2})



SOLUTION :

Applying work energy theorem for downward motion of the body W= Δ KE

$$mg\sin\theta\left(x+d\right)-f\times l_1-\frac{1}{2}Kx^2=\Delta KE$$

$$20\sin 37^0\left(5\right)-\mu\times 20\cos 37^0\times 5-\frac{1}{2}K\left(0.2\right)^2=0$$

$$80\mu+0.02K=60\rightarrow\left(1\right)$$

For the upward motion of the body

$$-mg\sin\theta l_2+\left(f\times l_2\right)+\frac{1}{2}Kx^2=\Delta KE$$

$$-2\times 10\sin 37^0\times 1-\mu\times 20\cos 37^0\times 1+\frac{1}{2}K\left(0.2\right)^2=0$$

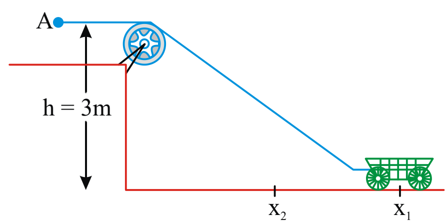
$$16\mu-0.02K=-12\rightarrow\left(2\right)$$

Adding equations (1) and (2), we get

$$96\mu=48\Rightarrow\mu=0.5$$

Now, use the value of μ in equation (1), we get K=1000 N/m

27. Figure shows a light, inextensible string attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the string is pulled over a pulley, of negligible mass and friction and fixed at height h = 3m from the ground level. The cart slides from $x_1 = 3\sqrt{3}$ m to $x_2 = 4$ m and during the move, tension in the string is kept constant 50 N. Find change in kinetic energy of the cart in joules. (Use $\sqrt{3} = 1.7$) in form of 10 x n, where n =



SOLUTION :

Change in kinetic energy = Work done by the force; so $W = 50\times 1$ Along the string);

so $W = 50 Joule$

28. In a molecule, the potential energy between two atoms is given by $U\left(x\right)=\frac{a}{x^{12}}-\frac{b}{x^6}$. Where 'a' and 'b' are positive constants and 'x' is the distance between atoms. Find the value of 'x' at which force is zero and minimum P.E at that point. (JEE MAIN 2010)

SOLUTION :

$$\text{Force is zero} \Rightarrow \frac{dU}{dx} = 0$$

$$\text{i.e., } a(-12)x^{-13} - b(-6)x^{-7} = 0$$

$$\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0 \Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7}$$

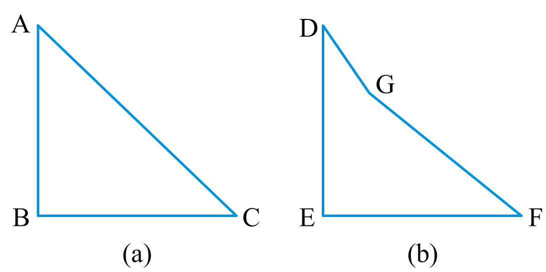
$$\Rightarrow x^6 \frac{2a}{b} \therefore x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

Substituting the value of x

$$\Rightarrow U_{\min} = a\left(\frac{b}{2a}\right)^{\frac{12}{6}} - b\left(\frac{b}{2a}\right)^{\frac{6}{6}}$$

$$U_{\min} = \left(\frac{b^2}{4a}\right) - \left(\frac{b^2}{2a}\right) \Rightarrow U_{\min} = \frac{-b^2}{4a}$$

29. In the figures A) and B) AC, DG and GF are fixed inclined planes, BC = EF = x and AB = DE = y. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_C. The small block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic frictions between the block and both the surfaces AC and DGF are m. Calculate V_C and V_F.



A) 1.7 m/s **B) 2.7 m/s** **C) 3.7 m/s** **D) 0.7 m/s**

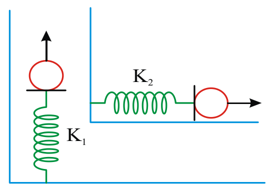
SOLUTION :

In both the cases work done by friction will be μMgx

$$\therefore \frac{1}{2}MV_C^2 = \frac{1}{2}MV_F^2 = Mgy - \mu Mgx$$

$$\therefore V_C = V_F = \sqrt{2gy - 2\mu gx}$$

30. Two balls of same mass are projected as shown, by compressing equally (say x) the springs of different force constants K_1 and K_2 by equal magnitude. The first ball is projected upwards along smooth wall and the other on the rough horizontal floor with coefficient of friction μ. If the first ball goes up by height h, then the distance covered by the second ball will be :



A) $\dfrac{2hK_2}{\mu K_1}$ B) $\dfrac{hK_1}{2\mu K_2}$ C) $\dfrac{3hK_2}{2\mu K_1}$ D) $\dfrac{hK_2}{\mu K_1}$

SOLUTION :

$$\frac{1}{2}K_1x^2 = mgh$$

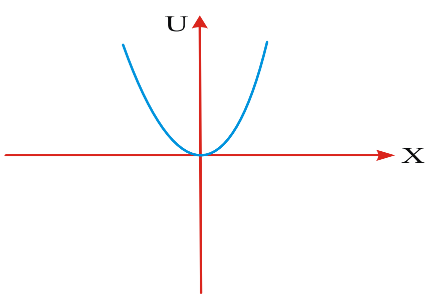
$$x^2 = \frac{2mgh}{K_1}$$

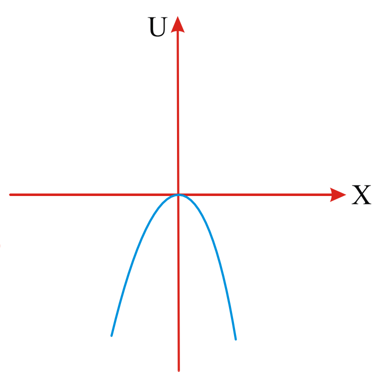
$$\frac{1}{2}K_2x^2 = \mu mgx_0$$

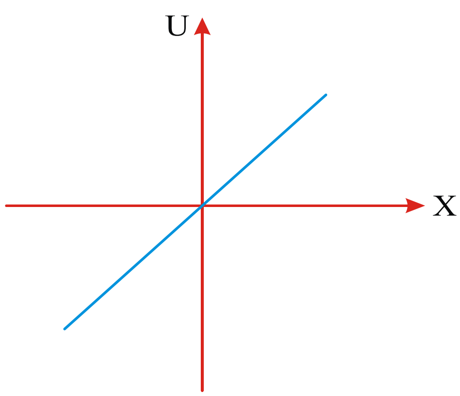
$$x^2 = \frac{2\mu mgx_0}{K_2}$$

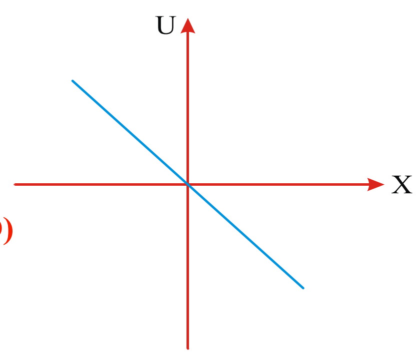
equating $x_0 = \dfrac{h}{\mu}.\dfrac{K_2}{K_1}$

31. A particle is acted by a force F=kx, where k is a +ve constant. Its potential energy at x=0 is zero. which curve correctly represents the variation of potential energy of the block with respect to x ? [IIT-2004]

A) 

B) 

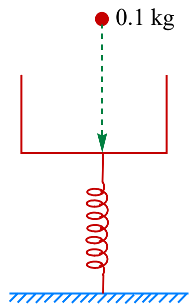
C) 

D) 

SOLUTION :

$$U=-\int F dx=-\int kx dx=-\frac{1}{2}kx^2$$

32. A massless platform is kept on a light elastic spring as shown in figure. When a sand particle of 0.1 kg mass is dropped on the pan from a height of 0.24m, the particle strikes the pan and the spring compresses by 0.01 m. From what height should particle be dropped to cause a compression of 0.04m.



SOLUTION :

By conservation of mechanical energy

$$mg\left(h+y\right)=\frac{1}{2}Ky^2$$

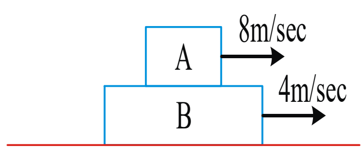
h=height of particle

y=compression of the spring

As here particle and spring remain same

$$\frac{h_1+y_1}{h_2+y_2}=\left(\frac{y_1}{y_2}\right)^2;\frac{0.24+0.01}{h_2+0.04}=\left(\frac{0.01}{0.04}\right)^2;h_2=3.96m$$

33. Block A of mass 1kg is placed on the rough surface of block B of mass 3 kg. Block B is placed on smooth horizontal surfac. Blocks are given the velocities as shown. Find net work done by the frictional force. [in (-) ve J]



SOLUTION :

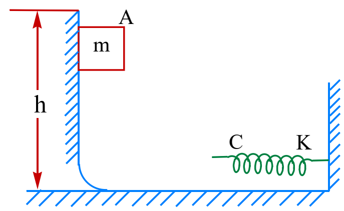
$$(1+3)v=(1)(8)+(3)(4)=20;\ v=5m/\text{sec}$$

$$\text{for block A, } W_f=\frac{1}{2}(1)\left(5^2-8^2\right)=-\frac{39}{2}\text{J}$$

$$\text{for block B, } W_f=\frac{1}{2}(3)\left(5^2-4^2\right)=+\frac{27}{2}\text{J}$$

net work done by friction = -6 J

34. A small mass 'm' is sliding down on a smooth curved incline from a height 'h' and finally moves through a horizontal smooth surface. A light spring of force constant K is fixed with a vertical rigid stand on the horizontal surface, as shown in the figure. Find the value for the maximum compression in the spring if mass 'm' is released from rest from height 'h' and hits the spring on the horizontal surface.



SOLUTION :

Conservation of energy b/w positions A and C

$$\left(PE_A\right)_{block} + KE_A = \left(PE_C\right)_{spring} + KE_C$$

$$mgh + 0 = \frac{1}{2}Kx^2 + 0; mgh = \frac{1}{2}Kx^2; x = \sqrt{\frac{2mgh}{K}}$$

35. A vehicle of mass 15 quintal climbs up a hill 200 m high. It then moves on a level road with a speed of 30ms^{-1}. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.

SOLUTION :

m= 15 quintal = 1500kg, g=9.8ms^{-2}, h=200m

P.E. gained,

U=mgh = 1500 x 9.8 x 200=2.94 x 10^{6}J

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\times 1500\times(30)^2 = 0.675\times 10^6 J$$

Total mechanical energy

$$E = K + U = (0.675 + 2.94)\times 10^6 = 3.615\times 10^6 J$$

36. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched Then,the maxumum extension in the spring is [IIT-2002]

A) $\frac{4Mg}{k}$ **B)** $\frac{2Mg}{k}$ **C)** $\frac{Mg}{k}$ **D)** $\frac{Mg}{2k}$

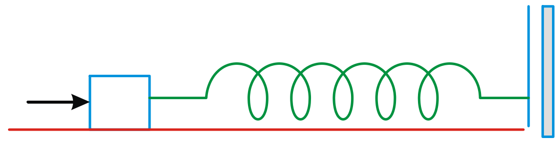
SOLUTION :

Loss in P.E = Gain in K.E + P.E stored in spring

$$MgX_{max} = 0 + \frac{1}{2}kX^2{}_{max} \;;$$

$$X_{max} = \frac{2Mg}{k}$$

37. A block of mass 0.18kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1 Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure.The block slides a distance of 0.06m and comes to rest for the first time . The initial velocity of the block in m/s is $V = N/10$. Then N is:
[IIT-2011]



SOLUTION :

Decrease in mechanical energy = work done against friction

$$\frac{1}{2}m\upsilon^2 - \frac{1}{2}kx^2 = (\mu mg)x$$

$$v = \sqrt{\frac{2\mu gx + k}{m}}$$

Putting $m = 0.18kg, x = 0.06m, k = 2Nm^{-1},$

$\mu = 01$ we get

$$\upsilon = 0.4m/s = \frac{4}{10}m/s$$

$$\therefore N = 4$$

38. A particle is released from height H. At certain height from the ground its kinetic energy is twice its gravitational potential energy. Find the height and speed of particle at that height

SOLUTION :

K.E= 2PE

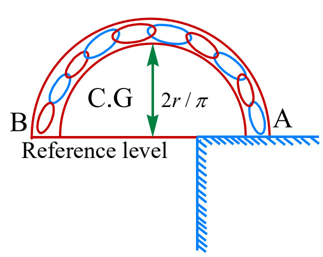
But KE=TE - PE

$$mg(H-h) = 2mgh; mgH = 3mgh$$

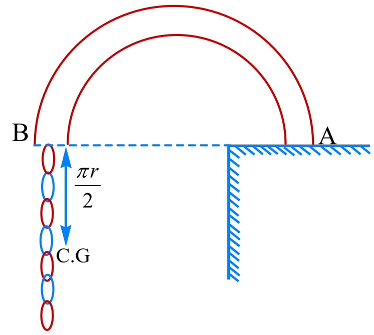
$$\Rightarrow h = \frac{H}{3}; \qquad Also\ K.E = 2P.E,$$

$$\frac{1}{2}mv^2 = 2mg\left(\frac{H}{3}\right) \Rightarrow v = 2\sqrt{\frac{gH}{3}}$$

39. A heavy flexible uniform chain of length πr and mass $\lambda \pi r$ lies in a smooth semicircular tube AB of radius 'r'. Assuming a slight disturbance to start the chain in motion, find the velocity v with which it will emerge from the end of the tube?



SOLUTION :



Centre of gravity of a semicircular arc is at a distance $\frac{2r}{\pi}$ from the centre.

Initial potential energy $U_i = (\lambda \pi r) g \left(\frac{2r}{\pi} \right)$

Final potential energy $U_f = (\lambda \pi r) g \left(\frac{-\pi r}{2} \right)$

When the chain is completely slipped off the tube, all the links of the chain have the same velocity v.

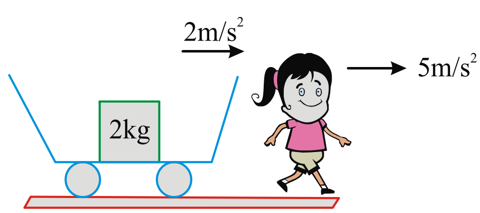
Kinetic energy of chain $k = \frac{1}{2} m v^2 = \frac{1}{2} (\lambda \pi r) v^2$

From conservation of energy,

$$\lambda \pi r g \left(\frac{2r}{\pi} \right) = (\lambda \pi r) g \left(\frac{-\pi r}{2} \right) + \frac{1}{2} (\lambda \pi r) v^2$$

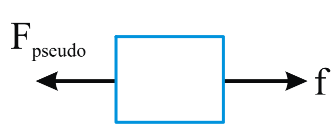
On solving we get, $v = \sqrt{2rg \left(\frac{2}{\pi} + \frac{\pi}{2} \right)}$

40. An observer and a vehicle, both start moving together from rest with accelerations 5 m/s^2 and 2 m/s^2, respectively. There is a 2 kg block on the floor of the vehicle, and $\mu = 0.3$ between their surfaces. Find the work done by frictional force on the 2 kg block as observed by the running observer, during first 2 seconds of the motion.



SOLUTION :

FBD of the block,



$f_L = 6N$, $F_{pseudo} = 4N$

$$\therefore \ f = 4N$$

acc. of the block with respect to observes $= 2 - 5 = -3\,m/s^2$

$\therefore$ displacement of the block w.r.to observes $= \frac{1}{2} \times -3 \times 4 = -6\,m$

$\therefore$ work done by friction w.r. to observers $= -24$ *Joule*

41. The potential energy of 1 kg particle free to move along X-axis is given by $U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$J. The total mechanical energy of the particle is 2 J. Find the maximum speed of the particle.

SOLUTION :

For maximum value of U, $\frac{dU}{dx} = 0$.

$$\therefore \frac{4x^3}{4} - \frac{2x}{2} = 0 \ or \ x = 0, \ x = \pm 1.$$

At $x = 0, \frac{d^2U}{dx^2} = -1$ and At $x = x \pm 1, \frac{d^2U}{dx^2} = 2$

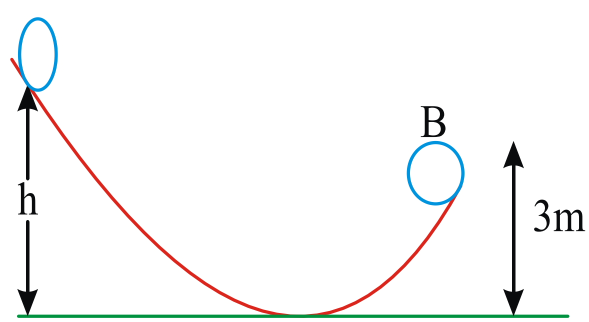
Hence U is minimum at $x = x \pm 1$ with value

$$U_{\min} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}J$$

$$K_{\max} + U_{\min} = E \ \ or \ \ K_{\max} - \frac{1}{4} = 2 \ or \ K_{\max} = \frac{9}{4}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{9}{4} \Rightarrow v_{\max} = \frac{3}{\sqrt{2}}, ms^{-1}$$

42.. A ball leaves the track at B which is at 3m height from bottom most point of the track. The ball further rises upto 4m height from the bottom most point before falling down. Find h (in m), if the track at B makes an angle 30^0 with horizontal.



SOLUTION :

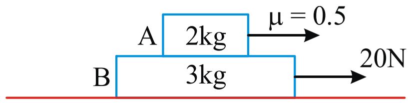
$$h_0 = 3 + \frac{u_B{}^2 \sin^2\theta}{2g} = 3 + \frac{2g\left(h - h_B\right)\sin^2 30^0}{2g}$$

$$4\left(\text{given}\right) = 3 + h\sin^2 30^0 - h_B \sin^2 30^0$$

$$= 3 + \frac{h}{4} - \frac{(3)}{4};$$

$$\frac{7}{4} = \frac{h}{4} \Rightarrow h = 7\text{ m}$$

43. Two blocks A and B are placed one over other. Block B is acted upon by a force of 20 N which displaces it through 5 m. Find work done by frictional force on block A

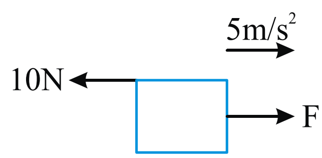


SOLUTION :

Limiting friction between the blocks $f_L = 10N$

$$[a_{\max}]_A = \frac{10}{2} = 5\,m/s^2$$

i.e., for shipping between A, B, Bmust move with $5\,m/s^2$

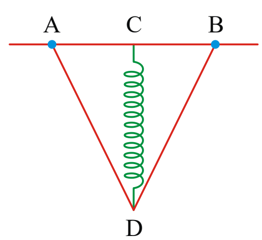


$$\Rightarrow F = 25N$$

but given $F = 20N$

44. A and B are smooth light hinges equidistant from C, which can slide on ABC. The spring of force constant K is fixed at its one end C and connected to light rods AD and BD at point D. A block of mass m is suspended at D. Find the velocity of the block, when $\angle CAD$ changes from $30°$ to $45°$.

$AD = BD = L$



A) $\left[gL - \frac{KL^2}{2m}\left(\sqrt{2}-1\right)^2\right]^{\frac{1}{2}}$

B) $\left[gL\sqrt{2} - \frac{KL^2}{2m}\left(\sqrt{2}-1\right)^2\right]^{\frac{1}{2}}$

C) $\left[gL\left(\sqrt{2}-1\right) - \frac{KL^2}{4m}\left(\sqrt{2}-1\right)^2\right]^{\frac{1}{2}}$

D) $\left[gL - \frac{KL^2}{2m}\right]^{\frac{1}{2}}$

SOLUTION :

$$\text{Inititally, } CD = L\sin 30° = \frac{L}{2}$$

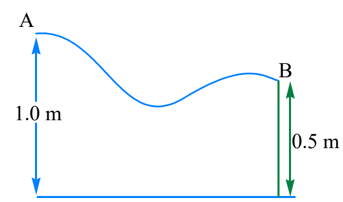
$$\text{Finally, } CD = L\sin 45° = \frac{L}{\sqrt{2}}$$

Increase in elastic PE + Increase in KE = Decrease in PE

$$\frac{1}{2}K\left(\frac{L}{\sqrt{2}} - \frac{L}{2}\right)^2 + \frac{1}{2}mv^2 = mg\left(\frac{L}{\sqrt{2}} - \frac{L}{2}\right)$$

$$\text{On solving, } v = \left[gL\left(\sqrt{2}-1\right) - \frac{KL^2}{4m}\left(\sqrt{2}-1\right)^2\right]^{\frac{1}{2}}$$

45. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



SOLUTION :

Applying the law of conservation of mechanical energy for the points A and B,

$$mgH = \frac{1}{2}mv^2 + mgh$$

$$g - \frac{v^2}{2} = \frac{g}{2} \text{ or } v^2 = g \Rightarrow v = \sqrt{g} = 3.1ms^{-1}$$

After point B the particle exhibits projectile motion with $\theta = 0^0$ and y=-0.5 m

Horizontal distance travelled by the body

$$R = u\sqrt{\frac{2h}{g}} = 3.1\times\sqrt{\frac{2\times 0.5}{9.8}} = 1m$$

Power:

The rate of doing work is called power.

Power or average power is given by

$$P_{avg} = \frac{work\ done}{time},$$

Power is a scalar

SI Unit: watt (W) (or) J/s,

CGS Unit: erg/sec

Other units: kilo watt, mega watt and horse power

One horse power (H.P)=746 watt.

Instantaneous Power:

$$P = \underset{\Delta t \to 0}{Lt}\left(\frac{\Delta W}{\Delta t}\right)$$

It is also calculated by $P = FV\cos\theta = \vec{F}.\vec{V}$

Relation Between P_{avg} and P_{ins}:

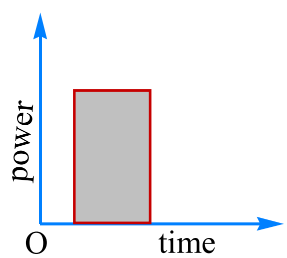
$$P_{ave} = \frac{W}{t} = \frac{mv^2}{2t} = \frac{1}{2}mv\left(\frac{v}{t}\right) = \frac{1}{2}mav = \frac{1}{2}\vec{F}.\vec{V}$$

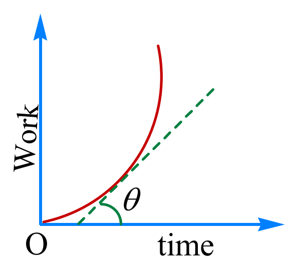
$$P_{ave} = \frac{1}{2}P_{inst}$$

The area under P - t graph gives work done

$$P = \frac{dW}{dt} \qquad \therefore W = \int P.dt$$

The slope of W - t curve gives instantaneous power $P = \frac{dW}{dt} = \tan\theta$





Applications on power:

♦ The power of a machine gun firing 'n' bullets each of mass 'm' with a velocity 'v' in a time interval 't' is given by

$$P = \frac{n\left(\frac{1}{2}mv^2\right)}{t} = \frac{nmv^2}{2t}$$

♦ A crane lifts a body of mass 'm' with a constant velocity v from the ground, its power is

P = Fv = m g v

♦ Power of lungs of a body blowing a whistle is $P = \frac{1}{2}$ (mass of air blown per sec) $(\text{velocity})^2$

♦ Power of a heart pumping blood = (pressure) (volume of blood pumped per sec)

♦ A conveyor belt is moving with a constant speed 'v' horizontally and gravel is falling on it at a rate of $\frac{dm}{dt}$.

Then additional force required to maintain speed v is $F = v\frac{dm}{dt}$

additional power required to drive the belt is, $P = Fv = v^2\frac{dm}{dt}$

♦ When a liquid of density $'\rho'$ coming out of a hose pipe of area of cross section 'A' with a velocity 'v' strikes the wall normally and stops dead.

The npower exerted by the liquid is $P=\frac{1}{2}\frac{mv^2}{t}=\frac{1}{2}\rho Av^3$

($\because$ mass = density x volume = m= $\rho \times A \times l$)

♦ A vehicle of mass 'm' is driven with constant acceleration along a straight level road against a constant external resistance 'R' when the velocity is 'v', power of engine is

P= Fv = (R + ma)v

♦ If P is a rated power of a device and if its efficiency is x%, useful power is (output power)

$$P^1 = \frac{x}{100}P$$

♦ If a motor lifts water from a well of depth 'h' and delivers with a velocity 'v' in a time t then power of the motor

$$P = \frac{mgh + \frac{1}{2}mv^2}{t}$$

♦ If a body of mass 'm' starts from rest and accelerated uniformly to a velocity v_0 in a time t_0, then the work done on the body in a time 't' is given by

$$W = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_o t}{t_o}\right)^2; a = \frac{v_0}{t_o}; \ v = at = \left(\frac{v_0}{t_o}\right)t$$

Instantaneous power, P = Fv = m a v

$$\therefore P = m\frac{v_0}{t_o}\left(\frac{v_0}{t_o}\right)t = m\frac{v_0^2}{t_0^2}t$$

♦ A motor pump is used to deliver water at a certain rate from a given pipe. To obtain 'n' times water from the same pipe in the same time by what among of (a) force and (b) power of the motor should be increased.

If a liquid of density $'\rho'$ is flowing through a pipe of cross section 'A' at speed 'v' the mass coming out per second will be $\frac{dm}{dt} = Av\rho$.

To get 'n' times water in the same time

$\left(\frac{dm}{dt}\right)^1 = n\left(\frac{dm}{dt}\right) \Rightarrow A'v'\rho' = n(Av\rho)$

As the pipe and liquid are not changed,

$$\rho^{|} = \rho; A' = A \,\&\, v' = nv$$

$$\text{as } F = v\frac{dm}{dt} \Rightarrow \frac{F'}{F}\frac{v^{|}\left(\frac{dm}{dt}\right)^{1}}{v\left(\frac{dm}{dt}\right)} = \frac{(nv)\left(n\frac{dm}{dt}\right)}{v\left(\frac{dm}{dt}\right)} = n^2$$

as $P = Fv \Rightarrow$

$$\frac{P'}{P} = \frac{F'v'}{Fv} = \frac{\left(n^2F\right)(nv)}{Fv} = n^3$$

$$\boxed{\therefore F' = n^2F} \qquad \boxed{\therefore P' = n^3P}$$

To get 'n' times of water force must be increased n^2 times while power n^3 times.

Position and velocity of an automobile w.r.t. time:

An automobile of mass 'm' accelerates starting from rest, while the engine supplies constant power, its position and velocity changes w.r.t. time as

Velocity:

As Fv = P = constant

i.e. $m\frac{dv}{dt}v = p \qquad \left(F = m\frac{dv}{dt}\right)$

or $\int v dv = \int \frac{P}{m}dt$ on integrating we get $\frac{v^2}{2} = \frac{P}{m}t + C_1$

As initially the body is at rest,

ie. v = 0 at t =0 $\Rightarrow C_1 = 0$;

$$v = \left(\frac{2Pt}{m}\right)^{1/2} \Rightarrow v \alpha t^{1/2}$$

Position:

From the above expression

$$v = \left(\frac{2Pt}{m}\right)^{1/2} \quad (or) \quad \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\int ds = \int\left(\frac{2Pt}{m}\right)^{1/2} dt = \left(\frac{2p}{m}\right)^{\frac{1}{2}} \int t^{\frac{1}{2}} dt$$

integrating on both sides we get $S = \left(\frac{2P}{m}\right)^{1/2}\frac{2}{3}.t^{3/2} + C_2$

Now at t = 0, S = 0 $\Rightarrow C_2 = 0$

$$S\left(\frac{8P}{9m}\right)^{1/2} t^{3/2}, \therefore S \alpha t^{3/2}$$

:: PROBLEMS ::

1. An automobile is moving at 100 kmph and is exerting attractive force of 3920 N. What horse power must the engine develop, if 20% of the power developed is wasted?

SOLUTION :

$$\text{Velocity} = 100 \text{ kmph } 100\times\frac{5}{18} m/s$$

$$\text{Force} = 3920 \text{ N;; Useful power} = 80\%$$

$$\text{Power} = \frac{W}{t} = \frac{F.S}{t} F.v \Rightarrow \frac{80}{100} P = 3920\times100\times\frac{5}{18}$$

$$P = \frac{100}{80}\times3920\times100\times\frac{5}{18} = 13.16\times10^4 W = 182.5hp$$

2. The coefficient of friction between a particle moving with some velcoity V_0 and the rough horizontal surface is $\left(\frac{V_0}{2gt_0}\right)$. Find how much kinetic energy is lost in time t_0 due to friction:

A) 1/4 **B) 1/2** **C) 3/4** **D) 2/3**

SOLUTION :

$$v^1 = v_0 - (\mu g)t_0$$

$$= v_0 - \left(\frac{v_0}{2gt_0}\right)gt_0 = v_0/2$$

$$\text{velocity left} = v_0/2$$

$$\text{K.E left} = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mv_0^{\ 2}\right)$$

$$= \frac{1}{4} \text{ of initial K.E}$$

3. The velocity of a particle is $\vec{v} = at\hat{i} + bt^2\hat{j}$, where t is the time in second. Match the columns for $t=1$ second:

Column-I

A) Acceleration of particle is

B) Tangential acceleration is

C) Radial acceleration is

D) Radius of curvature of path is

Column-II

p) less than $\left(a^2+b^2\right)^{3/2}$

q) less than ab

r) less than $\left(a^2+b^2\right)$

s) greater than 2b

SOLUTION :

A-s; B-r; C-q; D-p

$$\vec{v} = at\hat{i} + bt^2\hat{j} = a\hat{i} + b\hat{j} \text{ for } t=1 \text{ second}$$

$$\vec{a} = a\hat{i} + 2bt\hat{j}$$

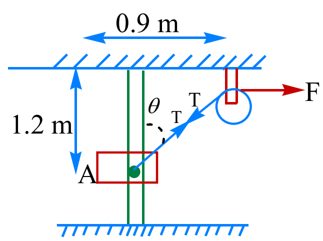
$$\left|\vec{a}\right| = \left(a^2 + 4b^2\right)^{1/2} > 2b$$

$$= \left[\left(a^2 + 4b^2\right) - \frac{\left(a^2 + 2b^2\right)^2}{\left(a^2 + b^2\right)}\right]^{\frac{1}{2}}$$

$$= \frac{ab}{\left(a^2 + b^2\right)^{1/2}} < ab$$

$$R = \frac{v^2}{a_n} = \frac{\left(a^2 + b^2\right)^{3/2}}{ab} < \left(a^2 + b^2\right)^{3/2}$$

4. The 50 N collar starts from rest at A and is lifted with a constant speed of 0.6 m/s along the smooth rod. Determine the power developed by the force F at the instant shown.



SOLUTION :

Since the collar is lifted with a constant speed

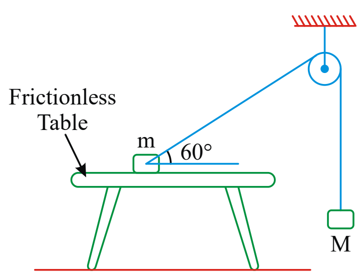
$$T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg = 5 \times 10$$

Now, $P = \vec{F}.\vec{v} = T\cos\theta \times v; Here T = F$

P=50 x v = 50 x 0.6 = 30W

5. What is the minimum value of the mass *M* so that the block is lifted off the table at the instant shown in the diagram ? Assume that the blocks are initially at rest.

A) $\frac{m}{\sin 60^0}$ **B)** $\frac{m}{\tan 60^0}$ **C)** $m\sin 60^0$ **D) none of these**



SOLUTION :

The accelerations of the blocks along the string are equal;
now apply F = ma for both the blocks.

6. **A machine delivers power to a body which is directly proportional to velocity of the body. If the body starts with a velocity which is almost negligible, find the distance covered by the body in attaining a velocity v.**

SOLUTION :

$$\text{Power } P = Fv\cos 0 = Fv = m\left(\frac{dv}{dt}\right)v \alpha v$$

$$mv\frac{dv}{dt} = K_0 v, Where\ K_0 = constant$$

$$m\frac{dv}{dt} = K_0;\ m\left(\frac{dv}{dx}\right)\frac{dx}{dt} = K_0$$

$$mv\frac{dv}{dx} = K_0;\ vdv = \left(\frac{K_0}{m}\right)dx$$

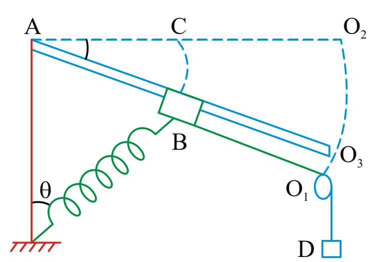
$$\text{Integrating} \int_0^v vdv = \int_0^x \left(\frac{K_0}{m}\right)dx;$$

$$\frac{v^2}{2} = \left(\frac{K_0}{m}\right)x \Rightarrow x = \frac{1}{2}\frac{mv^2}{K_0}$$

Passage

Rod AO_3 of length L can rotate about A. Initially rod was at potition AO_2, when spring OB of force constant K, attached to block B of mass m was at position OA with unstretched length L. The smooth block B can slide on rod when pulled by the block D of mass m through a massless spring and smooth pulley at O_1.

7. **Find the velocity of the block B, when the rod and spring at B make an angle of $30°$ with their respective initial positions : B is the middle point of the block)**



A) $\left[\frac{10mgL - KL^2\left(2-\sqrt{3}\right)^2}{8m}\right]^{\frac{1}{2}}$ **B)** $\left[\frac{2mgL - KL^2\left(\sqrt{2}-1\right)}{4m}\right]^{\frac{1}{2}}$ **C)** $\left[\frac{5mgL - KL^2\left(\sqrt{2}-1\right)}{4m}\right]^{\frac{1}{2}}$ **D)** $\left[\frac{6mgL - KL^2\left(\sqrt{2}-1\right)}{4m}\right]^{\frac{1}{2}}$

SOLUTION :

$$\angle ABO = 90°$$

(Since, $\angle AOB = 30°$ and $\angle OAB = 60°$)

$$OB = L\cos 30° = \frac{L\sqrt{3}}{2}\ ;\ \ AB = \frac{L}{2}$$

$$BC = BA\sin 30° = \frac{L}{4}$$

$$\text{Distance by which B ahs gone down} = BC = \frac{L}{4}$$

$$\text{Distance by which D has gone down} = AB = L\sin 30° = L$$

Decrease in PE = Increase in KE + Increase in elastic PE

$$mgL + mg\frac{L}{4} = \frac{1}{2}\times 2m\times v^2 + \frac{1}{2}K\left(L - L\frac{\sqrt{3}}{2}\right)^2$$

$$\text{on solving, } v = \left[\frac{10mgL - KL^2\left(2-\sqrt{3}\right)^2}{8m}\right]^{\frac{1}{2}}$$

8. Find the work done by the frictional force (if slider is rough) at the instant when rod and the spring attached at block B make an anlge of $30°$ with their respective initial positions.

A) $\frac{1}{2}KL^2\left(2-\sqrt{3}\right)^2 - mgL$ **B)** $KL^2\left(2-\sqrt{3}\right)^2 - \frac{mgL}{4}$

C) $\frac{1}{8}KL^2\left(2-\sqrt{3}\right)^2 - \frac{5}{4}mgL$ **D)** $\frac{1}{2}KL^2\left(\sqrt{2}-1\right)^2$

SOLUTION :

$$W = \Delta KE = 0$$

$$W_f + W_N + W_S + W_g = 0$$

W_N = Work done by normal reaction = 0 Acts perpendicular to displacement)

W_S = Work done by spring force

$$= 0 - \frac{1}{2}K\left(L - \frac{L\sqrt{3}}{2}\right)^2$$

W_g = Work done by force of gravity $= \frac{5}{4}mgL$

$$W_f = -\left[0 - \frac{1}{2}KL^2\left(\frac{2-\sqrt{3}}{2}\right)^2 + \frac{5}{4}mgL\right]$$

$$= \frac{1}{8}KL^2\left(2-\sqrt{3}\right)^2 - \frac{5}{4}mgL$$

9. Find the power of an engine which can draw a train of 400 metric ton up the inclined plane of 1 in 98 at the rate 10 ms^{-1}. The resistance due to friction acting on the train is 10 N per ton.

SOLUTION :

$$\text{Given } \sin\theta = \frac{1}{98}; m = 400\times 10^3\, kg$$

frictional force f = 10 × 400 = 4000N;

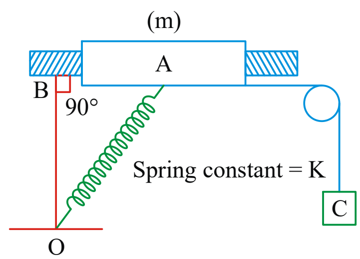
velocity v = 10 ms^{-1}

$$\therefore Power\ P = (mg\sin\theta + f)v$$

$$\therefore P = \left[\left(400\times10^3\times9.8\times\frac{1}{98}\right)+4000\right]\times10$$

=440000W = 440 KW

10. A block A of mass m slides on a smooth slider in the system as shown. A block c of same mass hanging from a pulley pulls block A. When the block A was at position B, the spring was unstretched. Find the speed of the block A when $AB = OB = L$



A) $\left[\frac{gL}{\sqrt{2}} - \frac{KL^2\sqrt{2}}{m}\right]^{\frac{1}{2}}$ **B)** $\left[gL - \frac{KL^2}{2m}(\sqrt{2}-1)^2\right]^{\frac{1}{2}}$ **C)** $\left[gL - \frac{2KL^2}{m}(\sqrt{2}-1)^2\right]^{\frac{1}{2}}$ **D)** $\left[\frac{gL}{2} - \frac{KL^2\sqrt{2}}{m}\right]^{\frac{1}{2}}$

SOLUTION :

Decrease in PE = Increase in KE + Increase in elastic PE

$$mgL = \frac{1}{2}\times(2m)v^2 + \frac{1}{2}Kx^2$$

$$s = mv^2 + \frac{1}{2}KL^2(\sqrt{2}-1)^2$$

$$v = \left[gL - \frac{KL^2}{2m}(\sqrt{2}-1)^2\right]^{\frac{1}{2}}$$

11. A hose pipe has a diameter of 2.5 cm and is required to direct a jet of water to a height of atleast 40m. Find the minimum power of the pump needed for this hose.

SOLUTION :

Volume of water ejected per sec

$$Av = \pi\left(\frac{d}{2}\right)^2 \times \sqrt{2gh}m^3/s; \therefore v = \sqrt{2gh}$$

Mass ejected per sec is $M = \frac{1}{4}\pi d^2 \times \sqrt{2gh}\rho\ Kg/s$

Kinetic energy of water leaving hose/sec $K.E = \frac{1}{2}mv^2 = \frac{1}{8}\pi d^2\times(2gh)^{\frac{3}{2}}\times\rho$

$$\frac{1}{8}\times3.14\times(2.5\times10^{-2})^2\times(2\times9.8\times40)^{\frac{3}{2}}\times1000 = 21.5KJ$$

12. **A body of mass m accelerates uniformly from rest to velocity v_0 in time t_0, find the instantaneous power delivered to body when velocity is $\frac{v_0}{2}$.**

SOLUTION :

$$\text{Acceleration } a = \frac{v_0}{t_0}; \text{Force} F = \frac{mv_0}{t_0}$$

$$\text{Instantaneous power P=F.} \frac{v_0}{2} = \left(\frac{mv_0}{t_0}\right)\frac{v_0}{2} = \frac{mv_0^2}{2t_0}$$

13. **A particle of 500 gm mass moves along a horizontal circle of radius 16m such that normal acceleration of particle varies with time as $a_n = 9t^2$**

Column - I	Column - II
A) Tangential force on particle at t = 1 second (in newton)	p) 72
B) Total force on particle at t = 1 second (in newton)	q) 36
C) Power delivered by total force at t = 1 sec (in watt)	r) 75
D) Averge power developed by total force over first one second (in watt)	s) 6

SOLUTION :

A-s,B-r,C-p,D-q

14. **A wind -powered genrator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy . For wind speed υ, the electrical powwer output will be proportional to**

[IIT-2008]

A) υ B) υ^2 C) υ^3 D) υ^4

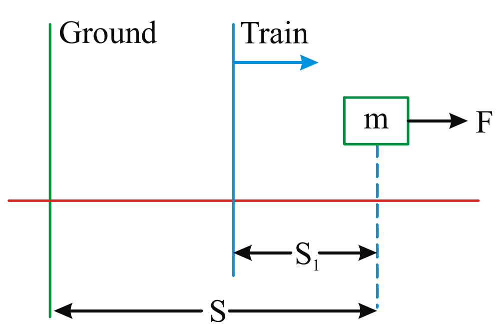
SOLUTION :

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = v(Apv) = Apv^2$$

$$\therefore P = Fv = (APv)^2 v = APv^3 \Rightarrow P \alpha v^3$$

Comprehension

A block of mass m sits at rest on a frictionless table in a rail car that is moving with speed v_c along a straight horizontal track (fig.) A person riding in the car pushes on the block with a net horizontal force F for a time t in the direction of the car's motion.



15. What is the final speed of the block according to a person in the car?

A) $\frac{Ft}{m}$ B) $\frac{2Ft}{m}$ C) $-\frac{Ft}{m}$ D) zero

SOLUTION :

$$V = u + at = 0 + \frac{Ft}{m}$$

16. According to a person standing on the ground outside the train?

A) $V_c + \frac{Ft}{m}$ B) $V_c - \frac{2Ft}{m}$ C) $\frac{Ft}{m} - V_c$ D) zero

SOLUTION :

$$V_g = V_0 + \frac{Ft}{m}$$

17. How much did K.E of the block change according to the person in the car?

A) $\frac{F^2t^2}{2m}$ B) $\frac{F^2t^2}{m}$ C) $\frac{2F^2t^2}{m}$ D) none of these

SOLUTION :

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{F^2t^2}{m^2} \quad = \frac{1}{2}\frac{F^2t^2}{m}$$

18. In terms of F, m, & t, how far did the force displace the object according to the person in car?

A) $\frac{Ft^2}{m}$ B) $\frac{Ft^2}{2m}$ C) $\frac{2Ft^2}{m}$ D) $\frac{4Ft^2}{m}$

SOLUTION :

$$S_{train} = 0t + \frac{1}{2}\frac{F}{m}t^2$$

19. According to the person on the ground. The displacement of block is

A) $\frac{Ft^2}{2m} + 2v_c t$ B) $\frac{Ft^2}{2m} + v_c t$ C) $\frac{Ft^2}{m} + v_c t$ D) $\frac{Ft^2}{2m} - v_c t$

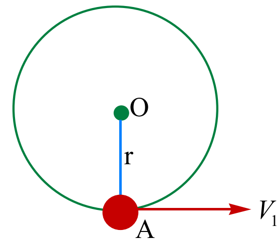
SOLUTION :

$$S_{ground} = V_C t + S_{train};$$

$$= V_C t + \frac{1}{2}\frac{F}{m}.t^2$$

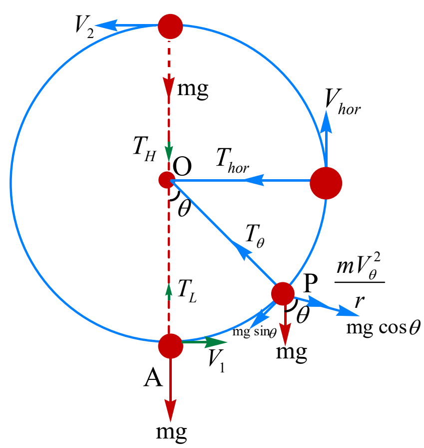
Vertical circular motion

Vertical circular motion with variable speed:



Consider a body of mass 'm' tied at one end of a string of length 'r' and is whirled in a vertical circle by fixing the other end at 'O'.

Let V_1 be the velocity of the body at the lowest point.



Velocity of the body at any point on the vertical circle:

$$TE_A = TE_P; \frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV_\theta^2 + mgh$$

$$V_\theta^2 = V_1^2 - 2gh, \; but \; h = r(1-\cos\theta)$$

$$V_\theta^2 = V_1^2 - 2gr(1-\cos\theta); V_\theta = \sqrt{V_1^2 - 2gr(1-\cos\theta)}$$

If V_2 is the velocity of the body at highest point $(\theta = 180^0)$

$$V_2 = \sqrt{V_1^2 - 2gr(1+1)} = \sqrt{V_1^2 - 4gr}$$

Tension in the string at any point:

Let T_θ be the tension in the string when the string makes an angle θ with vertical.

$$T_\theta = \frac{mV_\theta^2}{r} mg\cos\theta$$

1) At the lowest point $\theta = 0^0$ tension in the string is

$$T_L = \frac{mV_1^2}{r} + mg(\max imum.)$$

2) At the highest point $\theta = 180^0$.

The tension in the string is $T_H = \frac{mV_2^2}{r} - mg$ (min)

3) When the string is horizontal, $\theta = 90^0$,

tension in the string at this position is $T_{(hor)} = \frac{mV_{horz}^2}{r}$

4) The difference in maximum and minimum tension in the string is

$$T_{max} - T_{min} = \frac{mV_1^2}{r} + mg - \frac{mV_2^2}{r} + mg$$

$$= \frac{m}{r}\left(V_1^2 - V_2^2\right) + 2mg$$

$$= \frac{m}{r}(4gr) + 2mg = 4mg + 2mg = 6mg$$

5) Ratio of maximum tension to minimum tension in the string is

$$\frac{T_{max}}{T_{min}} = \frac{\frac{mV_1^2}{r} + mg}{\frac{mV_2^2}{r} - mg} = \frac{V_1^2 + rg}{V_2^2 - rg}$$

When the particle is at 'P'

a) Tangential force acting on the particle is $F_t = mg\sin\theta$.

Tangential acceleration $a_t = g\sin\theta$

b) Centripetal force acting on the particle is

$$F_e = \left(\frac{mV_\theta^2}{r}\right) T_\theta - mg\cos\theta.$$

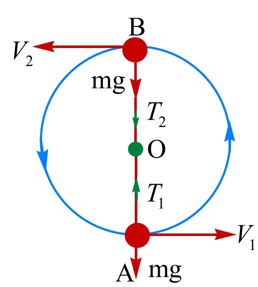
Centripetal acceleration $a_c = \frac{V_\theta^2}{r}$

c) Net acceleration of the particle at the point 'P' is $a = \sqrt{a_t^2 + a_c^2}$.

d) The net force acting on the particle at point 'P' is $F = \sqrt{F_t^2 + F_c^2}$

Angle made by net force or net acceleration with centripetal component is ϕ and tan $\phi = \frac{F_t}{F_c} \frac{a_t}{a_c}$

Condition for vertical circular motion of a body



We know that $T_2 = \frac{mV_2^2}{r} - mg$

The body will complete the vertical circular path when tension at highest point is such that

$$T_2 \geq 0, \frac{mV_2^2}{r} mg \geq 0; V_{2\min} = \sqrt{gr}$$

Hence the minimum speed at highest point to just complete the vertical circle is $\sqrt{gr}$

From the law of conservation of mechanical energy total energy at lowest point A=total energy at highest point B

$$U_A + KE_A = U_B + KE_B$$

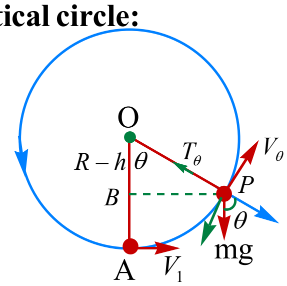
$$O + \frac{1}{2}mV_1^2 = mg(2r) + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}mV_1^2 = 2mgr + \frac{1}{2}mgr \quad \left[\because V_2 = \sqrt{gr}\right]$$

$$= \frac{5}{2}mgr \Rightarrow V_1 = \sqrt{5gr}$$

For the body to continue along a circular path the critical velocity at lowest point is $\sqrt{5gr}$

Critical velocity at any point on the vertical circle:



From the Law of conservation of energy total energy at point 'A'= total energy at point P

$$U_A + KE_A = U_P + KE_P$$

$$O + \frac{1}{2}mV_1^2 = mgh + \frac{1}{2}mV_\theta^2$$

$$\frac{1}{2}m(5gR) = mgR(1-\cos\theta) + \frac{1}{2}mV_\theta^2$$

$$\frac{5gmR}{2} = mgR - mgR\cos\theta + \frac{1}{2}mV_\theta^2$$

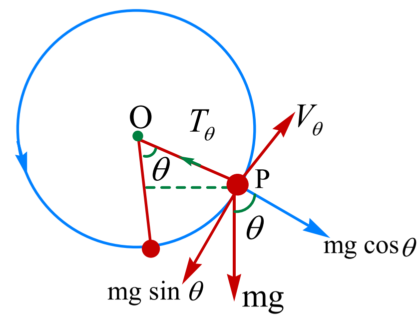
$$\frac{5gmR}{2} - mgR + mgR\cos\theta = \frac{1}{2}mV_\theta^2$$

$$\frac{mgR}{2}[3 + 2\cos\theta] = \frac{1}{2}mV_\theta^2$$

$$V_\theta = \sqrt{gR(3 + 2\cos\theta)}$$

Minimum tension in the string to just complete vertical circle:

Let T_θ be the tension in the string when the string is making an angle θ from lowest point



$$T_\theta = mg\cos\theta + \frac{mv_\theta^2}{R} = mg\cos\theta + \frac{m}{R}gR(3+2\cos\theta)$$

$$= mg\cos\theta + 3mg + 2mg\cos\theta$$

$$= 3mg\cos\theta + 3mg = 3mg(1+\cos\theta)$$

In case of non uniform circular motion in a vertical plane if velocity of the body at the lowest point is less than $\sqrt{5gr}$, the particle will not complete the circle in vertical plane, the particle may either oscillate about the lowest point or it leaves the circle with out looping.

Condition for oscillating about the lowest position:

1)If $0 < V_L < \sqrt{2gr}$. in this case, velocity becomes zero before tension vanishes and the particle oscillates about its lowest position with angular amplitude $0^0 < \theta < 90^0$

2)If velocity of the body at the lowest point $V_L < \sqrt{2gr}$, then the maximum height reached by the body just before its velocity becomes zero is given by $h = \frac{V_L^2}{2g}$

3) The angle made by the string with the vertical when its velocity becomes zero is given by $\cos\theta = 1 - \frac{V_L^2}{2gr}$

Note: If $0 < V_L \le \sqrt{2gr}$ then the particle oscillates such that $0^0 < \theta \le 90^0$

Condition for leaving the circular path without looping:

♦ If $\sqrt{2gr} < V_L < \sqrt{5gr}$.

the particle is not able to complete the vertical circle,

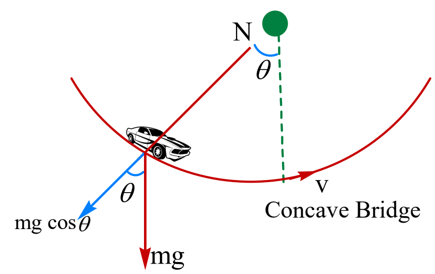
it goes to certain height and leaves the circular path $(90^0 < \theta < 180^0)$

while leaving the circular path $T = 0$ but $V \neq 0$

♦ The angle made by the string with downward vertical when the tension in the string becomes zero is given by $\cos\theta = \frac{2}{3} - \frac{V_L^2}{3gr}$

♦ The heigth at which the tension in the string becomes zero is given by $h = \frac{V_L^2 + gr}{3g}$

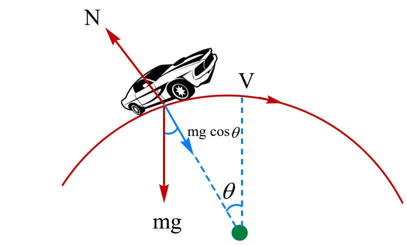
♦ **When car moves on a concave bridge of radius**



$$\text{Centripetal force} = N - mg\cos\theta = \frac{mv^2}{r}$$

$$\text{and normal reaction } N = mg\cos\theta + \frac{mv^2}{r}$$

♦ **When car moves on a convex-bridge of radius r**



$$\text{Centripetal for} = mg\cos\theta - N = \frac{mv^2}{r}$$

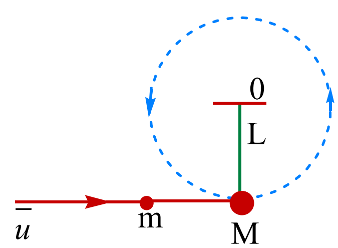
$$\text{and normal reaction } N = mg\cos\theta - \frac{mv^2}{r}$$

♦ A ball of mass 'M' is suspended vertically by a string of length 'L'.

A bullet of mass 'm' is fired horizontally with a velocity 'u' onto the ball, sticks to it.

For the system to complete the vertical circle, the minimum value of 'u' is given by

$$u = \frac{(M+m)}{m}\sqrt{5gL}$$

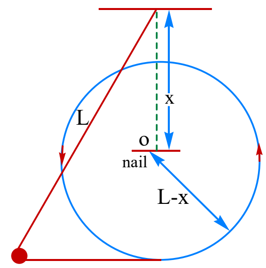


♦ A nail is fixed at a certain distance 'x' vertically below the point of suspension of a simple pendulum of length L.

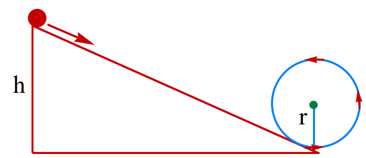
The bob is released when the string makes an angle θ with vertical.

The bob reaches the lowest position then describes a vertical circle whose centre coincides with the nail. Then

$$x_{\min} = \frac{L(3+2\cos\theta)}{5}$$



♦ A body of mass 'm' is allowed to slide down from rest, from the top of a smooth incline of height 'h'. For the body to move in loop of radius 'r' on arriving at the bottom.



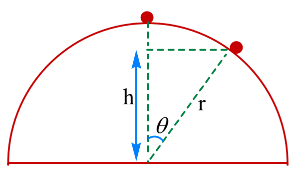
a) Minimum height of smooth incline $h = \left(\frac{5r}{2}\right)$

b)'h' is independent of mass of the body

♦ A small body is freely sliding down from the top of a smooth convex-hemisphere of radius r, placed on a table with its flat face on the table then

a) Normal reaction on the body is zero at the instant the body leaves the hemisphere.

b) The vertical height from table a which the body leaves the hemisphere is h = 2r/3



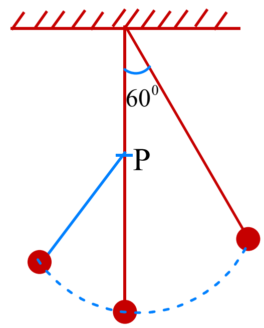
If the position vector of the body with respect to the centre of curvature makes an angle θ with vertical when the body leaves the hemisphere, then cos θ=2/3

d) velocity of block at that instant is $V = \sqrt{\frac{2gr}{3}}$

e) If the block is given a horizontal velocity 'u' from the top of the smooth convex-hemisphere then the angle θ with vertical at which the block leaves hemisphere is $\cos\theta = \frac{2}{3} + \frac{u^2}{3gr}$

: : PROBLEMS : :

1. **A nail is located at certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle 60^0 from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolutions with the nail as the centre. Assume the length of the pendulum to be 1m.**



SOLUTION :

Velocity of bob at lowest position

$$V = \sqrt{2g\ell(1-\cos\theta)}$$

$$= \sqrt{2g \times \ell(1-\cos 60^0)} = \sqrt{2g\frac{\ell}{2}} = \sqrt{g\ell} ...(1)$$

Let 'd' be the distance of nail from the point of suspension. The bob will have to complete the circle of radius r $= l - d$

To complete vertical circle

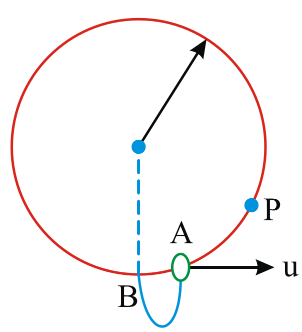
$$V_{\min} = \sqrt{5gr} = \sqrt{5g(1-d)} - (2)$$

Equating, equations (1) and (2), we get

$$\sqrt{gl} = \sqrt{5g(1-d)} \Rightarrow d = \frac{4l}{5} = \frac{4}{5} = 0.80m$$

PASSAGE

A bead of mass m is threaded on a smooth circular wire centre O, radius a, which is fixed in vertical plane. A light string of natural length 'a', elastic constant $= \frac{3mg}{a}$ and breaking strength 3mg connects the bead to the lowest point A of the wire. The other end of the string is fixed to ring at point B near point A. The string is slaked initially. The bead is projected from A with speed u.



2. The smallest value u_0 of u for which the bead will make complete revolutions of the wire will be

A) $u_0 = \sqrt{5ga}$ **B)** $u_0 = \sqrt{6ga}$ **C)** $u_0 = \sqrt{7ga}$ **D)** $u_0 = 2\sqrt{ga}$

SOLUTION :

When particle is at highest position, the elastic force is downwards

$$F_l = \frac{3mg}{a}(2a - a) = 3mg$$

it v is velocity at height point at B

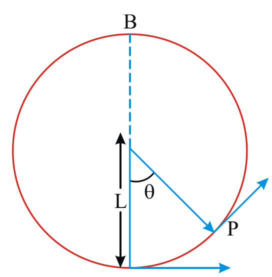
$$\frac{mu_0{}^2}{a} = F_l + mg - N$$

If V = 0, then KE at lowest point A will be $\frac{1}{2}mu_0^2 = [Elastic\ energy + gPE]at\ B$

$$= \frac{1}{2}\left(\frac{3mg}{a}\right)a^2 + mg2a$$

$$u_0^2 = 7ga$$

3. A bob of mass M is suspended by a massless string of length L. The horizontal velocity υ at just sufficient ot make it reach the point B.The angle θ at which the speed of the bob is half of that A, satisfies [IIT-2008]



A) $\theta = \frac{\pi}{4}$ **B)** $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ **C)** $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ **D)** $\frac{3\pi}{4} < \theta < \pi$

SOLUTION :

As the body just reaches the topmost point B, therefore , $\upsilon_A = \sqrt{5gL}$ and $\upsilon_B = \sqrt{gL}$

Let at point P having angular displacement θ the speed becomes half of the initial value υ_A.

Using the law of conservation of energy

Energy at A=Energy at B

$$\frac{1}{2}m\upsilon^2 A = \frac{1}{2}m\upsilon^2 P + mgL(1 - \cos\theta)$$

$$\frac{1}{2}m\left(\upsilon_A{}^2 = \upsilon_P{}^2\right) = mgL(1 - \cos\theta)$$

$$\frac{1}{2}m\left(5gL - \frac{5gL}{4}\right) = mgL(1 - \cos\theta)$$

$$\frac{15}{8} = (1 - \cos\theta) \quad ; \quad \cos\theta = -\frac{7}{8}$$

Which means $\frac{3\pi}{4} < \theta < \pi$

4. If $v = 2u_0$, the tension T in the elastic string when the bead is at the highest point B of the wire is

A) $\frac{3mu_0^2}{a}$ **B)** $4mg$ **C)** $2mg$ **D)** $\left(\frac{4u_0^2}{a} - g\right)m$

SOLUTION :

When $v = 2v_0 = 2\sqrt{7ga}$, For point B

$$\frac{m(2v_0)^2}{a} = 3mg + mg - N$$

$$\frac{4mv_0^2}{a} = 4mg - N$$

$$N = (4 - 28)mg = -24 \text{ mg}$$

negative sign denote it acts downwards and adds to tension total tension in string T = 3mg+N

$$T = \left(\frac{4v_0^2}{a} - g\right)m$$

5. The elastic energy stored in the string when the bead is at the highest point B will be

A) $\frac{3mga}{2}$ **B)** $2mga$ **C)** $4mga$ **D)** $\frac{2mga}{2}$

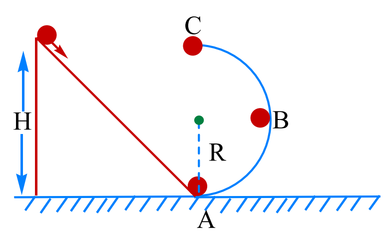
SOLUTION :

.

Elastic PE stored in the string

$$\frac{1}{2}\left(\frac{3mg}{a}\right)a^2 = \frac{3}{2}mga$$

6. A body slides without friction from a height H=60 cm and then loops the loop of radius R=20cm at the bottom of an incline. Find the ratio of forces exerted on the body by the track at the positions A,B and C ($g = 10\ ms^{-2}$)



SOLUTION :

From data H=3R

Velocity at A, $V_A = \sqrt{2gH} = \sqrt{2g(3R)} = \sqrt{6gR}$

Velocity at B, $V_B = \sqrt{4gR}$

$$\text{Velocity at C, } V_C = \sqrt{2gR}$$

$$\text{Reaction force at A} = R_1 = \frac{mV_A^2}{R} + mg\cos\left(0^0\right)$$

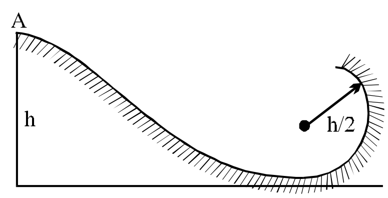
$$= \frac{m \times 6gR}{R} + mg = 7mg$$

$$\text{Reaction force at B} = R_2 = \frac{mV_B^2}{R} + mg\cos\left(90^0\right) = \frac{m \times 4gR}{R} + 0 = 4mg$$

$$\text{Reaction force at C} = R_3 = \frac{mV_C^2}{R} + mg\cos\left(180^0\right) = \frac{m \times 2gR}{R} - mg = mg$$

$$\therefore R_1 : R_2 : R_3 = 7 : 4 : 1$$

7. A small body A starts sliding from the height h down an inclined groove passing into a half - circle of radius h/2 (see figure). Assuming the friction to be negligible, find the velocity of the body at the highest point of its trajectory After breaking off the groove).

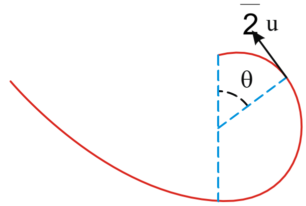


A) $\sqrt{\frac{9}{27}gh}$ **B)** $\sqrt{\frac{8}{27}gh}$ **C)** $\sqrt{\frac{27}{8}gh}$ **D)** $\sqrt{\frac{10}{27}gh}$

SOLUTION :

v^2 At end of track) = 2gh.

Let body break at angle θ the $\frac{mu^2}{\frac{h}{2}} = mg\cos\theta$. (1)



$$u^2 = v^2 - 2g\frac{h}{2}(1+\cos\theta)$$

solving $\cos\theta = \frac{2}{3}$ & $u = \sqrt{\frac{2}{3}gh}$.

v at highest pt is

$$u\cos\theta = \sqrt{\frac{2}{3}gh} \times \frac{2}{3} = \sqrt{\frac{8}{27}gh}$$

8. A hemispherical vessel of radius R moving with a constant velocity v_0 and containing a ball, is suddenly haulted. Find the height by which ball will rise in the vessel, provided the surface is smooth:

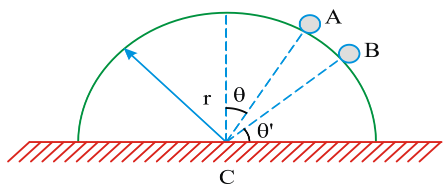
A) $\frac{v_0^2}{2g}$ **B)** $\frac{2v_0^2}{g}$ **C)** $\frac{v_0^2}{g}$ **D) none of these**

SOLUTION :

$$\frac{1}{2}mv_0^2 = mgR(1-\cos\theta)$$

$$\frac{v_0^2}{2g} = R - R\cos\theta = \text{required height}$$

9. A particle of mass m initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown. The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating θ and θ' is



A) $3\sin\theta = 2\cos\theta'$ **B)** $2\sin\theta = 3\cos\theta'$ **C)** $3\sin\theta' = 2\cos\theta$ **D)** $2\sin\theta = 3\cos\theta'$

SOLUTION :

Let v be the speed of particle at B when it is about to loose contact $\frac{mv^2}{r} = mg\sin\theta'$

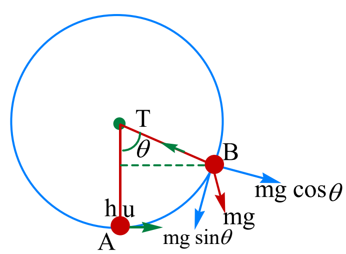
Applying conservation of energy

$$\frac{1}{2}mv^2 = mg(r\cos\theta - r\sin\theta')$$

$$3\sin\theta' = 2\cos\theta$$

10. A heavy particle hanging from a fixed point by a light inextensible string of length ℓ is projected horizontally with a speed of $\sqrt{g\ell}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion, when the tension in the string is equal to the weight of the particle.

SOLUTION :



$$\mu = \sqrt{gl}$$

Let T=mg at an angle 'θ' as shown in the

figure and $h = \ell(1-\cos\theta) \to (i)$

Applying law of conservation of mechanical energy between the points A and B,

$$\text{we get } \frac{1}{2}m\left(u^2 - v^2\right) = mgh$$

$$\text{Here } u^2 = g\ell \rightarrow (ii)$$

$$\text{and } v^2 = u^2 - 2gh \rightarrow (iii)$$

$$\text{Further } T - mg\cos\theta = \frac{mv^2}{\ell}\,(T = mg)$$

$$v^2 = g\ell(1-\cos\theta) \rightarrow (iv)$$

From equations (i), (iii) and (iv)

$$\text{we have } g\ell(1-\cos\theta) = g\ell - 2g\ell(1-\cos\theta)$$

$$\cos\theta = \frac{2}{3}; \qquad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\text{From equation (iv) } v = \sqrt{\frac{g\ell}{3}}$$

11. A bob of mass m is suspended from a fixed support with a light string and the system with bob and support is moving with a uniform horizontal acceleration. The breaking strength of the string is $mg\sqrt{2}$. Find the workdone by the tension in the string in the first one second:

A) $2mg^2$ **B)** $\frac{mg^2}{\sqrt{2}}$ **C)** $\frac{mg^2}{2}$ **D)** $mg^2\sqrt{2}$

SOLUTION :

$$T\sin\theta = ma \text{ and } T\cos\theta = mg$$

$$\text{So, } a = g\tan\theta$$

$$\text{Now, } T = \frac{mg}{\cos\theta} = mg\sqrt{2} \text{ (given)}$$

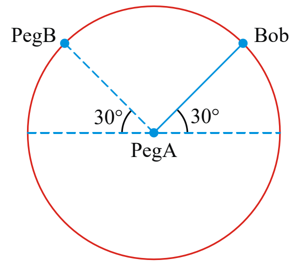
$$\cos\theta = \frac{1}{\sqrt{2}}, \text{ or } \theta = 45^0$$

$$\text{and } a = g\tan\theta = g\tan 45^0 = g$$

$$\text{workdone, } W = T.S = T\left(\frac{1}{2}at^2\right)\sqrt{2}$$

$$= \left(mg\sqrt{2}\right)\left(\frac{1}{2}\right)(g)(1)^2\sqrt{2} = mg^2/2$$

12. A bob attached to one end of a string, other end of which is fixed at peg A. The bob is taken to a position where string makes an angle of 30^0 with the horizontal. On the circular path of the bob in vertical plane there is a peg 'B' at a symmetrical position with respect to the position of release as shown in the figure. If V_c and V_a be the minimum speeds in clockwise and anticlock wise directions respectively, given to the bob in order to hit the peg 'B' then ratio $V_c : V_a$ is equal to



A) 1:1 B) $1:\sqrt{2}$ C) $1:2$ D) 1:4

SOLUTION :

For complete circular motion speed at highest point in $\sqrt{gR}$

Apply conservation energy $v_a = \sqrt{2gR}$

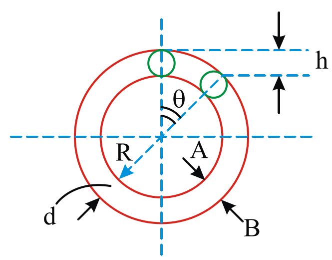
For clock wise motion

$$T + mg\cos 60 = \frac{mv_c^2}{R}$$

v_C to minimum T=0 ;

$$v_C = \sqrt{\frac{gR}{2}}$$

13. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B as shown. The sphere A has radius R and sphere B has a radius $(R+d)$. All surfaces are smooth. The diameter of ball is slightly less than d. The ball is given a gentle push so that angle made by radius vector of the ball with vertical is θ. N_A and N_B are the magnitudes of normal reaciton forces on the ball exerted by spheres A and B respectively:



Match the columns:

Column-I

A) $\theta \le \cos^{-1}\left(\frac{2}{3}\right)$ B) $\theta \le \cos^{-1}\left(\frac{3}{4}\right)$ C) $\theta \ge \cos^{-1}\left(\frac{3}{4}\right)$ D) $\theta \ge \cos^{-1}\left(\frac{2}{3}\right)$

Column-II

p) N_B =0 and $N_A = mg(3\cos\theta - 2)$ q) N_B =0 and $N_A = mg(4\cos\theta - 2)$

r) N_A =0 and $N_B = mg(2 - 3\cos\theta)$ s) none of these

SOLUTION :

A-P, B-S, C-S, D-R

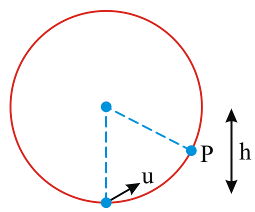
$h = \left(R + \frac{d}{2}\right)(1 - \cos\theta)$ $v^2 = 2gh$

or $N = mg(3\cos\theta - 2)$

Ball will lose contact with sphere A, when N = 0 $3\cos\theta - 2 = 0$

PASSAGE-

A particle of mass M attached to an inextensible string is moving in a vertical circle of radius. R about fixed point O. It is imparted a velocity u in horizontal direction at lowest position as shown in figure.



Following information is being given

i) Velocity at a height h can be calculated by using formula $v^2 = u^2 - 2gh$

ii) Particle will complete the circle if $u \geq \sqrt{5gR}$

iii) Particle will oscillates in lower half $(0^0 < \theta \leq 90^0)$ **if** $0 < u \leq \sqrt{2gR}$

iv) The magnitude of tension at a height 'h' is calculated by using formula $T = \frac{M}{R}\left[u^2 + [gR - 3gh]\right]$

14. If $R = 2m, M = 2kg$ **and** $u = 12m/s$**. Then value of tension at lowest position is**

A) 120 N   B) 164 N   C) 264 N   D) zero

SOLUTION :

Put h = 0   T = 164 N

15. Tension at highest point of its trajectory in above question will be

A) 100 N   B) 44 N   C) 144 N   D) 264 N

SOLUTION :

Put h = 2R T= 144 N

16. If $M = 2kg, R = 2m$ **and** $u = 10m/s$**. Then velocity of particle when** $\theta = 60^0$ **is**

A) $2\sqrt{5}\ m/s$ **B)** $4\sqrt{5}\ m/s$ **C)** $5\sqrt{2}\ m/s$ **D)** $5\ m/s$

SOLUTION :

At $\theta = 60^0$ $h = R - R\cos 60 = \frac{R}{2}$

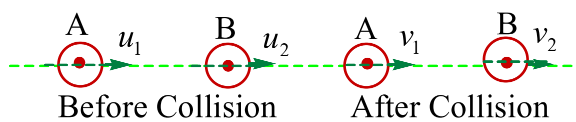
Put $h = \frac{R}{2}$ in $v^2 - u^2 = 2gh$

Collisions

- The strong interaction among bodies involving exchange of momentum in a short interval of time is called collision.
- During collision bodies may or may not come into physical contact.

  Ex:
  In the collision of α particle with nucleus, due to coulombic repulsive forces α particle is scattered away without any physical contact.
- Based on the direction of motion of colliding bodies, collisions are classified into
  (i) Head on or one dimensional collision
  (ii) oblique collision

Head on (or) one dimensional collision



- It is the collision in which the velocities of the colliding bodies are confined to same straight line before and after collision.

Oblique Collision:
- It is the collision in which the velocities of the colliding bodies are not confined to same straight line before and after collision.
- Oblique collision may be two dimensional or three dimensional.
- When a particle hits elastically and obliquely another stationary particle of same mass, then they move perpendicular to each other after collision.

Types of Collision: Based on conservation of kinetic energy collisions are classified into
(i) Elastic Collision
(ii) Inelastic collision

Elastic Collision:
It is the collision in which both momentum and kinetic energy are conserved. Forces involved during collision are conservative in nature

Ex:
1. Collision between atomic particles.
2. Collision between two smooth billiard balls.
3. Collision of α particle with nucleus.

Inelastic collision:
It is the collision in which momentum is conserved but not kinetic energy. Some or all the forces involved during collision are non conservative.

Ex:
Collision between two vehicles.

Perfectly inelastic collision:
It is the collision in which the colliding bodies stick together and move as a single body after collision
- In perfectly inelastic collision the momentum remains conserved but the loss of kinetic energy is maximum.

Ex:
A bullet is fired into a wooden block and remains embedded in it.

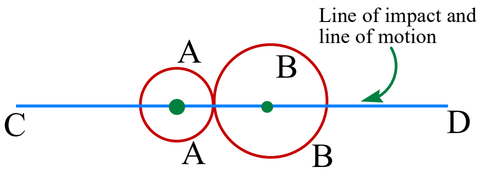
Line of impact:
The line passing through the common normal to the surfaces in contact during impact is called

line of impact.

The force during collision acts along this line on both bodies.

EX-1:

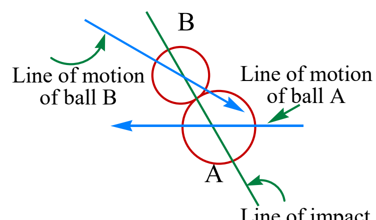
Two balls A and B are approaching each other such that their centres are moving along line CD.



Head on Collision

EX-2:

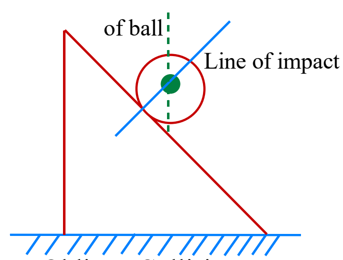
Two balls A and B are approaching each other such that their centres are moving along dotted lines as shown in figure.



Oblique collision

Ex3:

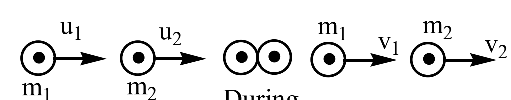
Ball is falling on a stationary wedge Line of motion



Oblique Collision

Elastic collision in one dimension:

When two particles of masses m_1 and m_2 are moving along the line joining their centers with velocities u_1 and u_2 ($u_1 > u_2$) before collision. Then v_1 and v_2 are their velocities after collision



From the conservation of linear momentum $m_1\left(\vec{u_1} - \vec{v_1}\right) = m_2\left(\vec{v_2} - \vec{u_2}\right)$

From Law of conservation of K.E $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$\therefore \vec{u_1} - \vec{u_2} = \vec{v_2} - \vec{v_1}$$

i.e Relative velocity of approach before collision = Relative velocity of separation after collision

Velocities after collision are

- $\vec{v_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{u_2}$
- $\vec{v_2} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{u_1} + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{u_2}$

Special cases:

1)If colliding particles have equal masses

i.e $m_1 = m_2 = m$;

$\boxed{\vec{v}_1 = \vec{u}_2}, \boxed{\vec{v}_2 = \vec{u}_1}$

2) If two bodies are of equal masses and the second body is at rest

ie., $m_1 = m_2 = m \ and \ \vec{u_2} = \vec{0}$

then $\boxed{\vec{v}_1 = \vec{0}}; \boxed{\vec{v}_2 = \vec{u}_1}$

3)A lighter particle collides with heavier particle which is at rest $m_1 <<< m_2$, $\vec{u_2} = \vec{0}$

$\boxed{\vec{v}_1 = \vec{u}_1}; \quad \boxed{\vec{v}_2 = \bar{0}}$

4)A heavier body collides with lighter body at rest

$m_1 >>> m_2$, $\vec{u}_2 = \vec{0}$;

$\boxed{\vec{v}_1 = \vec{u}_1}; \quad \boxed{\vec{v}_2 = 2\vec{u}_1}$

Applications:

- **A body of mass m_1 moving with a velocity v_1 collides elastically with a stationary mass m_2**

1)Velocity of first body after collision $\vec{v_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1}$

2)Velocity of second body after collision $\vec{v_2} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{u_1}$

3)KE of first body after collision (or) KE retained by first body

$$K.E_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$$

$$K.E_{net} = \frac{1}{2}m_1u_1^2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = KE_i\left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2$$

4)Fraction of KE retained by 1st body $\dfrac{K.E_{ret}}{K.E_i} = \left[\dfrac{m_1 - m_2}{m_1 + m_2}\right]^2$

5)KE of second body after collision (or) KE transferred to the second body

$$KE_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\left(\frac{2m_1}{m_1 + m_2}\right)^2 u_1^2$$

$$KE_2 = \left(\frac{4m_1m_2}{(m_1+m_2)^2}\right)\left(\frac{1}{2}m_1u_1^2\right)$$

$$KE_{tra} = \left(\frac{4m_1m_2}{(m_1+m_2)^2}\right)KE_i$$

6)Fraction of KE transferred form 1st body to second body (or) Fraction of KE lost by 1st body is

$$\frac{KE_{tra}}{KE_i} = \frac{4m_1m_2}{(m_1+m_2)^2}$$

7)Fraction of momentum retained by m_1 $\frac{P_1}{P_i} = \frac{m_1v_1}{m_1u_1} = \frac{m_1-m_2}{m_1+m_2}$

8)Fraction of momentum transferred from 1st body to second body

$$\frac{P_2}{P_i} = \frac{P_i - P_1}{P_i} 1 - \frac{P_1}{P_i} = 1 - \left(\frac{m_1-m_2}{m_1+m_2}\right) = \frac{2m_2}{m_1+m_2}$$

Coefficient of restitution

Newton introduced a dimensionless parameter called the coefficient of restitution (e) to measure the elasticity of collision.

It is defined as the ratio of the relative velocity of separation to the relative velocity of approach of the two colliding bodies

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{|\vec{v_2} - \vec{v_1}|}{|\vec{u_1} - \vec{u_2}|}$$

This formula is applied along the line of impact.

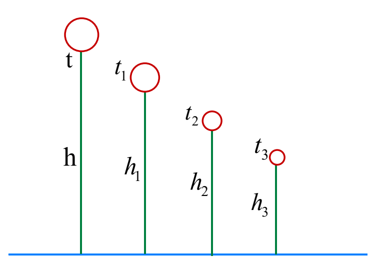
Here the velocities mentioned in the expression should be taken along the line of impact.

- ♦ For a perfectly elastic collision e = 1
- ♦ For an inelastic collision 0 < e < 1
- ♦ For completely inelastic collision e = 0

♦ A body dropped freely from a height ‘h’ strikes the floor the rebounds to a height h_1

$$e = \sqrt{\frac{h_1}{h}}$$

after nth rebound $h_n = e^{2n}h$



♦ When a freely falling ball strikes the ground with a velocity ‘v’ and rebounds with a velocity v_1 then

$$e = \frac{v_1}{v}$$

after n[th] rebound $V_n = e^n V$

♦ Total distance travelled by the ball before it stops bouncing

$$d = h + 2h_1 + 2h_2 + 2h_3 + ...$$

$$= h + 2e^2 h + 2e^4 h + 2e^6 h + ...$$

$$= h + 2e^2 h\left[1 + e^2 + e^4 + ...\right]$$

$$d = h\left[\frac{1+e^2}{1-e^2}\right]$$

♦ Total time taken by the ball to stop bouncing

$$T = t + 2t_1 + 2t_2 + 2t_3 + ...$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + 2\sqrt{\frac{2h_3}{g}} + ...$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\left[1 + e + e^2 + ...\right]$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\left[1 + e + e^2 + ...\right]$$

♦ Average speed of the ball during its entire journey is given by

$$Average\ speed = \frac{Total\ dis\tan ce\ travelled}{Total\ time\ taken}$$

$$= \frac{h\left[\frac{1+e^2}{1-e^2}\right]}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]} = \sqrt{\frac{gh}{2}}\frac{\left(1+e^2\right)}{\left(1+e\right)^2}$$

♦ Average velocity of the ball during its entire journey is given by

$$Average\ velocity = \frac{Net\ displacement}{Total\ time\ taken}$$

$$= \frac{h}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]} = \sqrt{\frac{gh}{2}}\frac{(1-e)}{(1+e)}$$

♦ Change in momentum in 1[st] collision

$$\Rightarrow mv_1 - \left(-mu\right) = \left(mv_1 + mu\right)$$

$$\Rightarrow meu + mu = mu\left(1+e\right)$$

Change in momentum in 2[nd] collision

$$\Rightarrow m\left(v_2 + v_1\right) = m\left(e^2 u + eu\right) = meu\left(1+e\right)$$

Total change in momentum before it stops is

$$\Delta p = mu\left(1+2e+2e^2+........\right) \quad \left[u=\sqrt{2gh}\right]$$

$$= mu\left[\frac{1+e}{1-e}\right] = m\sqrt{2gh}\left[\frac{1+e}{1-e}\right]$$

♦ Distance travelled before second impact is $d_2 = h + 2h_1 = h\left(1+2e^2\right)$

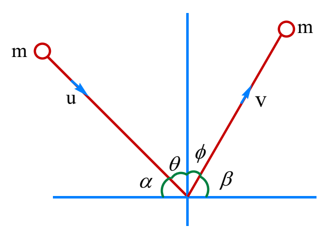
♦ Distance travelled before third impact is $d_3 = h + 2h_1 + 2h_2 = h\left(1+2e^2+2e^4\right)$

♦ Time taken for second impact is $t_2 = t + 2t_1 = \sqrt{\frac{2h}{g}}\left(1+2e\right)$

♦ Time taken for third impact is $t_3 = t + 2t_1 + 2t_2 = \sqrt{\frac{2h}{g}}\left(1+2e+2e^2\right)$

Application

♦ A particle of mass m moving with a speed u strikes a smooth horizontal surface at an angle α. The particle rebounds at an angle β with a speed v. The coefficient of restitution is 'e'.



Since no external impulse acts in the horizontal direction, momentum of the ball is conserved in the horizontal direction.

$$mu\cos\alpha = mv\cos\beta$$

$$u\cos\alpha = v\cos\beta..........(1)$$

By def of coefficient of restitution we get

$$eu\sin\alpha = v\sin\beta \quad -(2)$$

from (1) and (2), $\tan\beta = e\tan\alpha$ $\boxed{\tan\alpha = \frac{\tan\beta}{e}}$

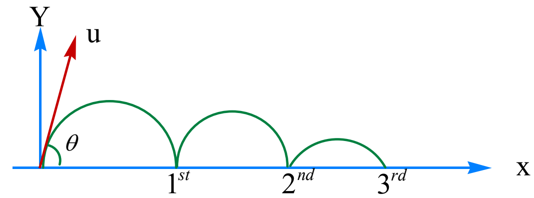
On squaring eq (1) and (2) and adding we get

$$v^2 = u^2\left(\cos^2\alpha + e^2\sin^2\alpha\right)$$

$$v = u\sqrt{\cos^2\alpha + e^2\sin^2\alpha}$$

♦ A ball is projected with an initial velocity u at an angle θ to the horizontal surface.

If 'e' is the coefficient of restitution between the ball and the surface then



1)Time taken for 1st collision, $T = \dfrac{2u\sin\theta}{g}$

2)Time interval between 1st and 2nd collisions,

$$T_1 = \frac{2v_1\sin\theta}{g} \quad (\because v_1 = eu)$$

$$T_1 \frac{2(eu)\sin\theta}{g} = eT$$

3)Time interval between 2nd and 3rd collisions,

$$T_2 = \frac{2v_2\sin g\theta}{g} = \frac{2(e^2u)\sin\theta}{g} = e^2T \quad (\because v_2 = e^2u)$$

4)The total time of flight is

$$T^1 = T + T_1 + T_2 + \ldots\ldots\ldots = T + eT + e^2T + e^3T + \ldots\ldots.$$

$$= T\left[1 + e + e^2 + e^3 + \ldots\ldots.\right]$$

$$\boxed{T^1 = \frac{T}{1-e}}$$

If collision is elastic, e = 1 the $T^1 = \infty$

5)The horizontal distance covered by the ball before 1st collision is

$$R = \frac{u^2\sin 2\theta}{g} = u\cos\theta \times T$$

6)The horizontal distance covered by it between 1st and 2nd collisions, $R_1 = u\cos\theta \times eT = eR$

7)horizontal distance covered between 2nd and 3rd collisions, $R_2 = u\cos\theta \times e^2T = e^2R$

8)Total horizontal distance covered by the ball is

$$R^1 = R_0 + R_1 + R_2 + R_3 + \ldots\ldots\ldots$$

$$= R + eR + e^2R + \ldots\ldots\ldots = R\left[1 + e + e^2 + \ldots..\right]$$

$$\boxed{R^1 = \frac{R}{1-e}}$$

For perfectly elastic collision e = 1 and $R^1 = \infty$

9)The maximum height reached by the ball before 1st collision

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g}$$

10)Maximum height it reaches between 1st and 2nd collisions is

$$H^1 = \frac{(eu\sin\theta)^2}{2g} = e^2 H$$

11)The sum of maximum heights reached by the ball is

$$H^1 = H + H_1 + H_2 + ... = H + e^2 H + e^4 H + ...$$

$$= H\left[1 + e^2 + e^4 + ...\right], \quad H^1 = \frac{H}{1-e^2}$$

If the collision is elastic e = 1 and H'= ∞

Head on inelastic collision

Two bodies of masses m_1 and m_2 moving with initial velocities $\vec{u_1}$ and $\vec{u_2}$ $(\vec{u_1} > \vec{u_2})$ collide.

After collision two bodies will move with velocities $\vec{V_1}$ and $\vec{V_2}$.

From Law of conservation of linear momentum $m_1(\vec{u}_1 - \vec{v}_1) = m_2(\vec{v}_2 - \vec{u}_2)$

By the definition of coefficient of restitution $\vec{v}_2 - \vec{v}_1 = e(\vec{u}_1 - \vec{u}_2)$

$$\vec{v}_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)\vec{u}_1 \left(\frac{(1+e)m_2}{m_1 + m_2}\right)\vec{u}_2$$

$$\vec{v}_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)\vec{u}_2$$

♦ If $m_1 = m_2 = m, u_2 = 0$ then

$$v_1 = (1-e)\frac{u_1}{2}; \quad v_2 = (1+e)\frac{u_1}{2}$$

$$\boxed{\frac{v_1}{v_2} = \frac{1-e}{1+e}}$$

Loss of kinetic energy of the system:

$$\Delta KE = KE_I - LE_F$$

$$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{u}_1 - \vec{u}_2)^2(1-e^2)$$

In case of perfectly in-elastic collision, e = 0

$\therefore$ loss in KE of system is

$$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{u}_1 - \vec{u}_2)^2$$

♦ If two bodies are approaching each other then loss in KE of the system is maximum

$$\Delta KE_{\max} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 + u_2)^2$$

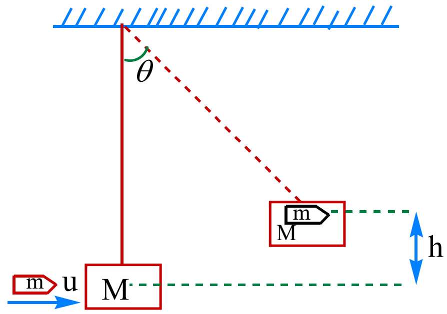
Ballistic pendulum:

It is an arrangement used to determine the velocities of bullets.

A log of wood of mass 'M' is suspended by a string of length $'l'$ as shown in the figure. A bullet of mass 'm' is fired horizontally into the wooden block with a velocity 'u'

Case I:

Let the bullet gets embedded in the block and system rises to a height 'h' as shown in the figure.



From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$mu + 0 = (m+M)v \Rightarrow v = \frac{mu}{m+M} \text{..........}(1)$$

KE of the system after collision is given by

$$KE = \frac{1}{2}(m+M)v^2$$

PE at highest point = (m + M) gh

$$\text{From LCE, } \frac{1}{2}(m+M)v^2 = (m+M)gh$$

$$v^2 = 2gh (or) v = \sqrt{2gh} \text{...........}(2)$$

From (1) and (2) velocity of the bullet

$$u = \frac{M+m}{m}\sqrt{2gh} = \frac{M+m}{m}\sqrt{2gl(1-\cos\theta)}$$

Loss in KE of the system = KE_1 - KE_2

$$\Delta KE = \frac{1}{2}mu^2 = \frac{1}{2}(m+M)v^2$$

$$\Delta KE = \frac{1}{2}\left[mu^2 - (m+M)\frac{m^2u^2}{(m+M)^2}\right]$$

$$\Delta KE = \frac{1}{2}\left[\frac{mM}{m+M}\right]u^2$$

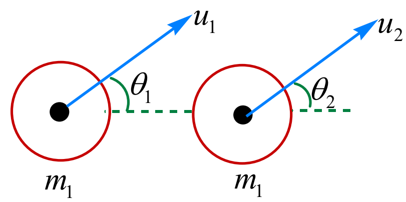
Case II:

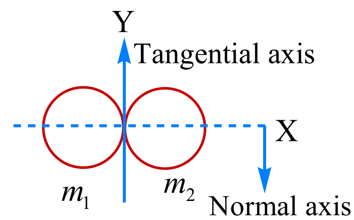
If the bullet emerges out of the block with velocity 'v' then

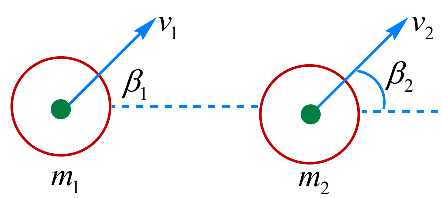
$$mu = mv + MV \ Where \ V = \sqrt{2gh}$$

Collisions in two dimensions (oblique collisions)

1. A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles changes along common normal direction.
2. No component of impulse acts along common tangent direction. Hence, linear momentum (or) linear velocity of individual particles remains unchanged along this direction.
3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.
4. Definition of coefficient of restitution can be applied along common normal direction.







From law of conservation of linear momentum along x - axis:

$$m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\beta_1 + m_2 v_2 \cos\beta_2$$

Along y - axis:

$$m_1 u_1 \sin\theta_1 + m_2 u_2 \sin\theta_2 = m_1 v_1 \sin\beta_1 + m_2 v_2 \sin\beta_2$$

Coefficient of restitution $e = -\dfrac{v_1 \cos\beta_1 - v_2 \cos\beta_2}{u_1 \cos\theta_1 - u_2 \cos\theta_2}$

: : PROBLEMS : :

1. A bullet of mass 'm' moving at a speed 'v' hits a ball of mass 'M' kept at rest. A small part having mass m_1 breaks from the ball and sticks to the bullet. The remaining ball is found to move at a speed v_2 in the direction of the bullet. Find the velocity of the bullet after the collision.

SOLUTION :

Mass of bullet = m and speed = v.

Mass of the ball M and fractional mass of the ball m_1 According to law of conservation of linear momentum

$$mv + 0 = (m + m_1)\, v_1 + (M - m_1)\, v_2$$

Where v_1 = final velocity of the (bullet + fractional mass)

$$v_1 = \frac{mv - (M - m_1)}{(m + m_1)} v_2$$

2. Two bodies of masses m_1 and m_2 are moving with velocities $1 ms^{-1}$ and $3 ms^{-1}$ respectively in opposite directions. If the bodies undergo one dimensional elastic collision, the body of mass m_1 comes to rest. Find the ratio of m_1 and m_2

SOLUTION :

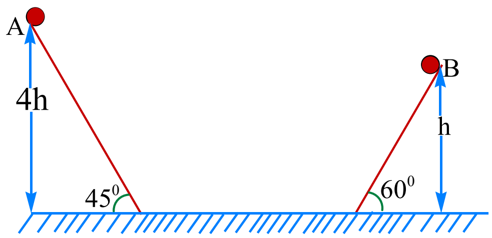
$$u_1 = 1 m/s, u_2 = -3 m/s, v_1 = 0$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

$$0 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) 1 + \left(\frac{2m_2}{m_1 + m_2}\right)(-3)$$

$$m_1 - m_2 = 6m_2; m_1 = 7m_2; \frac{m_1}{m_2} = \frac{7}{1}$$

3. Two identical balls A and B are released from the positions as shown in the figure. They collide elastically on the horizontal portion. The ratio of heights attained by A and B after collision (neglect friction



SOLUTION :

As mass of two balls are equal, they exchange their velocities after collision.

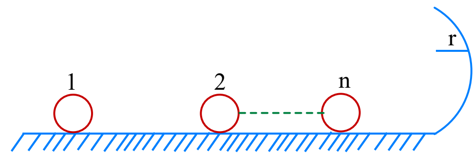
$$u_A = \sqrt{2gh}, u_B = \sqrt{2g(4h)} = \sqrt{8gh}; h_A = \frac{u_A^2}{2g} = h;$$

$$h_B = h + \frac{v_B^2 \sin^2 60^0}{2g} = h + \frac{9h}{4} = \frac{13h}{4}$$

$$\left(\therefore (v_B)^2 - u_B^2 = -2gh \Rightarrow V_B^2 = u_B^2 - 2gh \Rightarrow v_B^2 = 6gh\right)$$

$$\frac{h_A}{h_B} = \frac{4}{13}$$

4. n elastic balls are placed at rest on a smooth horizontal plane which is circular at the end with radius 'r' as shown in the figure. The masses of the balls are $m, \frac{m}{2}, \frac{m}{2^2} \cdots\cdots\cdots \frac{m}{2^{n-1}}$ respectively. Find the minimum velocity that should be imparted to the first ball of mass 'm' such that the 'n^th' ball will complete the vertical circle.



SOLUTION :

Let speed to be imparted to the first ball be v_0. Consider the impact between the first two balls and v_1 and v_2 be the velocities of balls 1 and 2 after the impact respectively.

According to law of conservation of linear momentum $mv_0 = mv_1 + \frac{m}{2}v_2 \to (1)$

According to law of conservation of kinetic energy $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{m}{2}\right)v_2^2 \to (2)$

Solving equations (1) and (2), we get $v_2 = \frac{4}{3}v_0$

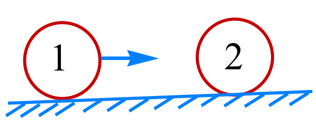
Similarly, for n^th ball $v_n = \left(\frac{4}{3}\right)^{n-1} v_0 \to (3)$

For the n^th ball to complete the vertical circular motion $v_n = \sqrt{5gr} \to (4)$

From equations (3) and (4), we have

$$\left(\frac{4}{3}\right)^{n-1} V_0 = \sqrt{5gr}; V_0 \left(\frac{3}{4}\right)^{n-1} \sqrt{5gr}$$

5. Ball 1 collides with an another identical ball 2 at rest as shown in the figure. For what value of coefficient of restitution e, the velocity of second ball become two time that of first ball after collision?



SOLUTION :

Here $m_1 = m_2 \ and \ u_2 = 0$

$$\text{After collision, } v_2 = \left(\frac{1+e}{2}\right)u \;\&\; v_1 = \left(\frac{1-e}{2}\right)u$$

$$\text{Given } v_2 = 2v_1; \left(\frac{1+e}{2}\right)u = 2\left(\frac{1-e}{2}\right)u$$

$$1+e = 2 - 2e \;;\; 3e = 1; \; e = \frac{1}{3}$$

6. A body 'A' with a momentum 'P' collides with another identical stationary body 'B' one dimensionally. During the collision, 'B' gives an impulse 'J' to the body 'A'. Then the coefficient of restitution is

SOLUTION :

From the law of conservation of linear momentum,

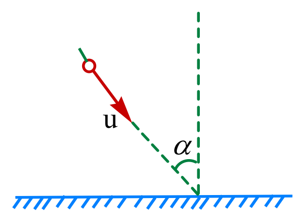
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$mu + m(0) = mv_1 + mv_2$$

$$\Rightarrow P - P_1 = P_2 \; where \; P_2 = J, (given)$$

$$\therefore e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{mv_2 - mv_1}{mu - 0} = \frac{P_2 - P_1}{P}$$

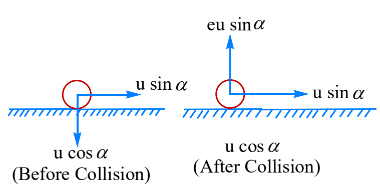
$$= \frac{P_2 - (P - P_2)}{P} = \frac{2P_2 - P}{P} = \frac{2J - P}{P} = \frac{2J}{P} - 1$$

7. A ball of mass m collides with the ground at an angle α with the vertical. If the collision lasts for time t, the average force exerted by the ground on the ball is: (e = coefficient of restitution between the ball and the ground)



SOLUTION :

Impulse = change in linear momentum.



$$\therefore Ft = m(eu\cos\alpha + u\cos\alpha) \; or \; F = \frac{mu\cos\alpha(1+e)}{t}$$

8. A ball strikes a horizontal floor at an angle $\theta = 45^0$ with the normal to floor. The coefficient of restitution between the ball and the floor is e = 1/2 . The fraction of its kinetic energy lost in the collision is

SOLUTION :

Let 'u' be the velocity of ball before collision. Speed of the ball after collision will become

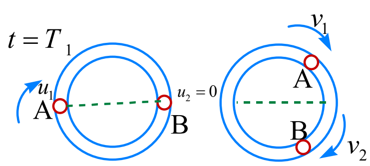
$$v=\sqrt{u^2\sin^2\theta+e^2u^2\cos^2}$$

$$=\sqrt{\left(\frac{u}{\sqrt{2}}\right)^2+\left(\frac{u}{2\sqrt{2}}\right)^2}=\sqrt{\frac{5}{8}}u$$

$\therefore$ Fraction of KE lost in collision

$$=\frac{\frac{1}{2}mu^2-\frac{1}{2}mv^2}{\frac{1}{2}mu^2}=1-\left(\frac{v}{u}\right)^2=1-\frac{5}{8}=\frac{3}{8}$$

9. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. At time t = 0, A is projected along the groove and it first impinges on B at time t = T_1 and again at time t = T_2. If 'e' is the coefficient of restitution, find the ratio of $\frac{T_2}{T_1}$



SOLUTION :

$$T_1=\frac{\pi R}{u_1}........(1)$$

$$\frac{v_2-v_1}{u_1}=e\Rightarrow v_2-v_1=eu_1$$

Time taken for A to collide with B again is

$$T_2-T_1=\frac{2\pi R}{v_2-v_1}\Rightarrow T_2-T_1=\frac{2\pi R}{eu_1}.......(2)$$

from (1) and (2) , $\frac{T_2}{T_1}=\frac{2+e}{e}$

10. After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the combined particle becomes half the initial speed of either particle. The angle between the velocities of the two before collision is

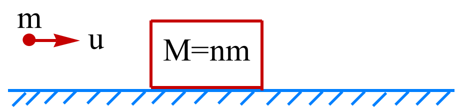
SOLUTION :

In perfectly inelastic collision between two particles, linear momentum is conserved. Let θ be the angle between the velocities of the two particles before collision. Then

$$P^2=P_1^2+P_2^2+2P_1P_2\cos\theta \qquad or$$

$$\left(2m\frac{v}{2}\right)^2=(mv)^2+(mv)^2+2(mv)(mv)\cos\theta \text{ or } 1=1+1+2\cos\theta \; or \; \cos\theta=-\frac{1}{2};(or)\theta=120^0$$

11. A bullet of mass 'm' moving with velocity 'u' passes through a wooden block of mass M= nm as shown in figure. The block is resting on a smooth horizontal floor. After passing through the block, velocity of the bullet becomes 'v'. Its velocity relative to the block is



SOLUTION :

Let v be the velocity of block. Then from conservation of linear momentum.

$$mu = mv + mnv \,(or)\, v = \left(\frac{u-v}{n}\right)$$

$\therefore$ Velocity of bullet relative to block will be

$$v_t = v - v' = v - \left(\frac{u-v}{n}\right) = \frac{(1+n)v-u}{n}$$

12. A block of mass 0.50 Kg is moving with a speed of 2.00 m/s on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. Find the energy loss during the collision (JEE MAIN 2008)

SOLUTION :

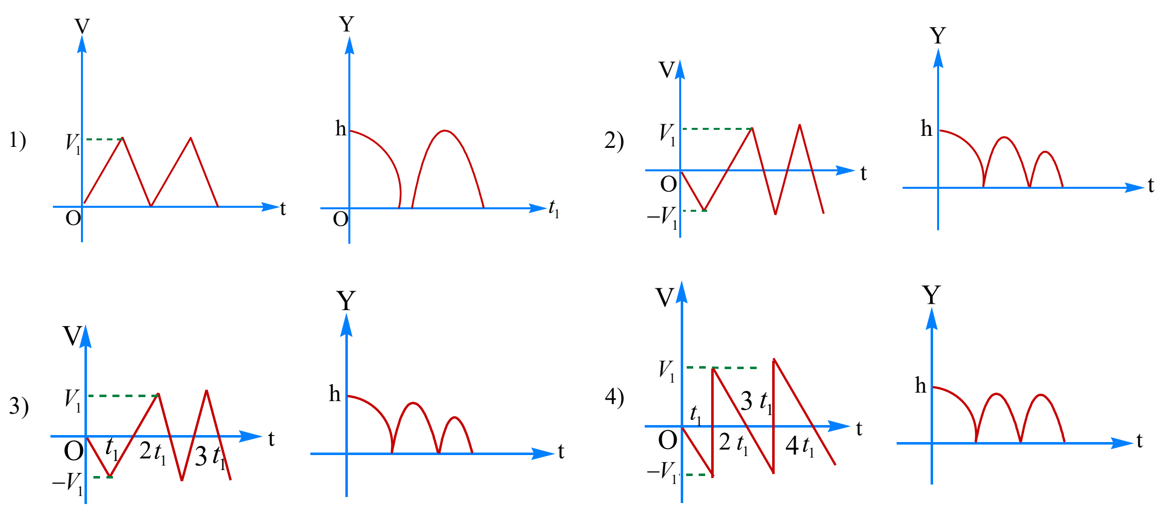
From LCLM, $m_1u_1 + m_2u_2 = (m_1+m_2)v$

$$0.50\times 2 + 1\times 0 = (0.5+1)v \Rightarrow v = \frac{2}{3}ms^{-1}$$

$\therefore$ Energy loss $\Delta KE = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1+m_2)v^2$

$$\Delta KE = \frac{1}{2}(0.5)(2)^2 - \frac{1}{2}(1.5)\left(\frac{2}{3}\right)^2 = 0.67J$$

13. Consider a rubber ball freely falling from a height h=4.9 m on a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be; (JEE MAIN 2009)



SOLUTION :

When ball strikes the surface its velocity will be reversed so correct option is (3)

14. A pendulum consists of a wooden bob of mass 'm' and of length *l*. A bullet of mass m_1 is fired towards the pendulum with a speed v_1 and it emerges out of the bob with a speed $\frac{v_1}{3}$. Find the initial speed of the bullet if the bob just completes the vertical circle.

SOLUTION :

From the Law of conservation of momentum

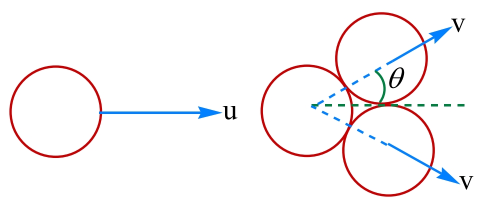
$$mv = m_1\left(v_1 - \frac{v_1}{3}\right) or\ v = \frac{m_1}{m} \times \frac{2v_1}{3}$$

To describe a vertical circle $v = \sqrt{5gl}$

$$\text{hence } \sqrt{5gl} = \frac{m_1}{m} \times \frac{2v_1}{3} \Rightarrow v_1 = \frac{m}{m_1} \times \frac{3\sqrt{5gl}}{2}$$

15. Two billiard balls of same size (radius r) and same mass are in contact on a billiard table. A third ball also of he same size and mass strikes them symmetrically and remains at rest after the impact. The coefficient of restitution between the balls is

SOLUTION :



$$\sin\theta = \frac{r}{2r} = \frac{1}{2}; \quad \therefore \theta = 30^0$$

From conservation of linear momentum

$$mu = 2mv\cos 30^0 \quad or \quad v = \frac{u}{\sqrt{3}}$$

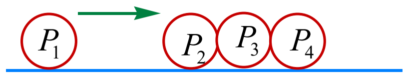
$$\text{Now } e = \frac{relative\ velocity\ of\ separation}{relative\ velocity\ of\ approach}$$

in common normal direction

$$\text{Hence, } e = \frac{v}{u\cos 30^0} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$$

: : THEORY BITS : :

1. A perfectly elastic ball P_1 of mass 'm' moving with velocity v collides elastically with three exactly similar balls P_1, P_2, P_3 lying on a smooth table, Velocity of the four balls after collision are



1) 0,0,0,0 2) v, v, v, v 3) v, v, v,0 4) 0, 0, 0, v

KEY:4

2. A bucket full of water is drawn up by a person. In this case the work done by the gravitational force is

1) negative because the force and displacement are in opposite directions
2) positive because the force and displacement are in the same direction
3) negative because the force and displacement are in the same direction
4) positive because the force and displacement are in opposite directions

KEY:1

3. A motor car of mass m travels with a uniform speed v on a convex bridge of radius r. When the car is at the middle point of the bridge, then the force exterted by the car on the bridge is

1) mg 2) $mg+\frac{mv^2}{r}$ 3) $mg-\frac{mv^2}{r}$ 4) $mg\pm\frac{mv^2}{r}$

KEY:3

4. A man is rowing a boat upstream and inspite of that the boat is found to be not moving with respect to the bank. The work done by the man is

1) zero 2) positive 3) negative 4) may be +ve or –ve

KEY:1

5. A ball of mass 'm' moving with a speed u undergoes a head – on elastic collision with a ball of mass 'nm' initially at rest. Find the fraction of the incident energy transferred to the heavier ball.

1) $\frac{n}{n+1}$ 2) $\frac{n}{(n+1)^2}$ 3) $\frac{2n}{(1+n)^2}$ 4) $\frac{4n}{(1+n)^2}$

KEY:4

6. A ball is thrown vertically upwards from the ground. Work done by air resistance during its time of flight is

1) positive during ascent and negative during descent
2) positive during ascent and descent
3) negative during ascent and positive during
4) negative during ascent and descent

KEY:4

7. Workdone by force of friction

1) can be zero 2) can be positive 3) can be negative 4) any of the above

KEY:4

8. Identify the non-conservative force in the following

1) weight of a body
2) force between two ions
3) magnetic force
4) air resistance

KEY:4

9. If x, F and U denote the displacement, force acting on and potential energy of particle, then

1) U= F 2) $F = +\frac{dU}{dx}$ 3) $F = -\frac{dU}{dx}$ 4) $F = \frac{1}{x}\left(\frac{dU}{dx}\right)$

KEY:3

10. In the case of conservative force

1) work done is independent of the path 2) work done in a closed loop is zero

3) work done against conservative force is stored is the form of potential energy

4) all the above

KEY:4

11. A body of mass 'm' moving with a constant velocity V hits another body of the same mass moving with the same velocity V but in opposite direction and sticks to it. The velocity of the compound body after the collision is

1) 2V 2) V 3) V/2 4) zero

KEY:4

12. When the momentum of a body is doubled, the kinetic energy is

1) doubled 2) halved

3) becomes four times 4) becomes three times

KEY:3

13. Internal forces can change

1) Kinetic Energy 2) Mechanical energy 3) Momentum 4) 1 and 2

KEY:4

14. If the momentum of a particle is plotted on X-axis and its kinetic energy on the Y-axis, the graph is a

1) straight line 2) parabola 3) rectangular hyperbola 4) circle

KEY:2

15. A 2 kg mass moving on a smooth frictionless surface with a velocity of $10ms^{-1}$ hits another 2kg mass kept at rest, in an inelastic collision. After collision, if they move together

1) they travel with a velocity of $5ms^{-1}$ in the same direction

2) they travel with a velocity of $10ms^{-1}$ in the same direction

3) they travel with a velocity of $10ms^{-1}$ in opposite direction

4) they travel with a velocity of $5ms^{-1}$ in opposite direction

KEY:1

16. When two identical balls are moving with equal speeds in opposite direction, which of the following is true? For the system of two bodies

1) momentum is zero, kinetic energy is zero 2) momentum is not zero, kinetic energy is zero

3) momentum is zero, kinetic energy is not zero 4) momentum is not zero, kinetic energy is not zero

KEY:3

17. About a collision which are not correct

1) physical contact is must 2) colliding particles cannot change their direction of motion

3) the effect of the external force is not considered 4) linear momentum does not conserved

KEY:1

18. The product of linear momentum and velocity of a body represents

1) half of the kinetic energy of the body 2) kinetic energy of the body

3) twice of the kinetic energy of the body 4) mass of the body

KEY:3

19. The K.E of a freely falling body

1) is directly proportional to height of its fall
2) is inversely proportional to height of its fall
3) is directly proportional to square of time of its fall
4) 1 and 3 are true

KEY:4

20. Choose the false statement

1) In a perfect elastic collision the relative velocity of approach is equal to the relative velocity of separation
2) In an inelastic collision the relative velocity of approach is less than the relative velocity of separation
3) In an inelastic collision the relative velocity of separation is less than the relative velocity of approach
4) In perfect inelastic collision relative velocity of separation is zero

KEY:2

21. Consider the following statements

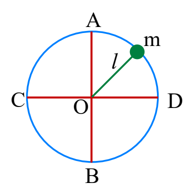
A) Linear momentum of a system of particles is zero

B) Kinetic energy of a system of particles is zero then

1) A does not imply B & B does not imply A
2) A implies B and B does not imply A
3) A does not imply B but B implies A
4) A implies B and B implies A

KEY:3

22. A small sphere of mass 'm' is attached to a cord and rotates in a vertical plane about a point O. If the averages speed of the sphere is increased, the cord is most likely to break at the orientation when the mass is at :



1) bottom point B
2) the point C
3) the point D
4) top point A

23. If force acting on a body is inversely proportional to its speed, then its kinetic energy is

1) Linearly related to time
2) Inversely proportional to time
3) Inversely proportional to the square of time
4) A constant

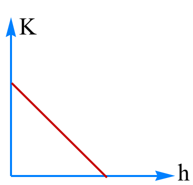
KEY:1

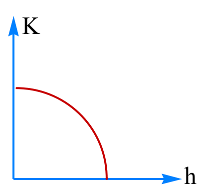
24. In one-dimensional elastic collision, the relative velocity of approach before collision is equal to

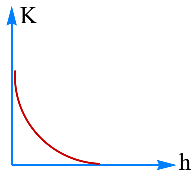
1) relative velocity of separation after collision
2) 'e' times relative velocity of separation after collision
3) '1/e' times relative velocity of separation after collision
4) sum of the velocities after collision

KEY:1

25. Which of the following graphs depicts the variation of K.E. of a ball bouncing on a horizontal floor with height? (Neglect air resistances)

1) 

2) 

3) 

4) None of these

KEY:1

26. Which of the following statement is correct?

1) KE of a system cannot be changed without changing its momentum
2) KE of a system cannot be changed without changing its momentum
3) Momentum of a system cannot be changed without changing its KE
4) A system cannot have energy without having momentum

KEY:1

27. Six steel balls of identical size are lined up along a straight frictionless groove. Two similar balls moving with speed v along the groove collide with this row on the extreme left end. Then

1) one ball from the right end will move on with speed v
2) two balls from the extreme right end will move on with speed v and the remaining balls will be at rest
3) all the balls will start moving to the right with speed v/8
4) all the six balls originally at rest will move on with speed v/6 and the incident balls will come to rest

KEY:2

28. Two bodies of masses m_1 **and** m_2 **have equal momentum. Their K.E. are in the ratio**

1) $\sqrt{m_2}:\sqrt{m_1}$ 2) $m_1:m_2$ 3) $m_2:m_1$ 4) $m_1^2:m_2^2$

KEY:3

29. A lighter body moving with a velocity v collides with a heavier body at rest. Then

1) the lighter body rebounced with twice the velocity of bigger body
2) the lighter body retraces its path with the same velocity in magnitude
3) the heavier body does not move practically
4) both (2) and (3)

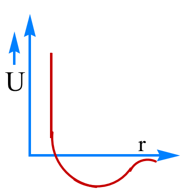
KEY:4

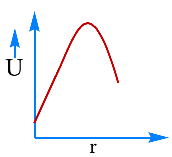
30. A body can have

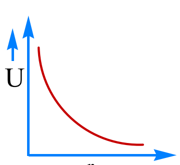
1) changing momentum and finite kinetic energy
2) zero kinetic energy and finite momentum
3) zero acceleration and increasing kinetic energy
4) finite acceleration and zero kinetic energy

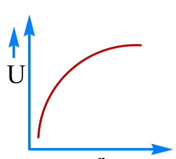
KEY:1

31. These diagrams represent the potential energy U of a diatomic molecule as a function of the inter-atomic distance r. The diagram corresponds to stable molecule found in nature is

1) 

2) 

3) 

4) 

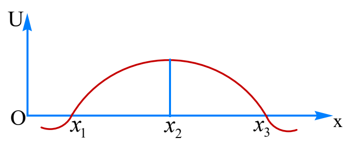
KEY:1

32. A man pushes a wall and fails to displace it. He does

1) negative work
2) positive but not maximum work
3) maximum work
4) no work at all

KEY:4

33. In the fig. the potential energy U of a particle plotted against its position *x* from origin. Which of the following statement is correct?



1) $at\ x_1$ is in stable equilibrium
2) $at\ x_2$ is in stable equilibrium
3) $at\ x_3$ is in stable equilibrium
4) at $x_{1,}\ x_2$ and x_3 particle is in unstable equilibrium

KEY:4

34. Two identical bodies moving in opposite direction with same speed, collided with each other. If the collision is perfectly elastic then

1) after the collision both comes to rest
2) after the collision first comes to rest and second moves in the opposite direction with same speed.
3) after collision they recoil with same speed
4) both and 1 and 2

KEY:3

35. When a spring is wound, a certain amount of PE is stored in it. If this wound spring is dissolved in acid the stored energy

1) In completely lost
2) Appears in the form of electromagnetic waves
3) Appears in the form of heat raising the temperature of the acid
4) Appears in the form of KE by splashing acid drops

KEY:3

36. A bottle of soda water is rotated in a vertical circle with the neck held in hand. The air bubbles are collected

1) near the neck
2) near the bottom
3) at the middle
4) uniformly in the bottle

KEY:1

37. Two springs have their force constants k_1 and k_2 and they are stretched to the same extension. If $K_2 > K_1$ work done is

1) same in both the springs
2) more in spring K_1
3) more in spring K_2
4) independent of spring constant K

KEY:3

38. Two springs have their force constants K_1 and K_2 ($K_2 > K_1$). When they are stretched by the same force, work done is

1) same in both the springs
2) more in spring K_1
3) more in spring K_2
4) independent of spring constant K

KEY:2

39. A lorry and a car moving with the same K.E. are brought to rest by applying the same retarding force. then

1) Lorry will come to rest in a shorter distance
2) Car will come to rest in a shorter distance
3) Both come to rest in same distance
4) any of above

KEY:3

40. A cricket ball and a ping-pong ball are dropped from the same height in a vacuum chamber from same height. When they have fallen half way down, they have the same

1) velocity 2) potential energy 3) kinetic energy 4) mechanical energy

KEY:1

41. A cyclist free-wheels from the top of a hill, gathers speed going down the hill, applies his brakes and eventually comes to rest at the bottom of the hill. Which one of the following energy changes take place.

1) Potential to kinetic to heat energy 2) Kinetic to potential to heat energy

3) chemical to heat to potential energy 4) Kinetic to heat to chemical energy

KEY:1

42. If 'E' represents total mechanical energy of a system while 'U' represents the potential energy, then E-U is

1) always zero 2) negative 3) either positive or negative 4) positive

KEY:4

43. For a body thrown vertically upwards, its direction of motion changes at the point where its total mechanical energy is

1) greater than the potential energy 2) less than the potential energy

3) equal to the potential energy 4) zero

KEY:3

44. Negative of work done by the conservation forces on a system is equal to

1) the change in kinetic energy of the system 2) the change in potential energy of the system

3) the change in total mechanical energy of the system 4) the change in the momentum of the system

KEY:2

45. Which of the following statements is wrong?

1) KE of a body is independent of the direction of motion

2) In an elastic collision of two bodies, the momentum and energy of each body is conserved

3) If two protons are brought towards each other, the P.E. of the system increases

4) A body can have energy without momentum

KEY:2

46. When a body falls from an aeroplane there is increase in its:

1) acceleration 2) potential energy 3) kinetic energy 4) mass

KEY:3

47. An agent is moving a positively charged body towards another fixed positive charge. The work done by the agent is

1) positive 2) negative 3) zero 4) may be positive or negative

KEY:1

48. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

1) $t^{1/2}$ 2) $t^{3/4}$ 3) $t^{3/2}$ 4) t^2

KEY:3

49. Potential energy is defined for

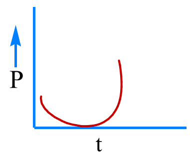
1) non-conservative forces only 2) conservative forces only

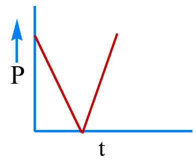
3) both conservative & non-conservative forces 4) neither conservative nor non-conservative forces

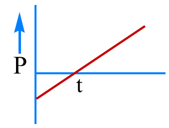
KEY:2

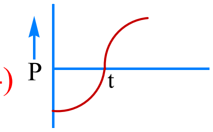
50. A particle is projected at t = 0 from a point on the ground with certain velocity at an angle with the horizontal. The power of gravitational force is plotted against time. Which of the following is the

best representation?

1) 

2) 

3) 

4) 

KEY:3

51. A body starts form rest and acquires a velocity V in time T. The instantaneous power delivered to the body in time 't' is proportional to

1) $\frac{V}{T}t$ 2) $\frac{V^2}{T}t^2$ 3) $\frac{V^2}{T^2}t$ 4) $\frac{V^2}{T^2}t^2$

KEY:3

52. Two bodies of different masses have same linear momentum. The one having more K.E. is

1) Lighter body 2) Heavier body 3) both 4) none

KEY:1

53. A car drives along a straight level frictionless road by an engine delivering constant power. Then velocity is directly proportional to

1) t 2) $\frac{1}{\sqrt{t}}$ 3) $\sqrt{t}$ 4) t^2

KEY:3

54. The change in kinetic energy per unit 'space' (distance) is equal to

1) power 2) momentum 3) force 4) pressure

KEY:3

55. A particle is projected with a velocity u making an angle θ with the horizontal. The instantaneous power of the gravitational force

1) varies linearly with time 2) Is constant throughout the path

3) Is negative for complete path 4) varies inversly with time

KEY:1

56. A gramphone record is revolving with an angular velocity ω. A coin is placed at a distance R from the centre of the record. the static coefficient of friction is μ. The coin will revolve with the record if

1) $R > \frac{\mu g}{\omega^2}$ 2) $R = \frac{\mu g}{\omega^2} only$ 3) $R < \frac{\mu g}{\omega^2}$ 4) $R \le \frac{\mu g}{\omega^2}$

KEY:4

57. A rock of mass m is dropped to the ground from a height h. A second rock with mass 2m is dropped from the same height. When second rock strikes the ground, its kinetic energy will be

1) twice that of the first rock 2) four times that of the first rock

3) the same as that of the first rock 4) half that of the first rock

KEY:1

58. A car is moving up with uniform speed along a fly over bridge which is part of a vertical circle. The true statement from the following is

1) Normal reaction on the car gradually decreases and becomes minimum at highest position of bridge

2) Normal reaction on the car gradually increases and becomes maximum at highest position

3) Normal reaction on car does not change

4) Normal reaction on the car gradually decreases and becomes zero at highest position

KEY:2

59. A vehicle is moving with uniform speed along horizontal, concave and convex surface roads. The surface on which, the normal reaction on the vehicle is maximum is

1) Concave 2) Convex 3) Horizontal 4) Same at all surfaces

KEY:1

60. A ball with initial momentum $\vec{P}$ collides with rigid wall elastically. If $\vec{P^1}$ be it's momentum after collision then

1) $\vec{P^1} = \vec{P}$ 2) $\vec{P^1} = -\vec{P}$ 3) $\vec{P^1} = 2\vec{P}$ 4) $\vec{P^1} = -2\vec{P}$

KEY:2

61. Internal force can change

1) Linear momentum as well as kinetic energy 2) linear momentum but not the Kinetic energy
3) the kinetic energy but not linear momentum 4) neither the linear momentum nor the kinetic energy

KEY:3

62. Which of the following forces is called a conservative force?

1) Frictional force 2) Air resistance 3) Electrostatic force 4) Viscous force

KEY:3

63. Two particles of different masses collide head on. Then for the system

1) loss of KE is zero, if it were perfect elastic collision
2) If it were perfect inelastic collision, the loss of KE of the bodies moving in opposite directions is more than that of the bodies moving in the same direction
3) loss of momentum is zero for both elastic and inelastic collision
4) 1, 2 and 3 are correct

KEY:4

64. For the same kinetic energy, the momentum shall be maximum for which of the following particle?

1) Electron 2) Proton 3) Deuteron 4) Alpha particle

KEY:4

65. In an inelastic collision, the kinetic energy after collision

1) is same as before collision 2) is always less than the before collision
3) is always greater than that before collision 4) may be less or greater than that before collision

KEY:2

66. A ball hits the floor and rebounds after an inelastic collision. In this case

1) the momentum of the ball just after the collision is same as that just before the collision
2) The mechanical energy of the ball remains the same in the collision
3) The total momentum of the ball and the earth is conserved
4) the total kinetic energy of the ball and the earth is conserved

KEY:3

67. A body of mass 'm' moving with certain velocity collides with another identical body at rest. If the collision is perfectly elastic and after the collision, if both the bodies moves

1) in the same direction 2) in opposite direction
3) in perpendicular direction 4) at 45° to each other

KEY:3

68. A heaver body moving with certain velocity collides head on elastically with a lighter body at rest, then

1) smaller body continues to be in the same state of rest

2) smaller body starts to move in the same direction with same velocity as that of bigger body
3) the smaller body start to move with twice the velocity of the bigger body in the same direction
4) the bigger body comes to rest

KEY:3

69. In which of the following, the work done by the mentioned force is negative? The work done by
1) the tension of the cable while the lift is ascending
2) the gravitational force when a body slides down an inclined place
3) the applied force to maintain uniform motion of a block on a rough horizontal surface
4) the gravitational force when a body is thrown up

KEY:4

70. Two bodies P and Q of masses m_1 and m_2 (m_2>m_1) are moving with velocity v_1 and v_2 respectively, collided with each other. Then the force exerted by P on Q during the collision is
1) greater that the force exerted by Q on
2) less than the force exerted by Q on
3) same as the force exerted by Q on P
4) same as the force exerted by P on Q but opposite in direction

KEY:4

71. The coefficient of restitution (e) for a perfectly elastic collision is
1) -1 2) 0 3) ∞ 4)1

KEY:4

72. A ball of mass M moving with a velocity V collides head on elastically with another of same mass but moving with a velocity v in the opposite direction. After collision.
1) the velocities are exchanged between the two balls
2) both the balls come to rest
3) both of them move at right angles to the original line of motion
4) one ball comes to rest and another ball travels back with velocity 2V

KEY:2

73. A small bob of a simple pendulum released from 30° to the vertical hits another bob of the same mass and size lying at rest on the table vertically below the point of suspension. After elastic collision, the angular amplitude of the bob will be
1) 30° 2) 60° 3) 15° 4) zero

KEY:4

74. Two spheres 'X' and 'Y' collide. After collision, the momentum of X is doubled . Then
1) the initial momentum of X and Y are equal
2) the initial momentum of X is greater then that of Y
3) the initial momentum of Y is double that of X
4) the loss in momentum of Y is equal to the initial momentum of X

KEY:4

75. A shell is fired into air at an angle θ with the horizontal from the ground. On reaching the maximum height,
1) its kinetic energy is not equal to zero 2) its kinetic energy is equal to zero
3) its potential energy is equal to zero 4) both its potential and kinetic energies are zero

KEY:1

76. A bullet is fired into a wooden block. If the bullet gets embedded in wooden block, then
1) momentum alone is conserved
2) kinetic energy alone is conserved
3) both momentum and kinetic energy are conserved

4) neither momentum nor kinetic energy are conserved

KEY:1

77. During collision, which of the following statements is wrong?

1) three is a change in momentum of individual bodies

2) the change in total momentum of the system of colliding particle is zero

3) the change in total energy is zero

4) they law of conservation of momentum is not valid

KEY:4

: : PRACTICE BITS : :

1. A man weighing 80 kg climbs a staircase carrying a 20 kg load. The staircase has 40 steps, each of 25 cm height. If he takes 20 seconds to climb, the work done is

1) 9800 J 2) 490 J 3) 98 x 10^5 J 4) 7840 J

KEY:1

HINT:

$$W = \vec{F}.\vec{S} = FS = (M+m)\,g\left(n \times h_{each\ step}\right)$$

2. A ball of mass 0.6kg attached to a light inextensible string rotates in a vertical circle of radius 0.75m such that it has speed is 5ms^{-1} when the string is horizontal. Tension in string when it is horizontal on other side is (g-10ms^{-2}) [2007M]

1) 30N 2) 26N 3) 20N 4) 6N

KEY:3

HINT:

$$T = \frac{mv_H^2}{r}\ where \quad v_H = \sqrt{3gr}$$

3. A body of mass 10 kg moving with a velocity of 5ms^{-1} hits a body of 1 gm at rest. The velocity of the second body after collision assuming it to be perfectly elastic is

1) 10 ms^{-1} 2) 5 ms^{-1} 3) 15 ms^{-1} 4) 0.10 ms^{-1}

KEY:1

HINT:

$$v_2\left(\frac{2m_1}{m_1+m_2}\right)u_1$$

4. A force $\vec{F} = (6\hat{i} - 8\hat{j})N$, acts on a particle and displaces it over 4 m along the X-axis and 6 m along the Y-axis. The total work done during the two displacements is

1) 72 J 2) 24 J 3) - 24 J 4) zero

KEY:3

HINT:

$$W = W_x + W_y \qquad W_x = \vec{F}.x\hat{i},\ W_y = \vec{F}.y\hat{j}$$

5. In the above problem the ratio of distance travelled in two consecutive rebounds

1) 1 : e 2) e : 1 3) 1 : e^2 4) e^2 : 1

KEY:3

HINT:

$$h_n e^{2n} h$$

6. An object has a displacement from position vector $\vec{r}_1 = \left(2\hat{i} + 3\hat{j}\right)m\ to\ \vec{r}_2 = \left(4\hat{i} + 6\hat{j}\right)m$ under a force $\vec{F} = \left(3x^2\hat{i} + 2y\hat{j}\right)N$, then work done by the force is

1) 24 J 2) 33 J 3) 83 J 4) 45 J

KEY:3

HINT:

$$W=\int dw\int_{x_1}^{x_2}F_x dx+\int_{y_1}^{y_2}F_y dy$$

7. A body starts from rest and is acted on by a constant force. The ratio of kinetic energy gained by it in the first five seconds to that gained in the next five second is

1) 2 : 1 2) 1 : 1 3) 3 : 1 4) 1 : 3

KEY:4

HINT:

$$KE=\frac{1}{2}mv^2=\frac{1}{2}m(gt)^2\ (\because v=gt)$$

$$\frac{K_1}{K_2}=\frac{t_1^2}{t_2^2-t_1^2}\ where\ t_1=5\sec\ and\ t_2=10\sec$$

8. A body starts from rest and moves with uniform acceleration. What is the ratio of kinetic energies at the end of 1st, 2nd and 3rd seconds of its journey?

1) 1 : 8 : 27 2) 1 : 2 : 3 3) 1 : 4 : 9 4) 3 : 2 : 1

KEY:3

HINT:

$$KE=\frac{1}{2}mv^2=\frac{1}{2}mg^2t^2\ (\because v=gt)$$

9. A liquid of specific gravity 0.8 is flowing in a pipe line with a speed of 2 m/s. The K.E. per cubic meter of it is

1) 160 J 2) 1600 J 3) 160.5 J 4) 1.6 J

KEY:2

HINT:

$$KE=\frac{1}{2}mv^2=\frac{1}{2}(\rho V)v^2;\frac{K.E}{V}=\frac{1}{2}\rho v^2$$

10. A 60 kg boy lying on a surface of negligible friction throws horizontally a stone of mass 1 kg with a speed of 12 m/s away from him. As a result with what kinetic energy he moves back?

1) 2.4 J 2) 72 J 3) 1.2 J 4) 36 J

KEY:3

HINT:

$$m_1v_1=m_2v_2;KE=\frac{1}{2}m_2v_2^2$$

11. Two stones of masses m and 2 m are projected vertically upwards so as to reach the same height. The ratio of the kinetic energies of their projection is

1) 2 : 1 2) 1 : 2 3) 4 : 1 4) 1 : 4

KEY:2

HINT:

$KE=\frac{1}{2}mv^2$, When two bodies reach the same

height, $v_1=v_2;\frac{KE_1}{KE_2}=\frac{m_1}{m_2}\left(\because v=\sqrt{2gh}\right)$

12. A tank of size 10 m × 10 m × 10 m is full of water and built on the ground. If g = 10 ms^{-2}, the potential energy of the water in the tank is

1) 5×10^7 J 2) 1×10^8 J 3) 5×10^4 J 4) 5×10^5 J

KEY:1

HINT:

$$P.E = mgh_1;\ here\ h_1 = \frac{h}{2}\ and\ m = \rho\times \text{V}$$

13. A spring when compressed by 4 cm has 2 J energy stored in it. The force required to extend it by 8 cm will be

1) 20 N 2) 2 N 3) 200 N 4) 2000 N

KEY:3

HINT:

$$U = \frac{1}{2}Kx_1^2 \Rightarrow K = \frac{2U}{x_1^2}\ and\ F = Kx_2$$

14. The elastic potential energy of a stretched spring is given by E = $50x^2$ where x is the displacement in meter and E is in joule, then the force constant of the spring is

1) 50 Nm 2) 100 N m^{-1} 3) 100 N/m^2 4) 100 Nm

KEY:2

HINT:

$$U = \frac{1}{2}Kx^2 --(1), U = 50x^2 --(2), \text{Compare equation (1) and (2) to find K}$$

15. A body of mass 2 kg is projected with an initial velocity of 5 m s^{-1} along a rough horizontal table. The work done on the body by the frictional forces before it is brought to rest is

1) 250 J 2) 2.5 J 3) -250 J 4) 25 J

KEY:4

HINT:

$$W_f\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$

16. A body of mass 20 gms is moving with a certain velocity. It collides with another body of mass 80 gm at rest. The collision is perfectly inelastic. The ratio of the kinetic energies before and after collision of the system is

1) 2 : 1 2) 4 : 1 3) 5 : 1 4) 3 : 2

KEY:3

HINT:

$$m_1u_1 = (m_1 + m_2)v$$

$$KE_i = \frac{1}{2}m_1u_1^2;\ KE_f = \frac{1}{2}(m_1 + m_2)v^2$$

17. An object is acted on by a retarding force of 10 N and at a particular instant its kinetic energy is 6 J. The object will come to rest after it has travelled a distance of

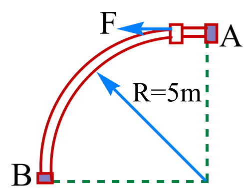
1) 3/5 m 2) 5/3 m 3) 4 m 4) 16 m

KEY:1
HINT:

According to work energy theorem

$$W_f\left(\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2\right)$$

18. A bead of mass $\frac{1}{2}kg$ starts from rest from "A" to move in a vertical plane along a smooth fixed quarter ring of radius 5m, under the action of a constant horizontal for F=5 N as shown. The speed of bead as it reaches point "B" is



1) 14.14 m/s 2) 7.07 m/s 3) 5 m/s 4) 25 m/s

KEY:1
HINT:

Applying the work - energy theorem, we get

$$\frac{1}{2}\times mv^2-0=W_1+W_2$$

=Horizontal force $\times$ displacement + Vertical force $\times$ displacement.

$$=F\times R+mg\times R$$

19. If $\vec{F}=2\hat{i}+3\hat{j}+4\hat{k}$ acts on a body and displaces it by $\vec{S}=3\hat{i}+2\hat{j}+5\hat{k}$ then the work done by the force is

1) 12 J 2) 20 J 3) 32 J 4) 64 J

KEY:3
HINT:

$$W=\vec{F}.\vec{S}$$

20. A cradle is 'h' meters above the ground at the lowest position and 'H' meters when it is at the highest point. If 'v' is the maximum speed of the swing of total mass 'm' the relation between 'h' and 'H' is

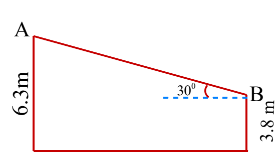
1) ½ mv^2 + h = H 2) (v^2/2g) + h = H 3) (v^2/g) + 2h = H 4) (v^2/2g) + H = h

KEY:2
HINT:

K. E at mean = P. E at extreme position

.

21. AB is a frictionless inclined surface making an angle of 30^0 with horizontal. A is 6.3 m above the ground while B is 3.8 m above the ground. A block slides down from A, initially starting from rest. Its velocity on reaching B is



1) 7 m s^{-} [1] 2)14 m s^{-1} 3) 7.4 m s^{-1} 4) 4.9 m s^{-1}

KEY:1

HINT:

Gain in K.E = Loss of P.E = mg (h_1-h_2)

22. A shot is fired at 30° with the vertical from a point on the ground with kinetic energy K. If air resistance is ignored, the kinetic energy at the top of the trajectory is

1) 3K/4 2) K/2 3) K 4) K/4

KEY:4

HINT:

$$\text{At maximum height, } K^1 = \frac{1}{2}mu^2\cos^2\theta$$

$$\text{At projection, } K = \frac{1}{2}mu^2$$

23. A stone of mass "m" initially at rest and dropped from a height "h" strikes the surface of the earth with a velocity "v". If the gravitational force acting on the stone is W, then which of the following identities is correct?

1) mv - mh = 0 2) ½ mv^2 - Wh^2 = 0 3) ½ mv^2 - Wh = 0 4) ½ mv^2 - mh = 0

KEY:3

HINT:

Gain in K.E = Loss of P.E

24. A ball is dropped on to a horizontal floor. It reaches a height of 144 cm on the first bounce and 81 cm on the second bounce. The height it attains on the third bounce is

1) 45.6 cm 2) 81 cm 3) 144 cm 4) 0 cm

KEY:1

HINT:

$$h_n = e^{2n}h$$

25. A motor boat is going in a river with a velocity $\vec{V} = \left(4\hat{i} - 2\hat{j} + \hat{k}\right)ms^{-1}$. If the resisting force due to stream is $\vec{F} = \left(5\hat{i} - 10 + 6\hat{k}\right)$N, then the power of the motor boat is

1) 100W 2) 50 W 3) 46W 4) 23W

KEY:3

HINT:

$$P = \vec{F}.\vec{V}$$

26. An 8 gm bullet is fired horizontally into a 9 kg block of wood and sticks in it. The block which is free to move, has a velocity of 40 cm/s after impact. The initial velocity of the bullet is

1) 450 m/s 2) 450 cm/s 3) 220 m/s 4) 220 cm/s

KEY:1

HINT:

$$mu = \left(m + M\right)v$$

27. Two riffles fire the same number of bullets in a given interval of time. The second fires bullets of mass twice that fired by the first and with a velocity that is half that of the first. The ratio of their power is

1) 1 : 4 2) 4 : 1 3) 1 : 2 4) 2 : 1

KEY:4
HINT:

$$\frac{P_1}{P_2}=\left(\frac{m_1}{m_2}\right)\left(\frac{v_1^2}{v_2^2}\right)$$

28. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is P, the energy of explosion is.

1) $\frac{p^2}{2(m_1+m_2)}$ 2) $\frac{p^2}{2\sqrt{m_1 m_2}}$ 3) $\frac{p^2(m_1+m_2)}{2m_1 m_2}$ 4) $\frac{p^2}{2(m_1-m_2)}$

KEY:3
HINT:

$$E=E_1+E_2=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m^2}$$

29. A force of 1200 N acting on a stone by means of a rope slides the stone through a distance of 10 m in a direction inclined at 60^0 to the force. The work done by the force is

1) $6000\sqrt{3}J$ 2) 6000 J 3) 12000 J 4) 8000 J

KEY:2
HINT:

W=FS $\cos\theta$

30. A crane can lift up 10,000 kg of coal in 1 hour from a mine of 180 m depth. If the efficiency of the crane is 80 %, its input power must be ($g = 10\ m\ s^{-2}$)

1) 5 kW 2) 6.25 kW 3) 50 kW 4) 62.5 kW

KEY:2
HINT:

$$\eta=\frac{P_{out}}{P_{in}}, where P_{out}=\frac{W}{t}=\frac{mgh}{t}$$

31. A man carries a load of 50 kg through a height of 40 m in 25 seconds. If the power of the man is 1568 W, his mass is

1) 5 kg 2) 1000 kg 3) 200 kg 4) 50 kg

KEY:4
HINT:

$$P=\frac{W}{t}=\frac{(m+M)gh}{t}$$

32. A body dropped freely from a height h on to a horizontal plane, bounces up and down and finally comes to rest. The coefficient of restitution is e. The ratio of velocities at the beginning and after two rebounds is

1) 1 : e 2) e : 1 3) 1 : e^2 4) e^2 : 1

KEY:3
HINT:

$$v_n=e^n v$$

33. An electric motor creates a tension of 4500 newton in a hoisting cable and reels it in at the rate of 2m/s. What is the power of the motor?

1) 15 kW 2) 9 kW 3) 225 W 4) 9000 kW

KEY:2

HINT:

$$P_{inst} = \vec{F}.\vec{V} = FV\cos\theta$$

34. The mass of a simple pendulum bob is 100 gm. The length of the pendulum is 1 m. The bob is drawn aside from the equilibrium position so that the string makes an angle of 60° with the vertical and let go. The kinetic energy of the bob while crossing its equilibrium position will be

1) 0.49 J 2) 0.94 J 3) 1 J 4) 1.2 J

KEY:1

HINT:

$$K.E_{mean} = P.E_{extreme} = mg\left(1-\cos\theta\right)$$

35. A juggler throws continuously balls at the rate of three in each second each with a velocity of 10 m s^{-1}. If the mass of each ball is 0.05 kg his power is

1) 2 W 2) 50 W 3) 0.5 W 4) 7.5 W

KEY:4

HINT:

$$P = \frac{W}{t} = \frac{n\left(\frac{1}{2}mv^2\right)}{t}$$

36. A body of mass 2 kg attached at one end of light string is rotated along a vertical circle of radius 2m. If he string can withstand a maximum tension of 140.6 N, the maximum speed with which the stone can be rotated is

1) 22 m/s 2) 44 m/s 3) 33 m/s 4) 11 m/s

KEY:4

HINT:

$$T_{\max} = m\left(\frac{v^2}{r} + g\right)$$

37. The work done by a force $\vec{F} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ displaces the body from a point (3,4,6) to a point (7,2,5) is

1) 15 units 2) 25 units 3) 20 units 4) 10 units

KEY:1

HINT:

$$W = \vec{F}.\vec{S} = \vec{F}.\left(\vec{r_2} - \vec{r_1}\right)$$

38. A lawn roller is pulled along a horizontal surface through a distance of 20 m by a rope with a force of 200 N. If the rope makes an angle of 60° with the vertical while pulling, the amount of work done by the pulling force is

1) 4000 J 2) 1000 J 3) $2000\sqrt{3}$ J 4) 2000 J

KEY:3

HINT:

$$W = \vec{F}.\vec{S} = FS\cos\theta$$

39. A pilot of mass m can bear a maximum apparent weight 7 times of mg. The aeroplane is moving in a vertical circle. If the velocity of aeroplane is 210 m/s while diving up from the lowest point of vertical circle, the minimum radius of vertical circle should be

1) 375 m 2) 420 m 3) 750 m 4) 840 m

KEY:3

HINT:

At lowest point of vertical circle,

$$T_{\max} = \frac{mv^2}{r_{\min}} + mg$$

40. A simple pendulum is oscillating with an angular amplitude 60^0. If mass of bob is 50 gram, the tension in the string at mean position is ($g = 10ms^{-2}$)

1) 0.5 N 2) 1 N 3) 1.5 N 4) 2N

KEY:2

HINT:

$$T = mg + \frac{mv^2}{r} = mg + \frac{m}{l}\left[2gl\left(1-\cos\theta\right)\right]$$

41. A body is moving in a vertical circle such that the velocities of body at different points are critical. The ratio of velocities of body at angular displacements 60^0, 120^0 from lowest point is

1) $\sqrt{5}:\sqrt{2}$ 2) $\sqrt{3}:\sqrt{2}$ 3) $\sqrt{3}:1$ 4) 2 : 1

KEY:4

HINT:

$$v = \sqrt{gR\left(3+2\cos\theta\right)} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{3+2\cos\theta_1}{3+2\cos\theta_2}}$$

42. A 6 kg mass travelling at $2.5\ \text{ms}^{-1}$ collides head on with a stationary 4 kg mass. After the collision the 6 kg mass travels in its original direction with a speed of $1\ \text{ms}^{-1}$. The final velocity of 4 kg mass is

1) 1 m/s 2) 2.25ms^{-1} 3) $2\ \text{ms}^{-1}$ 4) $0\ \text{ms}^{-1}$

KEY:2

HINT:

According to law of conservation of linear momentum, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

43. A block of mass 1 kg moving with a speed of $4ms^{-1}$, collides with another block of mass 2 kg which is at rest. The lighter block comes to rest after collision. The loss in the K.E of the system.

1) 8 J 2) 4×10^{-7} J 3) 4 J 4) 0 J

KEY:3

HINT:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Delta KE = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_2v_2^2$$

44. A bolt of mass 0.3kg falls from the ceiling of an elevator moving down with an uniform speed of 7m/s. It hits the floor of the elevator (length of the elevator = 3m) and does not rebound. What is the heat produced by impact.

1)8.82J 2)7.72J 3)6.62J 4)5.52J

KEY:1

HINT:

Heat produced = loss of potential energy = mgh

45. A marble going at a speed of $2ms^{-1}$ hits another marble of equal mass at rest. If the collision is perfectly elastic, thn the velocity of the first after collision is

1) $4\ ms^{-1}$ 2) $0ms^{-1}$ 3) $2ms^{-1}$ 4) $3ms^{-1}$

KEY:2

HINT:

$$\vec{v}_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u}_1 = \left(\frac{2m_2}{m_1 + m_2}\right)\vec{u}_2$$

46. By applying the brakes without causing a skid, the driver of a car is able to stop his car in a distance of 5 m, if it is going at 36 kmph. If the car were going at 72 kmph, using the same brakes, he can stop the car over a distance of

1) 10 m 2) 2.5 m 3) 20 m 4) 40 m

KEY:3

HINT:

$$W = \Delta KE; \frac{\Delta KE_1}{\Delta KE_2} = \frac{W_1}{W_2}\frac{FS_1}{FS_2}$$

47. A massive ball moving with a speed v collides head on with a fine ball having mass very much smaller than the mass of the first ball at rest. The collision is elastic and then immediately after the impact, the second ball will move with a speed approximately equal to

1) V 2) 2V 3) v/3 4) infinite

KEY:2

HINT:

$$m_2 <<< m_1 \ and$$

$$\vec{v}_2 = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{u}_2$$

48. A 1 kg ball moving at 12 m/s collides head on with a 2 kg ball moving in the opposite direction at 24 m/s. The velocity of each ball after the impact, if the coefficient of restitution if 2/3 is

1) -28 m/s; -4 m/s 2) 28 m/s; -4 m/s 3) 20 m/s; 24 m/s 4) -20m/s; -4 m/s

KEY:1

HINT:

$$\vec{v}_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{m_2(1+e)}{m_1 + m_2}\right)\vec{u}_2$$

$$\vec{v}_2 = +\left(\frac{m_1(1+e)}{m_1 + m_2}\right)\vec{u}_1\left(\frac{m_2 - em_1}{m_1 + m_2}\right)\vec{u}_2$$

49. A body of mass m moving at a constant velocity v hits another body of the same mass moving at the same velocity v/s but in the opposite direction and sticks to it. The common velocity after collision is

1) v 2) v/4 3) 2v 4) v/2

KEY:2

HINT:

$$m_1u_1 - m_2u_2 = \left(m_1 + m_2\right)v$$

50. A neutron, one of the constituents of a nucleus, is found to pass two points 60 meters apart in a time interval of 1.8×10^{-4} sec. The mass of a neutron is 1.7×10^{-27} kg. Assuming that the speed is constant, its kinetic energy is

1) 9.3×10^{-17} joule 2) 9.3×10^{-14} joule 3) 9.3×10^{-21} joule 4) 9.3×10^{-11} joule

KEY:1

HINT:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{s}{t}\right)^2$$

51. A block of wood of mass 9.8 kg is suspended by a string. A bullet of mass 200 gm strikes horizontally with a velocity of $100\ \text{ms}^{-1}$ and gets imbedded in it. The maximum height attained by the block is $\left(g = 10\ \text{ms}^{-2}\right)$

1) 0.1 m 2) 0.2 m 3) 0.3 m 4) 0

KEY:2

HINT:

$$mu = \left(m + M\right)v \ andh = \frac{v^2}{2g}$$

52. A 15 gm bullet is fired horizontally into a 3 kg block of wood suspended by a string. The bullet sticks in the block, and the impact causes the block to swing 10 cm above the initial level. The velocity of the bullet nearly is (in ms^{-1})

1) 281 2) 326 3) 184 4) 58

KEY:1

HINT:

$$mu = \left(m + M\right)v \ and \ h = \frac{v^2}{2g}$$

53. A car weighing 1000 kg is going up an incline with a slope of 2 in 25 at a steady speed of 18 kmph. If g = 10 m s^{-2}, the power of its engine is

1) 4 kW 2) 50 kW 3) 625 kW 4) 25 kW

KEY:1

HINT:

$$P = \overrightarrow{F}.\vec{v} = Fv = mg\sin\theta v$$

54. A rubber ball drops from a height h and after rebounding twice from the ground, it rises to h/2. The co - efficient of restitution is

1) $\frac{1}{2}$ 2) $\left(\frac{1}{2}\right)^{1/2}$ 3) $\left(\frac{1}{2}\right)^{1/4}$ 4) $\left(\frac{1}{2}\right)^{1/6}$

KEY:3

HINT: $h_n = e^{2n}h$

55. A bullet fired into a trunk of a tree loses 1/4 of its kinetic energy in traveling a distance of 5 cm. Before stopping it travels a further distance of

1) 150 cm 2) 1.5 cm 3) 1.25 cm 4) 15 cm

KEY:4

HINT:

$$W = \vec{F}.\vec{S} = \Delta KE; \frac{S_1}{S_2} = \frac{\Delta KE_1}{\Delta KE_2}$$

56. In the above problem, the ratio of times of two consecutive rebounds

1) 1 : e 2) e : 1 3) 1 : e² 4) e² : 1

KEY:1

HINT:

$$t_n = e^n t$$

57. A ball is dropped on to a horizontal floor. It reaches a height of 144 cm on the first bounce and 81 cm on the second bounce. The coefficient of restitution is

1) 0 2) 0.75 3) 81/144 4) 1

KEY:2

HINT:

$$e = \sqrt{\frac{h_2}{h_1}}$$

58. A ball is dropped from height 'H' on to a horizontal surface. If the coefficient of restitution is 'e' then the total time after which it comes to rest is

1) $\sqrt{\frac{2H}{g}}\left(\frac{1-e}{1+e}\right)$ 2) $\sqrt{\frac{2H}{g}}\left(\frac{1+e}{1-e}\right)$ 3) $\sqrt{\frac{2H}{g}}\left(\frac{1+e^2}{1-e^2}\right)$ 4) $\sqrt{\frac{2H}{g}}\left(\frac{1-e^2}{1+e^2}\right)$

KEY:2

HINT:

$$.t = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + --- and\ h_n = e^{2n}h$$

59. The length of a ballistic pendulum is 1 m and mass of its block is 0.98 kg. A bullet of mass 20 gram strikes the block along horizontal direction and gets embedded in the block. If block + bullet completes vertical circle of radius 1m, the striking velocity of bullet is

1) 280 m/s 2) 350 m/s 3) 420 m/s 4) 490 m/s

KEY:2

HINT:

According to law of conservation of linear momentum $mu = (M + m)\,v$

$$u = \frac{(M+m)\sqrt{5gr}}{m}$$

60. A 6 kg mass collides with a body at rest. After the collision, they travel together with a velocity one third the velocity of 6kg mass. The mass of the second body is

1) 6 kg 2) 3 kg 3) 12 kg 4) 18 kg

KEY:3

HINT:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$$

JEE MAINS PREVIOUS QUESTIONS AND SOLUTIONS

WORK ,ENERGY AND POWER

Work :

1. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement ofthe box? [4 Sep. 2020 (II)]

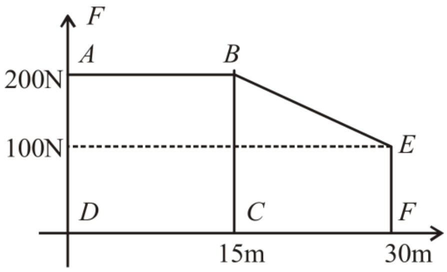
(a) 3280 J (b) 2780 J (c) 5690 J (d) 5250 J

SOLUTION : . (d)

The given situation can be drawn graphically as shown in figure.

Work done = Area under F-x graph

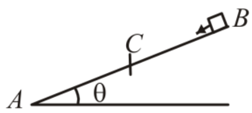
= Area ofrectangle $ABCD$ + Area oftrapezium $BCFE$



$$W=(200\times 15)+\frac{1}{2}(100+200)\times 15=3000+2250$$

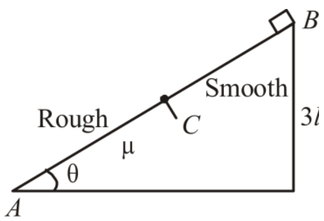
$$\Rightarrow W=5250\text{J}$$

2. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient offriction μ. It is found that the block comes to rest as it reaches the bottom (point A) ofthe inclinedplane. If$BC=2AC$, the coefficient offriction is given by $\mu=k\tan\theta$. The value of k is [NA 2 Sep. 2020 (I)]



SOLUTION : (3)

If$AC=l$ then according to question, $BC=2l$ and $AB=3l$.



$'\sin\theta$

Here, work done by all the forces is zero. Wffiction $+W_{mg}=0$

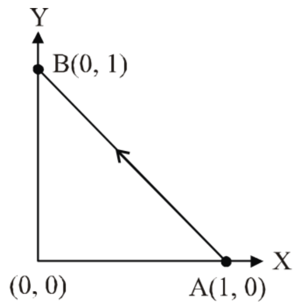
$$mg(3l)\sin\theta-\mu mg\cos\theta(l)=0$$

$$\Rightarrow \mu mg\cos\theta l=3mgl\sin\theta$$

$$\Rightarrow \mu=3\tan\theta=k\tan\theta$$

$$k=3$$

3. Consider a force $\vec{F}=-x\hat{i}+y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units) [9 Jan. 2020 I]



(a) 2J (b) $\frac{1}{2}$J (c) 1J (d) $\frac{3}{2}$J

SOLUTION : (c)

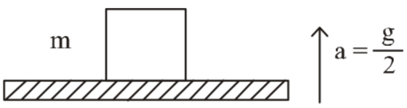
$$\text{Work done, } W=\int \vec{F}\cdot\overline{ds}$$

$$=(-x\hat{\imath}+y\hat{\jmath})\cdot(d\times\hat{\imath}+dy\hat{\jmath})$$

$$\Rightarrow W=-\int_1^0 x\,dx+\int_0^1 y\,dy$$

$$=\left(0+\frac{1}{2}\right)+\frac{1}{2}=1J$$

4. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. work done by normal reaction on block in time t is: [10 Jan. 2019 I]



(a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$ (c) 0 (d) $\frac{3mg^2t^2}{8}$

SOLUTION : . (d)

$$\text{Here, } N-mg=ma=\frac{mg}{2}\Rightarrow N=\frac{3mg}{2}N=\text{normal reaction}$$

Now, work done by normal reaction 'N' on block in time t, $W=\vec{N}\vec{S}=\left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/_2t^2\right)$

$$\text{or, } W=\frac{3mg^2t^2}{8}$$

5. A body of mass starts moving fi$_{i0}$m rest along x - axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant s and is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start ofthe motion is: [Online Apri116, 2018]

(a) $\frac{1}{8}ma^4t^2$ (b) $4ma^4t^2$ (c) $8ma^4t^2$ (d) $\frac{1}{4}ma^4t^2$

SOLUTION : (a)

From question, $v = a\sqrt{s} = \frac{ds}{dt}$

or, $2\sqrt{s} = at \Rightarrow S = \frac{a^2t^2}{4}$

$$\Gamma = m \times \frac{a^2}{2}$$

Work done $= \frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}ma^4t^2$

6. When a rubber - band is stretched by a distance x, it exerts restoring force ofmaguitude $\Gamma = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber - band by L is: [2014]

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$ (c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

SOLUTION : . (c)

Work done in stretching the rubber-band by a distance dx is

$$dW = Fdx = (ax + bx^2)dx$$

Integrating both sides,

$$W = \int_0^L a\,xdx + \int_0^L b\,x^2dx = \frac{aL^2}{2} + \frac{bL^3}{3}$$

7. A uniform chain oflength 2 m is kept on a table such that a length of60 cm hangs freely from the edge ofthe table. The total mass ofthe chain is 4 kg. What is the work done in pulling the entire chain on the table? [2004]

(a) 12 J (b) 3.6J (c) 7.2J (d) 1200 J

SOLUTION : . (b)

Mass of over hanging part of the chain

$$m' = \frac{4}{2} \times (0.6)\text{kg} = 1.2 \text{ kg}$$

Weight ofhanging part ofthe chain

$= 1.2 \times 10 = 12\text{N}$

C.M. ofhanging part $= 0.3\text{m}$ below the table

Workdone in putting the entire cha in on the table $= 12 \times 0.30 = 3.6$ J.

8 . A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})m$. The work done on the particle injoules is [2004]

(a) $+10$ (b) $+7$ (c) -7 (d) $+13$

SOLUTION : **(b)**

Given, Force, $\vec{F} = (5\hat{i}+3j+2k\hat{})$

Displacement, $x=(2\hat{i}-j)$

Work done,

$W = \vec{F} \cdot \vec{x} = (5\hat{i}+3j+2k\hat{}).(2\hat{i}-j)$

$= 10 - 3 = 7$joules

9. Aspring ofspring constant $5 \times 10^3 \mathrm{N/m}$ is stretched initially by 5cm from the unstretched position. Then the work required to stretch it further by another 5 cm [2003]

(a) 12.50 N - m (b) 18.75 N - m (c) 25.00 N - m (d) 6.25 N - m

SOLUTION : **. (b)**

Spring constant, $k = 5 \times 10^3 \mathrm{N/m}$

Let x_1 and x_2 be the initial and fmal stretched position of the spring, then

$$\text{Work done, } W = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{1}{2} \times 5 \times 10^3[(0.1)^2 - (0.05)^2]$$

$$= \frac{5000}{2} \times 0.15 \times 0.05 = 18.75 \text{ Nm}$$

10. Asprin gofforce constant 800N/mhasan extension of5 cm. The work done in extending it fi¡om5 cm to 15 cm is [2002]

(a) 16 J (b) 8J (c) 32 J (d) 24 J

SOLUTION : **(b)**

Small amount ofwork done in extending the spring by dx is

$$dW = kxdx$$

$$W = k\int_{0.05}^{0.15} x\,dx$$

$$= \frac{800}{2}[(0.15)^2 - (0.05)^2]$$

$$= 400[(0.15 + 0.05)(0.15 - 0.05)]$$

$$= 400 \times 0.2 \times 0.1 = 8\text{J}$$

ENERGY

11. A cricket ball ofmass 0.15 kg is thrown verticallyup by a bowling machine so that it rises to a maximum height of20 m after leaving the machine. Ifthe part pushing the ball applies a constant force F on the ball and moves horizontally a distance of0.2m while launching the ball, the value ofF (in N) is ($g = 10\text{ms}^{-2}$)

[NA 3 Sep. 2020 (I)]

SOLUTION : . (150.00)

From work energy theorem,

$$W = F \cdot s = \Delta KE = \frac{1}{2}mv^2$$

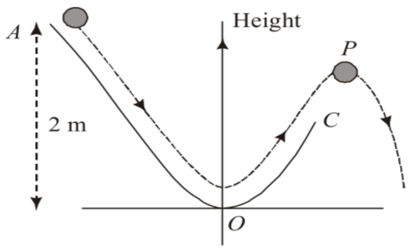
Here $V^2 = 2gh$

$$F \cdot s = F \times \frac{2}{10} = \frac{1}{2} \times \frac{15}{100} \times 2 \times 10 \times 20$$

$$F = 150 \text{ N.}$$

12. A particle ($m = 1$kg) slides down a fiictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move fieely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy ofthe particle (in J) is: (Figure drawn is schematic and not to scale; take $g = 10$ms 2)−.

[NA 7 Jan. 2020 I]



SOLUTION :

(10.00) Kinetic energy = change in potential energy of the particle,

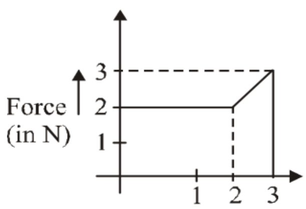
$$\text{KE} = \text{mg}\Delta\text{h}$$

Given, $m = 1$kg,

$$\Delta h = \text{h}_2 - \text{h}_1 = 2 - 1 = 1m$$

$$KE = 1 \times 10 \times 1 = 10J$$

13. A particle moves in one dimension from rest under the influence ofa force that varies with the distance travelled bythe particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is: [7 Jan. 2020 II]



Distance (in m) →

(a) 4 J (b) 2.5J (c) 6.5 J (d) 5 J

SOLUTION : . (c)

We know area under F-x graph gives the work done by the body

$$W=\frac{1}{2}\times(3+2)\times(3-2)+2\times 2=2.5+4=6.5\text{J}$$

Using work energy theorem,

Δ K. E = work done

Δ K.E = 6.5J

14. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths 1_1 and l_2 where, $l_1=nl_2$ and n is an integer. The ratio $k_1 l k_2$ of the corresponding force constants, k_1 and k_2 will be:

[12 April 2019 II]

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

SOLUTION : (c)

$$l_1+l_2=l \text{ and } l_1=nl_2$$

$$l_1=\frac{nl}{n+1} \text{ and } l_2=\frac{l}{n+1}$$

As $k\propto\frac{1}{l}$,

$$\frac{k_1}{k_2}=\frac{l/(n+1)}{(nl)/(n+1)}=\frac{1}{n}$$

15. Abodyofmass 1 kg falls fieely from aheight of 100m, on a platform of mass 3 kg which is mounted on a spring having spring constant $k=1.25\times 10^6$N/m. The body sticks to the platform and the spring' s maximum compression is found to be x. Given that $g=10\text{ms}^{-2}$, the value ofx will beclose to: [11 April 2019 I]

(a) 40 cm (b) 4 cm (c) 80 cm (d) 8 cm

SOLUTION : (b)

Velocity of 1 kg blockjust before it collides with 3 kg block = $\sqrt{2\text{gh}}=\sqrt{2000}\text{n}y_S$

Using principle of conservation oflinear momentumjust before andjust after collision, we get

$$1\times\sqrt{2000}=4\text{v}\Rightarrow \text{v}=\frac{\sqrt{2000}}{4}\text{m/s v}$$

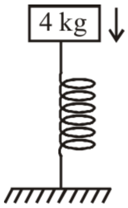
Initial compression ofspring

$$1.25\times 10^6\text{x}_0=30\Rightarrow \text{x}_0\approx 0$$

using work energy theorem,

$$\text{W}_\text{g}+\text{W}_\text{sp}=\Delta\text{KE}$$

$$\Rightarrow 40\times\text{x}+\frac{1}{2}\times 1.25\times 10^6(0^2-\text{x}^2)$$



$$= 0 - \frac{1}{2} \times 4 \times v^2$$

solving x ≈ 4 cm

16. Auniform cable ofmass M' and length L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge ofthe surface. To lift the hanging part of the cable upto the surface, the work done should be: [9 April 2019 I]

(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$ (c) $\frac{2MgL}{n^2}$ (d) nMgL

SOLUTION : . (a)

$$W = u_f - u_j$$

$$= 0 - \left(-\frac{mg}{n} \times \frac{L}{2n}\right) = \frac{MgL}{2n^2}.$$

17 . A wedge ofmass M = 4m lies on a fiictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by: [9 April 2019 II]

(a) $\frac{v^2}{g}$ (b) $\frac{2v^2}{7g}$ (c) $\frac{2v^2}{5g}$ (d) $\frac{v^2}{2g}$

SOLUTION : (c)

$$mv = (m + M)V'$$

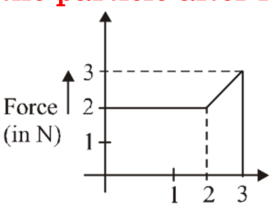
$$\text{or } v = \frac{mv}{m+M} = \frac{mv}{m+4m} = \frac{v}{5}$$

Using conservation ofME, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}(m + 4m)\left(\frac{v}{5}\right)^2 + mgh$$

$$\text{or } h = \frac{2v^2}{5g}$$

18. A particle moves in one dimension from rest under the influence ofa force that varies with the distance travelled bythe particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is: [8 April 2019 I]



Dist$_c$mce (in m) →

(a) 4 J (b) 2.5J (c) 6.5J (d) 5 J

SOLUTION : (c)

We know area under $\Gamma - x$ graph gives the work done by the body

$$W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$$

$$= 2.5 + 4$$

$$= 6.5J$$

Using work energy theorem,

$$\Delta\text{ K.E} = \text{work done}$$

$$\Delta\text{ K.E} = 6.5J$$

19. A particle which is experiencing a force, given by $\vec{\Gamma} = 3\hat{\imath} - 12\hat{\jmath}$, undergoes a displacement of $\vec{d} = 4\hat{\imath}$. If the particle had a kinetic energy of 3 J at the beginning ofthe displacement, what is its kinetic energy at the endof the displacement?

[10 Jan. 2019 II]

(a) 9 J (b) 12 J (c) 10 J (d) 15 J

SOLUTION : . (d)

$$\text{Work done} = \vec{\Gamma}.\vec{d} = (3\hat{\imath} - 12\hat{\jmath}).(4\hat{\imath}) = 12J$$

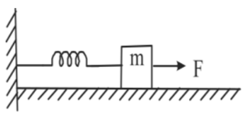
From work energy theorem,

$$w_{net} = \Delta K.E. = k_f - k_i$$

$$\Rightarrow 12 = k_f - 3$$

$$K_f = 15J$$

20. A block ofmass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. Ifnow the block is pulled with a constant force Γ, the maximum speed ofthe block is: [9 Jan. 2019 I]



(a) $\frac{2\Gamma}{\sqrt{mk}}$ (b) $\frac{\Gamma}{\pi\sqrt{mk}}$ (c) $\frac{\pi\Gamma}{\sqrt{mk}}$ (d) $\frac{\Gamma}{\sqrt{nk}}$

SOLUTION : . (d)

Maximum speed is at mean position or equilibrium At equilibrium Position

$$\Gamma = kx \Rightarrow x = \frac{\Gamma}{k}$$

From work-energy theorem,

$$W_\Gamma + W_{sp} = \Delta KE$$

$$\Gamma(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$\Gamma\left(\frac{\Gamma}{k}\right) - \frac{1}{2}k\left(\frac{\Gamma}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{\Gamma^2}{K} = \frac{1}{2}mv^2$$

$$\text{or, } v_{max} = \frac{\Gamma}{\sqrt{mk}}$$

21. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? [9 Jan. 2019 II]

(a) 850 J (b) 950 J (c) 875 J (d) 900 J

SOLUTION : (d)

$$\text{Position, } x = 3t^2 + 5$$

$$\text{Velocity, } v = \frac{dx}{dt} \Rightarrow v = \frac{d(3t^2+5)}{dt}$$

$$\Rightarrow v = 6t + 0$$

$$\text{At } t = 0\ v = 0$$

$$\text{And, at } t = 5 \text{ sec } v = 30 m/s$$

According to work-energy theorem, $w = \Delta KE$ or $^{W=\frac{1}{2}}$ mv2-0 $= \frac{1}{2}(2)\ (30)2 = 900J$

22 . Aparticle is moving in a circular path ofradius a under the action ofan attractive potential $U = -\frac{k}{2r^2}$. Its total energyis: [2018]

a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

SOLUTION : . (c)

$$\Gamma = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$$

Since particle is moving in circular path

$$\Gamma = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$$

$$K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

$$\text{Total energy} = P.E. + K.E.$$

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero } (\because P.E. = -\frac{K}{2r^2} \text{ given})$$

23. Two particles ofthe same mass m are moving in circular orbits because of force, given by $\Gamma(r) = -r^3 \underline{-16}$

r The first particle is at a distance = 1 , and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to

[Online April 16, 2018]

(a) 10^{-1} (b) 6×10^{-2} (c) 6×10^{2} (d) 3×10^{-3}

SOLUTION : . (b)

As the particles moving in circular orbits, So

mv^2 16 2

$$-=-+r$$
$$r\ r$$

Kinetic energy, $KE_0 = \frac{1}{2}mv^2 = \frac{1}{2}[16+r^4]$

For first particle, $= 1$, $K_1 = \frac{1}{2}m(16+1)$

Similarly, for second particle, $r = 4$, $K_2 = \frac{1}{2}m(16+256)$

$$\frac{K_1}{K_2} = -\frac{16+256}{2}\frac{16+1}{2} = \frac{17}{272} = 6\times10^{-2}$$

24. A body ofmass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional forc $\Gamma = -kv^2$. Its intial speed is $v_0 = 10ms^{-1}$. If, after 10 s, its energyis $\frac{1}{8}mv_0^2$, the value ofk will be : [2017]

(a) 10^{-4} kg m^{-1}
(b) 10^{-1} kg $m^{-1}s^{-1}$
(c) $10^{-3}kgm^{-1}$
(d) 10^{-3} kg s^{-1}

SOLUTION : (a)

Let V_f is the fmal speed ofthe body.

From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \Rightarrow V_f = \frac{V_0}{2} = 5\ mls$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2\ (10^{-2})\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100K\int_{0}^{10}d\,t$$

$$\frac{1}{5}-\frac{1}{10} = 100K(10) \text{ or, } K = 10^{-4}kgm^{-1}$$

25. An object is dropped fi$_{\text{i}}$om a height hfi$_{\text{i0}}$m the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as $t\to\infty$ is [Online April 8, 2017]

(a) 3 h
(b) ∞
(c) $\frac{5}{3}$h
(d) $\frac{8}{3}$h

SOLUTION : . (a)

(K.E.)' = 50% ofK.E. after hit i.e.,

$$\frac{1}{2}mv'^2 = \frac{50}{100}\times\frac{1}{2}mv^2 \Rightarrow v' = \frac{v}{\sqrt{2}}$$

Coefficient ofrestitution $= \frac{1}{\sqrt{2}}$

Now, total distance travelled by object is (1)

$$H = h(\quad)(\quad) = h\left(\frac{1+-2}{1-\frac{1}{2}}\right) = 3h$$

26. A time dependent force $F = 6t$ acts on a particle ofmass 1 kg. Ifthe particle starts fi: om rest, the work done by the force during the first 1 second will be [2017]

(a) 9 J (b) 18 J (c) 4.5J (d) 22 J

SOLUTION : . (c)

$$\text{Using, } F = \text{ma} = m\frac{dV}{dt}$$

$$6t = 1.\frac{dV}{dt} \; [m = 1 \text{ kg given}]$$

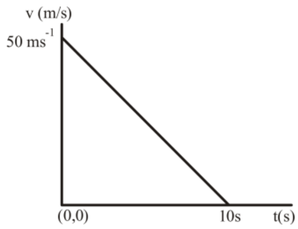
$$\int_0^v dV = \int 6\, tdtV = 6\left[\frac{t^2}{2}\right]_0^1 = 3\text{ms}^{-1}$$

$[t = 1 \text{ sec given}]$ From work-energy theorem,

$$\text{W} = \Delta\text{KE} = \frac{1}{2}m(V^2 - u^2) = \frac{1}{2} \times 1 \times 9$$

$$= 4.5\text{J}$$

27. Velocity - time graph for a body ofmass 10kg is shown in figure. Work - done on the body in first two seconds ofthe motion is: [Online Apri110, 2016]



(a) -9300J (b) 12000 J (c) -4500J (d) -12000J

SOLUTION : . (c)

$$\text{Acceleration (a)} = \frac{\text{v}-4}{\text{t}} = \frac{(0-50)}{(10-0)} = -5\text{m/s}^2$$

$$\text{u} = 50\text{m/s}$$

$$\text{v} = \text{u} + \text{at} = 50 - 5\text{t}$$

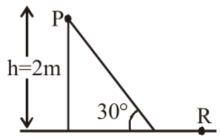
Veocity in first two seconds $\text{t} = 2$

$$\text{v}_{(\text{att}=2)} = 40\text{m/s}$$

From work-energy theorem,

$$\triangle \text{K.E.} = \text{W} = \frac{1}{2}(40^2 - 50^2) \times 10 = -4500\text{J}$$

28. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, fi_{i0}m rest fi_{i}om the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, ofthe track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.
The value ofthe coefficient of friction μ and the distance $(= \text{QR})$, are, respectively close to: [2016]



Horizontal → Q

Surface

(a) $0.29 and 3.5m$ (b) $0.29 and 6.5m$

(c) 0.2 and $6.5m$ (d) 0.2 and $3.5m$

SOLUTION : **(a)**

Work done by fiiiction at QR $= \mu mgx$

In triangle, $\sin 30^\circ = \frac{1}{2} = \frac{2}{PQ}$

$\Rightarrow PQ = 4m$

Work done by friction at PQ $= \mu mg \times \cos 30^\circ \times 4$

$$= \mu mg \times \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}\mu mg$$

Since work done by fiiction on parts PQ and QR are equal, $\mu mgx = 2\sqrt{3}$ umg

$\Rightarrow X = 2\sqrt{3} \cong 3.5m$

Usingwork energytheorem mg $\sin 30^\circ \times 4 = 2\sqrt{3}\mu mg + \mu mX$

$\Rightarrow 2 = 4\sqrt{3}\mu$

$\Rightarrow \mu = 0.29$

29. A person trying to lose weight byburning fat lifts a mass of10 kg upto a height ofl m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8ms^{-2}$: [2016]

(a) 9.89×10^{-3} kg (b) 12.89×10^{-3} kg

(c) 2.45×10^{-3} kg (d) 6.45×10^{-3} kg

SOLUTION : **(b)**

$$n = \frac{W}{input} = \frac{mgh\times1000}{input} = \frac{10\times9.8\times1\times1000}{input} \text{ Input} = \frac{98000}{0.2} = 49 \times 10^4 J$$

$$\text{Fat used} = \frac{49\times10^4}{3.8\times10^7} = 12.89 \times 10^{-3} kg.$$

30. A particle is moving in a circle ofradius r under the action of a force $\Gamma = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) ofthe particle is (take μ)tential energy = 0 for r = 0): [Online April 11, 2015]

(a) $\frac{1}{2}\alpha r^3$ (b) $\frac{5}{6}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) ff

SOLUTION : **(b)**

As we know, $dU = \Gamma . dr$

$$U = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3} \text{ (i) As, } \underline{mv^2}_{=\alpha r^2}$$

$$r$$

$$m^2v^2 = mar^3$$

or, $2m(KE) = \frac{1}{2}\alpha r^3$ (ii)

Total energy = Potential energy + kinetic energy Now, fi¡om eqn (i) and (ii)

Total energy = K. E. +P. E.

$$= \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$$

31. A block ofmass $m = 0.1$ kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity $3ms^{-1}$.
The total initial energy ofthe spring is: [Online April 10, 2015]

(a) 0.3 J (b) 0.6 J (c) 0.8J (d) 1.5J

SOLUTION : . (b)

Applying momentum conservation

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.1u + m(0) = 0.1(0) + m(3)$$

$$0.1u = 3m$$

$$\frac{1}{2}0.1u^2 = \frac{1}{2}m(3)^2$$

Solving we get, $u = 3$

$$\frac{1}{2}kx^2 = \frac{1}{2}K\left(\frac{x}{2}\right)^2 + \frac{1}{2}(0.1)32$$

$$\Rightarrow \frac{3}{4}kx^2 = 0.9$$

$$\Rightarrow \frac{3}{2} \times \frac{1}{2}kx^2 = 0.9$$

$\frac{1}{2}K\kappa^2 = 0.6$J (total initial energy of the spring)

32. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be: [Online Apri119, 2014]

(a) $\frac{n^2}{2n-1}$ (b) $\frac{2n^2}{n-1}$ (c) infinite (d) n

SOLUTION : (a)

Let u be the initial velocity ofthe bullet ofmass m. After passing through a plank of width x, its velocity decreases to v.

$$u - v = \frac{4}{n} \text{ or, } v = u - \frac{4}{n} = \frac{u(n-1)}{n}$$

If Γ be the retarding force applied by each plank, then using work- energy theorem,

$$\Gamma x = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$$

$$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$$

$$\Gamma x = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet.

Total distance travelled by the bullet before coming to rest = Px

Using work-energy theorem again,

$$\Gamma(Px) = \frac{1}{2}\text{mu2-0}$$

$$\text{or, } P(\Gamma x) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$$

$$P = \frac{n^2}{2n-1}$$

33. A spring ofunstretched length 1 has a mass m with one end fixed to a rigid support. Assuming spring to be made ofa uniform wire, the kinetic energypossessed by it ifits free end is pulled with uniform velocity v is: [Online Apri112, 2014]

(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{1}{3}mv^2$ (d) $\frac{1}{6}mv^2$

SOLUTION : . (d)

34. Two springs of force constants 300 N/m (Spring A) and 400 N/m(Spring B) arejoined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is $\frac{E_A}{E_B}$. Then $\frac{E_A}{E_B}$ is equalto: [Online April 9, 2014]

(a) $\frac{4}{3}$ (b) $\frac{16}{9}$ (c) $\frac{3}{4}$ (d) $\frac{9}{16}$

SOLUTION : . (a)

Given : $k_A = 300N/m, k_B = 400N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x.

Therefore compression in spring B

$$x_B = (8.75 - x)\text{ cm}$$

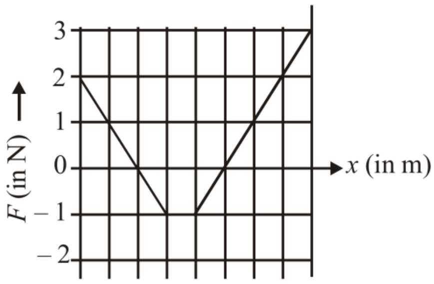
$$\Gamma = 300 \times x = 400(8.75 - x)$$

Solving we get, $x = 5$ cm

$$x_B = 8.75 - 5 = 3.75\text{cm}$$

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300 \times (.5)^2}{400 \times (375)^2} = \frac{4}{3}$$

35. The force $\vec{F} = F\hat{\imath}$ on a particle ofmass 2 kg, moving along the x - axis is given in the figure as a function ofits position x. The particle is moving with a velocity of5 m/s along the x - axis at $x = 0$. What is the kinetic energy ofthe particle at$x = 8$m? [Online May 26, 2012]



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(a) 34 J (b) 34. 5J (c) 4. 5J (d) 29. 4J

SOLUTION : (d)

36. A particle gets displaced by $\Delta\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$m under the action of a force $\vec{F} = (7\hat{\imath} + 4\hat{\jmath} + 3\hat{k})$. The change in its kinetic energy is [Online May 7, 2012]

(a) 38 J (b) 70 J (c) 52. 5J (d) 126 J

SOLUTION : (a)

According to work-energy theorem,

Change in kinetic energy = work done =→ $F.\Delta \rightarrow r$ =($7\hat{\imath}+4j+3\hat{k}$).($2\hat{\imath}+3j+4\hat{k}$)

$= 14 + 12 + 12 = 38$J

37. At time $t = 0$ a particle starts moving along the x - axis. If its kinetic energy increases uniformly with time 't' , the net force acting on it must be proportional to [2011 RS]

(a) constant (b) t (c) $\frac{1}{\sqrt{t}}$ (d) $\sqrt{t}$

SOLUTION : (c)

$$\text{K.E.} \propto t$$

$$\text{K.E.} = ct \text{ [Here, } c = \text{constant]}$$

$$\Rightarrow \frac{1}{2}mv^2 = ct$$

$$\Rightarrow \frac{(mv)^2}{2m} = ct$$

$$\Rightarrow \frac{p^2}{2m} = ct (\because p = mv)$$

$$\Rightarrow p = \sqrt{2ctm}$$

$$\Rightarrow F = \frac{dp}{dt} = \frac{d(\sqrt{2ctm})}{dt}$$

$$\Rightarrow F = \sqrt{2cm} \times \frac{1}{2\sqrt{t}}$$

$$\Rightarrow F \propto \frac{1}{\sqrt{t}}$$

38. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. Ifthe dissociation energy of the molecule is $D=[U(x=\infty)-U_{\text{at}}\text{ equilibrium}]$, D is [2010]

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

SOLUTION : . (d)

$$\text{At equilibrium: } F=\frac{-dU(x)}{dx}$$

$$\Rightarrow F=\frac{-d}{dx}\left[\frac{a}{x^{12}}-\frac{b}{x^6}\right]$$

$$\Rightarrow F=-\left[\frac{12a}{x^{13}}+\frac{6b}{x^7}\right]$$

$$\Rightarrow \frac{12a}{x^{13}}=\frac{6b}{x^7}\Rightarrow x=\left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

$$U_{\text{at}}\text{ equilibrium}=\frac{a}{\left(\frac{2a}{b}\right)^2}-\frac{b}{\left(\frac{2a}{b}\right)}=-\frac{b^2}{4a}\text{ and }U_{(x=\infty)}=0$$

$$D=0-\left(-\frac{b^2}{4a}\right)=\frac{b^2}{4a}$$

39. An athlete in the olympic games covers a distance of100 m in 10 s. His kinetic energy can be estimated to be in therange [2008]

(a) 200 J - 500J (b) $2\times10^5\text{J}-3\times10^5\text{J}$

(c) 20, 000 J - 50,000 J (d) 2,000 J - 5, 000 J

SOLUTION : . (d)

The average speed ofthe athelete

$$v=\frac{5}{t}=\frac{100}{10}=10\text{m/s}$$

$$\text{K. E.}=\frac{1}{2}mv^2$$

Assuming the mass of athelet to 40 kg his average K.E would be

$$\text{K. E}=\frac{1}{2}\times40\times(10)^2=2000\text{J}$$

Assuming mass to 100 kg average kinetic energy

$$\text{K.E.}=\frac{1}{2}\times100\times(10)^2=5000\text{J}$$

40. A2 kg block slides on ahorizontal floor with a speed of4m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic fiiction force is 15N and spring constant is 10,000 N/m. The spring compresses by [2007]

(a) 8.5 cm (b) 5.5 cm (c) 2.5 cm (d) 11.0 cm

SOLUTION : . (b)

Suppose the spring gets compressed by x before stopping.

kinetic energy of the block = P. E. stored in the spring + work done against fiiction.

$$\Rightarrow \frac{1}{2}\times 2\times(4)^2=\frac{1}{2}\times 10,000\times x^2+15\times x$$

$$\Rightarrow 10,000x^2+30x-32=0$$

$$\Rightarrow 5000x^2+15x-16=0$$

$$x=\frac{-15\pm\sqrt{(15)^2-4\times(5000)(-16)}}{2\times 5000}$$

$$=0.055\text{m}=5.5\text{cm}.$$

41. Aparticle isprojected at$60^{o}$to the horizontal with akinetic energy K. The kinetic energy at the highest point is

(a) K *12* (b) K [2007] (c) Zero (d) K *14*

SOLUTION : (d)

Let u be the velocitywith which the particle is thrown and m be the mass ofthe particle. Then

$$K=\frac{1}{2}mu^2\ldots(1)$$

At the highest point the velocity is u cos 60° (only the horizontal component remains, the vertical component being zero at the top-most point). Therefore kinetic energy at the highest point.

$$K'=\frac{1}{2}m(u\cos 60^\circ)^2=\frac{1}{2}mu^2\cos^2 60^\circ=\frac{K}{4}\text{ [From 1]}$$

42. A particle ofmass 100g is thrown verticallyupwards with a speed of 5 m/s. The work done by the force ofgravity during the time the particle goes up is [2006]

(a) -0.5J (b) -1.25J (c) 1.25J (d) 0.5J

SOLUTION : . (b)

Given, Mass ofthe particle, $m=100$g Initial speed ofthe particle, $\mu=5$m/s

Final speed of the particle, $v=0$

Work done by the force of gravity

= Loss in kinetic energy ofthe body.

$$=\frac{1}{2}m(v^2-u^2)=\frac{1}{2}\times\frac{100}{1000}(0^2-5^2)$$

$$=-1.25\text{J}$$

43. The potential energyofa 1 kg particle free to move along the x - axis is given by $V(x)=\left(\frac{x^4}{4}-\frac{x^2}{2}\right)J$. The total mechanical energy ofthe particle is 2 J. Then, the maximum speed (in m/s) is [2006]

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

SOLUTION : . (a)

Potential energy

$$V(x)=\frac{x^4-x^2}{42}\text{ joule}$$

Formaxima ofminima

$$\frac{dV}{dx}=0\Rightarrow x^3-x=0\Rightarrow x=\pm 1$$

$$\Rightarrow \text{ Min . P.E.}=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}\text{J}$$

$$\text{K. E}_{(\max .)}+\text{P. E}_{(\min .)}=2\text{ (Given)}$$

$$\text{K. E}_{(\max .)}=2+\frac{1}{4}=\frac{9}{4}$$

$$\text{K.E. } \max .=\frac{1}{2}mv^2_{\max .}$$

$$\Rightarrow \frac{1}{2}\times 1\times v^2_{\max .}=\frac{9}{4}\Rightarrow v_{\max .}=\frac{3}{\sqrt{2}}$$

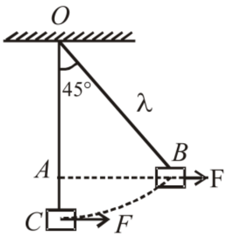
44. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is [2006]

(a) $Mg(\sqrt{2}+1)$ (b) $Mg^{\sqrt{2}}$ (c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2}-1)$

SOLUTION : (d)

Work done by tension + Work done by force (applied) + Work done by gravitational force = change in kinetic energy

Work done by tension is zero



$$\Rightarrow 0+F\times AB\text{-}Mg\times AC=0$$

$$\Rightarrow F=\text{Mg}\left(\frac{\text{AC}}{\text{AB}}\right)=\text{Mg}\left[\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right]$$

$$[\because AB=\ell\sin 45^{\circ}=\frac{\ell}{\sqrt{2}}\text{ and}$$

$$AC=OC-OA=\ell-\ell\cos 45^{\circ}=\ell\left(1-\frac{1}{\sqrt{2}}\right)$$

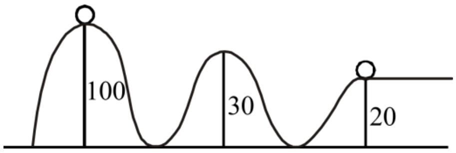
where ℓ = length ofthe string.]

$$\Rightarrow F=Mg(\sqrt{2}-1)$$

45 . A spherical ball ofmass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill ofheight 30 m and finally rolls down to a horizontal base at a height of20 m above the ground. The velocity attained by the ball is [2005]

(a) 20 m/s (b) 40 m/s (c) $\frac{10\sqrt{30} m}{s}$ (d) 10 m/s

SOLUTION : . (b)



$$mgH \frac{1}{2}mv^2 + mgh$$

Using conservation of energy,
Total energy at 100 m height
= Total energy at 20m height

$$m(10 \times 100) = m\left(\frac{1}{2}v^2 + 10 \times 20\right)$$

$$\text{or } \frac{1}{2}v^2 = 800 \text{ or } v = \sqrt{1600} = 40 m/s$$

Note :
Loss in potential energy = gain in kinetic energy

$$m \times g \times 80 = \frac{1}{2}mv^2$$

$$10 \times 80 = \frac{1}{2}v^2$$

$$v^2 = 1600 \text{ or } v = 40 m/s$$

46. A particle moves in a straight line with retardation proportional to its displacement. Its loss ofkinetic energy for any displacement x is proportional to [2004]

(a) x (b) e^x (c) *Reject* (d) $\log_e x$

SOLUTION : . (c)

Given: retardation ∝ displacement $i.e., a = -kx$ [Here, k = constant]

$$\text{But } a = v\frac{dv}{dx}$$

$$\frac{vdv}{dx} = -h \Rightarrow \int_{v_1}^{v_2} v\,dv = -\int_0^x \quad kxdx$$

$$\Rightarrow \left[\frac{v^2}{2}\right]_{v_1}^{v_2} = -k\left[\frac{x^2}{2}\right]_0^x$$

$$\Rightarrow (v_2^2 - v_1^2) = -\frac{kx^2}{2}$$

$$\Rightarrow \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}m\left(\frac{-kx^2}{2}\right)$$

Loss in kinetic energy, $\Delta K \propto x^2$

47. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity ofthe particle, the motion ofthe particles takes place in a plane. It follows that [2004]

(a) its kinetic energy is constant
(b) its acceleration is constant
(c) its velocity is constant
(d) it moves in a straight line

SOLUTION : . (a)

Work done by such force is always zero since force is acting in a direction perpendicular to velocity.

From work-energy theorem $= \Delta K = 0$

K remains constant.

48. A wire suspended vertically from one of its ends is stretched by attaching a weight of$200\mathrm{N}$ to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is [2003]

(a) $0.2\mathrm{J}$
(b) $10\mathrm{J}$
(c) $20\mathrm{J}$
(d) $0.1\mathrm{J}$

SOLUTION : **(d)**

The elastic potential energy

$$= \frac{1}{2} \times \text{Force} \times \text{extension}$$

$$= \frac{1}{2} \times F \times x$$

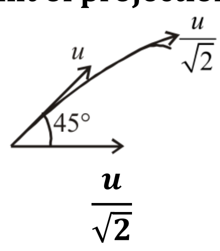
$$= \frac{1}{2} \times 200 \times 0.001 = 0.1\mathrm{J}$$

49. Aball whose kinetic energy is$E$, is projected at an angle of 45° to the horizontal. The kinetic energy ofthe ball at the highest point ofits flight will be [2002]

(a) E
(b) $El\sqrt{2}$
(c) $El2$
(d) zero

SOLUTION : . (c)

Let u be the speed with which the ball ofmass m is projected. Then the kinetic energy (E) at the point of projection is



$$E = \frac{1}{2} m u^2 \text{ (i)}$$

When the ball is at the highest point ofits flight, the speed of the ball is $\frac{u}{\sqrt{2}}$ (Remember that the horizontal component of velocity does not change during a projectile motion).

The kinetic energy at the highest point

$$= \frac{1}{2} m \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2} \frac{mu^2}{2} = \frac{E}{2} \text{ [From (i)]}$$

POWER

50. A body ofmass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) [5 Sep. 2020 (II)]

SOLUTION : **. (18)**

Given, Mass ofthe body, $m = 2$ kg Power delivered by engine, $P = 1 \mathrm{J/s}$

Time, $t = 9$ seconds

Power, $P = Fv$

$$\Rightarrow P = mav \; [\because F = ma]$$

$$\Rightarrow m\frac{dv}{dt}v = P \left(\because a = \frac{dv}{dt}\right)$$

$$\Rightarrow vdv = \frac{P}{m}dt$$

Integrating both sides we get

$$\Rightarrow \int_0^v v\,dv = \frac{P}{m}\int_0^t d\,t$$

$$\Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

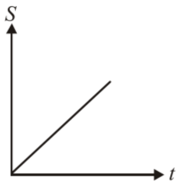
$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \left(\because v = \frac{dx}{dt}\right)$$

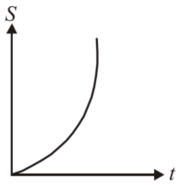
$$\Rightarrow \int_0^x d\,x = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

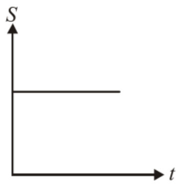
$$\text{Distance, } x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2}$$

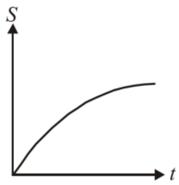
$$\Rightarrow x = \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18.$$

51. Aparticle is moving unidirectionally on ahorizontal plane under the action of a constant power supplying energy source. The displacement (s) - time(t) graph that describes the motion ofthe particle is(graphs are drawn schematically and are not to scale): [3 Sep. 2020 (II)]

(a) 

(b) 

(c) 

(d) 

SOLUTION : . (b)

We know that

Power, $P = Fv$

But $F = mav = m\frac{dv}{dt}v$

$$P = mv\frac{dv}{dt} \Rightarrow Pdt = mvdv$$

Integrating both sides $\int_0^t P\,dt = m\int_0^v v\,dv$

$$\text{P.t} = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

$$\text{Distance, } s = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt = \sqrt{\frac{2P}{m}}\cdot\frac{t^{3/2}}{3/2}$$

$$\Rightarrow s = \sqrt{\frac{8P}{9m}}\cdot t^{3/2} \Rightarrow s \propto t^{3/2}$$

So, graph (b) is correct.

52. A 60 HP electric motor lifts an elevator having a maximum total load capacity of200kg. Ifthe fiictional force on the elevator is 4000 N, the speed ofthe elevator at full load is close to: (1 HP = 746W, g = 10 ms 2) [7 Jan. 2020 I]

(a) 1.7 ms 1 (b) 1.9 ms 1 (c) 1.5 ms 1 (d) 2.0 ms 1

SOLUTION : . (b)

Total force required to lift maximum load capacity against fiictional force $= 400N$

$$F_{\text{tota1}} = Mg + \text{friction}$$

$$= 2000 \times 10 + 4000$$

$$= 20,000 + 4000 = 24000N$$

Using power, $P = F \times v$

$$60 \times 746 = 24000 \times v$$

$$\Rightarrow v = 1.86\text{m/s} \approx 1.9 mls$$

Hence speed ofthe elevator at full load is close to 1.9 ms 1

53. Aparticle ofmass Mis moving in a circle offixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is: [Online April 10, 2016]

(a) $\frac{1}{2}Mn^2R^2t^2$ (b) Mn^2R^2t (c) MnR^2t^2 (d) MnR^2t

SOLUTION : . (b)

$$\text{Centripetel acceleration } a_c = n^2Rt^2$$

$$a_c = \frac{v^2}{R} = n^2Rt^2$$

$$v^2 = n^2R^2t^2$$

$$v = nRt$$

$$a_c = \frac{dv}{dt} = nR$$

$$\text{Power} = ma_tv = m\, nRnRt = Mn^2R^2t.$$

54. A car ofweight W is on an inclined road that rises by 100 m over a distance of 1 Km and applies a constant fiictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 s^{-1} , the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is: [Online April 9, 2016]

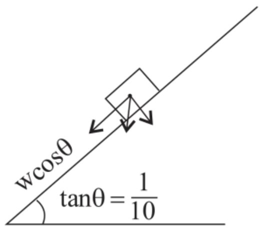
(a) 20 ms^{-1}(b)5ms^{-1} (c) 15 ms^{-1} (d) 10 ms^{-1}

SOLUTION : . (c)

While moving downhill power

$$P = \left(w \sin\theta + \frac{w}{20}\right)10$$

$$P = \left(\frac{w}{10} + \frac{w}{20}\right)10 = \frac{3w}{2}$$



$$\frac{P}{2} = \frac{3w}{4} = \left(\frac{w}{10} - \frac{w}{20}\right)V$$

$$\frac{3}{4} = \frac{v}{20} \Rightarrow v = 15 m/s$$

Speed ofcar while moving downhill v = 15m/s.

55. Awind - poweredgeneratorconvertswindenergyintoelectrical energy. Assume that the generator converts a fixed fiiaction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to [Online April 25, 2013]

(a) v^4 (b) v^2 (c) v (d) v

SOLUTION : . (d)

56. A70kgman leaps vertically into the air from acrouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The center ofgravity rises by $0.5\mathrm{m}$ before he leaps. After the leap the c.g. rises by another $1\ \mathrm{m}$. The maximum power delivered by the muscles is : (Take $\mathrm{g}=10\mathrm{ms}^{-2}$)

[Online April 23, 2013]

(a) 6.26×10^{3} Watts at the start
(b) 6.26×10^{3} Watts at take off
(c) 6.26×10^{4} Watts at the start
(d) 6.26×10^{4} Watts at take off

SOLUTION : . (b)

57. Abody ofmass m', accelerates uniformlyfrom rest to v_1' in time t_1'. The instantaneous power delivered to the body as a hnCt5$^{\dot{}onoftimet}$is [2004]

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$ (c) $\frac{mv_1t}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$

SOLUTION : (b)

Let a be the acceleration ofbody Using, $v=u+at$

$$v_1=0+at_1\Rightarrow a=\frac{v_1}{t_1}$$

Velocity ofthe body at instant t,

$$v=at$$

$$\Rightarrow v=\frac{v_1t}{t_1}$$

Instantaneous powr, $\mathrm{P}=\vec{F}\cdot\vec{v}=(m\vec{a})\cdot\vec{v}$

$$=\left(\frac{mv_1}{t_1}\right)\left(\frac{v_1t}{t_1}\right)=m\left(\frac{v_1}{t_1}\right)^2t$$

58. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t' is proportional to [2003]

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

SOLUTION : . (b)

Power, $P=Fv=ma.v$

$$\Rightarrow P=\frac{mdv}{dt}v=c=\text{contant}$$

$$\left(\because F=ma=\frac{mdv}{dt}\right)$$

$$mv_0v=cdt$$

Integrating both sides, we get

$$m\int_0^v v\,dv=c\int_0^t d\,t$$

$$\Rightarrow\frac{1}{2}mv^2=ct$$

$$\Rightarrow\frac{v^2}{2}=\frac{c.t}{m}$$

$$\Rightarrow v_2=\frac{2c.t}{m}$$

$$\Rightarrow v = \sqrt{\frac{2c}{m}} \times t^{1/2}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2c}{m}} \times t^{1/2} \text{ where } v = \frac{dx}{dt}$$

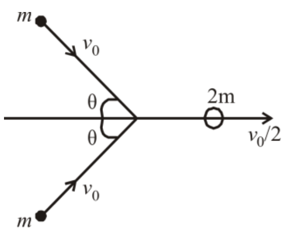
$$\Rightarrow \int_e^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2} dt$$

$$\Rightarrow x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \Rightarrow x \propto t^{3/2}$$

COLLISIONS

59. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is halfoftheir initial speed. The angle between the initial velocities of the two bodies (in degree) is . [NA 6 Sep. 2020 (I)]

SOLUTION : . (120)



Momentum conservation along x direction,

$$2mv_0 \cos\theta = 2m\frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

Hence angle between the initial velocities ofthe two bodies

$$= \theta + \theta = 60^\circ + 60^\circ = 120^\circ.$$

60. ParticleAofmass m movingwith velocity $(\sqrt{3}\hat{i} + \hat{j})\text{ms}^{-1}$ collides with another particle B ofmass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities ofparticles A and B after collision respectively. If$m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j})\text{ms}^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is : [6 Sep. 2020 (II)]

(a) 15^o (b) 60^o (c) -45^o (d) 105^o

SOLUTION : (d)

Before collision,

Velocity ofparticle A, $u_1 = (\sqrt{3}\hat{i} + \hat{j})$m/s

Velocity ofparticle B, $u_2 = 0$

After collision,

Velocity ofparticle A, $v_1 = (\hat{i} + \sqrt{3}\hat{j})$

Velocity of particle B, $v_2 = 0$

Using principal ofconservation of angular momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow 2m_2(\sqrt{3}\hat{\imath} + j) + m_2 \times 0 = 2m_2(\hat{\imath} + \sqrt{3}\hat{\jmath}) + m_2 \times \vec{v}_2$$

$$\Rightarrow 2\ 3\hat{\imath}+2j = 2\hat{\imath} + 2\sqrt{3}\hat{\jmath} + \vec{v}_2$$

$$\Rightarrow \vec{v}_2 = (\sqrt{3}-1)\hat{\imath} - (\sqrt{3}-1)j$$

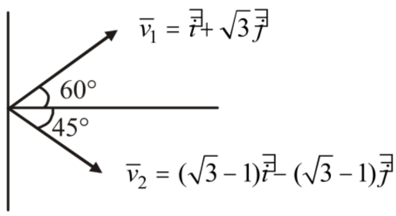
$$\Rightarrow \vec{v}_1 = \hat{\imath}+\sqrt{\ }3\hat{j}$$

For angle between $\vec{v}_1$ and $\vec{v}_2$,

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{\vec{v}_1\vec{v}_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \theta = 105^\circ$$

Angle between $\vec{v}_1$ and $\vec{v}_2$ is 105°



61. Blocks ofmasses m, $2m$, $4m$ and $8m$ are arranged in aline on a fiictionless floor. Another block ofmass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving the total energy loss is p% of the original energy. Value of p' is close to: [4 Sep. 2020 (I)]

(a) 77 (b) 94 (c) 37 (d) 87

SOLUTION : . (b)

According to the question, all collisions are perfectly inelastic, so after the final collision, all blocks are moving together.

m $2m$ $4m$ $8m$

Let the final velocity be v', using momentum conservation

$$mv = 16mv' \Rightarrow v^\dagger = \frac{v}{16}$$

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy: $E_f = \frac{1}{2} \times 16m \times \left(\frac{v}{16}\right)^2 = \frac{1}{2}\frac{mv^2}{16}$

Energy loss : $E_i - E_f = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$$\Rightarrow \frac{1}{2}mv^2\left[1-\frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$$

The total energy loss is $P^0/0$ of the original energy.

$$\%P = \frac{\text{Energy1oss}}{\text{Origina1energy}} \times 100$$

$$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

Hence, value of P is close to 94.

62. A block ofmass 1.9 kg is at rest at the edge ofa table, of height 1 m. Abullet ofmass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10\text{m/s}^2$. Assume there is no rotational motion and losss of energy after the collision is negligiable.] [3 Sep. 2020 (II)]

(a) 20 J (b) 21 J (c) 19 J (d) 23 J

SOLUTION : **(b)**

Given,

Mass ofblock, $m_1 = 1.9$ kg

Mass ofbullet, $m_2 = 0.1$ kg

Velocity of bullet, $v_2 = 20\text{m/s}$

Let v be the velocity of the combined system. It is an inelastic collision.

Using conservation oflinear momentum

$$m_1 \times 0 + m_2 \times v_2 = (m_1 + m_2)v$$

$$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times v$$

$$\Rightarrow v = 1\text{m/s}$$

Using work energy theorem

Work done = Change in Kinetic energy

Let K be the Kinetic energy of combined system.

$$(\mathrm{m_1 + m_2})\mathrm{gh}$$

$$= \mathrm{K} - \frac{1}{2}(\mathrm{m_1 + m_2})\mathrm{V}^2$$

$$\Rightarrow 2 \times g \times 1 = K - \frac{1}{2} \times 2 \times 1^2 \Rightarrow K = 21$$

63. A particle ofmass m with an initial velocity $u\,\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by:

[2 Sep. 2020 (I)]

(a) $v = \sqrt{\frac{2}{3}}u$ (b) $v = \frac{u}{\sqrt{3}}$ (c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{1}{\sqrt{6}}u$

SOLUTION : (c)

From conservation oflinear momentum

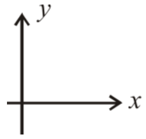
$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$$

$$-, u \wedge v \wedge$$

$$v = -i - -j33$$

$$m - u\ 3m\ m1^{v}\ 3m$$

$$\sim A_{v'}$$



Before

After collision

collision From kinetic energy conservation,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$$

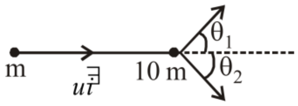
$$\text{or, } mu^2 = mv^2 + \frac{mu^2}{3} + \frac{mv^2}{3}$$

$$v = \frac{u}{\sqrt{2}}$$

64. Aparticle ofmass m is moving along the x - axis with initial velocity $u\hat{i}$. It collides elastically with a particle ofmass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is

[NA 2 Sep. 2020 (II)]

m.



10m

SOLUTION : . (10.00)

From momentum conservation in perpendicular direction of initial motion.

$$mu_1 \sin\theta_1 = 10mv_1 \sin\theta_2 \text{ (i)}$$

It is given that energy of m reduced by half. If u_1 be velocity of m after collision, then

$$\left(\frac{1}{2}mu^2\right)\frac{1}{2} = \frac{1}{2}mu_1^2$$

$$\Rightarrow u_1 = \frac{u}{\sqrt{2}}$$

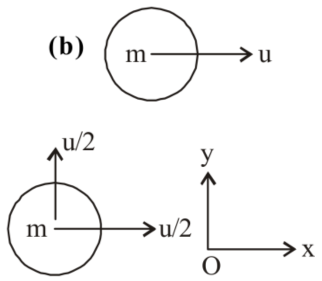
If v_1 be the velocity of mass 10 m after collision, then

$$\frac{1}{2} \times 10m \times v_1^2 = \frac{1u^2}{2^m2} \Rightarrow v_1 = \frac{u}{\sqrt{20}}$$

From equation (i), we have

$$\sin\theta_1 = \sqrt{10}\sin\theta_2$$

(b)



65. Two particles of equal mass m have respective initial $(\hat{\imath}+\hat{\jmath})$ velocities $u\hat{\imath}$ and $u(\overline{2})$. They collide completely inelastically. The energylost in the process is:

[9 Jan. 2020 I]

(a) $\frac{1}{3}mu^2$ (b) $\frac{1}{8}mu^2$ (c) $\frac{3}{4}mu^2$ (d) $\sqrt{\frac{2}{3}}mu^2$

SOLUTION : [b]

x-direction

$$mu + \frac{mu}{2} = 2mv_{\acute{x}} \Rightarrow V_{\acute{x}} = \frac{3u}{4}$$

$$\text{y-direction } 0 + \frac{mu}{2} = 2mv_{\acute{y}} \Rightarrow v_{\acute{y}} = \frac{u}{4} K.E.\, i = \frac{1}{2}mu^2 + \frac{1}{2}m\left[\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]$$

$$= \frac{1}{2}mu^2 + \frac{mu^2}{4} = \frac{3mu^2}{4}$$

$$\text{K.E. } f = \frac{1}{2}(2m)(v_x')^2 + \frac{1}{2}(2m)(v_y')^2$$

$$= \frac{1}{2}2m\left[\left(\frac{3u}{4}\right)^2 + \left(\frac{u}{4}\right)^2\right] = \frac{5}{8}mu^2$$

$$\text{Loss in } KE = KE_f - KE_j$$

$$= mu^2\left(\frac{6}{8} - \frac{5}{8}\right) = \frac{mu^2}{8}$$

66. Abα1yA, ofmass $m = 0.1$ kg has an initialvelocityof3 $\hat{\imath}$ ms 1. It collides elastically with another body, B of the same mass which has an initial velocity of 5 $\hat{j}$ ms 1. After collision, A moves with a velocity $\vec{v} = 4(\hat{\imath} + \hat{\jmath})$. The

energy of$B$ after collision is written as $\frac{x}{10}J$. The value of x is . [NA 8 Jan. 2020 I]

SOLUTION : . (a)

$$\text{For elastic collision } KE_{\rm i} = \mathfrak{M}_f$$

$$\frac{1}{2}m \times 25 + \frac{1}{2} \times m \times 9 = \frac{1}{2}m \times 32 + \frac{1}{2}mv_B^2$$

$$34 = 32 + V_B^2 \Rightarrow V_B = \sqrt{2}$$

$$KE_B = \frac{1}{2}mv_B^2 = \frac{1}{2} \times 0.1 \times 2 = 0.1J = \frac{1}{10}J \quad x = 1$$

67. Aparticle ofmass m is dropped fi¡om a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head - on completely inelastically, the time taken for the combined mass to reach

the ground, in units of $\sqrt{\frac{h}{g}}$ is: [8 Jan. 2020 II]

(a) $\sqrt{\frac{1}{2}}$ (b) $\sqrt{\frac{3}{4}}$ (c) $\frac{1}{2}$ (d) $\sqrt{\frac{3}{2}}$

SOLUTION : **(d)**

Let t be the time taken by the particle dropped from height h to collide with particle thrown upward.

Using,

$$S_1 \text{ T} \frac{h}{4}$$

$$s_2 \text{í T} \frac{3h}{4}$$

$$v^2 - u^2 = 2gh$$

$$\Rightarrow v^2 - 0^2 = 2gh$$

$$\Rightarrow v = \sqrt{2gh}$$

Downward distance travelled

$$S_1 = \frac{1}{2}gt^2 = \frac{1}{2}g.\frac{h}{2g} = \frac{h}{4}$$

Distance of collision point fi¡om ground

$$s_2 = h - \frac{h}{4} = \frac{3h}{4}$$

Speed of (A) just before collision

$$v_1 = gt = \sqrt{\frac{gh}{2}}$$

And speed of(B) just before collision

$$v_2 = \sqrt{2gh} - \sqrt{\frac{gh}{2}}$$

Using principle of conservation of linear momentum

$$mv_1 + mv_2 = 2mv_f$$

$$\Rightarrow v_f = m\left(\sqrt{2gh}\left(-\sqrt{\frac{gh}{2}}\right)\right) - m\sqrt{\frac{gh}{2}} = 0$$

$$2m$$

After collision, time taken (t_1) for combined mass to reach the ground is

$$\Rightarrow \frac{3h}{4} = \frac{1}{2}gt_1^2 \quad \Rightarrow t_1 = \sqrt{\frac{3h}{2g}}$$

68. A man (mass = 50kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms 1 with respect to the man. The speed ofthe man with respect to the surface is:

[12 April 2019 I]

(a) $0.28 \mathrm{ms}^{-1}$ (b) 0.20 ms 1 (c) 0.47 ms 1 (d) 0.14 ms 1

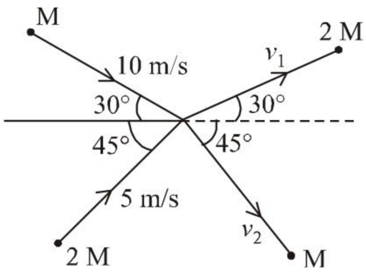
SOLUTION : **(b)**

$$P_i = P_f$$

$$\text{or } 0 = 20(0.7 - v) = 50v \text{ or } v = 0.2 \mathrm{m/s}$$

69. Two particles, ofmasses M and 2M, moving, as shown, with speeds of10 $\mathrm{m/s}$ and 5 $\mathrm{m/s}$, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values ofv_1 and v_2 are nearly:

[10 April 2019 I]



(a) 6.5 m/s and 6.3 m/s (b) 3.2 m/s and 6.3 m/s

(c) 6.5m/sand3.2m/s (d) 3.2m/s and 12.6m/s

SOLUTION : **. (a)**

Apply concervation oflinear momentum in X and Y direction for the system then

$$M(10 \cos 30^\circ) + 2M(5 \cos 45^\circ) = 2M(v_1 \cos 30^\circ)$$
$$+M(v_2 \cos 45^\circ)$$

$$5 + 5\sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} (1)$$

Also

$$2M(5 \sin 45^\circ) - M(10 \sin 30^\circ) = 2Mv_1 \sin 30^\circ - Mv_2 \sin 45^\circ 5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} (2)$$

Solving equation (1) and (2)

$$(\sqrt{3} + 1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5 \Rightarrow v_1 = 6.5 \mathrm{m/s}$$
$$v_2 = 6.3 \mathrm{m/s}$$

70. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth ofits original speed. What is the mass ofthe second body? [9 April 2019 I]

(a) 1.0kg (b) 1.5kg (c) 1.8kg (d) 1.2kg

SOLUTION : **. (b)**

$$2u + 0 = 2\left(\frac{u}{4}\right) + mv_2$$

$$\text{and } \frac{1}{2} \times 2 \times u^2 + 0 = \frac{1}{2} \times 2 \times \left(\frac{u}{4}\right)^2 + \frac{1}{2}mv_2^2 \text{ On solving, we get } m = 1.5k$$

71. A particle of mass 'm' is moving with speed '2v' and collides with a mass '$2m$' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each ofmass m', which move at angle 45° with respect to the original direction. [9 April 2019 II]

The speed of each ofthe moving particle will be:

(a) $\sqrt{2}v$ (b) $2\sqrt{2}v$ (c) $vl(2\sqrt{2})$ (d) $v/\sqrt{2}$

SOLUTION : . (b)

$$m(2v) + 2mv = 0 + 2mv'\cos 45^\circ$$

$$\text{or } v' = 2\sqrt{2}v$$

72. A body ofmass m_1 moving with an unknown velocity of $v_1\,\hat{\imath}$, undergoes a collinear collision with a body ofmass m_2 moving with a velocity $v_2\,\hat{\imath}$. After collision, m_1 and m_2 move with velocities of $v_3\,\hat{\imath}$ and $v_4\,\hat{\imath}$, respectively. If $m_2 = 0.5m_1$ and $v_3 = 0.5v_1$, then v_1 is: [8 April 2019 I]

(a) $v_4 - \frac{v_2}{2}$ (b) $v_4 - v_2$ (c) $v_4 - \frac{v_2}{4}$ (d) $v_4 + v_2$

SOLUTION : . (b)

$$m_1v_1 + m_2v_2 = m_1v_2 + m_2v_1$$

$$\text{or } m_1v_1 + (0.5m_1)v_2 = m_1 0.5v_1 + (0.5m_1)v_4$$

$$\text{On solving, } v_1 = v_4 - v_2$$

73. An alpha - particle ofmass m suffers 1 - dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass ofthe nucleus is: [12 Jan. 2019 II]

(a) 2m (b) 3.5m (c) 1.5m (d) 4m

SOLUTION : . (d)

Using conservation ofmomentum,

$$mv_0 = mv_2 - mv_1$$

$$\alpha V_0 \leftrightarrow$$

After collision

$$\frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv_0^2$$

$$\Rightarrow v_1 = 0.6v_0$$

The collision is elastic. So,

$$\frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mv_0^2 [\because M = \text{mass ofnucleus}]$$

$$\Rightarrow V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0$$

$$mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$$

$$\Rightarrow 1.6m = 0.8\sqrt{mM}$$

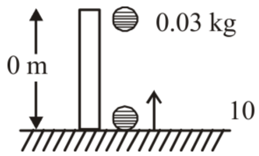
$$\Rightarrow 4m^2 = mM \;\; ; \; M = 4m$$

74. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet ofmass 0.02 kg is fired vertically upward, with a velocity 100 s^{-1} , from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: $(g = 10ms^{-2})$ [10 Jan. 2019 I]

(a) 20m (b) 30m (c) 40m (d) 10m

SOLUTION : (c)

10



0m/s

0.02 kg

Time taken for the particles to collide,

$$t = \frac{d}{V_{re1}} = \frac{100}{100} = 1 \text{ sec}$$

Speed of woodjust before collision = gt = 10m/s and speed of bullet just before collision

$= v - gt$

$= 100 - 10 = 90m/s$

$$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95m$$

Now, using conservation oflinear momentumjust before and after the collision

$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v \; v = 30m/s$

Max . height reached by body

$$h == \frac{30 \times 30}{2 \times 10} = 45m$$

Before After

0.03kg$\boldsymbol{Reject} \downarrow 10m/s \; \boldsymbol{Reject}^{V}$

$\boldsymbol{Reject}$

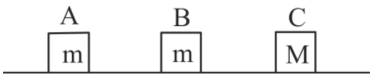
$\uparrow 90m/s$

0.02 kg

0.05 kg

Height above tower = 40m

5. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with



(a) 5 (b) 2 (c) 4 (d) 3

SOLUTION : (c)

Kinetic energy of block A

$$k_1 = \frac{1}{2}mv_0^2$$

From principle oflinear momentum conservation

$$mv_0 = (2m + M)v_f \Rightarrow v_f = \frac{mv_0}{2m + M}$$

According to question, of $\frac{5}{6}$th the initial kinetic energy is lost in whole process.

$$\frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m + M)\left(\frac{mv_0}{2m + M}\right)^2} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6 \therefore \frac{M}{m} = 4$$

76 . In a collinear collision, aparticle with an initial speed v_0 strikes a stationary particle ofthe same mass. Ifthe fmal total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: [2018]

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

SOLUTION . (b)

Before Collision After Collision

$$\infty mv_{00m} \Rightarrow \infty mv_1 \propto mv_2$$

Stationary

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \text{ (i)}$$

From momentum conservation

$$mv_0 = m(v_1 + v_2) \text{ (ii)}$$

Squarring both sides,

$$\Rightarrow v_1 + v_{2^2} + 2v_1v_2 = v_{0^2}(v_1 + v_@)^2 = v_{0^2}$$

$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$$

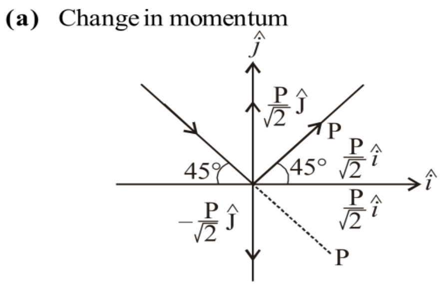
Solving we get relative velocity between the two particles

$$v_1 - v_{2=\sqrt{2}}v_0$$

77. The mass ofa hydrogen molecule is 3.32×10^{-27}kg. If10^{23} hydrogen molecules strike, per second, a fixed wall ofarea 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of103 m/s, then the pressure on the wall is nearly: [2018]

(a) $2.35 \times 10^3 N/m^2$ (b) $4.70 \times 10^3 N/m^2$
(c) $2.35 \times 10^2 N/m^2$ (d) $4.70 \times 10^2 N/m^2$

SOLUTION . (a)



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule}$$

$$\Rightarrow I_{wal1} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{\Gamma}{A} = \frac{\sqrt{2}p}{A} n \ (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}} = 2.35 \times 10^3 N/m^2$$

78. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss ofits energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values ofPdand P_c are respectively: [2018]

(a) $(\cdot 89, \cdot 28)$ (b)$(\cdot 28, \cdot 89)$ (c)$(0, 0)$ (d) $(0, 1)$

SOLUTION (a)

For collision ofneutron with deuterium:

$$m\ 2m\ \tilde{m}\ \overline{2m}$$

$$\underline{v}\ \cdot\ v_1\ v_2$$

Applying conservation ofmomentum:

$$mv + 0 = mv_1 + 2mv_2 \text{ (i)}$$

$$v_2 - v_1 = v \text{ (ii)}$$

Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

$$m\ 12m\ \overline{m}\ \overline{12m}$$

$$\underline{v} \cdot v_1\ v_2$$

Applying Conservation ofmomentum

$$mv + 0 = mv_1 + 12mv_2 \text{ (iii)}$$

$$v = v_2 - v_1 \text{ (iv)}$$

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13}v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

79. A proton ofmass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass ofunknown particle is: [Online April 15, 2018]

(a) $\frac{m}{\sqrt{3}}$ (b) $\frac{m}{2}$ (c) 2n (d) m

SOLUTION : . (d)

Applyprinciple ofconservation ofmomentum along x-direction,

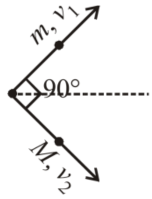
$$mu = mv_1 \cos 45^\circ + Mv_2 \cos 45^\circ$$

$$mu = \frac{1}{\sqrt{2}}(mvi + Mv_2) \text{ ..... (i)}$$

Along y-direction,

$$o = mv_1 \sin 45^\circ - Mv_2 \sin 45^\circ$$

$$o = (mv_1 - Mv_2)\frac{1}{\sqrt{2}} \text{ ..... (ii)}$$



Proton$m,u_1 = u$ Unknown mass$M,u_2 = 0$

Before collision

After collision

Coefficient of restution e $= 1 = \frac{v_2 - v_1 \cos 90}{u \cos 45}$

(Collision is elastic)

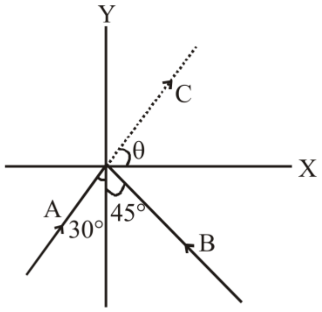
$$\Rightarrow \frac{v_2}{u} = 1$$

$$\sqrt{2}$$

$$\Rightarrow u = \sqrt{2} v_2 \text{ (iii)}$$

Solving eqs (i), (ii), & (iii), we get mass of unknown particle, $M = m$

80. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X - axis is [Online April 9, 2017] given by:



(a) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$ (b) $\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

(c) $\tan\theta = \frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$ (d) $\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{2}}$

SOLUTION : **(a)**

For particle C,

According to law of conservation of linear momentum, verticle component,

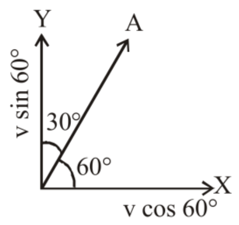
$$2\,mv' \sin\theta = mv \sin 60^\circ + mv \sin 45^\circ$$

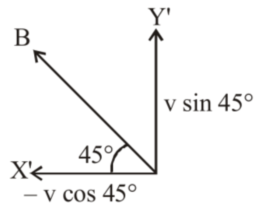
$$2mv' \sin\theta = \frac{mv}{\sqrt{2}} + \frac{mv\sqrt{3}}{2} \text{ ...... (i)}$$

Horizontal component,

$$2\,mv' \cos\theta = mv \sin 60^\circ - mv \cos 45^\circ$$

$$2mv^{\uparrow} \cos\theta = \frac{mv}{2} + \frac{mv}{\sqrt{2}} \text{ ...... (ii)}$$





For particle A For particle B

Dividing eq^n (i) by eq^n (ii),

$$\tan\theta = \frac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}+\sqrt{3}}{1-\sqrt{2}}$$

81. A neutron moving with a speed $v^{\uparrow}$ makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is : [Online Apri110, 2016]

(a) 20.4eV((b)10.2eV (c) 12.1eV (d) 16.8eV

SOLUTION : . (a)

For inelastic collision $v' = \frac{m_1}{(m_1+m_2)} v$

$$= \frac{1}{(1+1)} v = \frac{v}{2}$$

$n \rightarrow v(H)$ Before

$(n)\ (H) \rightarrow \frac{v}{2}$ After

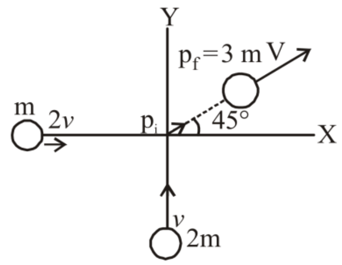
$$\text{Loss in K.E.} = \frac{1}{2} mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4} mv^2$$

K.E. lost is used tojump fiiom 1st orbit to 2nd orbit

$\Delta K.E. = 10.2ev$

Minimum K.E. ofneutron for inelastic collision

$$\frac{1}{2} mv^2 = 2 \times 10.2 = 20.4eV$$

(a) 

82. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : [2015]

(a) 56% (b) 62% (c) 44% (d) 50%

SOLUTION :[a]

Initial momentum of the system

$$p_i = \sqrt{[m(2V)^2 \times 2m(2V)^2]}$$

$$= \sqrt{2}m \times 2V$$

Final momentum of the system = 3mV

By the law of conservation of momentum

$$2\sqrt{2}mv = 3mV \Rightarrow \frac{2\sqrt{2}v}{3} = V_{combined}$$

Loss in energy

$$\Delta E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 - \frac{1}{2}(m_1 + m_2)V_{combined}^2$$

$$DE = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$$

Percentage loss in energy during the collision = 56%

83. A bullet of mass 4g is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of fiiction between the block and the table is 0.3, how far will the block slide approximately? [Online Apri112, 2014]

(a) 0.19m (b) 0.379m (c) 0.569m (d) 0.758m

SOLUTION : **(b)**

Given, $m_1 = 4g, u_1 = 300m/s$

$$m_2 = 0.8kg = 800g, u_2 = 0m/s$$

From law of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Let the velocity of combined system = vm/s then,

$$4 \times 300 + 800 \times 0 = (800 + 4) \times v$$

$$v = \frac{1200}{804} = 1.49m/s$$

Now, $\mu = 0.3$ **(given)**

$$a = \mu g$$

$$a = 0.3 \times 10 \text{ (take } g = 10m/s^2)$$

$$= 3m/s^2$$

then, from $v^2 = u^2 + 2as$

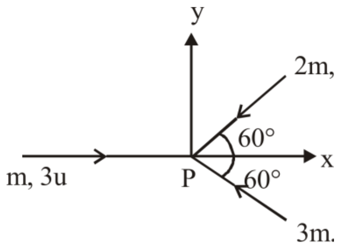
$$(1.49)2 = 0 + 2 \times 3 \times s$$

$$s = \frac{(1.49)^2}{6}$$

$$s = \frac{2.22}{6} = 0.379m$$

84. Three masses m, 2mand3mare moving in x - y plane with speed 3u, 2u and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity ofresulting mass will be: [Online Apri112, 2014]

2u



)u

(a) $\frac{u}{12}(\hat{i}+\sqrt{\ }3j)$ (b) $\frac{u}{12}(\hat{i}-\sqrt{\ }3j)$ (c) $\frac{u}{12}(-\hat{i}+\sqrt{\ }3j)$ d) $\frac{u}{12}(-\hat{i}-\sqrt{\ }3j)$

SOLUTION : . (d)

From the law of conservation ofmomentum we know that,

$$m_1u_1 + m_2u_2 + \cdots . = m_1v_1 + m_2v_2 + \cdots .$$

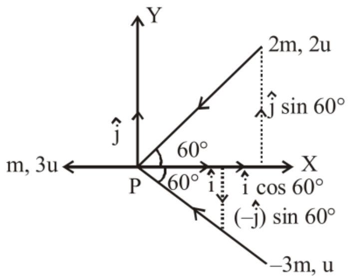
$$\text{Given } m_1 = m, m_2 = 2m \text{ and } m_3 = 3m$$

$$\text{and } u_1 = 3u, u_2 = 2u \text{ and } u_3 = u$$

$$\rightarrow$$

Let the velocity when they stick = v

Then, according to question,



$$m \times 3u\ (\hat{i}) + 2m \times 2u\ (-\hat{i}\cos 60^\circ - \hat{j}\sin 60^\circ)$$

$$+3m \times u\ (-\hat{i}\cos 60\circ + j\sin 60\circ) = (m + 2m + 3m) \rightarrow v$$

$$\Rightarrow 3mu\hat{i} - 4mu\frac{\hat{i}}{2} - 4mu(\quad)(\quad) - 3mu\frac{\hat{i}}{2}$$

$$+3mu\left(\frac{\sqrt{3}}{2}\hat{j}\right) = 6mv \rightarrow$$

$$\Rightarrow mu\hat{i} - \frac{3}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6mv \rightarrow$$

$$\Rightarrow -\frac{1}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6mv \rightarrow$$

$$\Rightarrow \rightarrow v = \frac{u}{12}(-\hat{i}-\sqrt{\ }3j)$$

85. This question has statement I and statement II. Ofthe four choices given after the statements, choose the one that best describes the two statements.

Statement - I: Apoint particle of mass **m** moving with speed **u** collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $=\left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result ofthe collision. [2013]

(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.

(b) Statement - I is true, Statment - II is true, StatementII is not the correct explanation of Statement - II.

(c) Statement - I is true, Statment - II is false.

(d) Statement - I is false, Statment - II is true.

SOLUTION : **. (d)**

$$\text{Maximum energy loss} = \frac{P^2}{2m} - \frac{P^2}{2(m+M)}$$

$$[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case ofperfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

86. A projectile of mass *M* is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses *M/4* and 3*M/*4 respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is : [Online April 23, 2013]

(a) 16 km (b) 1 km (c) 10 km (d) 2 km

SOLUTION : **. (c)**

87. A moving particle of mass *m*, makes a head on elastic collision with another particle ofmass 2*m*, which is initially at rest. The percentage loss in energy of the colliding particle on collision, is close to [Online May 19, 2012]

(a) 33% (b) 67% (c) 90% (d) 10%

SOLUTION : **. (c)**

Fractional decrease in kinetic energy of mass m

$$= 1 - (\quad)(\quad)^2 = 1 - \left(\frac{2-1}{2+1}\right)^2$$

$$= 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

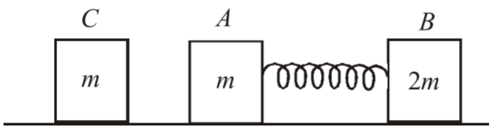
Percentage loss in energy

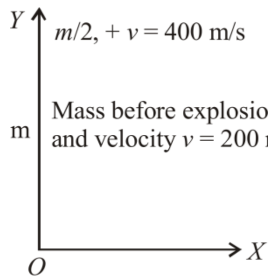
$$= \frac{8}{9} \times 100 = 90\%$$

88. Two bodies A and B of mass m and $2m$ respectively are placed on a smooth floor. They are connected by a spring of negligible mass. A third body C of mass m is placed on the floor. The body C moves with a velocity v_0 along the line joining A and B and collides elastically with A. At a certain time after the collision it is found that the instantaneous velocities of A and B are same and the compression of the spring is x_0. The spring constant k will be [Online May 12, 2012]

(a) $m\frac{v_0^2}{x_0^2}$ (b) $m\frac{v_0}{2x_0}$ (c) $2m\frac{v_0}{x_0}$ (d) $\frac{2}{3}m(\ \ )(\ \ )^2$

SOLUTION : **(d)**





$v_0 \rightarrow$

Initial momentum of the system block $(C) = mv_0$. After striking with A, the block C comes to rest and now both block A and B moves with velocity v when compression in sprin g is x_0.

By the law ofconservation oflinear momentum

$$mv_0 = (m + 2m)v \Rightarrow v = \frac{v_0}{3}$$

By the law of conservation of energy

K.E. ofblock C = K. E. of system +P. E. of system

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{6}mv_0^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2 - \frac{1}{6}mv_0^2 = \frac{mv_0^2}{3}$$

$$k = \frac{2}{3}m(\ \ )(\ \ )^2$$

89. A projectile moving vertically upwards with a velocity of 200 ms^{-1} breaks into two equal parts at a height of490 m. One part starts moving vertically upwards with a velocity of400 ms^{-1}. How much time it will take, after the break up with the other part to hit the ground? [Online May 12, 2012]

(a) $2\sqrt{10}$s (b) 5s (c) 10 s (d) $\sqrt{10}$s

SOLUTION : . (c)

$$n = m$$

490

m/s (vertically)

Momentum before explosion = Momentum after explosion

$$m \times 200\hat{j} = \frac{m}{2} \times 400\hat{j} + \frac{m}{2} v$$

$$= \frac{m}{2}(400\hat{j} + v)$$

$$\Rightarrow 400j - 400j = v$$

$$v = 0$$

i. e., the velocity ofthe other part ofthe mass, $v = 0$

Let time taken to reach the earth by this part be t

Applying formula, $h = ut + \frac{1}{2}gt^2$

$$490 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow t^2 = \frac{980}{9.8} = 100$$

$$t = \sqrt{100} = 10 \text{ sec}$$

90 . Statement - 1: Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement –2 : Principle of conservation of momentum holds true for all kinds of collisions. [2010]

(a) Statement - l is true, Statement - 2 is true; Statement –2 is the correct explanation of Statement - 1.

(b) Statement - l is true, Statement - 2 is true; Statement - 2 is not the correct explanation of Statement - l

(c) Statement - l is false, Statement - 2 is true.

(d) Statement - l is true, Statement - 2 is false.

SOLUTION : . (a)

In completely inelastic collision, all initial kinetic energy is not lost but loss in kinetic energy 15 as large as it can be. Linear momentum remain conserved in all types of collision. Statement -2 explains statement -1 correctly because applying the principle of conservation ofmomentum, we can get the common velocity and hence the kinetic energy of the combined body.

91. A block ofmass 0.50 kg is moving with a speed of $2.00 \mathrm{ms}^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is [2008]

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

SOLUTION : **(c)**

Initial kinetic energy of the system

$$\mathrm{K.E}_j = \frac{1}{2} m u^2 + \frac{1}{2} M (0)^2$$

$$= \frac{1}{2} \times 0.5 \times 2 \times 2 + 0 = 1\mathrm{J}$$

Momentum before collision

= Momentum after collision

$$m_1 u_1 + m_2 u_2 = (m + M) \times v$$

$$0.5 \times 2 + 1 \times 0 = (0.5 + 1) \times v \Rightarrow v = \frac{2}{3} \mathrm{m/s}$$

Final kinetic energy of the system is

$$\mathrm{K.E}\ f^{=\frac{1}{2}(m+M)v^2}$$

$$= \frac{1}{2}(0.5 + 1) \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3} \mathrm{J}$$

Energy loss during collision

$$= \left(1 - \frac{1}{3}\right) \mathrm{J} = 0.67\mathrm{J}$$

92. A bomb ofmass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velolcity ofthe 12 kg mass is 4 ms^{-1}. The kinetic energy ofthe other mass is [2006]

(a) 144 J (b) 288 J (c) 192 J (d) 96 J

SOLUTION : **. (b)**

Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.

Situation 1O16kg Initial momentum $= 0$

$$4\mathrm{kg}_{\mathrm{v}_1}^{=\mathrm{m}} \psi^1 * \mathrm{v}_2 \mathrm{m}_2 = 12\mathrm{kgFina1momentum} = \mathrm{m}_2 \mathrm{v}_2 - \mathrm{m}_1 \mathrm{v}_1$$

Applyi$^{\text{Situation2}}$ngconservation oflinear momentum

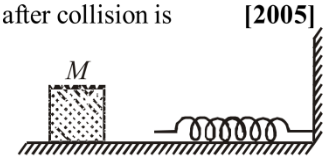
$$16 \times 0 = 4 \times v_1 + 12 \times 4$$

$$\Rightarrow v_1 = -\frac{12 \times 4}{4} = -12 m s^{-1}$$

Kinetic energy of 4 kg mass

$$K.E. = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times 4 \times 144 = 288J$$

93. The block ofmassMmoving on the fiiictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is [2005]



(a) $\frac{kL^2}{2M}$ (b) $\sqrt{MkL}$ (c) $\frac{ML^2}{k}$ (d) zero

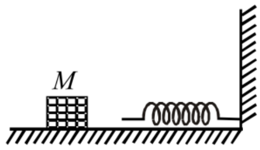
SOLUTION : . (b)

When the spring gets compressed by length L.

K.E. lost bymass M = P. E. stored in the compressed spring.

$$\frac{1}{2}Mv^2 = \frac{1}{2}kL^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{M}} \cdot L$$



Momentum ofthe block, $= M \times v$

$$= M \times \sqrt{\frac{k}{M}} \cdot L = \sqrt{kM} \cdot L$$

94. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision

the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction

ofmotion. Find the speed of the 2nd mass after collision. $[\leftrightarrow^{m} \cdot m \downarrow v/\sqrt{3}$ after before collision [2005]

(a) $\sqrt{3}v$ (b) v (c) $\frac{v}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}v$

SOLUTION : . (d)

Considering conservation ofmomentum along x-direction,

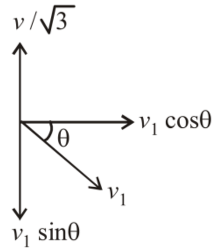
$$mv = mv_1 \cos\theta \text{ (1)}$$

where v_1 is the velocity of second mass

In y-direction,

$$0 = \frac{mv}{\sqrt{3}} - mv_1 \sin\theta$$

$$\text{or } m_1 v_1 \sin\theta = \frac{mv}{\sqrt{3}} \text{ (2)}$$



Squaring and adding eqns. (1) and(2) we get

$$v_{1^2} = v^2 + \frac{v^2}{\sqrt{3}} \Rightarrow v_1 = \frac{2}{\sqrt{3}} v$$

95. Consider the following two statements : [2003] A Linear momentum of a system of particles is zero B. Kinetic energy of a system of particles is zero.

Then

(a) A does not imply B and B does not imply A

(b) A implies B but B does not imply A

(c) A does not imply B but B implies A

(d) A implies B and B implies A

SOLUTION : **. (c)**

Kinetic energy of a system of particle is zero only when the speed of each particles is zero. This implies momentum ofeach particle is zero, thus linear momentum of the system of particle has to be zero.

Also iflinear momentum of the system is zero it does not mean linear momentum ofeach particle is zero. This is because linear momentum is a vector quantity. In this case the kinetic energy of the system ofparticles will not be zero.

A does not imply B but B implies A.

Given, force, $F = 200$N extension ofwire, $x = 1$mm.

SYSTERM OF PARTICLES AND ROTATIONAL MOTION

CENTRE OF MASS

C.M is the point which behaves as if total mass of the body is supposed to be concentrated at that point. This point may lie inside or outside the material of body, but always lies within the space occupied by the body.

Mass may exist or may not exist at the location of centre of mass.

Centre of gravity: The point through which weight of the body acts is called centre of gravity.

Coordinates of C.M.

Coordinates of C.M. of a system of 'n' particles

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + ...+m_nx_n}{m_1 + m_2 + ...+m_n}$$

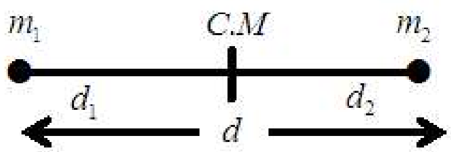
$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + ...+m_ny_n}{m_1 + m_2 + ...+m_n}$$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + ...+m_nz_n}{m_1 + m_2 + ...+m_n}$$

***Case-1:* Position of C.M. of two particle system**:

In case of two bodies, the ratio of distance of centre of mass from the bodies is in the inverseratio of their masses. If m_1 and m_2 are masses of two bodies separated by a distance 'd' then

$$m_1d_1 = m_2d_2 \ (\text{or}) \ \frac{d_1}{d_2} = \frac{m_2}{m_1}$$



Thus C.M. locates near to heavier body.

In figure, $d = d_1 + d_2$.

On solving, $m_1d_1 = m_2d_2$

$$m_1d_1 = m_2 \times (d - d_1)$$

$$\Rightarrow d_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } d_2 = \frac{m_1 d}{m_1 + m_2}$$

d_1, d_2 are the distances of CM from m_1, m_2.

1: If the distance between the centres of the atoms of potassium and bromine in KBr (potassium-bromide) molecule is $0.282 \times 10^{-9} m$, find the centre of mass of this two particle system from potassium mass of bromine = 80 u, and of potassium = 39 u)

Sol: Let position co-ordinate of potassium, $x_k = 0$

Position co-ordinate of bromine,

$x_{Br} = 0.282 \times 10^{-9} m$.

$\therefore$ Position co-ordinate of centre of mass.

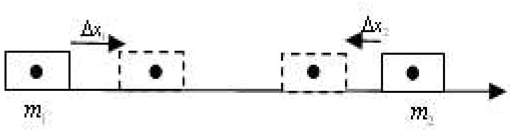
$$x_c = \frac{m_k x_k + m_{Br} x_{Br}}{m_k + m_{Br}}$$

$$\Rightarrow x_c = \frac{39 \times 0 + 80 \times 0.282 \times 10^{-9}}{39 + 80}$$

$$\Rightarrow x_c = 0.189 \times 10^{-9} m.$$

2: Two blocks of masses 10 kg and 30 kg are placed on x-axis. The first mass is moved on the axis by a distance of 2 cm right. By .what distance should the second mass be moved to keep the position of centre of mass unchanged.

Sol:



Mass of the first block, $m_1 = 10 kg$.

Mass of the second block, $m_2 = 30 kg$.

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$0 = \frac{10 \times 2 + 30 \Delta x_2}{40}.$$

$\therefore \Delta x_2 = -\frac{2}{3}$. Therefore the second block should be moved left through a distance of $\frac{2}{3} cm$ to keep the position of centre of mass unchanged.

3. When 'n' number of particles of masses $m, 2m, 3m, ...nm$ are at distances $x_1 = 1, x_2 = 2, x_3 = 3....x_n = n$ units respectively from origin on the x-axis, then find the

distance of centre of mass of the system from origin.

Sol:

$$x_{cm} = \frac{m(1)+2m(2)+3m(3)+...+(nm)n}{m+2m+3m+...+nm}$$

$$x_{cm} = \frac{m(1^2+2^2+3^2+....+n^2)}{m(1+2+3+....+n)}$$

$$x_{cm} = \frac{\left(\frac{n(n+1)(2n+1)}{6}\right)}{\left(\frac{n(n+1)}{2}\right)} = \frac{2n+1}{3}.$$

4 When'n' number of particles of masses $m, 2m, 3m, ...nm$ are at distances $x_1 = 1, x_2 = 4, x_3 = 9, ...x_n = n^2$ units respectively from origin on the x-axis, then find the distance of their centre of mass from origin.

Sol:

$$x_{cm} = \frac{m(1)+2m(4)+3m(9)+....+nm(n^2)}{m+2m+3m+....+nm}$$

$$= \frac{m(1+2^3+3^3+.....+n^3)}{m(1+2+3+....+n)}$$

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2}.$$

5.When 'n' number of particles each of mass ' m ' are at distances $x_1 = a, x_2 = ar, x_3 = ar^2....x_n = ar^n$ units from origin on the x-axis, then find the distance of their centre of mass from origin.

Sol: $x_{cm} = \frac{ma+m(ar)+m(ar^2)+....+m(ar^n)}{m+m+m+...+m(n\,terms)}$

$$x_{cm} = \frac{m(a+ar+ar^2+....+ar^n)}{mn}$$

If $r > 1$ then

$$x_{cm} = \frac{1}{n}\left[\frac{a\left(r^n - 1\right)}{r-1}\right] = \frac{a\left(r^n - 1\right)}{n\left(r-1\right)}$$

If $r < 1$ then

$$x_{cm} = \frac{1}{n}\left[\frac{a\left(1 - r^n\right)}{1-r}\right] = \frac{a\left(1 - r^n\right)}{n\left(1-r\right)}$$

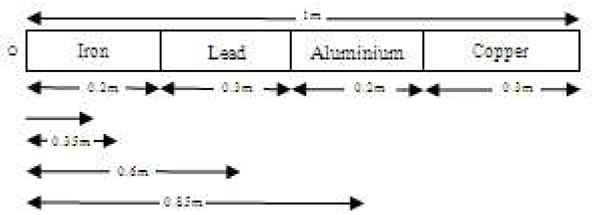
6.A 1m long rod having a constant cross sectional area is made of four materials. The first 0.2 m are made of iron, the next 0.3 m of lead, the following 0.2 m of aluminium and the remaining part is made of copper. Find the centre of mass of the rod. The densities of iron, lead, auminium and copper are

$7.9\times10^3 kg/m^3$, $11.4\times10^3 kg/m^3$,

$2.7\times10^3 kg/m^3$ and $8.9\times10^3 kg/m^3$

respectively.

Sol:



$$mass(m) = volume(v)\times density(d)$$

$$m = Area(A)\times length(l)\times density(d)$$

$$m = Ald.$$

Mass of iron part, $m_1 = A\times0.2\times7.9\times10^3 = 1.58\times10^3 A.$

Mass of lead part, $m_2 = A\times0.3\times11.4\times10^3 = 3.42\times10^3 A.$

Mass of aluminium part,

$$m_3 = A\times0.2\times2.7\times10^3 = 0.54\times10^3 A.$$

Mass of copper part,

$$m_4 = A\times0.3\times8.9\times10^3 = 2.67\times10^3 A.$$

Co-ordinate of iron part from end "O" of the rod $x_1 = 0.1m$.

Co-ordinate of lead part from end "O" of the rod $x_2 = 0.35m$.

Co-ordinate of aluminium part from end "O" of the rod, $x_3 = 0.6m$.

Co-ordinate of copper part from end "O" of the rod, $x_4 = 0.85m$.

$\therefore$ Centre of mass of the rod,

$$X_c = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow X_{cm} = \frac{\left(1.58\times10^3\times0.1+3.42\times10^3\times0.35+0.54\times10^3\times0.6+2.67\times10^3\times0.85\right)A}{\left(1.58\times10^3+3.42\times10^3+0.54\times10^3+2.67\times10^3\right)A}$$

$\Rightarrow X_{cm} = 0.481m$ from the end "O" of the rod.

7: If the centre of mass of three particles of masses of 1kg, 2kg, 3kg is at (2,2,2), then where should a fourth particle of mass 4kg be placed so that the combined centre of mass may be at (0,0,0).

Sol. Let (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) be the positions of masses 1kg, 2kg, 3kg and let the co-ordinates of centre of mass of the three particle system is (x_{cm}, y_{cm}, z_{cm}) respectively.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow 2 = \frac{1\times x_1 + 1\times x_2 + 1\times x_3}{1+2+3},$$

$(or)\ x_1 + 2x_2 + 3x_3 = 12$ ....(1)

Suppose the fourth particle of mass 4kg is placed at (x_4, y_4, z_4) so that centre of mass of new system shifts to (0,0,0).

For X co-ordinate of new centre of mass we have

$$0 = \frac{1\times x_1 + 2\times x_2 + 3\times x_3 + 4\times x_4}{1+2+3+4}$$

$\Rightarrow x_1 + 2x_2 + 3x_3 + 4x_4 = 0$ ....(2)

from equations (1) and (2)

$12 + 4x_4 = 0 \Rightarrow x_4 = -3$

similarly, $y_4 = -3$ *and* $z_4 = -3$

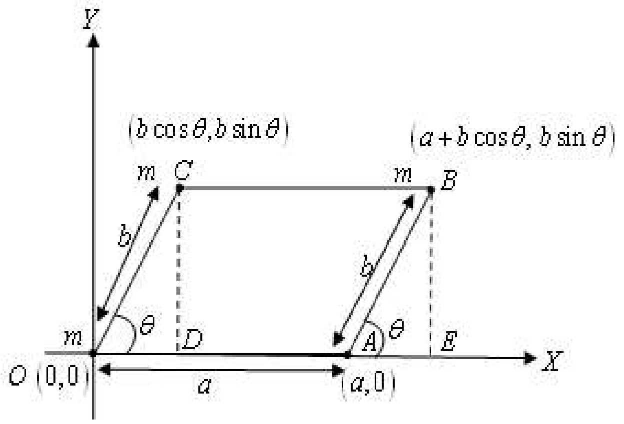
Therefore 4kg should be placed at (-3,-3,-3).

Case-3: **Center of mass of a system of particles in (two dimensional) Plane**:

Consider n-particles in x-y plane having masses $m_1, m_2, ..., m_n$ with co-ordinates $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ respectively. The distance of centre of mass from origin in a plane is $d = \sqrt{x_{cm}^2 + y_{cm}^2}$

where $x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$ and $y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$

8. Find position of centre of mass of four particle system, which are at the vertices of a parallelogram, as shown in figure.



Sol: From figure

$$DC = b\sin\theta, OD = b\cos\theta.$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$x_{cm} = \frac{[a + b\cos\theta]}{2}$$, similarly $y_{cm} = \frac{b\sin\theta}{2}$

$$\therefore [x_{cm}, y_{cm}] = \left[\frac{a + b\cos\theta}{2}, \frac{b\sin\theta}{2}\right]$$

.

Case-4: **Centre of mass of a system of 'n' particles in (Three dimensional) Space**:

Consider n-particles in space having masses with cordinates. $(x_1, y_1, z_1), (x_2, y_2, z_2)......(x_n, y_n, z_n)$ respectively, then distance of centre of mass from origin in space is $d = \sqrt{x_{cm}^2 + y_{cm}^2 + z_{cm}^2}$

Where $x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$, $y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$, $z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$

Case-5: **Position vector of Centre of mass:**

Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_n$ be the position vectors of n-particles having masses $m_1, m_2, ..., m_n$ respectively. If $\vec{r}_{c.m.}$ is position vector of their C.M., then

$$\vec{r}_{c.m.} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_1}{M}$$

Where $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, ..., $\vec{r}_n = x_n\hat{i} + y_n\hat{j} + z_n\hat{k}$

$$\therefore \vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

MOTION OF CENTRE OF MASS

(a) Velocity of centre of mass $(\bar{v}_{cm})$:

If $\bar{v}_1, \bar{v}_2, ..., \bar{v}_n$ are velocities of particles of masses $m_1, m_2, ...m_n$ respectively and $\bar{v}_{cm}$ is velocity of their centre of mass then

$$\bar{v}_{cm} = \frac{d\bar{r}_{cm}}{dt} = \frac{1}{M}\sum_{i=1}^{n} m_i\left(\frac{d\bar{r}_i}{dt}\right)$$

$$\bar{v}_{cm} = \frac{m_1\bar{v}_1 + m_2\bar{v}_2 + ... + m_n\bar{v}_n}{m_1 + m_2 + ... + m_n}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_i\bar{v}_i$$

where $M = m_1 + m_2 + ... + m_n$ = Total mass of the system.

When two particles of masses m_1 and m_2 aremoving from a point with velocities v_1 and v_2 at an angle 'θ' with each other, then the velocity of their centre of mass is given by

$$v_{cm} = \frac{\sqrt{m_1^2v_1^2 + m_2^2v_2^2 + 2(m_1v_1)(m_2v_2)\cos\theta}}{(m_1 + m_2)}$$

If they move in the same direction, then $\theta = 0°$ and $v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

f they move at right angles to each other, then $\theta = 90°$ and $v_{cm} = \frac{\sqrt{m_1^2v_1^2 + m_2^2v_2^2}}{m_1 + m_2}$

f they move in opposite directinos, then $\theta = 180°$ and $v_{cm} = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}$

(b)Linear momentum of centre of mass $(\bar{p}_{cm})$:

If $\bar{p}_1, \bar{p}_2, ..., \bar{p}_n$ are linear momenta of particles of masses $m_1, m_2, ...m_n$ respectively and $\bar{p}_{cm}$ is linear momentum of their centre of mass then $\bar{p}_{cm} = \bar{p}_1 + \bar{p}_2 + ... + \bar{p}_n = \sum_{i=1}^{n}\bar{p}_i$,

$$\bar{p}_{cm} = M\bar{v}_{cm} = \sum_{i=1}^{n} m_i\bar{v}_i = \bar{P}_{system}$$

$M = m_1 + m_2 + ... + m_n$ = Total mass of the system.

(c) Acceleration of centre of mass $(\bar{a}_{cm})$:

If $\bar{a}_1, \bar{a}_2, ..., \bar{a}_n$ are accelerations of particles of masses $m_1, m_2....m_n$ respectively and $\bar{a}_{cm}$ is the acceleration of their centre of mass then

$$\bar{a}_{cm} = \frac{d\bar{v}_{cm}}{dt} = \frac{1}{M}\sum_{i=1}^{n} m_i \frac{d\bar{v}_i}{dt}$$

$$\bar{a}_{cm} = \frac{m_1\bar{a}_1 + m_2\bar{a}_2 + ... + m_n\bar{a}_n}{m_1 + m_2 + ... + m_n} = \frac{1}{M}\sum_{i=1}^{n} m_i\bar{a}_i$$

Where $M = m_1 + m_2 + ... + m_n =$ Total mass of a system.

When two particles of masses m_1 and m_2 are moving from a point with accelerations a_1 and a_2 at an angle θ with each other, then the acceleration of their centre of mass is given by $\overline{a_{cm}} = \frac{\sqrt{m_1^2 a_1^2 + m_2^2 a_2^2 + 2(m_1 a_1)(m_2 a_2)\cos\theta}}{m_1 + m_2}$

f they move in the same direction, then $\theta = 0°$ and $a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

If they move at right angles to each other, then $\theta = 90^0$ and $a_{cm} = \frac{\sqrt{m_1^2 a_1^2 + m_2^2 a_2^2}}{m_1 + m_2}$

If they move in opposite directions, then $\theta = 180°$ and then $a_{cm} = \frac{m_1 a_1 - m_2 a_2}{m_1 + m_2}$

9.An object A is dropped from rest the top of a 30m high building and at the same moment another object B is projected vertically upwards with an inital speed of 15m/s from the base of the building. Mass of the object A is 2kg while mass of the object B is 4kg. Find the maximum height above the ground level attained by the centre of mass of the A and B system (take g = 10m/s²)

Sol. $m_1 = 4kg,\ m_2 = 2kg$

Initially 4kg is on the ground, therefore $x_1 = 0$

2kg is on top of the building, therefore $x_2 = 30m$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{0 + 2\times 30}{4+2} = 10m$$

Initital height of CM = 10m

Initial velocity of CM, $u_{cm} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

$$u_{cm} = \frac{4\times 15 + 0}{4+2} = 10m/s\ upward$$

Acceleration of CM, $a_{cm} = g = 10m/s^2\ downwards$

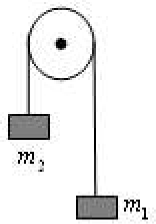
Maximum height attained by CM from initial position, $h_{cm} = \frac{u_{cm}^2}{2g} = \frac{10^2}{20} = 5m$

Maximum height attained by CM of 4kg and 2kg from the ground = 10+5=15m

10.Find the acceleration of centre of mass of the blocks of masses

m_1 *and* $m_2 (m_1 > m_2)$ ***in Atwood's machine.***

Sol. We know from Newton's laws of motion magnitude of acceleration of each block



$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad a_{cm} = \frac{m_1 a + m_2(-a)}{m_1 + m_2}$$

Acceleration of centre of mass $a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

Note: The magnitude of displacement of centre ofmass in time 't' is $\boxed{S_{cm} = \frac{1}{2} a_{cm} t^2}$

Effect of external forces on C.M.

We know $\bar{a}_{cm} = \frac{1}{M} \sum_i m_i \bar{a}_i$

Therefore $M\bar{a}_{cm} = \sum \bar{F}_{ext} + \sum \bar{F}_{int}$

But the internalforces are in the form of action - reaction pairs. Hence they cancel each other. Thus $\sum \bar{F}_{int} = \bar{0}$

$\therefore M\bar{a}_{cm} = \sum \bar{F}_{ext}$

Thus centre of mass is effected by only external force acting on the system. Internal forces will have no effect on the motion of centre of mass.

When no external force acts on the system then

a) acceleration of centre of mass is zero i.e.,

$\bar{F}_{ext} = M\bar{a}_{cm} \Rightarrow M\bar{a}_{cm} = \bar{O} \Rightarrow \bar{a}_{cm} = \bar{O}$.

b) Velocity of centre of mass is constant

$\bar{v}_{cm}$ = constant

c) Linear momentum of the system is constant

$\bar{p}_{cm}$ = constant. It is called the law of conservation of linear momentum.

Characteristics of centre of mass

1.Centre of mass of system of particles depend on mass of particles and their relative positions.
2. For continuous distribution of mass, centre of mass depends on mass distribution and shape of the body.

Sum of moments of masses about centre of mass is zero i.e., $\sum_i m_i \bar{r}_i = \bar{0}$

3. Centre of mass is independent of frame of reference chosen to locate it.
4.Mass need not be present at centre of mass.
5.The motion of centre of mass is purely translational.
6.**External forces only can effect the motion of C.M., but internal forces have no effect.**
7.The motion of centre of mass is according to Newton's 2^{nd} law.

Examples for the motion of centre of mass

(a) When a bomb at rest at origin of xyz-coordinate system explodes due to internal forces into many ragments. These fragments fly off randomly with different velocities in different directions. But C.M. is not effected and remains at rest at the origin.

$\therefore \sum m_i \bar{r}_i = \bar{0}$, where $\bar{r}_i$ is position vector of i^{th} particle about origin.

(b) A bomb is projected on the ground to follow parabolic path. When it explodes during the motion due to internal forces into many fragments, they move randomly in different directions. But the centre of mass follows the same parabolic path as unexploded bomb. So at any moment the vector sum of the moments of mass of all the fragments about centre of mass is zero.
(c) When a wheel is rolling on a road, then the paths of various particles are complicated as they are in combined motion (translational + rotational). But the motion of centre of mass is purely translational and it follows straight line path.

Note: Gravitational force between two masses, electric force between two charges are the examples of internal forces for the system, Which cannot produce acceleration in centre of mass of the system.

Mutual forces between two bodies :

When two particles approach each other due to their mutual interaction, then they always meet at their centre of mass.

To a system of particles $m_1(x_1y_1)$, $m_2(x_2y_2)$ another particle of mass m_3 is added so that centre of mass shifts to the origin then coordinates of third particle are $x_3 = -\frac{(m_1x_1 + m_2x_2)}{m_3}$;

$$y_3 = \frac{-(m_1y_1 + m_2y_2)}{m_3}$$

In a system of two particles of mass m_1 and m_2,

when m_1 is pushed towards m_2 through a distance d then shift in m_2 towards m_1 without altering C.M position is $\frac{-m_1}{m_2}d$.

A boy of mass m is at one end of a flat boat of mass M and length l which floats stationary on water. If boy moves to the other end, the boat moves through a distance d in the opposite direction with respect to ground (or shore), such that $d = \frac{ml}{(M+m)}$.

The displacement of boy with respect to ground is $d^1 = \frac{-Ml}{(M+m)}$

A boy of mass m is standing on a flat boat floating stationary on the surface of water. If the boy starts moving on the boat with velocity V_r with respect to boat, then

Velocity of the boat w.r.t. ground is $V = \frac{-mV_r}{M+m}$,

'–' indicates boat moves opposite to the velocity of the boy.

Velocity of boy w.r.t. ground is $V^1 = \frac{MV_r}{M+m}$

11: A 10kg boy standing in a 40kg boatfloating on water is 20m from the shore of the river. If he moves 8 meters on the boat towards the shore, then how far is he from the shore now?

Sol. Mass of the boy (m)=10kg

Mass of the boat (M)=40kg

Distance travelled by boy (*l*)=8m

Distance travelled by the boat in the opposite direction $= \frac{ml}{M+m} = \frac{10 \times 8}{10+40} = 1.6m$

Distance of the boy from the shore is (20-8+1.6) = 13.6m

Shift in centre of mass in different cases:

Shift is the distance of final location of centre of mass of the system from its initial location. Shift in the centre of mass generally occurs due to

a) Addition of matter

b) Removal of matter

c) Change in shape

d) Change in mass distribution

a) Addition of mass : Due to addition of mass, the C.M of a system generally shifts towards or into the region where mass is added. If C_1 is C.M before addition and C_2 is the C.M of added mass and $C_1C_2 = d$.,then

$$\Delta X_{shift} = \left[\frac{m_{added} \times d}{m_{initial} + m_{added}} \right]$$

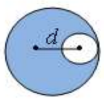
C.M shifts towards the side of added mass

b) Removal of mass : Due to removal of mass the C.M of a system shifts away from the region where mass is removed. If C_1 is C.M of the body before removal and C_2 is the C.M of the removed part and $C_1C_2 = d$ then

$$\Delta X_{shift} = \left[\frac{-m_{removed} \times d}{M_{initial} - m_{removed}}\right]$$

'$-$' indicates CM shifts opposite to the side of removed massOut of a uniform circular disc of radius R, if a circular sheet of radius 'r' is removed, then the centre of massof remaining part shifts by a distance $\frac{-r^2 d}{R^2 - r^2}$

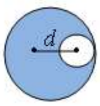
where 'd' is the distance of the C.M. of the removed part from the centre of the original disc. In this case the circular sheet is removed from the edge of disc, then the shift in centre of mass is maximum. Here $d = R - r$.



Maximum shift = $\frac{-r^2}{R + r}$

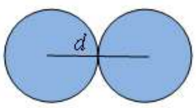
2.Out of a uniform solid sphere of radius R, if a sphere of radius 'r', is removed, then the centre of mass of the ramaining part shifts by $\frac{-r^3 d}{\left(R^3 - r^3\right)}$,

where 'd' isthe distance of the C.M. of removed part from thecentre of the original sphere. In this case spherical cavity is made at the edge of large sphere,



then shift in C.M. is maximum. It is given by $\frac{-r^3 (R - r)}{\left(R^3 - r^3\right)}$.

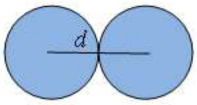
3.To a circular disc of radius R_1 another of radius R_2 and of the same material is added then shift



in the CM is x = $\frac{R_2^{\ 2}(R_1+R_2)}{R_1^{\ 2}+R_2^{\ 2}}$

4.If two spheres of same material and radii r_1 and r_2 are kept in contact, distance of centre of mass from the centre of the first sphere is equal to

$\frac{r_2^3}{r_1^3+r_2^3}(r_1+r_2)$.



Similarly distance of centre of mass from the centre of the second sphere is $\frac{r_1^3}{r_1^3+r_2^3}(r_1+r_2)$.

The location of C.M. of system depends on the mass distribution within the system. Due to this the location of C.M. changes whenever the shape of system changes and also the relative positions of particles change.

Methods to locate C.M:

Locating the Centre of Mass can be done in four different ways. They are

1) Method of symmetry
2) Method of Decomposition
3) Method using theorems of Pappu's
4) Method of integration

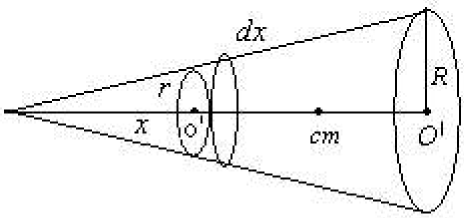
For continuous distribution of mass, the coordi inates of centre of mass are given by

$$x_{cm}=\frac{\int xdm}{\int dm};\ y_{cm}=\frac{\int ydm}{\int dm};\ z_{cm}=\frac{\int zdm}{\int dm}.$$

12. Distance of centre of mass of a uniform cone of height 'h' and base radius R, from the vertex on the line of symmetryis $\frac{3h}{4}$.

Sol. Consider a cone of height 'h' base radius 'R' and density ρ. To find centre of mass of the cone imaging a small element of radius 'r' and thickness 'dx' at a distance x from 'O'. Mass of small element, $dm=\left(\pi r^2\right)dx\,\rho$

from figure



$$\frac{r}{R}=\frac{x}{h}\Rightarrow r=\frac{Rx}{h}$$

$$x_{cm}=\frac{\int dm\ x}{\int dm}=\frac{\int_0^h \left(\pi r^2 \rho dx\right)x}{\int_0^h \pi r^2 dx\,\rho}$$

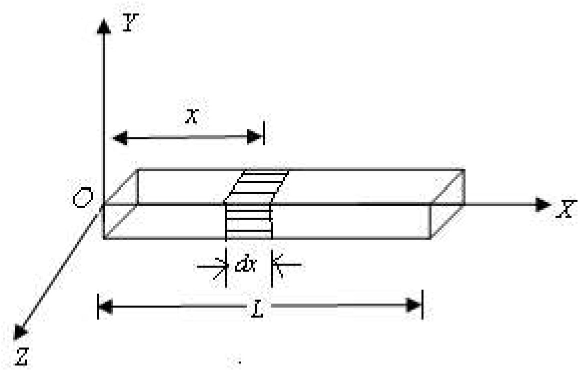
$$=\frac{\int_0^h \left(\frac{R^2x^2}{h^2}\right)x\,dx}{\int_0^h \frac{R^2x^2}{h^2}dx}=\frac{\int_0^h x^3\,dx}{\int_0^h x^2 dx}$$

$$=\frac{\left(\frac{x^4}{4}\right)_0^h}{\left(\frac{x^3}{3}\right)_0^h}=\frac{\frac{h^4}{4}}{\frac{h^3}{3}}=\frac{3h}{4}$$

Therefore, centre of mass of cone is at a distance $\frac{3h}{4}$ from vertex on its line of symmetry.

13: If the linear density of a rod of length L varies as $\lambda = A + Bx$, find the position of its centre of mass.

Sol. Let the x-axis be along the length of the rod and origin at one of its ends. As rod is along x-axis, for all points on it y and z coordinates are zero.

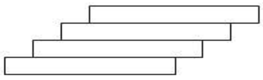


Centre of mass will be on the rod. Now consider an element of rod of length dx at a distance x from the origin, then $dm = \lambda\, dx = \left(A + Bx\right)dx$

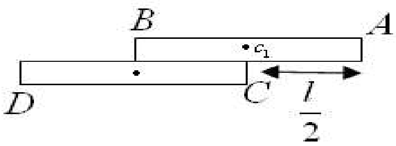
$$X_{cm}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\int_0^L x\left(A+Bx\right)dx}{\int_0^L \left(A+Bx\right)dx}\quad or$$

$$X_{cm} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{3AL + 2BL^2}{3(2A+BL)} = \frac{L(3A+2BL)}{3(2A+BL)}$$

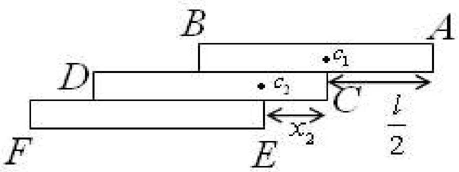
14: Identical blocks each of mass M and length L are placed one above the other such that each extends out by maximum length as shown in figure. Find the maximum extension of the n^{th} block from the top. So that the blocks will not fall.



Sol. For a two block system, the centre of mass (C_1) of upper block should be at the edge of lower block i.e. at $\frac{l}{2}$ distance. But if center of mass of upper block is not resting on the lower block then, the upper block falls down because of unbalaned torque created by gravitational force.



If a third block (EF) is arranged below the two blocks then

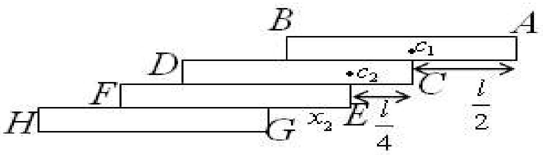


The centre of mass (C_2) of (AB) and (CD) block system must lie on the edge E of third block. To find x_2 consider C as origin. Then

$$x_2 = \frac{M(0) + M\left(\frac{l}{2}\right)}{2M} = \frac{l}{2}$$

$x_2 = \frac{l}{4}$ So, center of mass of upper two blocks is at $\frac{l}{4}$ distance from edge of lower block.

Also, ifanother block (GH) is placed below the three blocks in equilibrium, then



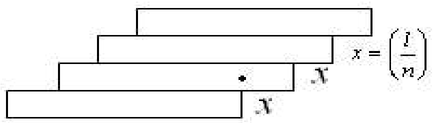
The center of mass (C_3) of the upper three block must lie on the edge of the lower fourth block i.e. at G. To find x_3 consider E as origin.

$$x_3 = \frac{2M(0) + M\left(\frac{l}{2}\right)}{3M} = \frac{l}{6} \qquad \therefore x_3 = \frac{l}{6}$$

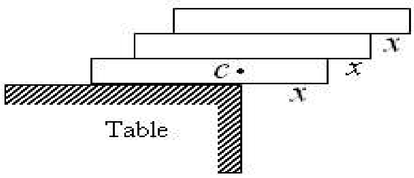
similarly $x_4 = \frac{l}{8},\ x_5 = \frac{l}{10}, \ldots\ldots$

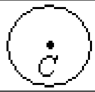
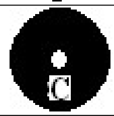
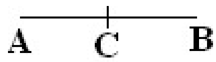
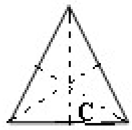
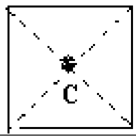
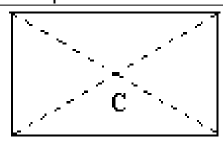
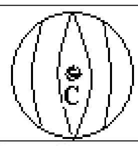
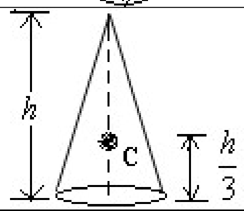
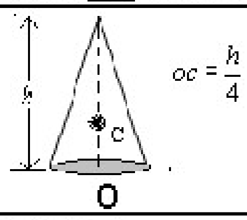
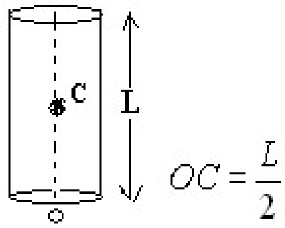
for n[th] block $\boxed{x_n = \frac{l}{2n}}$

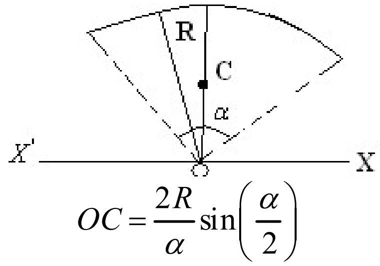
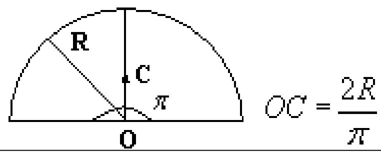
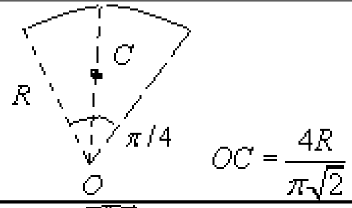
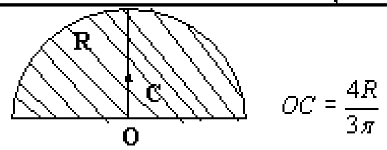
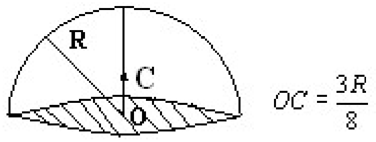
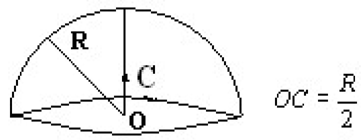
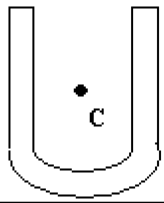
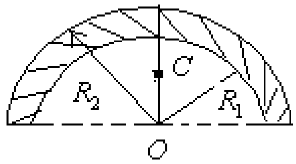
Note-1: When the above blocks are arranged in such a manner, that each block projects out by same distance, so that the blocks will not fall then the distance of projection of each block form the edge of its bottom block is $\left(\frac{l}{n}\right)$.



Note-2: If the entire system is placed at the edge of a table, so that the blocks will not fall then the equal distance of projection of each block from the edge of its bottom block is $\left(\frac{l}{n+1}\right)$



S.NO	SHAPE OF THE BODY	POSITION OF CENTRE OF MASS	FIGURE
1	Circular ring	At the centre of the ring	
2	Circular disc	A the centre of the disc	
3	Thin uniform straight rod	At the geometric centre	
4	Triangular plate	At the centroid	
5	Square plate	At the point of intersection of the diagonals	
6	Rectangular plate	At the point of intersection of the diagonals	
7	Hollow or solid sphere	At the centre of the sphere	
8	Hollow cone	At a height of $\frac{h}{3}$, from the base	
9	Solid cone or Pyramid	At a height of $\frac{h}{4}$ from the base	
10	Solid (or) hollow cylinder	At the mid-point of its own axis	

S.NO	SHAPE OF THE BODY	POSITION OF CENTRE OF MASS	FIGURE
11	An arc of radius R subtending an angle α at its centre Of curvature	At a distance of $\frac{2R}{\alpha}\sin\left(\frac{\alpha}{2}\right)$ from its centre of curvature on the axis of symmetry	
	i) A semi-circle of radius 'R'	At a distance of $\frac{2R}{\pi}$ from its centre on the axis of symmetry	
	ii) A quadrant of a circle of radius 'R'	At a distance of $\frac{4R}{\pi\sqrt{2}}$ from its centre 'o' on the axis of symmetry	
12	Semi-circular disc	At a distance of $\frac{4R}{3\pi}$ from its centre 'o' on the axis of symmetry	
13	Solid hemi-sphere	At a distance of $\frac{3R}{8}$ from its centre 'o' on the axis of symmetry	
14	Hollow hemi-sphere (or) Hemi-spherical shell	At a distance of $\frac{R}{2}$ from its centre 'o' on the axis of symmetry	
15	Horse-shoe magnet	At its centre within the boundary limits	
16	Semi-Circular annular plate	At a distance of $OC = \frac{4\left(R_1^2 + R_1R_2 + R_2^2\right)}{3\pi\left(R_1 + R_2\right)}$ from its centre of symmetry	

16. Find position vectors of mass center of a system of three particle of masses 1 kg, 2 kg and 3 kg located at position vectors $\vec{r}_1 = (4\hat{i} + 2\hat{j} - 3\hat{k})$ m, $\vec{r}_2 = (\hat{i} - 4\hat{j} + 2\hat{k})$ m and $\vec{r}_3 = (2\hat{i} - 2\hat{j} + \hat{k})$ m respectively.

Solution.

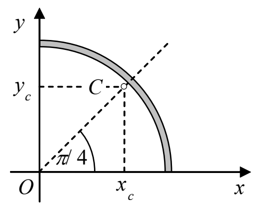
From eq. , we have

$$\vec{r}_c = \frac{\Sigma m_i \vec{r}_i}{M} \text{ ® } \quad \vec{r}_c = \frac{1(4\hat{i} + 2\hat{j} - 3\hat{k}) + 2(\hat{i} - 4\hat{j} + 2\hat{k}) + 3(2\hat{i} - 2\hat{j} + \hat{k})}{1 + 2 + 3} = 2\hat{i} - 2\hat{j} + \frac{2}{3}\hat{k}$$

17. Find coordinates of mass center of a quarter ring of radius *r* placed in the first quadrant of a Cartesian coordinate system, with centre at origin.

Solution.

Making use of the result obtained in the previous example, distance



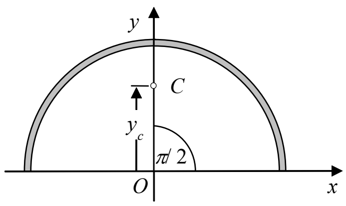
OC of the mass center form the center is $OC = \frac{r\sin(\pi/4)}{\pi/4} = \frac{2\sqrt{2}r}{\pi}$

Coordinates of the mass center (x_c, y_c) are $\left(\frac{2r}{\pi}, \frac{2r}{\pi}\right)$

18. Find coordinates of mass center of a semicircular ring of radius *r* placed symmetric to the *y*-axis of a Cartesian coordinate system.

Solution.

The *y*-axis is the line of symmetry, therefore mass center of the ring lies on it making *x*-coordinate zero.



Distance *OC* of mass center from center is given by the result obtained in example 4. Making use of this result, we have

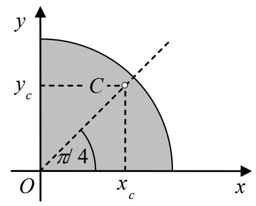
$$OC = \frac{r\sin\theta}{\theta} \text{ ® } \quad y_c = \frac{r\sin(\pi/2)}{\pi/2} = \frac{2r}{\pi}$$

19. Find coordinates of mass center of a quarter sector of a uniform disk of radius *r* placed in the first quadrant of a Cartesian coordinate system with centre at origin.

Solution.

Making use of the result obtained in the previous example, distance *OC* of the mass center form the center is

$$OC = \frac{2r\sin(\pi/4)}{3\pi/4} = \frac{4\sqrt{2}r}{3\pi}$$



Coordinates of the mass center (x_c, y_c) are $\left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$

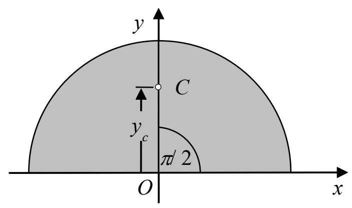
20.Find coordinates of mass center of a uniform semicircular plate of radius *r* placed symmetric to the *y*-axis of a Cartesian coordinate system, with centre at origin.

Solution.

The *y*-axis is the line of symmetry, therefore mass center of the plate lies on it making *x*-coordinate zero.

Distance *OC* of mass center from center is given by the result obtained in example 7. Making use of this result, we have

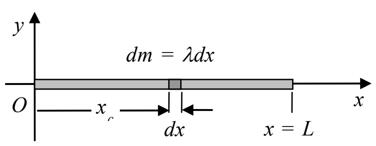
$$OC = \frac{2r\sin\theta}{3\theta} \text{ ® } y_c = \frac{2r\sin(\pi/2)}{3\pi/2} = \frac{4r}{3\pi}$$



21. Find coordinates of mass center of a non-uniform rod of length *L* whose linear mass density l varies as l=a+bx, where *x* is the distance from the lighter end.

Solution.

Assume the rod lies along the *x*-axis with its lighter end on the origin to make mass distribution equation consistent with coordinate system.



Making use of eq. , we have $x_c = \frac{\int x\,dm}{M}$ ® $x_c = \frac{\int_0^L x\lambda dx}{\int_0^L \lambda dx} = \frac{\int_0^L x(ax+b)dx}{\int_0^L (ax+b)dx} = \frac{(2aL+3b)L}{3(aL+2b)}$

22.A jeep of mass 2400 kg is moving along a straight stretch of road at 80 km/h. It is followed by a car of mass 1600 kg moving at 60 km/h.

(a) How fast is the center of mass of the two cars moving?

(b) Find velocities of both the vehicles in centroidal frame.

Solution.(a) Velocity of the mass center $\vec{v}_c = \dfrac{m_{jeep}\vec{v}_{jeep} + m_{car}\vec{v}_{car}}{m_{jeep} + m_{car}}$

Assuming direction of motion in the positive *x*-direction, we have

$\vec{v}_c = \dfrac{m_{jeep}\vec{v}_{jeep} + m_{car}\vec{v}_{car}}{m_{jeep} + m_{car}}$ ® $\vec{v}_c = \dfrac{2400 \times 80 + 1600 \times 60}{2400 + 1600} = 72$ km/h

(b) Velocity of the jeep in centroidal frame $v_{jeep/c} = 80 - 72 = 8$ km/h in positive *x*-direction.

Velocity of the car in centroidal frame $v_{car/c} = 60 - 72 = -12$ km/h 12 km/h negative *x*-direction direction.

23.A 2.0 kg particle has a velocity of $\vec{v}_1 = \left(2.0\hat{i} - 3.0\hat{j}\right)$ m/s, and a 3.0 kg particle has a velocity $\vec{v}_2 = \left(1.0\hat{i} + 6.0\hat{j}\right)$ m/s.

(a) How fast is the center of mass of the particle system moving?

(b) Find velocities of both the particles in centroidal frame.

Solution.

(a) Velocity of the mass center $\vec{v}_c = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$\vec{v}_c = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ ® $\vec{v}_c = \dfrac{2\left(2.0\hat{i} - 3.0\hat{j}\right) + 3\left(1.0\hat{i} + 6.0\hat{j}\right)}{2+3} = \left(1.4\hat{i} + 2.4\hat{j}\right)$ m/s

(b) Velocity of the first particle in centroidal frame

$\vec{v}_{1/c} = \vec{v}_1 - \vec{v}_c$ ® $\vec{v}_{1/c} = \left(2.0\hat{i} - 3.0\hat{j}\right) - \left(1.4\hat{i} + 2.4\hat{j}\right) = 0.6\left(\hat{i} + \hat{j}\right)$ m/s

Velocity of the second particle in centroidal frame

$\vec{v}_{2/c} = \vec{v}_1 - \vec{v}_c$ ® $\vec{v}_{2/c} = \left(1.0\hat{i} + 6.0\hat{j}\right) - \left(1.4\hat{i} + 2.4\hat{j}\right) = -\left(0.4\hat{i} + 3.6\hat{j}\right)$ m/s

24.Two particles of masses 2 kg and 3 kg are moving under their mutual interaction in free space. At an instant they were observed at points (-2 m, 1 m, 4 m) and (2 m, -3 m, 6 m) with velocities $\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$ m/s and $\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ m/s respectively. If after 10 sec, the first particle passes the point (6 m, 8 m, -6 m), find coordinate of the point where the second particle passes at this instant?

Solution.

System of these two particles is in free, therefore no external forces act on them. There total linear momentum remains conserved and their mass center moves with constant velocity relative to an inertial frame.

Velocity of the mass center

$$v_c = \frac{\Sigma m_i \vec{v}_i}{\Sigma m_i} = \frac{2\left(3\hat{i} - 2\hat{j} + \hat{k}\right) + 3\left(-\hat{i} + \hat{j} - 2\hat{k}\right)}{2+3} = \frac{3\hat{i} - \hat{j} - 4\hat{k}}{5} \text{ m/s}$$

Location $\vec{r}_{co}$ of the mass center at the instant $t = 0$ s

$$\vec{r}_c = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i} \rightarrow \qquad \vec{r}_{co} = \frac{2\left(-2\hat{i} + \hat{j} + 4\hat{k}\right) + 3\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{2+3} = \frac{2\hat{i} - 7\hat{j} + 26\hat{k}}{5}$$

New location $\vec{r}_c$ of the mass center at the instant $t = 10$ s

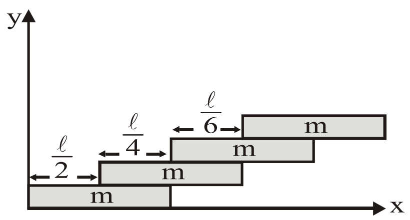
$$\vec{r}_c = \vec{r}_{co} + \vec{v}_c t \rightarrow \qquad \vec{r}_c = \frac{2\hat{i} - 7\hat{j} + 26\hat{k}}{5} + \frac{3\hat{i} - \hat{j} - 4\hat{k}}{5} \times 10 = \frac{32\hat{i} - 17\hat{j} - 14\hat{k}}{5}$$

New location (x, y, z) of the second particle.

$$\vec{r}_c = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i} \rightarrow \qquad \frac{32\hat{i} - 17\hat{j} - 14\hat{k}}{5} = \frac{2\left(6\hat{i} + 8\hat{j} - 6\hat{k}\right) + 3\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{2+3}$$

Solving the above equation, we obtain the coordinates of the second particle (20/3, –11, –2/3)

25. Find the x coordinate of the centre of mass of the bricks shown in figure :



Solution

$$X_{cm} = \frac{m\left(\frac{\ell}{2}\right) + m\left(\frac{\ell}{2} + \frac{\ell}{2}\right) + m\left(\frac{\ell}{2} + \frac{\ell}{4} + \frac{\ell}{2}\right) + m\left(\frac{\ell}{2} + \frac{\ell}{4} + \frac{\ell}{6} + \frac{\ell}{2}\right)}{m + m + m + m} = \frac{25}{24}\ell$$

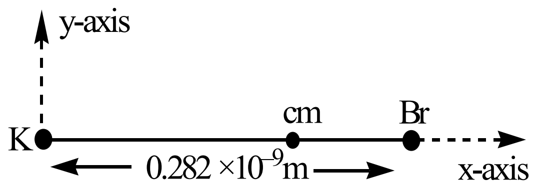
26. If the distance between the centres of the atoms of potassium and bromine in KBr (potassium -bromide) molecule is 0.282 ×10⁻⁹m, find the centre of mass of this two particle system from potassium (mass of bromine = 80 u, and of potassium = 39 u).

Solution: Mass of bromine, m_{Br} = 80 units

Mass of potassium, m_K = 39 units

Position co-ordinate of potassium, x_k = 0

Position co-ordinate of bromine, x_{Br} = 0.282 × 10^{-9}m



∴ Position co-ordinate of centre of mass,

$$x_c = \frac{m_k x_k + m_{Br} x_{Br}}{m_k + m_{Br}} \Rightarrow x_c = \frac{39 \times 0 + 80 \times 0.282 \times 10^{-9}}{39 + 80}$$

$\Rightarrow x_c = 0.189 \times 10^{-9}$ m

27.When 'n' number of particles of masses m, 2m, 3m,..... nm are at distances $x_1=1$, $x_2=2$, $x_3=3$, ... $x_n = n$ units respectively from origin on the x-axis, then find the distance of centre of mass of the system from origin.

Solution:

$$x_{cm} = \frac{m(1) + 2m(2) + 3m(3) + ... + (nm)n}{m + 2m + 3m + ... + nm}$$

$$x_{cm} = \frac{m(1^2 + 2^2 + 3^2 + ... + n^2)}{m(1 + 2 + 3 + ... + n)}$$

$$X_{cm} = \frac{\left(\frac{n(n+1)(2n+1)}{6}\right)}{\left(\frac{n(n+1)}{2}\right)} = \frac{2n+1}{3}$$

28.When 'n' number of particles each of mass 'm' are at distances $x_1=1$, $x_2=2$, $x_3=3$, ...$x_n=n$ units from origin on the x-axis, then find the distance of their centre of mass from origin.

Solution: $x_{cm} = \frac{m(1) + m(2) + m(3) + ... + m(n)}{m + m + m + ... + m(n \text{ terms})}$

$$= \frac{m(1 + 2 + 3 + ... + n)}{nm} = \frac{m\left(\frac{n(n+1)}{2}\right)}{nm}$$

$$x_{cm} = \frac{n+1}{2}$$

29.When 'n' number of particles each of mass 'm' are at distances $x_1=a$, $x_2 = ar$, $x_3 = ar^2$..... $x_n = ar^n$ units from origin on the x-axis, then find the distance of their centre of mass from origin.

Solution:

$$x_{cm} = \frac{ma + m(ar) + m(ar^2) + ... + m(ar^n)}{m + m + m + ... + m(n \text{ terms})}$$

$$x_{cm} = \frac{m(a + ar + ar^2 + ... + ar^n)}{m\,n}$$

If r > 1 then $x_{cm} = \frac{1}{n}\left[\frac{a(r^n - 1)}{r - 1}\right] = \frac{a(r^n - 1)}{n(r - 1)}$

If r < 1 then $x_{cm} = \frac{1}{n}\left[\frac{a(1 - r^n)}{1 - r}\right] = \frac{a(1 - r^n)}{n(1 - r)}$

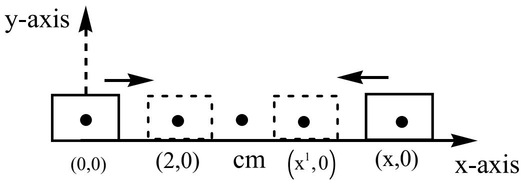
30.Two blocks of masses 10 kg and 30 kg are placed on x - axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of centre of mass unchanged.

Solution: mass of the first block, $m_1 = 10kg$

mass of the second block, $m_2 = 30$ kg

Let x_1 and x_1' are positions of m_1

x_2 adn x_1' are positions of m_2



In this case if 'x_{cm}' is the position of centre of mass then $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

then the new position of CM when blocks are shifted $x'_{cm} = \frac{m_1x_1' + m_2x_2'}{m_1 + m_2}$

subtracting the above equations

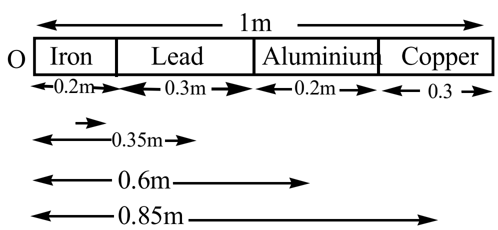
$$x'_{cm} - x_{cm} = \frac{m_1(x_1' - x_1) + m_2(x_2' - x_2)}{m_1 + m_2}$$

$$\Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$0 = \frac{10 \times 2 + 30\Delta x_2}{40}$ $\therefore \Delta x_2 = -2/3$ Therefore the second block should be moved left through a distance of $\frac{2}{3}$ cm to keep the position of centre of mass unchanged.

31.A 1 m long rod having a constant cross sectional area is made of four materials. The first 0.2 m are made of iron, the next 0.3 m of lead, the following 0.2m of aluminium and the remaining part is made of copper. Find the centre of mass of the rod. The densities of iron, lead, aluminium and copper are $7.9 \times 10^3 kg/m^3$, 11.4×10^3 kg/m³, 2.7×10^3 kg/m³ and 8.9×10^3 kg/m³ respectively.

Solution:



mass(m) = volume(v) x density (d)

m = Area (A) x length (l) x density (d)

m = Ald

Mass of iron part, $m_1 = A \times 0.2 \times 7.9 \times 10^3 = 1.58 \times 10^3 A$

Mass of lead part, $m_2 = A \times 0.3 \times 11.4 \times 10^3 = 3.42 \times 10^3 A$

Mass of aluminium part, $m_3 = A \times 0.2 \times 2.7 \times 10^3 = 0.54 \times 10^3 A$

Mass of copper part, $m_4 = A \times 0.3 \times 8.9 \times 10^3$

$= 2.67 \times 10^3 A$

Co - ordinate of iron part from end "O" of the rod, $x_1 = 0.1m$

Co - ordinate of lead part from end "O" of the rod, $x_2 = 0.35m$

Co - ordinate of aluminium part from end "O" of the rod, $x_3 = 0.6m$

Co - ordinate of copper part from end "O" of the rod, $x_4 = 0.85m$

$\therefore$ Centre of mass of the rod,

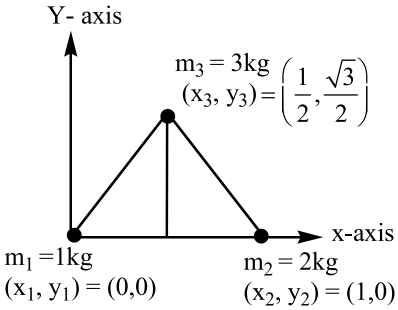
$$X_c = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow X_{cm} = \frac{\left(1.58\times10^3 \times 0.1 + 3.42\times10^3 \times 0.35 + 0.54\times10^3 \times 0.6 + 2.67\times10^3 \times 0.85\right)A}{\left(1.58\times10^3 + 3.42\times10^3 + 0.54\times10^3 + 2.67\times10^3\right)A}$$

$\Rightarrow X_{cm} = 0.481m$ from the end "O" of the rod.

32.Find the position of centre of mass of the system of 3 objects of masses 1 kg, 2kg and 3 kg located at the corner of an equilateral triangle of side 1 m. Take 1 kg mass object at the origin and 2 kg along x-axis.

Solution:



$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow x_{cm} = \frac{1\times0 + 2\times1 + 3\times\frac{1}{2}}{1+2+3} \Rightarrow x_{cm} = \frac{7}{12}m$$

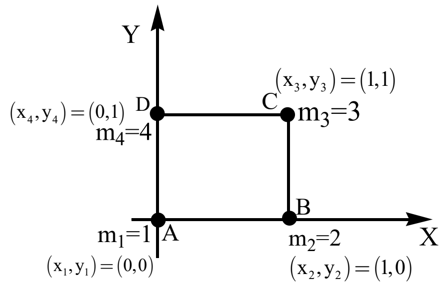
$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow Y_{cm} = \frac{1\times0 + 2\times0 + 3\times\frac{\sqrt{3}}{2}}{1+2+3} \Rightarrow Y_{cm} = \frac{\sqrt{3}}{4}m$$

$\therefore$ Co- ordinates of centre of mass $(x_{cm}, y_{cm}) = \left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

33.Four particles of masses 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A,B, C and D of the square of side 1 m. Find the position of centre of mass of the particles.

Solution: Assuming A as the origin, AB as x-axis and AD as y-axis we have



Co-ordinates of their CM are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(0) + 2(1) + 3(1) + 4(0)}{1 + 2 + 3 + 4} = 0.5\text{m}$$

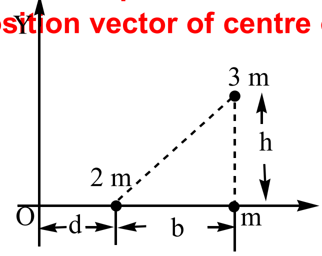
Similarly, $y_{CM} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$

$$= \frac{1(0) + 2(0) + 3(1) + 4(1)}{1 + 2 + 3 + 4} = 0.7\text{m}$$

\ Co-ordinates of centre of mass $(x_{CM}, y_{CM}) = (0.5\text{ m}, 0.7\text{ m})$

34.A system consists of three particles located at the corners of a right triangle as shown in the figure. Find the position vector of centre of mass of the system.

Solution :



Using the equation

$$X_c = \frac{\sum m_i x_i}{M} = \frac{2md + m(b+d) + 3m(d+b)}{6m} = d + \left(\tfrac{2}{3}\right)b$$

$$Y_c = \frac{\sum m_i y_i}{M} = \frac{2m(o) + m(o) + 3mh}{6m} = h/2$$

$Z_c = 0$; because the particles are in X - Y plane we can express the position of centre of mass from the origin using a position vector as

$$\vec{r}_c = X_c\hat{i} + Y_c\hat{j} + Z_c\hat{k}\ ,\ \vec{r}_c = \left(d + \frac{2}{3}b\right)\hat{i} + \frac{h}{2}\hat{j}$$

35.The position vectors of three particles of mass $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 4$ kg are $\vec{r}_1 = (\hat{i} + 4\hat{j} + \hat{k})m,\ \vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})m,$ and $\vec{r}_3 = (2\hat{i} - \hat{j} - 2\hat{k})m$ respectively. Find the position

vector of their centre of mass.

Solution :

The position vector of centre of mass of the three particles is given by

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$\vec{r}_c = \frac{1\left(\hat{i} + 4\hat{j} + \hat{k}\right) + 2\left(\hat{i} + \hat{j} + \hat{k}\right) + 4\left(2\hat{i} - \hat{j} - 2\hat{k}\right)}{1 + 2 + 4}$$

$$= \frac{\left(11\hat{i} + 2\hat{j} - 5\hat{k}\right)}{7} = \frac{1}{7}\left(11\hat{i} + 2\hat{j} - 5\hat{k}\right)m$$

36.Two 3 kg masses have velocities $\overline{v_1} = 2\hat{i} + 3\hat{j}\ m/s$ and $\overline{v_2} = 4\hat{i} - 6\hat{j}\ m/s$. Find a) velocity of centre of mass, b) the total momentum of the system, c) The velocity of centre of mass 5s after application of a constant force $\overline{F} = 24\hat{i}\,N$, d) position of centre of mass after 5s if it is at the origin at t = 0

Solution :

a) $\vec{v}_c = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$, $\vec{v}_c = \dfrac{3\left(2\hat{i} + 3\hat{j}\right) + 3\left(4\hat{i} - 6\hat{j}\right)}{6}$

$\therefore$ Velocity of centre of mass $\overline{V}_c = 3\hat{i} - 1.5\hat{j}\ ms^{-1}$.

b) The momentum of the system $= Mv_c = 6kg\left(3\hat{i} - 1.5\hat{j}\right)ms^{-1} = 18\hat{i} - 9\hat{j}\ kgms^{-1}$

c) To find the velocity of centre of mass after 5 s of application of the force $\vec{F} = 24\hat{i}N$ we first find the acceleration of the centre of mass. It is given by

$$\vec{a}_c = \frac{\vec{F}}{M} = \frac{24\hat{i}}{6} = 4\hat{i}\ \ ms^{-2}$$

The velocity of centre of mass before the force is applied is $\vec{v}_c$.

and from the equation $\vec{v}_c{}' = \vec{v}_c + \vec{a}_c.t$

$$\vec{v}_c{}' = \left(3\hat{i} - 1.5\hat{j}\right) + \left(4\hat{i}\right)5\ = \left(3\hat{i} - 1.5\hat{j} + 20\hat{i}\right)$$

$$\vec{V}_c^1 = \left(23\hat{i} - 1.5\hat{j}\right)ms^{-1}$$

37.Two 3 kg masses have velocities $\overline{v_1} = 2\hat{i} + 3\hat{j}\ m/s$ and $\overline{v_2} = 4\hat{i} - 6\hat{j}\ m/s$. Find a) velocity of centre of mass, b) the total momentum of the system, c) The velocity of centre of mass 5s after application of a constant force $\overline{F} = 24\hat{i}\,N$, d) position of centre of mass after 5s if it is at the origin at t = 0

Solution :

a) $\vec{v}_c = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$, $\vec{v}_c = \dfrac{3\left(2\hat{i} + 3\hat{j}\right) + 3\left(4\hat{i} - 6\hat{j}\right)}{6}$

$\therefore$ Velocity of centre of mass $\overline{V}_c = 3\hat{i} - 1.5\hat{j}\ ms^{-1}$.

b) The momentum of the system $= Mv_c = 6kg\left(3\hat{i} - 1.5\hat{j}\right)ms^{-1} = 18\hat{i} - 9\hat{j}\ kgms^{-1}$

c) To find the velocity of centre of mass after 5 s of application of the force $\vec{F} = 24\hat{i}N$ we first find the acceleration of the centre of mass. It is given by

$$\vec{a}_c = \frac{\vec{F}}{M} = \frac{24\hat{i}}{6} = 4\hat{i}\ \ ms^{-2}$$

The velocity of centre of mass before the force is applied is $\vec{v}_c$.

and from the equation $\vec{v}_c{}' = \vec{v}_c + \vec{a}_c.t$

$$\vec{v}_c{}' = \left(3\hat{i} - 1.5\hat{j}\right) + \left(4\hat{i}\right)5 \ = \left(3\hat{i} - 1.5\hat{j} + 20\hat{i}\right)$$

$$\vec{V}_c^1 = \left(23\hat{i} - 1.5\hat{j}\right)ms^{-1}$$

d) From the equation of the position vector $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ where $\vec{r}_0 = \vec{0}$ (origin at t = 0);

$\vec{v}_0 = \vec{v}_c;\ \vec{a} = \vec{a}_c$ and t = 5 s

$$\vec{r} = \left(3\hat{i} - 1.5\hat{j}\right)5 + \frac{1}{2}\left(4\hat{i}\right)25$$

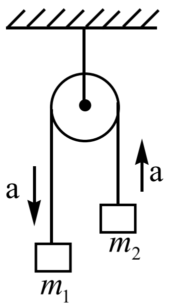
$$\vec{r} = \left(15\hat{i} - 7.5\hat{j} + 50\hat{i}\right) \therefore \vec{r} \ = \left(65\hat{i} - 7.5\hat{j}\right)m$$

The coordinates of the centre of mass after 5 s of application of the force $\vec{F}$ are (65 m, – 7.5 m)

38. Find the acceleration of center of mass of the blocks of masses m_1 and m_2 ($m_1 > m_2$) in Atwood's machine:

Solution: We know from Newton's laws of motion magnitude of acceleration of each block

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$



$$a_{cm} = \frac{m_1 a + m_2(-a)}{m_1 + m_2}$$

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)a$$

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

$\therefore$ Accleration of centre of mass $a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

d) From the equation of the position vector $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ where $\vec{r}_0 = \vec{0}$ (origin at t = 0);

$\vec{v}_0 = \vec{v}_c;\ \vec{a} = \vec{a}_c$ and t = 5 s

$$\vec{r} = \left(3\hat{i} - 1.5\hat{j}\right)5 + \frac{1}{2}\left(4\hat{i}\right)25$$

$$\vec{r} = \left(15\hat{i} - 7.5\hat{j} + 50\hat{i}\right) \therefore \vec{r} \ = \left(65\hat{i} - 7.5\hat{j}\right)m$$

The coordinates of the centre of mass after 5 s of application of the force $\overline{F}$ are (65 m, – 7.5 m)

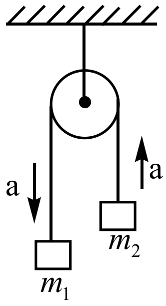
39.Find the acceleration of center of mass of the blocks of masses m_1 and m_2 ($m_1 > m_2$) in Atwood's machine:

Solution: We know from Newton's laws of motion magnitude of acceleration of each block

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$a_{cm} = \frac{m_1 a + m_2(-a)}{m_1 + m_2}$$

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) a$$



$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

∴ Accleration of centre of mass $a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

40.An object A is dropped from rest the top of a 30m high building and at the same moment another object B is projected vertically upwards with an initial speed of 15 m/s from the base of the building. Mass of the object A is 2 kg while mass of the object B is 4 kg. Find the maximum height above the ground level attained by the centre of mass of the A and B system (take g = 10 m/s²)

Solution : m_1 = 4kg, m_2 = 2 kg

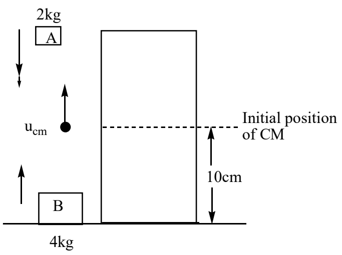
Initially 4 kg is on the ground ∴ $x_1 = 0$

2 kg is on top of the building ∴ x_2 = 30 m

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{0 + 2 \times 30}{4 + 2}$$

= 10 m



∴ Initial height of CM = 10m.

Initial velocity of CM, $u_{cm} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

$u_{cm} = \frac{4 \times 15 + 0}{4 + 2}$ = 10 m/s upward.

Acceleration of CM, a_{cm} = g = 10 m/s² downwards

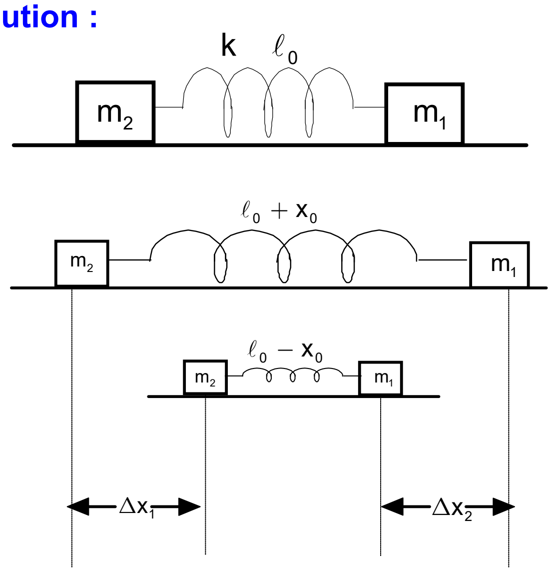
∴ Maximum height attained by CM from initial position, $h_{cm} = \frac{u_{cm}^2}{2g} = \frac{10^2}{20}$ = 5 m

∴ Maximum height attained by CM of 4 kg and 2 kg from the ground = 10 + 5 = 15 m

.

41. Two masses m_1 and m_2 are connected by a spring of force constant k and are placed on a frictionless horizontal surface. Initially the spring is stretched through a distance x_0, when the system is released from rest. Find the distance moved by two masses before they again comes to rest.

Solution :



Blocks again come to rest when spring is compressed by x_0. Since no external force is acting on the system, so there is no change in the position of c.m. of the system. i.e. $\Delta x_{cm} = 0$.

Let mass m_1 displaces by Δx_1 and m_2 displaces by Δx_2, then

We have $\Delta x_1 + \Delta x_2 = 2x_0$ .... (i)

and $\Delta x_{cm} = \dfrac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$

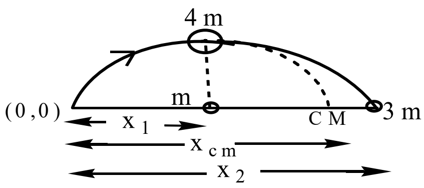
As $\Delta x_{cm} = 0 \therefore \dfrac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} = 0$ ....(ii)

After solving equation (i) & (ii), we get

$$\Delta x_1 = \frac{2m_2 x_0}{m_1 + m_2}, \Delta x_2 = \frac{2m_1 x_0}{m_1 + m_2}$$

42.**A projectile is fired at a speed of 100 m/s at an angle of 37^0 above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the smaller piece coming to rest. Find the distance from the launching point to the point where the heavier piece lands.**

Sol : Internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at a position where the original projectile would have landed. The range of the original projectile is



$$x_{CM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times10^4 \times \frac{3}{5}\times\frac{4}{5}}{10} m = 960m$$

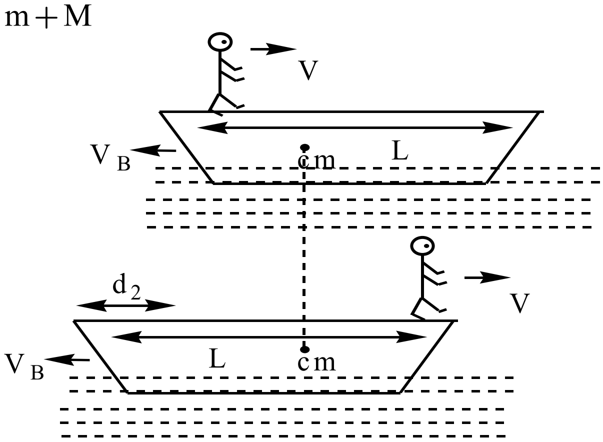
The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at x = 480 m. If the heavier block hits the ground at x_2, then

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \Rightarrow 960 = \frac{(m)(480)+(3m)(x_2)}{(m+3m)}$$

$$\therefore x_2 = 1120m$$

43. A man of mass 'm' is standing on a boat of mass M which is at rest in still water. If the man walks a distance L on the boat towards the shore the boat moves back through a distance, $d_2 = \frac{mL}{m+M}$

Proof :



As the system is initially at rest and no external force acts on the system (horizontally) $\therefore \vec{v}_{CM} = 0$.

(or) $\frac{m\vec{v}_1 + M\vec{v}_2}{m+M} = 0$ (or) $m\vec{v}_1 + M\vec{v}_2 = 0$

or $m\frac{\Delta\vec{r}_1}{dt} + M\frac{\Delta\vec{r}_2}{dt} = 0$ (or) $m\Delta\vec{r}_1 + M\Delta\vec{r}_2 = 0$

or $md_1 - Md_2 = 0$ [as $\vec{d}_2$ is opposite to $\vec{d}_1$]

$md_1 = Md_2$

Now when man moves a distance L towards the shore relative to boat, the boat will shift a distance d_2 relative to shore opposite to the displacement of man. The displacement of man relative to shore (towards shore) will be

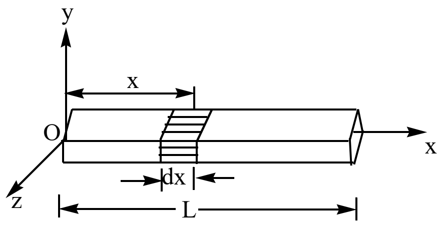
$d_1 = L - d_2$ (i.e., $d_1 + d_2 = d_{rel} = L$)

so, $md_1 = Md_2 \Rightarrow m(L - d_2) = Md_2$

hence $\boxed{d_2 = \frac{mL}{M+m}}$

44.If the linear density of a rod of length L varies as $\lambda = A + Bx$, find the position of its centre of mass.

Solution: Let the x-axis be along the length of the rod and origin at one of its ends. As rod is along x–axis, for all points on it y and z co-ordinates are zero.



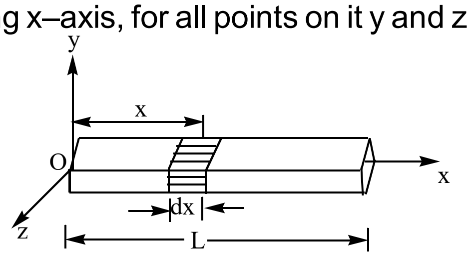
Centre of mass will be on the rod. Now consider an element of rod of length dx at a distance x from the origin, then $dm = \lambda\, dx = (A+Bx)dx$

$$X_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx}$$

or $X_{CM} = \dfrac{\dfrac{AL^2}{2} + \dfrac{BL^3}{3}}{AL + \dfrac{BL^2}{2}} = \dfrac{L(3A+2BL)}{3(2A+BL)}$

45.If the linear density of a rod of length L varies as $\lambda = \frac{kx^2}{L}$ where k is a constant and x is the distance of any point from one end, then find the distance of centre of mass from the end at x = 0.

Solution: Let the x-axis be along the length of the rod and origin at one of its ends. As rod is along x–axis, for all points on it y and z co-ordinates are zero.



Centre of mass will be on the rod. Now consider an element of rod of length dx at a distance x from the origin, then $dm = \lambda\, dx = \frac{Kx^2}{L}dx$

so, $X_{CM} = \dfrac{\int_0^L x\,dm}{\int_0^L dm} = \dfrac{\int_0^L x\dfrac{kx^2}{L}dx}{\int_0^L \dfrac{kx^2}{L}dx}$

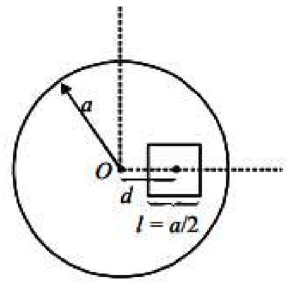
$$= \frac{\int_0^L x^3 \, dx}{\int_0^L x^2 \, dx} = \frac{\frac{L^4}{4}}{\frac{L^3}{3}} = \frac{3L}{4}$$

PREVIOUS JEEMAINS QUESTIONS

46.The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is________ [6SEP 2020 MAINS]

Solution: Centre of mass of solid hemisphere of radius *R* lies at a distance $\frac{3R}{8}$ above the centre of flat side of hemisphere.

47.A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is________



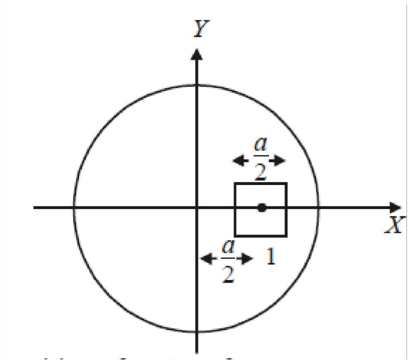
[2 SEP 2020 MAINS]

Solution: Let σ be the mass density of circular disc.

Original mass of the disc, $m_0 = \pi a^2 \sigma$

Removed mass, $m = \frac{a^2}{4}\sigma$

Remaining, mass, $m' = \left(\pi a^2 - \frac{a^2}{4}\right)\sigma = a^2\left(\frac{4\pi - 1}{4}\right)\sigma$



New position of centre of mass

$$X_{cm} = \frac{m_0x_0 - mx}{m_0 - m} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-a^3/8}{\left(\pi - \frac{1}{2}\right)a^2} = \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2} = -\frac{a}{23}$$

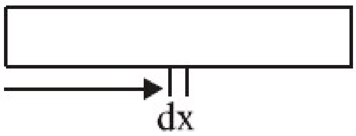
$\therefore x = 23$

48.A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:

(1) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (2) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$ [9JAN 2020 MAINS]

(3) $\frac{3}{4}\left(\frac{a+b}{2a+3b}\right)L$ (4) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

solution Linear mass density, $\rho(x) = a + b\left(\frac{x}{L}\right)^2$



$$X_{CM} = \frac{\int x\,dm}{\int dm}$$

$$\int dm = \int_0^L \rho(x)\,dx$$

$$\int dm = \int_0^L \rho(x)\,dx$$

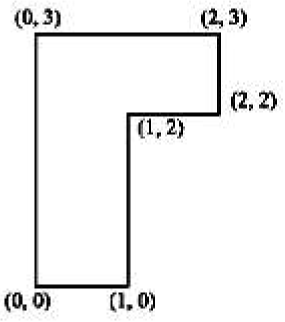
$$\int_0^L \left[a + b\left(\frac{x}{L}\right)^2\right]dx = aL + \frac{bL}{3}$$

$$\int_0^L x\,dm \int_0^L \left(ax + \frac{bx^3}{L^2}\right)dx = \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$$

$$\therefore X_{CM} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)}{aL + \frac{bL}{3}}$$

$$\Rightarrow X_{CM} = \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$$

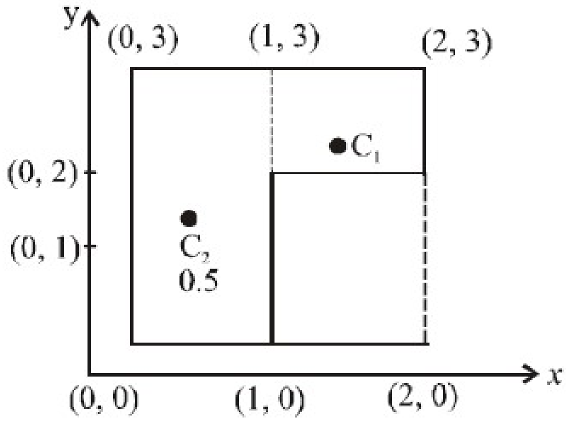
49.The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plale)of mass 4kg. (The coordinates of the same are shown in figure) are



[8 JAN 2020 MAINS]

(1) (1.25, 1.50 m) **(2) (0.75 m, 1.75 m)**
(3) (0.75 m, 0.75 m) **(4) (1 m, 1.75 m)**

(Solution: **4)**



For given Lamina

$$\quad\quad x \quad y$$

$$m_1 = 1, C_1 = (1.5, 2.5)$$

$$m_2 = 3, C_2 = (0.5, 1.5)$$

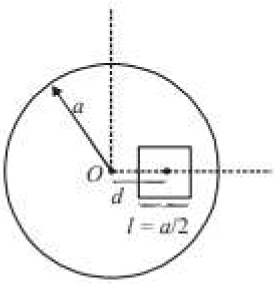
$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of centre of mass of flag shaped lamina

(0.75, 1.75)

50. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at (C), the centre of maa ss of remaining (shaped) part of sphere in at G, i.e. on the surface of the cavity. R and be determined by the equation [8 JAN 2020 MAINS]

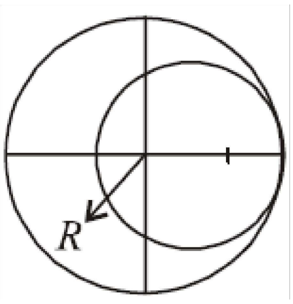


(1) $(R^2 + R + 1)(2 - R) = 1$

(2) $(R^2 - R - 1)(2 - R) = 1$

Solution **(1)** Mass of sphere = volume of sphere x density of sphere

$= \frac{4}{3}\pi R^3 \rho$



Mass of cavity $M_{cavity} = \frac{4}{3}\pi(1)^3 \rho$

$M_{(Remaining)} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi(1)^3 \rho$

Centre of mass of remaining part,

$X_{COM} = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$

$$\Rightarrow -(2-R) = \frac{\left[\frac{4}{3}\pi R^3 \rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R-1]}{\frac{4}{3}\pi R^3 \rho + \frac{4}{3}\pi(1)^3(-\rho)} \Rightarrow \frac{(R-1)}{(R^3-1)} = 2-R$$

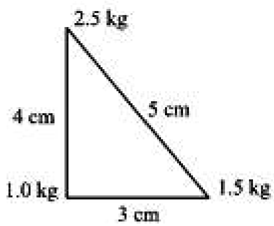
$$\Rightarrow \frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$$

$$\Rightarrow (R^2+R+1)(2-R) = 1$$

(3) $(R^2-R+1)(2-R)=1$

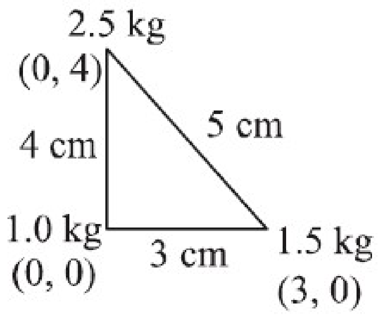
(4) $(R^2+R-1)(2-R)=1$

51. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at thre corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: [7 JAN 2020MAINS]



(1) 0.6 cm right and 2.0 cm above 1 kg mass
(2) 1.5 cm right and 1.2 cm above 1 kg mass
(3) 2.0 cm right and 0.9 cm above 1 kg mass
(4) 0.9 cm right and 2.0 cm above 1 kg mass [7 JAN 2020 MAINS]

Solution



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

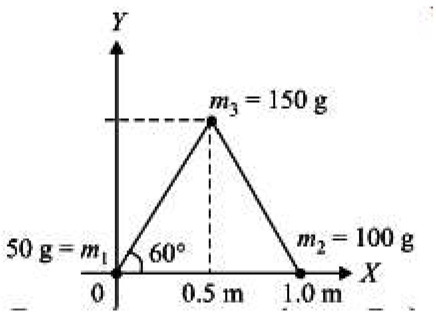
$$X_{cm} = \frac{1\times0 + 1.5\times3 + 2.5\times0}{1 + 1.5 + 2.5} = \frac{1.5\times3}{5} = 0.9cm$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{1\times0 + 1.5\times0 + 2.5\times4}{1 + 1.5 + 2.5} = \frac{2.5\times4}{5} = 2cm$$

Hence, centre of mass of system is at point (0.9, 2)

52. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1m (as shown in the figure). The (x,y) coordinates of the centre of mass will be

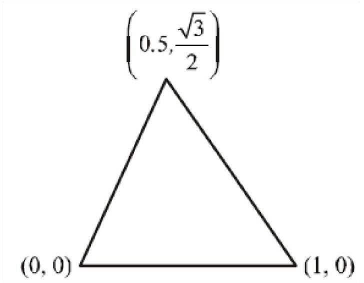


[APR 12 2019 MAINS]

(1) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$ (2) $\left(\frac{7}{12}m, \frac{3}{8}m\right)$

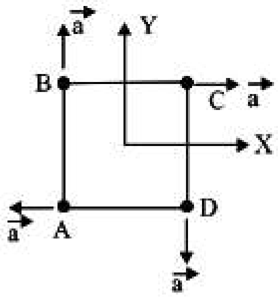
(3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (4) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

Solution (3) $x_{cm}=\frac{50\times 0+100\times 1+150\times 0.5}{50+100+150}=\frac{7}{12}m$



$$y_{cm}=\frac{50\times 0+100\times 0+150\times\frac{\sqrt{3}}{2}}{50+100+150}=\frac{\sqrt{3}}{4}m$$

53.Four particles A, B, C and D with masses $m_A=m, m_a=2m, m_c=3m$ and $m_D=4m$ are at the comers of a square. They have accelerations of equal magnitud with directions as shown. The acceleration of the centre of mass of the particles is



[APR8 2019 MAINS]

(1) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$ (2) a

(3) Zero (4) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$

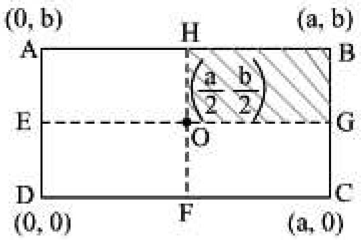
Solution Acceleration of centre of mass (a_{cm}) is given by

$$\therefore \vec{a}_{cm}=\frac{m_1\vec{a}_1+m_2\vec{a}_2+........}{m_1+m_2+........}$$

$$=\frac{(2m)a\hat{j}+3m\times a\hat{i}+ma\left(-\hat{i}\right)+4m\times a\left(-\hat{j}\right)}{2m+3m+4m+m}$$

$$=\frac{2a\hat{i}-2a\hat{j}}{10}=\frac{a}{5}\left(\hat{i}-\hat{j}\right)$$

54 A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be [8APR 2019 MAINS]



(1) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (2) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(3) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (4) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Solution. With respect to point θ, the CM of the cut-off portion

$$\left(\frac{a}{4}, \frac{b}{4}\right)$$

Using, $x_{CM} = \frac{MX - mx}{M - m}$

$$= \frac{M \times 0 - \frac{M}{4} \times \frac{a}{4}}{M - \frac{M}{4}} = -\frac{a}{12}$$

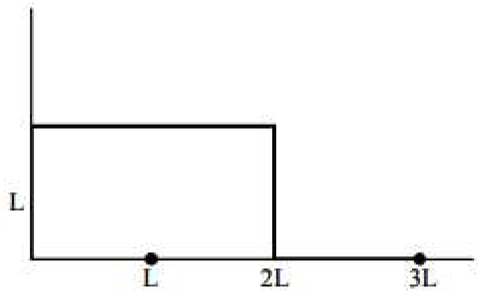
and $y_{CM} = -\frac{b}{12}$

So CM coordinates one

$$x_0 = \frac{a}{2} - \frac{a}{12} = \frac{5a}{12}$$

and $y_0 = \frac{b}{2} - \frac{b}{12} = \frac{5b}{12}$

55. The position vector of the centre of mass r_{cm} of an asymmetric uniform bar of negligible area of crosssection as shown in figure is: [12 JAN 2019 MAINS]



(1) $\vec{r}cm = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(2) $\vec{r}cm = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

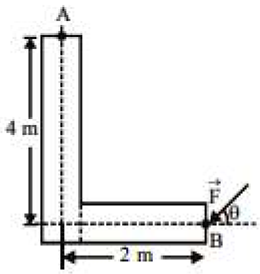
(3) $\vec{r}cm = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

(4) $\vec{r}cm = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

Solution. x-coordinate of centre of mass is

$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

56. A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle that will produce maximum moment of the force about point A is given by: [APR15 2018 MAINS]

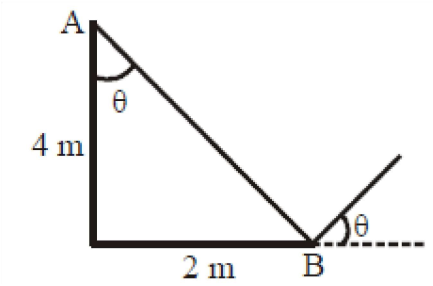


(1) $\tan\theta = \frac{1}{4}$ (2) $\tan\theta = 2$

(3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = 4$

Solution.

To produce maximum moment of force line of action of force must be perpendicular to line AB.



$\therefore \tan\theta = \frac{2}{4} = \frac{1}{2}$

57. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

(1) Left arm is longer than the right arm [APR8 2017 MAINS]

(2) Both the arms are of same length

(3) Left arm is shorter than the right arm

(4) Every object that is weighed using this balance appears lighter than its actual weight

Solution.

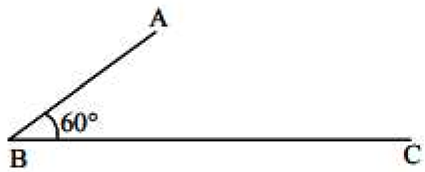
According to principle of moments when a system is stable or balance, the anti-clockwise moment is equal to clockwise moment.

i.e., load × load arm = effort × effort arm

When 5 mg weight is placed, load arm shifts to left side,

hence left arm becomes shorter than right arm.

58.In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to :

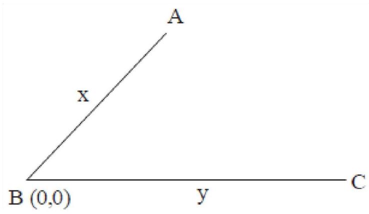


[APR11 2016 MAINS]

(1) 1.85 (2) 1.5 (3) 1.37 (4) 3

Solution.Centre of mass $x_{cm} = \frac{x}{2} = \frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho y^{y/2}}{\rho(x+y)}$

$\Rightarrow \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$



$\therefore \frac{Bc}{A}\frac{y}{x} = \frac{1+\sqrt{3}}{2} = 1.37$

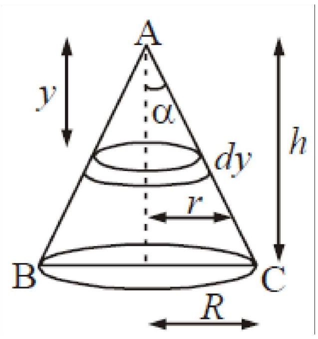
59. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$ [APR11 2015 MAINS]

Solution. Let density of cone $= \rho$.

Centre of mass, $ycm = \frac{\int ydm}{\int dm}$

$$= \frac{\int_0^h y\pi r^2 dy\rho}{\frac{1}{3}\pi R^2 h\rho} = \frac{\int_0^h r^2 ydy}{\frac{1}{3}R^2 h} \quad ...(i)$$



For a cone, we know that

$\frac{r}{R} = \frac{y}{n} \qquad \therefore r = \frac{y}{n}R$

$$ycm = \frac{\int_0^h 3y^3 dy}{h^3} = \frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3} = \frac{3}{4}h$$

60 A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as:

(1) a = 2b (2) 2a = b (3) a = b (4) 3a= 2b [APR 2015 MAINS]

Solution. Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L (ax + \frac{bx^2}{L})dx}{\int_0^L (a + \frac{bx}{L})dx}$$

$$\frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$

On solving we get, $b = 2a$

61. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_O, then the distance of the centre of mass from the lighter end is

(1) $\frac{L}{2} - \frac{\lambda_0 L^2}{4M}$ (2) $\frac{L}{3} + \frac{\lambda_0 L^2}{8M}$ [APR 2014 MAINS]

(3) $\frac{L}{3} + \frac{\lambda_0 L^2}{4M}$ (4) $\frac{2L}{3} - \frac{\lambda_0 L^2}{6M}$

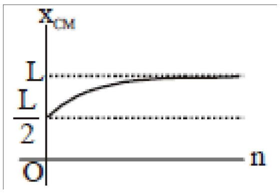
Solution. **(3)**

62. A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be

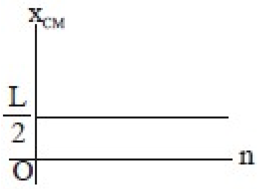
(1) 15 m (2) 12.5 m (3) 15.5 m (4) 17 m [APR 2013 MAINS]

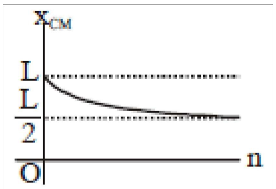
Solution. **(4)**

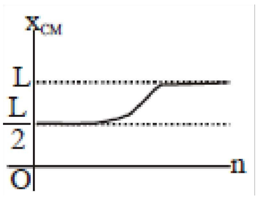
63. A thin rod of length 'L' is lying along the x-axis with its ends at x = 0 and x=L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the follwing graphs best aproximates the dependence of x_{CM} on n?

(1)


[MAINS 2008]

(2)


(3)


(4)


Solution. The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$

Here $\frac{x}{L} \le 1$

With increase in the value of *n*, the centre of mass shift towards the end *x* = *L* This is satisfied by only option (a).

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n .x dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{kx^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0, x_{CM} = \frac{L}{2}; n = 1,$

$x_{CM} = \frac{2L}{3}; n = 2, x_{CM} = \frac{3L}{4};.....$

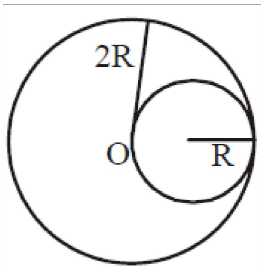
For $n \to \infty x_{cm} = L$

Moment of inertia of a square plate about an axis through its centre and perpendicular to its plane is.

64. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is α / R from the centre of the bigger disc. The value of α is

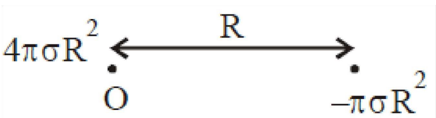
(1) 1/4 (2) 1/3 (3) 1/2 (4) 1/6 **[MAINS 2007]**

Solution. Let σ be the mass per unit area of the disc. Then the mass of the complete disc $= \sigma\left(\pi(2R)^2\right)$



The mass of the removed disc $= \sigma\left(\pi R^2\right) = \pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius 2*R* on which a disc of radius *R* of negative mass is superimposed. Let *O* be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{cm} = \frac{\left(6\pi(2R)^2\right)\times 0 + \left(-6\left(\pi R^2\right)\right)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

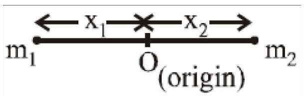
$$\therefore x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore x_{c.m} = -\frac{R}{3} = \alpha R \Rightarrow \alpha = \frac{1}{3}$$

65 Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, so as to keep the centre of mass at the same position?

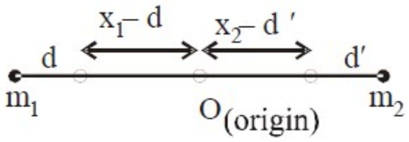
(1) $\frac{m_2}{m_1}d$ (2) $\frac{m_1}{m_1+m_2}d$ **[MAINS 2006]**

(3) $\frac{m_1}{m_2}d$ (4) d

Solution. Initially, 

$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2} \Rightarrow m_1 x_1 = m_2 x_2 \text{.......(1)}$$

Let the particles is displaced through distanced away from centre of mass



$$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$$

$$\Rightarrow 0 = m_1 d - m_1 x_1 + m_2 x_2 - m_2 d'$$

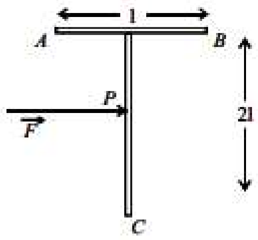
$$\Rightarrow d' = \frac{m_1}{m_2}d$$

21. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}$ M and a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(1) does not shift
(2) depends on height of breaking **[MAINS 2005]**
(3) body B
(4) body C

Solution.**(1)** The centre of mass of bodies B and C taken together does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

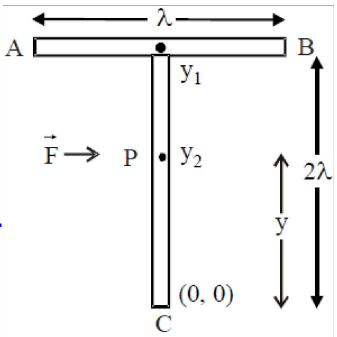
22. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C



[MAINS 2005]

(1) $\frac{3}{2}\ell$ (2) $\frac{2}{3}\ell$ (3) ℓ (4) $\frac{4}{3}\ell$

Solution.



To have translational motion without rotation, the force $\vec{F}$ has to be applied at centre of mass. i.e. the point 'P' has to be at the centre of mass Taking point C at the origin position, positions of y, and y_2 are $r_1 = 2l, r_2 = l$ and $ml = m$ and $m_2 = 2m$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$$

ROTATIONAL DYNAMICS

Rigid body : If there is no relative motion between any two particles of the body along the line joining them by the application of external force, then that body is called rigid body.

A No real body is truly rigid, since real bodies deform under the influence of external forces.

Types of motion of a rigid body

A **Translational motion :** All particles of the body move in parallel paths such that displacements of all the particles are same as that of the body then its motion is said to be translational.

A **Rotational motion :** A body is said to be in pure rotation if every particle of the body moves in a circle and the centres of all the particles lie on a straight line called the axis of rotation.

A **Rolling motion :** The combination of rotational and translational motion with regard to certain constraints is called rolling motion.

Kinematics of rotational motion about a fixed axis :

A The kinetic equations for rotational motion with uniform angular acceleration

1) $\omega = \omega_0 + \alpha t$ 2) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

3) $\omega^2 = \omega_0^2 + 2\alpha\theta$ 4) $\theta_n = \omega_0 + \alpha\left(n - \frac{1}{2}\right)$

1: The motor of an engine is rotating about its axis with angular velocity of 120 rpm. It comes to rest in 10s, after being switched off. Assuming constant deceleration, calculate the number of revolutions made by it before coming to rest.

Sol. Here n=120rpm=2rps

$\omega_0 = 2\pi n = 4\pi \ rad \ s^{-1}; \omega = 0$ and $t = 10s$

$\omega = \omega_0 + \alpha t$

$0 = 4\pi + \alpha \times 10 \ \ or \ \alpha = -0.4\pi rad \ s^{-2}$

Also, the angle covered by the motor,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$\therefore \theta = 4\pi \times 10 + \frac{1}{2} \times (-0.4\pi) \times 10^2 = 40\pi - 20\pi = 20\pi \, rad$ Hence, the number of revolutions completed,

$$N = \frac{\theta}{2\pi} = \frac{20\pi}{2\pi} = 10$$

2: A wheel rotates with an angular acceleration given by $\alpha = 4at^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, write the equations for the (a) angular speed (b) angular displacement

Sol. (a) Since $\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha dt$

Integrating both sides, we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha dt = \int \left(4at^3 - 3bt^2\right) dt$$

$$\omega - \omega_0 = 4\alpha \frac{t^4}{4} - 3b\frac{t^3}{3} \Rightarrow \omega = \omega_0 + at^4 - bt^3$$

(b) Since $\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega dt$

On integrating both sides, we get

$$\int_0^{\theta} d\theta = \int_0^t \omega dt = \int_0^t \left(\omega_0 + at^4 - bt^3\right) dt$$

$$\Rightarrow \theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$$

Calculation of Angular velocityof a Rigid Body :

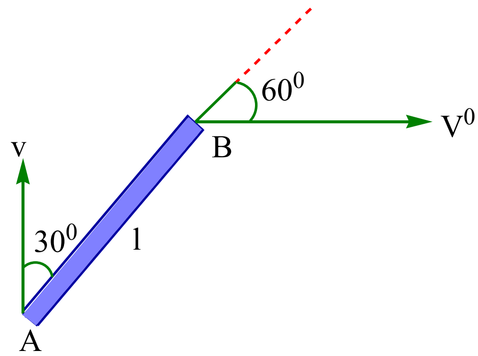
A The angular velocity of the rigid body can be given as the relative angular velocity between any two points of the rigid bodies,

$$\omega\left(=\left|\vec{\omega}_{AB}\right| = \left|\vec{\omega}_{BA}\right|\right) = \frac{v_{AB}}{l}$$

Where l = distance of separation between A and B and v_{AB} = magnitude of relative velocity between A and B. The vectorial representation of above equation can be given as $\vec{v}_{AB} = \vec{\omega}_{AB} \times \vec{r}_{AB}$

Where, $\vec{r}_{AB}$ = position vector of A relative to B.

3.The velocities at the ends of rod of length l are given as $\vec{v}$ and $\vec{v}'$, making angles 30^0 and 60^0 with rod. Find

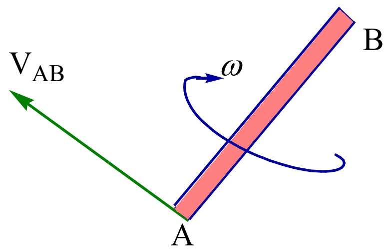


(a) $\vec{v}'$ (b) angular velocity of the rod

Sol. Since rod is rigid, equate the velocities of the points A and B along the rod to obtain,

$$v_A \cos 30^0 = v' \cos 60^0$$

or $v\frac{\sqrt{3}}{2} = \frac{v'}{2}$ or $v' = \sqrt{3}v$



(b) the angular velocity of the rod is $\omega = \frac{v_{AB}}{l}$

$$= \frac{\frac{1}{\sqrt{2}}v + \frac{\sqrt{3}}{2}v'}{l} = \frac{v + \sqrt{3}\left(\sqrt{3}v\right)}{2l}$$

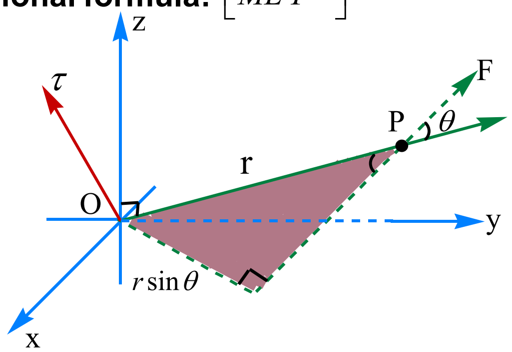
$$= 2\frac{v}{l}(\text{clock wise})$$

MOMENT OF FORCE (TORQUE):

A Torque is the turning effect of a force about a fixed point.

A Magnitude of the torque is given as the product of magnitude of force and perpendicular distance of line of action of a force from the fixed point. $\tau = F\left(r\sin\theta\right) \Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$.

S.I. Unit : Nm **Dimensional formula:** $\left[ML^2T^{-2}\right]$



Application: A force of given magnitude applied at right angles to the door at its outer edge is most effective in producing rotation.

A The moment of a force vanishes if either the magnitude of the force is zero, or if the line action of the force pass through the fixed point.

A If the direction of **F** is reversed, the direction of the moment of force is also reversed.

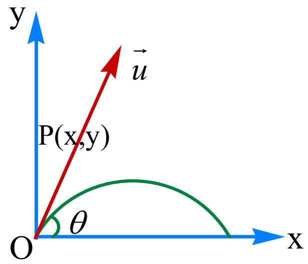
A If directions of both **r** and **F** are reversed, the direction of the moment of force remains the same.

Sign convention : Torque that produces anti clockwise rotation is taken as positive and clockwise rotation taken as negative.

4.A particle is projected at time t=0 from a point 'O' with a speed 'u' at an angle 'θ' to horizontal. Find the torque of a gravitational force on projectile about the origin at time 't'. (x, y) plane is vertical plane)

Sol. $\vec{r} = \left(u\cos\theta\right)t\hat{i} + \left(\left(u\sin\theta\right)t - \frac{1}{2}gt^2\right)\hat{j}$

$\vec{F} = -(mg)\hat{j};\ \vec{\tau} = \vec{r} \times \vec{F}$



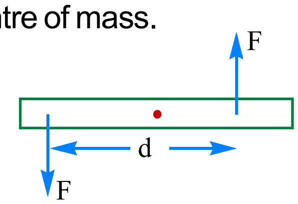
$$\vec{\tau} = \left[(u\cos\theta)t\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}\right] \times mg(-\hat{j})$$

$$\vec{\tau} = (u\cos\theta)t(mg)(\hat{i} \times -\hat{j})$$

$$\vec{\tau} = -mg(u\cos\theta)t(\hat{k})$$

MOMENT OF COUPLE:

A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation. If an object is not on pivot (unconstrained) a couple causes the object to rotate about its centre of mass.



This couple can produce turning effect (or) torque on the body. Moment of couple is the measure of turning effect (τ).

$\therefore \tau =$ moment of couple = magnitude of either force $\times$ perpendicular distance between the forces

$\therefore \tau = \text{Fd}$

Mechanical Equilibrium of a rigid body:

A A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time, or equivalently, body has neither linear acceleration nor angular acceleration.

Condition for translational equilibrium

A The vector sum of the forces, on the rigid body is zero; $\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_1 = 0$

If the total force on the body is zero, then the total linear momentum of the body does not change with time. P=constant

Condition for rotational equilibrium :

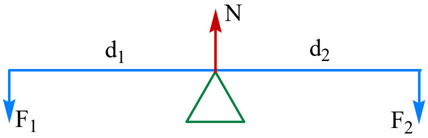
A The vector sum of the the torques on the rigid body is zero; $\vec{\tau}_1 + \vec{\tau}_2 + ... + \vec{\tau}_n = \sum_{i=1}^{n} \vec{\tau}_1 = 0$

If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time.

A The rotational equilibrium condition is independent of the location of the origin about which the torques are taken.

Principle of moments :

A An ideal is lever essentially a light rod pivoted at a point along its length. This point is called the fulcrum. Two forces F_1 and F_2, parallel to each other and usually perpendicular to the lever act on the lever at distances d_1 and d_2 respectively from the fulcrum. N is directed opposite to the forces F_1 and F_2. (N=Reaction at fulcrum) For translational equilibrium. $N-F_1-F_2=0$



A For rotational equilibrium take the moments about the fulcrum; the sum of moments must be zero, $d_1F_1=d_2F_2=0$

N acts at the fulcrum itself and has zero moment about the fulcrum.

A In the case of the lever force F_1 is usually some weight to be lifted. It is called the load and its distance from the fulcrum d_1 is called the load am. Force F_2 is the effort applied to lift the load; distance d_2 of the effort from the fulcrum is the effort arm.

Principle of moments for a lever

Load arm x load = effort arm x effort

Mechanical advantage :

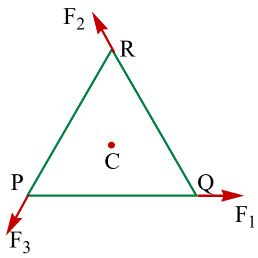
A The ratio F_1/F_2 is called the Mechanical Advantage

$$M.A. = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

If the effort arm d_2 is larger than the load arm, the mechanical advantages is greater than one. It means that a small effort can be used to lift a large load.

5. PQR is a rigid equilateral triangle frame of a side length 'L'. Forces F_1,F_2 and F_3 are acting along PQ, QR, PR. If the system is in rotational equilibrium find the relation between the forces.

Sol. Perpendicular distance of any force from centroid 'C' of triangle is $L/2\sqrt{3}$. The forces F_1, F_2 produce anti-clockwise turning effect where as F_3 clock wise turning effect about 'C'.

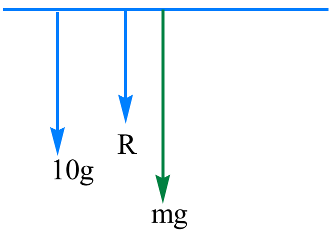


Since the system is in rotational equilibrium the total torque acting on the system about the centroid is zero

$$F_1 \times \frac{L}{2\sqrt{3}} + F_2 \times \frac{L}{2\sqrt{3}} - F_3 \times \frac{L}{2\sqrt{3}} = 0$$

Hence $F_1 + F_2 - F_3 = 0; \therefore F_3 = F_1 + F_2$

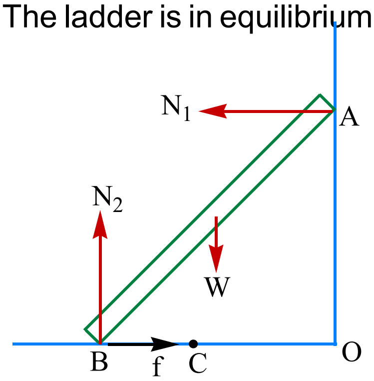
6.A meter stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12cm mark, the stick is found to be balanced at 45cm. What is the mass of the metre stick?

Sol. 

Since the stick is in rotational equilibrium, the total torque of all the forces about the resultant 'R' is zero. Taking the turning effects about the point of action of the resultant R we have 10g x 33=mg x 5; m= 66g

7.A uniform ladder of mass 10 Kg leans against a smooth vertical wall making an angle 53^0 with it. The other end rests on a rough horizontal floor. Find the normal force and the frictional force that the floor exerts on the ladder.

Sol. The ladder is in equilibrium.



$\therefore N_1 = f \;\; and \;\; N_2 = W$

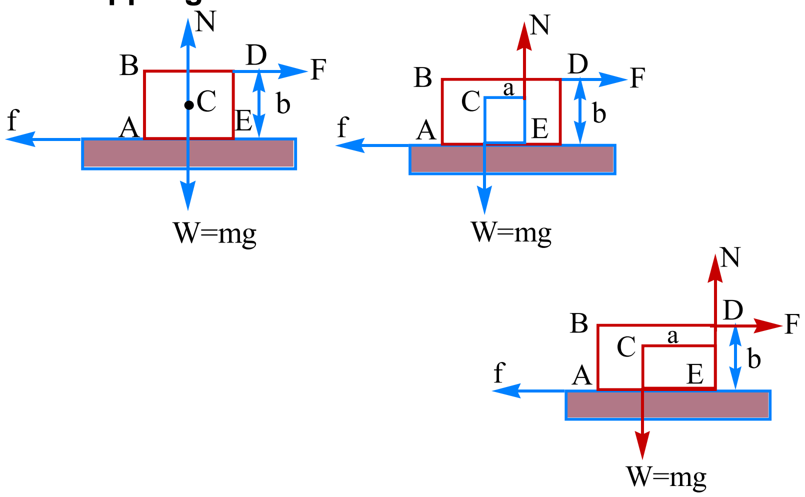
Taking torque about 'B'; N_1(AO)=W(CB)

$N_1(AB)\cos 53^0 = W\left(\frac{AB}{2}\right)\sin 53^0$

$N_1 = \frac{2}{3}W$ and $N_2 = W = 10 \times 9.8 = 98N$.

The frictional force is $f = N_1 = \frac{2}{3}W = 65N$

Toppling :



Suppose a force F is applied at a height b above the base AE of the block. Further, suppose the friction 'f' is sufficient to prevent sliding. In this case if the normal reaction N also passes through C then despite the fact that the block is in translational equilibrium (F=f and N=mg) an unbalanced torque (due to the couple of forces F and f) is there. This torque has tendency to topple the block about point E. To cancel the effect of this unbalanced torque the normal reaction N is shifted towards right a distance ‘a’ such that, net anti clock wise torque is equal to the net clock wise torque.

$$Fb = mg(a) \Rightarrow a = \frac{Fb}{mg}$$

Now, as F(or) b (or) both are increased distance a also increases. But it can not go beyond the right edge of the block. So in extreme case the normal reaction passes through E. Now if F or b are further increased, the block will topple down. This is why the block having the broader base has less chances of toppling in comparison to a block of smaller base.

8 A uniform cylinder of height h and radius r is placed with its circular face on a rough inclined plane and the inclination of the plane to the horizontal gradually in creased. If μ is the coefficient of friction, then under what conditions the cylinder will

a) slide before toppling

b) topple before sliding

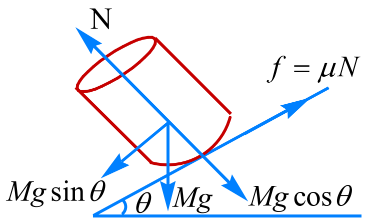
Sol.

a) The cylinder will slide if

$$Mg\sin\theta > \mu Mg\cos\theta \Rightarrow \tan\theta > \mu....(1)$$

The cylinder will topper if

$$(Mg\sin\theta)\frac{h}{2} > (Mg\cos\theta)r \Rightarrow \tan\theta > \frac{2r}{h}....(2)$$

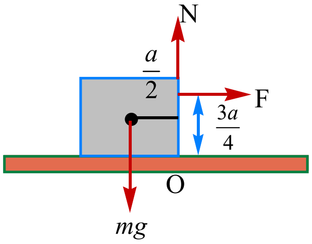


Thus, the condition of sliding is $\tan\theta > \mu$ and condition of toppling is $\tan\theta > \frac{2r}{h}$. Hence, the cylinder will slide before toppling if $\mu < \frac{2r}{h}$

b) The cylinder will topple before sliding if $\mu > \frac{2r}{h}$

9.A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the face at a point directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. What is the minimum value of F for which the cube begins to topple about an edge?

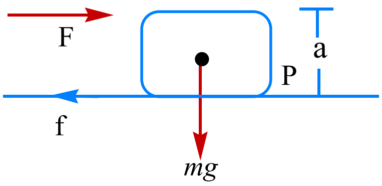
Sol. In the limiting case normal reaction will pass through O. The cube will topple about O if torque of F exceeds the torque of mg.



$$\Rightarrow F\left(\frac{3a}{4}\right) > mg\left(\frac{a}{2}\right); \Rightarrow F > \frac{2}{3}mg$$

So, the minimum value of F is $\frac{2}{3}mg$

10. A force F is applied on the top of a cube as shown in the figure. The coefficient of friction between the cube and the ground is μ. If F is gradually increased, find the value of μ for which the cube will topple before sliding.



Sol. Let m be the mass of the cube and 'a' be the side of the cube.

The cube will slide if $F > \mu mg$ ......(1)

and it will topple if torque of F about P is greater than torque of 'mg' about P i.e,

$$Fa > \left(\frac{a}{2}\right)mg \text{ or } F > \frac{1}{2}mg.....(2)$$

From equations (1) and (2) we see that cube will topple before sliding if $\mu > \frac{1}{2}$.

MOMENT OF INERTIA [ROTATIONAL INERTIA]

A A body at rest cannot start rotating itself or a rotating body cannot stop rotating on its own. Hence, a body has inertia of rotational motion.

A The quantity measuring the inertia of rotational motion is known as moment of inertia.

A Moment of inertia of a particle of mass m is

$I = mr^2$

Where r=perpendicular distance of particle from axis of rotation.

S.I Unit : kgm^2; Its D.F-ML^2

Dimensional formula : $\left[ML^2\right]$

A Moment of inertia of a group or system of particles is $I = m_1r_1^2 + m_2r_2^2 + ... + m_nr_n^2$ $I = \sum mr^2$

Where $m_1, m_2, ..., m_n$ are masses of particles and $r_1, r_2, ..., r_n$ are their perpendicular distance from axis of rotation.

A Motional or Inertia in rotational motion is analogous (similar) to mass in translatory motion.

A Moment of inertia of a rigid body depends on the following three factors.
a) mass of the body b) position of axis of rotation
c) Nature of distribution of mass.

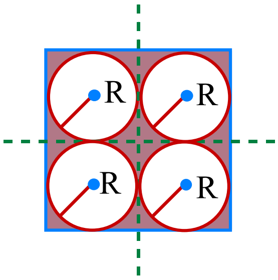
Note-1 : Moment of inertia of a rotating rigid body is independent of its angular velocity.

Note-2 : Moment of inertia of a metallic body depends on its temperature

11.Four holes of radius R are cut from a thin square plate of the side 4R and mass M in XY plane as shown. Then moment of inertia of the remaining portion about z-axis is

Sol. M is the mass of the square plate before cutting the holes.

Mass of one hole $m=\left[\frac{M}{16R^2}\right]\pi R^2=\frac{\pi}{16}M$



moment of inertia of remaining portion

$I=I_{square}-4I_{hole}$

$$I=\frac{M}{12}\left[16R^2+16R^2\right]-4\left[\frac{mR^2}{2}+m\left(2R^2\right)\right]$$

$$=\frac{8}{3}MR^2-10mR^2 \qquad =\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$$

Radius of Gyration(K) : Radius of gyration of a rigid body about an axis of rotation is distance between the axis of rotation and a point at which the whole mass of the body can be supposed to be concentrated so that its moment of inertia would be the same with the actual distribution of mass.

A Moment of inertia of a rigid body of mass M is $I=MK^2$
Where K=radius of gyration

$$K=\sqrt{\frac{r_1^2+r_2^2+......+r_n^n}{n}}$$

When n is total number of particles in the body and r_1, r_2,.....,r_n are their perpendicular distances from axis of rotation.

S.I. Unit : metre **CGS Unit:** cm

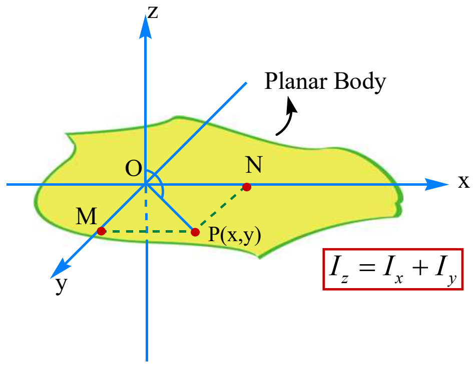
Dimensional formula : $[M^0LT^0]$

Note : K is not the distance of centre of mass of body from the axis considered.

A **Radius of gyration of a rigid body depends on the following two factors**
a) Position of axis of rotation
b) Nature of distribution of mass.

PERPENDICULAR AXES THEOREM

Statement : It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two mutually perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

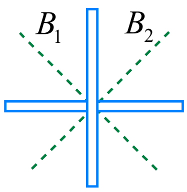


A This theorem is applicable to bodies which are planar.

A This theorem applies to flat bodies whose thickness is very small compared to their other dimensions.

A $K_z = \sqrt{K_x^2 + K_y^2}$

12. Two identical rods each of mass M and length L are joined in cross position as shown in figure. The moment of inertia of a system about a bisector would be.



Sol. Moment of inertia of a system about an axis which is perpendicular to plane of rods and passing through the common centre of rods

$$I_z = \frac{ML^2}{12} + \frac{ML^2}{12} = \frac{ML^2}{6}$$

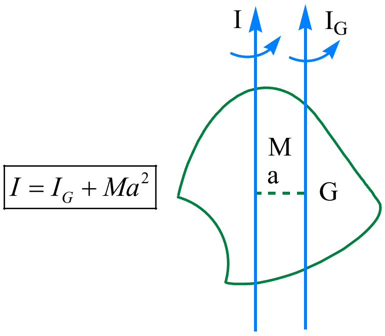
Again from perpendicular axes theorem

$$I_z = I_{B_1} + I_{B_2} = 2I_{B_1} = 2I_{B_2} \left[as\ I_{B_1} = I_{B_2}\right]$$

$$\therefore I_{B_1} = I_{B_2} = \frac{I_z}{2} = \frac{ML^2}{12}$$

PARALLEL AXES THEOREM

Statement: The moment of inertia of a body about an axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of gravity and the product of its mass and the square of the distance between the two parallel axes.



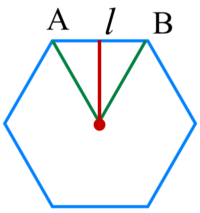
A This theorem is applicable to a body of any shape.

A $K = \sqrt{K_G^2 + a^2}$

13. The moment of inertia of a rod of length *l* about an axis passing through its centre of mass and perpendicular to rod is I. The moment of inertia of hexagonal shape formed by six such rods, about an axis passing through its centre of mass and perpendicular to its plane will be

Sol. M.I. of rod AB about its centre and

perpendicular to length$=\frac{ml^2}{12} = I \Rightarrow ml^2 = 12I$



Now moment of inertia of rod about the axis which is passing through O and perpendicular to the pane of hexagon

$I_{rod} = \frac{ml^2}{12} + mx^2$ [from parallel axes theorem]

$= \frac{ml^2}{12} + m\left[\frac{\sqrt{3}}{2}l\right]^2 = \frac{5ml^2}{6}$

Now moment of inertia of system

$I_{system} = 6 \times I_{rod} = 5ml^2 = 5 \times 12I = 60I$

14.The radius of gyration of a body about an axis at a distance of 12cm from its centre of mass is 13cm. Find its radius of gyration about a parallel axis through its centre of mass.

Sol. By parallel axes theorem

$M(13)^2 = I_0 + M(12)^2$

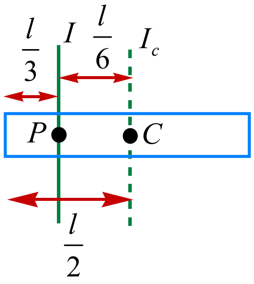
$I_0 = M(13^2 - 12^2) = M(25)$

Its radius of gyration about a parallel axes through its centre of mass $K = \sqrt{\frac{I_0}{M}} = \sqrt{25} = 5cm$

15. Find the moment of inertia of a thin uniform rod about an axis perpendicular to its length and passing through a point which is at a distance of $\frac{l}{3}$ from one end. Also find radius of gyration about that axis.

Sol. i) Let P be the point at a distance $\frac{l}{3}$ from one end.

It is a distance of $\left(\frac{l}{2}-\frac{l}{3}\right)=\frac{l}{6}$ from the centre as shown in the figure.



By parallel axes theorem $I=I_C+Mr^2$

$$=\frac{Ml^2}{12}+M\left(\frac{l}{6}\right)^2=\frac{Ml^2}{9}$$

ii) The radius of gyration, $K=\sqrt{\frac{I}{M}}=\sqrt{\frac{Ml^2}{9M}}=\frac{l}{3}$

16..A uniform cylinder has radius R and length L. If the moment of inertia of this cylinder about an axis passing through its centre and normal to its circular face is mg equal to the moment of inertia of the same cylinder about an axis passing through its centre and normal to its length, then

Sol. Moment of inertia of a cylinder about an axis passing through centre and normal to circular face = $\frac{MR^2}{2}$

Moment of inertia of a cylinder about an axis passing through centre and normal to its length

$$=M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$$

But $\frac{MR^2}{2}=M\left[\frac{L^2}{12}+\frac{2}{4}\right]$

$$\frac{R^2}{2}=\frac{L^2}{12}+\frac{R^2}{4}\Rightarrow\frac{R^2}{4}=\frac{L^2}{12};$$

$$\therefore L=\sqrt{3}R$$

17. A metal piece of mass 120g is stretched to form a plane rectangular sheet of area of cross section 0.54m². If length and breadth of this sheet are in the ratio 1:6, find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

Sol. Mass M=120g=120x10^{-3} kg

Area = $lb = 0.54m^2 \Rightarrow \frac{b}{6}b = 0.54\left(\because l = \frac{b}{6}\right)$

$b^2 = 0.54 \times 6 \Rightarrow b = \sqrt{3.24} = 1.8m$

$I = \frac{M\left(l^2 + b^2\right)}{12} = 33.3 \times 10^{-23} kgm^2$

18.: The moment of inertia of HCl molecule about an axis passing through its centre of mass and perpendicular to the line joining the H^+ and Cl^- ions will be (if the inter atomic distance is $1A^0$).

Sol. $r = 1A^0 = 10^{-10}m; m_1 = 1amu; m_2 = 35.5amu$

Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2} = 0.9726amu$

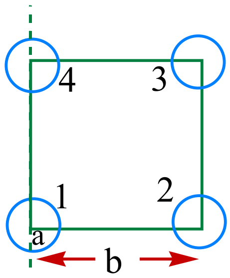
$\cong 1.624 \times 10^{-27} kg\left[\because 1amu = 1.67x10^{-27} kg\right]$

Moment of inertia about an axis passing through centre of mass of two particle system and perpendicular to the line joining them is

$1 = \mu r^2 = 1.624 \times 10^{-47} kgm^2$

19. Four solid spheres each of diameter 2a and mass m are placed with their centers on the four corners of a square of side b. Calculate the moment of inertia of the system about any side of the square.

Sol.



$I_1 = \frac{2}{5}ma^2; I_2 = \frac{2}{5}ma^2 + mb^2$

$I_3 = \frac{2}{5}ma^2 + mb^2;\ I_4 = \frac{2}{5}ma^2$

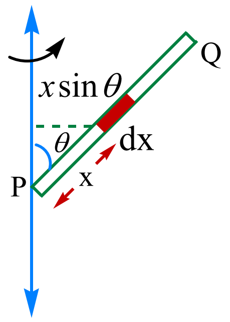
Moment of inertia of the system $I = I_1 + I_2 + I_3 + I_4$

$= \frac{2}{5}ma^2 + \frac{2}{5}ma^2 + \frac{2}{5}ma^2 + ma^2 + \frac{2}{5}ma^2\theta$

$$I = \frac{8}{5}ma^2 + 2mb^2$$

20.A rod PQ of mass 'm' and length L is rotated about an axis through 'P' as shown in figure. Find the moment of inertia of the rod about the axis of rotation.

Sol. Consider a small element 'dx' of the rod which is at a distance 'x' from the end 'P'. If 'θ' is the inclination of rod w.r.t the axis of rotation, the radius of the circle in which the element rotates is given by $\sin\theta = \frac{r}{x} \Rightarrow r = x\sin\theta$



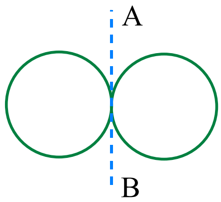
M.I. of the element about the axis of rotation is $dI = dm.r^2$

where dm is the mass of element $dm = \frac{m}{L}dx$

$dI = \frac{mL}{dx}(x\sin\theta)^2$. Total M.I. of the rod is given by $I = \int dI = \int_0^L \frac{m}{L}\sin^2\theta x^2 dx, I = \frac{mL^2}{3}\sin^2\theta$

21. Two uniform circular disc, each of mass 1kg and radius 20cm, are kept in contact about the tangent passing through the point of contact. Find the moment of inertia of the system about the tangent passing through the point of contact.

Sol.



Mass m+1kg, r=20x10^{-2}m

$$I_1 = \frac{MR^2}{4} + MR^2 = \frac{5MR^2}{4}$$

Similarly $I_2 = \frac{5MR^2}{4}, I = I_1 + I_2$

$$\therefore I = \frac{10MR^2}{4} = \frac{10\times1\times(20\times10^{-2})^2}{4} = 0.1kgm^2$$

22. Two solid sphere (A and B) are made of metals of different densities ρ_A and ρ_B respectively. If their masses are equal, the ratio of their moments of inertia (I_A/I_B)

about their respective diameter is [E-2007]

Sol. As two solid spheres are equal in masses, so

$$m_A = m_B \Rightarrow \frac{4}{3}\pi R_A^3 \rho_A = \frac{4}{3}\pi R_B^3 \rho_B \Rightarrow \frac{R_A}{R_B} = \left(\frac{\rho_B}{\rho A}\right)^{\frac{1}{3}}$$

The moment of inertia of sphere about diameter

$$I = \frac{2}{5} mR^2 \Rightarrow \frac{I_A}{I_B} = \left(\frac{R_A}{R_B}\right)^2 \Rightarrow \frac{I_A}{I_B} = \left(\frac{\rho_B}{\rho_A}\right)^{2/3}$$

23. The moment of inertia of a then circular disc about an axis passing through its center and perpendicular to its plane is I. Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is [E-2008]

Sol. $I = \frac{MR^2}{2} \Rightarrow MR^2 = 2I$

M.I. of the disc about tangent in a plane

$$= \frac{5}{4} MR^2 = \frac{5}{2} I$$

24. The moment of inertia of a disc, of mass M and radius R, about an axis which is a tangent and parallel to its diameter is [E-2010]

Sol. About the tangent parallel to the diameter

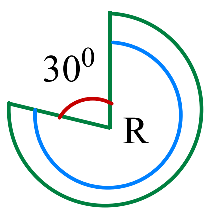
$$I = I_g + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4} MR^2$$

25.0Two solid spheres A and B each of radius R are made of materials of densities ρ_A and ρ_B respectively. Their moments of inertia about a diameter are I_A and I_B respectively. The value of I_A/I_B is [E-2012]

Sol. $$\frac{I_A}{I_B} = \frac{\frac{4}{3}\pi R^3 \rho_A}{\frac{4}{3}\pi R^3 \rho_3} = \frac{\rho_A}{\rho_B}$$

26.From a complete ring of mass M and radius R, an are making 30^0 at centre is removed. What is the moment of inertia of the incomplete ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring.

Sol. Mass of incomplete ring= $M - \frac{M}{2\pi} \times \frac{\pi}{6} = \frac{11M}{12}$



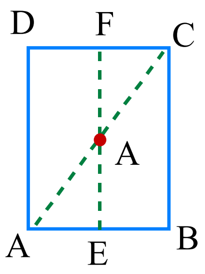
M.I. of incomplete ring $I=\left[\frac{11M}{12}\right]R^2=\frac{11}{12}MR^2$

Note : If a sector of mass m, rotates about its natural axis then its M.I. is mR^2

27. A thin wire of length l having density ρ is bent into a circular loop with C as its centre, as shown in figure. The moment of inertia of the loop about the line AB is [E-2014]

Sol. $I=\frac{3}{2}MR^2=\frac{3}{2}\times l\rho\times\left(\frac{l}{2\pi}\right)^2=\frac{3\rho l^3}{8\pi^2}$

28. For the given uniform square lamina ABCD, whose centre is O. Its moment of inertia about an axis AD is equal to how many times its moment of inertia about an axis EF? [AIEEE-2007]



1) $\sqrt{2}I_{AC}=I_{EF}$ 2) $I_{AD}=3I_{EF}$

3) $I_{AC}=4I_{EF}$ 4) $I_{AC}=\sqrt{2}I_{EF}$

Sol. $I_{EF}=I_{GH}$ $(due\ to\ symmetry)$

$I_{AC}=I_{BD}$ $(due\ to\ symmetry)$

$I_{AC}+I_{BD}=I_0$

$\Rightarrow 2I_{AC}=I_0....(1)$ and $I_{EF}+I_{GH}=I_0$

$\Rightarrow 2I_{EF}=I_0.......(2)$

From Eqs (1) and (2), we get

$I_{AC}=I_{EF}$

$\therefore I_{AD}=I_{EF}+\frac{md^2}{4}+\frac{md^2}{12}+\frac{md^2}{4}$

$I_{AD}=\frac{md^2}{3}=4I_{EF}$

29. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is [AIEEE-2008]

1) $\frac{5}{6}ma^2$ 2) $\frac{1}{12}ma^2$ 3) $\frac{7}{12}ma^2$ 4) $\frac{2}{3}ma^2$

Sol. Using parallel axes theorem,

$I=I_G+Mr^2=\frac{Ml^2}{12}+\frac{Ml^2}{2}=\frac{7Ml^2}{12}$

30. A disc of moment of inertia $4kgm^2$ is spinning freely at $3rads^{-1}$. A second disc of moment of inertia $2kgm^2$ slides down the spindle and they rotate together. a) What is the

angular velocity of the combination?

b) What is the change in kinetic energy of the system?

Sol. a) Since there are no external torques acting, we may apply the conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i \Rightarrow (6kgm^2)\omega_f = (4kgm^2)(3rad\ s^{-1})$$

Thus $\omega_f = 2rads^{-2}$

b) The kinetic energies before and after the collision are

$$K_i = \frac{1}{2}I_i\omega_i^2 = 18J; K_f = \frac{1}{2}I_f\omega_f^2 = 12J$$

The change is $\Delta K = K_f - K_i = -6J$

In order for the two discs to spin together at the same rate, there had to be friction between them. The loss in kinetic energy is converted into thermal energy.

Angular momentum of a particle

Definition : The moment of linear momentum of a body w.r.t. an axis of rotation is known as **angular momentum**.

A The angular momentum $\vec{L}$ of the particle with respect to the origin O is represented as $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$.

A The magnitude of the angular momentum vector is $L = r\,p\sin\theta$, where p is the magnitude of $\vec{p}$ and θ is the angle between $\vec{r}$ and $\vec{p}$.

A It is always directed perpendicular to the plane of rotation and along the axis of rotation.

Angular momentum of rigid body:

When a rigid body is rotating, then the vector sum of angular momenta of all the particles of body about the axis of rotation is called angular momentum of rigid body. It is equal to the product of moment of inertia and angular velocity.

$$\therefore \bar{L} = \sum_i (\bar{r}_i \times m_i\bar{v}_i) = I\bar{\omega}$$

S.I. Unit : kgm^2/sec

Dimensional formula : ML^2T^{-1}

When a body is rolling its total angular momentum is the vector sum of its angular momentum about centre of mass and the angular momentum of centre of mass about a fixed point on the ground.

Moment of inertia of some regular rigid bodies			
Rigid Body	**Axis Rotation**	**Moment of Inertia (I)**	**Radius of Gyration (K)**
1) Circular ring of mass M and radius R.	1) $\perp^r$ to the plane of ring and passing through its centre	MR^2	R
	2) $\perp^r$ to the plane of ring and passing through its rim (or) passing through any tangent $\perp^r$ to the plane of ring	$2MR^2$	$\sqrt{2}R$
	3) In the plane of the ring and passing through its centre (or) passing through any diameter of ring	$MR^2/2$	$R/\sqrt{2}$
	4) In the plane of the ring and passing through its edge (or) passing through any tangent of ring in its plane.	$3MR^2/2$	$\sqrt{3/2}R$
2) Thin circular plane of mass M and radius R	1) $\perp^r$ to the plane of plate and passing through its centre	$MR^2/2$	$R/\sqrt{2}$
	2) $\perp^r$ to the plane of plate and passing through its edge (or) passing through any tangent $\perp^r$ to its plane.		$\sqrt{32}R$
	3) In the plane of plate and passing through its centre (or) passing through any diameter of plate	$MR^2/4$	$R/2$
	4) In the plane of the plate and passing through its edge (or) passing through any tangent of plate in its plane.	$5MR^2/4$	$\sqrt{5}R/2$
3) Thin hollow sphere of mass M and radius R	1) Passing through its centre or any diameter	$2MR^2/3$	$\sqrt{2}R/\sqrt{3}$
	2) Passing through any tangent	$5MR^2/3$	$\sqrt{5}R/\sqrt{3}$
4) Solid sphere of mass M and radius R	1) Passing through its centre or any diameter	$2MR^2/5$	$\sqrt{2}R/\sqrt{5}$
	2) Passing through any tangent	$7MR^2/5$	$\sqrt{7}R/\sqrt{5}$
5) Thin uniform rod of mass M and L	1) $\perp^r$ to the length of rod and passing through its centre	$ML^2/12$	$L/2\sqrt{3}$
	2) $\perp^r$ to the length of rod and passing through its end	$ML^2/3$	$L/\sqrt{3}$

Moment of inertia of some regular rigid bodies			
Rigid Body	**Axis Rotation**	**Moment of Inertia (I)**	**Radius of Gyration (K)**
	4) $\perp^r$ to the axis of cylinder and passing through one end	$M\left(\frac{L^3}{3}+\frac{R^2}{2}\right)$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{2}}$
9) Solid cylinder of Mass M radius R and length L	1) About geometrical or natural axis	$MR^2/2$	$R/\sqrt{2}$
	2) Parallel to the length of cylinder and touching its surface (or) passing through line of contact of cylinder with floor when it is rolling.	$3MR^2/2$	$\sqrt{3}R/\sqrt{2}$
	3) $\perp^r$ to the axis of cylinder and passing through its centre	$M\left(\frac{L^2}{12}+\frac{R^2}{4}\right)$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{4}}$
	4) $\perp^r$ to the axis of cylinder and passing through one end	$M\left(\frac{L^2}{3}+\frac{R^2}{4}\right)$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{4}}$

Law of conservation of angular momentum

If there is no external torque acting on the rotating body (or system of particles), then its angular momentum is conserved.

If $\bar{\tau}_{ext}=\bar{0}$ then $\frac{d\bar{L}}{dt}=\bar{0}\left[\because \frac{d\bar{L}}{dt}=\bar{\tau}_{ext}\right]$

$\Rightarrow \bar{L}=I\omega=\text{constant} \therefore I_1\omega_1=I_2\omega_2$

31. A ballet dancer spins about a vertical axis at 60 rpm with arms outstretched. When her arms are folded the angular frequency increases to 90 rms. Find the change in her moment of inertia

Sol. By the principle of conservation of angular momentum $I\times 60=I_2\times 90$

Final moment of inertia, $I_2=\frac{2I}{3}$

Change in moment of inertia = $I-\frac{2I}{3}=\frac{I}{3}$

32.A horizontal disc is freely rotating about a vertical axis passing through its centre at the rate of 100 rpm. A bob of wax of mass 20g falls on the disc and sticks to it a distance of 5 cm from the axis. If the moment of inertia of the disc about the given axis is 2x10⁻⁴ kgm², find new frequency of rotation of the disc.

Sol. I_1=Moment of inertia of disc = 2×10^{-4} kgm^2

I_2=moment of inertia of the disc + moment of inertia of the bob of wax on the disc

$=2\times10^{-4}+mr^2=2\times10^{-4}+20\times10^{-3}(0.05)^2$

$=2\times10^{-4}+0.5\times10^{-4}=2.5\times10^{-4}kgm^2$

By the principle of conservation of angular momentum

$I_1n_1=I_2n_2\Rightarrow 2\times10^{-4}\times100=2.5\times10^{-4}n_2$

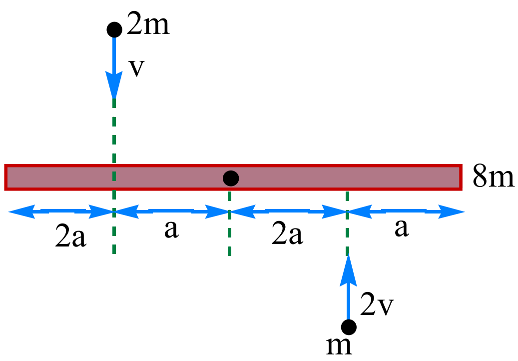
$$n_2 = \frac{100 \times 2}{2.5} = 80 rpm$$

33.A circular platform is mounted on a vertical frictionless axle. Its radius is r=2m and its moment of inertia is I=200kg-m². It is initially at rest. A 70kg man stands on the edge of the platform and begins to walk along the edge at speed. V_0=1.0 m/s relative to the ground. Find the angular velocity of the platform.

Sol. Angular momentum of man=angular momentum of platform in opposite direction.

$$mv_0 r = I\omega \Rightarrow \omega = 0.7 rad/s$$

EX-52: A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v and v respectively, strike the bar (as shown in fig) and stick to the bar after collisions. Calculate (a) velocity of the centre of mass (b) angular velocity about centre of mass and (c) total kinetic energy, just after collision.



Sol. (a) As F_{ext}=0 linear momentum of the system is conserved, i.e.,

$$-2m \times v + m \times 2v + 0 = (2m + m + 8m) \times V$$

or V=0 i.e. velocity of centre of mass is zero.

(b) As $\tau_{ext} = 0$ angular momentum of the system is conserved, i.e.

$$m_1 v_1 r_1 + m_2 v_2 r_2 = (I_1 + I_2 + I_3)\omega$$

$$2mva + m(2v)(2a) = \left[2m(a)^2 + m(2a)^2 + 8m \times (6a)^2/12\right]\omega$$

i.e. $6mva = 30ma^2\omega \qquad \Rightarrow \omega = \left(\frac{v}{5a}\right)$

(c) From (a) and (b) it is clear that, the system has no translatory motion but only rotatory motion.

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(30ma^2)\left[\frac{v}{5a}^2\right] = \frac{3}{5}mv^2$$

34.A hoop of radius r and mass m rotating with an angular velocity 'ω_0' is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip. (JEE-2013)

Sol. $mr^2\omega_0 = mvr + mr^2 \times \frac{v}{r} \Rightarrow v = \frac{\omega_0 r}{2}$

Rotational dynamics

Relation between Torque and angular momentum of a rigid body:

The vector sum of torques acting on various particles of rigid body gives the net torque acting on body.

$\bar{\tau} = \sum \bar{\tau}_i$ and $\bar{\tau} = \frac{d\bar{L}}{dt}$, $\bar{L}$ is total angular momentum of body.

The time rate of change of the angular momentum of a particle is equal to the torque acting on it.

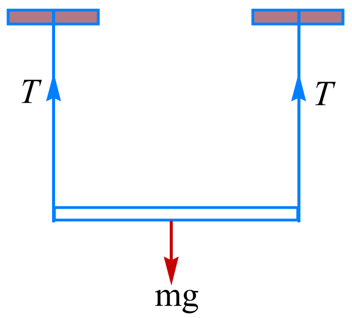
Relation between torque and angular acceleration:

$\bar{\tau} = \frac{d\bar{L}}{dt}$ But $\bar{L} = I\bar{\omega}$ $\therefore \bar{\tau} = I\frac{d\bar{\omega}}{dt} \Rightarrow \bar{\tau} = I\bar{\alpha}$

This equation is called equation of rotatory motion and analogous to Newton's 2^{nd} law in dynamics.

35.A uniform rod of mass 'm' and length '*l*' is suspended by means of two light inextensible strings at the ends of a rod. Tension in one string immediately after the other string is cut is

Sol. $mg - T = ma.....(1)$



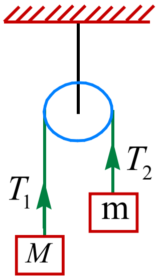
$$\alpha = \frac{\tau}{I} = \frac{mg\frac{\lambda}{2}}{\frac{ml^2}{3}} = \frac{3g}{2l}......(2)$$

$$a = \frac{1}{2}\alpha......(3)$$

solving eq (1), (2) and (3) we get, $T = \frac{mg}{4}$

36. The pulley of Atwoods machine has a moment of inertia 'I' about its axis and its radius is 'R'. Find the magnitude of acceleration of the two blocks assuming the string is light and does not slip on the pulley.

Sol.



Suppose the block of mass 'M' goes down with an acceleration 'a'. The angular acceleration of the pulley is, $\alpha = \frac{a}{R}$

$$Mg - T_1 = Ma; T_2 - mg = ma$$

and $T_1R - T_2R = I\alpha = I\frac{a}{R}$

Solving the equation, $a = \frac{(M-m)gR^2}{I + (M+m)R^2}$

ROTATIONAL KINETIC ENERGY

The sum of the kinetic energies of various particles of rotating body is called rotational kinetic energy.

$$KE_{rot} = \frac{L^2}{2I} = \frac{1}{2}I\omega^2 = \frac{1}{2}\omega L$$

37. The angular momentum of rotating body is increased by 20%. What will be the increase in its rotational kinetic energy?

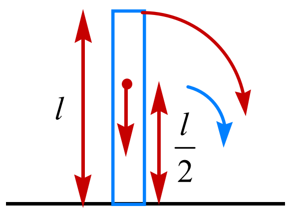
Sol. Kinetic energy KE = $\frac{L^2}{2I} \Rightarrow E \propto L^2$

$$\frac{\Delta E}{E} = \left(\frac{120}{100}\right)^2 (or) \frac{\Delta E}{E} = 0.44$$

$$\frac{\Delta E}{E} \times 100 = 44\%$$

38. A uniform rod of length 'l' is held vertically on a horizontal floor fixing its lower end, the rod is allowed to fall onto the ground. Find (i) its angular velocity at that instant of reaching the ground (ii) The linear velocity with which the tip of rod hits the floor.

Sol. The rod rotates about an axis through one end. From the principle of conservation of mechanical energy. Loss of P.E. of the rod is equal to its gain in rotational K.E.



$$\therefore mg\frac{l}{2}=\frac{1}{2}I\omega^2 \Rightarrow mg\frac{l}{2}=\frac{1}{2}.\frac{ml^2}{3}\omega^2$$

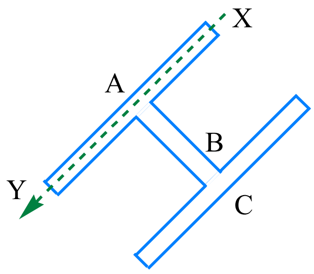
on solving $\omega=\sqrt{\frac{3g}{l}}$

iii) $V=r\omega \text{ or } V=l\omega=l\sqrt{3g/l}=\sqrt{3gl}$

39. A rigid body is made of three identical thin rods, each of length 'L' fastened together in the form of the letter 'H'. The body is free to rotate about horizontal axis that runs along the length of one of the legs of 'H'. The body is allowed to fall from rest from a position in which the plane of 'H' is horizontal. What is the angular speed of the body when the plane of 'H' is vertical?

Sol. The moment of inertia of the system about one rod as axis $I=\frac{mL^2}{3}+mL^2; I=\frac{4}{3}mL^2$

Potential energy decreases for B and C



$$\frac{mgL}{2}+mgL=\frac{3}{2}mgL$$

By conservation of mechanical energy, the loss in PE of body is equal to the gain in rotational KE

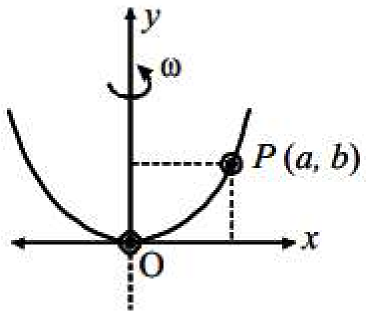
$\therefore \frac{3}{2}mgL=\frac{1}{2}\left(\frac{4}{3}mL^2\right)\omega^2$ on solving $\omega=\frac{3}{2}\sqrt{\frac{g}{L}}$

40. A uniform rod AB of mass 'm' length '2a' is allowed to fall under gravity with AB in horizontal. When the speed of the rod is 'v' suddenly the end 'A' is fixed. Find the angular velocity with which it begins to rotate.

PREVIOUS MAINS QUESTIONS

Topic-2: Angular Displacement, Velocity and Aceleration

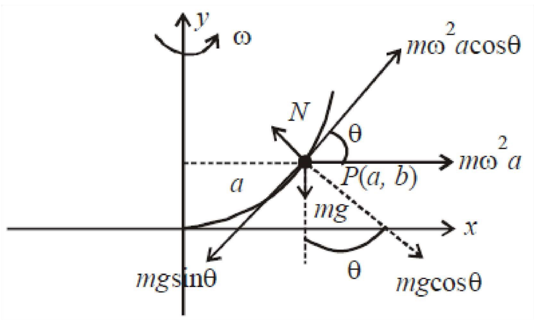
42. A bead of mass m stays at point $P(a,b)$ on a wire bent in the shape of a parabola $y=4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction **[sep2 2020 MAINS]**



(1) $2\sqrt{2gC}$ (2) $2\sqrt{gC}$ (3) $\sqrt{\frac{2gC}{ab}}$ (4) $\sqrt{\frac{2g}{C}}$

solution:. $y=4Cx^2 \Rightarrow \frac{dy}{dx}=\tan\theta=8Cx$

At P, $\tan\theta=8Ca$



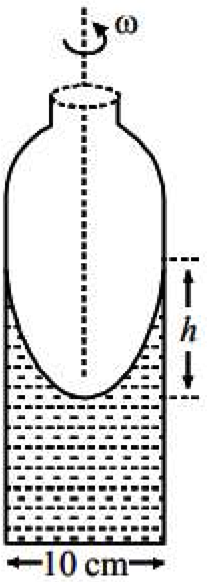
For steady circular motion

$$m\omega^2 a\cos\theta = mg\sin\theta$$

$$\Rightarrow \omega=\sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega=\sqrt{\frac{g\times 8aC}{a}}=2\sqrt{2gC}$$

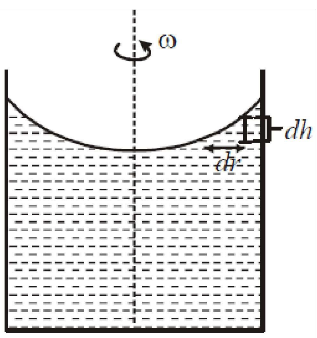
43. A cylindrical vessel containing a liquid is rotated about its axis so that the the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, *h* (in cm)of liquid at the centre of vessel and at the side will be [sep2 2020 MAINS]



(1) $\frac{2\omega^2}{25g}$ (2) $\frac{5\omega^2}{2g}$ (3) $\frac{25\omega^2}{2g}$ (4) $\frac{2\omega^2}{5g}$

solution:. (3) Here, $\rho dr\omega^2 r = \rho g dh$

$$\Rightarrow \omega^2 \int_0^R r dr = g \int_0^h dh$$



$$\Rightarrow \frac{\omega^2 R^2}{2} = gh$$ (Given R = 5 cm)

$$\therefore h = \frac{\omega^2 R^2}{2g} = \frac{25\omega^2}{2g}$$

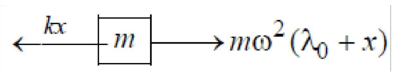
44 A spring mass system (mass m, spring constant k and natural length *l*) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If thedisc together with spring mass system, rotates about it's axis with an angular velocity ω, $\left(k >> m\omega^2\right)$ the relative change in the lenght of the spring is best given by the option : **[9 JAN2020 MAINS]**

(1) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (2) $\frac{2m\omega^2}{k}$

(3) $\frac{m\omega^2}{k}$ (4) $\frac{m\omega^2}{3k}$

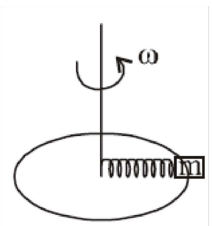
solution:.

25. (3) Free body diagram in the frame of disc



$$\therefore m\omega^2\left(\ell_0 + x\right) = kx$$

$$\Rightarrow x = \frac{m\ell_0\omega^2}{k - m\omega^2}$$



For $k >> m\omega^2$

$$\Rightarrow \frac{x}{\ell_0} = \frac{m\omega^2}{k}$$

45.A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length *l*. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is **[8JAN2020 MAINS]**

(1) $\frac{ml\omega^2}{k-\omega m}$ (2) $\frac{ml\omega^2}{k-m\omega^2}$ (3) $\frac{ml\omega^2}{k+m\omega^2}$ (4) $\frac{ml\omega^2}{k+m\omega}$

solution:.26. (2) At elongated position (x),

$$F_{radial} = \frac{mv^2}{r} = mr\omega^2$$

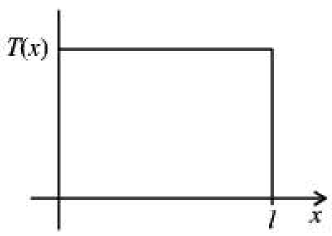
$$\therefore kx = m(\ell + x)\omega^2$$

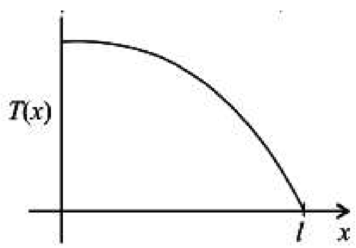
$$(\because r = \ell + x \ here)$$

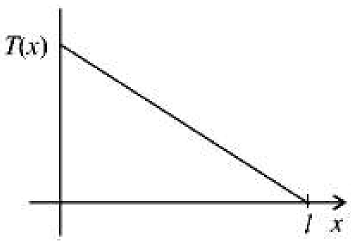
$$kx = m\ell\omega^2 + mx\omega^2$$

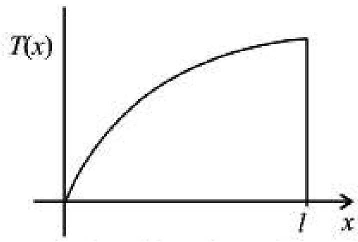
$$\therefore x = \frac{m\ell\omega^2}{k - m\omega^2}$$

46. A uniform rod of length *l* is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely? **[120APR 2019 MAINS]**

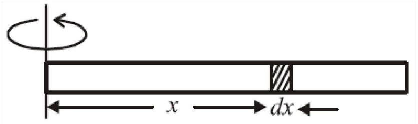
(1)


(2)


(3)


(4)


solution:. (4) $\int_0^T(-dT)=\int_l^x(dm)\omega^2x$

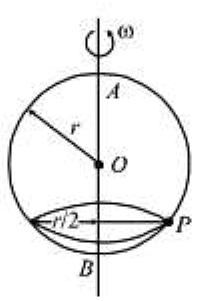


$$-T=\int_l^x\left(\frac{m}{l}dx\right)\omega^2x$$

Or $T=\frac{m\omega^2}{l}\left(l^2-x^2\right)$

It is a parabola between T and x.

47 **A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:**

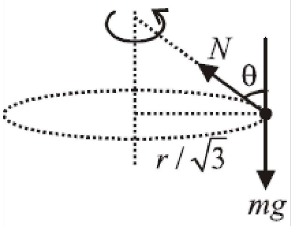


[12 APR 2019 MAINS]

(1) $\frac{\sqrt{3}g}{2r}$ (2) $2g/\left(r\sqrt{3}\right)$

(3) $\left(r\sqrt{3}\right)/r$ (4) $2g/r$

solution:(2) $N\sin\theta=m\omega^2\left(r/2\right)$ .....(i)



$\sin\theta = \frac{r/2}{r} = \frac{1}{2} \Rightarrow \theta = 30^0$

and $N\cos\theta = mg$ ......(ii)

or $\tan\theta = \frac{\omega^2 r}{2g}$

or $\tan 30^0 = \frac{\omega^2 r}{2g}$

or $\frac{1}{\sqrt{3}} = \frac{\omega^2 r}{2g}$

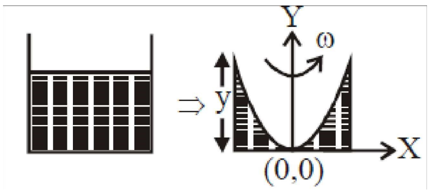
$\therefore \omega^2 = \frac{2g}{r\sqrt{3}}$

48. A long cylindrical vessel is half filled is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heigtths between the centre and the sides, in cm, will be

(1) 2.0 (2) 0.1 (3) 0.4 (4) 1.2 **[12 JAN 2019 MAINS]**

solution:((1) Using $v^2 = u^2 + 2gy$ [$\therefore u = 0$ at (0, 0)]

$v^2 = 2gy$ [$\therefore v = \omega x$]



$\Rightarrow y = \frac{\omega^2 x^2}{2g} = \frac{(2\times 2\pi)^2 \times (0.05)^2}{20} = 2cm$

49.. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^th^ power of R. If the period of rotation of the particle is T, then **[2018 MAINS]**

(1) $T \propto R^{3/2}$ for any n (2) $T \propto R^{n/2+1}$

(3) $T \propto R^{(n+1)/2}$ (4) $T \propto R^{n/2}$

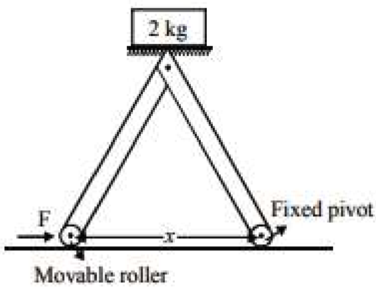
solution:.(3) $m\omega^2 R = Force \propto \frac{1}{R^n}$ (Force $= \frac{mv^2}{R}$)

$$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

Time period $T = \frac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

50.The machine as shown has 2 rods of length 1m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a:



[2017 MAINS]

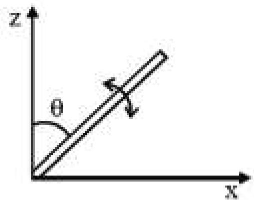
(1) constant speed (2) decreasingspeed (3) increasing speed

(4) speed which is $\frac{3}{4}th$ of that of the roller when the weight is 0.4 m above the ground

solution:.

50. (2) decreasing speed

51.A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angualr acceleration of the rod when it makes an angle with the vetical is

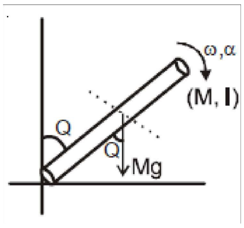


[2017MAINS]

(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{2\ell}\sin\theta$

solution:.(3) Torques at angle θ

$\tau = Mg\sin\theta.\frac{l}{2}$ Also $\tau = I\alpha$ $\therefore I\alpha = Mg\sin\theta\frac{l}{2}$



$\frac{Ml^2}{3}.\alpha = Mg\sin^\theta\frac{l}{2}$ $\left[\because I_{rod} = \frac{Ml^2}{3}\right]$

$\Rightarrow \frac{l\alpha}{3} = g\frac{\sin\theta}{2}$ $\therefore \alpha = \frac{3g\sin\theta}{2l}$

52.Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too faste, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of te drum in revolutions per minute (rpm) to ensure proper mixing is close to: (Take the radius of the drum to be 1.25 m and its axle to be horizontal):

(1) 27.0 (2) 0.4 (3) 1.4 (4) 8.0 **[2016MAINS]**

solution:.(1) For just complete rotation

$v = \sqrt{Rg}$ at top point

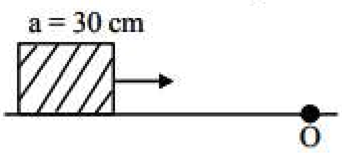
The rotational speed of the drum

$$\Rightarrow \omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}}$$

The maximum rotational speed of the drum in revolutions per minute

$$\omega(rpm) = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} = 27$$

53.A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is



[APR9 2016 MAINS]

(1) 13.3 (2) 5.0 (3) 9.4 (4) 6.7

solution:.(2) Angular momentum, $mvr = I\omega$

Moment of Inertia (I) of cubical block is given by

$$I = m\left(\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right) \quad \therefore \omega = \frac{m.2\frac{R}{2}}{m\left[\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right]}$$

$$\Rightarrow \omega = \frac{12}{8R} = \frac{3}{2\times 0.3} = \frac{10}{2} = 5\,rad/s$$

54.Two point masses of mass $m_1 = fM$ and $m_2 = (1-f)M\,(f<1)$ are in outer space (far from gravitational influence of other objects) at a distance R from each other. They move in circular orbits about their centre of mass with angular velocities ω_1 for m_1 and ω_2 for m_2. In the case **[MAY9 2016 MAINS]**

(1) $(1-f)\omega_1 = f\omega$

(2) $\omega_1 = \omega_2$ and independent of f

(3) $f\omega_1 = (1-f)\omega_2$

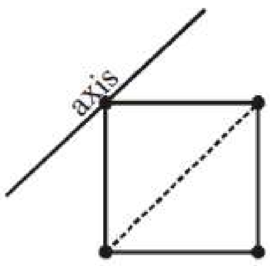
(4) $\omega_1 = \omega_2$ and depend on f

solution:.(2) Angular velocity is the angular displacement per unit time i.e., $\omega = \dfrac{\Delta\theta}{\Delta t}$

Here $\omega_1 = \omega_2$ and independent of f.

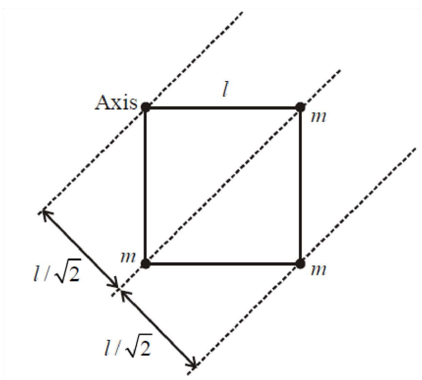
Topic-3: Torque, Couple and Angular Momentum

55.Four point masses, each of mass m, are fixed at the corners of a square of side *l*. The square is rotating with angular frequency ω, aqbout an ax is passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular moemtnum of the square about this axis is: [SEP 6 2020 MAINS]



(1) $ml^2\omega$ (2) $4ml^2\omega$ (3) $3ml^2\omega$ (4) $2ml^2\omega$

solution:.(3) Angular momentum, $L = I\omega$



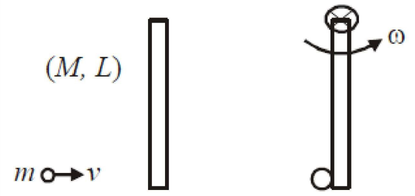
$$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m\left(\sqrt{2}l\right)^2$$

$$= \frac{2ml^2}{2} + 2ml^2 = 3ml^2$$

Angular momentum $L = I\omega = 3ml^2\omega$

56.A thin rod of mass 0.9 kg and length 1m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. Aparticle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ________ [SEP 6 2020 MAINS]

solution:.(20)



Before collision After collision

Using principal of conservation of angular momentum we have

$$\vec{L}_i = \vec{L}_f \Rightarrow mvL = I\omega$$

$$\Rightarrow mvL\left(\frac{ML^2}{3} + mL^2\right)\omega$$

$$\Rightarrow 0.1\times 80\times 1 = \left(\frac{0.9\times 1^2}{3} + 0.1\times 1^2\right)\omega$$

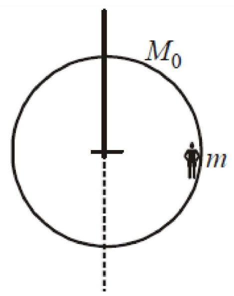
$$\Rightarrow 8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega \Rightarrow 8 = \frac{4}{10}\omega$$

$\Rightarrow \omega = 20$ rad/sec

57.A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the plastform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre__________[SEP3 2020 MAINS]

solution:. Here $M_0 = 200kg,\ m = 80\,kg$

Using conservation of angular momentum, $L_i = L_f$



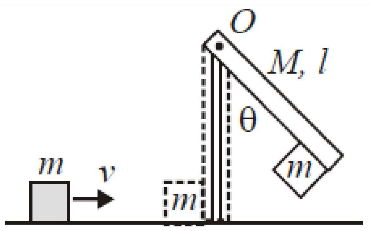
$I_1\omega_1 = I_2\omega_2$ $\qquad$ $I_1 = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$

$I_2 = \frac{1}{2}M_0R^2$ and $\omega_1 = 5\,rpm$

$$\therefore \omega_2 = \left(\frac{M_0R^2}{2} + mR^2\right) \times \frac{5}{\frac{M_0R^2}{2}}$$

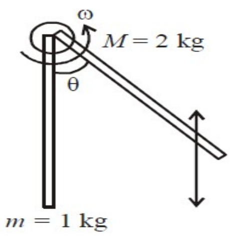
$$= \frac{5R^2}{R^2} \times \frac{(80+100)}{100} = 9\,rpm$$

58..A block of mass *m* = 1 kg slides with velocity *v* = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about *O* and swings as a result of the collision making angle θ q before momentarily coming to rest. If the rod has mass *M* = 2 kg, and length *l* = 1 m, the value of q is approximately: (take *g* = 10 m/s2) **[SEP3 2020 MAINS]**



(1) 63^0 (2) 55^0 (3) 69^0 (4) 49^0

solution:.(1) Using conservation of angular momentum



Now using energy conservation, after collision

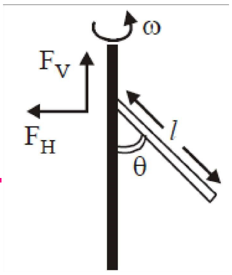
$$\frac{1}{2}I\omega^2 = 2mg\frac{l}{2}(1-\cos\theta) + mgl(1-\cos\theta) \quad \Rightarrow \frac{1}{2}\left(\frac{5}{3}ml^2\right)\frac{9v^2}{25l^2} = 2mgl(1-\cos\theta)$$

$$\Rightarrow \frac{3}{5\times 2}mv^2 = 2mgl(1-\cos\theta)$$

$$\frac{3}{10}\times\frac{36}{2\times 10} = 1-\cos\theta \Rightarrow 1-\frac{27}{50} = \cos\theta$$

Or, $\cos\theta = \dfrac{23}{50}$ $\quad\quad \therefore \theta \approx 63^0$

$$mvl = \left(ml^2 + \frac{2ml^2}{3}\right)\omega \Rightarrow mvl = \frac{5}{3}ml^2\omega \Rightarrow \omega = \frac{3v}{5l}$$ or $\omega = \dfrac{3\times 6}{5\times 1} = \dfrac{18}{5}$ rad/s



59. **A uniform rod of length '*l*' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of q is then such that :** **[SEP3 2020 MAINS]**

(1) $\cos\theta = \dfrac{2g}{3l\omega^2}$ (2) $\cos\theta = \dfrac{g}{2l\omega^2}$

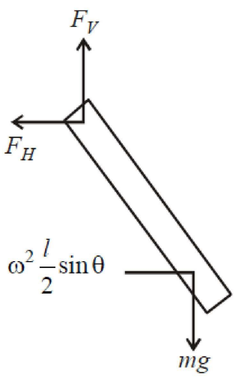
(3) $\cos\theta = \dfrac{g}{l\omega^2}$ (4) $\cos\theta = \dfrac{3g}{2l\omega^2}$

solution:.(4) Vertical force = mg

Horizontal force = Centripetal force $= m\omega^2 \frac{l}{2}\sin\theta$

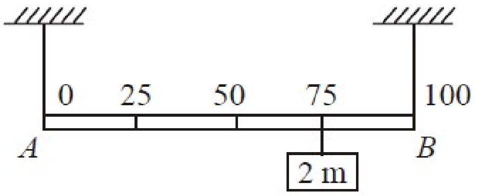
Torque due to vertical force $= mg\frac{l}{2}\sin\theta$

Torque due to horizontal force $= m\omega^2 \frac{l}{2}\sin\theta \frac{l}{2}\cos\theta$



Net Torque = Angular momentum $mg\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{2}\frac{g}{\omega^2 l}$

60. 

Shown in the figure is rigid and uniform one meter long rod *AB* held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass '*m*' and has another weight of mass 2 m hung at a distance of 75 cm from *A*. The tension in the string at *A* is : **[SEP2 2020 MAINS]**

(1) 0.5 mg (2) 2 mg

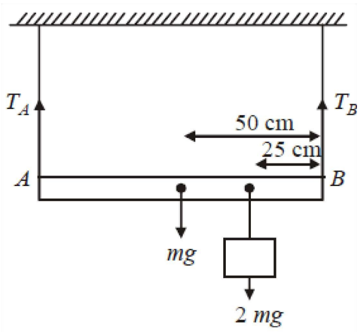
(3) 0.75 mg (4) 1 mg

solution:.(4) Net torque, τ_{net} about B is zero at equilibrium

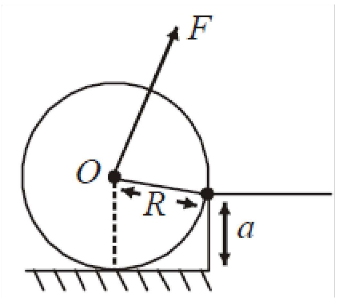
$$\therefore T_A \times 100 - mg \times 50 - 2mg \times 25 = 0$$

$$\Rightarrow T_A \times 100 = 100mg$$

$$\Rightarrow T_A = 1mg$$ (Tension in the string at A)



61.A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :

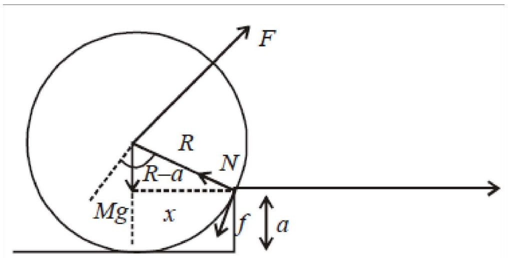


[SEP 2 2020 MAINS]

(1) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(3) $Mg\frac{a}{R}$ (4) $Mg\sqrt{1-\frac{a^2}{R^2}}$

solution:.(1)



For step up, $F \times R \geq Mg \times x$

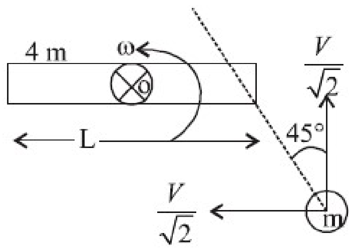
$x = \sqrt{R^2 - (R-a)^2}$ from figure

$$F_{\min} = \frac{Mg}{R} \times \sqrt{R^2 - (R-a)^2} = Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

62.Consider a uniform rod of mass M = 4m and length l pivoted about its centre. A mass m moving with velocity v making angle to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass $\theta = \frac{\pi}{4}$ system just after the collision is: **[8 JAN 2020 MAINS]**

(1) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (2) $\frac{3}{7}\frac{v}{l}$ (3) $\frac{3\sqrt{2}}{7}\frac{v}{l}$ (4) $\frac{4}{7}\frac{v}{l}$

solution:.(3)



About point O angular momentum

$L_{initial} = L_{final}$

$$\Rightarrow \frac{mV}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$

$$\therefore \omega = \frac{6V}{7\sqrt{2L}} = \frac{3\sqrt{2}V}{7L}$$

63.A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$ $\quad$ $y = y_0 + b\cos\omega_2 t$ $\quad$ **[10 APR 2019 MAINS]**

The torque, acting on the particle about the origin, at *t* = 0 is:

(1) $m\left(-x_0 b + ya\right)\omega_1^2\hat{k}$ (2) $+my_0 a\omega_1^2\hat{k}$ (3) zero (4) $-m\left(x_0 b\omega_2^2 - y_0 a\omega_1^2\right)\hat{k}$

solution:.(2) Given that, $x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

$$\frac{dx}{dt} = V_x$$

$\Rightarrow v_x = -a\omega_1\sin\left(\omega_1 t\right)$, and $\frac{dy}{dt} = v_y = b\omega_2\cos\left(\omega_2 t\right)$

$\frac{dv_x}{dt} = a_x = -a\omega_1^2\cos\left(\omega_1 t\right)$, $\frac{dv_y}{dt} = a_y = -a\omega_2^2\sin\left(\omega_2 t\right)$

At $t = 0, x = x_0 + a, y = y_0$

$$a_x = -a\omega_1^2, a_y = 0$$

Now, $\vec{\tau} = \vec{r}\times\vec{F} = m\left(\vec{r}\times\vec{a}\right)$

$$= \left[\left(x_0 + a\right)\hat{i} + y_0\hat{j}\right]\times m\left(-a\omega_1^2\hat{i}\right) = +my_0 a\omega_1^2\hat{k}$$

64.The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}\left(t\right) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is

(1) $48\left(\hat{i}+\hat{j}\right)$ (2) $36\hat{k}$ (3) $-34\left(\hat{k}-\hat{i}\right)$ (4) $-48\hat{k}$

[10 APR 2019 MAINS]

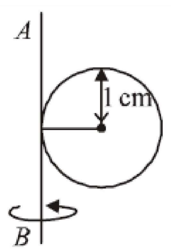
solution:. We have given $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$ $\vec{r}\left(at\ t = 2\right) = 4\hat{i} - 12\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$

$\vec{v}(at\ t = 2) = 2\hat{i} - 12\hat{j}$

$\vec{L} = mvr\sin\theta\hat{n} = m(\vec{r} \times \vec{v}) \qquad = 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$

65. metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to :



[10 APR 2019 MAINS]

(1) $4.0 \times 10^{-6}\,Nm$ (2) $1.6 \times 10^{-5}\,Nm$

(3) $7.9 \times 10^{-6}\,Nm$ (4) $2.0 \times 10^{-5}\,Nm$

solution:.(4) Angular acceleration,

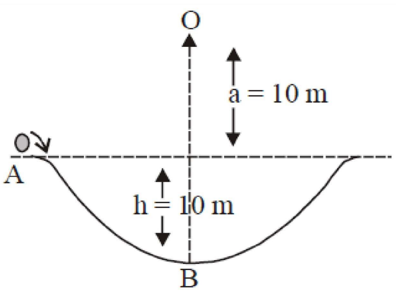
$$\alpha = \frac{\omega - \omega_0}{t} = \frac{25 \times 2\pi - 0}{5} = 10\pi\, rad / s^2$$

$$\tau = I\alpha$$

$$= \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$$= 2.0 \times 10^{-5}\,Nm$$

66.A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g = 10 m/s2)[12 JAN 2019 MAINS]



(1) 2 kg-m²/s (2) 8 kg-m²/s (3) 6 kg-m²/s (4) 3 kg-m²/s

solution:.(3) According to work-energy theorem

$$mgh = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$2gh = v_B^2 - v_A^2$$

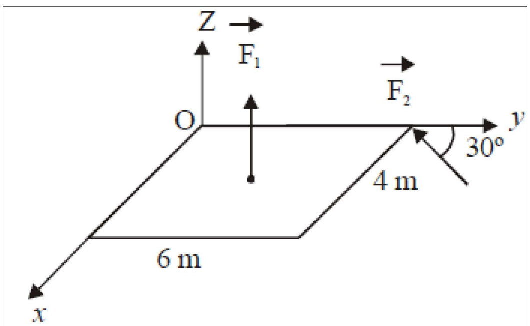
$$2\times 10\times 10 = v_B^2 - 5^2 \quad \Rightarrow v_B = 15\,m/s$$

Angular momentum about O,

$$L_0 = mvr$$

$$L_0 = 6kg.m^2/s$$

67.A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along *z*-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be : [11 JAN 2019 MAINS]



(1) $(3\hat{i}-2\hat{j}+3\hat{k})F$ (2) $(3\hat{i}-2\hat{j}-3\hat{k})F$

(3) $(3\hat{i}+2\hat{j}-3\hat{k})F$ (4) $(3\hat{i}+2\hat{j}+3\hat{k})F$

solution:.(1) Given, $\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$

$$\vec{r}_1 = 0\hat{i} + 6\hat{j}$$

Torque due to F_1 force

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 6\hat{j} \times \left(\frac{F}{2}\left(-\hat{i}\right) + \frac{F\sqrt{3}}{2}\left(-\hat{j}\right)\right) = 3F\left(\hat{k}\right)$$

Torque due to F_2 force

$$\vec{\tau}_{F_2} = \left(2\hat{i} + 3\hat{j}\right) \times F\hat{k} = 3F\hat{i} + 2F\left(-\hat{j}\right)$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} = 3F\hat{i} + 2F\left(-\hat{j}\right) + 3F\left(\hat{k}\right)$$

$$= \left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)F$$

68.The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians):

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{4}$ **[11 JAN 2019 MAINS]**

solution:.(1) Torque about the origin $= \vec{\tau} = \vec{r} \times \vec{F}$

$$= rF\sin\theta \Rightarrow 2.5 = 1 \times 5\sin\theta$$

$$\sin\theta = 0.5 = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

69.To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is m, the torque, applied by the machine on the mop is: **[10JAN 2019 MAINS]**

(1) $\mu FR/3$ (2) $\mu FR/6$

(3) $\mu FR/2$ (4) $\frac{2}{3}\mu\,\mu FR$

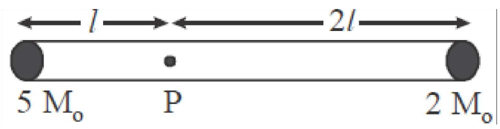
solution:.(4) Consider a strip of radius x and thickness dx, Torque due to friction on this strip

Net torque = $\sum$ Torque on ring

$$\int d\tau = \int_0^R \frac{\mu F.2\pi x dx}{\pi R^2} \qquad \Rightarrow \tau = \frac{2\mu F}{R^2}.\frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

70.A rigid massless rod of length 3*l* has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be: **[10 JAN 2019 MAINS]**



(1) $\frac{g}{13l}$ (2) $\frac{g}{3l}$ (3) $\frac{g}{2l}$ (4) $\frac{7g}{3l}$

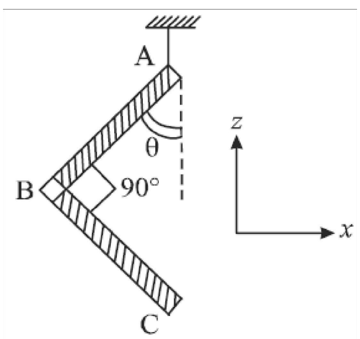
solution:.(1) Applying torque equation about point P.

$$\tau = I\alpha = \left[2M_0(2\ell)^2 + 5M_0\ell^2\right]\alpha$$

$$\Rightarrow 5M_0 g\ell - 4M_0 g\ell = \left[2M_0(2\ell)^2 + 5M_0\ell^2\right]\alpha$$

$$\Rightarrow M_0 g\ell = \left(13M_0 g\ell^2\right)\alpha$$

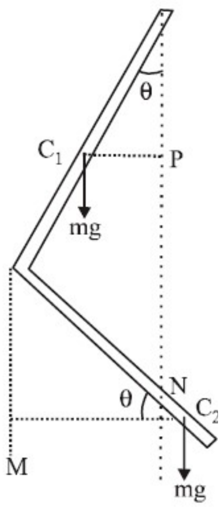
71.An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: **[9 JAN 2019 MAINS]**



(1) $\tan\theta = \frac{1}{2\sqrt{3}}$ (2) $\tan\theta = \frac{1}{2}$

(3) $\tan\theta = \frac{2}{\sqrt{3}}$ (4) $\tan\theta = \frac{1}{3}$

solution:.(4) Given that, the rod is of uniform mass density and AB = BC



Let mass of one rod is m

Balancing torque about hinge point

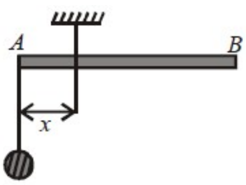
$$mg(C_1P) = mg(C_2N)$$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3} \text{ or, } \tan\theta = \frac{1}{3}$$

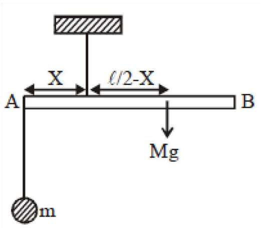
72.



A uniform rod *AB* is suspended from a point *X*, at a variable distance from *x* from *A*, as shown. To make the rod horizontal, a mass *m* is suspended from its end *A*. A set of (*m*, *x*) values is recorded. The appropriate variable that give a straight line, when plotted, are:

(1) $m, \frac{1}{x}$ (2) $m, \frac{1}{x^2}$ (3) m, x (4) m, x^2 [15 APR 2018 MAINS]

solution:.(1) Balancing torque w.r.t. point of suspension



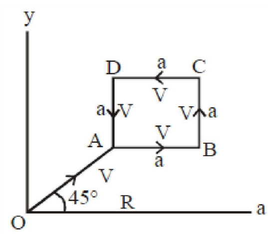
$$mgx = Mg\left(\frac{\ell}{2} - x\right)$$

$$\Rightarrow mx = M\frac{\ell}{2} - Mx$$

$$m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$$

$$y = \alpha\frac{1}{x} - C$$ Straight line equation

73.A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure : [11 JAN 2016 MAINS]



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

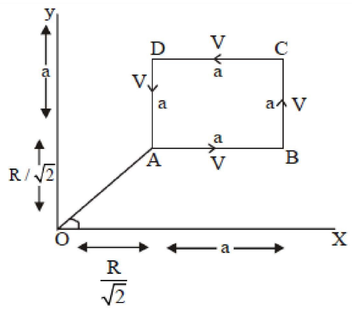
(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(2) $\vec{L} = \frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A.

(3) $\vec{L} = -\frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B.

(4) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

solution:.(1) We know that $|L| = mvr_{\perp}$



In none of the cases, the perpendicular distance $r_{\perp}$ is $\left(\frac{R}{\sqrt{2}} + a\right)$

74.A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is : [11 APR 2015 MAINS]

(1) 14.4 kg m^{2}s^{-1} (2) 8.64 kg m^{2}s^{-1}

(3) 20.16 kg m^{2}s^{-1} (4) 11.52 kg m^{2}s^{-1}

solution:.(1) Angular momentum, $L_0 = mvr\sin 90^0$

$= 2 \times 0.6 \times 12 \times 1 \times 1$

[As $V = r\omega, \sin 90^0 = 1$]

So, $L_0 = 14.4\, kgm^2/s$

75.A bob of mass m attached to an inextensible string of length *l* is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: [11 JAN 2014 MAINS]

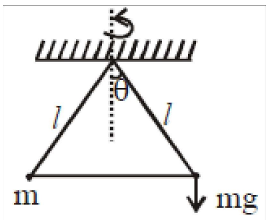
(1) angular momentum is conserved

(2) angular momentum changes in magnitude but not in direction

(3) angular momentum changes in direction but not in magnitude

(4) angular momentum changes both in direction and magnitude.

solution:.(c) Torque working on the bob of mass m is, $\tau = mg \times \ell \sin\theta$. (Direction parallel to plane of rotation of particle)



As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same

76.A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly (g = 10 ms^{-2})

(1) 1.73 kg m^2/s (2) 3.0 kg m^2/s **[19 APR 2014 MAINS]**

(3) 3.46 kg m^2/s (4) 6.0 kg m^2/s

solution:..(3) Given : $m = 0.160\, kg$

$\theta = 60^0$

$v = 10\ m/s$

Angular momentum $L = \vec{r} \times m\vec{v}$

$= H\, mv \cos\theta$

$$= \frac{v^2 \sin^2\theta}{2g}\cos\theta \qquad \left[H = \frac{v^2\sin^2\theta}{2g}\right]$$

$$= \frac{10^2 \times \sin^2 60^0 \times \cos 60^0}{2\times 10}$$

$$= 3.46\, kg\, m^2 / s$$

77.A particle of mass 2 kg is moving such that at time *t*, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at *t* = 2*s* about the origin in kg m^{-2} s^{-1} is : **[23 APR 2013 MAINS]**

(1) $-80\hat{k}$ (2) $\left(10\hat{i} - 16\hat{j}\right)$

(3) $-40\hat{k}$ (4) $40\hat{k}$

solution:.(1) Angular momentum $L = m(v \times r)$

$$= 2kg\left(\frac{dr}{dt} \times r\right) = 2kg\left(4tJ \times 5i - 2t^2\hat{j}\right) = 2kg\left(-20t\hat{k}\right) = 2kg \times -20 \times 2m^{-2}s^{-1}\hat{k} \qquad = -80\hat{k}$$

78.A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc. **[9APR 2013 MAINS]**

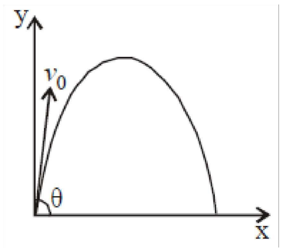
(1) continuously decreases (2) continuously increases

(3) first increases and then decreases (4) remains unchanged

solution:.(3) Angular momentum, $L = I\omega \Rightarrow L = mr^2\omega$

As insect moves along a diameter, the effective mass and hence moment of inertia (I) first decreases then increases so from principle of conservation of angular momentum, angular speed ω first increases then decreases

79.A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity n0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[23 APR 2010MAINS]**



(1) $-mgv_0t^2\cos\theta\hat{j}$ (2) $mgv_0t\cos\theta\hat{k}$ (3) $-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$ (4) $\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$

Where $\hat{i},\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively

solution:.(3) $\vec{L}=m(\vec{r}\times\vec{v})$

$$\vec{L}=m\left[v_0\cos\theta t\hat{i}+\left(v_0\sin\theta t-\frac{1}{2}gt^2\right)\hat{j}\right]$$

$$\times\left[v_0\cos\theta\hat{i}+(v_0\sin\theta-gt)\hat{j}\right]$$

$$=mv_0\cos\theta t\left[-\frac{1}{2}gt\right]\hat{k} \qquad =-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

80.Angular montum of of particle rotating with a central force is constant due to

(1) constant torque **[2007 MAINS]**

(2) constant force

(3) constant linear momentum

(4) zero torque

solution:.(4) We know Torque $\vec{\tau}_c=\frac{d\vec{L}_c}{dt}$

Where $\vec{L}_c=$ Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therfore torque about center of mass is zero

$\therefore\tau=\frac{dL}{dt}=0 \quad \Rightarrow \vec{L}_c=$ constt

81.A thin circular ring of mass *m* and radius *R* is rotating about its axis with a constant angular velocity ω. Two objects each of mass *M* are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

(1) $\frac{\omega(m+2M)}{m}$ (2) $\frac{\omega(m-2M)}{(m+2M)}$ (3) $\frac{\omega m}{(m+M)}$ (4) $\frac{\omega m}{(m+2M)}$ **[2006 MAINS]**

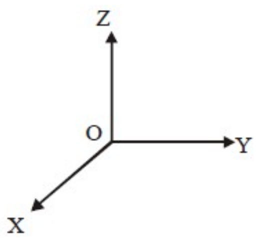
solution:.(4) Applying conservation of angular momentum $I'\omega' = I\omega$

$$\left(mR^2 + 2MR^2\right)\omega' = mR^2\omega$$

$$\Rightarrow (m+2m)R^2\omega' = mR^2\omega$$

$$\Rightarrow \omega' = \omega\left[\frac{m}{m+2M}\right]$$

82.A force of $-F\hat{k}$ acts on *O*, the origin of the coordinate system. The torque about the point (1, –1) is **[2006 MAINS]**



(1) $F(\hat{i}-\hat{j})$ (2) $-F(\hat{i}+\hat{j})$

(3) $F(\hat{i}+\hat{j})$ (4) $-F(\hat{i}-\hat{j})$

solution:.(3) Torque $\vec{\tau} = \vec{r}\times\vec{F} = (\hat{i}-\hat{j})\times(-F\hat{k})$

$$= F\left[-\hat{i}\times\hat{k}+\hat{j}+\hat{k}\right] = F(\hat{j}+\hat{i}) = F(\hat{i}+\hat{j})$$

[Since $\hat{k}\times\hat{i} = \hat{j}$ and $\hat{j}\times\hat{k} = \hat{i}$]

83. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ?[2004 MAINS]

(1) Angular velocity (2) Angular momentum

(3) Moment of inertia (4) Rotational kinetic energy

solution:.(2) Angular momentum will remain the same since no external torque act in free space.

84.Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then [2003 MAINS]

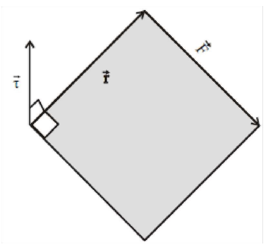
(1) $\vec{r}.\vec{T}=0$ and $\vec{F}.\vec{T}\neq 0$

(2) $\vec{r}.\vec{T}\neq 0$ and $\vec{F}.\vec{T}=0$

(3) $\vec{r}.\vec{T}\neq 0$ and $\vec{F}.\vec{T}\neq 0$

(4) $\vec{r}.\vec{T}=0$ and $\vec{F}.\vec{T}=0$

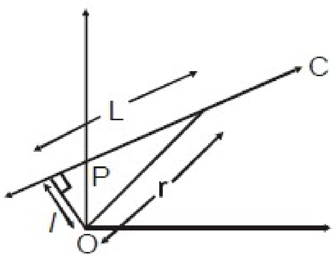
solution:.(4) We know that $\hat{\tau}=\vec{r}\times\vec{F}$



Vector $\hat{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$. We also know that the dot product of two vectors which have an angle of 90^0 between them is zero.

$\therefore \vec{r}.\vec{T}=0$ and $\vec{F}.\vec{T}=0$

85.A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P? [2002 MAINS]



(1) *mvL* (2) *mvl* (3) *mvr* (4) zero

solution:.(4) Angular momentum (L)

= (linear momentum) x (perpendicular distance of the line of motion passes through

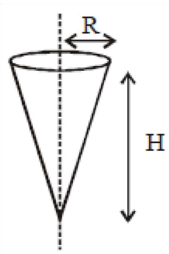
$\therefore L = mv \times r$

$= mv \times 0$

$= 0$

Topic-4: Moment of Inertia and Rotational K.E.

86.hown in the figure is a hollow icecream cone (it is open at the top). If its mass is *M*, radius of its top, *R* and height, *H*, then its moment of inertia about its axis is :



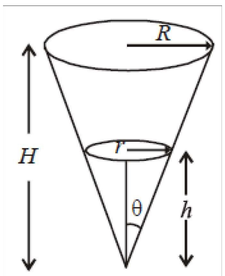
[SEP 6 2020 MAINS]

(1) $\frac{MR^2}{2}$ (2) $\frac{M\left(R^2+H^2\right)}{4}$

(3) $\frac{MH^2}{3}$ (4) $\frac{MR^2}{3}$

solution:.(4) Hollow ice-cream cone can be assume as several parts of discs having different radius, so

$I = \int dI = \int dm\left(r^2\right)$ .....(i)



From diagram,

$\frac{r}{h}=\tan\theta=\frac{R}{H}$ or $r=\frac{R}{H}h$

Mass of element, $dm=\rho\left(\pi r^2\right)dh$ ....(iii)

From eq. (i), (ii) and (iii),

From eq. (i), (ii) and (iii),

Area of element, $dA=2\pi rdl=2\pi r\frac{dh}{\cos\theta}$

Mass of element, $dm=\frac{2Mh\tan dh}{R\sqrt{R^2+H^2}\cos\theta}$

(here, $r-h\tan\theta$)

$$I=\int dI=\int_0^H dm\left(r^2\right)=\int_0^H \rho\left(\pi r^2\right)dh\left(\frac{R}{H}.h\right)^2$$

$$=\int_0^H \rho\left(\pi\left(\frac{R}{H}.h\right)^4\right)dh$$

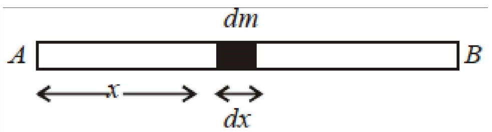
Solving we get, $I=\frac{MR^2}{2}$

87.The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x)=\lambda_0\left(1+\frac{x}{L}\right)$, where *x* is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is :

(1) $\frac{5}{12}ML^2$ (2) $\frac{7}{18}ML^2$ **[SEP 6 2020 MAINS]**

(3) $\frac{2}{5}ML^2$ (4) $\frac{3}{7}ML^2$

solution:.(2)



Mass of the small element of the rod

$dm = \lambda.dx$

Moment of inertia of small element

$$dI = dm.x^2 = \lambda_0\left(1+\frac{x}{L}\right).x^2dx$$

Moment of inertia of the complete rod can be obtained by integration

$$I = \lambda_0\int_0^L\left(x^2+\frac{x^3}{L}\right)dx$$

$$= \lambda_0\left|\frac{x^3}{3}+\frac{x^4}{4L}\right|_0^L = \lambda_0\left[\frac{L^3}{3}+\frac{L^3}{4}\right]$$

$$\Rightarrow I = \frac{7\lambda_0L^3}{12} \qquad ......(i)$$

Mass of the thin rod,

$$M = \int_0^L\lambda dx = \int_0^L\lambda_0\left(1+\frac{x}{L}\right)dx = \frac{3\lambda_0L}{2}$$

$$\therefore \lambda_0 = \frac{2M}{3L}$$

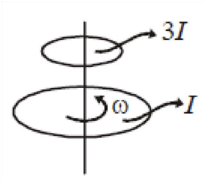
$$\therefore I = \frac{7}{12}\left(\frac{2M}{3L}\right)L^3 \Rightarrow I = \frac{7}{18}ML^2$$

88.A wheel is rotating freely with an angular speed w on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

(1) $\frac{5}{6}$ (2) $\frac{1}{4}$ (3) 0 (4) $\frac{3}{4}$ **[SEP 6 2020 MAINS]**

solution:.(4) By angular momentum conservataion, $L_c = L_f$

$\omega I + 3I + 0 = 4I\omega' \Rightarrow \omega' = \dfrac{\omega}{4}$



$(KE)_i = \dfrac{1}{2} I\omega^2$

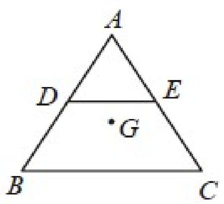
$(KE)_f = \dfrac{1}{2}(3I + I)\omega'^2$

$= \dfrac{1}{2} \times (4I) \times \left(\dfrac{\omega}{4}\right)^2 = \dfrac{I\omega^2}{8}$

$\Delta KE = \dfrac{1}{2} I\omega^2 = \dfrac{3}{8} I\omega^2$

$\therefore$ Fractional loss in K.E. $= \dfrac{\Delta KE}{KE_{l_i}} = \dfrac{\frac{3}{8} I\omega^2}{\frac{1}{2} I\omega^2} = \dfrac{3}{4}$

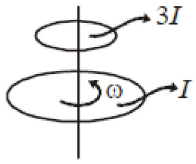
89.*ABC* is a plane lamina of the shape of an equilateral triangle. *D*, *E* are mid points of *AB, AC* and *G* is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through *G* and perpendicular to the plane *ABC* is I_0. If part *ADE* is removed, the moment of inertia of the remaining part about the same axis is $\dfrac{NI_0}{16}$ where *N* is an integer. Value of *N* is _______.



[SEP 4 2020 MAINS]

solution:.(4) By angular momentum conservataion, $L_c = L_f$

$$\omega I + 3I + 0 = 4I\omega' \Rightarrow \omega' = \frac{\omega}{4}$$



$$(KE)_i = \frac{1}{2}I\omega^2$$

$$(KE)_f = \frac{1}{2}(3I + I)\omega'^2$$

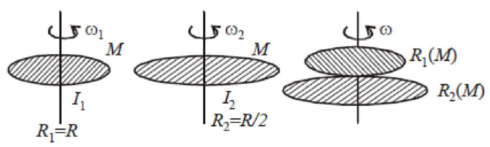
$$= \frac{1}{2}\times(4I)\times\left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$$

$$\Delta KE = \frac{1}{2}I\omega^2 = \frac{3}{8}I\omega^2$$

$\therefore$ Fractional loss in K.E. $= \dfrac{\Delta KE}{KE_{l_i}} = \dfrac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \dfrac{3}{4}$

90.A circular disc of mass *M* and radius *R* is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass *M* is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is *p*% of the initial energy. Value of *p* is ___________.

solution:



[SEP4 2020 MAINS]

Using angular momentum conservation

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\times\omega_f$$

$$\frac{MR^2}{2}\times\omega+0=\left(\frac{MR^2}{2}+\frac{MR^2}{8}\right)\omega_f\Rightarrow\omega_f=\frac{4}{5}\omega$$

Initial K.E., $K_i=\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2=\frac{MR^2\omega^2}{4}$

Final K.E., $K_f=\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2=\frac{MR^2\omega^2}{4}$

Percentage loss in kinetic energy % loss

$$=\frac{\frac{MR^2\omega^2}{4}-\frac{MR^2\omega^2}{5}}{\frac{MR^2\omega^2}{4}}\times100=20\%=P\%$$

Hence, value of P = 20

91.Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of a is :

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3) 2 (4) 4 **[SEP 4 2020 MAINS]**

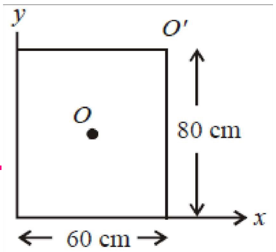
solution:.(3) Let p be the density of the discs and t is the thickness of discs

Moment of inertia of disc is given by

$$I=\frac{MR^2}{2}=\frac{\left[\rho\left(\pi R^2\right)t\right]R^2}{2}$$

$I\propto R^4$ (As ρ and t are same)

$$\frac{I_2}{I_1}=\left(\frac{R_2}{R_1}\right)^4\Rightarrow\frac{16}{1}=\alpha^4\alpha=2$$

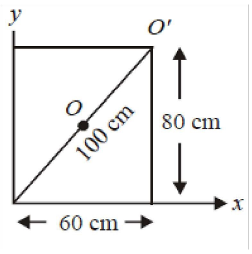
92. 

For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through *O* (the centre of mass) and *O'* (corner point) is : **[SEP4 2020 MAINS]**

(1) 2/3 (2) 1/4 (3) 1/8 (4) 1/2

solution:(b) Moment of inertia of rectangular sheet about an axis passing through O,

$$I_0 = \frac{M}{12}\left(a^2 + b^2\right) = \frac{M}{12}\left[(80)^2 + (60)^2\right]$$



From the parallel axis theorem, moment of inertia about O',

$$I_{O'} = I_O + M(50)^2$$

$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{2}\left(80^2 + 60^2\right)}{\frac{M}{12}\left(80^2 + 60^2\right) + M(50)^2} = \frac{1}{4}$$

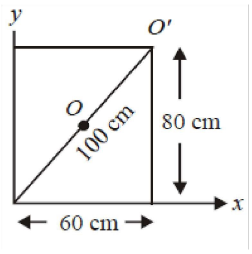
93.Moment of inertia of a cylinder of mass *M*, length *L* and radius *R* about an axis passing through its centre and perpendicular to the axis of the cylinder is

$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio *L/R* for it to have minimum possible *I* is : **[SEP 3 2020 MAINS]**

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\sqrt{\frac{2}{3}}$

solution:(b) Moment of inertia of rectangular sheet about an axis passing through O,

$$I_0 = \frac{M}{12}\left(a^2+b^2\right) = \frac{M}{12}\left[(80)^2+(60)^2\right]$$

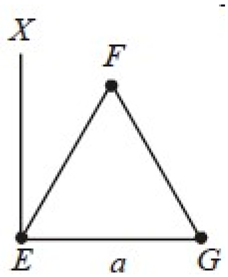


From the parallel axis theorem, moment of inertia about O',

$$I_{O'} = I_O + M(50)^2$$

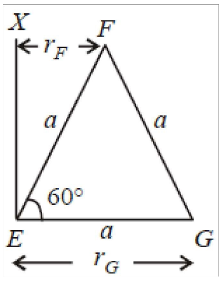
$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{2}\left(80^2+60^2\right)}{\frac{M}{12}\left(80^2+60^2\right)+M(50)^2} = \frac{1}{4}$$

.94. An massless equilateral triangle ***EFG*** **of side '*a*' (As shown in figure) has three particles of mass *m* situated at its vertices. The moment of inertia of the system about the line *EX* perpendicular to *EG* in the plane of *EFG* is** $\frac{N}{20}ma^2$ **where *N* is an integer. The value of *N* is __________. [SEP 3 2020 MAINS]**



solution(25)

Moment of inertia of the system about axis XE



$$I = I_E + I_F + I_G$$

$$\Rightarrow I = m \times 0^2 + m\left(\frac{a}{2}\right)^2 + ma^2 = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$$

$$\therefore N = 25$$

95.Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m2 and 10 rad s–1 respectively while those for the second one are 0.2 kg-m2 and 5 rad s–1 respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is :

(1) $\frac{10}{3}J$ (2) $\frac{20}{3}J$ (3) $\frac{5}{3}J$ (4) $\frac{2}{3}J$ **[SEP2 2020 MAINS]**

solution:.(2) Initial angular momentum $= I_1\omega_1 + I_2\omega_2$

Let ω be angular speed of the combined system

Final angular momentum $= I_1\omega + I_2\omega$

According to conservation of angular momentum $(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$

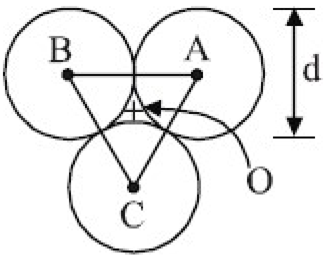
$$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_2 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{20}{3}$$

Final rotational kinetic energy

$$K_f = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(0.1+0.2)\times\left(\frac{20}{3}\right)^2$$

$$\Rightarrow K_f = \frac{20}{3}J$$

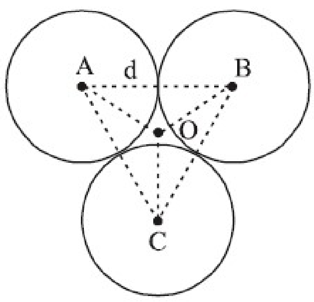
96.Three solid spheres each of mass *m* and diameter *d* are stuck together such that the lines connecting the centres form an equilateral triangle of side of length *d*. The ratio $\frac{I_0}{I_A}$ of moment of inertia I0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is:



[9 JAN 2020 MAINS]

(1) $\frac{13}{23}$ (2) $\frac{15}{13}$ (3) $\frac{23}{13}$ (4) $\frac{13}{15}$

solution:.[1)]Moment of inertia



$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$

And $AO = \frac{d}{\sqrt{3}}$

Moment of inertia about 'O'

$$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$$

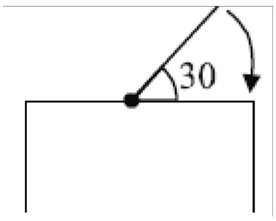
$$\Rightarrow I_0 = \frac{13}{10}Md^2$$

And $I_A = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + Md^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2$

$$\Rightarrow I_A = \frac{23}{10}Md^2$$

$$\therefore \frac{I_O}{I_A} = \frac{\frac{13}{10}Md^2}{\frac{23}{10}Md^2} = \frac{13}{23}$$

97.One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{ns}^{-1}$, where *n* is an integer. The value of *n* is _________ **[9 JAN 2020 MAINS]**



solution:.(15) Here, length of bar, 1 = 1m

Angle, $\theta = 30^0$

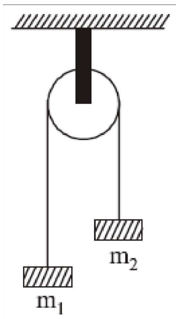
$\Delta PE = \Delta KE$ or $mgh = \frac{1}{2}I\omega^2$

$$\Rightarrow (mg)\frac{l}{2}\sin 30^0 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow mg\frac{l}{2}\times\frac{1}{2}=\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \omega=\sqrt{15}\,rad/s$$

98.A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is: [9 JAN 2020 MAINS]



(1) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$

(2) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$

(3) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+1}\right]^{1/2} gh$

(4) $\left[\frac{(m_1+m_2)}{(m_1+m_2)R^2+1}\right]^{1/2} gh$

solution:.(1) Using principal of conservation of energy

$$(m_1-m_2)gh=\frac{1}{2}(m_1+m_2)v^2+\frac{1}{2}I\omega^2$$

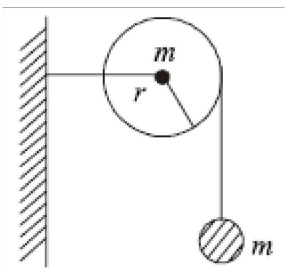
$$\Rightarrow (m_1 - m_2) gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2} I \omega^2$$

$$(\because v = \omega R)$$

$$\Rightarrow (m_1 - m_2) gh = \frac{\omega^2}{2}\left[(m_1 + m_2) R^2 + I\right]$$

$$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2) gh}{(m_1 + m_2) R^2 + I}}$$

99.As shown in the figure, a bob of mass *m* is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be:



[7JAN 2020 MAINS]

(1) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (2) $r\sqrt{\frac{3}{2gh}}$

(3) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (4) $r\sqrt{\frac{3}{4gh}}$

solution:.(1) When the bob covered a distance 'h'

Using $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2} m (\omega r)^2 + \frac{1}{2} \times \frac{m r^2}{2} \times \omega^2 \, (\because v = \omega r \, no \, slipping)$$

$$\Rightarrow mg = \frac{3}{4} m \omega^2 r^2$$

$$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

100.The radius of gyration of a uniform rod of length *l*, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is:

(1) $\frac{1}{4}l$ (2) $\frac{1}{8}l$ (3) $\sqrt{\frac{7}{48}}l$ (4) $\sqrt{\frac{3}{8}}l$ **[7 JAN 2020 MAINS]**

solution:.(3) Moment inertia of the rod passing through a point $\frac{\ell}{4}$ away from the centre of the rod

$$I = Ig + m\ell^2$$

$$\Rightarrow I = \frac{MI^2}{12} + M \times \left(\frac{I^2}{16}\right) = \frac{7MI^2}{48}$$

Using $I = MK^2 = \frac{7MI^2}{48}$ (K = radius of gyration)

$$\Rightarrow K = \sqrt{\frac{7}{48}}I$$

101.Mass per unit area of a circular disc of radius a depends on the distance *r* from its centre as σ (*r*) = *A* + *Br*. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:[7 JAN 2020 MAINS]

(1) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (2) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$

(3) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (4) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

solution:.(1) Given,

Mass per unit area of circular disc, $\sigma = A + Br$

Area of the ring $= 2\pi r dr$

Mass of the ring, $dm = \sigma 2\pi r dr$

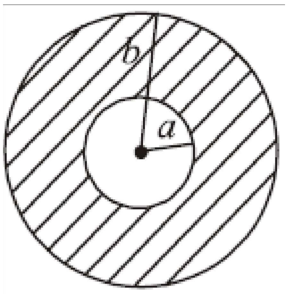
Moment of inertia,

$$I = \int dmr^2 = \int \sigma 2\pi r dr.r^2$$

$$\Rightarrow I = 2\pi \int_0^a (A + Br) r^3 dr = 2\pi \left[\frac{Aa^4}{4} + \frac{Ba^5}{5} \right]$$

$$\Rightarrow I = 2\pi a^4 \left[\frac{A}{4} + \frac{Ba}{5} \right]$$

102.A circular disc of radius *b* has a hole of radius *a* at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is : **:[7 JAN 2020 MAINS]**



(1) $\sqrt{\frac{a^2 + b^2 + ab}{2}}$ (2) $\frac{a+b}{2}$

(3) $\sqrt{\frac{a^2 + b^2 + ab}{3}}$ (4) $\frac{a+b}{3}$

solution:.(3) $I = \int_a^b (dm) r^2$

$$= \int_a^b \left(\frac{\sigma_0}{r} \times 2\pi r\, dr \right) r^2 = \frac{2\pi\sigma_0}{3} \left| r^3 \right|_a^b$$

$$= \frac{2\pi\sigma_0}{3} \left(b^3 - a^3 \right)$$

Mass of the disc,

$$m=\int_a^b \frac{\sigma_0}{r}\times 2\pi r\,dr = 2\pi\sigma_0\left(b-a\right)$$

Radius of gyration,

$$k=\sqrt{\frac{I}{m}}$$

$$=\sqrt{\frac{\left(2\pi\sigma_0/3\right)\left(b^3-a^3\right)}{2\pi\sigma_0\left(b-a\right)}}=\sqrt{\frac{a^2+b^2+ab}{3}}$$

103.Two coaxial discs, having moments of inertia I1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then ($E_f - E_i$) is ::[10 APR 2019MAINS]

(1) $-\frac{I_1\omega_1^2}{12}$ (2) $\frac{I_1\omega_1^2}{6}$ (3) $\frac{3}{8}I_1\omega_1^2$ (4) $-\frac{I_1\omega_1^2}{24}$

solution:.(d) As no external torque is acting so angular momentum should be conserved

$\left(I_1+I_2\right)\omega = I_1\omega_1 + I_2\omega_2$ [ω_c = common angular velocity of the system, when discs are in contact]

$$\omega_c=\frac{I_1\omega_1+\frac{I_1\omega_1}{4}}{I_1+\frac{I_1}{2}}\left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\omega_c=\frac{5\omega_1}{6}$$

$$E_f-E_i=\frac{1}{2}\left(I_1+I_2\right)\omega_c^2-\frac{1}{2}I_1\omega_1^2-\frac{1}{2}I_1\omega_2^2$$

Put $I_1/2$ and $\omega_c=\frac{5\omega_1}{6}5\omega_1/6$

We get:

$$E_f - E_i = -\frac{I_1\omega_1^2}{24}$$

104.A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where *r* is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :: [10 APR 2019 MAINS]

(1) $\frac{MR^2}{3}$ (2) $\frac{2MR^2}{3}$ (3) $\frac{MR^2}{6}$ (4) $\frac{MR^2}{2}$

solution:.(2) As from the question density $(\sigma) = kr^2$

Mass of disc $M = \int_0^R (kr^2)2\pi r dr = 2\pi k \frac{R^4}{4} = \frac{\pi k R^4}{2}$

$\Rightarrow k = \frac{2M}{\pi R^4}$ ....(i)

$\therefore$ Moment of inertia about the axis of the disc

$1 = \int d1 = \int (dm) r^2 \int \delta dA r^2$

$= \int (kr^2)(2\pi r dr) r^2$

$$= \int_0^R 2\pi k\, r^5\, dr = \frac{\pi k R^6}{3} = \frac{\pi \times \left(\frac{2M}{\pi R^4}\right) \times R^6}{3} = \frac{2}{3}MR^2$$

[putting value of k from equ .....(i)]

105.A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : [10 APR 2019 MAINS]

(1) 185 (2) 140 (3) 285 (4) 65

solution:.2)

$$I_1 = \left(\frac{7M}{8}\right)(2R)^2 \frac{1}{2} = \left(\frac{7}{16} \times 4\right) MR^2 = \frac{14}{8} mR^2$$

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right) r^2$$

$$\Rightarrow I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$$

$$\left[\begin{array}{l} \frac{4}{3}\pi r^3 \rho = \frac{1}{8}\frac{4}{3}\pi R^3 \times \rho \\ \Rightarrow r = R/2 \end{array}\right]$$

$$\frac{I_1}{I_2} = \frac{14 \times 80}{8} = 140$$

106.A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as kθ^2. If its moment of inertia is I then the angular acceleration of the disc is: **[9 APR 2019 MAINS]**

(1) $\frac{k}{4I}\theta$ (2) $\frac{k}{I}\theta$ (3) $\frac{k}{2I}\theta$ (4) $\frac{2k}{I}\theta$

solution:.92. (4) $\frac{1}{2} I\omega^2 = kQ^2$

Or $\omega = \left(\sqrt{\frac{2k}{I}}\right) Q$

Or $\alpha = \frac{d\omega}{\pi} = \sqrt{\frac{2K}{I}}\left(\frac{dQ}{dt}\right) = \left(\sqrt{\frac{2k}{I}}\right)\omega$

$$\left(\sqrt{\frac{2k}{I}}\right)\left(\sqrt{\frac{2k}{I}}\right)\theta = \frac{2k\theta}{I}$$

107.Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of:

(1) 2.5 g (2) 2s (3) 5s (4) 3s **[9APR 2019 MAINS]**

solution:.(2) $\omega = \alpha t = 20t$

Given, $\frac{1}{2} I\omega^2 = 1200$

Or $\frac{1}{2} \times 1.5 \times (20t)^2 = 1200$

Or $t = 2s$

108.A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be: [9 APR 2019 MAINS]

(1) $\frac{M\omega_0}{M+m}$ (2) $\frac{M\omega_0}{M+3m}$ (3) $\frac{M\omega_0}{M+6m}$ (4) $\frac{M\omega_0}{M+2m}$

solution:.(c) $I_i\omega_i = I_f\omega_f$

Or $\left(\frac{ML^2}{12}\right)\omega_0 = \left(\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right)\omega_f$

$\therefore \omega_f = \left(\frac{M\omega_0}{M+6m}\right)$

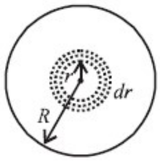
109.A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and *r* is the distance from its center. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is *I* = *a* MR². The value of the coefficient *a* is:

(1) $1/2$ (2) $3/5$ (3) $8/5$ (4) $3/2$ **[8APR 2019 MAINS]**

solution:.(3) Taking a circular ring of radius r and thickness dr as a mass element, so total mass,

$$M = \int_0^R \rho_0 r \times 2\pi r dr = \frac{2\pi\rho_0 R^3}{3}$$

$$I_C = \int_0^R \rho_0 r \times 2\pi r dr \times r^2 = \frac{2\pi\rho_0 R^5}{5}$$

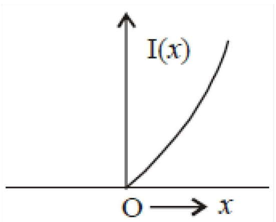


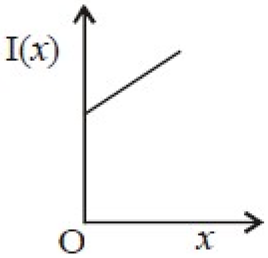
Using parallel axis theorem

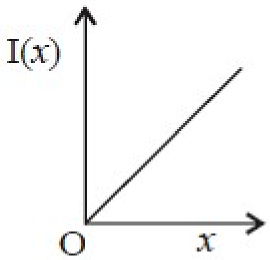
$$\therefore I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3}+\frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$$

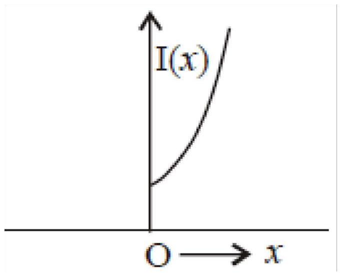
$$= \frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2 = \frac{8}{5}MR^2$$

110. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of *x* from it, is 'I(*x*)'. Which one of the graphs represents the variation of I(*x*) with *x* correctly? [12 JAN 2019 MAINS]

(1) 

(2) 

(3) 

(4) 

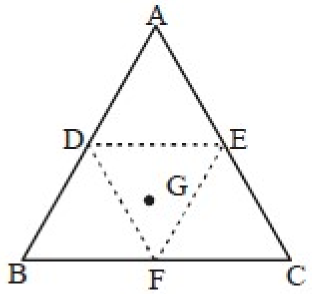
solution:.(4) According to parallel axes theorem

$$I = \frac{2}{5}mR^2 + mx^2$$

Hence graph (4) correctly depicts I vs x.

111.An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then :

[11 JAN 2019 MAINS]



(1) $I = \frac{15}{16} I_0$ (2) $I = \frac{3}{4} I_0$ (3) $I = \frac{9}{16} I_0$ (4) $I = \frac{I_0}{4}$

solution:.(1) Let mass of the larger triangle = M

Side of larger triangle = ℓ

Moment of inertia of larger triangle = ma^2

Mass of smaller triangle $= \frac{M}{4}$

Length of smaller traiangle $= \frac{\ell}{2}$

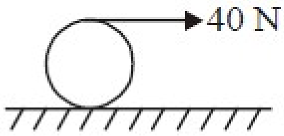
Mometn of removed triangle $= \frac{M}{4}\left(\frac{a}{2}\right)^2$

$$\therefore \frac{I_{removed}}{I_{original}} = \frac{\frac{M}{4}}{M} \cdot \frac{\left(\frac{a}{2}\right)^2}{(a)^2}$$

$$I_{removed} = \frac{I_0}{16}$$

So, $I = I_0 - \frac{I_0}{16} = \frac{15 I_0}{16}$

112.a string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)

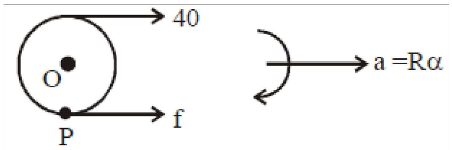


[11 JAN 2019 MAINS]

(1) 20 rad/s^2 (2) 16 rad/s^2

(3) 12 rad/s^2 (4) 10 rad/s^2

solution:.(2)



From newton's second law

$40 + f = m(R\alpha)$ .....(i)

Taking torque about 0 we get

$$40\times R - f\times R = I\alpha$$

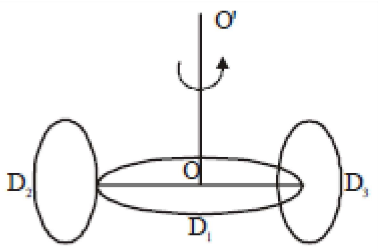
$$40\times R - f\times R = mR^2\alpha$$

$40 - f = mR\alpha$ .....(ii)

Solving equation (i) and (ii)

$$\alpha = \frac{40}{mR} = 16\ rad/s^2$$

113. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be : **[11 JAN 2019 MAINS]**



(1) MR^2 (2) $3MR^2$ (3) $\frac{4}{5}MR^2$ (4) $\frac{2}{3}MR^2$

solution:. (2) Moment of inertia of disc D_1 about $OO' = I_1 = \frac{MR^2}{2}$

$$= I_2 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$$

M.O.I of D_3 about OO'

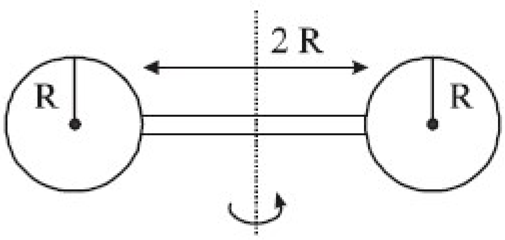
$$= I_3 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$$

So, resultant M.O.I about OO' is $I = I_1 + I_2 + I_3$

$$\Rightarrow I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$\Rightarrow \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3MR^2$$

114.Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:



(1) $\frac{137}{15}MR^2$ (2) $\frac{17}{15}MR^2$ (3) $\frac{209}{15}MR^2$ (4) $\frac{152}{15}MR^2$ **[10 JAN 2019 MAINS]**

solution:.(1) For Ball

Using parallel axes theorem, for ball moment of inertia,

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2$$

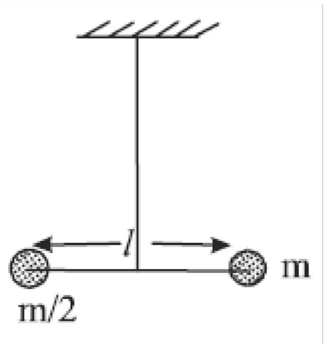
For two balls $I_{ball} = 2 \times \frac{22}{5}MR^2$

Irod $= \frac{M(2R)^2}{12} = \frac{MR^2}{3}$

$$I_{system} = I_{balls} + I_{rod}$$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3} = \frac{137}{15}MR^2$$

115.Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant *k* at the centre of mass of the rod-mass system (see figure). Because of torsional constant *k*, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:

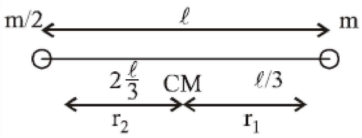


[9JAN 2019 MAINS]

(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{2k\theta_0^2}{l}$ (3) $\frac{k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{2l}$

solution:.(3) As we know, $\omega = \sqrt{\frac{k}{I}}$

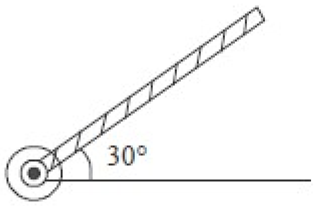
$$\omega = \sqrt{\frac{3k}{m\ell^2}} \left[\because I_{rod} = \frac{1}{3}m\ell^2\right]$$



Tension when it passes through the mean position,

$$= m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

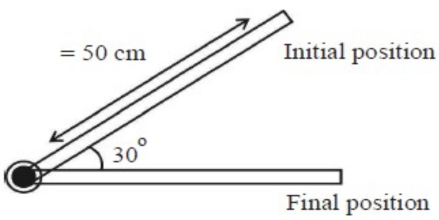
116.A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads^{-1}) will be (g = 10 ms^{-2})



[9 JAN 2019 MAINS]

(1) $\sqrt{\frac{30}{7}}$ (2) $\sqrt{30}$ (3) $\sqrt{\frac{20}{3}}$ (4) $\sqrt{\frac{30}{2}}$

solution:.(4)



By the low of conservation of energy,

P.E. of rod = Rotational K.E.

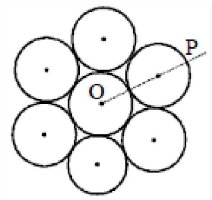
$$mg\frac{\ell}{2}\sin\theta = \frac{1}{2}I\omega^2$$

$$\Rightarrow mg\frac{\ell}{2}\sin 30^0 = \frac{1}{2}\frac{m\ell^2}{3}\omega^2 \Rightarrow mg\frac{\ell}{2}\times\frac{1}{2} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$

For complete length of rod,

$$\omega = \sqrt{3g/2(2\ell)} = \sqrt{\frac{30}{2}}\,rod\,s^{-1}$$

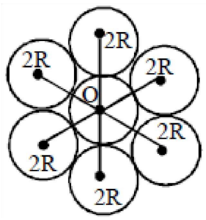
117.Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is: **[2018 MAINS]**



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$ (3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

solution:.4) Using parallel axes theorem, moment of inertia about 'O' $I_0 = I_{cm} + md^2$

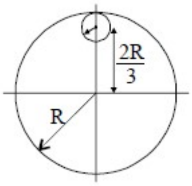
$$= \frac{7MR^2}{2} + 6\left(M\times(2R)^2\right) = \frac{55MR^2}{2}$$



Again, moment of inertia about point P, $I_p = I_0 + md^2$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

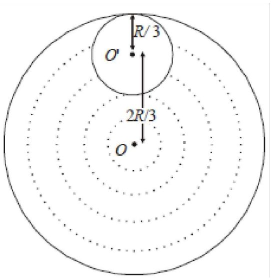
118.From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure.The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing,through centre of disc is :



[2018 MAINS]

(1) 4 MR^2 (2) $\frac{40}{9}MR^2$ (3) $10MR^2$ (4) $\frac{37}{9}MR^2$

solution:.1) Let σ be the mass per unit area.



The total mass of the disc $= \sigma \times \pi R^2 = 9M$

Let us consider the above system as a complete disc of mass 9M and a negative mass M super imposed on it.

Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc $= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$

$\therefore M.I.(I_2)$ of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$=\left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2+M\times\left(\frac{2R}{3}\right)^2\right]$$

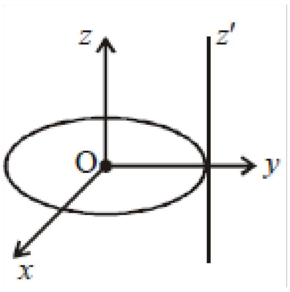
[Using perpendicular axis theorem]

$\therefore$ The total M.I. of the system about an axis passing through O and perpendicular to the plane of the disc is $I=I_1+I_2$

$$=\frac{1}{2}9MR^2-\left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2+M\times\left(\frac{2R}{3}\right)^2\right]$$

$$=\frac{9MR^2}{2}-\frac{9MR^2}{18}=\frac{(9-1)MR^2}{2}=4MR^2$$

119.A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z˘ axes will be



[2018 MAINS]

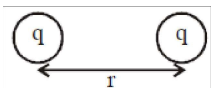
(1) 1 : 2 (2) 1 : 4 (3) 1 : 3 (4) 1 : 5

solution:.(3) As we know, moment of inertia of a disc about an axis passing through C.G. and perpendicular to its plane,

$$I_z=\frac{mR^2}{2}$$

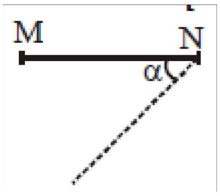
Moment of inertia of a disc about a tangential axis perpendicular to its own plane,

$$I_z^{'}=\frac{3}{2}mR^2$$



$$\therefore I_z / I_z' = \frac{mR^2}{2} / \frac{3mR^2}{2} = 1/3$$

120.A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle a with the horizontal, will be proportional to: (see figure)



[2018 MAINS]

(1) $\sqrt{\cos\alpha}$ (2) $\cos\alpha$ (3) $\sin\alpha$ (4) $\sqrt{\sin\alpha}$

solution:.(1) When the rod makes an angle α Displacement of ventre of mass $= \frac{l}{2}\cos\alpha$

$$mg\frac{l}{2}\cos\alpha = \frac{l}{2}I\omega^2$$

$mg\frac{l}{2}\cos\alpha = \frac{ml^2}{6}\omega^2$ ($\because$ M.I. of thin uniform rod about an axis passing through its centre of mass and perpendicular to the rod $I = \frac{ml^2}{12}$)

$$\Rightarrow \omega = \sqrt{\frac{3g\cos\alpha}{l}}$$

Speed of end $= \omega \times l = \sqrt{3g\cos\alpha l}$

i.e., speed of end,

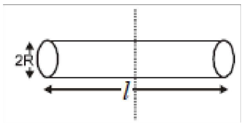
121.The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ?

(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$ **[2017 MAINS]**

solution:.(3) As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3}\right]$$

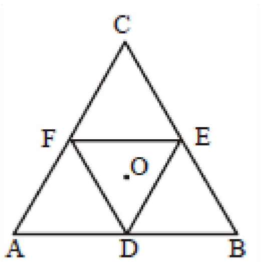


$$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3}\right] \Rightarrow \frac{dl}{dl} = \frac{m}{4}\left[\frac{-V}{\pi l^2} + \frac{2l}{3}\right] = 0$$

$$\frac{V}{\pi l^2} = \frac{2l}{3} \quad \Rightarrow V = \frac{2\pi l^3}{3}$$

$$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2} \text{ or, } \frac{l}{R} = \sqrt{\frac{3}{2}}$$

122.Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_o as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is :



[APR 8 2017 MAINS]

(1) $\frac{7}{8}I_o$ (2) $\frac{15}{16}I_o$ (3) $\frac{3I_o}{4}$ (4) $\frac{31I_o}{32}$

solution:.(2) According to theorem of perpendicular axes, moment of inertia of triangle (ABC)

$I_0 = km\, l^2$ .....(i)

BC=1

Moment of inertia of a cavity DEF

$$I_{DEF} = K\frac{m}{4}\left(\frac{l}{2}\right)^2$$

$$= \frac{k}{16}ml^2$$

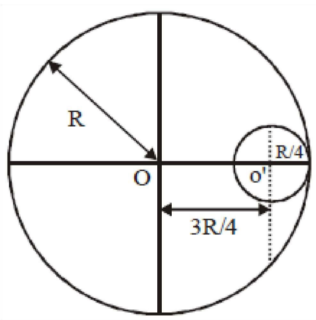
From equation (i),

$$I_{DEF} = \frac{I_0}{16}$$

Moment of inertia of remaining part

$$I_{remain} = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

123.A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is :



[APR92017 MAINS]

(1) $\frac{219MR^2}{256}$ (2) $\frac{237MR^2}{512}$ (3) $\frac{19MR^2}{512}$ (4) $\frac{197MR^2}{256}$

solution:.(2) Moment of Inertia of complete disc about 'O' point

$$I_{total} = \frac{MR^2}{2}$$

Radius of removed disc $= R/4$

$\therefore$ Mass of removed disc $= M/16$

$\left[\text{As } M \propto R^2\right]$

M.I. of removed disc about its own axis (O')

$$= \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{MR^2}{512}$$

M.I. of removed disc about O

$$I_{removed\ disc} = I_{cm} + mx^2$$

$$= \frac{MR^2}{512} + \frac{M}{16}\left(\frac{3R}{16}\right)^2 = \frac{19MR^2}{512}$$

M.I. of remaining disc

$$I_{remaining} = \frac{MR^2}{2} + \frac{19}{512}MR^2 = \frac{237}{512}MR^2$$

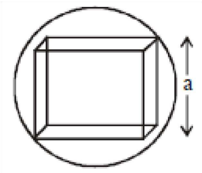
124.From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is : **[APR 8 2015 MAINS]**

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

solution:.(1) Here $a = \frac{2}{\sqrt{3}}R$

Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.$$



$$M' = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about the given axis,

$$I = \frac{M'a^2}{6}$$

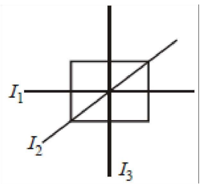
$$\frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}} R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

125.Consider a thin uniform square sheet made of a rigid material. If its side is '*a*' mass *m* and moment of inertia I about one of its diagonals, then:[APR 2015 MAINS]

(1) $I > \frac{ma^2}{12}$ (2) $\frac{ma^2}{12} < I < \frac{ma^2}{12}$ (3) $I = \frac{ma^2}{24}$ (4) $I = \frac{ma^2}{12}$

solution:.d) For a thin uniform square sheet

$$I_1 = I =_2 = I_3 \frac{ma^2}{12}$$



126. A ring of mass M and radius R is rotating about its axis with angular velocity w. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be:

[APR 8 2013MAINS]

(1) $\frac{m(M+2m)}{M}\omega^2 R^2$ (2) $\frac{Mm}{(M+m)}\omega^2 R^2$ (3) $\frac{Mm}{(M+2m)}\omega^2 R^2$ (4) $\frac{(M+m)M}{(M+2m)}\omega^2 R^2$

solution:.(3) Kinetic energy$_{(\text{rotational})}$ $K_R = \frac{1}{2}I\omega^2$

Kinetic energy$_{(\text{translational})}$ $K_T = \frac{1}{2}Mv^2 \quad (v = R\omega)$

$M.I._{(initial)} I_{ring} = MR^2; \omega_{initial} = \omega$

$M.I._{(new)} I'_{(system)} = MR^2 + 2MR^2 \quad \omega'_{(system)} = \frac{M\omega}{M+2m}$

Solving we get loss in K.E.

$= \frac{Mm}{(M+2m)}\omega^2 R^2$

127. This question has Statement 1and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: When moment of inertia *I* of a body rotating about an axis with angular speed ω increases, its angular momentum *L* is unchanged but the kinetic energy *K* increases if there is no torque applied on it. **[2012 MAINS]**

Statement 2: *L* = *I*w, kinetic energy of rotation $= \frac{1}{2}I\omega^2$

(1) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(2) Statement 1 is false, Statement 2 is true.

(3) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation of the Statement 1.

(4) Statement 1 is true, Statement 2 is false

solution:.(2) As $I\omega$ so L increases with invrease in ω.

depends on an angular velocity and moment of inertia of the body

SIMPLE HARMONIC MOTION (S.H.M.)

PERIODIC MOTION

(i) Any motion which repeats itself after regular interval of time is called periodic motion or harmonic motion.

(ii) The constant interval of time after which the motion is repeated is called time period.

Examples : (i) Motion of planets around the sun.

(ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

(i) The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.

(ii) The fixed point about which the body oscillates is called mean position or equilibrium position.

Examples : (i) Vibration of the wire of 'Sitar'.

(ii) Oscillation of the mass suspended from spring.

***Note :** Every oscillatory motion is periodic but every periodic motion is not oscillatory.*

SIMPLE HARMONIC MOTION (S.H.M.)

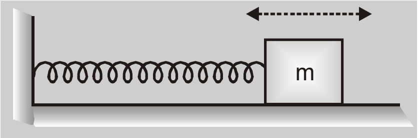
Simple harmonic motion is the simplest form of vibratory or oscillatory motion.

(i) S.H.M. are of two types

(a) Linear S.H.M.

When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.

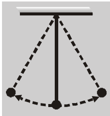
Example : Motion of a mass connected to spring.



(b) Angular S.H.M.

When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.

Example :- Motion of a bob of simple pendulum



(ii) Necessary Condition to execute S.H.M.

(a) *Motion of particle should be oscillatory.*

(b) *Total mechanical energy of particle should be conserved (Kinetic energy + Potential energy = constant)*

(c) *extreme position should be well defined.*

(d) ***In linear S.H.M.***

The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

$$F \propto -y \quad \text{or} \quad a \propto -y$$

Negative sign shows that direction of force and acceleration is towards equilibrium position and y is displacement of particle from equilibrium position.

(e) ***In angular S.H.M.***

The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

$$\tau \propto -\theta \quad \text{or} \quad a \propto -\theta$$

(iii) Comparision between linear and angular S.H.M.

Linear S.H.M.	Angular S.H.M.
$F \propto -x$	$\tau \propto -\theta$
$F = -kx$	$\tau = -C\theta$
Where k is the restoring force constant	*Where C is the restoring torque constant*
$a = -\frac{k}{m}x$	$\alpha = -\frac{C}{I}\theta$
$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$	$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$
It is known as differential equation of linear S.H.M.	*It is known as differential equation of angular S.H.M.*
$x = A \sin wt$	$\theta = \theta_0 \sin wt$

$\alpha = -w^2 x$

where w is the angular frequency

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi n$

where T is time period and n is frequency

$T = 2\pi\sqrt{\frac{m}{k}}$

$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

This concept is valid for all types of linear S.H.M.

$\alpha = -w^2 \theta$

$\omega^2 = \frac{C}{I}$

$\omega = \sqrt{\frac{C}{I}} = \frac{2\pi}{T} = 2\pi n$

$T = 2\pi\sqrt{\frac{I}{C}}$

$n = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$

This concept is valid for all types of angular S.H.M.

SOME BASIC TERMS

Mean Position

The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

Restoring Force

(a) The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.

(b) This force is always directed towards the mean position.

(c) Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.

(d) It is given by $F = -kx$ and has dimension MLT^{-2}.

Amplitude

The maximum displacement of particle from mean position is define as amplitude.

Time period (T)

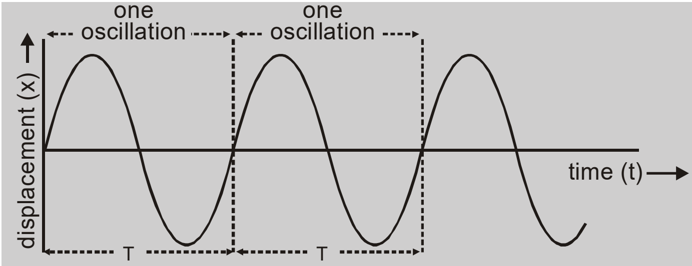
(a) The time after which the particle keeps on repeating its motion is known as time period.

(b) The smallest time taken to complete one oscillation or vibration is also define as time period.

(c) It is given by $T = \frac{2\pi}{\omega}$, $T = \frac{1}{n}$ where w is angular frequency and n is frequency.

Oscillation or Vibration

When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



Frequency (n or f)

(a) The number of oscillations per second is define as frequency.

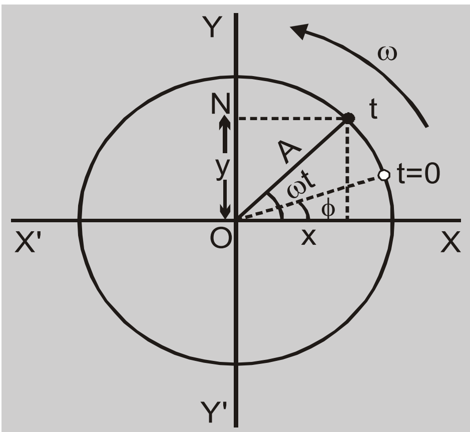
(b) It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$

(c) **SI UNIT :** hertz (Hz)

1 hertz = 1 cycle per second (cycle is a number not a dimensional quantity).

(d) **Dimension :** $M^0 L^0 T^{-1}$.

Phase



(a) Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

(b) Projection of particle's position on Y-axis.

$y = A\sin(wt + \phi)$ or $y = A\cos(wt + \phi')$

The quantity (wt + ϕ) represents the phase angle at that instant.

(c) The phase angle at time t = 0 is known as **initial phase or epoch.**

(d) The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.

(e) If the phase angles of two particles executing S.H.M. are (wt + ϕ_1) and (wt + ϕ_2) respectively, then the phase difference between two particles is given by

$$\Delta\phi = (wt + \phi_2) - (wt + \phi_1) \quad \text{or} \quad \Delta\phi = \phi_2 - \phi_1$$

(f) Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$ Same phase.

(g) Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ opposite phase.

Angular frequency (w)

(a) The rate of change of phase angle of a particle with respect to time is define as its angular frequency.

(b) **SI UNIT :** radian/second **Dimension :** $M^0 L^0 T^{-1}$

(c) $\omega = \sqrt{\frac{k}{m}}$

Instantaneous displacement

(a) The displacement of the particle from mean position in a particular direction at any instant of time is known as instantaneous displacement.

(b) At time t the instantaneous displacement

$$x = A \sin (wt + \phi),$$

where ϕ is initial phase and A is amplitude.

→ Every peroidic motion can be resolved into a number of simple harmonic motions.

→ Oscillatory motion can be treated as simple harmonic motion only in the limit of small amplitudes because in this limit the restoring force (or torque) becomes linear.

→ Harmonic oscillations is that oscillations which can be expressed in terms of single harmonic function.(i.e. sine functions or cosine function)

→ The motion of the molecules of a solid, the vibration of the air columns and the vibration of string of music instruements are either simple harmonic or superposition of simple harmonic motions.

DISPLACEMENT, VELOCITY AND ACCELERATION IN S.H.M.

DISPLACEMENT IN S.H.M.

(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

(ii) It can be given by relation $y = A\sin wt$ or $x = A\cos wt$.

The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

Note : *(i) The direction of displacement is always away from the mean position whether the particle is moving from or coming towards the mean position.*

(ii) In linear S.H.M. the length of S.H.M. path = 2A

(iii) In. S.H.M. total work done in one complete oscillation is zero but total covered length is 4A

VELOCITY IN S.H.M.

(i) It is define as the time rate of change of the displacement of the particle at the given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A \sin \omega t)$ $\Rightarrow$ *v = Awcoswt*

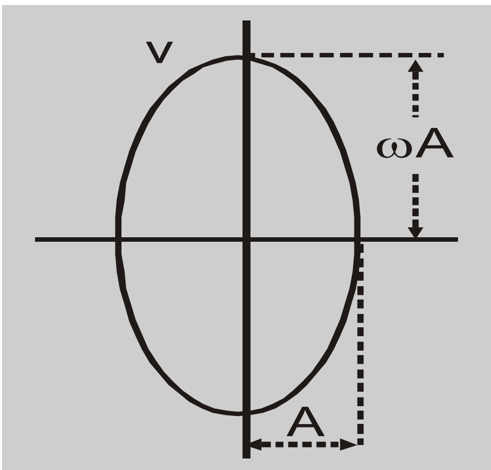
$v = \pm A\omega\sqrt{1 - \sin^2 \omega t}$ $\Rightarrow$ $v = \pm A\omega\sqrt{1 - \frac{x^2}{A^2}}$ $[\because\ x = A \sin \omega t]$

$v = \pm \omega\sqrt{(A^2 - x^2)}$

Squaring both the sides $v^2 = w^2 (A^2 - x^2)$ $\Rightarrow$ $\frac{v^2}{\omega^2} = A^2 - x^2$

$$\frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.



(iii) The graph between velocity and displacement is shown in figure.
If particle oscillates with unit angular frequency (w = 1) then curve between V and x will be circular.

Note: (i) *The direction of velocity of a particle in S.H.M. is either towards or away from the position*

(ii) *At mean position (x = 0), velocity is maximum (=Aw) and at extreme position (x = $\pm$ A), the velocity of particle executing S.H.M. is zero*

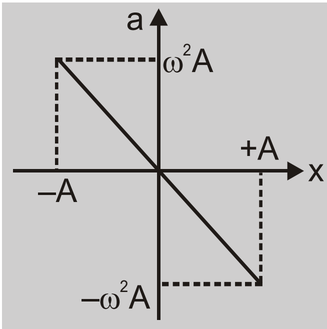
ACCELERATION IN S.H.M.

(i) *It is define as the time rate of change of the velocity of the particle at given instant.*

(ii) *Acceleration in S.H.M. is given by* $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t)$

$a = -w^2 A \sin wt \quad \Rightarrow \quad a = -w^2 x$

(iii) *The graph between acceleration and displacement as shown in figure*



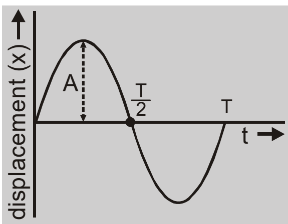
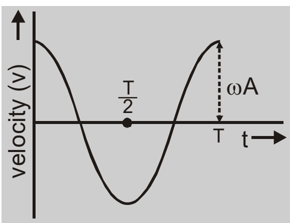
Note :

(i) *The acceleration of a particle executing S.H.M. is always directed towards the mean position.*

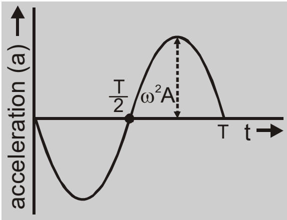
(ii) *The acceleration of the particle executing S.H.M. is maximum at extreme position (= w^2A) and minimum at mean position*

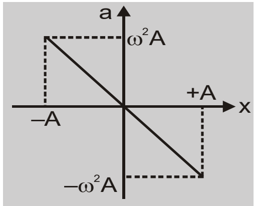
GRAPHICAL REPRESENTATION

Graphical study of displacement, velocity, acceleration and force in S.H.M.

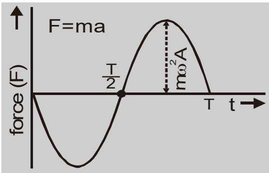
S. No.	Graph	In form of t	In from of x	Maximum value
1.	Displacement 	x = Asinwt	x = x	x = $\pm$ A
2.	Velocity 	v = Awcoswt	$v = \pm\omega\sqrt{A^2 - x^2}$	$v = \pm\omega A$

3. *Acceleration* $a = -w^2 A\sin wt$ $a = -w^2 x$ $a = \pm w^2 A$





4. *Force (F = ma)* $F = -mw^2 A\sin wt$ $F = -mw^2 x$ $F = \pm mw^2 A$



→ *In linear S.H.M., the length of S.H.M. path = 2A*

→ *In S.H.M., the total work done and displacement in one complete oscillation is zero but total traversed length is 4A.*

→ *In S.H.M., the velocity and acceleration varies simple harmonically with the same frequency as displacement.*

→ *Velocity is always ahead of displacement by phase angle* $\frac{\pi}{2}$ *radian*

→ *Acceleration is ahead of displacement by phase angle* π *radian i.e., opposite to displacement.*

→ *Acceleration leads the velocity by phase angle* $\frac{\pi}{2}$ *radian.*

→ *The velocity of a particle in S.H.M. at position* x_1 *and* x_2 *are* v_1 *and* v_2 *respectively. Determine value of time period and amplitude.*

$v = \omega\sqrt{A^2 - x^2}$ ⇒ $v^2 = w^2 (A^2 - x^2)$

At position x_1 *velocity* $v_1^2 = w^2 (A^2 - x_1^2)$ ... *(i)*

At position x_2 *velocity* $v_2^2 = w^2 (A^2 - x_2^2)$ ... *(ii)*

Subtracting (ii) from (i) $v_1^2 - v_2^2 = \omega^2 (x_2^2 - x_1^2)$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

Time period $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

Dividing (i) by (ii) $$\frac{v_1^2}{v_2^2} = \frac{A^2 - x_1^2}{A^2 - x_2^2}$$

$$v_1^2 A^2 - v_1^2 x_2^2 = v_2^2 A^2 - v_2^2 x_1^2$$

So $$A^2 (v_1^2 - v_2^2) = v_1^2 x_2^2 - v_2^2 x_1^2$$

$$A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$$

PROBLEMS

1. ***A particle executes SHM represented bythe equation, y=0.02sin $\left(3.14t + \frac{\pi}{2}\right)$ metre.***

Find (i) amplitude (ii) time period (iii) frequency (iv) epoch (v) maximum velocity and (vi) maximum acceleration.

SOLUTION : Compare the equation y=0.02sin $\left(3.14t + \frac{\pi}{2}\right)$ *with the general form of the equation,* $y = A\sin\left(\omega t + \phi_0\right)$

i) *Amplitude : A = 0.02m*

ii) *Time period (T) is given by*

$T = \frac{2\pi}{\omega}$ *or* $T = \frac{2\pi}{3.14} = 2s$

iii) *Frequency* $v = \frac{1}{T} = \frac{1}{2}Hz = 0.5Hz$

iv) *Epoch* $\varphi_0 = \frac{\pi}{2} = \frac{3.14}{2} = 1.57rad$

v) *Maximum velocity* $v_{max} = A\omega = 0.02 \times 3.14$ *= 0.68* ms^{-1}

vi) *Maximum acceleration*

$a_{max} = -A\omega^2 = -0.02 \times (3.14)^2$ *= 0.1972* ms^{-2}

2. The equation of a simple harmonic wave is given by :$y = 3 \sin \frac{\pi}{2}(50t - x)$,where x and y are in metres and t is in seconds. The ratio of maximum particle velocity to the wave velocity is :-

(1) 3π (2) $\frac{2}{3}\pi$ (3) 2π (4) $\frac{3}{2}\pi$

SOLUTION :..

$y = 3\sin \frac{\pi}{2}(50t - x)$

Particle velocity = $\frac{\partial y}{\partial t} = 3\left(\frac{\pi}{2} \times 50\right)\cos\frac{\pi}{2}(50t - x)$

Maximum particle velocity = 75π m/s

Wave velocity $v = \frac{\omega}{k} = \frac{50}{1} = 50$ m/s

Required ratio = $\frac{75\pi}{50} = \frac{3}{2}\pi$

3. A particle executing simple harmonic motionof amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of oscillation is :

(1) 1 Hz (2) 3 Hz (3) 2 Hz (4) 4 Hz

SOLUTION :

$V_{max} = Aw$

$V_{max} = A(2\pi n)$

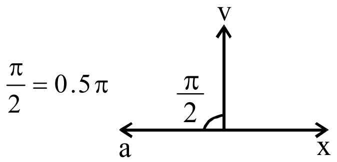
$31.4 = 5\ (2 \times 3.14)\ n$

$n = \frac{31.4}{31.4} = 1\text{Hz}$

4.. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is :-

(1) Zero (2) 0.5π (3) π (4) 0.707π

SOLUTION :



5.. *A particle executes simpe harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is :-*

(1) T/2 *(2) T/4* *(3) T/8* *(4) T/12*

SOLUTION :

. ***.DISPLACEMENT*** *x = A sin wt from mean position*

$$\frac{A}{2} = A\sin\frac{2\pi t}{T} \qquad x = \frac{A}{2}$$

$$\frac{\pi}{6} = \frac{2\pi t}{T} \qquad t = \frac{T}{12}\ sec.$$

6. *A point performs simple harmonic oscillation of period T and the equation of motion is given by x = a sin(wt + π/6). After the elapse of what fraction of the time period the velocity of the point will be equal to half of tis maximum velocity ?*

(1) T/3 *(2) T/12* *(3) T/8* *(4) T/6*

SOLUTION :.

$$x = a\ \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$\omega t = \frac{\pi}{6} \qquad \boxed{\therefore t = \frac{T}{12}}$$

7.. *The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be :-*

(1) $\sqrt{2}T$ *(2) T* *(3)* $\frac{T}{\sqrt{2}}$ *(4) 2T*

SOLUTION : Time period $T = 2\pi\sqrt{\frac{m}{K}}$

$$T' = 2\pi\sqrt{\frac{2m}{K}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{K}}\right) = \sqrt{2}T$$

8. The instantaneous displacement of a simple pendulum oscillator is given by $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$. Its speed will be maximum at time

(a) $\frac{\pi}{4\omega}$ (b) $\frac{\pi}{2\omega}$ (c) $\frac{\pi}{\omega}$ (d) $\frac{2\pi}{\omega}$

SOLUTION :

$x = A\cos\left(\omega t + \frac{\pi}{4}\right)$ and $v = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \frac{\pi}{4}\right)$

For maximum speed, $\sin\left(\omega t + \frac{\pi}{4}\right) = 1$

$\omega t + \frac{\pi}{4} = \frac{\pi}{2}$ or $\omega t = \frac{\pi}{2} - \frac{\pi}{4}$

$t = \frac{\pi}{4\omega}$

9. A particle in S.H.M. is described by the displacement function $x(t) = a\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is $\pi\, cm/s$. The angular frequency of the particle is $\pi\, rad/s$, then it's amplitude is

(a) 1 cm (b) $\sqrt{2}\, cm$ (c) 2 cm (d) 2.5 cm

SOLUTION : $x = a\cos(\omega t + \theta)$ ....(i)

and $v = \frac{dx}{dt} = -a\omega\sin(\omega t + \theta)$ ....(ii)

Given at $t = 0$, $x = 1\, cm$ and $v = \pi$ and $\omega = \pi$

Putting these values in equation (i) and (ii) we will get $\sin\theta = \frac{-1}{a}$ and $\cos\theta = \frac{1}{a}$

$$\sin^2\theta + \cos^2\theta = \left(-\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2$$

$$a = \sqrt{2}\, cm$$

10. *A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude*

(a) T / 2 (b) T / 4 (c) T / 8 (d) T / 12

SOLUTION : $y = A\sin\omega t = \frac{A\sin 2\pi}{T}t$

$$\frac{A}{2} = A\sin\frac{2\pi t}{T}$$

$$t = \frac{T}{12}.$$

11. *A particle executing S.H.M. of amplitude 4 cm and T = 4 sec. The time taken by it to move from positive extreme position to half the amplitude is*

(a) 1 sec (b) 1/3 sec (c) 2/3 sec (d) $\sqrt{3/2}$ sec

SOLUTION :

Equation of motion $y = a\cos\omega t$

$$\frac{a}{2} = a\cos\omega t \Rightarrow \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3}\times T}{2\pi} = \frac{4}{3\times 2} = \frac{2}{3}\sec$$

12. *Two simple harmonic motions are represented by the equations*

$y_1 = 0.1\sin\left(100\,\pi\, t + \frac{\pi}{3}\right)$ *and* $y_2 = 0.1\cos\pi t.$ *The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is*

(a) $\frac{-\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) $\frac{\pi}{3}$

SOLUTION :

$$v_1 = \frac{dy_1}{dt} = 0.1\times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$$

$$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$$

Phase difference of velocity of first particle with respect to the velocity of 2nd particle at t = 0 is

$$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}.$$

13. *A particle has simple harmonic motion. The equation of its motion is* $x = 5\sin\left(4t - \frac{\pi}{6}\right)$*, where x is its displacement. If the displacement of the particle is 3 units, then it velocity is*

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) 20 (d) 16

SOLUTION : From the given equation, a = 5 and w = 4

$$\therefore v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$$

14. The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are $2\,m/s$ and $4\,m/s^2$. Then angular velocity will be

(a) 3 rad/sec (b) 0.5 rad/sec
(c) 1 rad/sec (d) 2 rad/sec

SOLUTION :

$v_{\max} = a\omega$ and $A_{\max} = a\omega^2 \Rightarrow \omega = \frac{A_{\max}}{v_{\max}} = \frac{4}{2} = 2\,rad/sec$

15. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is

(a) $\sqrt{3}\ cm$ (b) $\sqrt{5}\ cm$ (c) $2(\sqrt{3})\,cm$ (d) $2(\sqrt{5})\,cm$

SOLUTION : $v_{\max} = a\omega \Rightarrow \omega = \frac{v_{\max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2)$

$$y^2 = a^2 - \frac{v^2}{\omega^2}$$

$$y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}\ \text{cm}$$

16. Two particles P and Q start from origin and execute Simple Harmonic Motion along X-axis with same amplitude but with periods 3 seconds and 6 seconds respectively. The ratio of the velocities of P and Q when they meet is

(a)1 : 2 (b)2 : 1(c)2 : 3 (d)3 : 2

SOLUTION :SOL..The particles will meet at the mean position when P completes one oscillation and Q completes half an oscillation

So $\frac{v_P}{v_Q} = \frac{a\omega_P}{a\omega_Q} = \frac{T_Q}{T_P} = \frac{6}{3} = \frac{2}{1}$

17. The displacement of a particle moving in S.H.M. at any instant is given by $y = a \sin \omega t$. The acceleration after time $t = \frac{T}{4}$ is (where T is the time period)

(a) $a\omega$ (b) $-a\omega$ (c) $a\omega^2$ (d) $-a\omega^2$

SOLUTION :. $-a\omega^2$ when it is at one extreme point.

18. The amplitude of a particle executing S.H.M. with frequency of 60 Hz is 0.01 m. The maximum value of the acceleration of the particle is

(a) $144\pi^2 m/sec^2$ (b) $144 m/sec^2$ (c) $\frac{144}{\pi^2} m/sec^2$ (d) $288\pi^2 m/sec^2$

SOLUTION : (a) Maximum acceleration $= a\omega^2 = a \times 4\pi^2 n^2$

19. A small body of mass 0.10 kg is executing S.H.M. of amplitude 1.0 m and period 0.20 sec. The maximum force acting on it is

(a) 98.596 N (b) 985.96 N (c) 100.2 N (d) 76.23 N

SOLUTION : Maximum acceleration

$$A_{\max} = a\omega^2 = \frac{a \times 4\pi^2}{T^2} = \frac{1 \times 4 \times (3.14)^2}{0.2 \times 0.2}$$

$$F_{\max} = m \times A_{\max} = \frac{0.1 \times 4 \times (3.14)^2}{0.2 \times 0.2} = 98.596 \ N$$

20.. A particle of mass 10 grams is executing simple harmonic motion with an amplitude of 0.5 m and periodic time of $(\pi/5)$ seconds. The maximum value of the force acting on the particle is

(a) 25 N (b) 5 N (c) 2.5 N (d) 0.5 N

SOLUTION :Maximum force $= m(a\omega^2) = ma\left(\frac{4\pi^2}{T^2}\right)$

$= 0.5\left(\frac{4\pi^2}{\pi^2/25}\right) \times 0.01 = 0.5\,N$

21. A body is executing simple harmonic motion with an angular frequency $2\,rad/s$. The velocity of the body at 20 mm displacement, when the amplitude of motion is 60 mm, is .

(a) 40 mm /s (b) $60\,mm/s$ (c) $113\,mm/s$ (d) $120\,mm/s$

SOLUTION :. $v = \omega\sqrt{(a^2 - y^2)} = 2\sqrt{60^2 - 20^2} = 113\,mm/s$.

22. A body of mass 5 gm is executing S.H.M. about a point with amplitude 10 cm. Its maximum velocity is 100 cm/sec. Its velocity will be 50 cm/sec at a distance **[CPMT 1976]**

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $10\sqrt{2}$

SOLUTION : It is given $v_{\max} = 100\,cm/sec$, a = 10 cm.

$v_{\max} = a\omega$

$\omega = \frac{100}{10} = 10\,rad/sec$

Hence $v = \omega\sqrt{a^2 - y^2}$ Ţ $50 = 10\sqrt{(10)^2 - y^2}$

$y = 5\sqrt{3}\,cm$

23. A simple harmonic oscillator has a period of 0.01 sec and an amplitude of 0.2 m. The magnitude of the velocity in $m\,\sec^{-1}$ at the centre of oscillation is

(a) 20π (b) 100 (c) 40p (d) 100π

SOLUTION : At centre $v_{\max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$

24. A simple harmonic oscillator has a period of 0.01 sec and an amplitude of 0.2 m. The magnitude of the velocity in $m\,\sec^{-1}$ at the centre of oscillation is

(a) 20π (b) 100 (c) 40p (d) 100π

SOLUTION : At centre $v_{\max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$

25. A body executes SHM, such that its velocity at the mean position is 1 ms^{-1} and acceleration at extreme position is 1.57 ms^{-2}. Calculate the amplitude and the time period of oscillation.

SOLUTION : $\frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \frac{1.57}{1}$

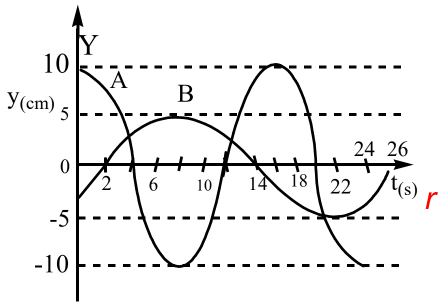
or $\omega = 1.57 \Rightarrow \frac{2\pi}{T} = 1.57$ rad.

$\therefore$ Time period $T = \frac{2\pi}{1.57} = \frac{2(3.14)}{1.57} = 4s.$

but $A\omega = 1$ i.e., A(1.57) = 1 or $A = \frac{1}{1.57}$

$\therefore$ Amplitude A = 0.637 m.

26. **Figure given below shows the displacement versus time graph for two particles A and B executing simple harmonic motions. Find the ratio of their maximum velocities,**



SOLUTION :For A, time peroid T_A = 16s [Distance between two adjacent crets]

for B, time period T_B = (26–2) = 24s

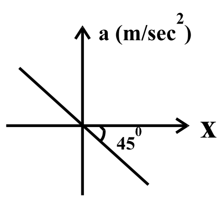
[length between the crest and trough shown =20s –8s = 12s]

Also, amplitudes a_A = 10cm; a_B = 5cm

Ratio of maximum velocities

$$\frac{V_A}{V_B} = \frac{a_A\omega_A}{a_B\omega_B} = \frac{a_A T_B}{a_B T_A} = \frac{10\times24}{5\times16} = \frac{3}{4}$$

27. **Acceleration displacement graph of a particle executing S.H.M is as shown in given figure. Find the time period of its oscillation (in sec)**



SOLUTION :

Acceleration= $-\omega^2 x$, i.e., ω^2 =tan 45^0 =1

or $\frac{2\pi}{T} = 1$ or $T = 2\pi s$

28. .Two particles execute SHM of same amplitude and frequency on parallel lines. They cross each another when moving in opposite directions each time their displacement is half their amplitude. What is the phase difference between them ?

SOLUTION :

If we assume that the particles are initially at the mean position, their equation for displacement.

$x = A\sin\omega t$ *But* $x = \frac{A}{2}$

$\therefore \frac{A}{2} = A\sin\omega t$ *(or)* $\sin\omega t = \frac{1}{2}$

Phase = $\omega t = 30^0, 150^0$

$\left(\because \sin\left(180^0 - \theta\right) = \sin\theta; \sin\left(180^0 - 30^0\right) = \sin 30^0\right)$

One of the particles has phase of 30^0 and the other has phase of 150^0

Phase difference between them = 120^0 = $\frac{2\pi}{3}$ *radian*

29. The displacement of SHO at which its velocity is half of maximum velocity is...

SOLUTION : $v = v_{\max}\sqrt{1-\frac{y^2}{A^2}} \Rightarrow \frac{v_{\max}}{2} = v_{\max}\sqrt{1-\frac{y^2}{A^2}}$

$$\frac{1}{4} = 1 - \frac{y^2}{A^2} \Rightarrow y = \pm\frac{\sqrt{3}}{2}A$$

30. What is average speed and average velocity of SHO in one oscillation

SOLUTION :

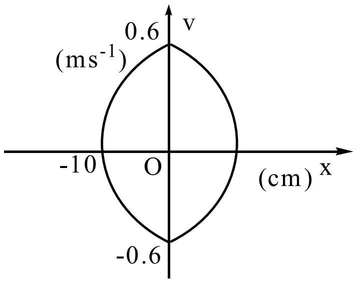
In one oscillation SHO travels a distance of 4A

$\therefore$ ***average speed in one oscillation =*** $\frac{4A}{T} = \frac{4A}{\frac{2\pi}{\omega}} = 2.\frac{A\omega}{\pi} = \frac{2}{\pi}v_{\max}$

Average velocity = 0 (as displacement is 0)

31. Figure shows the graph of velocity versus displacement of a particle executing simple harmonic motion. Find the period of oscillation of the particle

SOLUTION :



x_{max} = A=10 cm and v_{max} = $\omega A = 0.6 ms^{-1}$

$$\therefore \omega = \frac{v_{max}}{x_{max}} = \frac{6\times10^{-1}}{10\times10^{-2}} = 6 rad\, s^{-1}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3} sec$$

32. *A particle is performing simple harmonic motion along x-axis with amplitude 4 cm and time period 1.2 sec. The minimum time taken by the particle to move from x =2 cm to x = + 4 cm and back again is given by*

(a) 0.6 sec *(b) 0.4 sec* *(c) 0.3 sec* *(d) 0.2 sec*

SOLUTION :.

Time taken by particle to move from x=0 (mean position) to x = 4 (extreme position) $= \frac{T}{4} = \frac{1.2}{4} = 0.3\ s$

Let t be the time taken by the particle to move from x=0 to x=2 cm

$$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$$

$$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1\ s.$$

Hence time to move from x = 2 to x = 4 will be equal to 0.3 - 0.1 = 0.2 s

Hence total time to move from x = 2 to x = 4 and back again $= 2\times0.2 = 0.4\ sec$

33. *A large horizontal surface moves up and down in SHM with an amplitude of 1 cm. If a mass of 10 kg (which is placed on the surface) is to remain continually in contact with it, the maximum frequency of S.H.M. will be*

(a) 0.5 Hz *(b) 1.5 Hz* *(c) 5 Hz* *(d) 10 Hz*

SOLUTION : For body to remain in contact $a_{max} = g$

$$\therefore \omega^2 A = g \Rightarrow 4\pi^2 n^2 A = g$$

$$\Rightarrow n^2 = \frac{g}{4\pi^2 A} = \frac{10}{4(3.14)^2 0.01} = 25 \quad \Rightarrow n = 5\ Hz$$

34. Due to some force F_1 a body oscillates with period 4/5 sec and due to other force F_2 oscillates with period 3/5 sec. If both forces act simultaneously, the new period will be

(a) 0.72 sec (b) 0.64 sec (c) 0.48 sec (d) 0.36 sec

SOLUTION :

Under the influence of one force $F_1 = m\omega_1^2 y$ and under the action of another force, $F_2 = m\omega_2^2 y$.

Under the action of both the forces $F = F_1 + F_2$

$$\Rightarrow \quad m\omega^2 y = m\omega_1^2 y + m\omega^2 y$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$

$$\Rightarrow T = \sqrt{\frac{T_1^2 T_2^2}{T_1^2 + T_2^2}} \sqrt{\frac{\left(\frac{4}{5}\right)^2 \left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}} = 0.48S$$

35. An object performs S.H.M. of amplitude 5 cm and time period 4 s. If timing is started when the object is at the centre of the oscillation i.e., x = 0 then calculate.

(i) Frequency of oscillation

(ii) The displacement at 0.5 sec.

(iii) The maximum acceleration of the object.

(iv) The velocity at a displacement of 3 cm.

SOLUTION :

(i) Frequency $f = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

(ii) The displacement equation of object x = Asinwt

so at t = 0.5 s $\quad$ x = 5sin(2p × 0.25 × 0.5) = $5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}}$ cm

(iii) Maximum acceleration a_{max} = w^2A = $(0.5\ p)^2$ × 5 = 12.3 cm/s^2

(iv) Velocity at x = 3 cm is $v = \pm\omega\sqrt{A^2 - x^2}$ $= \pm 0.5\pi\sqrt{5^2 - 3^2} = \pm 6.28$ cm/s

36. A particle executes S.H.M. from extreme position and covers a distance equal to half of its amplitude in 1 s. Determine the time period of motion.

SOLUTION :

For particle starts S.H.M. from extreme position $y = A\cos wt$

$$\frac{A}{2} = A\cos(\omega \times 1)$$

$$\cos\omega = \cos\frac{\pi}{3}$$

$$\omega = \frac{\pi}{3}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi \times 3}{\pi} = 6\,s$$

37. A particle executing S.H.M. having amplitude 0.01 m and frequency 60 Hz. Determine maximum acceleration of particle.

SOLUTION : Maximum acceleration $a_{max.} = w^2A = 4p^2n^2A$

$= 4p^2(60)^2 \times (0.01)$

$= 144\ p^2\ m/s^2$

38. A particle is executing S.H.M. of frequency 300 Hz and with amplitude 0.1 cm. Its maximum velocity will be :

(1) 60 π cm/s (2) 0.6π cm/s (3) 0.50 π cm/s (4) 0.05π cm/s

SOLUTION :.V_{max} = AW= $2\pi nA = 2\pi(300)0.1 = 60\pi$ cm/s

39. Which of the following equation does not represent a simple harmonic motion :

(1) y = asinwt (2) y = bcoswt

(3) y = asinwt + bcoswt (4) y = atanwt

SOLUTION :

.y = atanwt is not a periodic function

40. A particle moves according to the equation $x = a\cos\left(\frac{\pi t}{2}\right)$. The distance covered by it in the time interval between $t = 0$ to $t = 3\ s$ is

1) 2a 2) 3a 3) 4a 4) a

SOLUTION :

Here $\omega = \frac{\pi}{2}, T = 4\sec$;

Amplitude = a

Between $t = 0$ to $t = 3\ s$ It Covers a distance 3a

ENERGY OF PARTICLE IN S.H.M.POTENTIAL ENERGY (U OR P.E.)

In terms of displacement

The potential energy is related to force by the relation

$$F = -\frac{dU}{dx} \quad \Rightarrow \quad \int dU = -\int F dx$$

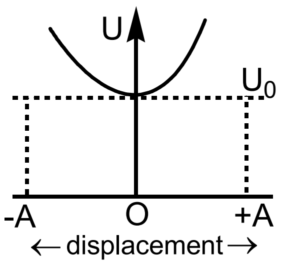
for S.H.M. $F = -kx$

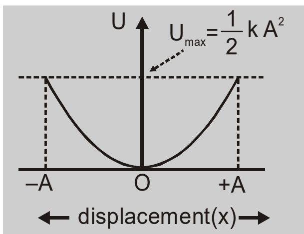
$$\int dU = -\int (-kx)dx = \int kx\,dx \quad \Rightarrow \quad U = \frac{1}{2}kx^2 + C$$

At $x = 0, U = U_0$ *r* $C = U_0$ *So* $U = \frac{1}{2}kx^2 + U_0$

the potenital energy at equilibrium position $= U_0$

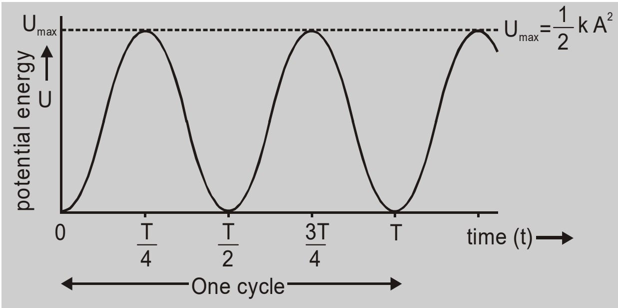
When $U_0 = 0$ *then* $U = \frac{1}{2}kx^2$





In terms of time

Since $x = A\sin(wt + \phi)$



P. E. $U = \frac{1}{2} kA^2 \sin^2(wt + \phi)$

If initial phase (ϕ) is zero $U = \frac{1}{2} kA^2 \sin^2 wt$

$$\Rightarrow \quad U = \frac{1}{2} mw^2 A^2 \sin^2 wt$$

Note :

(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position (x = 0) and maximum at extreme position ($x = \pm A$)

(ii) The potential energy is the periodic function of time.

It is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2} \ldots$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} \ldots$

KINETIC ENERGY (K. E.)

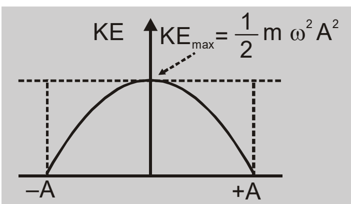
In terms of displacement

If mass of the particle executing S.H.M. is m and

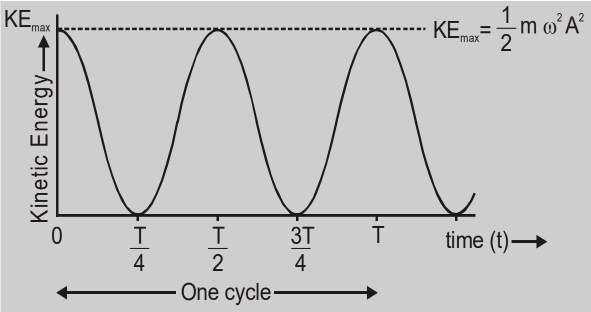
Its velocity is v then kinetic energy at any instant.

$$K.E. = \frac{1}{2} mv^2 = \frac{1}{2} mw^2 (A^2 - x^2)$$

$$K.E. = \frac{1}{2} k(A^2 - x^2)$$



In terms of time



$v = Aw\cos(wt + \phi)$

$K.\ E. = \frac{1}{2} mw^2A^2 \cos^2 (wt + \phi)$

If initial phase ϕ is zero

$K.\ E. = \frac{1}{2} mw^2A^2 \cos^2 wt$

Note :

(i) *In S.H.M. the kinetic energy is a inverted parabolic function of displacement.*

The kinetic energy is maximum ($\frac{1}{2} kA^2$) at mean position (x = 0) and minimum (zero) at extreme position ($x = \pm A$)

(ii) *The kinetic energy is the periodic function of time. It is maximum at $t = 0, \frac{T}{2}, T,$ ....and minimum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$...*

TOTAL ENERGY (E)

Total energy in S.H.M. is given by ; E = P. E. + K. E.

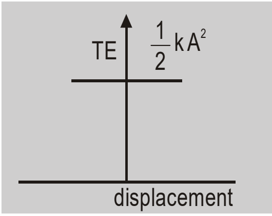
(i) w.r.t. position

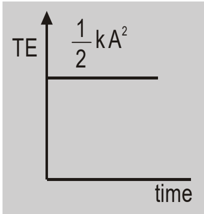
$E = \frac{1}{2} kx^2 + \frac{1}{2} k (A^2 - x^2) \Rightarrow E = \frac{1}{2} kA^2$

(ii) w.r.t. time

$E = \frac{1}{2} mw^2A^2 \sin^2 wt + \frac{1}{2} mw^2A^2 \cos^2 wt$

$E = \frac{1}{2} mw^2A^2 \Rightarrow E = \frac{1}{2} kA^2$





Note

(i) *Total energy of a particle in S.H.M. is same at all instant and at all displacement.*

(ii) *Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.*

AVERAGE ENERGY IN S.H.M.

(i) *The time average of P.E. and K.E. over one cycle is*

(a) $\langle K.E.\rangle_t = \frac{1}{4}kA^2$ *(b)* $\langle P.E.\rangle_t = \frac{1}{4}kA^2 + U_0$ *(c)* $\langle T.E.\rangle_t = \frac{1}{2}kA^2 + U_0$

$\rightarrow$ *Both K. E. and P. E. varies periodically but the variation is not simple harmonic.*

$\rightarrow$ *The frequency of oscillation of P. E. and K. E. is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.*

$\rightarrow$ *Frequency of total energy is zero*

PROBLEMS

1. A linear harmonic oscillator of force constant $2\times10^6\,Nm^{-1}$ ***and amplitude 0.01 m has a total mechanical energy of 160J. Then find maximum and minimum values of P.E and K.E?***

(1) 60 J ,60 J *(2) 0 J ,60 J* *(3) 60J , 0 J* *(4) 0 J , 0 J*

SOLUTION : $\frac{1}{2}KA^2 = \frac{1}{2}\times2\times10^6\times(0.01)^2 = 100J$

Since total energy is 160J. Maximum P.E. is 160 J.

From this it is understood that at the mean position potential energy of the simple harmonic oscillator is minimum which need not be zero.

$PE_{\min} = TE - KE_{\max} = 160 - 100 = 60J$

$KE_{\min} = 0$

2. A particle of mass 1 kg is executing SHM with an amplitude of 1m and time period π ***s. Calculate kinetic energy of the particle at the moment when the displacement is 0.8m from mean position***

(1) 0.72J *(2) 72 J* *(3) 7.2 J* *(4) 0 J*

SOLUTION :

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2s$$

We have, $v = \omega\sqrt{A^2 - x^2}$

$$v = 2\sqrt{(1)^2 - (0.8)^2} = 2\times0.6 = 1.2m/s$$

Kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2}\times1\times(1.2)^2 = 0.72J$

3. A particle of mass 10 g executes a linear SHM of amplitude 5 cm with a period of 2s. Find the P.E. and K.E. $\frac{1}{6}$ s after it has passed through the mean position.

(1) $9.25\times10^{-5}J$, $3.085\times10^{-6}J$
(2) $3.085\times10^{-6}J$, $9.25\times10^{-5}J$
(3) $9.25\times10^{-6}J$, $3.085\times10^{-5}J$
(4) $8.5\times10^{-5}J$, $9\times10^{-6}J$

SOLUTION :

Mass of particle $m=10g=10^{-2}kg$

Time period T =2s, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi rad/s$

Amplitude A=5cm = $5\times10^{-2}m$

K.E. = $\frac{1}{2}mA^2\omega^2\cos^2\omega t$ when $t = \frac{1}{6}s$

K.E.= $\frac{1}{2}\times1\times10^{-2}\times\left(5\times10^{-2}\right)^2\left(\pi^2\right)\cos^2\frac{\pi}{6}$

$=\frac{25\times10^{-6}}{2}\times\pi^2\times\left(\frac{\sqrt{3}}{2}\right)^2$ = $3.085\times10^{-6}J$

$$PE = \frac{1}{2}mA^2\omega^2\sin^2\omega t$$

$= \frac{1}{2}\times1\times10^{-2}\times\left(5\times10^{-2}\right)^2\pi^2\sin^2\frac{\pi}{6}$

$=\frac{25\times10^{-6}}{2}\times\pi^2\times\left(\frac{1}{2}\right)^2$ = $9.25\times10^{-5}J$

4. The potential energy of a harmonic oscillator of mass 2 kg at its mean position is 5J. If its total energy is 9J and its amplitude is 0.01m, find its time period

(1) $\frac{\pi}{100}$ s (2) $\frac{\pi}{10}$ s (3) π s (4) $\frac{100}{\pi}$ s

SOLUTION :

$$\frac{1}{2}kA^2 = (9-5) = 4J,$$

$$k = \frac{8}{(0.01)^2} = 8\times10^4 \text{ N/m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{8\times10^4}} = \frac{100}{\pi} \text{ s}$$

5. A particle is describing SHM with amplitude 'a'. When the potential energy of particle is one fourth of the maximum energy during oscillation, then its displacement from mean position will be:

(1) $\frac{a}{4}$ (2) $\frac{a}{3}$ (3) $\frac{a}{2}$ (4) $\frac{2a}{3}$

SOLUTION :

SOL.. $\text{P.E}(U) = E_0 \sin^2 wt = \frac{E_0}{4}$

$where E_0 = \frac{1}{2}KA^2$

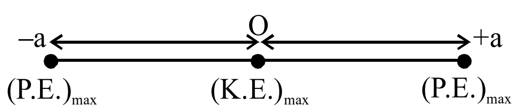
$\sin wt = \frac{1}{2}$

$x = a\sin wt = \frac{a}{2}$

6.. Displacement between max. P.E. position and max. K.E. position for a particle excuting simple harmonic motion is :

(1) $\pm\frac{a}{2}$ (2) + a (3) $\pm a$ (4) – 1

SOLUTION :



the displacement between max P.E and max K.E is $\pm\frac{a}{2}$

7. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\frac{25}{\pi}$ Hz. At position x = 0.04m, the object has kinetic energy 0.5J and (potential energy is zero at mean position) .Find its amplitude of vibration.

(1) 6 cm (2) 9 cm (3) 3 cm (4) 0.6 cm

SOLUTION :

$$U = \frac{1}{2}m\omega^2 x^2 = 0.4\,J$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad \therefore \quad k = (2\pi f)^2 m$$

Total energy of oscillation is (0.5+0.4) = 0.9 J

$$\therefore \quad 0.9 = \frac{1}{2}kA^2 \text{ or } A = \sqrt{\frac{1.8}{k}} = \sqrt{\frac{1.8}{(2\pi f)^2 m)}}$$

$$= \frac{1}{2\pi f}\sqrt{\frac{1.8}{0.2}} = \frac{1}{2\pi\left(\frac{25}{\pi}\right)}\sqrt{\frac{1.8}{0.2}} = \frac{3}{50} \text{ m} = 6 \text{ cm}$$

8. The displacement of two identical particles executing SHM are represented by equations $x_1 = 4\sin\left(10t + \frac{\pi}{6}\right)$ and $x_2 = 5\cos\omega t$ For what value of ω energy of both the particles is same

(1) 6 unit (2) 9 unit (3) 3 unit (4) 8 unit

SOLUTION :

$$E = \frac{1}{2}mA^2\omega^2$$

i.e., $E \propto (A\omega)^2$

$$(A_1\omega_1)^2 = (A_2\omega_2)^2$$

$$A_1\omega_1 = A_2\omega_2$$

$4 \times 10 = 5 \times \omega$

$\omega = 8$ *unit*

9. ***A particle of mass 'm' is executing oscillation about the origin on the x -axis. Its P.E.varies with position as $U(x) = K|x|^3$, here K is constant. The amplitude of oscillation is 'a', then how does its time period T vary with amplitude.***

(1) $\sqrt{a}$ *(2)* $\frac{1}{\sqrt{a}}$ *(3)* a^2 *(4)* a

SOLUTION :

$U(x) = K|x|^3$,

but $U = \frac{1}{2} m A^2 \omega^2 \sin^2 \omega t$

$m a^2 \omega^2 \propto K a^3$

$\omega^2 \propto a$

$\therefore T \propto \frac{1}{\sqrt{a}}$

Alternate method:

$K = \frac{U}{x^3} = M L^{-1} T^{-2}$

Now, time period may depend on

T α $(mass)^x (amplitude)^y (K)^z$

$\left[M^0 L^0 T^1\right] \alpha M^x L^y \left[M L^{-1} T^{-2}\right]^z$

on solving x= 1/2, y = z = -1/2

Hence T $\propto (amplitude)^{-1/2}$

$\therefore T \propto \frac{1}{\sqrt{a}}$

10. An object of mass 4 kg is moving along X-axis and its potential energy as a functionof x varies as $U(x) = 4(1 - \cos 2x)$ J then time period for small oscillation is :

(1) π seconds (2) $\frac{1}{\pi}$ seconds

(3) 2 seconds (4) π^2 seconds

SOLUTION : -

$$F = -\frac{dU}{dx} = -\frac{d}{dx}[4 - 4\cos 2x]$$

$F = +4(2)(-\sin 2x) = -8\sin 2x$

here 'x' is small

$\therefore \sin 2x = 2x$

So $F = -16x \Rightarrow m\omega^2 = 16 \Rightarrow 4\omega^2 = 16$

$\Rightarrow \omega = 2$

$T = 2\pi / \omega = \pi$ seconds

11. The displacement of SHO is , $y = 6\sin(\pi t + \pi/3)$ find 1) Instants at which PE is min. (or) KE max (or) velocity is max.2) Instants at which PE is max (or) KE is zero (or) velocity is zero

SOLUTION : *PE is min (or) KE is max (or) velocity is max*

when SHO is at mean position. i.e., y = 0

$y = 0 = 6\sin(\pi t + \pi/3)$

$\Rightarrow \pi\left(t + \frac{1}{3}\right) = n\pi$ here n = 1,2,3 ....

$t = n - \frac{1}{3}$

PE is max (or) KE is min (or) velocity is zero

12.. *In case of simple harmonic motion –*

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude.

(b) At what displacement the kinetic and potential energies are equal.

SOLUTION :

In S.H.M. $K.E. = \frac{1}{2}k(A^2 - x^2)$ $P.E. = \frac{1}{2}kx^2$ $T.E. = \frac{1}{2}kA^2$

(a) $f_{K.E.} = \frac{K.E.}{T.E.} = \frac{A^2 - x^2}{A^2}$ $f_{P.E.} = \frac{P.E.}{T.E} = \frac{x^2}{A^2}$

at $x = \frac{A}{2}$ $f_{K.E.} = \frac{A^2 - A^2/4}{A^2} = \frac{3}{4}$ *and* $f_{P.E.} = \frac{A^2/4}{A^2} = \frac{1}{4}$

(b) $K.E. = P.E.$ r $\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$ r $2x^2 = A^2$ r $x = \pm\frac{A}{\sqrt{2}}$

13. *The total energy of a particle executing S.H.M. is proportional to*

(a) Displacement from equilibrium position

(b) Frequency of oscillation

(c) Velocity in equilibrium position

(d) Square of amplitude of motion

SOLUTION :

$$E = \frac{1}{2}m\omega^2 a^2 \quad Ţ \quad E \propto a^2$$

14. *The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement X from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is*

(a) $X^2\omega^2/(a^2 - X^2\omega^2)$ (b) $X^2/(a^2 - X^2)$

(c) $(a^2 - X^2\omega^2)/X^2\omega^2$ (d) $(a^2 - X^2)/X^2$

SOLUTION : .

$E = \frac{1}{2}m\omega^2 a^2$ Ţ $E \propto a^2$ and potential energy, $V = \frac{1}{2}m\omega^2 x^2 \quad \therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

15.. *When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is*

(a) $a/4$ (b) $a/3$ (c) $a/2$ (d) $2a/3$

SOLUTION :

$$\frac{U}{U_{\max}} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2}$$

$$\frac{1}{4} = \frac{y^2}{a^2}$$

$$y = \frac{a}{2}$$

16. *A particle of mass 10 gm is describing S.H.M. along a straight line with period of 2 sec and amplitude of 10 cm. Its kinetic energy when it is at 5 cm from its equilibrium position is*

(a) $37.5\pi^2 ergs$ (b) $3.75\pi^2 ergs$ (c) $375\pi^2 ergs$ (d) $0.375\pi^2 ergs$

SOLUTION :

Kinetic energy $K = \frac{1}{2}m\omega^2(a^2 - y^2)$

$$= \frac{1}{2} \times 10 \times \left(\frac{2\pi}{2}\right)^2 [10^2 - 5^2] = 375\ \pi^2 ergs$$

17. *When the displacement is half the amplitude, the ratio of potential energy to the total energy is*

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{8}$

SOLUTION :

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a} = \frac{1}{4}$$

18. *For any S.H.M., amplitude is 6 cm. If instantaneous potential energy is half the total energy then distance of particle from its mean position is*

(a) 3 cm (b) 4.2 cm (c) 5.8 cm (d) 6 cm

SOLUTION : If at any instant displacement is y then it is given that $U = \frac{1}{2} \times E$ Ţ

$$\frac{1}{2} m\omega^2 y^2 = \frac{1}{2} \times \left(\frac{1}{2} m\omega^2 a^2\right)$$

$$y = \frac{a}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 4.2\, cm$$

19. A body of mass $1\,kg$ is executing simple harmonic motion. Its displacement $y(cm)$ at t seconds is given by $y = 6\sin(100t + \pi/4)$. Its maximum kinetic energy is

(a) 6 J (b) 18 J (c) 24 J (d) 36

SOLUTION :

So $a = 6cm,\ \omega = 100\, rad/\sec$

$$K_{\max} = \frac{1}{2} m\omega^2 a^2 = \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2 = 18\ J$$

20. A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy change into potential energy is

(a) f/2 (b) f (c) 2 f (d) 4 f

SOLUTION :

n S.H.M., frequency of K.E. and P.E. = 2 ´ (Frequency of oscillating particle)

21. There is a body having mass m and performing S.H.M. with amplitude a. There is a restoring force $F = -Kx$, where x is the displacement. The total energy of body depends upon

(a) K, x (b) K, a (c) K, a, x (d) K, a, v

SOLUTION : .

Total energy $U = \frac{1}{2} Ka^2$

22. The total energy of a particle executing S.H.M. is 80 J. What is the potential energy when the particle is at a distance of 3/4 of amplitude from the mean position

(a) 60 J (b) 10 J (c) 40 J (d) 45 J

SOLUTION :

$$\frac{U}{E}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}=\frac{y^2}{a^2}$$

$$\frac{U}{80}=\frac{\left(\frac{3}{4}a\right)^2}{a^2}=\frac{9}{16}$$

$$U=45\ J$$

23. When a mass M is attached to the spring of force constant k, then the spring stretches by l. If the mass oscillates with amplitude l, what will be maximum potential energy stored in the spring

(a) $\frac{kl}{2}$ (b) $2kl$ (c) $\frac{1}{2}Mgl$ (d) Mgl

SOLUTION :

$$Mg=Kl$$

$$U_{\max}=\frac{1}{2}Kl^2=\frac{1}{2}mgl$$

24. A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, is its energy half potential and half kinetic

(a) 1 cm (b) $\sqrt{2}$ cm (c) 3 cm (d) $2\sqrt{2}$ cm

SOLUTION :

Let x be the point where K.E. = P.E.

Hence $\frac{1}{2}m\omega^2(a^2-x^2)=\frac{1}{2}m\omega^2x^2$

$$2x^2=a^2$$

$$x=\frac{a}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}\ cm$$

25. For a particle executing simple harmonic motion, the kinetic energy K is given by $K=K_o\cos^2\omega t$. The maximum value of potential energy is

(a) K_0 (b) Zero (c) $\frac{K_0}{2}$ (d) Not obtainable

SOLUTION :

Since maximum value of $\cos^2 \omega t$ is 1.

$\therefore K_{\max} = K_o \cos^2 \omega t = K_o$

Also $K_{\max} = PE_{\max} = K_o$

26. The potential energy of a particle with displacement X is U(X). The motion is simple harmonic, when (K is a positive constant)

(a) $U = -\frac{KX^2}{2}$ (b) $U = KX^2$ (c) $U = K$ (d) $U = KX$

SOLUTION :

$F = -kx$

$dW = Fdx = -kxdx$

So $\int_0^W dW = \int_0^x -kx\, dx$

$W = U = -\frac{1}{2}kx^2$

27. The kinetic energy and potential energy of a particle executing simple harmonic motion will be equal, when displacement (amplitude = a) is

(a) $\frac{a}{2}$ (b) $a\sqrt{2}$ (c) $\frac{a}{\sqrt{2}}$ (d) $\frac{a\sqrt{2}}{3}$

SOLUTION :

Suppose at displacement y from mean position potential energy = kinetic energy

$\frac{1}{2}m(a^2 - y^2)\omega^2 = \frac{1}{2}m\omega^2 y^2$

$\therefore a^2 = 2y^2$

$y = \frac{a}{\sqrt{2}}$

28. The total energy of the body executing S.H.M. is E. Then the kinetic energy when the displacement is half of the amplitude, is

(a) $\frac{E}{2}$ (b) $\frac{E}{4}$ (c) $\frac{3E}{4}$ (d) $\frac{\sqrt{3}}{4}E$

SOLUTION :

Total energy in SHM $E = \frac{1}{2}m\omega^2 a^2$ *; (where a = amplitude)*

Potential energy $U = \frac{1}{2}m\omega^2(a^2 - y^2) = E - \frac{1}{2}m\omega^2 y^2$ *When* $y = \frac{a}{2}$

$$U = E - \frac{1}{2}m\omega^2\left(\frac{a^2}{4}\right) = E - \frac{E}{4} = \frac{3E}{4}$$

29. *The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of amplitude. The total energy of the particle be*

(a) 18 J *(b) 10 J* *(c) 12 J* *(d) 2.5 J*

SOLUTION : .

$$\frac{2.5}{E} = \frac{\left(\frac{a}{2}\right)^2}{a^2}$$

$$E = 10\,J$$

30. *A particle starts oscillating simple harmonically from its equilibrium position with time period T. Determine ratio of K.E. and P.E. of the particle at time t =* $\frac{T}{12}$.

(a) 1:3 *(b) 3:1* *(c) 9:1* *(d) 16:9*

SOLUTION :

at t = $\frac{T}{12}$

x = $A\sin\frac{2\pi}{T}\times\frac{T}{12} = A\sin\frac{\pi}{6} = \frac{A}{2}$

so *K.E.* = $\frac{1}{2}k(A^2 - x^2) = \frac{3}{4}\times\frac{1}{2}kA^2$ *and*

P.E. = $\frac{1}{2}kx^2 = \frac{1}{4}\times\frac{1}{2}kA^2$

\ $\frac{\text{K.E.}}{\text{P.E.}} = \frac{3}{1}$

31.. *The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of the amplitude, then determine total energy of particle.*

(a) 8 J *(b) 16 J* *(c) 12 J* *(d) 10 J*

SOLUTION : .

$$P.E. = \frac{1}{2}kx^2$$

$$\frac{1}{2}k\frac{A^2}{4} = 2.5$$

$$\text{total energy } \frac{1}{2}kA^2 = 2.5 \times 4 = 10 \text{ J}$$

32. *A harmonic oscillator of force consant 4×10^6 Nm and amplitude 0.01 m has total energy 240 J. What is maximum kinetic energy and minimum potential energy ?*

(a) 18 J *(b) 26 J* *(c) 40 J* *(d) 20 J*

SOLUTION :

$k = 4 \times 10^6$ N/m,

$a = 0.01$ m, T.E. = 240 J,

As $\omega^2 = \frac{k}{m}$

Maximum kinetic energy = $\frac{1}{2}m\omega^2a^2 = \frac{1}{2}ka^2 = \frac{1}{2} \times 4 \times 10^6 \times (0.01)^2 = 200$ J

Minimum potential energy = T.E. – maximum kinetic energy = 40 J

33. *When a spring is stretch by 4 cm then its potential energy is U. If the spring is stretched by 20 cm then determine its potential energy.*

(a) 18 U *(b) 25U* *(c) 40U* *(d) 2U*

SOLUTION :

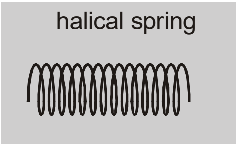
$$U = \frac{1}{2}kx^2$$

$$\frac{U'}{U} = \frac{x'^2}{x^2} = \frac{(20)^2}{(4)^2} = 25$$

$$U' = 25U$$

SPRING SYSTEM AND SPRING PENDULUM

SPRING SYSTEM

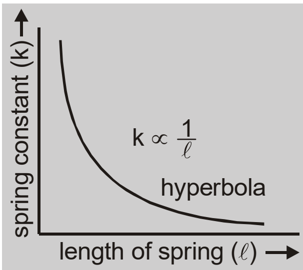


(i) *When spring is given small displacement by stretching or compressing it, then restoring elastic force is daveloped in it because it obeys Hook's law.*

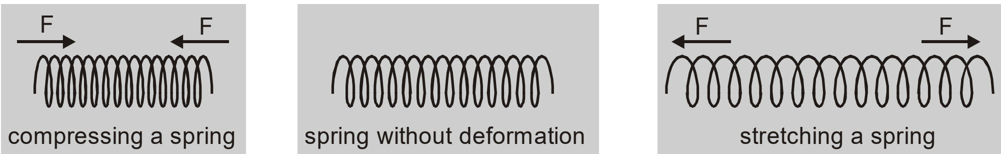
$F \propto -x \quad \Rightarrow \quad F = -kx$ *Here k is spring constant*

(ii) *Spring is assumed massless, so restoring elastic force in spring is assumed same everywhere.*

(iii) *Spring constant (k) depends on length, radius and material of wire used in spring.*



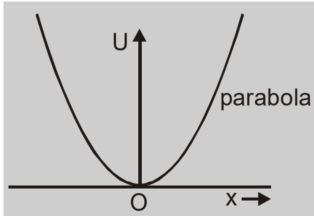
(iv) *When spring is stretched or compressed then work done on it is stored as elastic potential energy.*



$W = \int F dx = \int kx \, dx$ *and* $U = W = \frac{1}{2} kx^2$

When spring is stretched from l_1 *to* l_2 *then*

Work done $W = \frac{1}{2}k(X_2^2 - X_1^2)$



(v) *If there are two springs of force constant k_1 and k_2 with $k_1 > k_2$ then work done :*

(a) *When they are stretched by same amount ($x_1 = x_2$)*

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1}{k_2} > 1 \quad \Rightarrow \quad W_1 > W_2$$

(b) *When they are stretched by same force*

$$W = \frac{F^2}{2K} \quad \text{as } (F_1 = F_2) \qquad \frac{W_1}{W_2} = \frac{k_2}{k_1} < 1 \quad \Rightarrow \quad W_1 < W_2$$

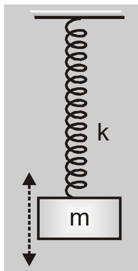
SPRING PENDULUM

(i) *When a small mass is suspended from a mass less spring then this arrangement is known as spring pendulum.*

For small linear displacement the motion of spring pendulum is simple harmonic.

(ii) *For a spring pendulum*

$$F = -kx \quad \Rightarrow \quad m\frac{d^2x}{dt^2} = -kx \qquad \left[F = ma = m\frac{d^2x}{dt^2}\right]$$



$$\text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x \quad \text{or} \quad \frac{d^2x}{dt^2} = -w^2x \quad \text{with } w^2 = \frac{k}{m}$$

This is standard equation of linear S.H.M.

Time period $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$ *Frequency* $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(iii) *Time period of a spring pendulum is independent of acceleration due to gravity. This is why a clock based on oscillation of spring pendulum will keep proper time everywhere on a hill or moon or in a satellite or different places of earth.*

(iv) *By increasing the mass, time period of spring pendulum increases* $(T \propto \sqrt{m})$*, but by increasing the force constant of spring (k). Its time period decreases* $\left[T \propto \frac{1}{\sqrt{k}}\right]$ *whereas frequency increases* $(n \propto \sqrt{k})$

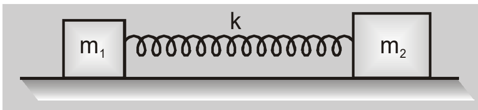
(v) *If the spring has mass M and mass m is suspended from it then effective mass is given by*

$m_{eff.} = m + \frac{M}{3}$ *and* $T = 2\pi\sqrt{\frac{m_{eff}}{k}} = 2\pi\sqrt{\frac{m + M/3}{k}}$

If spring oscillates by its own weight then $T = 2\pi\sqrt{\frac{M}{3k}}$

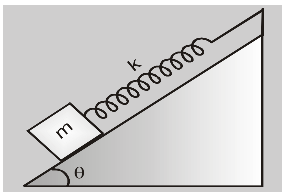
(vi) *If two masses* m_1 *and* m_2 *are connected by a spring and made to oscillate then time period*

$T = 2\pi\sqrt{\frac{\mu}{k}}$



Here, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ *= reduced mass*

(vii) *If a spring pendulum oscillates in a vertical plane is made to oscillate on a horizontal surface or on an inclined plane then time period will remain unchanged.*



(viii) When body of mass m attached to spring then its elongation *is* y_0 $ky_0 = mg$ *i.e.,*

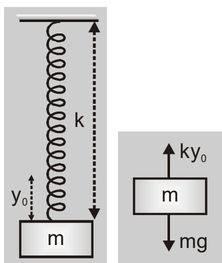
$\frac{m}{k} = \frac{y_0}{g}$ *then it is further streched by y*

If the stretch in a vertically loaded spring is y then for equilibrium of mass m.

Net force = $mg - K(y + y_0)$ *= ma*

$a = -\frac{K}{m}y$

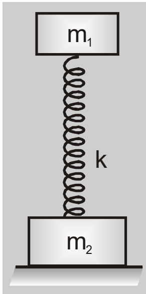
So, time period $T = 2\pi\sqrt{\frac{m}{k}}$



$$T = 2\pi\sqrt{\frac{y_0}{g}}$$

but remember time period of spring pendulum is independent of acceleration due to gravity.

(ix) *If two particles are attached with spring in which only one is oscillating*



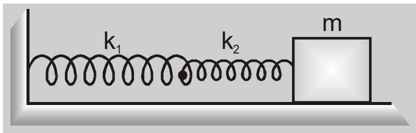
Time period $= 2\pi\sqrt{\frac{\text{mass of oscillating particle}}{\text{force constant}}}$

$$T = 2\pi\sqrt{\frac{m_1}{k}}$$

VARIOUS SPRING ARRANGEMENTS

Series combination of springs

Total displacement $x = x_1 + x_2$



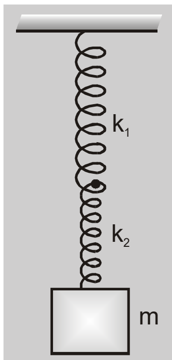
Force acting on both springs

$F = -k_1x_1 = -k_2x_2$

$x_1 = -\frac{F}{k_1}$ *and* $x_2 = -\frac{F}{k_2}$ $\qquad x = -\left[\frac{F}{k_1} + \frac{F}{k_2}\right]$ ....*(i)*

If equivalent force constant is k_s *then* $F = -k_sx$

so by equation (i) $\qquad -\frac{F}{k_s} = -\frac{F}{k_1} - \frac{F}{k_2} \Rightarrow \frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$



$$\Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$$

Time period $\quad T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

Frequency $\quad n = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}}$, $\qquad$ *Angular frequency* $\omega = \sqrt{\frac{k_s}{m}}$

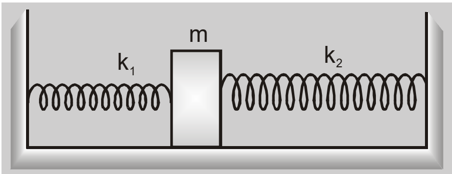
→ *In series combination same force exerts in all springs but extension will be different.*

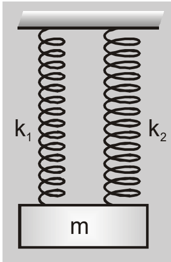
→ *In series combination extension of spring will be reciprocal of its spring constant.*

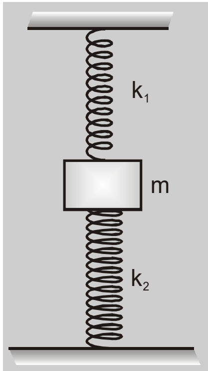
→ *Spring constant of spring is reciprocal of its length*

$\Rightarrow k \propto \frac{1}{\ell}$ $\Rightarrow$ $k_1 l_1 = k_2 l_2 = k_3 l_3$

Parallel Combination of springs







In this arrangement displacement on each spring is same but restoring force is different.

Force acting on the system $F = F_1 + F_2$ $\Rightarrow$ $F = -k_1 x - k_2 x$ ...(i)

If equivalent force constant is k_P *then,* $F = -k_P x$

so by equation (i) $-k_P x = -k_1 x - k_2 x$ $\Rightarrow$ $k_P = k_1 + k_2$

Time period $T = 2\pi\sqrt{\frac{m}{k_P}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k_P}{m}}$ *Angular frequency* $\omega = \sqrt{\frac{k_1 + k_2}{m}}$

→ *In parallel combination, different forces exerts in all springs but extension will be same.*

→ *In parallel combination, forces on spring will be proportional of its spring constant.*

→ *If the length of the spring is made n times then effective force constant becomes* $\frac{1}{n}$ *times and the time period becomes* $\sqrt{n}$ *times.*

→ *If a spring of spring constant k is divided into n equal parts, the spring constant of each part becomes nk and time period becomes* $\frac{1}{\sqrt{n}}$ *times.*

→ *In case of a loaded spring the time period comes out to be the same in both horizontal and vertical arrangement of spring system*

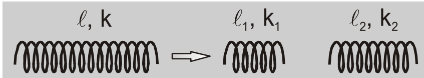
→ *The force constant k of a stiffer spring is higher than that of a soft spring. So the time period of a stiffer spring is less than that of a soft spring.*

→ *A spring of force constant k is cut into two unequal parts* l_1 *and* l_2*. Where* $\frac{\ell_1}{\ell_2} = \frac{n_1}{n_2}$ *then determine force constant of each part.*

Initial length of the spring = l and cut into two pieces of length l_1 *and* l_2*.*

since force constant $\propto \dfrac{1}{\text{length of spring}}$ so $k \propto \dfrac{1}{\ell} \Rightarrow k = \dfrac{C}{\ell}$ (C = constant)

$k_1 = \dfrac{C}{\ell_1}$, $k_2 = \dfrac{C}{\ell_2}$ and $k = \dfrac{C}{\ell} = \dfrac{C}{\ell_1 + \ell_2}$



$\dfrac{k_1}{k} = \dfrac{\ell_1 + \ell_2}{\ell_1} \Rightarrow k_1 = k\left[1 + \dfrac{\ell_2}{\ell_1}\right]$ and $\dfrac{k_2}{k} = \dfrac{\ell_1 + \ell_2}{\ell_2} \Rightarrow k_2 = k\left[1 + \dfrac{\ell_1}{\ell_2}\right]$

$\dfrac{\ell_1}{\ell_2} = \dfrac{n_1}{n_2}$ So, $k_1 = k\left[1 + \dfrac{n_2}{n_1}\right] \Rightarrow k_1 = k\left[\dfrac{n_1 + n_2}{n_1}\right]$

and $k_2 = k\left[1 + \dfrac{n_1}{n_2}\right] \Rightarrow k_2 = k\left[\dfrac{n_1 + n_2}{n_2}\right]$

PROBLEMS

1. A spring of force constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3.0 kg is attached to the free end of the spring, pulled side ways to a distance 2.0 cm and released. Determine

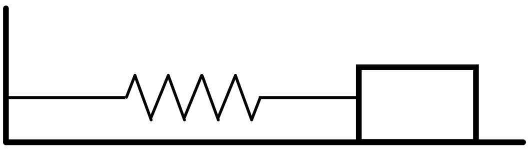
(a) the frequency of oscillation of the mass.

(b) the maximum acceleration of the mass.

(c) the maximum speed of the mass.

(a) 3.2 Hz ,8 m s^{-2} , 0.4 ms^{-1} (b) 2 Hz ,8.5 m s^{-2} , 4 ms^{-1}

(c) 3 Hz 1,8 m s^{-2} , 0.4 ms^{-1} (d) 32 Hz ,80 m s^{-2} , 4 ms^{-1}



SOLUTION :

Here, $k = 1200\, Nm^{-1}$; $m = 3.0\, kg$,

$A = 2.0\, cm = 0.02\, m$

(a) Frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{6.28}\sqrt{\frac{1200}{3}} = 3.2\, Hz$$

(b) Acceleration $a = \omega^2 y = \dfrac{k}{m} y$

Acceleration will be maximum when y is maximum i.e. y = A $\therefore$ Max.acceleration,

$$a_{max} = \frac{kA}{m} = \frac{1200 \times 0.02}{3} = \mathbf{8\,ms^{-2}}$$

(c) Max. speed of the mass will be when it is passing through the mean position, given by

$$V_{max} = A\omega = A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = \mathbf{0.4\,ms^{-1}}$$

2. *A body of mass m attached to a spring which is oscillating with time period 4 seconds. If the mass of the body is increased by 4 kg, its timer period increases by 2 sec. Determine value of initial mass m.*

1) 6.4Kg   2) 3.2Kg   3) 1.6Kg   4) 32Kg

SOLUTION :

$T = 2\pi\sqrt{\frac{m}{k}}$

1st case $4 = 2\pi\sqrt{\frac{m}{k}}$ *...(i) and*

2nd case $6 = 2\pi\sqrt{\frac{m+4}{k}}$ *...(ii)*

divide (i) by (ii) $\frac{4}{6} = \sqrt{\frac{m}{m+4}}$

$$\frac{16}{36} = \frac{m}{m+4}$$

m = 3.2 kg

3. *One body is suspended from a spring of length l, spring constant k and has time period T. Now if spring is divided in two equal parts which are joined in parallel and the same body is suspended from this arrangement then determine new time period.*

1) $\frac{T}{2}$ *2)* $\frac{T}{\sqrt{2}}$ *3)* $\frac{T}{4}$ *4)* $\frac{T}{6}$

SOLUTION :

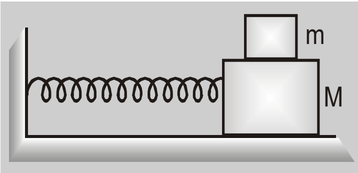
Spring constant in parallel combination

k' = 2k + 2k = 4k

$$T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}}$$

and $T' = 2\pi\sqrt{\frac{m}{k} \times \frac{1}{\sqrt{4}}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$

4. *A block is on a horizontal slab which is moving horizontally and executing S.H.M. The coefficient of static friction between block and slab is m. If block is not separated from slab then determine angular frequency of oscillation.*



1) $\sqrt{\frac{\mu g}{A}}$ 2) $\sqrt{\frac{\mu}{gA}}$ 3) $\sqrt{\frac{\mu}{g}}$ 4) $\sqrt{\frac{\mu}{A}}$

SOLUTION :

If block is not separated from slab then restoring force due to S.H.M. should be less than frictional force between slab and block.

$F_{restoring} = F_{friction}$

$m\, a_{max.} = mmg$

$a_{max.} = mg$

$w^2 A = mg$

$w = \sqrt{\frac{\mu g}{A}}$

5. A light vertical spring is stretched by 0.2 cm when a weight of 10 g is attached to its free end. The weight is further down by 1.0 cm and released. Compute the frequency and maximum velocity of load.

1) $\frac{1}{700} ms^{-1}$ 2) $\frac{1}{7000} ms^{-1}$ 3) $7000 ms^{-1}$ 4) $2000 ms^{-1}$

SOLUTION :

i) Force constant of the spring.

$$k = \frac{\text{Restoring Force}}{\text{Increase in length}} = \frac{\text{mg}}{\text{Increase in length}}$$

$$= \frac{10^{-2} \text{x} 9.8}{2.0 \text{x} 10^{-3}} = 49 Nm^{-1}.$$

Frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{49}{10^{-2}}} = \frac{70}{2\pi} Hz$

ii) amplitude of motion = distance through which the weight is further pulled down = 1.0 cm

i.e., A = 1.0 cm. $\Rightarrow$ *Maximum velocity*

$$= A\omega = 10^{-2} m \text{ x } \frac{1}{70} rads^{-1} = \frac{1}{7000} ms^{-1}$$

6. A mass m = 8kg is attached to a spring passing over a pulley whose other end is fixed to ground and held in position so that the spring remains unstretched. The spring constant is 200 N/ m. The mass m is then released and begins to undergo small oscillations. Find the maximum velocity of the mass (g =10 m/s²)

1) 0.5 m 2) 0.6 m 3) 0.4 m 4) 4 m

SOLUTION :

Mean position will be at kx =mg

or $x = \frac{mg}{k} = \frac{8 \times 10}{200} = \frac{2}{5} = 0.4m$

This is also the amplitude of oscillation A= 0.4m

7. *The spring constant of two springs are K_1 and K_2 respectively springs are stretch up to that limit when potential energyof both becomes equal. The ratio of applied force (F_1 and F_2) on them will be :*

(1) $K_1 : K_2$ (2) $K_2 : K_1$ (3) $\sqrt{K_1} : \sqrt{K_2}$ (4) $\sqrt{K_2} : \sqrt{K_1}$

SOLUTION :.

$$U_1 = U_2$$

$$\frac{1}{2}K_1 x_1^2 = \frac{1}{2}K_2 x_2^2$$

$$\frac{x_1}{x_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{F_1}{F_2} = \frac{K_1}{K_2} \times \sqrt{\frac{K_2}{K_1}}$$

8. *Force constant of a spring is K. one fourth part is detach then force constant of remaining spring will be :*

(1) $\frac{3}{4}K$ (2) $\frac{4}{3}K$ (3) K (4) 4K

SOLUTION :

$$k \infty \frac{1}{l} as length becomes \frac{3l}{4} spring cons \tan t becomes \frac{4k}{3}$$

$$v_{max} = A\omega = A\sqrt{\frac{k}{m}} = (0.4)\sqrt{\frac{200}{8}} = 2\ m/s$$

9. *The spring constant of a spring is K. When it is divided into n equal parts, then what is the spring constant of one piece :*

(1) nK (2) K/n (3) $\frac{nK}{(n+1)}$ (4) $\frac{(n+1)K}{n}$

SOLUTION :

$$K \propto \frac{1}{\ell}$$

$$\ell' = \frac{\ell}{n}$$

$$\therefore K' = nK$$

10. *The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be*

(1) $\frac{T}{4}$ (2) T (3) $\frac{T}{2}$ (4) 2T

SOLUTION :.

we know timeperiod $T = 2\pi\sqrt{\frac{m}{K}}$

when the spring made in to 4 equal parts springconstantof each part becomes 4k

$$T' = 2\pi\sqrt{\frac{m}{4K}} = \frac{2\pi}{2}\sqrt{\frac{m}{K}} = \frac{T}{2}$$

11. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be :

(1) 0.12 m *(2) 1.5 m* *(3) 0.5 m* *(4) 0.15m*

SOLUTION :.

Kinetic energy of mass particle will convert into potential energy of spring.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{mv^2}{k}} = v\sqrt{\frac{m}{k}} = 1.5\sqrt{\frac{0.5}{50}} = \frac{1.5}{\sqrt{100}} = 0.15 \text{ m}$$

12. Frequency of a particle executing SHM is 10 Hz. The particle is suspended from a vertical spring. At the highest point of its oscillation the spring is unstretched. Find the maximum speed of the particle : (g = 10 m/s²)

1) $\frac{1}{2\pi}$ *2)* 20π *3)* $\sqrt{2}\pi$ *4)* π

SOLUTION :

Mean position of the particle is $\frac{mg}{k}$ *distance below unstretched position of spring. therefore, amplitude of oscillation is*

$$A = \frac{mg}{k}$$

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = 20\pi$$

$$\therefore \frac{m}{k} = \frac{1}{400\pi^2}$$

Therefore, the maximum speed of particle will be

$$v_{\max} = A\omega = \left(\frac{g}{400\pi^2}\right)(20\pi) = \frac{1}{2\pi} \text{ m/s}$$

13. A15gm ball is shot from a spring gun whose spring has a force constant 600 N/ m .The spring is compressed by 5cm. The greatest possible horizontal range of the ball for this compression is (g = 10 m/sec^2)

1) 20 m 2) 10 m 3) 1 m 4) 100 m

SOLUTION :

$$R_{max} = \frac{U^2}{g} \quad \text{— (1)}$$

But K.E acquired by ball = P.E of spring gun

$$\Rightarrow \frac{1}{2} m U^2 = \frac{1}{2} kx^2$$

$$\Rightarrow U^2 = \frac{kx^2}{m} \quad \text{— (2)}$$

From equations (1) and (2)

$$R_{max} = \frac{kx^2}{mg} = \frac{600 \times (5 \times 10^{-2})^2}{15x10^{-3} \times 10} = 10m$$

SIMPLE PENDULUM

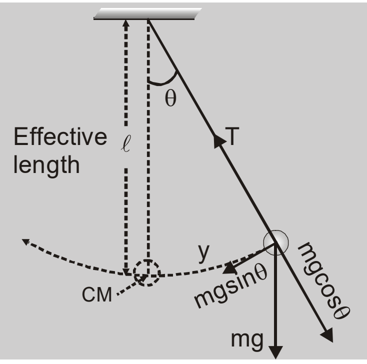
If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid su pport, then this arrangement is called a simple pendulum

Expression for time period

For small angular displacement, sin $\theta \approx \theta$, so that

$F = -mg \sin\theta$

$= -mg\ \theta$



$= -\left(\frac{mg}{\ell}\right)y$

$= -ky$

(because $y = \ell\theta$), Thus, the time period of the simple pendulum is

$T = 2\pi\sqrt{\frac{m}{k}}$

or $T = 2\pi\sqrt{\frac{\ell}{g}}$

Time period is independent of mass of the pendulum.

→ *If angular amplitude (θ_0) is large ($\theta_0 > 15°$) then time period is given by*

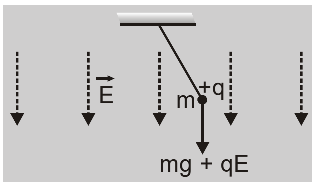
$$T = 2\pi\sqrt{\frac{\ell}{g}}\left[1 + \frac{\theta_0^2}{16}\right]$$ *here θ_0 is in radian.*

→ *If a simple pendulum of density d_s is made to oscillate in a liquid of density d_L then its time period will increase as compare to that of air and is given by*

$$T = 2\pi\sqrt{\frac{\ell}{\left[1 - \frac{d_L}{d_S}\right]g}}$$

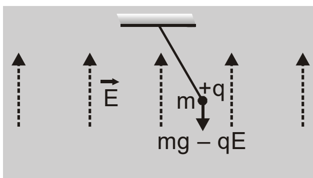
→ *If the bob of simple pendulum has positive charge q and pendulum is placed in uniform electric field which is in downward direction then time period decreases*

$$T = 2\pi\sqrt{\frac{\ell}{g + \frac{qE}{m}}}$$



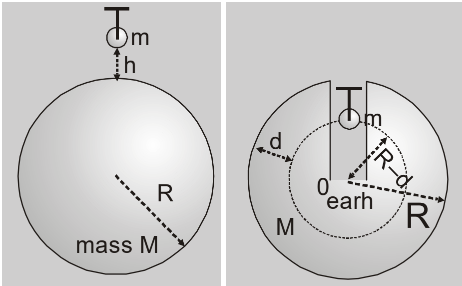
→ *If the bob of simple pendulum has positive charge q and is made is oscillate in uniform electric field acting in upward direction then time period increases*

$$T = 2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}}$$



→ *If simple pendulum is taken above or below the surface of earth then value of gravitational acceleration decreases and time period increases.*

$T \propto \frac{1}{\sqrt{g}}$



At height h $g_h = g_s\left(1 - \frac{2h}{R}\right)$

At depth d $g_d = g_s\left(1 - \frac{d}{R}\right)$

SECONDS PENDULUM

*A simple pendulum whose time period of oscillation is equal to two seconds, is called **"seconds pendulum"**.*

i.e., T = 2s

or $2\pi\sqrt{\frac{\ell}{g}} = 2 \quad or \quad \ell = \frac{g}{\pi^2}$

This is the length of a seconds pendulum and it changes from place to place.

If "g" is taken as 9.8ms⁻², then the length of the second's pendulum is equal to

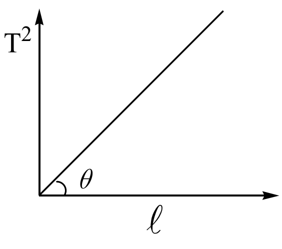
$\ell = \frac{9.8}{(3.14)^2} = 0.994m \quad (\because \pi^2 = 9.86)$ *(or)* $\ell \approx 1m$

GRAPHS RELATED TO SIMPLE PENDULUM:

we have $T = 2\pi\sqrt{\frac{\ell}{g}}$ *or* $T^2 = 4\pi^2\frac{\ell}{g}$

and $\frac{l}{T^2}$ *= constant at a given place.*

If a graph is drawn taking length "l" on X - axis and time period "T^2" on Y - axis it will be a straight line passing through origin as shown.

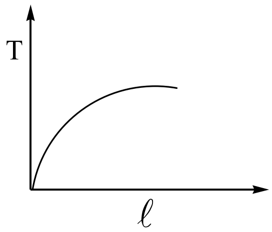


Slope of the graph "m" $= \tan\theta = \frac{T^2}{\ell}$.

from $T = 2\pi\sqrt{\frac{\ell}{g}}$

slope = $\frac{T^2}{\ell} = \frac{4\pi^2}{g}$. From this the value of 'g' at a given place can be found out.

If a graph is drawn taking length "l" on X - axis and time period T on Y - axis it will be a parabola as shown in figure.

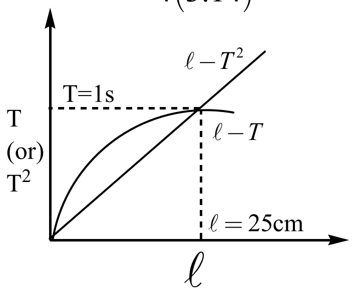


When l – T & l – T^2 graphs are plotted on the same graph paper they intersect at a point as shown in the diagram. At the point of intersection,

$T = T^2$ $\therefore$ T = 1s.

The length of the pendulum is given by

$\ell = \frac{gT^2}{4\pi^2} = \frac{9.8(1)^2}{4(3.14)^2}$ (or) $\ell \approx 25cm$.



Therefore for T = 1 s and $\ell \approx 25cm$, $\ell - T$ and $\ell - T^2$ graphs intersect.

PERIODIC TIME OF SIMPLE PENDULUM IN REFERENCE SYSTEM

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

where, $g_{eff.}$ = effective gravity acceleration in reference system

or total downward acceleration.

(a) If reference system is lift

(i) If velocity of lift v = constant

acceleration a = 0 and $g_{eff.} = g$ $T = 2\pi\sqrt{\frac{\ell}{g}}$

(ii) If lift is moving upwards with acceleration a

$g_{eff.} = g + a$

$$T = 2\pi\sqrt{\frac{\ell}{g+a}}$$ *T decreases*

(iii) *If lift is moving downwards with acceleration a*

$g_{eff.} = g - a$

$$T = 2\pi\sqrt{\frac{\ell}{g-a}}$$ *T increases*

(iv) *If lift falls downwards freely*

$g_{eff.} = g - g = 0$

T = 0 ∴ simple pendulum will not oscillate

(b) *A simple pendulum is mounted on a moving truck*

(i) *If truck is moving with constant velocity, no pseudo force acts on the pendulum and time period remains same*

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

(ii) *If truck accelerates forward with acceleration a then a pseudo force acts in opposite direction.*

So effective acceleration, $g_{eff.} = \sqrt{g^2 + a^2}$ *and*

$$T' = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$$

Time period $T' = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+a^2}}}$ *T' decreases*

vi) *Consider a simple pendulum carrying a bob of mass 'm' and of length "ℓ" which is very large and comparable to the radius "R" of earth. Let it be oscillating with small angular amplitude.*

The time period of oscillation of the pendulum is given by

$$\boxed{T = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{\ell}+\frac{1}{R}\right]}}}$$

a) If the pendulum is infinitely long, then using $l = \infty$ *in the above expression we get,*

$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$ *Taking g = 9.8m/s² and radius of earth as 6400km, the time period in this case is nearly equal to 84.6 minutes* *b) If the length of the pendulum is equal to radius of earth, then l = R and we get,*

$$\boxed{T = 2\pi\sqrt{\frac{R}{2g}}}$$

and this value is nearly equal to 60 minutes.

c) If $\ell << R, T = 2\pi\sqrt{\frac{\ell}{g}}$

vii) *Consider a simple pendulum of length "l" suspended inside a trolley which is coming down on an inclined plane of inclination "θ".*

$|\bar{a}|$ *= acceleration of the point of suspension =* $g\sin\theta$ *(down the plane)*

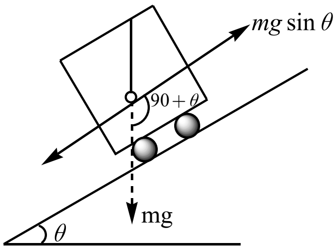
$$\therefore g_{eff} = \sqrt{g^2 + (g\sin\theta)^2 + 2(g)(g\sin\theta)\cos(90+\theta)}$$

$$= \sqrt{g^2 + g^2\sin^2\theta - 2g^2\sin^2\theta}$$

[$\because \cos(90+\theta) = -\sin\theta$*]*

$g_{eff} = \sqrt{g^2(1-\sin^2\theta)} = g\cos\theta$

But time period T = $2\pi\sqrt{\frac{\ell}{g_{eff}}}$



$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}}$$

viii) *Consider two simple pendulums of different lengths. Let, l_L and l_S are the lengths of longer and shorter pendulums and T_L and T_S are their time periods of oscillation respectively.*

They are made to oscillate at the same instant starting from the same phase. By the time they are again at the same phase, if "n" is the number of oscillations made by the longer pendulum, the shorter pendulum completes (n+1) oscillations.

If "t" is the minimum time interval after which the pendulums are again in the same phase, then

$t = n\,T_L = (n+1)T_S$ ....(1)

→ *Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as simple S.H.M.*

→ *If time period of clock based on simple pendulum increases then clock will be slow if* time period decreases then clock will be fast.

→ If Δl is change in length and Δg is the change in acceleration then for small variation (up to 5%) change in time period (ΔT) will be

$$\frac{\Delta T}{T}\times 100=\left[\frac{1}{2}\frac{\Delta \ell}{\ell}-\frac{1}{2}\frac{\Delta g}{g}\right]\times 100$$

→ Due to change in shape of earth (not shperical but ellipitical) gravitational acceleration is different at different places. So time period of simple pendulum varies with variation of g.

→ The time period of simple pendulum is independent of mass of bob.

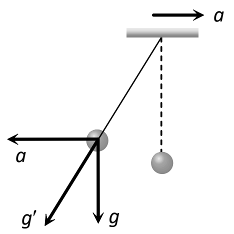
1. If the length of the pendulum is increased by 2%. What is the percentage change in time period of pendulum.

SOLUTION : $\frac{\Delta T}{T}\times 100=\frac{1}{2}\frac{\Delta l}{l}\times 100=1\%$

2. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T=2\pi\sqrt{\frac{l}{g'}}$, where g' is equal to

(a) g (b) $g-a$ (c) $g+a$ (d) $\sqrt{g^2+a^2}$

SOLUTION : d) $g'=\sqrt{g^2+a^2}$



3. A second's pendulum is placed in a space laboratory orbiting around the earth at a height 3R, where R is the radius of the earth. The time period of the pendulum is

(a) Zero (b) $2\sqrt{3}\ sec$ (c) 4 sec (d) Infinite

SOLUTION :

In the given case effective acceleration $g_{eff.} = 0$

$$T = \infty$$

4. The bob of a simple pendulum of mass m and total energy E will have maximum linear momentum equal to

(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$ (c) $2mE$ (d) mE^2

SOLUTION

$$p_{\max} = \sqrt{2m\, E_{\max}}$$

5.. The length of the second pendulum on the surface of earth is 1 m. The length of seconds pendulum on the surface of moon, where g is 1/6th value of g on the surface of earth, is

(a) 1 / 6 m (b) 6 m (c) 1 / 36 m (d) 36 m

SOLUTION

$T = 2\pi\sqrt{\frac{l}{g}}$ T $\sqrt{\frac{l}{g}}$ = constant

$$\Rightarrow l \propto g;$$

$$\frac{l_m}{1} = \frac{1}{6}\frac{g}{g} \Rightarrow l_m = \frac{1}{6}m$$

6. If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day

(a) 3927 sec (b) 3727 sec (c) 3427 sec (d) 864 sec

SOLUTION

$$T \propto \sqrt{l}\ T$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01$$

$$\Delta T = 0.01\, T$$

Loss of time per day $= 0.01 \times 24 \times 60 \times 60 = 864$ sec

7. The period of simple pendulum is measured as T in a stationary lift. If the lift moves upwards with an acceleration of 5g, the period will be

(a) The same (b) Increased by 3/5

(c) Decreased by 2/3 times (d) None of the above

SOLUTION

$$\frac{T'}{T} = \sqrt{\frac{g}{g'+a}} = \sqrt{\frac{g}{g+5g}} = \sqrt{\frac{1}{6}}$$

$$T' = \frac{T}{\sqrt{6}}$$

8. *The length of a simple pendulum is increased by 1%. Its time period will*

(a) Increase by 1% *(b) Increase by 0.5%*

(c) Decrease by 0.5% *(d) Increase by 2%*

SOLUTION

$$T \propto \sqrt{l}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2} \times 1\% = 0.5\%$$

9.. *A pendulum has time period T. If it is taken on to another planet having acceleration due to gravity half and mass 9 times that of the earth then its time period on the other planet will be*

(a) $\sqrt{T}$ *(b)* *T* *(c)* $T^{1/3}$ *(d)* $\sqrt{2}$ *T*

SOLUTION

If initial length $l_1 = 100$ *then* $l_2 = 121$

By using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

Hence, $\frac{T_1}{T_2} = \sqrt{\frac{100}{121}} \Rightarrow T_2 = 1.1\, T_1$

% increase = $\frac{T_2 - T_1}{T_1} \times 100 = 10\,\%$

10. *A simple pendulum is executing simple harmonic motion with a time period T. If the length of the pendulum is increased by 21%, the percentage increase in the time period of the pendulum of increased length is*

(a) 10% *(b) 21%* *(c) 30%* *(d) 50%*

SOLUTION

$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{100}{400}}$ *(If* $l_1 = 100$ *then* $l_2 = 400$ *)*

$\Rightarrow \; T_2 = 2T_1$

Hence % increase $= \frac{T_2 - T_1}{T_1} \times 100 = 100\,\%$

11. *The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)*

(a) $\frac{1}{\sqrt{2}}$ sec (b) $2\sqrt{2}$ sec (c) 2 sec (d) $\frac{1}{2}$ sec

SOLUTION

As we know $g = \frac{GM}{R^2}$

$$\Rightarrow \frac{g_{\text{earth}}}{g_{\text{planet}}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1} \quad \text{Also } T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$$

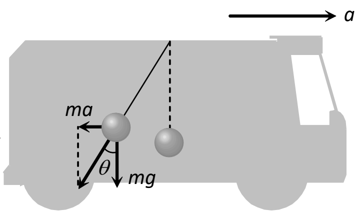
12. A simple pendulum is set up in a trolley which moves to the right with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle θ with the vertical

(a) $\tan^{-1}\frac{a}{g}$ in the forward direction (b) in the backward direction

(c) in the backward direction (d) in the forward direction

SOLUTION

In accelerated frame of reference, a fictitious force (pseudo force) ma acts on the bob of pendulum as shown in figure

.


Hence, $\tan\theta = \frac{ma}{mg} = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$ in the backward direction.

13. The time period of a second's pendulum is 2 sec. The spherical bob which is empty from inside has a mass of 50 gm. This is now replaced by another solid bob of same radius but having different mass of 100 gm. The new time period will be

(a) 4 sec (b) 1 sec (c) 2 sec (d) 8 sec

SOLUTION

$T = 2\pi\sqrt{\frac{l}{g}}$ (Independent of mass)

14. If the length of pendulum is increased by 44% find the percentage change in time period of simple pendulum

1) 24 % 2)36 % 3)14 % 4)20 %

SOLUTION

Percentage change in time period = $\frac{\sqrt{l_2}-\sqrt{l_1}}{\sqrt{l_1}}\times 100$

= $\frac{\sqrt{1.44 l_1}-\sqrt{l_1}}{\sqrt{l_1}}\times 100$ = *20%*

15. Two pendulums of lengths 1.69 m and 1.44 m start swinging together. After how many vibrations will they again start swinging together?

1) *they swing together after the shorter pendulum completes 13 oscillations*

2) *they swing together after the shorter pendulum completes 12 oscillations*

3) *they swing together after the shorter pendulum completes 10oscillations*

4) *they swing together after the shorter pendulum completes 8 oscillations*

SOLUTION -

$$\frac{n}{n-1}=\sqrt{\frac{l_L}{l_S}}$$

$$\frac{n}{n-1}=\sqrt{\frac{1.69}{1.44}}=\frac{1.3}{1.2}=\frac{13}{12}$$

$$n=13$$

So they swing together after the shorter pendulum completes 13 oscillations or longer pendulum completes 12 oscillations.

16. Weight of the bob of a simple pendulum is W. Length of the pendulum if l and it is vibrating with an amplitude A. Maximum tension in the string during oscillation will be:

1) $=W\left(1+\frac{A^2}{l^2}\right)$ 2) $W\left(1+\frac{A}{l}\right)$ 3) $W\left(1+\frac{l^2}{A^2}\right)$ 4) $W\left(\frac{A^2}{l^2}\right)$

SOLUTION

$$T=mg+\frac{mv^2}{l}$$

here $v = A\omega$ $\quad v = A\sqrt{\frac{g}{l}}$

T$= mg + \frac{mA^2 g}{l.l}$

T$=W\left(1+\frac{A^2}{l^2}\right)$ here $W = mg$

17. A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to $\frac{T}{2}$

SOLUTION

In stationary lift $T = 2\pi\sqrt{\frac{\ell}{g}}$ ...(i)

In accelerated lift $\frac{T}{2} = T' = 2\pi\sqrt{\frac{\ell}{g+a}}$ ...(ii)

Divide (i) by (ii) 2 = $\sqrt{\frac{g+a}{g}}$

g + a = 4g

a = 3g

18. The length of a second's pendulum at the surface of earth is 1m. Determine the length of second's pendulum at the surface of moon.

1) $\ell_m = \frac{\ell_e}{5}$ 2) $\ell_m = \ell_e$ 3) $\ell_m = \frac{\ell_e}{\sqrt{2}}$ 4) $\ell_m = \frac{\ell_e}{6}$

SOLUTION

For second's pendulum at the surface of earth $2 = 2\pi\sqrt{\frac{\ell_e}{g_e}}$ ...(i)

For second's pendulum at the surface of moon $2 = 2\pi\sqrt{\frac{\ell_m}{g_m}}$ ...(ii)

From (i) and (ii) $\frac{\ell_e}{g_e} = \frac{\ell_m}{g_m}$

$$\ell_m = \left[\frac{g_m}{g_e}\right]\ell_e$$

$$\ell_m = \frac{\ell_e}{6} \qquad \left[\because \quad g_m = \frac{g_e}{6}\right]$$

19.. If length of a simple pendulum is increased by 4%. Then determine percentage change in time period.

1) 20 % 2) 2% 3) 0.2% 4) 4%

SOLUTION

Percentage change in time period $\frac{\Delta T}{T}\times 100\% = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$ *[Q Dg = 0]*

According to question $\frac{\Delta\ell}{\ell}\times 100 = 4\%$

$$\frac{\Delta T}{T}\times 100\% = \frac{1}{2}\times 4\% = 2\%$$

20. A simple pedulum has time period 'T_1'. The point of suppension is now moved upwards according to the relation $y = kt^2$, ($k = 1m/sec^2$) where y is the vertical displacement. The time period now becomes 'T_2' then find the ratio of $\frac{T_2^2}{T_1^2}$

1) 3:5 2) 6:7 3) 6: 5 4) 5 : 6

SOLUTION

$y = kt^2 = 1/2\ at^2$

$$\Rightarrow \frac{1}{2}a = k = 1$$

$$\Rightarrow a = 2m/\sec^2 \quad (\textit{acceleration}\,\}$$

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{\ell}{g+a}}$$

$$\frac{T_1^2}{T_2^2} = \frac{g+a}{g} = \frac{10+2}{10} = \frac{6}{5}$$

21. A child swinging on a swing in sitting position, stands up, then the period of the swing will be:

(1) Increase (2) Decrease

(3) Remain same (4) Increase if child is long and decrease if child is short

SOLUTION .

When the child stands up the length of pendulum increases.then timeperiod also increases

22. A simple pendulum 4 m long swing with an amplitude of 0.2 m. What is its acceleratin at the ends of its path?

1) $0.5 m/s^2$ 2) $0.6 m/s^2$ 3) $5 m/s^2$ 4) $2 m/s^2$

SOLUTION

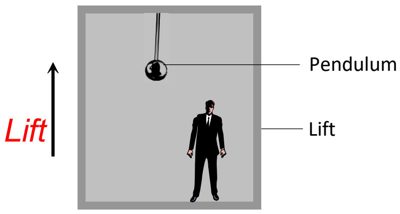
$T = mg \cos\theta$

$\therefore \quad F_{net} = mg\sin\theta$ and

acceleration = $g\sin\theta$

$= (10)\dfrac{(0.2)}{4} = 0.5\, m/s^2$

23. --A man measures the period of a simple pendulum inside a stationary lift and finds it to be T sec. If the lift accelerates upwards with an acceleration $g/4$, then the period of the pendulum will be



(a) T (b) $\frac{T}{4}$ (c) $\frac{2T}{\sqrt{5}}$ (d) $2T\sqrt{5}$

SOLUTION In stationary lift $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift $T' = 2\pi\sqrt{\frac{l}{(g+a)}}$

($a =$ Acceleration of lift)

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{\left(g+\frac{g}{4}\right)}} = \sqrt{\frac{4}{5}} \Rightarrow T' = \frac{2T}{\sqrt{5}}$$

SOME SPECIAL CASES

$\rightarrow$ *Consider a tunnel drilled along the diameter of the earth of radius "R". Let "ρ" be the mean density of earth and "g" is the acceleration due to gravity on its surface. Let a body of mass 'm' be released at one end of the tunnel.*

$$F = -GM_x \frac{m}{x^2} \text{ but } M_x = \frac{M}{R^3}x^3 \therefore F = -\frac{GM}{R^2}\left(\frac{mx}{R}\right)$$

$$F = -\frac{g}{R}(mx) \qquad \therefore K = \frac{g}{R}(m) \text{ but } T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Hence, time period of oscillation of the body is given by,

$$\boxed{\therefore T = 2\pi\sqrt{\frac{R}{g}}}$$

Taking R = 6400Km = 64 x 10^5m and g = 9.8 ms^{-2}

$$T = 2\pi\sqrt{\frac{64\times10^5}{9.8}} \text{ or } T \approx 84.6 \text{ min.}$$

Thus, the body will be in SHM with a time period of nearly 84.6min. It can be recalled that this is equal to time period of oscillation of a simple pendulum of infinite length.

Note :

1) The above expression for time period is valid even if the tunnel is dug along any chord of the earth.

2) The above mentioned expression is not valid if the body is dropped into the tunnel from certain height above the ground. In this case, the body executes oscillatory motion, but it is not simple harmonic.

$\rightarrow$ *Consider a body of mass "m" and cross sectional area "A" floating in a liquid of density "σ" with a height "h" submerged in the liquid.*

Under equilibrium,

Weight of the body = Buoyancy

mg = Ah σ g

(or) m = Ah σ .... (1)

When the body is slightly pushed down, through a small distance "y" and released, the restoring force on it due to extra thrust is

F = – A σ gy.

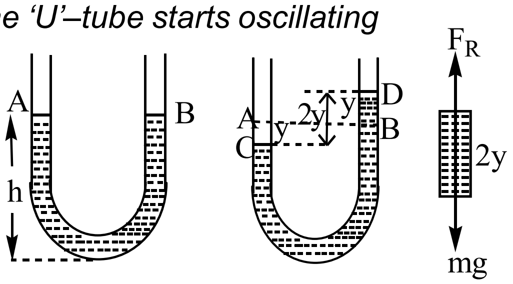
Which is in the form F = – Ky with K = A σ g $\therefore$ The body executes SHM with time period

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{A\sigma g}} \;\therefore\; \boxed{T = 2\pi\sqrt{\frac{m}{A\sigma g}}}$$

for this floating body mass m = Ah σ .

$$\therefore \boxed{T = 2\pi\sqrt{\frac{h}{g}}}$$

$\rightarrow$ *Consider a liquid of density 'σ' taken into a 'U'–tube such that the total length of the liquid column is 'L' and height of the liquid column in each limb is 'h' (L=2h). The liquid column in one of the limbs is depressed down by a small distance 'y' and released. Now the liquid in the 'U'–tube starts oscillating simple harmonically.*



In the new position, the difference in the liquid levels in the two limbs is '2y'.

The restoring force on the liquid is given by

F = Pressure x area = $(2y\rho g)A$

$F = -A(2y)\sigma g$ *........... (1)*

The negative sign indicates that the direction of restoring force is opposite to the displacement.

$F = -A(2y)\sigma g \;\Rightarrow K = 2A\sigma g$

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2Ah\sigma}{2A\sigma g}} = 2\pi\sqrt{\frac{h}{g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{h}{g}}}$$

$\rightarrow$ *Consider a small sphere of radius 'r' released from the position shown on a bowl of radius of curvature 'R' The motion of the sphere is analogus to oscillations of a simple pendulum of length (R–r). At one instant of time let the imaginary pendulum is making a small angle 'θ' with the vertical. At that instant the restoring force acting on the sphere is given by,*

$F = -mg\sin\theta$ .... *(1)*

The negative sign indicates that the restoring force acts in a direction opposite to the displacement.

F = mg θ *(* $\because \sin\theta \approx \theta$ *for small values of* θ *)*

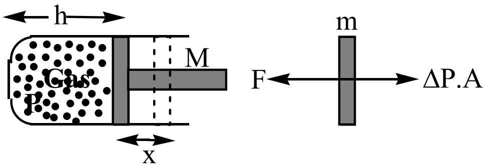
$= -\ mg\frac{y}{(R-r)}$

This is in the form F = –Ky with $K = \frac{mg}{R-r}$

$\therefore$ *The time period of oscillation of the sphere is given by, (or)* $\boxed{T = 2\pi\sqrt{\frac{(R-r)}{g}}}$

$\rightarrow$ *Consider a uniform cylinder of area of cross–section 'A' arranged horizontally, filled with a gas at a pressure 'P' and enclosed by means of a piston of mass 'M' as shown in the figure. Let, 'h' be the length of the gas column.*

When the piston is pushed inwards through a small distance 'x', due to increase in pressure, there acts a restoring force on the piston.



from ideal gas equation PV = constant

$\therefore P\Delta V + V\Delta P = 0$ *(or)* $\Delta P = -\left(\frac{\Delta V}{V}\right)P$

If F is the restoring force acting on the piston of area of cross section 'A', then

$\frac{F}{A} = -\left[\frac{Ax}{Ah}\right]P$ *(or)* $F = -\left[\frac{AP}{h}\right]x$ .... *(1)*

$\Rightarrow\ K = \frac{AP}{h} \qquad \therefore T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{Mh}{AP}}$

$\therefore$ *The time period of oscillation of the piston can be obtained from*

$\therefore \boxed{T = 2\pi\sqrt{\frac{Mh}{AP}}}$

$\rightarrow$ *Consider a massless elastic wire of length 'L', area of cross–section 'A' and Young's modulus 'Y' suspended from a rigid support. A small mass M is attatched at free end. When the string is pulled down through a small distance '* ΔL *' and released the string oscillates up and down. In equilibrium, the restoring force developed in it is given by*

$F = -\frac{YA}{L}\Delta L$ .... *(1)* $\left(\because Y = \frac{Fl}{A\Delta l}\right)$

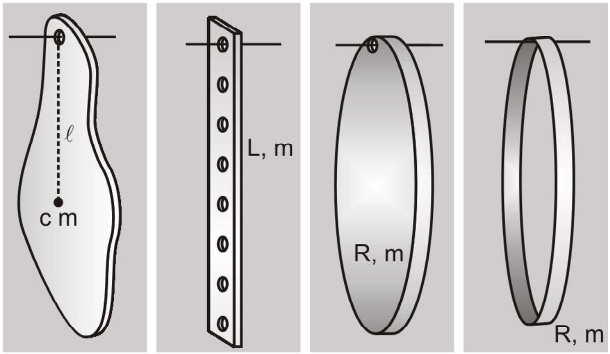
The negative sign shows that, the restoring force acts in a direction opposite to the elongation.

$\Rightarrow K = \frac{YA}{L} \qquad \therefore T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{ML}{YA}}$

$\therefore$ *The time period of oscillation can be obtained from*

$$\therefore \boxed{T = 2\pi\sqrt{\frac{ML}{YA}}}$$

Expression for time period compound pendulum



Ą Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is define compound pendulum

Torque acting on a body $\tau = -mgl\sin\theta$

if angle is very small $\sin\theta \approx \theta$

$$\tau = -mgl\theta \quad ...(i) \qquad \text{and} \quad \tau = I_s a \quad ...(ii)$$

m = mass of the body

l = distance between point of suspension and centre of mass

I_s *= moment of inertia about horizontal axis passes through point of suspension*

from equation (i) and (ii) $I_s a = -mgl\theta$

$$I_s\frac{d^2\theta}{dt^2} + mgl\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{mg\ell}{I_s}\theta = 0 \quad ...(iii)$$

$$\because \quad \frac{d^2\theta}{dt^2} + w^2\theta = 0 \quad ...(iv)$$

compare equation (iii) and (iv) $w^2 = \frac{mg\ell}{I_s}$

$$\Rightarrow \quad w = \sqrt{\frac{mg\ell}{I_s}} \qquad \Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{mg\ell}{I_s}}$$

Time period of compound pendulum $T = 2\pi\sqrt{\frac{I_s}{mg\ell}}$

Applying parallel axis theorm $I_s = I_{CM} + ml^2 \Rightarrow I_s = mk^2 + ml^2$

$$T = 2\pi\sqrt{\frac{I_s}{mg\ell}} = 2\pi\sqrt{\frac{mk^2 + m\ell^2}{mg\ell}}$$

Time period $T = 2\pi\sqrt{\frac{\frac{k^2}{\ell} + \ell}{g}}$

S = point of suspension ; O = point of oscillation ; k = radius of gyaration

l = distance between point of suspension and centre of mass

$L = \frac{k^2}{\ell} + l$ *= equivalent length of simple pendulum = distance between point of suspension and point of oscillation*

Time Period $T = 2\pi\sqrt{\frac{\frac{k^2}{\ell} + \ell}{g}}$

For maximum time period $\ell = 0$ *maximum time period* $T_{max} = \infty$

For minimum time period $\frac{dT}{d\ell} = 0$ *then* $k = \ell$

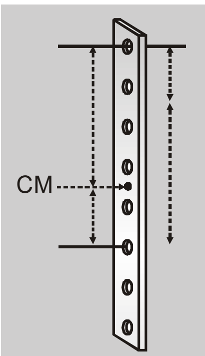
$$T = 2\pi\sqrt{\frac{\frac{k^2}{k} + k}{g}}$$

Minimum time period of compound pendulum $T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

$\frac{k^2}{\ell}$ *= distance from centre of mass to point of oscillation (standardresult)*

Bar pendulum

It is a steel bar of 1 metre length with hole at regular interval for suspension. Time period is measured by taking different value of l.



→ *There are maximum four points for which time period of compound pendulum is same.*

→ *Minimum time period is obtained at two points*

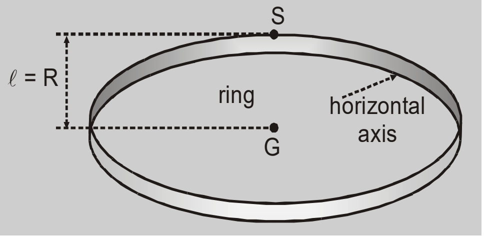
→ *The point of suspension and point of oscillation are mutually interchangeable.*

→ *Maximum time period will obtain at centre of gravity, which is infinite means compound pendulum will not oscillate at this point.*

→ *Compound pendulum executes angular S.H.M. about its mean position. Here restoring torque is provided by gravitational force.*

PROBLEMS

1.. *A ring is oscillating about a horizontal axis passes through its rim. Determine time period of oscillation.*



(1) $T = 2\pi\sqrt{\frac{2R}{g}}$ (2) $T = 2\pi\sqrt{\frac{R}{g}}$

(3) $T = 2\pi\sqrt{\frac{R}{2g}}$ (4) $T = \pi\sqrt{\frac{2R}{g}}$

SOLUTION :

For ring $I = Mk^2 = MR^2$

So, $k = R$

$$L = \ell + \frac{k^2}{\ell} = R + \frac{R^2}{R} = 2R$$

$$T = 2\pi\sqrt{\frac{2R}{g}}$$

2. *A sphere is made to oscillate about a horizontal tangential axis. Determine time period.*

(1) $T = 2\pi\sqrt{\frac{7R}{5g}}$ (2) $T = 2\pi\sqrt{\frac{5R}{7g}}$

(3) $T = 2\pi\sqrt{\frac{R}{g}}$ (4) $T = \pi\sqrt{\frac{2R}{g}}$

SOLUTION :

For sphere $I = Mk^2 = \frac{2}{5}MR^2$

$k^2 = \frac{2}{5}R^2$

$L = \ell + \frac{k^2}{\ell} = R + \frac{\frac{2}{5}R^2}{R} = \frac{7}{5}R$

$T = 2\pi\sqrt{\frac{7R}{5g}}$

3. *A disc is made to oscillate about a horizontal axis passing through mid point of its radius. Determine time period.*

(1) $T = 2\pi\sqrt{\frac{3R}{2g}}$ (2) $T = 2\pi\sqrt{\frac{5R}{7g}}$

(3) $T = 2\pi\sqrt{\frac{R}{g}}$ (4) $T = \pi\sqrt{\frac{2R}{g}}$

SOLUTION :

For disc $I = Mk^2 = \frac{MR^2}{2}$

$\Rightarrow k = \frac{R}{\sqrt{2}}$

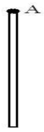
$L = \ell + \frac{k^2}{\ell} = \frac{R}{2} + \frac{R^2}{2\left[\frac{R}{2}\right]} = \frac{3R}{2}$

$\Rightarrow T = 2\pi\sqrt{\frac{3R}{2g}}$

4. *A uniform rod of mass 'm' and length 'l' is hinged at one end 'A'. It can rotate freely about a horizontal axis passing through A. If it is given a slight angular displacement and left to itself then it oscillate. Find the time period.*

(1) $T = 2\pi\sqrt{\frac{l}{g}}$ (2) $T = 2\pi\sqrt{\frac{3l}{2g}}$

(3) $T = 2\pi\sqrt{\frac{2l}{3g}}$ (4) $T = 2\pi\sqrt{\frac{3l}{g}}$



SOLUTION :

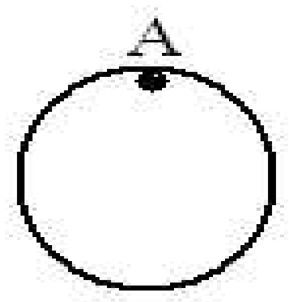
here $I = \frac{ml^2}{3},\ d = \frac{l}{2}$.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{ml^2}{3.mg\frac{l}{2}}} = 2\pi\sqrt{\frac{2l}{3g}}$$

5. *A uniform ring of radius 'R' is suspended from a horizontal nail 'A' as shown. Find time period of its small oscillations.*

(1) $T = 2\pi\sqrt{\frac{3R}{2g}}$ (2) $T = 2\pi\sqrt{\frac{5R}{7g}}$

(3) $T = 2\pi\sqrt{\frac{R}{g}}$ (4) $T = \pi\sqrt{\frac{2R}{g}}$



SOLUTION :

here $I = 2mR^2, d = R$

, $T = 2\pi\sqrt{\frac{I}{mgd}}$

$$T = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

** 6.17. Damped Oscillation :

So far we have dealt oscillation of a body which are simple harmonic, in which total energy of oscillation is constant.

*But when the oscillation are in a medium which offer some resistance force, in which the energy gradually decreases with time, and finally the oscillation will be stopped, such oscillations are called **"Damped oscillations".***

The damping force depends on velocity as f = –bv;

Hear, b = damping co efficient

Hence the total restoring force is

F_R = – (kx + bv) $\Rightarrow$ ma = –(kx + bv)

$$\therefore \boxed{\frac{d^2x}{dt^2} + \frac{b}{m}v + \frac{k}{m}x = 0} \Rightarrow \boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0}$$

for which the angular frequency is

$$\boxed{\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

→ ***Composition of two SHM's of equal frequency in mutually perpendicular directons.***

Let the two SHM's be

(i) $x = a\sin\omega t \Rightarrow \frac{x}{a} = \sin\omega t$

(ii) $y = b\sin(\omega t + \phi) \Rightarrow \frac{y}{b} = \sin(\omega t + \phi)$

$$\sin\omega t\cos\phi + \cos\omega t\sin\phi = \frac{y}{b}$$

$$\frac{x}{a}\cos\phi + \sqrt{1 - \frac{x^2}{a^2}}\sin\phi = \frac{y}{b}$$

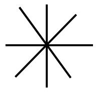
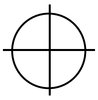
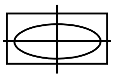
Squaring on both sides

$$\sqrt{1 - \frac{x^2}{a^2}}\sin^2\phi = \left(\frac{y}{b} - \frac{x}{a}\cos^2\phi\right)$$

$$\left(1 - \frac{x^2}{a^2}\right)\sin\phi = \frac{y^2}{b^2} + \frac{x^2}{a^2}\cos\phi - \frac{2xy}{ab}\cos\phi$$

$$\therefore \boxed{= \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi\right) = \sin^2\phi}$$

This is the equation representing resultant SHM. The path traversed by the particle, depends on the values of a, b and ϕ as shown in the table

Condition	Resultant SHM	Shape
$a = b, \phi = 0^0, 180^0$	Straight line $y = \pm x$	
$\phi = 90^0$	Circle $x^2 + y^2 = a^2$	
$a \neq b,\ \phi = 90^0,$ $180^0,\ Q = 90^0$	Straight line $y = \pm \frac{b}{a}x$ Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	

→ ***Vector method of adding two linear SHM's in a line :***

Suppose two SHM's of same frequencies are represented by $Y_1 = a_1 \sin \omega t$ and $Y_2 = a_2 \sin(\omega t + \phi)$ when these are super imposed on a particle, to get the resultant SHM consider a_1 and a_2 as vectors and the phase difference ϕ as the angle between them. Now applying parallelogram law to get the resultant amplitude and epoch

The resultant SHM equation is

$$Y = a \sin(\omega t + \delta)$$

The resultant amplitude

$$\boxed{a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}}$$

and epoch δ

$$\boxed{\tan\delta = \frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}}$$

EXERCISE PROBLEMS

1. *When an oscillator completes 100 oscillation its amplitude reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillation*

(1) $\frac{1}{8}$ *(2)* $\frac{2}{3}$ *(3)* $\frac{1}{6}$ *(4)* $\frac{1}{9}$

SOLUTION :

\ *After 100 oscillation amplitude(vk;ke) become* $\frac{A}{3}$

Q *After 200 oscillation it become* $\frac{1}{3}$ *of* $\frac{A}{3} = \frac{A}{9}$

2. A body of mass 1.0kg is suspended from a weightless spring having force constant 600 N/m . Another body of mass 0.5 kg moving vertially upwards hits the suspended body with velocity of 3.0 m/sec and gets embededin it. Find

the amplitude of oscillation (neglect gravity)

(1) 10 cm *(2) 5cm* *(3) 3 cm* *(4) 0.2 cm*

SOLUTION :

By conservation of linear momentum in the collision

mv = (m+ M) V

$$\Rightarrow V = \frac{mv}{m+M} = \frac{0.5 \times 3}{(1+0.5)} = 1m/sec$$

Now just after collision the system will have

KE = $\frac{1}{2}$ *(m+M) V^2 at equislibrium position .*

So after collision by conservation of mechanical energy $KE_{max} = PE_{max}$

$$\frac{1}{2}(m+M)v^2 = \frac{1}{2}KA^2$$

$$\Rightarrow A = V\sqrt{\left(\frac{m+M}{k}\right)} = 1\sqrt{\frac{1.5}{600}} = \frac{1}{20}m = 5cm$$

3. A block is kept on a rough horizontal plank. The coefficient of friction between block and the plank is 1/2. plank is undegoing SHM of angular frequency 10 rad/s.Find the maximum amplitude of plank in which the block does not slip over the plank(g = 10 m/s²).

(1) 0.5m *(2) 5cm* *(3) 5 m* *(4) 0.2 cm*

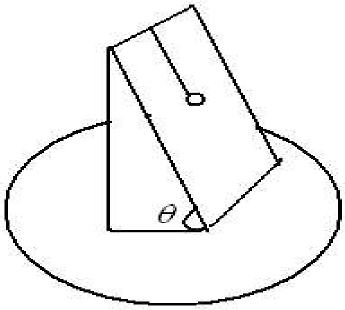
SOLUTION :

Maximum acceleration in SHM is $a_{\max} = \omega^2 A$ this will be provided to the block by friction .

Hence, $a_{\max} = \mu g$ or $\omega^2 A = \mu g$

or $a = \frac{\mu g}{\omega^2} = \frac{\left(\frac{1}{2}\right)(10)}{(10)^2}$ = 0.05m = 5 cm

4. *In the diagram shown find the time period of pendulum for small oscillations*



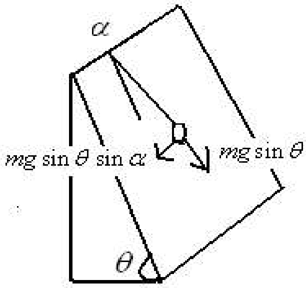
(1) $T = \pi\sqrt{\dfrac{l}{g\sin\theta}}$

(2) $T = 2\pi\sqrt{\dfrac{l}{g\tan\theta}}$

(3) $T = 2\pi\sqrt{\dfrac{l}{g\sin\theta}}$

(4) $T = 2\pi\sqrt{\dfrac{l}{g\cos\theta}}$

SOLUTION :



For smaller values of α, $\sin\alpha \approx \alpha$

$$f = mg\sin\theta.\alpha \Rightarrow f = -mg\sin\theta\frac{y}{l}$$

$$a = -\frac{g\sin\theta}{l}.y$$

$$\omega = \sqrt{\frac{g\sin\theta}{l}}$$

$$\Rightarrow \frac{T}{T_o} = \sqrt{\frac{l}{\mu^2 + 1}}$$

5. The trolly car having simple pendulum decelerated by friction. In consequence, the pen dulum has time period T. If T_o is time period of the simple pendulum in the absence of any acceleration of the trolly car, the value of $\frac{T}{T_o}$ is ---

(1) $\frac{T}{T_o} = \sqrt{\frac{l}{\sqrt{\mu^2+1}}}$ (2) $\frac{T}{T_o} = \sqrt{\frac{l}{\mu}}$

(3) $\frac{T}{T_o} = \sqrt{\frac{l}{\mu+1}}$ (4) $\frac{T}{T_o} = \sqrt{\frac{l}{\mu^2+1}}$

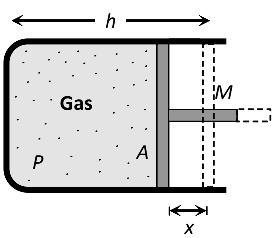
SOLUTION :

$$T = 2\pi\sqrt{\frac{l}{\sqrt{a^2+g^2}}} \;;\; a = \mu g$$

$$T = 2\pi\sqrt{\frac{l}{g\sqrt{\mu^2+1}}}$$

$$T = T_o\sqrt{\frac{1}{\sqrt{\mu^2+1}}} \Rightarrow \frac{T}{T_o} = \sqrt{\frac{l}{\sqrt{\mu^2+1}}}$$

6. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$ (b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$ (d) $T = 2\pi\sqrt{MPhA}$

SOLUTION :

Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (i.e. isothermal)

$$P_1V_1 = P_2V_2 \Rightarrow PV = (P+\Delta P)(V-\Delta V)$$

$$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$$

$$\Rightarrow \Delta P.V - P.\Delta V = 0 \text{ (neglecting } \Delta P.\Delta V)$$

$$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M.

Hence $F = \Delta P.A = \frac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$$

7.. *Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, and each time their displacement is half of their amplitude. The phase difference between them is*

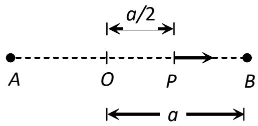
(a)	30°	(b)	60°
(c)	90°	(d)	120°

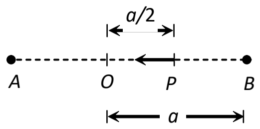
ANSWER : (d)

SOLUTION :

. According to the question of SHM $y = a\sin(\omega t + \varphi_0)$

Given $y = \frac{a}{2}$ $\Rightarrow$ $\frac{a}{2} = a\sin(\omega t + \phi_0) \Rightarrow (\omega t + \phi_0) = \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

if $\phi = \frac{\pi}{6}$ $\Rightarrow$ 

$\Rightarrow$ *Particle is at point P and it is going towards B* if $\phi = \frac{5\pi}{6}$ $\Rightarrow$ 

$\Rightarrow$ Particle is at point Pand it is going towards O

So phase difference $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

8.. The function $\sin^2(\omega t)$ represents

(a) A simple harmonic motion with a period $2\pi/\omega$

(b) A simple harmonic motion with a period π/ω

(c) A periodic but not simple harmonic motion with a period $2\pi/\omega$

(d) A periodic but not simple harmonic, motion with a period π/ω

ANSWER : (d)

SOLUTION : Given $y = \sin^2 \omega t$ $\Rightarrow$ $y = \frac{1 - \cos 2\omega t}{2}$

$\Rightarrow$ $y = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

Angular velocity $= 2\omega$

Period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

The given function is not satisfying the standard differential equation of S.H.M.

$\frac{d^2y}{dx^2} = -\omega^2 y$.

Hence it represents periodic motion but not S.H.M.

9. Which of the following function represents a simple harmonic oscillation

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^2 \omega t$

(c) $\sin \omega t + \sin 2\omega t$ (d) $\sin \omega t - \sin 2\omega t$

ANSWER : (a)

SOLUTION :

The function $\sin \omega t - \cos \omega t$. only satisfies the standard differential equation of S.H.M.

$$\frac{d^2y}{dx^2} = -\omega^2 y.$$

Hence it $\sin\omega t - \cos\omega t.$ represents S.H.M.

10. The displacement of a particle varies with time as $x = 12\sin\omega t - 16\sin^3\omega t$ (in cm). If its motion is S.H.M., then its maximum acceleration is

(a) $12\,\omega^2$ (b) $36\,\omega^2$ (c) $144\,\omega^2$ (d) $\sqrt{192}\,\omega^2$

ANSWER : (b)

SOLUTION :

Displacement $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$

By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

Displacement $x = 4[\sin 3\omega t]$

equation for maximum acceleration $|A_{max}| = \omega^2 a$

$\therefore$ maximum acceleration $A_{max} = (3\omega)^2 \times 4 = 36\omega^2$

11. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k[x]^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

(a) Proportional to $\frac{1}{\sqrt{a}}$ (b) Independent of a

(c) Proportional to $\sqrt{a}$ (d) Proportional to $a^{3/2}$

ANSWER : (a)

SOLUTION :

For concervSOLUTION :atieve force $F = -\frac{dU}{dx}$

Given Potential Energy $U = k|x|^3$

Then $F = -\frac{dk|x|^3}{dx}$

$F = -3k|x|^2$ ...(i)

Also, for SHM $F = -m\omega^2 x$ ...(ii)

From equation (i) & (ii) $\omega = \sqrt{\frac{3kx}{m}}$

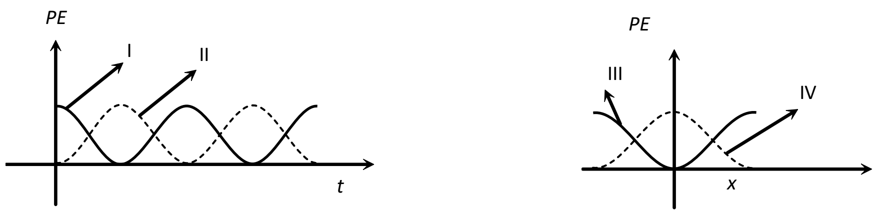
Time period $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{m}{3kx}}$$

$$T = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}}$$

$$T \propto \frac{1}{\sqrt{a}}.$$

12. *For a particle executing S.H.M. the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time t and displacement x*



(a) I, III *(b) II, IV* *(c) II, III* *(d) I, IV*

*ANSWER : **(a)***

SOLUTION :

Here displacement x is given by $x = A\cos\omega t$.

So Potential energy is

1) minimum (in this case zero) at mean position (x = 0)

2) maximum at extreme position $(x = \pm A)$.

From

At time t = 0, x = A,

hence potential should be maximum. Therefore graph I is correct. Further in graph III. Potential energy is minimum at x = 0, hence this is also correct

13. A point mass oscillates along the x-axis according to the law $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then:-

(1) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$
(2) $A = x_0, \delta = -\frac{\pi}{4}$
(3) $A = x_0\omega^2, \delta = \frac{\pi}{4}$
(4) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$

ANSWER : **(1**)

SOLUTION : Acceleration of SHM particle $a = \frac{d^2x}{dt^2}$

But displacement $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

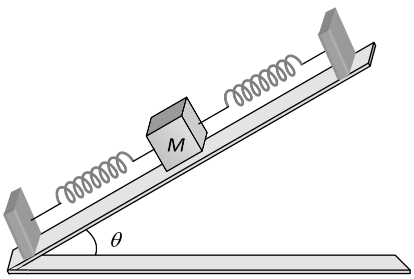
$$a = \frac{d^2\left(x_0 \cos\left(\omega t - \frac{\pi}{4}\right)\right)}{dt^2}$$

On solving we get $a = \omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$ ....(1)

But given acceleration $a = A\cos(\omega t + \delta)$ ....(2)

From (1) and (2) $A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$

14. On a smooth inclined plane, a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant K, the period of oscillation of the body (assuming the springs as massless) is



(a) $2\pi\left(\frac{m}{2K}\right)^{1/2}$ (b) $2\pi\left(\frac{2M}{K}\right)^{1/2}$ (c) $2\pi\frac{Mg\sin\theta}{2K}$ (d) $2\pi\left(\frac{2Mg}{K}\right)^{1/2}$

ANSWER : **(a**)

SOLUTION :

Time period doesnot depends on inclination ….

Hence time period of arrangement is $T = 2\pi\left(\frac{M}{K_{eff}}\right)^{1/2}$

Here two springs are arranged in parellel combination

Hence $K_{eff} = 2K$

Time period becomes $T = 2\pi\left(\frac{m}{2K}\right)^{1/2}$

15. The displacement of a particle executing S.H.M. is given by $x = 0.01\, sin\, 100\pi(t + 0.05)$. The time period is

(1) 0.01 s (2) 0.02 s (3) 0.1 s (4) 0.2 s

ANSWER : **(2**)

SOLUTION :

$x = 0.01\, sin\, 100\pi(t + 0.05)$

Here w = 100 π s

Time period $T = \frac{2\pi}{\omega}$ $\Rightarrow$ $T = \frac{2\pi}{100\pi}$ $\Rightarrow$ $T = \frac{1}{50} \Rightarrow$ T= 0.02 s

PREVIOUS MAINS QUESTIONS

1. The position co-ordinates of a particle moving in a 3-D coordinate system is given by **[9 Jan 2019, II]**

$x = a\cos\omega t$ $\quad y = a\sin\omega t$ $\quad$ **and** $z = a\omega t$

The speed of the particle is:

(1) $\sqrt{2}a\omega$ (2) $a\omega$

(3) $\sqrt{3}a\omega$ (4) $2a\omega$

SOLUTION :**(1) Here,** $v_x = -a\omega\sin\omega t, v_y = a\cos\omega t$ and $v_z = a\omega \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$$\Rightarrow v = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$$

2. Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.

$x(t) = A\sin(at+\delta) \quad y(t) = B\sin(bt)$

Identify the correct match below **[Online April 15, 2018]**

SOLUTION

(3) From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$$x = A\sin(at+\delta)$$

$$y = B\sin(bt+r)$$

Clearly $A \neq B$ hence ellipse.

(1) Parameters: $A = B, a = 2b; \delta = \frac{\pi}{2};$ Curve: Circle

(2) Parameters: $A = B, a = b; \delta = \frac{\pi}{2};$ Curve: line

(3) Parameters: $A \neq B, a = b; \delta = \frac{\pi}{2};$ Curve: Ellipse

(4) Parameters: $A \neq B, a = b; \delta = 0;$ Curve: Parabola

3. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $10 s^{-1}$. At, t = 0 the displacement is 5 m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$ **[Online April 8, 2017]**

(1) 500 m/s^2 (2) $500\sqrt{2} m/s^2$ (3) $750 m/s^2$ (4) $750\sqrt{2} m/s^2$

SOLUTION

(2) Maximum velocity in SHM, $v_{max} = a\omega$

Maximum acceleration in SHM, $A_{max} = a\omega^2$

where a and ω are maximum amplitude and angular frequency.

Given that, $\frac{A_{max}}{v_{max}} = 10$

i.e., $\omega = 10 s^{-1}$

Displacement is given by

$x = a\sin(\omega t + \pi/4)$

At $t = 0, x = 5$

$5 = a\sin \pi/4$

Maximum acceleration $A_{max} - a\omega^2 = 500\sqrt{2} m/s^2$

4. A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : [2016]

(1) $A\sqrt{3}$ (2) $\frac{7A}{3}$ (3) $\frac{A}{3}\sqrt{41}$ (4) 3A

SOLUTION (2) We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A' = final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

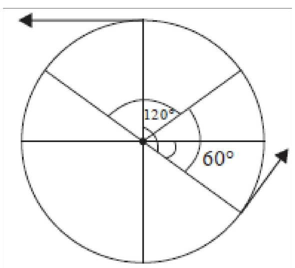
$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9} \qquad \therefore A' = \frac{7A}{3}$$

5. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time t = 0 one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is: **[Online April 9, 2016]**

(1) $\frac{5T}{6}$ (2) $\frac{T}{3}$ (3) $\frac{T}{4}$ (4) $\frac{T}{6}$

SOLUTION (at time t = 0)



Angel covered to meet $\theta = 60^0 = \frac{\pi}{3}$ rad.

If they cross each other at time t then

$$t = \frac{\theta}{2\pi} = \frac{\pi}{3\times 2\pi}T = \frac{T}{6}$$

6. **A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force F = sin *t* N. If the oscillator is at rest in its equilibrium position at *t* = 0, its position at later times is proportional to : [Online April 10, 2015]**

(1) $\sin t + \frac{1}{2}\cos 2t$ (2) $\cos t + \frac{1}{2}\sin 2t$

(3) $\sin t + \frac{1}{2}\sin 2t$ (4) $\sin t + \frac{1}{2}\sin 2t$

SOLUTION (3) As we know,

$$F = ma \Rightarrow a \propto F$$

Or, $a \propto \sin t$

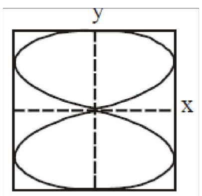
$$\Rightarrow \frac{dv}{dt} \propto \sin t$$

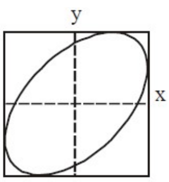
$$\Rightarrow \int_0^0 dv \propto \int_0^t \sin t dt$$

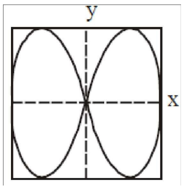
$$V \propto \cos t + 1$$

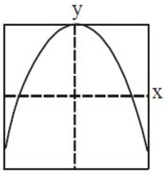
$$\int_0^x dx = \int_0^t (-\cot t + 1)\, dt$$

7. ***x* and *y* displacements of a particle are given as ω (*t*) = *a* sin ωt and *y* (*t*) = *a* sin 2ω*t*. Its trajectory will look like : [Online April 10, 2015]**

(1) 

(2) 

(3) 

(4) 

SOLUTION (3) At $t=0, x(t)=0; y(t)=0$

$x(t)$ is a sinusoidal funciton

At $t=\frac{\pi}{2\omega}; x(t)=a$ and $y(t)=0$

Hence trajectory of particle will look like as (3)

8. A body is in simple harmonic motion with time period half second (T = 0.5 s) and amplitude one cm (A = 1 cm). Find the average velocity in the interval in which it moves form equilibrium position to half of its amplitude.

[Online April 19, 2014]

(1) 4 cm/s (2) 6 cm/s

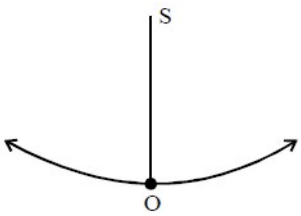
(3) 12 cm/s (4) 16 cm/s

SOLUTION

8. (3) Given: Time period, T = 0.5 sec

Amplitude, A = 1 cm

Average velocity in the interval in which body moves from equilibrium to half of its amplitude, v = ?



Time taken to a displacement A/2 where A is the amplitude of oscillation from the mean position 'O' is $\frac{T}{12}$

Therefore, $s=\frac{A}{2}=\frac{1}{2}cm$

$\therefore$ Average velocity, $v=\frac{\frac{A}{2}}{t}=\frac{\frac{1}{2}}{\frac{0.5}{12}}=12\,cm/s$

9.

9.Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants? [Online April 12, 2014]

(1) $a+bx-cx^2$ (2) bx^2

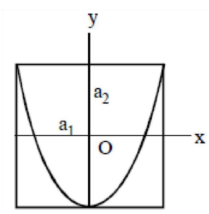
(3) $a-bx+cx^2$ (4) $-bx$

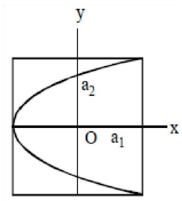
SOLUTION (4) In linear S.H.H., the restoring force acting on particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

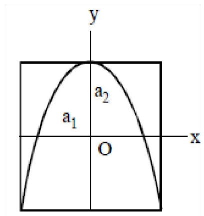
i.e., $F \propto x$

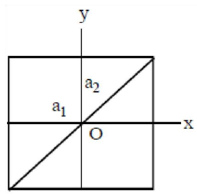
or F = - bx where b is a positive constant.

10. A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by; x = a1 cos ωt and y = a2 cos 2 ωt traces a curve given by: [Online April 9, 2014]

(1) 

(2) 

(3) 

(4) 

SOLUTION (2) Two perpendicular S.H.Ms are $x = a_1 \cos \omega t$ .....(1)

And $y = a_2 2 \cos \omega t$ .....(2)

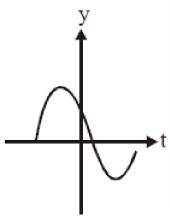
Form eqation (1)

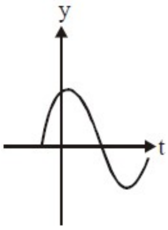
$$\frac{x}{a_1} = \cos wt$$

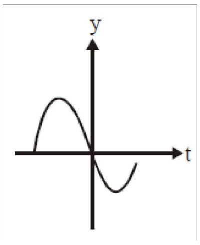
And form equation (2)

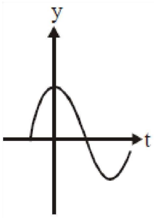
$$\frac{y}{a_2} = 2\cos \omega t \quad \therefore y = 2\frac{a_2}{a_1}x$$

11. The displacement $y(t) = A\sin(\omega t+\phi)$ of a pendulum for $\phi = \frac{2\pi}{3}$ is correctly represented by [Online May 19, 2012]

(1) 

(2) 

(3) 

(4) 

SOLUTION (a) Displacement $y(t) = A\sin(wt+\phi)[Given]$

For $\phi = \frac{2\pi}{3}$

At $t = 0; y = A\sin\phi = A\sin\frac{2\pi}{3}$

$= A\sin 120^0 = 0.87A\left[\because \sin 120^0 \approx 0.866\right]$

Graph (1) depicts $y = 0.87A$ at $t = 0$

12. Two particles are executing simple harmonic motion of the same amplitude *A* and frequency w along the *x*-axis. Their mean position is separated by distance X_0(X_0 > *A*). If the maximum separation between them is (X_0 + *A*), the phase difference between their motion is: [2011]

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

SOLUTION 1) Let, $x_1 = A\sin\omega t$ and $x_2 = A\sin(\omega t+\phi)$

$x_2 - x_1 = 2A\cos\left(\omega t+\frac{\phi}{2}\right)\sin\frac{\phi}{2}$

The above equation is SHM with amplitude $2A\sin\frac{\phi}{2}$

$$\therefore 2A\sin\frac{\phi}{2} = A$$

$$\Rightarrow \sin\frac{\phi}{2} = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$$

13. A mass *M*, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass *M* passes through its mean position then a smaller mass *m* is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(1) $\frac{M+m}{M}$ (2) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$ (3) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ (4) $\frac{M}{M+m}$

SOLUTION (3) At mean position, F net = 0

Therefore, by principal of conseruation of linear momentum

$$\therefore Mv_1 = (M+m)v_2$$

$$Mw,a,=(M+m)w_2a_2$$

$$MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{m+M}}$$

$$\therefore \left(V = A\sqrt{\frac{k}{M}}\right)$$

$$\Rightarrow A_1\sqrt{M} = A_2\sqrt{M+m}$$

$$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$$

14. A point mass oscillates along the x-axis according to the law *x* = *x*0 cos(ω*t* - p / 4) . If the acceleration of the particle is written as *a* = *A* cos(ω*t* + δ) ,then

[2007]

(1) $A = x_0\omega^2, \delta = 3\pi/4$ (2) $A = x_0, \delta = -\pi/4$

(3) $A = x_0\omega^2, \delta = \pi/4$ (4) $A = x_0\omega^2, \delta = -\pi/4$

SOLUTION

(1) Given,

Displacement, $x = x_0 \cos(\omega t - \pi/4)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration,

$$a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0\omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= x_0\omega^2 \cos\left(\omega t + \frac{\pi}{4}\right) \qquad ....(1)$$

Acceleration, $a = A\cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$

15. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency w. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time (a) at the mean position of the platform [2006]

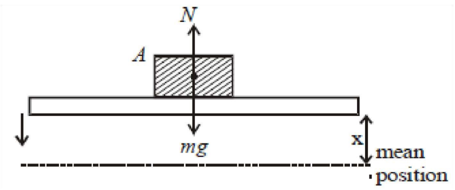
(1) at the mean position of the platform

(2) for an amplitude of $\frac{g}{\omega^2}$

(3) for an amplitude fo $\frac{g^2}{\omega^2}$

(4) at the highest position of the plastform

SOLUTION (2) For block A to move in SHM



$$mg - N = m\omega^2 x$$

Where x is the distance from mean position

For block to leave contact $N = 0$

$$\Rightarrow mg = m\omega^2 x \Rightarrow x = \frac{g}{\omega^2}$$

16. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is [2006]

(1) 0.01 s (2) 10s (3) 0.1 s (4) 100 s

SOLUTION (1) Maximum velocity,

$$v_{\max} = a\omega$$

Here, a = amplitude of SHM

ω = angular velocity of SHM

$$v_{\max} = a \times \frac{2\pi}{T} \therefore \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\Rightarrow T = \frac{2\pi a}{v_{\max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01 s$$

17. The function sin2 (ωt) represents **[2005]**

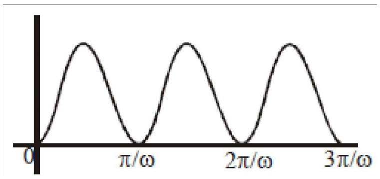
(1) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$

(2) a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$

(3) a simple harmonic motion with a period $\frac{\pi}{\omega}$

(4) a simple harmonic motion with a period $\frac{2\pi}{\omega}$

SOLUTION (1) Clearly $\sin 2\omega t$ is a periodic function with period $\frac{\pi}{\omega}$



For SHM $\frac{d^2y}{dt^2} \propto -y$

$$y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t$$

$$v = \frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t = 2\omega \sin \omega t \cos \omega t$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t$$

$$v = \frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t = 2\omega \sin \omega t \cos \omega t$$

$$= \omega \sin 2\omega t$$

Aceleration, $a = \frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$ which is not proportional to –y. Hence, it is not in SHM.

18. Two simple harmonic motions are represented by the equations y_1 = 0.1 sin $\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(1) $\frac{\pi}{3}$ (2) $\frac{-\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{-\pi}{3}$

SOLUTION (2) Velocity of particle 1,

$$v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$$

Velocity of particle 2,

$$v_2 = \frac{dy_2}{dt} = -0.1\pi \sin \pi t = 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$$

$\therefore$ Phase difference of velocity of particle 1 with respect to the velocity of particle 2 is

$$= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$$

19. Two particles *A* and *B* of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of *A* and *B* is [2003]

(1) $\sqrt{\frac{k_1}{k_2}}$ (2) $\frac{k_2}{k_1}$ (3) $\sqrt{\frac{k_2}{k_1}}$ (4) $\frac{k_1}{k_2}$

SOLUTION (3) Maximum velocity during SHM, $v_{max} = A\omega$

But $k = m\omega^2$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

$\therefore$ Maximum velocity ofh te body is SHM $= A\sqrt{\frac{k}{m}}$

As maximum velocities are equal

$$\therefore A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\Rightarrow A_1\sqrt{k_1} = A_2\sqrt{k_2} \qquad \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

20. The displacement of a particle varies according to the relation *x* = 4(cosp*t* + sin p*t*). The amplitude of the particle is [2003]

(1) – 4 (2) 4 (3) $4\sqrt{2}$ (4) 8

SOLUTION (3) Diplacement, $x = 4(\cos \pi t + \sin \pi t)$

$$= \sqrt{2} \times 4\left(\frac{\sin \pi t}{\sqrt{2}} + \frac{\cos \pi t}{\sqrt{2}}\right)$$

$$= 4\sqrt{2}\left(\sin \pi t \cos 45^0 + \cos \pi t \sin 45^0\right)$$

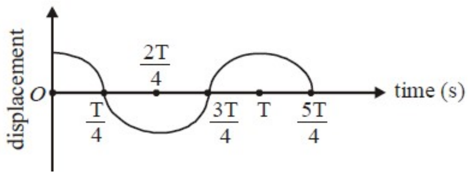
$x = 4\sqrt{2}\sin\left(\pi t + 45^0\right)$

On comparing it with standard equation $x = A\sin(\omega t + \phi)$

We get $A = 4\sqrt{2}$

Topic-2: Energy in Simple Harmonic Motion

21. The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion? [Sep. 02, 2020 (II)]

(1) The force is zero at $t = \frac{3T}{4}$

(2) The acceleration is maximum at t = T

(3) The speed is maximum at $t = \frac{T}{4}$

(4) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (1), (2) and (4) (2) (2), (3) and (4)
(3) (1), (2) and (3) (4) (1) and (4)

SOLUTION(3) From graph equation of SHM

$X = A\cos\omega t$

(1) At $\frac{3T}{4}$ particle is at mean position.

$\therefore$ Acceleration = 0, Force = 0
(2) At T particle again at extreme position so acceleration ismaximum.

(3) At $t = \frac{T}{4}$, particle again at extreme position so velocity is maximum .

Acceleration = 0
(4) When KE = PE

$$\Rightarrow \frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

Here, A = amplitude of SHM
x = displacement from mean postion

$$\Rightarrow A^2 = 2x^2 \Rightarrow x = \frac{+A}{\sqrt{2}}$$

$$\Rightarrow \frac{A}{\sqrt{2}} = A\cos\omega t \Rightarrow t = \frac{T}{2}$$

$\therefore x = -A$ which is not possible

$\therefore x = -A$ and 3 are correct.

22. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The t = 210s will be: [11 Jan 2019, I]

(1) $\frac{1}{9}$ (2) 1 (3) 2 (4) $\frac{1}{3}$

SOLUTION (4) Kinetic energy, $k = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$

Potential energy, $U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$

$$\frac{k}{U} = \cot^2 \omega t = \cot^2 = \cot^2 \frac{\pi}{90}(210) = \frac{1}{3}$$

23. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2.

(1) $K_2 = 2K_1$ (2) $K_2 = \frac{K_1}{2}$

(3) $K_2 = \frac{K_1}{4}$ (4) $K_2 = K_1$

SOLUTION

(1) $K = \frac{1}{2}m\omega^2 x^2$

$$\Rightarrow K_{max} = \frac{1}{2}m\omega^2 A^2$$

$$A = L\theta$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\Rightarrow K = \frac{1}{2m.\frac{g}{L}.L^2\theta^2}$$

$$= \frac{1}{2}mgL\theta^2$$

$$\therefore \frac{K_1}{K_2} = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$$

$$\therefore \frac{K_1}{K_2} = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$$

24. A particle is executing simple harmonic motion (SHM) of amplitude A, along the *x*-axis, about *x* = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be:

[9 Jan 2019, II]

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

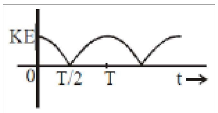
SOLUTION (3) Potential energy $(U) = \frac{1}{2}kx^2$

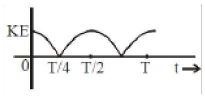
Kinetic energy $(K) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

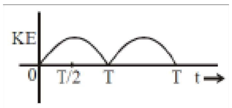
$$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

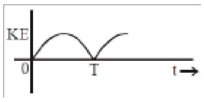
$$\Rightarrow x^2 = A^2 \quad \text{or,} \quad x = \pm\frac{A}{\sqrt{2}}$$

25. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like: [2017]

(1) 

(2) 

(3) 

(4) 

SOLUTION (2) For a particle executing SHM

At mean postion; $t=0, \omega t=0, y=0, V=V_{\max}=a\omega$

$$\therefore K.E.=KE_{\max}=\frac{1}{2}m\omega^2 a^2$$

At extreme postion : $t=\frac{T}{4}, \omega t=\frac{\pi}{2}, y=A, V=V_{\min}=0$

$$\therefore K.E.=KE_{\min}=0$$

Kinetic energy in SHM, $KE=\frac{1}{2}m\omega^2\left(a^2-y^2\right)$

$$=\frac{1}{2}m\omega^2 a^2\cos^2\omega t$$

Hence graph (2) correctly depicts kinetic energy time graph.

26. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm–1 and oscillates in a medium of constant 10^{-2} kg s^{-2}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to : [Online April 9, 2017]

(1) 2s (2) 3.5 s (3) 5 s (4) 7s

SOLUTION(2) Since system dissipates its energy gradually, and hence amplitude will also decreases with time according to

$$a=a_0 e^{-bt/m} \qquad ....(i)$$

$\because$ Energy of virbration drop to half of its initial value

$\left(E_0\right)$, as $E\propto a^2 \Rightarrow a\propto\sqrt{E}$

$$a=\frac{a_0}{\sqrt{2}} \Rightarrow \frac{bt}{m}=\frac{10^{-2}t}{0.1}=\frac{t}{10}$$

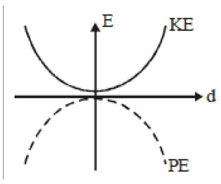
From equation (i),

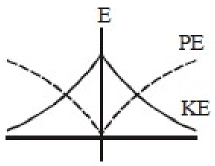
$$\frac{a_0}{\sqrt{2}} = a_0 e^{-t/10}$$

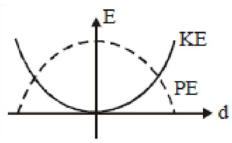
$$\frac{1}{\sqrt{2}} = e^{-t/10} \text{ or } \sqrt{2} = e^{\frac{t}{10}}$$

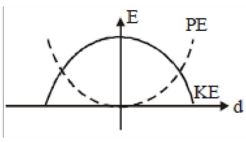
In $\sqrt{2} = \frac{t}{10}$ $\quad \therefore\ t = 3.5$ seconds

27. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*)
[2015]

(1) 

(2) 

(3) 

(4) 

SOLUTION (4) $K.E = \frac{1}{2}k\left(A^2 - d^2\right)$

And P.E. $= \frac{1}{2}kd^2$

At mean position d = 0. At extreme positions d = A

28. A pendulum with time period of 1s is losing energy. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s–1) is :
[Online April 11, 2015]

(1) $\frac{1}{2}$ (2) $\frac{1}{30}\ln 3$ (3) 2 (4) $\frac{1}{15}\ln 3$

SOLUTION (4) As we know, $E = E_0 e^{-\frac{bt}{m}}$

$$15 = 45e^{\frac{b15}{m}}$$

[As no. of oscillations = 15 so t = 15 sec]

$$\frac{1}{3} = e^{-\frac{b15}{m}}$$

Taking log on both sides

$$\frac{b}{m} = \frac{1}{15}\ell n 3$$

29. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements. If two springs S_1 and S_2 of force constants k_1 and k_2 respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount work done on S_1

Statement 2 : $k_1 < k_2$ **[2012]**

(1) Statement 1 is false, Statement 2 is true.

(2) Statement 1 is true, Statement 2 is false.

(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1

(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

SOLUTION (2) Work done, $w = \frac{1}{2}kx^2$

Work done by spring $S_1, w_1 = \frac{1}{2}k_1x^2$

Work done by spring $S_2, w_2 = \frac{1}{2}k_2x^2$

Since $w_1 > w_2$ Thus $(k_1 > k_2)$

30. A particle of mass m executes simple harmonic motion with amplitude *a* and frequency n. The average kinetic energy during its motion from the position of equilibrium to the end is [2007]

(1) $2\pi^2 ma^2v^2$ (2) $\pi^2 ma^2v^2$

(3) $\frac{1}{4}ma^2v^2$ (4) $4\pi^2 ma^2v^2$

SOLUTION (2) The kinetic energy of a particle executing S.H.M. at any instant t is given by

$K = \frac{1}{2} ma^2 \omega^2 \sin^2 \omega t$

Where, m = mass of particle
A = amplitude
ω = angular frequency
T = time

The average value of $\sin^2 \omega t^2$ over a cycle is $\frac{1}{2}$

$\therefore KE = \frac{1}{2} m\omega^2 a^2 \left(\frac{1}{2}\right) \quad \left(\because \langle \sin^2 \rangle = \frac{1}{2}\right)$

$= \frac{1}{4} m\omega^2 a^2 = \frac{1}{4} ma^2 (2\pi v)^2 \quad (\because \omega = 2\pi v)$

or, $\langle K \rangle = \pi^2 ma^2 v^2$

31. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? [2006]

(1) $\frac{1}{6} s$ (2) $\frac{1}{4} s$ (3) $\frac{1}{3} s$ (4) $\frac{1}{12} s$

SOLUTION (1) K.E. of a body undergoing SHM is given by $K.E. = \frac{1}{2} ma^2 \omega^2 \cos^2 \omega t$

Here, a = amplitude of SHM
$\omega =$ Angular velocity of SHM

Total energy in S.H.M $= \frac{1}{2} ma^2 \omega^2$

Given K.E. = 75% T.E.

$\frac{1}{2} ma^2 \omega^2 \cos^2 \omega t = \frac{75}{100} \times \frac{1}{2} ma^2 \omega^2$

$\Rightarrow 0.75 = \cos^2 \omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6} s$

32. The total energy of a particle, executing simple harmonic motion is [2004]

(1) independent of x (2) $\propto x^2$

(3) $\propto x$ (4) $\propto x^{1/2}$

SOLUTION (1) At any instant the total energy in SHM is $\frac{1}{2}kA_0^2$ = constant,

Where A_0 = amplitude

k = spring constant
hence total energy is independent of x.

33. A body executes simple harmoni c motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement *x*. Which of the following statements is true? [2003]

(1) K.E. is maximum when $x = 0$
(2) T.E is zero when $x = 0$
(3) K.E is maximum when x is maximum
(4) P.E is maximum when $x = 0$

SOLUTION (1) K.E. of simple harmonic motion $= \frac{1}{2}m\omega^2\left(a^2 - x^2\right)$

34. In a simple harmonic oscillator, at the mean position [2002]

(1) kinetic energy is minimum, potential energy is maximum
(2) both kinetic and potential energies are maximum
(3) kinetic energy is maximum, potential energy is minimum
(4) both kinetic and potential energies are minimum

SOLUTION (3) The kinetic energy (K.E.) of particle in SHM is given by, $K.E = \frac{1}{2}k\left(A^2 - x^2\right);$

Potential energy of particle in SHM is $U = \frac{1}{2}kx^2$

Where A = amplitude and $k = m\omega^2$

x = displacement from the mean postion

At the mean position $x = 0$

$\therefore K.E. = \frac{1}{2}kA^2$ = Maximum

And U = 0

Topic-3: Time Period, Frequency, Simple Pendulum and Spring Pendulum

35. An object of mass *m* is suspended at the end of a massless wire of length *L* and area of cross-section, *A*. Young modulus of the material of the wire is *Y*. If the

mass is pulled down slightly its frequency of oscillation along the vertical direction is: [Sep. 06, 2020 (I)]

(1) $f=\frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (2) $f=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

(3) $f=\frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (4) $f=\frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

SOLUTION (2) An elastic wqire can be treated as a spring and its spring constant.

$$k=\frac{YA}{L} \qquad \left[\because Y=\frac{F}{A}\bigg/\frac{\Delta l}{l_0}\right]$$

Frequency of oscillation,

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

36. When a particle of mass m is attached to a vertical spring of spring constant *k* and released, its motion is described by *y* (*t*) = y_0 sin2wt, where '*y*' is measured from the lower end of unstretched spring. Then w is : [Sep. 06, 2020 (II)]

(1) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{y_0}}$ (3) $\sqrt{\frac{g}{2y_0}}$ (4) $\sqrt{\frac{2g}{y_0}}$

SOLUTION (3) $y=y_0\sin^2\omega t$

$$\Rightarrow y=\frac{y_0}{2}(1-\cos 2\omega t)\left(\because \sin^2\omega t=\frac{1-\cos 2\omega t}{2}\right)$$

$$\Rightarrow y-\frac{y_0}{2}=\frac{-y_0}{2}\cos 2\omega t$$

$$\Rightarrow y=\frac{y_0}{2}=\frac{-y_0}{2}\cos 2\omega t$$

$$\Rightarrow y=A\cos 2\omega t$$

$\therefore$ Amplitude $=\frac{y_0}{2}$

Angular velocity $=2\omega$

For equilibrium of mass, $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$$

37. A block of mass *m* attached to a massless spring is performing oscillatory motion of amplitude '*A*' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become *fA*. The value of *f* is : [Sep. 03, 2020 (II)]

(1) $\frac{1}{\sqrt{2}}$ (2) 1 (3) $\frac{1}{2}$ (4) $\sqrt{2}$

SOLUTION(1) Potential energy of spring $= \frac{1}{2}kx^2$

Here, x = distance of block from mena postion,
k = spring constant

At mean postion, potential energy $= \frac{1}{2}kA^2$

At equilibrium position, half of the mass of block breaks off, so its potential energy becomes half

Remaining energy $= \frac{1}{2}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}kA'^2$

Here, A'= New distance of block from mena postion $\Rightarrow A' = \frac{A}{\sqrt{2}}$

39. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : [9 April 2019 I]

(1) $2T\sqrt{\frac{1}{10}}$ (2) $2T\sqrt{\frac{1}{14}}$

(3) $4T\sqrt{\frac{1}{15}}$ (4) $4T\sqrt{\frac{1}{14}}$

SOLUTION (3) $T = 2\pi\sqrt{\frac{l}{g}}$

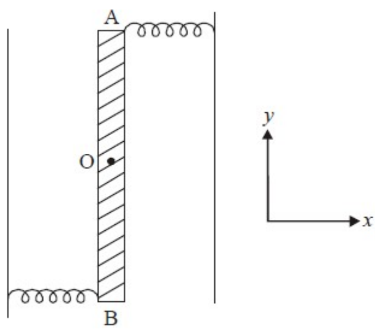
When immersed non viscous liquid

$$a_{mt} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$$

Now $T' = 2\pi\sqrt{\frac{l}{0_{net}}} = 2\pi\sqrt{\frac{l}{\sqrt{\frac{15g}{16}}}} = \frac{4}{\sqrt{15}}T$

40. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length *l* and mass m. The rod is pivoted at its centre 'O' and can rotate frreely in horizontal plane. The other ends of two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:

[12 Jan 2019, I]

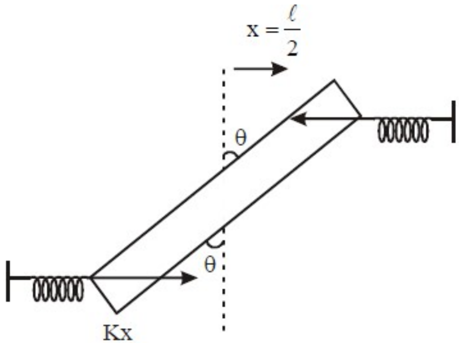


(1) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

SOLUTION (3) Net torque due to spring force:

$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$



$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta \quad \left[let\ C = \frac{K\ell^2}{2}\right]$$

$\Rightarrow$ So, frequency of resulting oscillations

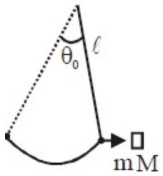
$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

41. A simple pendulum, made of a string of length *l* and a bob of mass m, is released from a small angle θ_1. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. The M is given by : [12 Jan 2019, I]

(1) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (2) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

(3) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (4) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

SOLUTION (2)



Velocity before collision $v=\sqrt{2g\ell\left(1-\cos\theta_0\right)}$

Velocity after colistion

$v_1=\sqrt{2g\ell\left(1-\cos\theta_1\right)}$

Using momentum conservation

$mv = MV_m - mV_1$

$$m\sqrt{2g\ell\left(1-\cos\theta_0\right)} = MV_m - m\sqrt{2g\ell\left(1-\cos\theta\right)}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right\} = MV_m$$

And $e = 1 = \dfrac{V_m + \sqrt{2g\ell\left(1-\cos\theta_1\right)}}{\sqrt{2g\ell\left(1-\cos\theta_0\right)}}$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right) = V_m \quad \text{...(i)}$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right) = MV_M \quad \text{...(ii)}$$

Divinding (ii) by (i) we get

$$\frac{\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)} = \frac{M}{m}$$

By compounendo and dividend rule

$$\frac{m-M}{m+M} = \frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m} = \frac{\theta_0-\theta_1}{\theta_0+\theta_1} \Rightarrow M = m\frac{\theta_0-\theta_1}{\theta_0+\theta_1}$$

42. A simple harmonic motion is represented by : $y=5\left(\sin 3\pi t+\sqrt{3}\cos 3\pi t\right)cm$ The amplitude and time period of the motion are :
[12 Jan 2019, II]

(1) $10cm, \frac{2}{3}s$ (2) $10cm, \frac{3}{2}s$ (3) $5cm, \frac{3}{2}s$ (4) $5cm, \frac{2}{3}s$

SOLUTION(1) Given: $y=5\left[\sin(3\pi t)+\sqrt{3}\cos(3\pi t)\right]$

$$\Rightarrow y=10\sin\left(3\pi t+\frac{\pi}{3}\right)$$

$\therefore$ Amplitude = 10 cm

Time period, $T=\frac{2\pi}{\omega}=\frac{2\pi}{3\pi}=\frac{2}{3}s$

43. A simple pendulum of length 1m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by: [11 Jan 2019, II]

(1) $10^{-3} rad/s$ (2) $1 rad/s$

(3) $10^{-1} rad/s$ (4) $10^{-5} rad/s$

SOLUTION (1) Angular frequency of pendulum $\omega=\sqrt{\frac{g}{\ell}}$

$\therefore$ Relative change in angular frequency

$\frac{\Delta\omega}{\omega}=\frac{1}{2}\frac{\Delta g}{g}$ [as length rmains constant]

$\Delta g=2A\omega_s^2$ [ω_s = angular frequency of support and , A = amplitude]

$$\frac{\Delta\omega}{\omega}=\frac{1}{2}\times\frac{2A\omega_s^2}{g}$$

$\Delta\omega=\frac{1}{2}\times\frac{2\times1^2\times10^{-2}}{10}=10^{-3}$ **rad/sec**

44. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be:[11 Jan 2019, II]

(1) $\frac{\sqrt{3}}{2}s$ (2) $\frac{2}{\sqrt{3}}s$ (3) $\frac{3}{2}s$ (4) $2\sqrt{3}s$

SOLUTION4) Acceleration due to gravity $g=\frac{GM}{R^2}$

$$\frac{g_p}{g_e}=\frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)=3\left(\frac{1}{3}\right)^2=\frac{1}{3}$$

Also $T\propto\frac{1}{\sqrt{g}}\Rightarrow\frac{T_p}{T_e}=\sqrt{\frac{g_e}{g_p}}=\sqrt{3}$

$$\Rightarrow T_p=2\sqrt{3}\,s$$

45. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is: [10 Jan 2019, II]

(1) $\frac{4\pi}{3}$ (2) $\frac{3}{8}\pi$ (3) $\frac{8\pi}{3}$ (4) $\frac{7}{3}\pi$

SOLUTION (3) Velocity, $v=\omega\sqrt{A^2-x^2}$ ...(i)

Acceleration, $a=-\omega^2x$ ...(ii)

And according to question,

$$|v|=|a|$$

$$\Rightarrow\omega\sqrt{A^2-x^2}=\omega^2x$$

$$\Rightarrow A^2-x^2=\omega^2x^2$$

$$\Rightarrow 5^2-4^2=\omega^2\left(4^2\right)$$

$$3=\omega\times4\Rightarrow\omega=\frac{3}{4}$$

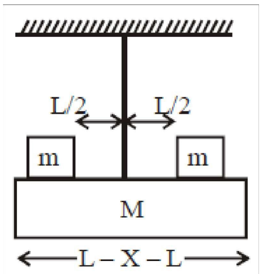
$$\therefore T = 2\pi / \omega = \frac{2\pi}{3/4} = \frac{8\pi}{3}$$

47. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

[9 Jan 2019, II]

(1) 0.77 (2) 0.57 (3) 0.37 (4) 0.17

SOLUTION (3)



$$f_1 = \frac{1}{2\pi}\sqrt{\frac{C}{1}} \quad \text{...(i)}$$

$$= \frac{1}{2}\sqrt{\frac{3C}{ML^2}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{C}{L^2\left(\frac{M}{3}+\frac{M}{2}\right)}} \quad \text{...(ii)}$$

As frequncey reduces by 80%

$$\therefore f_2 = 0.8 f_1 \Rightarrow \frac{f_2}{f_1} = 0.8 \quad \text{...(iii)}$$

Solving equations (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

As frequency reduces by 80%

$$\therefore f_2 = 0.8 f_1 \Rightarrow \frac{f_2}{f_1} = 0.8 \quad \text{...(iii)}$$

Solving equation (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

48. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 6.4 N/m (2) 7.1 N/m

(3) 2.2 N/m (4) 5.5 N/m

SOLUTION (2) As we know, frequency in SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

Where m = mass of one atom

Mass of one atom of silver, $= \frac{108}{\left(6.02\times10^{23}\right)} \times 10^{-3} kg$

$$\frac{1}{2\pi}\sqrt{\frac{k}{108\times10^{-3}} \times 6.02\times10^{23}} = 10^{12}$$

Solving we get, spring constant,
K = 7.1 N/m

49. A particle executes simple harmonic motion and is located at x = a, b and c at times t0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is [Online April 16, 2018]

(1) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ (2) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{3c}\right)$

(3) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$ (4) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

SOLUTION (4) Using $y = A\sin\omega t$

$$a = A\sin 2\omega t_0$$

$$c = A\sin 3\omega t_0$$

$$a + c = A\left[\sin\omega t_0 + A\sin 3\omega t_0\right]$$

$$= 2A\sin 2\omega t_0 \cos\omega t_0$$

$$\frac{a+c}{b} = 2\cos\omega t_0$$

$$\Rightarrow \omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

50. In an experiment to determine the period of a simple pendulum of length 1 m, it is attached to different spherical bobs of radii r_1 and r_2. The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5 × 10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given by:

[Online April 9, 2017]

(1) 1 cm (2) 0.1 cm (3) 0.5 cm (4) 0.01 cm

SOLUTION (2) As we know, Time-perid of simple pendulum, $T \propto \sqrt{l}$

Differentiating both side, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

$\because$ Change in length $\Delta l = r_1 - r_2$

$$5 \times 10^{-4} = \frac{1}{2}\frac{r_1 - r_2}{1}$$

$$r_1 - r_2 = 10 \times 10^{-4}$$

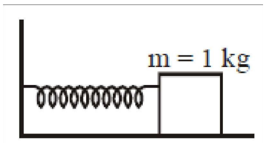
$$10^{-3} m = 10^{-1} cm = 0.1 cm$$

51. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is : [Online April 8, 2017]

(1) $\frac{1}{4} Hz$ (2) $\frac{1}{2\sqrt{2}} Hz$ (3) $\frac{1}{2} Hz$ (4) 2 Hz

SOLUTION (3) Frequency of spring $(f) = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1 Hz$

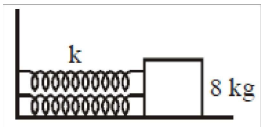
$$\Rightarrow 4\pi^2 = \frac{k}{m}$$



If block of mass m = 1 kg is attached then, $k = 4\pi^2$

Now, identical springs are attached in parallel with mass m = 8 kg Hence,

$k_{eq} = 2k$



$$F = \frac{1}{2\pi}\sqrt{\frac{k \times 2}{g}} = \frac{1}{2} Hz$$

52. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20° C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively : [2016]

(1) 30°C; α = 1.85 × 10–3/°C
(2) 55°C; = 1.85 × 10–2/°C
(3) 25°C; = 1.85 × 10–5/°C
(4) 60°C; = 1.85 × 10–4/°C

SOLUTION 3) Time lost/gained per day $= \frac{1}{2} \propto \Delta\theta \times 86400$ second

$$12 = \frac{1}{2}\alpha(40 - \theta) \times 86400 \quad ....(i)$$

$$4 = \frac{1}{2}\alpha(\theta - 20) \times 86400 \quad ....(ii)$$

On dividing we get, $3 = \frac{40 - \theta}{\theta - 20}$

$3\theta - 60 = 40 - \theta$

$4\theta - 100 \Rightarrow \theta = 25^0 C$

53. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor

speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to :

[Online April 10, 2016]

(1) 0.7 Hz (2) 1.9 Hz (3) 1.2 Hz (4) 0.1 Hz

SOLUTION (2) Washer contact with piston $\Rightarrow N = 0$

Given Amplitude A = 7 cm = 0.07 m.

$$a_{\max} = g = \omega^2 A$$

The frequency of piston

$$f = \frac{\omega}{2\pi} = \sqrt{\frac{g}{A}}\frac{1}{2\pi} = \sqrt{\frac{1000}{7}}\frac{1}{2\pi} = 1.9Hz$$

55. A particle moves with simple harmonic motion in a straight line. In first ts, after starting from rest it travels a distance a, and in next t s it travels 2a, in same direction, then: [2014]

(1) amplitude of motion is 3a

(2) time period of oscillations is 8

(3) amplitude of motion is 4a

(4) time period of oscillations is 6

SOLUTION

55. (d) In simple harmonic motion, starting from rest,

At t = 0 , x =A

x = A cos ωt ...(i)

When $t = \tau, x = A - a$

When $t = 2\tau, x = A - 3a$

From equation (i)

$$A - a = A\cos\omega\tau \quad \text{....(ii)}$$

$$A - 3a = A\cos 2\omega\tau \quad \text{....(iii)}$$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$ ...(iv)

From equation (ii), (iii), and (iv)

$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$\Rightarrow 2a^2 = aA$$

$\Rightarrow A = 2a$

$\Rightarrow \frac{a}{A} = \frac{1}{2}$

Now, $A - a = A\cos\omega\tau$

$\Rightarrow \cos\omega\tau = \frac{A-a}{A}$

$\Rightarrow \cos\omega\tau = \frac{1}{2}$ or $\frac{2\pi}{T}\tau = \frac{\pi}{3}$

$\Rightarrow T = 6\tau$

56. In an experiment for determining the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure. **[Online April 19, 2014]**

What is the value of g at the place?

(1) 9.81 m/s^2 (2) 9.87 m/s^2

(3) 9.91 m/s^2 (4) 10.0 m/s^2

SOLUTION (2) From graph it is clear that when

$L = 1m, T^2 = 4s^2$

As we know,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2}$$

$$\Rightarrow 4\times\left(\frac{22}{7}\right)^2\times\frac{1}{4} = \left(\frac{22}{7}\right)^2$$

$$\therefore g = \frac{484}{49} = 9.87 m/s^2$$

57. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3. The time

in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbon dioxide will be close to (*In* 5 = 1.601, *In* 2 = 0.693). [Online April 9, 2014]

(1) 231 s (2) 208 s (3) 161 s (4) 142 s

SOLUTION (4) As we know,

$$x = x_0 e^{-bt/2m}$$

From question,

$$8 = 10e^{-\frac{40b}{2m}} \qquad ....(i)$$

Similarly, $5 = 10e^{-\frac{bt}{2m}}$ ...(ii)

Solving equation (i) and (ii) we get

$t \cong 142s$

58. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take g = 10 ms–2) [Online April 9, 2014]

(1) 20 N (2) 10 N (3) 60 N (4) 40 N

SOLUTION (c) Mass of bigger body $M = 4kg$

Mass of smaller body m = 1 kg

Smaller mass (m = 1kg) executes S.H.M of angular frequency $\omega = 25$ rad/s

Amplitude x = 1.6 cm=1.6 x 10^{-2}

As we know,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

or, $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

or, $\frac{1}{25} = \sqrt{\frac{1}{K}}$ [$\because m = 1kg; \omega = 25$ rad/s]

or K =625 Nm^{-1}

The maximum force exerted by the system on the floor $= Mg + Kx + mg$

$$= 4\times10 + 625\times1.6\times10^{-2} + 1\times10$$

$$= 40 + 10 + 10$$

= 60 N

Topic-4: Damped, Forced Oscillations and Resonance

76. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to of the original amplitude is close to : [8 April 2019, II]

(1) 50 s (2) 100s (3) 20 s (4) 10 s

SOLUTION:(3) Time of half the amplitude is = 2s

Using, $A = A_0 e^{-kt}$

$$\frac{A_0}{2} = A_e e^{-kx2}$$

And $\frac{A_0}{40} = A_e e^{+kx2}$

Divining (i) and (ii) and soving, we get $t = 20s$

77. The displacement of a damped harmonic oscillator is given by . Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to: [9 Jan 2019, II]

(1) 4s (2) 7s (3) 13s (4) 27s

SOLUTION:(2) Amplitude of vibration at time t = 0 is given by

$$A = A_e e^{-0.1 \times 0} = 1 \times A_0 = a_0$$

Also at $A = t = t$, if $A = \frac{A_0}{0}$

$$\Rightarrow \frac{1}{2} = e^{-0.lt}$$

$t = 10$ In $2 \square 7s$

79. The angular frequency of the damped oscillator is given by, where k is the spring constant, m is the mass of the oscillator and r is the damping constant. If the ratio is 8%, the change in time period compared to the undamped oscillator is approximately as follows:

[Online April 11, 2014]

(1) increases by 1% (2) increases by 8%

(3) decreases by 1% (4) decreases by 8%

SOLUTION:(2) The change in time period compared to the undamped oscillator increases by 8%.

80. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to a times its original magnitude, where a equals [2013]

(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

SOLUTION:

(3) $\because A = A_0 e^{\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)
According to the quations, after 5 second,

$$0.9 A_0 = A_e^{\frac{b(5)}{2m}} \quad \ldots\text{(i)}$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots\text{(ii)}$$

From eqation (i) and (ii)

$$A = 0.729 A_0$$

$$\therefore \alpha = 0.729$$

82. Bob of a simple pendulum of length *l* is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then :

[Online April 23, 2013]

(1) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

(2) $T = 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

(3) $T > 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

(4) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

SOLUTION:4) When the pendulum is oscillating over a current carrying coil, and when the direction of oscillating pendulum bob is opposite to the direction of current. Its instantaneous acceleration increases.

Hence time period $T < 2\pi\sqrt{\frac{\ell}{g}}$

and damping is larger than in air alone due energy dissipatio

83. In forced oscillation of a particle the amplitude is maximum for a frequency of the force while the energy is maximum for a frequency of the force; then [2004]

(1) $\omega_1 < \omega_2$ when damping is small and when damping is large $\omega_1 > \omega_2$

(2) $\omega_1 > \omega_2$ (3) $\omega_1 = \omega_2$ (4) $\omega_1 < \omega_2$

SOLUTION: 3) As energy $\propto$ (Amplitude)^2, the maximum for both of them occurs at the same frequency and this is only possible in case of resonance

In resonance state $\omega_1 = \omega_2$

84. A particle of mass m is attached to a spring (of spring constant *k*) and has a natural angular frequency . An external force *F*(*t*) proportional to cosw*t* is applied to the oscillator. The displacement of the oscillator will be proportional to
[2004]

(1) $\frac{1}{m\left(\omega_0^2+\omega^2\right)}$ (2) $\frac{1}{m\left(\omega_0^2-\omega^2\right)}$ (3) $\frac{m}{\omega_0^2-\omega^2}$ (4) $\frac{m}{\left(\omega_0^2+\omega^2\right)}$

SOLUTION: :(2) Equation of displacement in forced oscillation is given by

$$y = \frac{F_0}{m\left(\omega_0^2-\omega^2\right)^2}$$

$$= \frac{F_0}{m\left(\omega_0^2-\omega^2\right)}$$

Here damping effect is considered to be zero

Gravitation

Basic Forces in Nature :

Basic forces are classified into four categories

a) Gravitational Force

b) Electromagnetic Force

c) Strong nuclear Force

d) Weak nuclear Force

Gravitational Force:

- This force between any two massive particles.
- It is always attractive force.
- It is a conservative force.
- It is independent of medium present between the masses.
- It can provide radial acceleration.
- It is communicated through a particle called as **Gravitation.**

Electro Magnetic Force :

- This force exists between any two charged particles.
- This force is either attractive or repulsive.
- It is communicated through **Photons**.

Strong Nuclear Force :

- This force may act between a pair of nucleons in the nucleus.
- It is charge independent.
- It is spin dependent.
- It is communicated through π **mesons.**

Weak Nuclear Force :

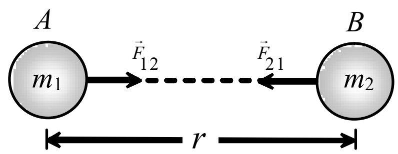
- They are responsible for radioactive decay like β-decay.
- They acts between all leptons, positrons, μ-mesons, neutrinos and Hadrons etc.
- It is communicated through **weak bosons**.

Relative strengths of basic forces between protons :

Basic force	Range	Relative strength
Gravitational	Long range (upto infinity)	1
Weak nuclear	Short range (<< 1 fm)	10^{31}
Electromagnetic	Long range (upto infinity)	10^{36}
Strong nuclear	Short range (1 fm)	10^{38}

Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.



Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 are separated by a distance r exert on each other is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

or $$F = G\frac{m_1 m_2}{r^2}$$

Vector form :

According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12} = \frac{Gm_1m_2}{r^2}\hat{r}_{21}$ $[\because \hat{r}_{12} = -\hat{r}_{21}]$

$\hat{r}_{12}$ = unit vector from A to B

$\hat{r}_{21}$ = unit vector from B to A,

$\vec{F}_{12}$ = gravitational force exerted on body A by body B

$\vec{F}_{21}$ = gravitational force exerted on body B by body A

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Universal gravitational constant:

$$F = G\frac{m_1 m_2}{r^2}$$

Here G is constant of proportionality which is called 'Universal gravitational constant'.

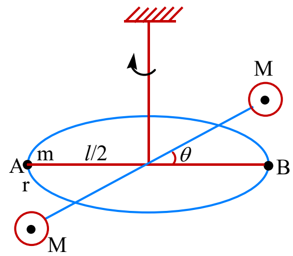
If $m_1 = m_2$

$r = 1$

then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of 'unit mass whose centres are placed unit distance apart.

- The value of G in the laboratory was first determined by Cavendish using the torsional balance.
- The value of G is 6.67×10^{-11} $N\text{–}m^2\ kg^{-2}$
- Dimensional formula $[M^{-1}L^3T^{-2}]$.



From Cavendish experiment the value of universal gravitational constant (G) can be calculatedby

$$G = \frac{k\theta r^2}{Mml}$$

M-Mass of heavier sphere
m-Mass of lighter sphere
k-Torsion constant ; θ-Angle of twist

- ♦ The value of G does not depend upon the nature and size of the bodies.
- ♦ It also does not depend upon the nature of the medium between the two bodies.
- ♦ As G is very small, hence gravitational forces are very small, unless one (or both) of the mass is huge.

Properties of Gravitational Force

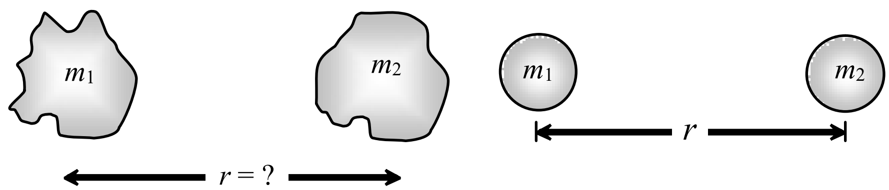
- ♦ It is always attractive in nature while electric and magnetic force can be attractive or repulsive.
- ♦ It is independent of the medium between the particles while electric and magnetic force depend on the nature of the medium between the particles.
- ♦ It holds good over a wide range of distances. It is found true for interplanetary to inter atomic distances.
- ♦ It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.
- ♦ It is a two-body interaction *i.e.* gravitational force between two particles is independent of the presence or absence of other particles; so the principle of superposition is valid *i.e.* force on a particle due to number of particles is the resultant of forces due to individual particles

  i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$
- ♦ While nuclear force is many body interaction
- ♦ It is the weakest force in nature : As $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$.
- ♦ The ratio of gravitational force to electrostatic force between two electrons is of the order of 10^{-43} .
- ♦ It is a conservative force *i.e.* work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.
- ♦ It is an action reaction pair *i.e.* the force with which one body (say earth) attracts the second body (say moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

NOTE :

The law of gravitation is stated for two point masses, therefore for any two arbitrary finite size bodies, as shown in the figure, It can not be applied as there is not unique value for the separation.

But if the two bodies are uniform spheres then the separation r may be taken as the distance between their centres because a sphere of uniform mass behave as a point mass for any point lying outside it.



PROBLEMS

1. Two identical spheres are placed in contact with each other. The force of gravitation between the spheres will be proportional to (R = radius of each sphere)

(a) R (b) R^2 (c) R^4 (d) None of these

SOLUTION :

From law of gravitation $F = G\frac{M \times M}{R^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{(2R)^2} \Rightarrow F \propto \frac{R^6}{R^2} \propto R^4$

2. The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth

(a) Is the same (b) Is smaller (c) Is greater (d) Varies with its phase

SOLUTION :

Force between earth and moon $F = \frac{Gm_m m_e}{r^2}$

This amount of force, both earth and moon will exert on each other

i.e. they exert same force on each other.

3. If the distance between two masses is doubled, the gravitational attraction between them

(a) Is doubled (b) Becomes four times

(c) Is reduced to half (d) Is reduced to a quarter

SOLUTION :

$F \propto \frac{1}{r^2}$.

If r becomes double then F reduces to $\frac{F}{4}$

4 . The gravitational force between two stones of mass 1 *kg* each separated by a distance of 1 *metre* in vacuum is

(a) Zero (b) 6.675×10^{-5} *newton* (c) 6.675×10^{-11} *newton* (d) 6.675×10^{-8} *newton*

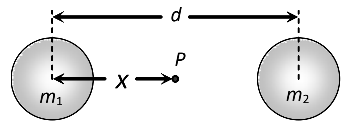
SOLUTION :

$F = G\frac{m_1 m_2}{r^2} = 6.675 \times \frac{1 \times 1}{1^2} \times 10^{-11} = 6.675 \times 10^{-11}\, N$

5 . The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero

(a) $\frac{D}{2}$ (b) $\frac{2D}{3}$ (c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

SOLUTION :



Force will be zero at the point of zero intensity

$$x = \frac{\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}} d$$

$$= \frac{\sqrt{81M}}{\sqrt{81M} + \sqrt{M}} D = \frac{9}{10} D.$$

6 . Mass M is divided into two parts xM and $(1-x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) 2

SOLUTION :

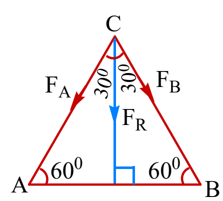
$$F \propto xm \times (1-x)m = xm^2(1-x)$$

For maximum force $\frac{dF}{dx} = 0$

$$\Rightarrow \frac{dF}{dx} = m^2 - 2xm^2 = 0 \Rightarrow x = 1/2$$

7. If two particles each of mass 'm' are placed at the two vertices of an equilateral triangle of side 'a', then the resultant gravitational force on mass m placed at the third vertex is

SOLUTION :



$$F_R = \sqrt{F_A^2 + F_B^2 + 2F_AF_B\cos 60^0}$$

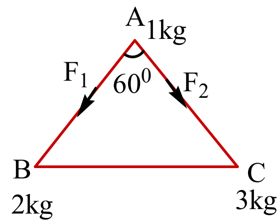
$$= \sqrt{3}F \left[\because F_A = F_B = F\right]$$

$$F_R = \sqrt{3}\left[\frac{Gm^2}{a^2}\right]$$

8. Three spherical balls of masses 1kg, 2kg and 3kg are placed at the corners of an equilateral triangle of side 1m. Find the magnitude of the gravitational force exerted by 2 kg and 3 kg masses on 1 kg mass.

SOLUTION :

If F_1 is the force of attraction between 1kg, 2kg masses, then, $F_1 = G \times \frac{1 \times 2}{(1)^2} \Rightarrow F_1 = 2G$



If F_2 is the force of attraction between 1kg, 3kg masses, then, $F_2 = G \times \frac{1 \times 3}{(1)^2} \Rightarrow F_2 = 3G$

The angle between the forces F_1 and F_2 is 60^0. If 'F_R' is the resultant of these two forces then

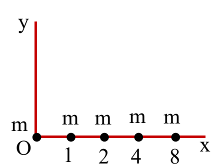
$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow F_R = \sqrt{(2G)^2 + (3G)^2 + 2 \times 2G \times 3G \times \cos 60^0}$$

$$\Rightarrow F_R = \sqrt{19}G$$

9. An infinite number of particle each of mass m are placed on the positive X-axis at 1m,2m,4m,8m,.... from the origin. Find the magnitude of the resultant gravitational force on mass 'm' kept at the origin.

SOLUTION :



The resultant gravitational force

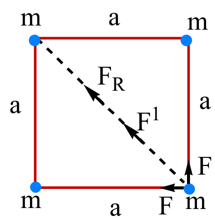
$$F = \frac{Gm^2}{1} + \frac{Gm^2}{4} + \frac{Gm^2}{16} + ...$$

$$= Gm^2\left(1 + \frac{1}{4} + \frac{1}{16} + ...\right)$$

$$= Gm^2\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{4}{3}Gm^2 \left(\because S_\infty = \frac{a}{1-r}\right)$$

10. If four identical particles each of mass m, are kept at the four vertices of a square of side length a, the gravitational force of attraction on any one of the particle is

SOLUTION :

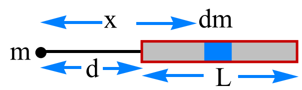


$$F_R = \sqrt{2}F + F' = \frac{\sqrt{2}Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$$F_R = \frac{Gm^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

along the diagonal towards the opposite corner.

11. A particle of mass m is situated at a distance d from one end of a rod of mass M and length L as shown in fig. Find the magnitude of the gravitational force between them.



SOLUTION :

Consider an element of mass 'dm' and length 'dx' at a distance 'x' from the point mass.

Mass of the element $dm = \frac{M}{L}dx$.

Gravitational force on 'm' due to this element is

$$dF = \frac{Gm\left(\frac{M}{L}dx\right)}{x^2} \; ; \; F = \int_d^{(d+L)} \frac{Gm\left(\frac{M}{L}\right)dx}{x^2}$$

$$\Rightarrow F = \frac{GmM}{L}\int_d^{(d+L)} x^{-2}dx = \frac{GmM}{L}\left[\frac{x^{-1}}{-1}\right]^{(d+L)}$$

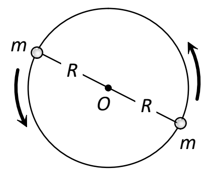
$$\Rightarrow F = \frac{GmM}{L}\left[\frac{-1}{x}\right]_a^{(d+L)} = \frac{GmM}{L}\left[\frac{1}{d} - \frac{1}{(d+L)}\right]$$

$$F = \frac{GmM}{L}\left[\frac{d+L-d}{(d+L)d}\right] = \frac{GmM}{d(d+L)}$$

12. Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is **[CBSE PMT 1995; RPMT 2003]**

(a) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (b) $v = \sqrt{\frac{Gm}{2R}}$ (c) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v = \sqrt{\frac{4Gm}{R}}$

SOLUTION :



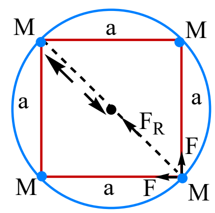
Centripetal force provided by the gravitational force of attraction between two particles

$$i.e.\ \frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

13. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is (2014A)

SOLUTION :

Let a be the distance between two particles.



The resultant gravitational force on any one of the particle is given by

$$F_R = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right).$$

Which provides necessary centripetal force for motion of mass M in circle, so

$$\left(\sqrt{2} + \frac{1}{2}\right)\left(\frac{GM^2}{a^2}\right) = \frac{MV^2}{\frac{a}{\sqrt{2}}}$$

$$V^2 = \left(\frac{2\sqrt{2}+1}{2\sqrt{2}}\right)\frac{GM}{a} = \left(\frac{2\sqrt{2}+1}{2\sqrt{2}}\right)\frac{GM}{\sqrt{2}R}$$

$$\Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

14. Two particles of masses 1Kg and 2Kg are placed at a distance of 50cm. Find the initial acceleration of the first particle due to gravitational force.

SOLUTION :

Gravitational force between two particles is

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 2}{(0.5)^2} = 5.3 \times 10^{-10}\,N$$

The acceleration of 1Kg particle is

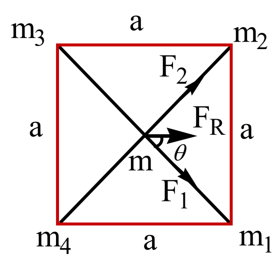
$$a_1 = \frac{F}{m_1} = \frac{5.3 \times 10^{-10}}{1} = 5.3 \times 10^{-10}\,ms^{-2}$$ towards the 2Kg mass

15. If four different masses m_1,m_2,m_3 and m_4 are placed at the four corners of a square of side 'a' the resultant gravitational force on mass m kept at the centre is

SOLUTION :

The force on m due to m_1 and m_3 is $F_1 = \frac{2Gm}{a^2}(m_1 - m_3)$

along the diagonal towards $m_1 [if\ m_{12} > m_3]$



The force on m due to m_2 and m_4 is $F_2 = \frac{2Gm}{a^2}(m_2 - m_4)$

along the diagonal towards $m_2 [if\ m_2 > m_4]$.

The resultant force is $\sqrt{F_1^2 + F_2^2} = F$

$$F = \frac{2Gm}{a^2}\sqrt{(m_1 - m_3)^2 + (m_2 - m_4)^2}$$

and the resultant force makes an angle θ with F_1

where, $\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right)$.

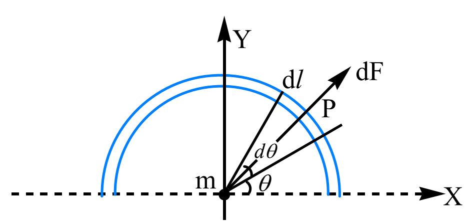
16. A thin rod of mass M and length L is bent into semicircle as shown in figure. What is gravitational force on a particle with mass m at the centre of curvature?

SOLUTION :

Consider an element of rod of length d*l* as shown in figure
treat it as a small particle of mass (M/L) d*l*
situated at a distance R from P.
Then gravitational force due to the element on the particle will be

$$dF = \frac{Gm(M/L)(Rd\theta)}{R^2} \text{ along OP}$$

[as $dl = Rd\theta$]



So the components of this force along x and y axes will be

$$dF_x = dF\cos\theta = \frac{GmM\cos\theta d\theta}{LR}$$

$$dF_y = dF\sin\theta = \frac{GmM\sin\theta d\theta}{LR}$$

So that

$$F_x = \frac{GmM}{LR}\int_0^{\pi}\cos\theta d\theta = \frac{GmM}{LR}[\sin\theta]_0^{\pi} = 0$$

$$F_y = \frac{GmM}{LR}\int_0^{\pi}\sin\theta d\theta = \frac{GmM}{LR}[-\cos\theta]_0^{\pi}$$

$$= \frac{2\pi GmM}{L^2} \qquad \left[as R = \frac{L}{\pi}\right]$$

$$F = \sqrt{F_x^2 + F_y^2} = F_y = \frac{2\pi GmM}{L^2} \text{ (as } F_x\text{=zero)}$$

17. Mass M is split into two parts m and (M-m), which are then separated by a certain distance. What is the ratio of (m/M) which maximises the gravitational force between the parts?

SOLUTION :

If r is the distance between m and (M-m), the gravitational force between them will be

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r^2}(mM - m^2)$$

For F to be maximum dF/dm=0 as M & r are constants.

$$\Rightarrow \frac{d}{dm}\left[\frac{G}{r^2}(mM - m^2)\right] = 0; \Rightarrow M - 2m = 0$$

$$\Rightarrow \frac{m}{M} = \frac{1}{2}.$$

So the force will be maximum when the parts are equal.

18. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. On what power of 'r' will the square of time period depend on the gravitational force of attraction between the planet and the star is proportional to $r^{-5/2}$?

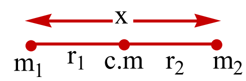
SOLUTION :

The gravitational force provides necessary centripetal force $\frac{mV^2}{r} = \frac{K}{r^{5/2}} \Rightarrow V^2 = \frac{K}{mr^{3/2}}$

But $T = \frac{2\pi r}{V} = 2\pi r\sqrt{\frac{mr^{3/2}}{K}}$; $\quad \therefore T^2 \ \alpha \ r^{7/2}$

19. In a double star system, two stars of masses m_1 and m_2 separated by a distance 'x' rotates about their centre of mass. Find the common angular velocity and Time period of revolution.

SOLUTION :



The gravitational force between the masses provides the necessary centripetal force.

i.e. $$\frac{Gm_1m_2}{x^2} = m_1r_1\omega^2 \qquad .....(1)$$

The distance of centre of mass from m_1 is

$$r_1 = \frac{m_2x}{m_1 + m_2} \qquad ....(2)$$

From (1) and (2) $$\frac{Gm_1m_2}{x^2} = \frac{m_1m_2x}{m_1 + m_2}\omega^2$$

$$\omega^2 = \frac{G(m_1 + m_2)}{x^2} \Rightarrow \omega = \sqrt{\frac{G(m_1 + m_2)}{x^3}}$$

$$T = \frac{2\pi}{\omega}; T = 2\pi\sqrt{\frac{x^2}{G(m_1 + m_2)}}$$

20. In Cavendish's experiment, let each small mass be 20 kg and each large mass be 5 kg. The rod connecting the small masses is 50 cm long, while the small and the large spheres are separated by 10.0 cm. The torsion constant is 4.8×10^{-8} kgm^2s^{-2} and the resulting angular deflection is 0.4^0. Calculate the value of universal gravitational constant G from this data.

SOLUTION :

Here, m=20kg=0.02kg, M=5 kg

r=10cm=0.1m, *l*=50cm=0.5m

$$\theta = 0.4^0 = (0.4^0)(2\pi / 360^0) = 0.007 rad,$$

$k=4.8 \times 10^{-8} kgm^2s^{-2}$

Thus, from $$G = \frac{k\theta r^2}{Mml}$$

$$G = \frac{(4.8\times10^{-8})(0.007)(0.1)^2}{5\times0.02\times0.5}$$

$=6.72 \times 10^{-11}$ Nm^2kg^{-2}.

21. The mean orbital radius of the Earth around the Sun is 1.5x10^8 km. Estimate the mass of the Sun.

SOLUTION :

As the centripetal force is provided by the gravitational pull of the Sun on the Earth

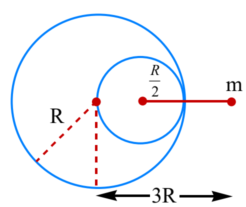
$$\frac{GM_sM_e}{r^2} = M_e r\omega^2 = M_e r\frac{4\pi^2}{T^2} (or) M_S = \frac{4\pi^2 r^3}{GT^2}$$

given, r=1.5x10^8 km=1.5x10^{11}m;

T=365 days=365x24x60x60s

$$M_S = \frac{4\times(22/7)^2 \times (1.5\times10^{11})^3}{(6.67\times10^{11})\times(365\times24\times60\times60)^2} \approx 2\times10^{30} kg$$

22. The gravitational force acting on a particle, due to a solid sphere of uniform density and radius R, at a distance of 3R from the centre of the sphere is F$_1$. A spherical hole of radius (R/2) is now made in the sphere as shown in the figure. The sphere with hole now exerts a force F$_2$ on the same particle. Ratio F$_1$ to F$_2$ is (2013E)



SOLUTION :

Let mass of the removed sphere = M.

Then mass of the original sphere = 8M (since mass $\propto R^3$)

$$F_1 = \frac{8GMm}{9R^2}$$

$$F_2 = \frac{8GMm}{9R^2} - \frac{GMm}{\left(\frac{5R}{2}\right)^2}$$

$$\text{Therefore, } \frac{F_1}{F_2} = \frac{50}{41} \qquad (\text{on simplifying})$$

Acceleration Due to Gravity

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

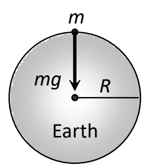


Fig. 8.4

Consider a body of mass m is lying on the surface of earth then gravitational force on the body is given by

$$F = \frac{GMm}{R^2} \qquad \ldots\text{(i)}$$

Where M = mass of the earth

R = radius of the earth.

If g is the acceleration due to gravity,

then the force on the body due to earth is given by Force = mass × acceleration

$$F = mg \qquad \ldots\text{(ii)}$$

From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

$$\therefore \quad g = \frac{GM}{R^2} \qquad \ldots\text{(iii)}$$

$$\Rightarrow \quad g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

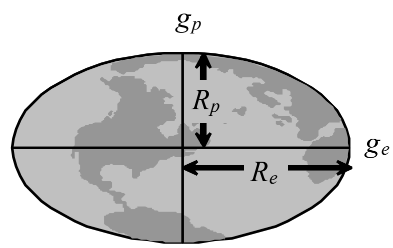
[As mass (M) = volume ($\frac{4}{3}\pi R^3$) × density (ρ)]

$$\therefore \quad g = \frac{4}{3}\pi\rho G R \qquad \ldots\text{(iv)}$$

- ♦ From the expression $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ it is clear that its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. *i.e.* a given planet (reference body) produces same acceleration in a light as well as heavy body.
- ♦ The greater the value of (M/R^2) or ρR, greater will be value of g for that planet.
- ♦ Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.
- ♦ Dimension $[g] = [LT^{-2}]$

- it's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2 or 32 $feet/sec^2$, on the surface of the earth at mean sea level.
- The value of acceleration due to gravity vary due to the following factors :
  (a) Shape of the earth,
  (b) Height above the earth surface,
  (c) Depth below the earth surface
  (d) Axial rotation of the earth.

Variation in g Due to Shape of Earth:



Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 *km* longer than polar radius, from $g = \frac{GM}{R^2}$

At equator $g_e = \frac{GM}{R_e^2}$ ...(i)

At poles $g_p = \frac{GM}{R_p^2}$ ...(ii)

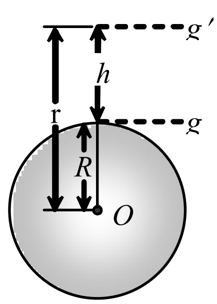
From (i) and (ii) $\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$

Since $R_{equator} > R_{pole}$

$\therefore\ g_{pole} > g_{equator}$ and $g_p = g_e + 0.018\ ms^{-2}$

Therefore the weight of body increases as it is taken from equator to the pole.

Variation in *g* With Height



Acceleration due to gravity at the surface of the earth $g = \frac{GM}{R^2}$ ...(i)

Acceleration due to gravity at height *h* from the surface of the earth $g' = \frac{GM}{(R+h)^2}$ ...(ii)

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ ...(iii)

$$= g\frac{R^2}{r^2} \quad \text{...(iv)}$$

[As $r = R + h$]

♦ As we go above the surface of the earth, the value of *g* decreases because $g' \propto \frac{1}{r^2}$.

♦ If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of *g* becomes zero.

♦) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$= g\left(1 + \frac{h}{R}\right)^{-2}$$

$$= g\left[1 - \frac{2h}{R}\right] \quad [\text{As } h << R]$$

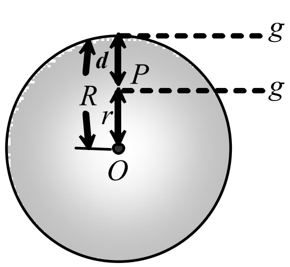
♦ If $h << R$ then decrease in the value of *g* with height :

♦ Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

♦ Fractional decrease $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{2h}{R}$

♦ Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} \times 100\%$

Variation in *g* With Depth:



Acceleration due to gravity at the surface of the earth $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ ...(i)

Acceleration due to gravity at depth *d* from the surface of the earth $g' = \frac{4}{3}\pi\rho G(R - d)$ ...(ii)

From (i) and (ii) $g' = g\left[1 - \frac{d}{R}\right]$

- ♦ The value of g decreases on going below the surface of the earth.
  From equation (ii) we get $g' \propto (R - d)$.
  So it is clear that if d increase, the value of g decreases.
- ♦ At the centre of earth $d = R$
  $\therefore \; g' = 0$,
  i.e., the acceleration due to gravity at the centre of earth becomes zero.
- ♦ Decrease in the value of g with depth
- ♦ Absolute decrease $\Delta g = g - g' = \frac{dg}{R}$
- ♦ Fractional decrease $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{d}{R}$
- ♦ Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\%$
- ♦ The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

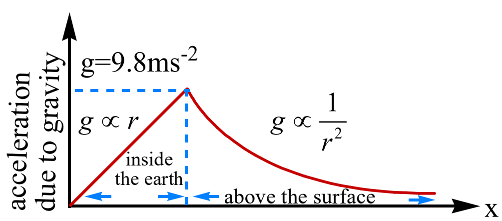
Graphical representation of variation of 'g' with height and depth:

The variation of g with the distance r from the centre of the earth is shown below

i) Above the earth :

$$g_h = \frac{gR^2}{(R+h)^2} \; : \qquad g_h = \frac{gR^2}{r^2} \qquad (\because R + h = r) \Rightarrow \boxed{g_h \propto \frac{1}{r^2}}$$

$\Rightarrow g_h$ versus r graph is a curve a shown.



i) Inside the earth :

$$g_d = \frac{g}{R}(R - d)$$

$$g_d = \frac{g}{R}(r)(\because R - d = r) \Rightarrow \boxed{(g_d \propto r)}$$

$\Rightarrow g_d$ versus r graph is a straight line passing through the origin as shown in fig.

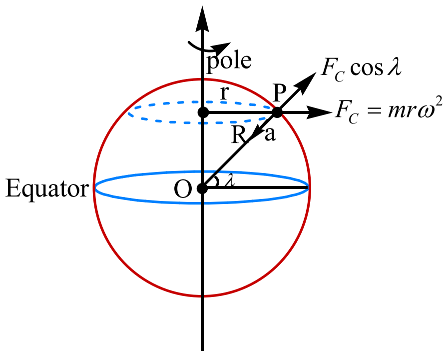
NOTE: Lines joining the places on the earth having same values of g are called isograms.

Variation of 'g' with latitude :

Consider an object of mass m at latitude λ of the earth due to rotation of a earth, the vale of acceleration due to gravity g_λ at a given place is given by

$$g_\lambda = g - r\omega^2 \cos\lambda$$

where $r\omega^2 \cos\lambda$ is the component of centrifugal acceleration along the radius of the Earth.



As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.

Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass *m* lying at point *P*, whose latitude is *l*, then due to rotation of earth its apparent weight can be given by

$$\overrightarrow{mg'} = \overrightarrow{mg} + \overrightarrow{F_c}$$

$$mg' = \sqrt{(mg)^2 + (F_c)^2 + 2mg\ F_c\ \cos(180 - \lambda)}$$

$$mg' = \sqrt{(mg)^2 + (m\omega^2 R\cos\lambda)^2 + 2mg\ m\omega^2 R\cos\lambda\,(-\cos\lambda)}$$

[As $F_c = m\omega^2 r = m\omega^2 R\cos\lambda$]

By solving we get $g' = g - \omega^2 R\cos^2\lambda$

where r is the radius of the circle in which the object is revolving.

Here $r = R\cos\lambda$

$$\therefore g_\lambda = g - \omega^2 R\cos^2\lambda$$

where ω is the angular velocity. R is radius of the earth and λ is latitude of the place

- The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.
- For the poles $\lambda = 90^o$
  for equator $\lambda = 0^o$

Special cases :

♦ **At the poles** $\lambda = 90^0$

$$\therefore g_{\lambda(=90^0)} = g - \omega^2 R(0)^2 \quad (\because \cos 90^0 = 0)$$

$$\therefore g_{\lambda(=90^0)} = g --(\text{maximum})$$

♦ **At the equator** $\lambda = 0^0$

$$\therefore g_{\lambda(=0^0)} = g - \omega^2 R(1)^2 \quad (\because \cos 0^0 = 1)$$

$$\therefore g_{\lambda(=0^0)} = g - \omega^2 R.....(\text{minimum})$$

From equation (i) and (ii) $\quad g_{pole} - g_{equator} = R\omega^2 = 0.034\, m/s^2$

♦ Here , $R\omega^2 = 0.034 ms^{-2}$ for the Earth.

♦ The value of 'g' at poles does not depend on the speed of rotation of the earth, but at the equator 'g' decreases with the increase of speed of rotation of earth.

♦ If earth suddenly stops its rotation, then the acceleration due to gravity at poles remains constant, and acceleration due to gravity at equator increases by $\omega^2 R$

Weightlessness:

Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$$g' = g - \omega^2 R \cos^2 \lambda$$

$$\Rightarrow 0 = g - \omega^2 R \cos^2 0^o \quad [\text{As } \lambda = 0^o \text{ for equator}]$$

$$\Rightarrow \quad g - \omega^2 R = 0$$

$$\therefore \quad \omega = \sqrt{\frac{g}{R}}$$

or time period of rotation of earth $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

Substituting the value of $R = 6400 \times 10^3 m$ and $g = 10 m/s^2$ we get

$$\omega = \frac{1}{800} = 1.25 \times 10^{-3} \frac{rad}{sec}$$

$$T = 5026.5\ sec = 1.40\ hr.$$

- ❑ This time is about $\frac{1}{17}$ times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.
- ❑ If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.
- ❑ After considering the effect of rotation and elliptical shape of the earth, acceleration due to gravity at the poles and equator are related as

$$g_p = g_e + 0.034 + 0.018\, m/s^2$$

$\therefore g_p = g_e + 0.052\, m/s^2$

Mass and Density of Earth

Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \frac{GM}{R^2}$,

so we have $M = \frac{gR^2}{G}$

$$\therefore \quad M = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.98 \times 10^{24}\, kg \approx 10^{25}\, kg$$

and as we know $g = \frac{4}{3}\pi\rho GR$,

so we have $\rho = \frac{3g}{4\pi GR}$

$$\therefore \quad \rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^6} = 5478.4\, kg/m^3$$

PROBLEMS

1. R is the radius of the earth and ω is its angular velocity and g_p is the value of g at the poles. The 'effective value of g at the latitude $\lambda = 60°$ will be equal to

(a) $g_p - \frac{1}{4}R\omega^2$ (b) $g_p - \frac{3}{4}R\omega^2$ (c) $g_p - R\omega^2$ (d) $g_p + \frac{1}{4}R\omega^2$

SOLUTION :

$$g = g_p - R\omega^2 \cos^2 \lambda$$

$$= g_p - \omega^2 R \cos^2 60°$$

$$= g_p - \frac{1}{4}R\omega^2$$

2. What is the time period of rotation of the earth around its axis so that the objects at the equator becomes weightless?(g=9.8m/s^2, Radius of earth = 6400km)

SOLUTION :'

When earth is rotating the apparent weight of a body at the equator is given by

$$W_{app} = mg - mR\omega^2$$

If bodies are weightless at the equator

$$0 = mg - mR\omega^2 \Rightarrow g = R\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{g}{R}}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

$$T = 2\pi\sqrt{\frac{6.4x10^6}{9.8}} = 5078s = 82 \text{ minute } 32\text{s}$$

3 . The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

SOLUTION :

$$g' = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1-\frac{d}{R}\right)$$

$$\Rightarrow d = \left(\frac{n-1}{n}\right)R$$

4. The height at which the acceleration due to gravity becomes g/9 (where g is the acceleration due to gravity on the surface of the earth) in terms of the radius of the earth (R) is(2009 A)

SOLUTION :

$$\text{Given } \frac{g}{9} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$$3R = R + h \Rightarrow 2R = h$$

5. Find the percentage decrease in the weight of the body when taken for a depth of 32Km below the surface of earth.

SOLUTION :

Weight of the body at depth d is

$$mg' = mg\left(1-\frac{d}{R}\right)$$

$$\%\text{ decrease in weight } = \frac{mg-mg'}{mg}\times 100$$

$$= \frac{d}{R}\times 100 = \frac{32}{6400}\times 100 = 0.5\%$$

6. A man can jump 1.5 m on the Earth. Calculate the approximate height he might be able to jump on a planet whose density is one-quarter that of the Earth and whose radius is one-third that of the Earth.

SOLUTION :

We know that, it case of Earth,

$$g = \frac{GM}{R^2} = G\times\frac{(4\pi/3)R^3\rho}{R^2} = \left(\frac{4}{3}\pi G\right)R\rho$$

Similarly, for the other planet whose radius $\frac{R}{3}$ and density $\frac{\rho}{4}$ is,

$$g' = \left(\frac{4\pi G}{3}\right)\left(\frac{R}{3}\right)\left(\frac{\rho}{4}\right)$$

$$g' = \frac{1}{12}\left(\frac{4\pi G}{3}\right)R\rho = \frac{1}{12}g \Rightarrow \frac{g}{g'} = 12$$

$$h_{\max} = \frac{u^2}{3g} \Rightarrow h_{\max} \propto \frac{1}{g} \text{ (here u is constant)}$$

$$\frac{h'}{h} = \frac{g}{g'} = 12 \Rightarrow h' = 12h = 12 \times 1.5 = 18m$$

7. If the change in the value of 'g' at a height h above the surface of the earth is the same as at a depth x below it, then (both x and h being much smaller than the radius of the earth)

(a) $x = h$ (b) $x = 2h$ (c) $x = \frac{h}{2}$ (d) $x = h^2$

SOLUTION :

The value of g at the height h from the surface of earth $g' = g\left(1 - \frac{2h}{R}\right)$

The value of g at depth x below the surface of earth $g' = g\left(1 - \frac{x}{R}\right)$

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

8. The time period of a simple pendulum on a freely moving artificial satellite is

(a) Zero (b) 2 sec (c) 3 sec (d) Infinite

SOLUTION :

Time period of simple pendulum $T = 2\pi\sqrt{\frac{l}{g'}}$

In artificial satellite $g' = 0$

$\therefore$ T = infinite.

9. A body weighs 700 *gm wt* on the surface of the earth. How much will it weigh on the surface of a planet whose mass is $\frac{1}{7}$ and radius is half that of the earth

(a) 200 *gm wt* (b) 400 *gm wt* (c) 50 *gm wt* (d) 300 *gm wt*

SOLUTION :

We know that $g = \frac{GM}{R^2}$

On the planet $g_p = \frac{GM/7}{R^2/4} = \frac{4g}{7} = \frac{4}{7}g$

Hence weight on the planet $= 700 \times \frac{4}{7} = 400$ *gm wt*

10. A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to

(a) GM_0/D_0^2 (b) $4mGM_0/D_0^2$ (c) $4GM_0/D_0^2$ (d) GmM_0/D_0^2

SOLUTION :

$$g = \frac{GM}{R^2} = \frac{GM_0}{(D_0/2)^2} = \frac{4GM_0}{D_0^2}$$

11. The mass and diameter of a planet have twice the value of the corresponding parameters of earth. Acceleration due to gravity on the surface of the planet is

(a) $9.8\ m/\sec^2$ (b) $4.9\ m/\sec^2$ (c) $980\ m/\sec^2$ (d) $19.6\ m/\sec^2$

SOLUTION :

$$\frac{g'}{g} = \frac{M'}{M}\left(\frac{R}{R'}\right)^2 = \left(\frac{2M}{M}\right)\left(\frac{R}{2R}\right)^2 = \frac{1}{2}$$

$$\Rightarrow g' = \frac{g}{2} = \frac{9.8}{2} = 4.9\ m/s^2$$

12. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is

(a) $4\pi G/3gR$ (b) $3\pi R/4gG$ (c) $3g/4\pi RG$ (d) $\pi RG/12G$

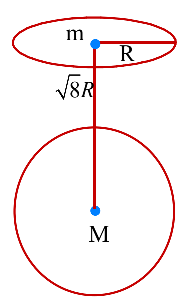
SOLUTION :

$$g = \frac{GM}{R^2}$$

$$M = \frac{4}{3}\pi R^3 \times \rho$$

$$\therefore g = \frac{4}{3}\frac{\pi R^3 \times G\rho}{R^2} \Rightarrow \rho = \frac{3g.}{4\pi RG}$$

13. The centres of a ring of mass m and a sphere of mass M of equal radius R, are at a distance $\sqrt{8}$ R apart as shown in fig. The force of attraction between the ring and the sphere is



1) $\frac{2\sqrt{2}}{27}\frac{GmM}{R^2}$ 2) $\frac{GmM}{8R^2}$ 3) $\frac{GmM}{9R^2}$ 4) $\frac{\sqrt{2}}{9}\frac{GmM}{9R^2}$

SOLUTION :

$$\text{S}\ dF = \frac{GMdm}{3R^2}; F = \sum dF\cos\theta$$

14. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$ (c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$ (d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

SOLUTION :

$$g = \frac{4}{3}\pi\rho GR$$

$$\therefore \frac{g_1}{g_2} = \frac{R_1\rho_1}{R_2\rho_2},$$

15. The mass of the earth is 81 times that of the moon and the radius of the earth is 3.5 times that of the moon. The ratio of the acceleration due to gravity at the surface of the moon to that at the surface of the earth is **[MP PMT 1994]**

(a) 0.15 (b) 0.04 (c) 1 (d) 6

SOLUTION :

$$g = \frac{GM}{R^2} \quad \text{(Given } M_e = 81M_m,\ R_e = 3.5R_m\text{)}$$

Substituting the above values,

$$\frac{g_m}{g_e} = 0.15$$

16. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M and E_1 and E_2 respectively. Then :

1) $\frac{E_1}{E_2} = \frac{r_1}{r_2}$ if $r_1<R$ and $r_2<R$

2) $\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$ if $r_1<R$ and $r_2<R$

3) $\frac{E_1}{E_2} = \frac{r_1^3}{r_3^2}$ if $r_1<R$ and $r_2<R$

4) $\frac{E_1}{E_2} = \frac{r_1^2}{r_2^2}$ if $r_1<R$ and $r_2<R$

SOLUTION :

If $r \le R$,

then $E = \frac{GM}{R^3}(r) \Rightarrow E \propto r$

$$\Rightarrow \frac{E_1}{E_2} = \frac{r_1}{r_2} \text{ if } r_1 < R \text{ and } r_2 < R$$

If $r \ge R$, then

$$E = \frac{GM}{r^2} \Rightarrow E \propto \frac{1}{r^2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{r_2^2}{r_1^2} \text{ if } r_1 > R \text{ and } r_2 > R$$

17. The value of g on the earth's surface is $980\ cm/\sec^2$. Its value at a height of 64 km from the earth's surface is(Radius of the earth $R = 6400$ *kilometers*)

(a) $960.40\ cm/\sec^2$ (b) $984.90\ cm/\sec^2$ (c) $982.45\ cm/\sec^2$ (d) $977.55\ cm/\sec^2$

SOLUTION :

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{6400}{6400+64}\right)^2 \Rightarrow g' = 960.40\ cm/s^2$$

18. The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is

(a) $g/4$ (b) $g/5$ (c) $g/6$ (d) $g/8$

SOLUTION :

Using $g = \frac{GM}{R^2}$ we get $g_m = g/5$

19. How much above the surface of earth does the acceleration due to gravity reduce by 36% of its value on the surface of earth.

SOLUTION :

Since g reduces by 36%, the value of g there is 100-36=64%.

It means, $g' = \frac{64}{100}g$.

If h is the height of location above the surface of earth, then,

$$g' = g\frac{R^2}{(R+h)^2} \Rightarrow \frac{64}{100}g = g\frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{8}{10} = \frac{R}{R+h} \Rightarrow h = \frac{R}{4} = \frac{6.4\times10^6}{4} = 1.6\times10^6 m$$

20. Weight of 1kg becomes 1/6 on moon. If radius of moon is $1.768\times10^6\ m$, then the mass of moon will be

(a) $1.99\times10^{30}\ kg$ (b) $7.56\times10^{22}\ kg$ (c) $5.98\times10^{24}\ kg$ (d) $7.65\times10^{22}\ kg$

SOLUTION :

$$g_m = \frac{GM_m}{R_m^2}$$

$$g_m = \frac{g_e}{6} = \frac{9.8}{6}\ m/s^2$$

$$= 1.63\ m/s^2$$

Substituting $R_m = 1.768\times10^6 m$,

$g_m = 1.63\ m/s^2$

and $G = 6.67\times10^{-11}\ N\text{-}m^2/kg^2$

We get $M_m = 7.65\times10^{22}\ kg$

21. Let g be the acceleration due to gravity at earth's surface and K be the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2% keeping all other quantities same, then

(a) g decreases by 2% and K decreases by 4% (b) g decreases by 4% and K increases by 2%

(c) g increases by 4% and K increases by 4% (d) g decreases by 4% and K increases by 4%

SOLUTION :

$$g = \frac{GM}{R^2} \text{ and } K = \frac{L^2}{2I}$$

If mass of the earth and its angular momentum remains constant then $g \propto \frac{1}{R^2}$

$$K \propto \frac{1}{R^2}$$

i.e. if radius of earth decreases by 2% then g and K both increases by 4%.

22. What should be the velocity of earth due to rotation about its own axis so that the weight at equator become 3/5 of initial value. Radius of earth on equator is 6400 *km*

(a) 7.4×10^{-4} *rad* / sec (b) 6.7×10^{-4} *rad* / sec (c) 7.8×10^{-4} *rad* / sec (d) 8.7×10^{-4} *rad* / sec

SOLUTION :

Weight of the body at equator = $\frac{3}{5}$ of initial weight

$\therefore \quad g' = \frac{3}{5}g$ (because mass remains constant)

$$g' = g - \omega^2 R \cos^2 \lambda$$

$$\Rightarrow \quad \frac{3}{5}g = g - \omega^2 R \cos^2 (0°)$$

$$\Rightarrow \quad \omega^2 = \frac{2g}{5R}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$$

$$= 7.8 \times 10^{-4} \frac{rad}{\sec}$$

23. Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface, what is the maximum range on the surface of the moon for the same velocity of projection

(a) $0.2\,R_e$ (b) $2\,R_e$ (c) $0.5\,R_e$ (d) $5\,R_e$

SOLUTION :

Range of projectile $R = \frac{u^2 \sin 2\theta}{g}$

if u and θ are constant then $R \propto \frac{1}{g}$

$$\frac{R_m}{R_e} = \frac{g_e}{g_m} \Rightarrow \frac{R_m}{R_e} = \frac{1}{0.2} \Rightarrow R_m = \frac{R_e}{0.2} \quad \Rightarrow \quad R_m = 5R_e$$

24. At what distance from the centre of the earth, the value of acceleration due to gravity g will be half that on the surface (R = radius of earth)

(a) $2R$ (b) R (c) $1.414\,R$ (d) $0.414\,R$

SOLUTION :

$$g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$$

$$\Rightarrow \quad R + h = \sqrt{2}\,R$$

$$\Rightarrow h = (\sqrt{2} - 1)R = 0.414\,R$$

25. A star 2.5 times the mass of the sun is reduced to a size of 12km and rotates with a speed of 1.5 rps. Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun=2x10^{30} kg).

SOLUTION :

$$\text{Acceleration due to gravity, } g = \frac{GM}{R^2}$$

$$= \frac{6.67\times10^{-11}\times2.5\times2\times10^{30}}{(12000)^2} = 2.3\times10^{12}\,ms^{-2}$$

$$\text{Centrifugal acceleration} = r\omega^2 = 2(2\pi f)^2 = 12000(2\pi\times1.5)^2 = 1.1\times10^{-6}\,ms^{-2}$$

Since, $g > r\omega^2$, the body will remain stuck with the surface of star.

26. A man can jump to a height of 1.5 m on a planet A. What is the height he may be able to jump on another planet whose density and radius are, respectively, one-quarter and one-third that of planet A

(a) 1.5 m (b) 15 m (c) 18 m (d) 28 m

SOLUTION :

$$H = \frac{u^2}{2g} \Rightarrow H \propto \frac{1}{g}$$

$$\Rightarrow \quad \frac{H_B}{H_A} = \frac{g_A}{g_B}$$

$$\text{Now } \quad g_B = \frac{g_A}{12} \text{ as } g \propto \rho R$$

$$\therefore \frac{H_B}{H_A} = \frac{g_A}{g_B} = 12$$

$$\Rightarrow H_B = 12\times H_A = 12\times1.5 = 18\,m$$

27. Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is

(a) 2% decrease (b) 0.5% decrease (c) 1% increase (d) 0.5% increase

SOLUTION :

$$\text{For height } \frac{\Delta g}{g}\times100\% = \frac{2h}{R} = 1\%;$$

$$\text{For depth } \frac{\Delta g}{g}\times100\% = \frac{d}{R} = \frac{h}{R} = \frac{1}{2}\% = 0.5\%$$

28. At what depth below the surface of the earth, acceleration due to gravity g will be half its value 1600 km above the surface of the earth

(a) $4.2 \times 10^6\ m$ (b) $3.19 \times 10^6\ m$ (c) $1.59 \times 10^6\ m$ (d) None of these

SOLUTION :

Radius of earth $R = 6400\ km$

$$\therefore \quad h = \frac{R}{4}$$

Acceleration due to gravity at a height h

$$g_h = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+\frac{R}{4}}\right)^2 = \frac{16}{25}g$$

At depth 'd' value of acceleration due to gravity

$$g_d = \frac{1}{2}g_h \text{ (According to problem)}$$

$$\Rightarrow \quad g_d = \frac{1}{2}\left(\frac{16}{25}\right)g$$

$$\Rightarrow g\left(1 - \frac{d}{R}\right) = \frac{1}{2}\left(\frac{16}{25}\right)g$$

By solving we get $d = 4.3 \times 10^6 m$

29. If earth is supposed to be a sphere of radius R, if g_{30} is value of acceleration due to gravity at latitude of 30° and g at the equator, the value of $g - g_{30^o}$ is

(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$ (c) $\omega^2 R$ (d) $\frac{1}{2}\omega^2 R$

SOLUTION :

Acceleration due to gravity at latitude λ is given by

$$g' = g - R\omega^2 \cos^2 \lambda$$

At 30^o, $g_{30^\circ} = g - R\omega^2 \cos^2 30^o = g - \frac{3}{4}R\omega^2$

‘

$$\therefore \quad g - g_{30} = \frac{3}{4}\omega^2 R.$$

30. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2 / T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

SOLUTION :

If acceleration due to gravity is g at the surface of earth then at height R it value becomes

$$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$$

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{l}{g/4}}$$

$$\frac{T_2}{T_1} = 2$$

31. Consider earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a balloon. The value of g measured by

(a) A goes on decreasing and that by B goes on increasing

(b) B goes on decreasing and that by A goes on increasing

(c) Each decreases at the same rate

(d) Each decreases at different rates

SOLUTION :

For scientist A which goes down in a mine $g' = g\left(1 - \frac{d}{R}\right)$

For scientist B, which goes up in a air $g' = g\left(1 - \frac{2h}{R}\right)$

So it is clear that value of g measured by each will decreases at different rates.

32. A clock S is based on oscillation of a spring and a clock P is based on pendulum motion. Both clocks run at the same rate on earth. On a planet having the same density as earth but twice the radius

(a) S will run faster than P

(b) P will run faster than S

(c) They will both run at the same rate as on the earth

(d) None of these

SOLUTION :

$$g = \frac{4}{3}\pi\rho GR\ .$$

If density is same then $g \propto R$

According to problem $R_p = 2R_e$

$\therefore\ g_p = 2g_e$

For clock P (based on pendulum motion) $T = 2\pi\sqrt{\frac{l}{g}}$

Time period decreases on planet so it will run faster because $g_p > g_e$

For clock S (based on oscillation of spring) $T = 2\pi\sqrt{\frac{m}{k}}$

So it does not change.

Gravitational Field:

It is the region or space around a massive particle in which its gravitational influence is left.

Gravitational field strength (or) Intensity of Gravitational Field:

Gravitational field strength at any point in a gravitational field is defined as the gravitational force experienced by a unit mass placed at that point.

$\therefore$ Gravitational field strength, $\overrightarrow{E_g} = \dfrac{\vec{F}}{m_0}$

- Units : Nkg^{-1} or ms^{-2}
- Dimensional formula: LT^{-2}
- It is a vector quantity.
- It is always directed radially towards the centre of mass of the body producing the field.

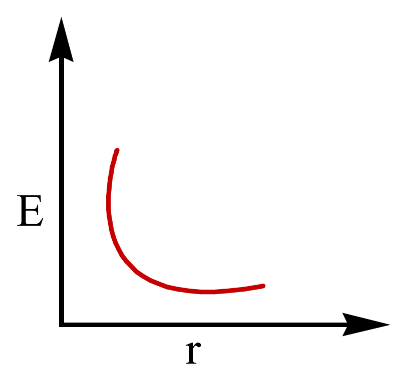
Note:

In the earth's gravitational field, $\overrightarrow{E_g} = \dfrac{\vec{F}}{m_0} = \dfrac{m_0\vec{g}}{m_0} = \vec{g}$.

Hence in the earth's gravitational field, the intensity of gravitational field is equal to acceleration due to gravity 'g'.

- The intensity of gravitational field at a distance r from a point mass 'M' is given by $E_g = \dfrac{GM}{r^2}$
- The direction of the force F and hence of E is from P to O as shown in fig.

' In vector form the above formula is $\overrightarrow{E_g} = \dfrac{GM}{r^3}\overrightarrow{(r)}$.



- Theoretically gravitational field due to a particle extends upto infinite distance around it
- The value of E_g is zero at $r = \alpha$.
- If the system has a number of masses, then resultant gravitational field intensity can be found out by using the principle of super-position.

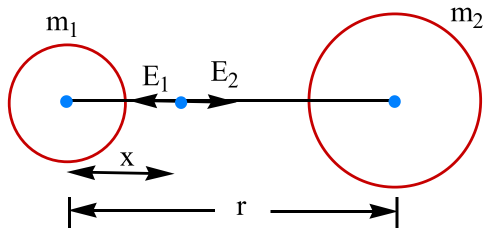
i.e. $\overrightarrow{E_g} = \overrightarrow{E_{g_1}} + \overrightarrow{E_{g_2}} + \overrightarrow{E_{g_3}} + ...$

Null Point :

It is the point in a gravitational field at which resultant field intensity is zero.

If two particles of mass m_1 & m_2 are separated by a distance r, the distance of null point from m_1 is given by

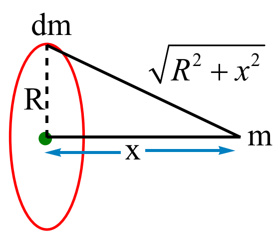
$$E_1 - E_2 = 0 \Rightarrow \frac{Gm_1}{x^2} = \frac{Gm_2}{(r-x)^2}; x = \frac{r}{\sqrt{\frac{m_2}{m_1+1}}}$$



Force and Field due to Circular Uniform Ring:

♦ A point mass m is at a distance x from the centre of the ring of mass M and radius R on its axis. Gravitational force between the two is

$$F = \frac{GMmx}{(R^2+x^2)^{3/2}}$$



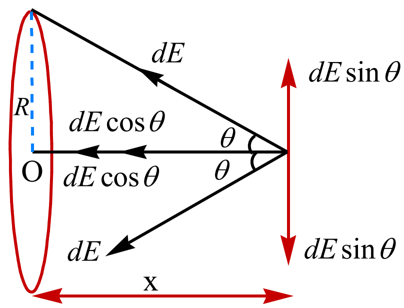
♦ If x>>>R $\Rightarrow F = \frac{GMm}{x^2}$, then for a distant point, ring behaves as point mass.

♦ If x<<<R $\Rightarrow F = \frac{GMm}{R^3}$, then force varies linearly as distance 'x'

♦ Force is maximum, at $x = \pm\frac{R}{\sqrt{2}}$ maximum force $F_{\max} = \frac{2GMm}{3\sqrt{3}R^2}$

♦ Gravitational field intensity due to a uniform circular ring of mass M at any point at a distance 'x' (from the centre of the ring) on its axis is

$$E_g = \frac{GMx}{(X^2+R^2)^{3/2}} \text{ along } \overrightarrow{PO}$$

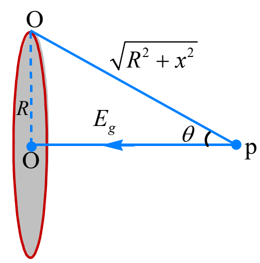


- Gravitational field intensity is directed towards the centre of the circular ring.
- At the centre of the circular ring, $E_g = 0$
- E_g is maximum, at $x = \dfrac{R}{\sqrt{2}}$ and

$$E_{max} = \frac{2GM}{3\sqrt{3}R^2}$$

Field due to Circular Disc:

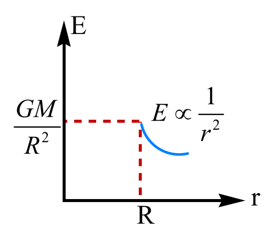
Gravitational field intensity due to a circular disc of mass M at any point on the axial line



or $E_g = \dfrac{2GM}{R^2}(1 - Cos\theta)$ (in terms of 'θ')

Field due to Hollow sphere (or)Spherical Shell (E or I):

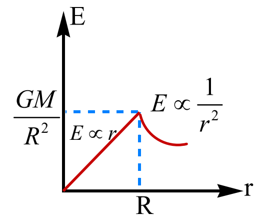
- Gravitational field intensity due to a uniform spherical shell



- At a point inside the spherical shell, $\left(E_g\right)_{inside} = 0, \left(E_g\right)_{centre} = zero$
- At a point on surface of the spherical shell, $\left(E_g\right)_{surface} = \dfrac{GM}{R^2}$ (here r=R)
- At a point outside the spherical shell, $\left(E_g\right)_{outside} = \dfrac{GM}{r^2}$ (here r>R)

Field due to Solid Sphere (uniform mass density):

Gravitational field intensity due to a solid sphere



- $E_g = 0$ (at the centre of solid sphere)
- $\left(E_g\right)_{inside} = \frac{GMr}{R^3}\left(for \;\; r < R\right)$
- At a point on the surface of the solid sphere, $\left(E_g\right)_{surface} = \frac{GM}{R^2}\left(for \; r = R\right)$
- At a point outside the solid sphere, $\left(E_g\right)_{outside} = \frac{GM}{r^2} \;\; \left(for \; r > R\right)$
- $E_g = 0$ (at infinite distance)

Field due to Straight Rod:

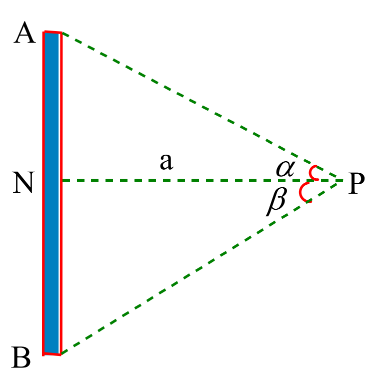
A rod of length 2l, density ρ ,placed along x- axis, such that mid point of rod is coincides with origin. The gravitational field intensity at a point P(a,o) is

$$E_g = \frac{2G\rho}{a}\frac{l}{\sqrt{l^2 + a^2}} = \frac{2G\rho}{a}\frac{1}{\sqrt{1+\frac{a^2}{l^2}}}$$

$$E_g = \frac{2G\rho}{a}\left(1 - \frac{a^2}{2l^2} + high \;\; powers \;\; of \;\; \frac{a^2}{l^2} + - - - - -\right)$$

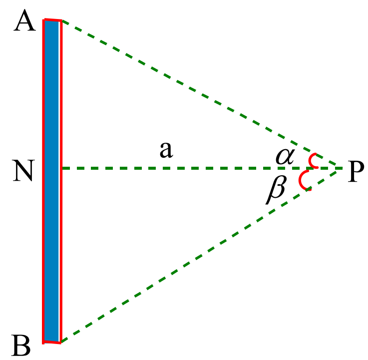
EXAMPLE:

Calculate the gravitational field due to a uniform rod AB at a point P at perpendicular distance a from the rod as shown in figure. Assume that the rod has a linear mass density λ .



Solution :

Here it is very improtant to note that the unsymmetrical placement of the point P and hence we must calculate the components of the gravitational fields E_x and E_y separately. For this, let us consider an infinitesimal element of length dx at a distance x as shown



The net gravitation field at a point P due to this infinitesimal element is dE. This dE is resolved into components.

(a) $dE_x = dE\cos\theta$ (b) $dE_y = dE\sin\theta$

The net field E can be calculated b finding E_x and E_y from the above expressions. so

$E_x = \int dE\cos\theta$, where $dE = \frac{G(\lambda dx)}{r^2}$

$$\Rightarrow E_x = G\lambda\int\frac{dx\cos\theta}{r^2} = G\lambda\int\frac{dx\cos\theta}{(a^2+x^2)}$$

Also, we observe that $\tan\theta = \frac{x}{a}$

$$\Rightarrow x = a\tan\theta$$

$$\Rightarrow dx = a\sec^2\theta d\theta$$

$$\Rightarrow E_x = G\lambda\int\frac{a\sec^2\theta\cos\theta d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow E_x = \frac{G\lambda}{a}\int_{-\beta}^{\alpha}\cos\theta d\theta$$

$$\Rightarrow E_x = \frac{G\lambda}{a}(\sin\beta+\sin\alpha)$$

Similarly, let us calculate the value of E_y, given by

$$E_y = \int dE_y = \int dE\sin\theta$$

$$\Rightarrow E_y = G\lambda\int\frac{dx\sin\theta}{(a^2+x^2)}$$

Since $x = a\tan\theta$

$$\Rightarrow dx = a\sec^2\theta d\theta$$

$$\Rightarrow E_y = G\lambda\int\frac{a\sec^2\theta\sin\theta d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow E_y = \frac{G\lambda}{a}\int_{-\beta}^{\alpha}\sin\theta d\theta$$

$$\Rightarrow E_y = \frac{G\lambda}{a}\left|-\cos\theta\right|_{-\beta}^{\alpha}$$

So, the gravitational field due to a rod of length having uniform mass density λ at a point P, that subtends and angle α at one end and β at the other is given by

$$E_x = \frac{G\lambda}{a}(\sin\beta + \sin\alpha)$$

$$E_y = \frac{G\lambda}{a}(\cos\beta - \cos\alpha)$$

♦ A thin rod of mass M and length L is bent into a complete circle, then resultant force on a particle placed at its centre is zero.

Work Done Against Gravity:

If the body of mass m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow \quad W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \quad [\text{As } r_1 = R \text{ and } r_2 = R + h]$$

$$\Rightarrow \quad W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}} \quad [\text{As } \frac{GM}{R^2} = g]$$

♦ When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

♦ If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

♦ If $h = R$ then $W = \frac{1}{2}mgR$

♦ If h is very small as compared to radius of the earth then term h/R can be neglected

From $W = \frac{mgh}{1 + h/R} = mgh \qquad \left[\text{As } \frac{h}{R} \to 0\right]$

Gravitational Potential:

The amount of work done in bringing a unit mass from infinity to a certain point in the gravitational field of another massive object is called gravitational potential at that point due to massive object .

Let W is the work done and m_0 is the test mass

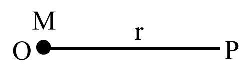
then $V = \frac{W}{m_0}$

As this work is negative, the gravitational potential is negative

S.I unit : J/Kg

Dimensional formula : $\left[M^0 L^2 T^{-2}\right]$

Potential due to a point mass:



The gravitational potential at a point p which is at a distance r from a point mass M is given by

$$V = -\frac{GM}{r}$$

In the system has a number of masses $m_1, m_2, m_3 .... m_n$ at distances $r_1, r_2, r_3 .... r_n$ from the point 'p', the resultant gravitational potential at a point 'p' can be written as

$$V = V_1 + V_2 + V_3 + ...V_n$$

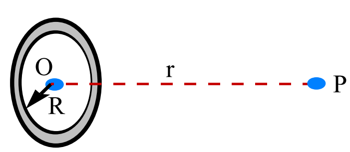
$$\Rightarrow V = -G\left[\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + ... + \frac{m_n}{r_n}\right]$$

$$\Rightarrow V = -G\sum_{i=1}^{n}\frac{m_i}{r_i}$$

Potential due to Circular Ring:

Gravitational potential due to a circular ring, at a distance r from the centre and on the axis of a ring of mass M and radius x is given by

$$V = \frac{-GM}{\sqrt{R^2 + x^2}}$$



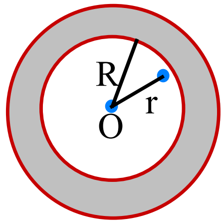
♦ At r=0, $v = -\frac{GM}{R}$,

i.e., at the centre of the ring gravitational potential is $-\frac{GM}{R}$

Gravitational potential due to a spherical shell:

Let M be the mass of spherical shell and R is its radius $V = \frac{-GM}{r}$

- At a point inside the spherical shell, (If r<R) $V_{inside} = \frac{-GM}{R}$

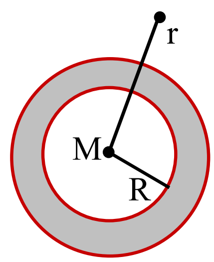


- At a point on the surface of the spherical shell, $V_{surface} = \frac{-GM}{R}$ (If r=R)

$v_{centre} = -\frac{GM}{R}$ (r=0 at centre)

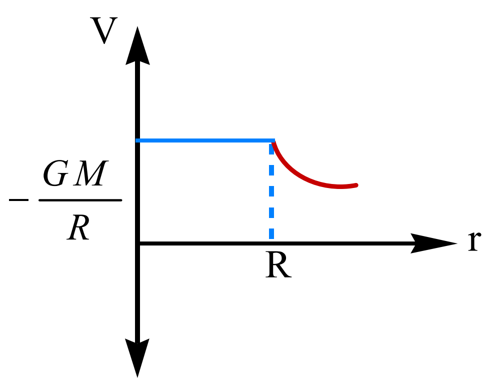
$v_{inside} = v_{surface} = v_{centre} = -\frac{GM}{R}$,

- At a point outside the spherical shell, $V_{outside} = \frac{-GM}{r}$ (If r>R)



- At infinity, $V_{\infty} = 0$

graphical



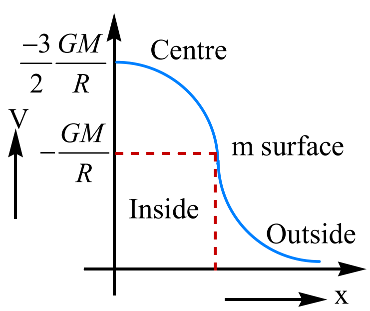
Gravitational potential due to a solid sphere:

At a point inside the solid sphere,

$$V_{inside} = \frac{-GM}{2R^3}(3R^2 - x^2)$$

$$V_{inside} = -GM\left(\frac{3}{2R} - \frac{r^2}{2R^3}\right) (\text{if } x<R)$$

- At a point on the surface of the solid sphere, $V_{surface} = \frac{-GM}{R}$ (If x=R)
- At a point outside the solid sphere, $V_{outside} = \frac{-GM}{x}$ (If x>R)
- At the centre, x=0 $\Rightarrow V_c = -\frac{3}{2}\frac{GM}{R} = \frac{3}{2}V_{surface}$.
- In case of solid sphere potential is maximum at centre.



Newton's Shell Theorem :

Gravitational potential at a point outside of a solid (or) hollow sphere of mass M is same as potential at that point due to a point mass of M separated by same distance. Hence, the sphere can be replaced by a point mass.

Gravitational potential difference :

The amount of work done in bringing a unit mass between two points in the gravitational field is called as the gravitational potential difference between the two points.

$$\Delta V = V_b - V_a = -\left(\frac{W_b - W_a}{m_0}\right)$$

$$W_{ab} = -m_0(V_b - V_a) = -Gmm_0\left(\frac{1}{r_0} - \frac{1}{r_a}\right)$$

Relation Between Gravitational Field and Potential:

- Gravitational field and the gravitational potential are related $\vec{E} = -\text{gradient}\,V = -\text{grad}\,V$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\dfrac{\partial V}{\partial x}$ = Partial derivative of potential function V with respect to x, i.e., differentiate V wrt x assuming y and z to be constant.

- The above equation can be written in the following forms.
- $E = \dfrac{-dV}{dx}$, If gravitational field is along x-direction only.
- $dV = -\vec{E}.d\vec{r}$, (where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$ and $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$)

Note :

a) If $\vec{E}$ is given V can be calculated by the formula $V = \int_{\infty}^{r} dV = -\int_{\infty}^{r} \vec{E}.\vec{dr}$

b) The negative slope of V-r curve gives E

Gravitational Potential Energy:

The amount of work done by the gravitational force in bringing a body from infinity to any point in the gravitational field is defined as the gravitational potential energy at that point.

For a conservative field, $F = -\dfrac{dU}{dr}$

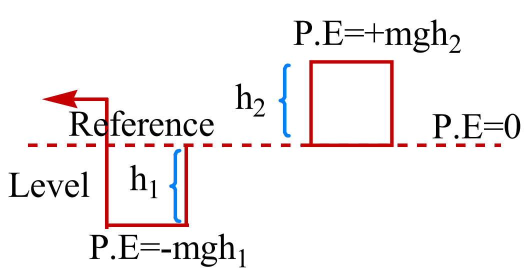
$$\Rightarrow dU = -\vec{F}.d\vec{r}$$

$$\Rightarrow \int_{u_0}^{u} dU = -\int_{r_0}^{r} \vec{F}.d\vec{r} \Rightarrow U - U_0 = -\int_{r_0}^{r} \vec{F}.d\vec{r}$$

We generally choose the reference point at infinity and assume potential energy to be zero there. If we take $r_0 = \infty$ and $U_0 = 0$ then

$$\Rightarrow U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W \left[as \int_{\infty}^{r} \vec{F}.d\vec{r} = W \right]$$

Potential energy of a body or system is the negative work done by the conservative forces in bringing it from infinity to present position.



- If a particle moves opposite to the field direction then work done by the field will be negative. So potential energy will increase and change in potential energy will be positive.
- If a particle moves in the direction of the field work done is positive, so potential energy decreases and change in potential energy is negative.

- potential energy exist for only conservative forces and it does not exist for non conservative forces.
- By the definition of gravitational potential,

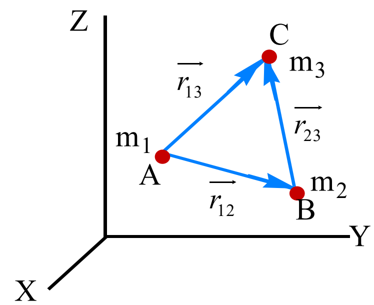
$$V = -\frac{W}{m} = \frac{U}{m} \Rightarrow U = mV$$

Gravitational Potential Energy of Two Particle System:

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

Gravitational Potential Energy of Three Particle System



Consider a system consists of three particles of masses m_1, m_2 and m_3 located at A, B and C respectively. Total potential energy 'U' of the system is

$$U = -G\left|\frac{m_1m_2}{|\vec{r}_{12}|} + \frac{m_2m_3}{|\vec{r}_{23}|} + \frac{m_1m_2}{|\vec{r}_{13}|}\right|$$

If a body is moving only under the influence of gravitational force, from law of conservation of mechanical energy

$$U_1 + K_1 = U_2 + K_2$$

Gravitational Potential Energy For a System of Particles:

The gravitational potential energy for a system of n particles is given by

$$U = \sum U_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \text{...........}\right]$$

For a n particle system there are $\frac{n(n-1)}{2}$ pairs and the potential energy is calculated for each pair and added to get the total potential energy of the system.

Gravitational Potential Energy of a Body in Earth's Gravitational Field:

- If a point mass 'm' is at a distance r from the centre of the earth. $U = -\frac{GMm}{r}$

- On the surface of earth, $U_{surface} = -\frac{GMm}{R} = -mgR\left(\because g = \frac{GM}{R^2}\right)$

- At a height 'h' above the surface of earth, $U_h = -\frac{GMm}{R+h}$
- The difference in potential energy of the body of mass m at a height h and on the surface of earth is

$$\Delta U = U_h - U_{surface}$$

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{r}\right)$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= \frac{GMmh}{(R+h)R} = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)}$$

$$\Rightarrow \Delta U = \frac{mgh}{1+\frac{h}{R}}$$

If $h << R,\ \Delta U \approx mgh$

- Work done in lifting a body of mass m from earth surface to a height h above the earths surface is

$$W = U_h - U_{surface};$$

$$W = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= \frac{mgh}{1+\frac{h}{R}}$$

- Gravitational potential energy at the centre of the earth is given by $U_c = mv_c = -\frac{3}{2}\frac{GMm}{R}$

Here, $V_c = \frac{3}{2}V_s = \frac{-3GM}{2R}$

(It is minimum but not zero. However 'g' at centre of earth is zero)

Self potential energy of a uniform sphere of mass 'M' and radius 'R' :

It is the amount of work done to bring identical massive particles to construct a sphere of mass M radius R and density ρ

For a sphere of radius 'x', mass of the sphere $=\frac{4}{3}\pi x^3\rho$,

where ρ =density of sphere

Gravitational potential on the surface $=\frac{-4}{3}\pi G\rho x^2$

(since gravitational potential $=-\frac{Gm}{x}=\frac{-G}{x}\times\frac{4}{3}\pi x^3\rho=\frac{-4}{3}\pi Gx^2\rho$)

Work done by the agent in increasing the surface from x to x+dx is

$\frac{-Gm(dm)}{x}$ =Gravitational potential x dm

$$=\left(\frac{-4}{3}\pi Gx^2\rho\right)\left(4\pi x^2dx\rho\right)=\frac{16\pi r^2}{3}G\rho^2x^4dx$$

Therefore, total work done $=\frac{-16\pi^2}{3}G\rho^2\int_0^R x^4dx=\frac{-16\pi^2G\rho^2R^5}{15}$

Therefore, total work done$=\frac{-16\pi^2}{3}G\rho^2\int_0^R x^4dx=\frac{-16\pi^2G\rho^2R^5}{15}$

$$=\frac{-16\pi^2GR^5}{15}\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2=\frac{-3}{5}\frac{GM^2}{R}$$

=Gravitational self potential energy of a sphere.

- **Self potential energy of a thin uniform shell of mass 'm' and radius 'R' is** $-\frac{Gm^2}{2R}$

Change in the gravitational potential energy:

Change in the gravitational potential energy in lifting a body from the surface of the earth to a height equal to 'nR' from the surface of the earth

$$\Delta U=\frac{GMmh}{R(R+h)}=\frac{GMm(nR)}{R(R+nR)}=\frac{GMmm}{R(n+1)}=\frac{mgRn}{n+1}$$

PROBLEMS

1. Two bodies of masses 100Kg and 10,000Kg are at a distance of 1m apart. At what distance from 100kg on the line joining them will the resultant gravitational field intensity be zero?

SOLUTION:

$$\frac{G\times 100}{x^2}=\frac{G\times 10,000}{(1-x)^2}$$

$$\Rightarrow 100x^2=(1-x)^2 \Rightarrow x=\frac{1}{11}$$

2. The potential energy of a body of mass 'm' is given by U=px+qy+rz. The magnitude of the acceleration of the body will be

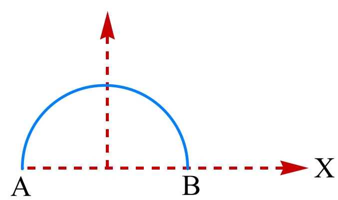
1) $\frac{p+q+r}{m}$ 2) $\frac{\sqrt{p^2+q^2+r^2}}{m}$ 3) $\frac{\sqrt{p^3+q^3+r^3}}{m}$ 4) $\frac{\sqrt{p^4+q^4+r^4}}{m}$

SOLUTION:

$$F=\frac{-dU}{dr};F_x=-p,F_y=-q,F_z=-r$$

$$\left|\vec{F}\right|=\sqrt{p^2+q^2+r^2}\Rightarrow ma=\sqrt{p^2+q^2r^2}$$

3. Gravitational field intensity at the centre of the semi circle formed by a thin wire AB of mass 'm' and length 'L' is



1) $\frac{Gm^2}{L^2}(\hat{i})$ 2) $\frac{Gm^2}{\pi L^2}(\hat{j})$ 3) $\frac{2\pi Gm}{L^2}(\hat{i})$ 4) $\frac{2\pi Gm}{L^2}(\hat{j})$

SOLUTION:

$$\lambda=\frac{m}{L};L=\pi r;dm=\lambda dl=\lambda(rd\theta)$$

$$E=\frac{G\lambda}{r}\left[\int_0^\pi \cos\theta d\theta\hat{i}+\int_0^\pi \sin\theta d\theta\hat{j}\right]$$

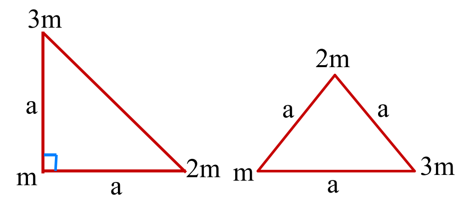
4. The gravitational field due to a mass distribution is given by E=-K/x³ in x-direction. Taking the gravitational potential to be zero at infinity, find its value at a distance x.

SOLUTION:

The potential at a distance x is

$$V=-\int Edx=\int_\infty^x \frac{K}{x^3}dx=\left[\frac{-K}{2x^2}\right]_\infty^x=\frac{-K}{2x^2}$$

5. Consider two configurations in fig(i) and fig(ii)



fig(i) fig(ii)

The work done by external agent in changing the configuration from fig(i) to fig(ii) is

1) Zero 2) $-\frac{6Gm^2}{a}\left(1+\frac{1}{\sqrt{2}}\right)$ 3) $-\frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$ 4) $-\frac{6Gm^2}{a}\left(2-\frac{1}{\sqrt{2}}\right)$

SOLUTION:

$$GPE = \frac{-Gm_1m_2}{r}; W = GPE_2 - GPE_1$$

6. A particle of mass m is placed at the centre of a uniform spherical shell of equal mass and radius a. Find the gravitational potential at a point P at a distance a/2 from the centre.

SOLUTION:

The gravitational potential at P due to particle at centre is $V_1 = \frac{-Gm}{a/2} = \frac{-2Gm}{a}$

The potential at P due to shell is $V_2 = \frac{-Gm}{a}$

The net potential at P is $V_1 + V_2 = \frac{-3Gm}{a}$

7. The gravitational field in a region is given by $\vec{E} = -\left(20Nkg^{-1}\right)\left(\vec{i}+\vec{j}\right)$. Find the gravitational potential at the origin (0, 0)

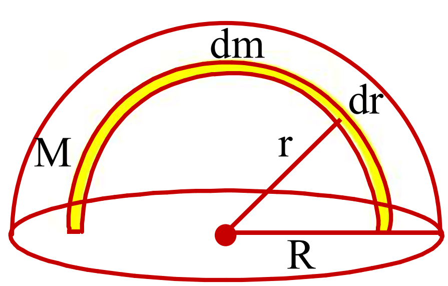
SOLUTION:

$$V = -\int \vec{E}.\vec{dr} = -\left[\int E_x dx + \int E_y dy\right] = 20x + 20y$$

$\Rightarrow V = 0$ at the origin (0, 0).

8. Calculate the gravitational potential at the centre of base of a solid hemisphere of mass M, radius R.

SOLUTION:



Consider a hemispherical shell of radius r and thickness dr. Its mass is given by

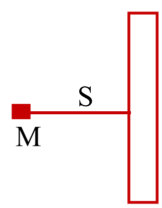
$$dm = \frac{M}{\frac{2}{3}\pi R^3}\left(2\pi r^2 dr\right) = \frac{3Mr^2 dr}{R^3}$$

Since all points of this hemispherical shell are at the same distance r from centre O, potential at O due to it is

$$dV = \frac{-Gdm}{r} = \frac{-3GMrdr}{R^3}$$

$$\therefore V = \int_0^R dV = \frac{-3GM}{2R}.$$

9. A point mass M is at a distance S from an infinitely long and thin rod of linear density D. If G is the gravitational constant then gravitational force between the point mass and the rod is



1) $2\frac{MGD}{S}$ 2) $\frac{MGD}{S}$ 3) $\frac{MGD}{2S}$ 4) $\frac{2}{3}\frac{MGD}{S}$,

SOLUTION:

$$dm = D \times dl = D \times \frac{Sd\alpha}{\cos\alpha}$$

Gravitational force, $DF = \frac{GMdm}{\left(\frac{S}{\cos\alpha}\right)}\cos\alpha$

total force $F = \int_{-\pi/2}^{\pi/2} \frac{MGD}{S}\cos\alpha \; d\alpha = \frac{2MGD}{S}$

10. The gravitational field in a region is given by the equation $E = \left(5\mathbf{i} + 12\mathbf{j}\right)N/kg$. If a particle of mass 2kg is moved from the origin to the point (12m, 5m) in this region, the change in the gravitational potential energy is (2012 E)

SOLUTION:

$$dV = -\vec{E}.\vec{dr}$$

$$= -\left(5\mathbf{i} + 12\mathbf{j}\right).\left(12\mathbf{i} + 5\mathbf{j}\right) = -\left(60 + 60\right) = -120$$

Change in gravitational potential energy

dU=mdV

=2(-120)=-240 J

11. **A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of sphere. The gravitational force on a particle of mass m at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is (opposite to the centre of cavity) [here $g=\frac{GM}{R^2}$ where M is the mass of the sphere].**

1) $\frac{mg}{2}$ 2) $\frac{3mg}{2}$ 3) $\frac{mg}{16}$ 4) $\frac{mg}{4}$

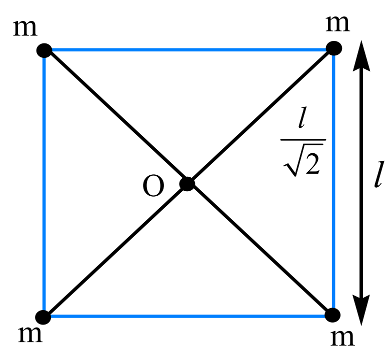
SOLUTION:

$$E_1=\frac{\rho R}{6\varepsilon_0}, E_e=\frac{-\rho(R/2)^3}{3\varepsilon_0 R^2}$$

$$E_{net}=E_1+E_e; \rho\frac{M}{\frac{4}{3}\pi R^3}; \varepsilon_0=\frac{1}{4\pi G}$$

12. **Find the gravitational potential energy of a system of four particles, each of mass m placed at the vertices of a square of side *l*. Also obtain the gravitational potential at centre of the square.**

SOLUTION:



The system has four pairs with distance *l* and two diagonal pairs with distance $\sqrt{2}l$.

$$U=-4\frac{Gm^2}{l}-2\frac{Gm^2}{\sqrt{2}l}=\frac{2Gm^2}{l}\left(2+\frac{1}{\sqrt{2}}\right)$$

The gravitational potential the centre of the square is

V=Algebraic sum of potential due to each particle

$$V=-\frac{4Gm}{r}=\frac{4\sqrt{2}Gm}{l}\quad\left(\because r=\frac{\sqrt{2}l}{2}\right)$$

13. **A mass *m* extends a vertical helical spring of spring constant *k* by *x* m at the surface of earth. Extension in spring by the same mass at height *h* meter above the surface of earth is**

1) $\frac{GMm}{k(R+h)}$ 2) $\frac{GMm}{kR^2}$ 3) $\frac{(R+h)^2}{R^2}x$ 4) $\frac{R^2}{(R+h)^2}x$

SOLUTION:

Let the extension at height h be x' then

$$x = \frac{GMm}{kR^2}\left(\therefore F = kx \text{ or } x = \frac{F}{k}\right)$$

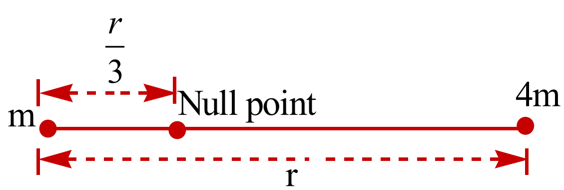
$$\text{then } \frac{x'}{x} = \frac{R^2}{k(R+h)^2}; \quad x' = \frac{R^2}{(R+h)^2}x$$

14. Two bodies of mass m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where gravitational field is zero is (2011A)

SOLUTION:

Position of null point from mass m is

$$x = \frac{r}{\sqrt{\frac{4m}{m}}+1} = \frac{r}{3}$$



$$\therefore \text{ potential } V = -Gm\left(\frac{3}{r}+\frac{12}{2r}\right) = \frac{-9Gm}{r}$$

15. The change in potential energy, when a body of mass m is raised to a height nR from the earth's surface is (R = Radius of earth)

(a) $mgR\frac{n}{n-1}$ (b) $nmgR$ (c) $mgR\frac{n^2}{n^2+1}$ (d) $mgR\frac{n}{n+1}$

SOLUTION:

$$\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mg\quad nR}{1+\frac{nR}{R}} = \frac{nm\quad gR}{n+1}$$

16. Four particles each of mass M are located at the vertices of a square with side L. The gravitational potential due to this at centre of square is

1) $-\sqrt{32}\frac{GM}{L}$ 2) $-\sqrt{64}\frac{GM}{L^2}$ 3) Zero 4) $-\sqrt{16}\frac{GM}{L}$

SOLUTION: $U = -4\frac{GM}{L/\sqrt{2}} = -\sqrt{32}\frac{GM}{L}$

17 . A body of mass m rises to height $h = R/5$ from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is

(a) mgh (b) $\frac{4}{5}mgh$ (c) $\frac{5}{6}mgh$ (d) $\frac{6}{7}mgh$

SOLUTION:

$$\Delta U = \frac{mgh}{1 + h/R}$$

Substituting $R = 5h$

we get $\Delta U = \frac{mgh}{1 + 1/5} = \frac{5}{6}mgh$

18. In a certain region of space, the gravitational field is given by -k/r, where *r* is the distance and *k* is a constant. If the gravitational potential at $r=r_0$ be V_0, then what is the expression for the gravitational potential V?

1) $k\log(r/r_0)$ 2) $k\log(r_0/r)$ 3) $V_0 + k\log(r/r_0)$ 4) $V_0 + k\log(r_0/r)$

SOLUTION:

Here, $I = -\frac{dV}{dr} = -k/r \ (or) dV = k\frac{dr}{r}$ then integrate

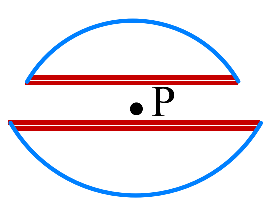
19. Two masses 90kg and 160kg are 5 m apart. The gravitational field intensity at a point 3m from 90kg and 4m from 160kg is

1) 10G 2) 5 G 3) $5\sqrt{2}G$ 4) $10\sqrt{2}G$

SOLUTION:

$$E_R = \sqrt{E_1^2 + E_2^2}$$

20'. A spherical shell is cut into two pieces along a chord as shown in the figure. P is a point on the plane of the chord. The gravitational field at P due to the upper part I_1 and that due to the lower part is I_2. What is the relation between them?



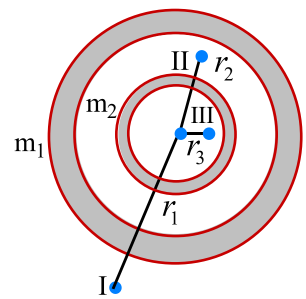
1) $I_1 > I_2$ 2) $I_1 < I_2$ 3) $I_1 = I_2$ 4) no definite relation

SOLUTION:

At the point P, we have I_1-I_2=0 (because the gravitational field inside a shell it is zero).

'Hence I_1=I_2

21. Two concentric shells of different masses m_1 and m_2 are having a sliding particle of mass m. The forces on the particle at position I, II and III are



1) $0, \frac{Gm_1}{r_2^2}, \frac{G(m_1+m_2)m}{r_1^2}$

2) $\frac{Gm_2}{r_2^2}, 0, \frac{Gm_1}{r_1^2}$

3) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{Gm_2}{r_2^2}, 0$

4) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{G(m_2)m}{r_2^2}, 0$

SOLUTION:

Position I. $F = \frac{Gm(m_1+m_2)}{r_1^2}$

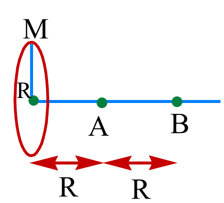
(here the particle lies outside of both the shells)

Position II. $F = \frac{Gm_2}{r_2^2}$

here the particle lies outside of the shell of mass m_1

Position III. Here the particle lies inside of both of the shells so F=0.

22. A ring has non-uniform distribution of mass having mass 'M' and radius 'R'. A point mass m_0 is moved from A to B along the axis of the ring. The work done by external agent against gravitational force of ring is



1) $\frac{GMm_0}{\sqrt{2}R}$

2) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$

3) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right]$

4) $\frac{GMm_0}{\sqrt{5}R}$

SOLUTION:

$$W = m[V_B - V_A];$$

23. The gravitational field due to a mass distribution is $E = K/x^3$ in the x-direction. (K is a constant). Taking the gravitational potential to be zero at infinity, its value at a distance x is

(a) K/x (b) $K/2x$ (c) K/x^2 (d) $K/2x^2$

SOLUTION:

Gravitational potential $= \int I\,dx = \int_x^\infty \frac{K}{x^3}dx$

$$= K\left(\frac{x^{-3+1}}{-3+1}\right)_x^\infty = \left|\frac{-K}{2x^2}\right|_x^\infty = \frac{K}{2x^2}$$

24. The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio 3:4. If the two shells coalesce into single one such that surface density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

1) 3:2 2) 4:3 3) 5:3 4) 5:6

SOLUTION:

$$4\pi r^2\rho = 4\pi r_1^2\rho + 4\pi\rho_2^2\rho \Rightarrow r^2 = r_1^2 + r_2^2$$

$$V = \frac{-GM}{r} = -\frac{G4\pi r^2\rho}{r}$$

$$V = -4\pi rG\rho \Rightarrow V \propto r$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{r_1}{r_2} = \frac{3}{4} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{9}{16}$$

$$r_1^2 : r_2^2 : r^2 = r_1^2 : r_2^2 : \left(r_1^2 + r_2^2\right) = 9:16:(19+16)$$

$$\Rightarrow r_1 : r_2 r = 3:4:5 = V_1 : V_2 : V_3$$

25. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are distance d apart. The minimum velocity with which a particle of mass m should be projected from a point midway between their centres so that it escapes to infinity is

(a) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (b) $2\sqrt{\frac{2G}{d}(M_1+M_2)}$ (c) $2\sqrt{\frac{Gm}{d}(M_1+M_2)}$ (d) $2\sqrt{\frac{Gm(M_1+M_2)}{d(R_1+R_2)}}$

SOLUTION:

Gravitational potential at mid point $V = \frac{-GM_1}{d/2} + \frac{-GM_2}{d/2}$

Now, $PE = m \times V = \frac{-2Gm}{d}(M_1 + M_2)$

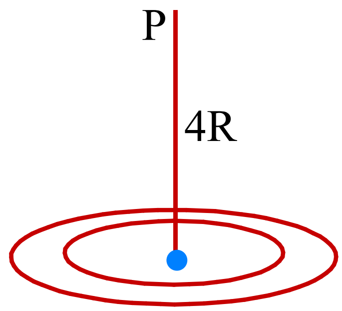
m = mass of particle

So, for projecting particle from mid point to infinity $KE = |PE|$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\Rightarrow v = 2\sqrt{\frac{G(M_1+M_2)}{d}},$$

26. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



1) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ 2) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ 3) $\frac{GM}{2R}$ 4) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

SOLUTION:

$$dm = \frac{M\left(2\pi r\right)dr}{\pi\left(16R^2-9R^2\right)4R}$$

$$DV = \frac{-G\left(dm\right)}{\sqrt{r^2+16R^2}}; V = \int_{3R}^{4R} dV,$$

$$W = m\left[V_\infty - V\right]$$

27. The gravitational potential energy of a body of mass 'm' at the earth's surface $-mgR_e$. Its gravitational potential energy at a height R_e from the earth's surface will be (Here R_e is the radius of the earth)

(a) $-2mgR_e$ (b) $2mgR_e$ (c) $\frac{1}{2}mgR_e$ (d) $-\frac{1}{2}mgR_e$

SOLUTION:

$$\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$$

$$\Rightarrow U_2 - (-mgR_e) = \frac{mgR_e}{2}$$

$$\Rightarrow U_2 = -\frac{1}{2}mgR_e$$

28. A body of mass m is placed on the earth's surface. It is taken from the earth's surface to a height $h = 3R$. The change in gravitational potential energy of the body is

(a) $\frac{2}{3}mgR$ (b) $\frac{3}{4}mgR$ (c) $\frac{mgR}{2}$ (d) $\frac{mgR}{4}$

SOLUTION:

$$\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mg \times 3R}{1+\frac{3R}{R}} = \frac{3}{4}mgR$$

29. The gravitational force in a region is given by, $\vec{F} = ay\hat{i} + ax\hat{j}$. The work done by gravitational force to shift a point mass m from (0, 0, 0) to (x_0, y_0, z_0) is

1) $max_0\, y_0 z_0$ 2) $max_0\, y_0$ 3) $-max_0\, y_0$ 4) 0

SOLUTION:

$$W = \int \vec{F}\,dr = ma \int_0^{x_0 y_0 z_0} \left(y\hat{i} + x\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$$

$$‘= ma\int d\left(xy\right) = ma\left(xy\right)$$

30. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

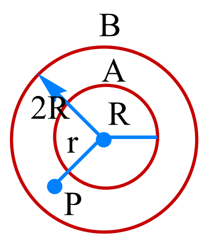
(a) $GMm/12R^2$ (b) $GMm/3R^2$ (c) $GMm/8R$ (d) $GMm/6R$

SOLUTION:

Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{GMm}{6R}$$

31. Two concentric spherical shells A and B of radii R and 2R and masses 4M and M respectively are as shown. The gravitational potential at point ’p’ at distance ‘r’ (R<r<2R) from centre of shell is (r=1.5R)



1) $-\frac{4GM}{R}$ 2) $-\frac{9GM}{2R}$ 3) $-\frac{4GM}{3R}$ 4) $-\frac{19GM}{6R}$

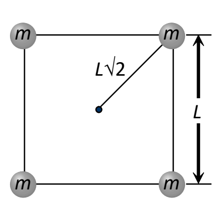
SOLUTION:

$$V = \frac{-G4M}{\frac{3}{2}R} - \frac{Mg}{2R}$$

32. Four particles each of mass M, are located at the vertices of a square with side L. The gravitational potential due to this at the centre of the square is

(a) $-\sqrt{32}\,\frac{GM}{L}$ (b) $-\sqrt{64}\,\frac{GM}{L^2}$ (c) Zero (d) $\sqrt{32}\,\frac{GM}{L}$

SOLUTION:

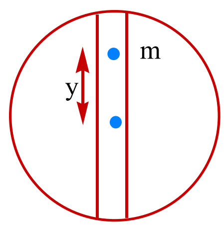


Potential at the centre due to single mass = $\frac{-GM}{L/\sqrt{2}}$

Potential at the centre due to all four masses = $-4\frac{GM}{L/\sqrt{2}}$

$$-4\sqrt{2}\frac{GM}{L} = -\sqrt{32}\times\frac{GM}{L}.$$

33. Suppose a vertical tunnel is dug along the diameter of earth assumed to be a sphere of uniform mass having density ρ. If a body of mass m is thrown in this tunnel, its acceleration at a distance y from the centre is given by



1) $\frac{4\pi}{3}G\rho ym$ 2) $\frac{3}{4}\pi G\rho y$ 3) $\frac{4}{3}\pi\rho y$ 4) $\frac{4}{3}\pi G\rho y$

SOLUTION:

Mass of the sphere is given by $M = \frac{4}{3}\pi y^3\rho$

Gravitational force. $F = \frac{G\left(\frac{4}{3}\pi y^3\rho\right)m}{y^2} \Rightarrow a = \frac{F}{m}$,

34. The magnitudes of the gravitational force at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$ (b) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$ (d) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 < R$ and $r_2 < R$

SOLUTION:

KEY: (a, b)

- $g = \frac{4}{3}\pi\rho Gr$ $\therefore$ $g \propto r$ if $r < R$
- $g = \frac{GM}{r^2}$ $\therefore$ $g \propto \frac{1}{r^2}$ if $r > R$

If $r_1 < R$ and $r_2 < R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

If $r_1 > R$ and $r_2 > R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

35. A particle is placed in a field characterized by a value of gravitational potential given V=-kxy, where 'k' is a constant. If $\overrightarrow{E_g}$ is the gravitational field then

1) $\vec{E}_g = k\left(x\hat{i} + y\hat{j}\right)$ and is conservative in nature

2) $\vec{E}_g = k\left(y\hat{i} + x\hat{j}\right)$ and is conservative in nature

3) $\vec{E}_g = k\left(x\hat{i} + y\hat{j}\right)$ and is non conservative in nature

4) $\vec{E}_g = k\left(y\hat{i} + x\hat{j}\right)$ and is non conservative in nature

SOLUTION:

$$E_g = \left(-\frac{\partial}{\partial x}\hat{i} - \frac{\partial}{\partial y}\hat{j}\right)(-kxy)$$

36. Two identical thin rings each of radius 'R' are co-axially placed at a distance 'R'. If the rings have a uniform mass distribution and each has mass m_1 and m_2 respectively, then the work done in moving a mass 'm' from the centre of one ring to that of the other is:

1) Zero 2) $\frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$ 3) $\frac{Gm\sqrt{2}(m_1 + m_2)}{R}$ 4) $\frac{Gm_1 m(\sqrt{2}+1)}{m_2 R}$

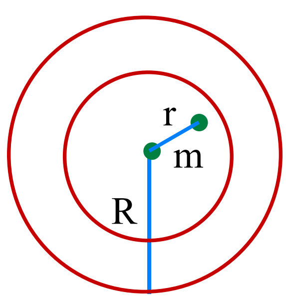
SOLUTION:

$$V_1 = \frac{-Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and } V_2 = \frac{-Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

$$\Delta V = V_2 - V_1 = \frac{-Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R} + \frac{-Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} = G(m_1 - m_2)\left(\frac{1}{R} - \frac{1}{\sqrt{2}R}\right)$$

$$\text{Hence } W = m(\Delta V) = \frac{mG(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$$

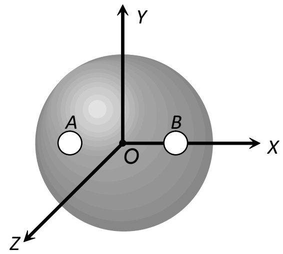
37. A mass m is placed in the cavity inside a hollow sphere of mass M as shown in the figure. The gravitational force on m is



1) $\frac{GMm}{R^2}$ 2) $\frac{GMm}{r^2}$ 3) $\frac{GMm}{(R-r)^2}$ 4) Zero

SOLUTION: Gravitational force is zero due to symmetry

38. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively are taken out of the solid leaving behind spherical cavities as shown in figure



(a)The gravitational force due to this object at the origin is zero

(b)'The gravitational force at the point B (2, 0, 0) is zero

(c)The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$

(d)The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

KEY: (a, c, d)

SOLUTION:

Since cavities are symmetrical *w.r.t. O*. So the gravitational force at the centre is zero.

The radius of the circle $z^2 + y^2 = 36$ is 6.

For all points for $r \geq 6$, the body behaves as if whole of the mass is concentrated at the centre.

So the gravitational potential is same.

Above is true for $z^2 + y^2 = 4$ as well.

39. A homogeneous spherical heavenly body has a uniform and very narrow frictionless duct along its diameter. Let mass of the body be M and diameter be D. A point mass m moves smoothly inside the duct. Force exerted on this mass when it is at a distance s from the centre of the body is

1) $\frac{GMm}{s^2}$ 2) $\frac{\pi GMm}{(D/2)^3}s$ 3) $\frac{8GMm}{D^3}s$ 4) $\frac{GMm}{(R-s)^2}$

SOLUTION:

$$\frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} = \frac{M_s}{\frac{4}{3}\pi S^3}; \; F = -\frac{GmM_s}{S^2}$$

Escape Velocity:

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^\infty \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow \qquad W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity,

then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \qquad \frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$\Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad v_e = \sqrt{2gR} \qquad \text{[As } GM = gR^2\text{]s}$$

$$\text{or } v_e = \sqrt{2 \times \frac{4}{3}\pi\rho GR \times R}$$

$$\Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho} \qquad \left[\text{As } g = \frac{4}{3}\pi\rho GR\right]$$

- ♦ Escape velocity is independent of the mass and direction of projection of the body.
- ♦ Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.
- ♦ For the earth as $g = 9.8 m/s^2$ and $R = 6400\ km$

  $\therefore v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km/\sec$
- ♦ A planet will have atmosphere if the velocity of molecule in its atmosphere $\left[v_{rms} = \sqrt{\frac{3RT}{M}}\right]$ is lesser than escape velocity.

  This is why earth has atmosphere (as at earth $v_{rms} < v_e$) while moon has no atmosphere (as at moon $v_{rms} > v_e$)
- ♦ If a body projected with velocity lesser than escape velocity ($v < v_e$), it will reach a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.
- ♦ Maximum height attained by body : Let a projection velocity of body (mass m) is v, so that it attains a maximum height h. At maximum height, the velocity of particle is zero, so kinetic energy is zero.

  By the law of conservation of energy

  Total energy at surface = Total energy at height h.

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$$\Rightarrow \frac{v^2}{2} = GM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= \frac{GMh}{R(R+h)}$$

$$\Rightarrow \frac{2GM}{v^2R} = \frac{R+h}{h} = 1 + \frac{R}{h}$$

$$\Rightarrow h = \frac{R}{\left(\frac{2GM}{v^2R} - 1\right)} = \frac{R}{\frac{v_e^2}{v^2} - 1} = R\left[\frac{v^2}{v_e^2 - v^2}\right]$$

[As $v_e = \sqrt{\frac{2GM}{R}}$ $\therefore$ $\frac{2GM}{R} = v_e^2$]

♦) If a body is projected with velocity greater than escape velocity then by conservation of energy.
Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R}$

$\Rightarrow$ $v'^2 = v^2 - v_e^2$ [As $\frac{2GM}{R} = v_e^2$]

$\therefore$ $v' = \sqrt{v^2 - v_e^2}$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

♦ Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

Total energy at the surface of the earth $= KE + PE = 0 - \frac{GMm}{R}$

$\therefore$ Escape energy $= \frac{GMm}{R}$

♦ If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called black holes.

The radius of a black hole is given as $R = \frac{2GM}{C^2}$

[As $C = \sqrt{\frac{2GM}{R}}$, where C is the velocity of light]

Orbital Velocity of Satellite:

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

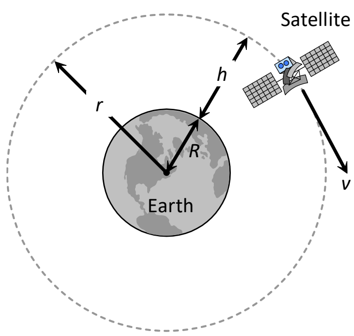


Fig. 8.26

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

As $GM = gR^2$ and $r = R + h$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

♦ Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit

i.e., satellites of diferent masses have same orbital velocity, if they are in the same orbit.

♦ Orbital velocity depends on the mass of central body and radius of orbit.

♦ For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v \propto 1/\sqrt{r}\right)$.

♦ Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$

As $h = 0$ and $GM = gR^2$

$$\therefore \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

For the earth $v = \sqrt{9.8 \times 6.4 \times 10^6}$

$$= 7.9\, km/s \approx 8\, km/\text{sec}$$

♦ Close to the surface of planet $v = \sqrt{\frac{GM}{R}}$

As $v_e = \sqrt{\frac{2GM}{R}}$

$$\therefore \quad v = \frac{v_e}{\sqrt{2}}$$

$$i.e.,\ v_{escape} = \sqrt{2}\ v_{orbital}$$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

♦ If the gravitational force of attraction of the sun on the planet varies as

$$F \propto \frac{1}{r^n}$$

then the orbital velocity varies as $v \propto \frac{1}{\sqrt{r^{n-1}}}$.

Time Period of Satellite:

It is the time taken by satellite to go once around the earth.

$$\therefore \quad T = \frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$$\Rightarrow \quad T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} \qquad [\text{As } v = \sqrt{\frac{GM}{r}}]$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}} \qquad [\text{As } GM = gR^2]$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \qquad [\text{As } r = R+h]$$

♦ From $T = 2\pi\sqrt{\frac{r^3}{GM}}$,

it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

♦ $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$\Rightarrow \quad T^2 = \frac{4\pi^2}{GM}r^3$$

i.e., $T^2 \propto r^3$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

♦ Time period of nearby satellite,

From $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h = 0$ and $GM = gR^2$]

For earth $R = 6400\ km$ and $g = 9.8 m/s^2$

$$T = 84.6 \text{ minute} \approx 1.4\ hr$$

‘ ♦ Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi\left(R^3\right)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

♦ If the gravitational force of attraction of the sun on the planet varies as then the time period varies as $T \propto r^{\frac{n+1}{2}}$

♦ If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$.

The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_s - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$

i.e. satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Height of Satellite:

As we know, time period of satellite $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2} = (R+h)^3$

$$\Rightarrow \quad h = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period we can calculate the height of satellite from the surface of the earth.

Geostationary Satellite:

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

♦ A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity *w.r.t.* that place on earth.

♦ The orbit of a geostationary satellite is known as the parking orbit.

♦ It should revolve in an orbit concentric and coplanar with the equatorial plane.

♦ Its sense of rotation should be same as that of earth about its own axis

i.e., in anti-clockwise direction (from west to east).

♦ Its period of revolution around the earth should be same as that of earth about its own axis.

$\therefore \; T = 24 \; hr = 86400 \; \sec$

♦ Height of geostationary satellite

As $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 24\,hr$

Substituting the value of G and M we get $R + h = r = 42000\; km = 7R$

height of geostationary satellite from the surface of earh $h = 6R = 36000\ km$

♦ Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\frac{GM}{r}}$

Substituting the value of and we get $v = 3.08\ km/\sec$

Energy of Satellite:

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

(1) Potential energy : $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$ $\left[\text{As } V = \frac{-GM}{r}, L^2 = m^2 GMr\right]$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$ $\left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$

(3) Total energy : $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$

♦ Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

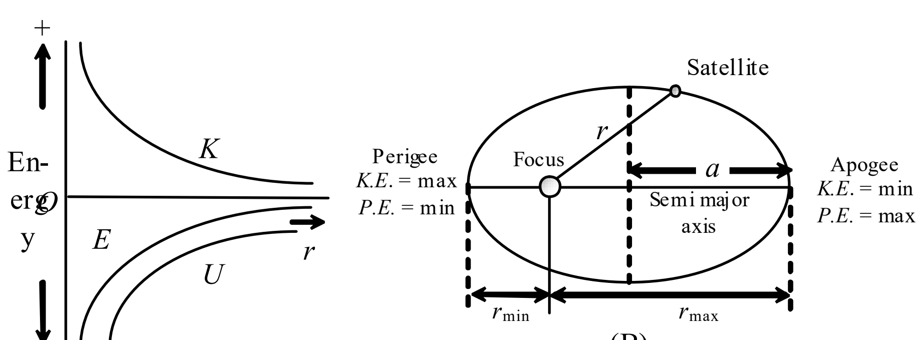
♦ From the above expressions we can say that

Kinetic energy $(K) = -$ (Total energy)

Potential energy $(U) = 2$ (Total energy)

Potential energy $(K) = -2$ (Kinetic energy)

♦ Energy graph for a satellite & Energy distribution in elliptical orbit



(A) (B)

♦ If the orbit of a satellite is elliptic then

(a) Total energy $(E) = \frac{-GMm}{2a} =$ constant ;

where a is semi-major axis .

(b) Kinetic energy (K) will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee)

(c) Potential energy (U) will be minimum when kinetic energy = maximum

i.e., the satellite is closest to the central body (at perigee)

Potential energy (U) will be maximum when kinetic energy = minimum

i.e., the satellite is farthest from the central body (at apogee).

Binding Energy :

The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system,

Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity.

i.e., Binding Energy (B.E.) $= -E = \dfrac{GMm}{2r}$

Weightlessness:

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

(1) **When objects fall freely under gravity** :

For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.

(2) **When a satellite revolves in its orbit around the earth** :

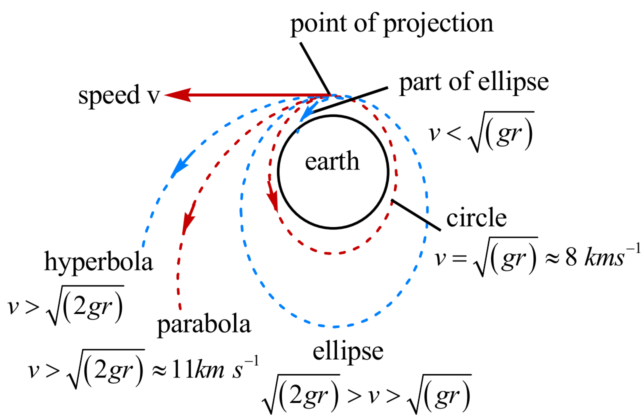
Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

(3) **When bodies are at null points in outer space** :

On a body projected up, the pull of the earth goes on decreasing, but at the same time the gravitational pull of the moon on the body goes on increasing. At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body in question is said to appear weightless.

Trajectories of a body projected with different velocities :

An object revolves around a planet only when it is projected with sufficient velocity in a direction perpendicular to the gravitational force of attraction of the planet on the object.



❈If $v < \sqrt{gr}$ object falls on the surface of earth

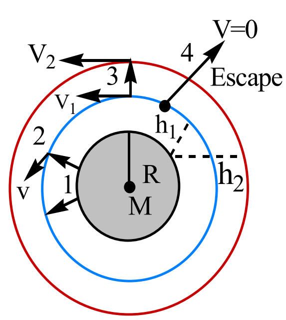
❈If $v = \sqrt{gr}$ object revolve in a circular orbit.

❈If $\sqrt{gr} < v < \sqrt{2gr}$ object revolves in an elliptical orbit.

❈If $v = \sqrt{2gr}$ object escapes from the field and follows parabolic path.

❈If $v > \sqrt{2gr}$ object escapes from the field and follows hyperbolic path.

Special Cases :



Case I:

Work done to lift an object at rest from the surface of a planet to a height h is

$$TE_i = TE_{surface} = -\frac{GMm}{R} + 0 = \frac{-GMm}{R}$$

$$TE_i = TE_{height} = -\frac{GMm}{R+h} + 0 = \frac{-GMm}{R+h}$$

Work done $W = TE_f - TE_i = \frac{GMm}{R} - \frac{GMm}{R+h}$

$$\Rightarrow W = \frac{GMmh}{R(R+h)} = \frac{mgh}{1+\frac{h}{R}}$$

Case II:

Work done to shift an object at rest from the surface of planet in to an orbit in which object revolves around the planet is

$$TE_i = TE_{surface} = \frac{-GMm}{R} + 0 = \frac{-GMm}{R}$$

$$TE_f = TE_{orbit} = \frac{-GMm}{R+h} + \frac{1}{2}mv_0^2 = \frac{-GMm}{2(R+h)}$$

Work done $W = TE_f - TE_i = \frac{GMm}{R} - \frac{GMm}{2(R+h)}$

$$W = GMm\left[\frac{R+2h}{2R(R+h)}\right]$$

Case III:

Work done to shift an object revolving around the planet from one orbit in to another orbit is

$$TE_i = (TE)_{h_1} = \frac{-GMm}{R+h_1} + \frac{1}{2}mv_1^2 = \frac{-GMm}{2(R+h_1)}$$

$$TE_f = (TE)_{h_2} = -\frac{GMm}{R+h_2} + \frac{1}{2}mv_2^2 = \frac{-GMm}{2(R+h_2)}$$

Work done $W = TE_f - TE_i = \frac{GMm}{2(R+h_1)} - \frac{GMm}{2(R+h_2)}$

$$\Rightarrow W = \frac{GMm}{2}\left[\frac{h_2 - h_1}{(R+h_1)(R+h_2)}\right]$$

Case IV :

Work done (or) additional energy to be imparted for an object to just escape an object with is initially revolving around the planet close the surface is

$$TE_i = \frac{-GMm}{R} + \frac{GMm}{2R} = -\frac{GMm}{2R}$$

TE_f=O(object escapes only when its TE becomes zero (or) positive)

Work done (or) additional energy imparted to the object is

$E = \Delta E = TE_f - TE_i = \frac{GMm}{2R} = KE$ of the object

Hence, an object (satellite) revolving around the planet escapes when

1) It's KE is doubled (increases by 100%)

2) It's velocity is increased to $\sqrt{2}$ times of present value (increases by 41.4%)

Additional velocity imparted to the body $= v_e - v_0 = \sqrt{2}v_0 - v_0$

‘ $$\left(\sqrt{2}-1\right)v_0 = 3.2km/s\left(\text{nearly}\right)$$

Note :

In the above case if the object initially revolves around the planet at a highest h from the surface then it’s

$$TE = \frac{-GMm}{2\left(R+h\right)}$$

Additional energy required to escape the object is $\frac{GMm}{2\left(R+h\right)}$ ”

Quantities	Variation	Relation with r
✡Orbital Velocity	Decreases	$v\alpha\frac{1}{\sqrt{r}}$
✡Time period	Increases	$T\alpha r^{3/2}$
✡Linear momentum	Decreases	$p\alpha\frac{1}{\sqrt{r}}$
✡Angular momentum	Increases	$L\alpha\sqrt{r}$
✡Kinetic energy	Decreases	$K\alpha\frac{1}{r}$
✡Potential energy	Increases	$U\alpha-\frac{1}{r}$
✡Total energy	Increases	$E\alpha-\frac{1}{r}$
✡Binding energy	Decreases	$BE\alpha\frac{1}{r}$

::PROBLEMS::

1 . Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[2G\frac{(m_1-m_2)}{r}\right]^{1/2}$ (b) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$ (c) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ (d) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

SOLUTION :

Let velocities of these masses at r distance from each other be v_1 and v_2 respectively.

By conservation of momentum $m_1v_1 - m_2v_2 = 0$

$$\Rightarrow m_1v_1 = m_2v_2 \quad \text{... (i)}$$

By conservation of energy change in $P.E.$=change in $K.E.$

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2\,Gm_1m_2}{r} \quad \text{...(ii)}$$

On solving equation (i) and (ii)

$$v_1 = \sqrt{\frac{2\,Gm_2^2}{r(m_1+m_2)}}$$

$$v_2 = \sqrt{\frac{2\,Gm_1^2}{r(m_1+m_2)}}$$

$$\therefore\ v_{\text{app}} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}(m_1+m_2)}$$

2 . If v_e and v_o represent the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius R, then

(a) $v_e = v_o$ (b) $\sqrt{2}v_o = v_e$ (c) $v_e = v_0/\sqrt{2}$ (d) and are not related

SOLUTION :

$$v_e = \sqrt{2gR}$$

$$v_0 = \sqrt{gR}$$

$$\therefore\ \sqrt{2}\,v_0 = v_e$$

3 . A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k<1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be : (R = radius of earth)

(a) $\frac{R}{k^2+1}$ (b) $\frac{R}{k^2-1}$ (c) $\frac{R}{1-k^2}$ (d) $\frac{R}{k+1}$

SOLUTION :

Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow \frac{1}{2}mk^2 2gR = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow h = \frac{Rk^2}{1-k^2}$$

Height of Projectile from the earth's surface = h

Height from the centre $r = R + h = R + \frac{Rk^2}{1-k^2}$

By solving $r = \frac{R}{1-k^2}$

4. If Earth has mass nine times and radius twice that of the planet mars, calculate the velocity required by a rocket to pull out of the gravitational force of Mars. Take escape speed on surface of Earth to be 11.2 km/s

SOLUTION :

Here, $M_e = 9M_m$, and $R_e = 2R_m$

v_e (escape speed on surface of Earth)=11.2 km/s

Let V_m be the speed required to pull out of the gravitational force of mars.

We know that

$$v_e = \sqrt{\frac{2GM_e}{R_e}} \text{ and } v_m = \sqrt{\frac{2GM_m}{R_m}}$$

Dividing, we get $\frac{v_m}{v_e} = \sqrt{\frac{2GM_m}{R_m} \times \frac{R_e}{2GM_e}}$

$$= \sqrt{\frac{M_m}{M_e} \times \frac{R_e}{R_m}} = \sqrt{\frac{1}{9} \times 2} = \frac{\sqrt{2}}{3}$$

$$\Rightarrow v_m = \frac{\sqrt{2}}{3}(11.2 km/s) = 5.3 km/s$$

5. A rocket is fired with a speed $v = 2\sqrt{gR}$ near the earth's surface and directed upwards.

(a) Show that it will escape from the earth.

(b) Show that it interstellar space its speed is $v = 2\sqrt{gR}$.

SOLUTION :

(a) As PE of the rocket at the surface of the earth is (-GMm/R) and at infinity is zero,

energy required for escaping from earth $= 0 - \left(\frac{GMm}{R}\right) = mgR \quad \left[\because g = \frac{GM}{R^2}\right]$

And as initial KE of the rocket $\frac{1}{2}mv^2 = 2mgR$ is greater than the energy required for escaping (=mg R), the rocket will escape.

(b) If υ is the velocity of the rocket in interstellar space (free from gravitational effects) then by conservation of energy,

$$\frac{1}{2}m\left(2\sqrt{gR}\right)^2 - \frac{1}{2}m\left(\sqrt{2gR}\right)^2 = \frac{1}{2}mv^2$$

$$v^2 = 4gR - 2gR \ \ or \ \ v = \sqrt{2gR}$$

6. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that escape velocity from the earth is 11km/s, the escape velocity from the surface of the planet is

SOLUTION : Given $M_P = 10M_e; R_P - \frac{R_e}{10}$ $(2008A)$

We know that $v_e = \sqrt{\frac{2GM}{R}}$

$$\therefore v_P = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{100 \times 2GM_e}{R_e}} = 10v_e$$

=10 11 = 110 km/s

7. The ratio of the K.E. required to be given to the satellite to escape earth's gravitational field to the K.E. required to be given so that the satellite moves in a circular orbit just above earth atmosphere is

(a) One (b) Two (c)Half (d)Infinity

SOLUTION :

K.E. required for satellite to escape from earth's gravitational field

$$\frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2 = \frac{GMm}{R}$$

K.E. required for satellite to move in circular orbit

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2 = \frac{GMm}{2R}$$

The ratio between these two energies = 2

8. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is *v*. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(a) $\frac{3}{2}v$ (b) $\sqrt{\frac{3}{2}}v$ (c) $\sqrt{\frac{2}{3}}v$ (d) $\frac{2}{3}v$

SOLUTION :

$$v = \sqrt{\frac{GM}{R+h}}$$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v$$

9. In a satellite if the time of revolution is T, then K.E. is proportional to

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$ (c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

SOLUTION :

$$v = \sqrt{\frac{GM}{r}}$$

$$\therefore\ K.E. \propto v^2 \propto \frac{1}{r}$$

$$T^2 \propto r^3$$

$$\therefore\ K.E. \propto T^{-2/3}$$

10. Two identical satellites are at R and $7R$ away from earth surface, the wrong statement is (R = Radius of earth)

(a) Ratio of total energy will be 4

(b) Ratio of kinetic energies will be 4

(c) Ratio of potential energies will be 4

(d) Ratio of total energy will be 4 but ratio of potential and kinetic energies will be 2

SOLUTION :

Orbital radius of satellites $r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$$U_1 = \frac{-GMm}{r_1} \text{ and } U_2 = \frac{-GMm}{r_2}$$

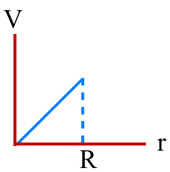
$$K_1 = \frac{GMm}{2r_1} \text{ and } K_2 = \frac{GMm}{2r_2}$$

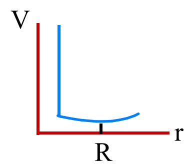
$$E_1 = \frac{GMm}{2r_1} \text{ and } E_2 = \frac{GMm}{2r_2}$$

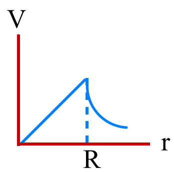
$$\therefore \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$$

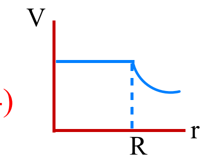
11. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & for\ r \le R \\ 0 & for\ r > R \end{cases}$.

Where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as function of distance *r* from the centre of system is represented by (2008 I)

1) 

2) 

3) 

4) 

SOLUTION :

For $r \le R;\ \dfrac{mv^2}{r} = \dfrac{GmM}{r^2} \rightarrow (1)$

here, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

substituting in Eq(1)

we get $v \propto r$

i.e., v-r graph is a straight line passing through origin,

for r>R

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{3}{4}\pi R^3\right)\rho_0}{r^2} \qquad or \quad v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (3)

12. In the following four periods

(i)Time of revolution of a satellite just above the earth's surface (T_{st})

(ii)Period of oscillation of mass inside the tunnel bored along the diameter of the earth (T_{ma})

(iii)Period of simple pendulum having a length equal to the earth's radius in a uniform field of $9.8\ N/kg$ (T_{sp})

(iv)Period of an infinite length simple pendulum in the earth's real gravitational field (T_{is})

(a) $T_{st} > T_{ma}$ (b) $T_{ma} > T_{st}$ (c) $T_{sp} < T_{is}$ (d) $T_{st} = T_{ma} = T_{sp} = T_{is}$

SOLUTION :

(i) $T_{st} = 2\pi\sqrt{\dfrac{(R+h)^3}{GM}} = 2\pi\sqrt{\dfrac{R}{g}}$ [As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\dfrac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$ [As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\frac{R}{g}}$

13. A geo-stationary satellite is orbiting the earth at a height of 6 R above the surface of earth, R being the radius of earth. The time period of another satellite at a height of 2.5 R from the surface of earth is

(a) 10 hr (b) $(6/\sqrt{2})\,hr$ (c) 6 hr (d) $6\sqrt{2}\,hr$

SOLUTION :

Distances of the satellite from the centre are $7R$ and $3.5R$ respectively.

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{3.5R}{7R}\right)^{3/2} = 6\sqrt{2}\,hr$$

14. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to

(a) $1/R^2$ (b) R^0 (c) R^1 (d) $1/R$

SOLUTION :

Gravitational force provides the required centripetal force for orbiting the satellite

$$\frac{mv^2}{R} = \frac{K}{R} \quad \text{because} \left(F \propto \frac{1}{R}\right)$$

$$\therefore\ v \propto R^\circ$$

15. Potential energy of a satellite having mass 'm' and rotating at a height of $6.4 \times 10^6\,m$ from the earth surface is

(a) $-0.5\,mgR_e$ (b) $-mgR_e$ (c) $-2\,mgR_e$ (d) $4\,mgR_e$

SOLUTION :

$$\text{Potential energy} = \frac{-GMm}{r} = \frac{GMm}{R_e + h} = \frac{-GMm}{2R_e}$$

$$= -\frac{gR_e^2 m}{2R_e}$$

$$= -\frac{1}{2}mgR_e$$

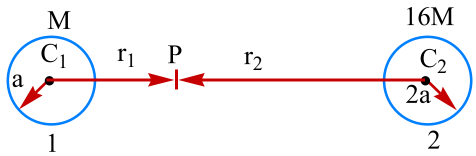
$$= -0.5\,mgR_e$$

16. Distance between the centre of two stars is 10a. the masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger start towards the surface of the smaller star. What should be its minimum initial speed to reach the surface of the smaller star?

1) $\sqrt{\frac{GM}{a}}$ 2) $\frac{1}{2}\sqrt{\frac{5GM}{a}}$ 3) $\frac{3}{2}\sqrt{\frac{GM}{a}}$ 4) $\frac{3\sqrt{5}}{2}\sqrt{\frac{GM}{a}}$

SOLUTION :

Let there are two stars 1 and 2 as shown below.



Let P is a point between C_1 and C_2, where gravitational field strength is zero.

$$\text{Hence } \frac{GM}{r_1^2}=\frac{G(16M)}{r_2^2};\frac{r_2}{r_1}=4,\quad r_1+r_2=10a$$

$$\therefore r_2=\left(\frac{4}{4+1}\right)(10a)=8a$$

$$r_1=2a$$

Now, the body of mass m is projected from the surface of large star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1. Therefore, the body should be projected to just cross point P because beyond that the particle is attracted towards the smaller star itself.

From conservation of mechanical energy $\frac{1}{2}mv^2$ = potential energy of the body at P

= potential energy at the surface of larger star.

$$\therefore \frac{1}{2}mv_{\min}^2=\left[\frac{GMm}{r_1}-\frac{16GMm}{r_2}\right]-\left[-\frac{GMm}{10a-2a}-\frac{16GMm}{2a}\right]$$

$$\frac{1}{2}mv_{\min}^2=\left(\frac{45}{8}\right)\frac{GMm}{a}$$

$$v_{\min}=\frac{3\sqrt{5}}{2}\left(\sqrt{\frac{GM}{a}}\right)$$

17. If $g\propto\frac{1}{R^3}$ (instead of $\frac{1}{R^2}$), then the relation between time period of a satellite near earth's surface and radius R will be

(a) $T^2\propto R^3$ (b) $T\propto R^2$ (c) $T^2\propto R$ (d) $T\propto R$

SOLUTION :

Gravitational force provides the required centripetal force

$$m\omega^2R=\frac{GMm}{R^3}$$

$$\Rightarrow \frac{4\pi^2}{T^2}=\frac{GM}{R^4}$$

18. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be (2010I)

1) 3 2) 6 3) 9 4) 12

SOLUTION :

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$

$$g \propto \rho R; \quad R \propto \frac{g}{\rho}$$

Now escape velocity, $v_e = \sqrt{2gR}$

$$v_e \propto \sqrt{gR}; \quad v_e \propto \sqrt{g \times \frac{g}{\rho}} \propto \sqrt{\frac{g^2}{\rho}}$$

$$(v_e)_{planet} = \left(11kms^{-1}\right)\sqrt{\frac{6}{121} \times \frac{3}{2}} = 3kms^{-1}$$

19. A satellite moves in a circle around the earth. The radius of this circle is equal to one half of the radius of the moon's orbit. The satellite completes one revolution in

(a) $\frac{1}{2}$ lunar month (b) $\frac{2}{3}$ lunar month (c) $2^{-3/2}$ lunar month (d) $2^{3/2}$ lunar month

SOLUTION :

Time period of revolution of moon around the earth

= 1 lunar month.

$$\frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$$

$\Rightarrow T_s = 2^{-3/2}$ lunar month.

20. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is (M_P+M_Q). The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then (2012I)

1) $V_Q > V_R > V_P$ 2) $V_R > V_Q > V_P$ 3) $V_R / V_P = 3$ 4) $V_P / V_Q = \frac{1}{2}$

SOLUTION :

$$V_P = \sqrt{\frac{2GM}{R}}, V_Q = \sqrt{\frac{2G8M}{2R}} = 2V_P$$

$$V_R = \sqrt{\frac{2G9M}{9^{1/3}R}} = 9^{1/3}V_P$$

21 . A geostationary satellite is revolving around the earth. To make it escape from gravitational field of earth, is velocity must be increased

(a) 100% (b) 41.4% (c) 50% (d) 59.6%

$$v_e = \sqrt{2}v_0 = 1.414\ v_0$$

Fractional increase in orbital velocity $\left(\frac{\Delta v}{v}\right)$

$$= \frac{v_e - v_0}{v_0} = 0.414$$

$\therefore$ Percentage increase $= 41.4\%$

22. There is a crater of depth R/100 on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile. (2003A)

1) 90R 2) 95R 3) 99.5R 4) 50R

SOLUTION :

Speed of particle at A, v_A = escape velocity on the surface of earth $= \sqrt{\frac{2GM}{R}}$

At highest point B, v_B=0

Applying conservation of mechanical energy,

decrease in kinetic energy=increase in gravitational potential energy

$$= \frac{1}{2}mv_A^2 = U_B - U_A = m(V_B - V_A)$$

$$\frac{v_A^2}{2} = V_B - V_A$$

$$\frac{GM}{R} = -\frac{GM}{R+h}\left[\frac{-GM}{R^3}\left[(1.5R^2) - 0.5\left(R - \frac{R}{100}\right)^2\right]\right]$$

$$\frac{1}{R} = \frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^2\frac{1}{R}$$

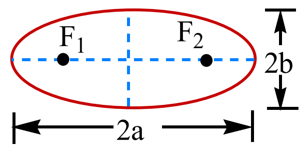
Solving this equation,

we get h=99.5R

Kepler's Laws :

Kepler's first law or laws of orbits:

'Every planet revolves around the sun in elliptical orbit with the sun is at one of its focii.

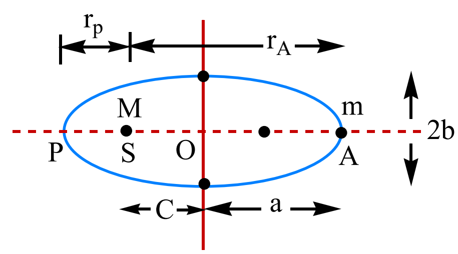


As shown in fig., sun may be at F_1 or F_2. Here a and b denote the lengths of semi major and semi minor axes.

♦ The nearest position of the planet from the sun is called perihelion.'

♦ The farthest position of the planet from the sun is called aphelion.

♦ A planet of mass m is moving in an elliptical orbit around the sun(S) of mass 'M', at one of its focii.



♦ Eccentricity of the elliptical path $e = \dfrac{SO}{OA}$ $e = \dfrac{e}{a} \Rightarrow c = ea$

From fig, $r_p = a - c = a - ea = a(1-e)$

Similarly $r_a = a + c = a + ea = a(1+e)$

♦ From conservation of angular momentum at A and P, we have $mV_P r_P = mV_A r_A$

$$\frac{V_P}{V_A} = \frac{r_A}{r_P} = \frac{1+e}{1-e}$$

♦ From conservation of energy, we have $V_A = \sqrt{\dfrac{GM}{a}\left(\dfrac{1-e}{1+e}\right)}$ *and* $V_P = \sqrt{\dfrac{GM}{a}\left(\dfrac{1+e}{1-e}\right)}$

❄If e>1 and total energy (K.E+P.E)>0,

the path of the satellite is hyperbolic and it escapes from its orbit.

❄If e<1 and total energy is negative, it moves in an elliptical path.

❄If e=0 and total energy is negative, it moves in an elliptical path.

❄If e=1 and total energy is zero, it will take parabolic path.

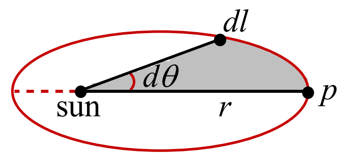
♦ The path of the projectile thrown to lower heights is parabolic and thrown to greater heights is elliptical.

Kepler's second law or Laws of Areas:

The radius vector joining the planet to the sun sweeps out equal areas in equal intervals of time.

♦ Areal Velocity of radius vector $\left(\frac{dA}{dt}\right)$ joining the planet to sun remains constant.

Mathematically $\frac{dA}{dt} = \text{constant}$



But $A = \frac{1}{2}(dl)r = \frac{1}{2}(ed\theta)r = \frac{1}{2}r^2 d\theta$

So, $\frac{d}{dt}\left(\frac{1}{2}r^2\theta\right) = \text{constant}$

$$\Rightarrow \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega = \text{constant}$$

$$\Rightarrow \frac{1}{2}\frac{mr^2 d\theta}{m} = \frac{I\omega}{2m} = \frac{L}{2m} = \text{constant}$$

L=constant

♦ As the gravitational force on planet by sun is central, torque is zero and hence angular momentum of the planet is constant.

♦ This law is consequence of law of conservation of angular momentum.

$$\frac{dA}{dt} = \frac{L}{2m} = \frac{mVr}{2m} = \frac{Vr}{2}$$

♦ Areal velocity of radius vector of the planet is independent of mass of the satellite.

♦ As angular momentum is conserved,

$$m(V_{\max})(r_{\min}) = m(V_{\min})(r_{\max})$$

$$\Rightarrow \frac{V_{\max}}{V_{\min}} = \frac{1+e}{1-e}$$

Here $V_{perihelion} = V_{\max}$

and $V_{apehelion} = V_{\min}$

♦ Kepler's laws can be applied to natural and artificial satellites as well.

Kepler's third law or Law of periods :

The square of period of revolution of a planet around the sun is proportional to cube of the average distance of planet (i.e., semi major axis of elliptical orbit) from the sun.

$$r_{mean} = \frac{r_{\max} + r_{\min}}{2} = \frac{(1+e)a + (1-e)a}{2} = a$$

Hence $T^2 \propto a^3$

where 'a' is length of semi major axis of ellipse

♦ The gravitational force between the planet and the Sun provides the necessary centripetal force for the planet to go round the Sun.

If M = mass of Sun,

m = mass of planet

'r = average distance of the planet from the Sun

then, $$F = \frac{GmM}{r^2} = mr\omega^2$$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2} \quad \left(as\ \omega = \frac{2\pi}{T}\right)$$

$$T^2 = 4\pi^2 \frac{r^3}{GM} \Rightarrow T^2 \propto r^3$$

NOTE: The gravitational force is a central force so torque on planet relative to sun is always zero, hence angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

PROBLEMS

1. An artificial satellite is in an elliptical orbit around the earth with aphelion of 6R and perihelion of 2R where R is Radius of the earth = 6400Km. Calculate the eccentricity of the elliptical orbit.

SOLUTION :

We know that

perigee $(r_p) = a(1-e) = 2R$ ....(1)

apogee $(r_a) = a(1+e) = 6R$.......(2)

Solving (1) & (2),

eccentricity $(e) = 0.5$

2. The mean distance of a planet from the sun is approximately 1/4 times that of earth from the sun. Find the number of years required for planet to make one revolution about the sun.

SOLUTION :

$$\text{Given } r_P = \frac{1}{4} r_E \text{ and } T_E = 1yr$$

For Kepler's third law, $T^2 \alpha r^3$

$$\left(\frac{T_P}{T_E}\right)^2 = \left(\frac{r_P}{r_E}\right)^3 \Rightarrow T_P = T_E\left(\frac{r_P}{r_E}\right)^{\frac{3}{2}}$$

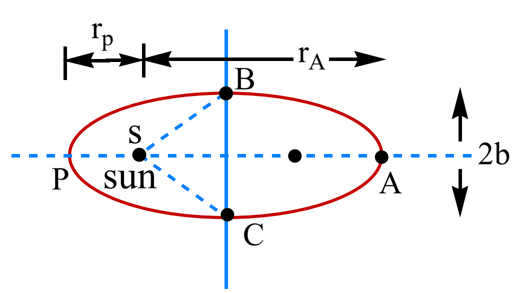
$$T_P = (1)\left(\frac{r_E}{4r_E}\right)^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{3/2} = 0.125Yrs$$

3. The speed of the planet at the perihelion P be V_P and the Sun-planet distance SP be r_P as shown in Fig. Relate $\{t_P, V_P\}$ to the corresponding quantities at the aphelion $\{r_A, V_A\}$. Will the planet take equal times to traverse BAC and CPB ?

SOLUTION :

The magnitude of the angular momentum at P is $L_P = m_P V_P r_P$,

The magnitude of the angular momentum at A is $L_A = m_A V_A r_A$



According to law of conservation of angular momentum,

$$m_P r_P V_P = m_P r_A V_A \text{ or } \frac{V_P}{V_A} = \frac{r_A}{r_P}$$

Here $r_A > r_P$ hence $V_P > V_A$.

The area SBAC bounded by the ellipse and the radius vectors SB and SC is larger than SBPC in Fig.
From Kepler's second law, equal areas are swept in equal times.
Hence, the planet will take a longer time to traverse BAC than CPB.

4. Let us consider that our galaxy consists of $2.5\text{x}10^{11}$ stars each of one solar mass. How long will this star at a distance of 50,000 light years from the galastic entre take to complete one revolution? Take the diameter of the Milky way to be 10^5ly.G=$6.67\text{x}10^{-11}$ Nm^2Kg^{-2}. (1 ly = $9.46\text{x}10^{15}$m)

SOLUTION :

Here M=$2.5\text{x}10^{11}$ solar mass=$2.5\text{x}10^{11}\text{x}(2\text{x}10^{30})$kg=$5.0\text{x}10^{41}$kg

r=50,000 ly=50,000x$9.46\text{x}10^{15}$m=$4.73\text{x}10^{20}$m

We know that, $M = \frac{4\pi^2 r^3}{GT^2}$

$$T = \left(\frac{4\pi^3 r^3}{GM}\right)^{\frac{1}{2}} = \left[\frac{4\times(22/7)^2\times(4.73\times10^{20})^3}{(6.67\times10^{-11})\times(5.0\times10^{41})}\right]^{\frac{1}{2}}$$

=$3.53\text{x}10^{14}$s.

5. If the distance between the earth and the sun becomes half its present value, the number of days in a year would have been

(a) 64.5 (b) 129 (c) 182.5 (d) 730

SOLUTION:

According to Kepler's third law, the ratio of the squares of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun

$$i.e. \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left[\frac{r_1}{\frac{1}{2}r_1}\right]^3 = 8$$

$$\Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$$

$$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \ \ days}{2\sqrt{2}} = 129 \ \ days$$

6. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 *km*. Then, the time period of a satellite orbiting a few hundred kilometres above the earth's surface $(R_{\text{Earth}} = 6400 \ km)$ will approximately be

(a)1/2 *h* (b) 1 *h* (c)2 *h* (d)4 *h*

SOLUTION:

$$\frac{T_2}{T_1}=\left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{6400}{36000}\right)^{3/2} \cong 2\,hour$$

7 . The distance of neptune and saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio

(a) $\sqrt{10}$ (b) 100 (c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

SOLUTION:

$$\frac{T_1}{T_2}=\left(\frac{R_1}{R_2}\right)^{3/2}=\left(\frac{10^{13}}{10^{12}}\right)^{3/2}=(1000)^{1/2}$$

$$=10\sqrt{10}$$

8 . Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-\frac{5}{2}}$, then T^2 is proportional to

(a) R^3 (b) $R^{7/2}$ (c) $R^{5/2}$ (d) $R^{3/2}$

SOLUTION:

For revolution of planet centripetal force is provided by gravitational force of attraction

$$m\omega^2 R \propto R^{-5/2}$$

$$\frac{1}{T^2} \propto R^{-7/2}$$

$$\Rightarrow T^2 \propto R^{7/2}$$

9 . Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

SOLUTION:

$$m\omega^2 R \propto \frac{1}{R^n}$$

$$\Rightarrow m\left(\frac{4\pi^2}{T^2}\right)R \propto \frac{1}{R^n}$$

$$\Rightarrow T^2 \propto R^{n+1}$$

$$\therefore T \propto R^{\left(\frac{n+1}{2}\right)}$$

10. An artificial satellite revolves around earth in circular orbit of radius r with time period T. The satellite is made to stop in the orbit which makes it fall onto earth. Time of fall of the satellite on to earth is given by

1) $\sqrt{3}\frac{T}{6}$ 2) $\frac{\sqrt{2}}{8}T$ 3) $\frac{T}{\sqrt{3}}$ 4) $\sqrt{\frac{2}{3}}\frac{T}{\pi}$

SOLUTION:

On stopping, the satellite will fall along the radius r of the orbit which can be regarded as a limiting case of an ellipse with semi major axis r/2

Using Kepler's third law $T^2 \propto r^3$

$$\text{time of fall} = \frac{T'}{2} - \frac{T}{2\sqrt{8}} = \frac{\sqrt{2}T}{8}$$

11. The period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius $4R$ is

(a) $4T$ (b) $T/4$ (c) $8T$ (d) $T/8$

SOLUTION:

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} \Rightarrow T_2 = 8T_1$$

12. A planet moves around the sun. At a given point P, it is closest from the sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$ (c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

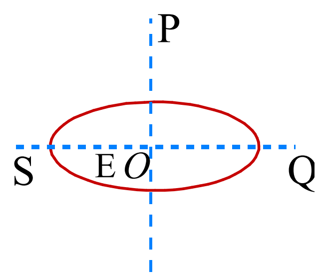
SOLUTION:

Angular momentum remains constant

$$mv_1 d_1 = mv_2 d_2 \Rightarrow v_2 = \frac{v_1 d_1}{d_2}$$

13. A satellite moving in elliptical orbit around earth as shown. The minimum and maximum distance of the satellite from earth are 3 units 5 units respectively. The distance of satellite from earth when it is at 'P' is ______ (units)

1) 4 2) 3 3) 3.75 4) 6



SOLUTION:

$$\text{Semi major axis} = 4 \Rightarrow ae = 1 \Rightarrow e = \frac{1}{4}$$

Semi minor axis = b

$$b = a\sqrt{1-e^2} = \sqrt{1-\frac{1}{16}} = \sqrt{15} = \sqrt{15}$$

$$\text{Required distance} = \sqrt{b^2+1} = 4$$

14. The largest and the shortest distance of the earth from the sun are r_1 and r_2, its distance from the sun when it is at the perpendicular to the major axis of the orbit drawn from the sun

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 r_2}{r_1 + r_2}$ (c) $\frac{2r_1 r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$

SOLUTION:

The earth moves around the sun is elliptical path. so by using the properties of ellipse

$$r_1 = (1+e)a$$

$$r_2 = (1-e)a$$

$$\Rightarrow a = \frac{r_1 + r_2}{2}$$

$$r_1 r_2 = (1-e^2)a^2$$

where a = semi major axis

b = semi minor axis

e = eccentricity

Now required distance = semi latusrectum $= \frac{b^2}{a}$

$$= \frac{a^2(1-e^2)}{a} = \frac{(r_1 r_2)}{(r_1 + r_2)/2} = \frac{2r_1 r_2}{r_1 + r_2}$$

15. A satellite of mass *m* is circulating around the earth with constant angular velocity. If radius of the orbit is R_0 and mass of the earth *M*, the angular momentum about the centre of the earth is

(a) $m\sqrt{GMR_0}$ (b) $M\sqrt{GmR_0}$ (c) $m\sqrt{\frac{GM}{R_0}}$ (d) $M\sqrt{\frac{GM}{R_0}}$

SOLUTION:

Angular momentum = Mass ×Orbital velocity ×Radius

$$= m \times \left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = m\sqrt{GMR_0}$$

16. The maximum and minimum distances of a comet from the sun are $8 \times 10^{12}\ m$ and $1.6 \times 10^{12}\ m$. If its velocity when nearest to the sun is 60 *m/s*, what will be its velocity in *m/s* when it is farthest

(a) 12 (b) 60 (c)112 (d)6

SOLUTION:

By conservation of angular momentum mvr = constant

$$v_{\min} \times r_{\max} = v_{\max} \times r_{\min}$$

$$\therefore v_{\min} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12\ m/s$$

17. A body revolved around the sun 27 times faster then the earth what is the ratio of their radii

(a) 1/3 (b) 1/9 (c)1/27 (d)1/4

SOLUTION:

$$\omega_{\text{body}} = 27\omega_{\text{earth}}$$

$$T^2 \propto r^3 \Rightarrow \omega^2 \propto \frac{1}{r^3} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \therefore r \propto \frac{1}{\omega^{2/3}}$$

$$\Rightarrow \frac{r_{\text{body}}}{r_{\text{earth}}} = \left(\frac{\omega_{\text{earth}}}{\omega_{\text{body}}}\right)^{2/3}$$

$$= \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}$$

18. The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then the new angular momentum will be

(a) $16L$ (b) $64L$ (c) $\frac{L}{4}$ (d) $4L$

SOLUTION:

$$L = mvr = m\sqrt{\frac{GM}{r}}r = m\sqrt{GMr} \quad \therefore \ L \propto \sqrt{r}$$

19. The longest and the shortest distance of a planet from sun is R_1 and R_2. Distance from sun when it is normal to major axis of orbit is

1) $\frac{R_1 + R_2}{2}$ 2) $\sqrt{\frac{R_1^2 + R_2^2}{2}}$ 3) $\frac{R_1 R_2}{R_1 + R_2}$ 4) $\frac{2R_1 R_2}{R_1 + R_2}$

SOLUTION:

$$R_1 = (1+e)a; R_2 = (1-e)a$$

$$a = \frac{R_1 + R_2}{2}; R_1 R_2 = (1-e^2)a^2$$

$$\text{semi-latus rectum} = \frac{b^2}{a}$$

$$= \frac{a^2(1-e^2)}{a} = \frac{R_1 R_2}{\frac{R_1 + R_2}{2}} = \frac{2R_1 R_2}{R_1 + R_2}$$

20. The mass of a planet that has a moon whose time period and orbital radius are T and R respectively can be written as

(a) $4\pi^2 R^3 G^{-1} T^{-2}$ (b) $8\pi^2 R^3 G^{-1} T^{-2}$ (c) $12\pi^2 R^3 G^{-1} T^{-2}$ (d) $16\pi^2 R^3 G^{-1} T^{-2}$

SOLUTION:

$$m\omega^2 R = \frac{GMm}{R^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 R = \frac{GM}{R^2}$$

$$\Rightarrow M = \frac{4\pi^2 R^3}{GT^2}$$

21. A point mass is orbiting a significant mass M lying at the focus of the elliptical orbital having major and minor axes given by 2a and 2b respectively. Let r be the distance between the mass M and the end point of major axis. Velocity of the particle can be given as

1) $\frac{ab}{r}\sqrt{\frac{GM}{a^3}}$ 2) $\frac{ab}{r}\sqrt{\frac{GM}{b^3}}$ 3) $\frac{ab}{2r}\sqrt{\frac{GM}{r^3}}$ 4) $\frac{2ab}{r}\sqrt{\frac{GM}{\left(\frac{a+b}{2}\right)^3}}$

SOLUTION:

Gravitational force = Centripetal force

$$\Rightarrow \frac{GM}{r^2} = \frac{v^2}{r'}$$

here r is the radius of curvature. From Kepler's law, time period is given by,

$$T = 2\pi\sqrt{\frac{a^3}{GM}} = \frac{2\pi ab}{rv}$$

$$\left(\frac{dA}{dt} = \frac{vr}{2} \Rightarrow dt = T = \frac{2dA}{vr} = \frac{2}{rv}\pi ab\right)$$

$$v = \frac{ab}{r}\sqrt{\frac{GM}{a^3}}$$

22. A planet of mass m revolves in elliptical orbit around the sun so that its maximum and minimum distances from the sun are equal to r_a and r_p respectively. The angular momentum of this planet relative to the sun is

1) $L = m\sqrt{\frac{GMr_Pr_a}{(r_P + r_a)}}$ 2) $L = m\sqrt{\frac{2GMr_Pr_a}{(r_P + r_a)}}$ 3) $L = M\sqrt{\frac{Gmr_Pr_a}{(r_P + r_a)}}$ 4) $L = M\sqrt{\frac{(r_P + r_a)}{Gmr_Pr_a}}$

SOLUTION:

From conservation of energy $-\frac{GMm}{r_P} + \frac{1}{2}mv_P^2 = \frac{-GMm}{r_a} + \frac{1}{2}mv_a^2$

$$L = mv_Pr_P = mv_ar_a$$

23. A satellite is orbiting just above the surface of a planet of average density D with period T. If G is the universal gravitational constant, the quantity $\frac{3\pi}{G}$ is equal to

1) T^2D 2) $3\pi T^2 D$ 3) $3\pi D^2 T$ 4) D^2T

SOLUTION:

Using $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G \times \frac{4}{3}\pi R^3 D}}$: $T^2 = \frac{4\pi^2 R^3}{G\frac{4}{3}\pi R^3 D} = \frac{3\pi}{DG} \Rightarrow \frac{3\pi}{G} = T^2 D$

24. A planet revolves around sun in an elliptical orbit of eccentricity 'e7'. If 'T' is the time period of the planet then the time spent by the planet between the end of the minor axis and close to sun is

1) $T\left(\frac{1}{4} - \frac{e}{2\pi}\right)$ 2) $\frac{Te}{\pi}$ 3) $\left(\frac{e}{\pi} - 1\right)$ 4) $\frac{\pi T}{e}$

SOLUTION:

$$\frac{dA}{dt}=\text{constant};$$

$$\frac{t_{AB}}{T}=\frac{(Area)SAB}{(Area)ellipse}=\frac{\frac{\pi ab}{4}-\frac{1}{2}b(ea)}{\pi ab}=\frac{\frac{\pi}{4}-\frac{1}{2}e}{\pi}=\text{T}\left[\frac{1}{4}-\frac{e}{2\pi}\right]$$

Polar Satellites:

▷These are low altitude (500 km to 800 km) satellites

▷They go round the poles of earth in north-south direction

▷Polar satellites have a time period of 100 minutes nearly

▷These satellites can view polar and equatorial regions at close distances with good resolution.

▷ These satellites are useful for remote sensing, meteorology and environmental studies of earth.

Condition of Weightlessness in a Satellite:

▷The force acting on the astronaut of mass 'm' is $\frac{GMm}{r^2}-F_R=\frac{mv_0^2}{r}$ here F_R is the reactional force

▷The reactional force on the floor of the satellite is zero, hence there is the state of weight lessness in a satellite.

i.e., $\frac{GMm}{r^2}=\frac{mv_0^2}{r}$

▷As the frame of reference attached to the satellite is an accelerated frame, whose acceleration towards the centre of the earth is $a=\frac{v_0^2}{r}=\frac{GM}{r^2}=g$

GAUSS THEOREM IN GRAVITATION IS:

$$\phi_g=-4\pi G\left[M_{enclosed}\right]=\int\vec{E}_g.\overrightarrow{dA}$$

:: THEORY BITS ::

1. If the earth is at one-fourth of its present distance from the sun, the duration of the year would be

1) Half the present year 2) One-eight the present year
3) One-fourth the present year 4) One -sixteenth the present year

KEY :2

2. The radius vector drawn from the sun to a planet sweeps out ___ areas in equal time

1) equal 2) unequal 3) greater 4) less

KEY :1

3. Feeling of weightlessness in a satellite is due to

1) absence of inertia 2) absence of gravity
3) absence of accelerating force 4) free fall of satellite

KEY :4

4. According to Kepler's second law, line joining the planet to the sun sweeps out equal areas in equal intervals. This suggests that for the planet

1) radial acceleration is zero 2) tangential acceleration is zero
3) transverse acceleration is zero 4) All

KEY :3

5. If F_g and F_e are gravitational and electrostatic forces between two electrons at a distance 0.1 m then F_g / F_e is in the order of

1) 10^{43} 2) 10^{-43} 3) 10^{35} 4) 10^{-35}

KEY :2

6. $F = \frac{Gm_1m_2}{r^2}$ is valid

1) Between bodies with any shape 2) Between particles
3) Between any bodies with uniform density 4) Between any bodies with same shape

KEY :2

7. F_g, F_e and F_n represent the gravitational, electro-magnetic and nuclear forces respectively, then arrange the increasing order of their strengths

1) F_n, F_e, F_g 2) F_g, F_e, F_n 3) F_e, F_g, F_n 4) F_g, F_n, F_e

KEY :2

8. The time period of revolution of geo-stationary satellite with respect to earth is

1) 24 hrs 2) 1 year 3) Infinity 4) zero

KEY :3

9. Find the false statement

1) Gravitational force acts along the line joining the two interacting particles
2) Gravitational force is independent of medium
3) Gravitational force forms an action-reaction pair
4) Gravitational force does not obey the principle of superposition.

KEY :4

10. A relay satellite transmits the television programme from one part of the world to another part continuously because its period

1) is greater than period of the earth about its axis
2) is less than period of rotation of the earth about its axis.
3) has no relation with the period of rotation of the earth about its axis.

4) is equal to the period of rotation of the earth about its axis.

KEY :4

11. Attractive Force is exists between two protons inside the Nucleus this is due to

1) Gravitational Forces 2) Electro magnetic Forces

3) Weak Nuclear Forces 4) Strong Nuclear Forces

KEY :4

12. Two equal masses separated by a distance d attract each other with a force (F). If one unit of mass is transferred from one of them to the other, the force

1) does not change 2) decreases by (G/d^2)

3) becomes d^2 times 4) increases by $(2G/d^2)$

KEY :2

13. When a satellite falls into an orbit of smaller radius its speed

1) decreases 2) increases 3) does not change 4) zero

KEY :2

14. Which of the following is the evidence to show that there must be a force acting on earth and directed towards Sun?

1) Apparent motion of sun around the earth 2) Phenomenon of day and night

3) Revolution of earth round the Sun 4) Deviation of the falling body towards earth

KEY :3

15. Polar satellite go round the poles of earth in

1) South-east direction 2) north-west direction

3) east-west direction 4) north-south direction

KEY :4

16. If suddenly the gravitational force of attraction between earth and satellite revolving around it becomes zero, then the satellite will (AIEEE 2002)

1) Continue to move in its orbit with same velocity

2) Move tangential to the original orbit with the same velocity

3) Becomes stationary in its orbit 4) Move towards the earth

KEY :2

17. A synchronous satellite should be at a proper height moving

1) From west to East in equatorial plan 2) From South to North in polar plane

3) From East to west in equatorial plan 4) From North to South in polar plane

KEY :1

18. If the speed of rotation of earth about its axis increases, then the weight of the body at the equator will

1) increase 2) decrease

3) remains unchanged 4) some times decrease and sometimes increase

KEY :2

19. A body is dropped from a height equal to radius of the earth. The velocity acquired by it before touching the ground is

1) $V=\sqrt{2gR}$ 2) V=3gR 3) $V=\sqrt{gR}$ 4) V=2gR

KEY :3

20. The ratio of acceleration due to gravity at a depth 'h' below the surface of earth and at a height 'h' above the surface for h<<R

1) constant only when h<<R 2) increases linearly with h

3) increases parabolically with h 4) decreases

KEY :2

21. A person will get more quantity of matter in Kg-Wt at

1) poles 2) a latitude of 60^0 3) equator 4) satellite

KEY :3

22. A geo-stationary satellite has an orbital period of

1) 2 hours 2) 6 hours 3) 24 hours 4) 12 hours

KEY :3

23. A pendulum clock which keeps correct time at the surface of the earth is taken into a mine, then

1) it keeps correct time 2) it gains time 3) it loses time 4) none of these

KEY :3

24. A satellite is revolving in a elliptical orbit in free space; then the false statement is

1) its mechanical energy is constant 2) its linear momentum is constant
3) its angular momentum is constant 4) its areal velocity is constant

KEY :2

25. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth

1) the acceleration of S is always directed towards the centre of the earth
2) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
3) the total mechanical energy of S varies periodically with time
4) the linear momentum of S remains constant in magnitude

KEY :1

26. Assuming the earth to be a sphere of uniform density the acceleration due to gravity

1) at a point outside the earth is inversely proportional to the square of its distance from the centre
2) at a point outside the earth is inversely proportional to its distance from the centre
3) at a point inside is zero
4) at a point inside is inversely proportional to its distance from the centre.

KEY :1

27. An artificial satellite of mass m is revolving round the earth in a circle of radius R. Then work done in one revolution is

1) mgR 2) $\frac{mgR}{2}$ 3) $2\pi R \times mg$ 4) Zero

KEY :4

28. If earth were to rotate faster than its present speed, the weight of an object

1) increase at the equator but remain unchanged at poles
2) decrease at the equator but remain unchanged at the poles
3) remain unchanged at the equator but decrease at the poles
4) remain unchanged at the equator but increase at the poles

KEY :2

29. A satellite is revolving round the earth. Its kinetic energy is E_k. How much energy is required by the satellite such that it escapes out of the gravitational field of earth

1) $2E_k$ 2) $3E_k$ 3) $\frac{E_k}{2}$ 4) infinity

KEY :1

30. If R=radius of the earth and g =acceleration due to gravity on the surface of the earth, the

acceleration due to gravity at a distance (r<R) from the centre of the earth is proportional to

1) r 2) r^2 3) r^{-2} 4) r^{-1}

KEY :1

31. Earth is flattened at poles and bulged at equators this is due to

1) revolution of earth around the sun is an elliptical orbit

2) angular of velocity of spinning about its axis is more at equator

3) centrifugal force is more at equator than poles

4) more centrifugal force at poles than equator

KEY :3

32. Intensity of gravitational field inside the hollow spherical shell is

1) Variable 2) minimum 3) maximum 4) zero

KEY :4

33. If the universal gravitational constant increases uniformly with time, then a satellite in orbit will still maintain its

1) weight 2) tangential speed 3) period of revolution 4) angular momentum

KEY :4

34. The work done by an external agent to shift a point mass from infinity to the centre of the earth is 'W'. Then choose the correct relation.

1) W=0 2) W>0 3) W<0 4) $W \leq 0$

KEY :3

35. When a satellite in a circular orbit around the earth enters the atmospheric region, if encounters small air resistance to its motion. Then

1) its angular momentum about the earth decreases 2) its kinetic energy decreases

3) its kinetic energy remains constant 4) its period of revolution around the earth increases

KEY :1

36. The intensity of the gravitational field of the earth maximum?

1) centre of earth 2) equator 3) poles 4) same everywhere

KEY :3

37. Let V_G and E_G denote gravitational potential and field respectively, then choose the wrong statement.

1) $V_G = 0, E_G = 0$ 2) $V_G \neq 0, E_G = 0$ 3) $V_G = 0, E_G \neq 0$ 4) $V_G \neq 0, E_G \neq 0$

KEY :3

38. The time period of an earth's satellite in circular orbit is independent of

1) the mass of the satellite 2) radius of its orbit

3) both the mass and radius of the orbit 4) neither the mass of the satellite nor the radius of its orbit

KEY :1

39. A thin spherical shell of mass 'M' and radius 'R' has a small hole. A particle of mass 'm' is released at the mouth of them. Then

1) the particle will execute S.H.M inside the shell

2) the particle will oscillate inside the shell, but the oscillations are not simple harmonic

3) the particle will not oscillate, but the speed of the particle will go on increasing

4) none of these

KEY :4

40. A hole is drilled through the earth along a diameter and a stone is dropped into it. When the stone is at the centre of the earth, it has finite a) weight b) acceleration c) P.E. d) mass

1) a & b 2) b & c 3) a, b & c 4) c & d

KEY :4

41. A body has weight (w) on the ground. The work which must be done to lift it to a height equal to the radius of the earth is

1) equal to WR 2) greater than WR 3) less than WR 4) we can't say

KEY :3

42. The time period of a simple pendulum at the centre of the earth is

1) zero 2) infinite 3) less than zero 4) none of these

KEY :2

43. If the gravitational force of earth suddenly disappears, then,

1) weight of the body is zero 2) mass of the body is zero
3) both mass and weight become zero 4) neither the weight nor the mass is zero

KEY :1

44. A hallow spherical shell is compressed to half it radius. The gravitational potential at the centre

1) increases 2) decreases 3) remains same
4) during the compression increases then returns to the previous value

KEY :2

45. A satellite is moving with constant speed 'V' is a circular orbit about earth. The kinetic energy of the satellite is

1) $\frac{1}{2}mV^2$ 2) mV^2 3) $\frac{3}{2}mV^2$ 4) $2mV^2$

KEY :1

46. For a satellite projected from the earth's surface with a velocity greater than orbital velocity the nature of the path it takes when its energy is negative, zero and positive respectively is

1) Elliptical, parabolic and hyperbolic 2) Hyperbolic, parabolic and elliptical
3) Elliptical, circular and parabolic 4) Parabolic, circular and Elliptical

KEY :1

47. If the mean radius of earth is R, its angular velocity is ω and the acceleration due to gravity at the surface of the earth is 'g' then the cube of the radius of the orbit of a satellite will be

1) $\frac{Rg}{\omega^2}$ 2) $\frac{R^2g}{\omega}$ 3) $\frac{R^2g}{\omega^2}$ 4) $\frac{R^2\omega}{g}$

KEY :3

48. If satellite is orbiting in space having air and no energy being supplied, then path of that satellite would be

1) circular 2) elliptical
3) spiral of increasing radius 4) spiral of decreasing radius

KEY :4

49. The earth retains its atmosphere, due to

1) The special shape of the earth
2) The escape velocity being greater than the mean speed of the molecules of the atmospheric gases.
3) The escape velocity being smaller than the mean speed of the molecules of the atmospheric gases.
4) The sun's gravitational effect.

KEY :2

50. Ratio of the radius of a planet A to that of planet B is 'r'. The ratio of accelerations due to

gravity for the two planets is x. The ratio of the escape velocities from the two planets is

1) $\sqrt{rx}$ 2) $\sqrt{r/x}$ 3) $\sqrt{r}$ 4) $\sqrt{x/r}$

KEY :1

51. The ratio of the escape velocity and the orbital velocity is

1) $\sqrt{2}$ 2) $1/\sqrt{2}$ 3) 2 4) 1/2

KEY :1

52. The escape velocity from the earth for a rocket is 11.2 km/sec. Ignoring the air resistance, the escape velocity of 10 mg grain of sand from the earth will be

1) 0.112 km/sec 2) 11.2 km/sec 3) 1.12 km/sec 4) None

KEY :2

53. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45^0 with the vertical, the escape velocity will be

1) $11\sqrt{2}\,km/s$ 2) $22\,km/s$ 3) $11\,km/s$ 4) $11\sqrt{2}\,km/s$

KEY :3

54. Following physical quantity is constant when a planet that revolves around Sun in an elliptical orbit.

1) Kinetic energy 2) Potential energy 3) Angular momentum 4) Linear velocity

KEY :3

55. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energies is

1) Positive 2) Negative 3) Zero
4) May be positive or negative depending upon its initial velocity

KEY :2

56. The escape velocity of a body depends upon its mass as

1) m^0 2) m^1 3) m^3 4) m^2

KEY :1

57. If the universal gravitational constant decreases uniformly with time, then a satellite in orbit will still maintain its

1) weight 2) tangential speed 3) period of revolution 4) angular momentum

KEY :4

58. The magnitude of potential energy per unit mass of the object at the surface of earth is 'E'. Then escape velocity of the object is

1) $\sqrt{2E}$ 2) $4E^2$ 3) $\sqrt{E}$ 4) $\sqrt{E/2}$

KEY :1

59. Tidal waves in the sea are primarily due to

1) the gravitational effect of the moon on the earth
2) the gravitational effect of the sun on the earth
3) the gravitational effect of the venus on the earth
4) the atmospheric effect of the earth itself

KEY :1

60. A space station is set up in space at a distance equal to earth's radius from earth's surface. Suppose a satellite can be launched from space station. Let V_1 and V_2 be the escape velocities of the satellite on earth's surface and space station respectively. Then

1) $V_2 = V_1$ 2) $V_2 < V_1$ 3) $V_2 > V_1$ 4) No relation

KEY :2

61. The minimum number of geo-stationary satellites required to televise a programme all over the earth is

1) 2 2) 6 3) 4 4) 3

KEY :4

62. When a satellite going around the earth in a circular orbit of radius r and speed v loses some of its energy , then

1) r and v both increase
2) r and v both decrease
3) r will increase and v will decrease
4) r will decrease and v will increase

KEY :4

63. The satellite is orbiting a planet at a certain height in a circular orbit. If the mass of the planet is reduced to half, the satellite would

1) fell on the planet
2) go to orbit of smaller radius
3) go to orbit of higher radius
4) escape from the planet

KEY :4

64. If R = radius of the earth and g = acceleration due to gravity on the surface of the earth, the acceleration due to gravity at a distance (r>R) from the centre of the earth is proportional to

1) r 2) r^2 3) r^{-2} 4) r^{-1}

KEY :3

65. A satellite is revolving round the earth in an elliptical orbit. Its speed will be

1) same at all points of the orbit
2) different at different points of the orbit
3) maximum at the farthest point
4) minimum at the nearest point

KEY :2

66. A satellite is moving in a circular orbit round the earth. If any other planet comes in between them, it will

1) Continue to move with the same speed along the same path
2) Move with the same velocity tangential to original orbit.
3) Fall down with increasing velocity.
4) Come to rest after moving certain distance along original path.

KEY :2

67. A space-ship entering the earth's atmosphere is likely to catch fire. This is due to

1) The surface tension of air
2) The viscosity of air
3) The high temperature of upper atmosphere
4) The greater portion of oxygen in the atmosphere at greater height.

KEY :2

68. An astronaut orbiting the earth in a circular orbit 120 km above the surface of earth, gently drops a ball from the space-ship. The ball will

1) Move randomly in space
2) Move along with the space-ship
3) Fall vertically down to earth
4) Move away from the earth

KEY :2

69. The energy required to remove an earth satellite of mass ‘m’ from its orbit of radius ‘r’ to infinity is

1) $\frac{GMm}{r}$ 2) $\frac{-GMm}{2r}$ 3) $\frac{GMm}{2r}$ 4) $\frac{Mm}{2r}$

KEY :3

70. Pseudo force also called fictitious force such as centrifugal force arises only in

1) Inertial frames 2) Non-intertial frames
3) Both inertial and non-inertial frames 4) Rigid frames

KEY :2

71 A satellite launching station should be L

1) Near the equatorial region 2) Near the polar region
3) On the polar axis 4) At any place

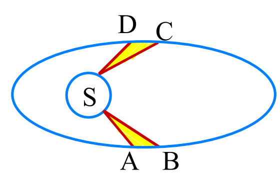
KEY :1

72. The period of a satellite moving in circular orbit near the surface of a planet is independent of

1) mass of the planet 2) radius of the planet
3) mass of the satellite 4) density of planet

KEY :3

73. The motion of a planet around sun in an elliptical orbit is shown in the following figure. Sun is situated on one focus. The shaded areas are equal. If the planet takes time 't_1' and 't_2' in moving from A to B and from C to D respectively, then



1) $t_1 > t_2$ 2) $t_1 < t_2$ 3) $t_1 = t_2$ 4) incomplete information

KEY :3

74. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B moves from west to east. Which train will exert greater force on the track?

1) A 2) B 3) they will exert equal force
4) The mass and the speed of each train must be known to reach a conclusion.

KEY :1

75. Out of the following statements, the one which correctly describes a satellite orbiting about the earth is

1) There is no force acting on the satellite
2) The acceleration and velocity of the satellite are roughly in the same direction
3) The satellite is always accelerating about the earth
4) The satellite must fall, back to earth when its fuel is exhausted.

KEY :3

76. Out of the following interactions weakest is

1) gravitational 2) electromagnetic 3) nuclear 4) electrostatic

KEY :1

77. When an astronaut goes out of his space-ship into the space he will

1) Fall freely on the earth 2) Go upwards
3) Continue to move along with the satellite in the same orbit. 4) Go spiral to the earth

KEY :3

78. If a satellite is moved from one stable circular orbit to a farther stable circular orbit, then the following quantity increases

1) Gravitational force 2) Gravitational P.E.
3) linear orbital speed 4) Centripetal acceleration

KEY :2

79. When the height of a satellite increases from the surface of the earth.

1) PE decreases, KE increases 2) PE decreases, KE decreases
3) PE increases, KE decreases 4) PE increases, KE increases

KEY :3

80. Neutron changing into Proton by emitting electron and anti neutrino this due to

1) Gravitational Forces 2) Electro magnetic Forces
3) Weak Nuclear Forces 4) Strong Nuclear Forces

KEY :3

81. If S_1 is surface satellite and S_2 is geostationary satellite, with time periods T_1 and T_2, orbital velocities V_1 and V_2,

1) $T_1 > T_2$; $V_1 > V_2$ 2) $T_1 > T_2$; $V_1 < V_2$
3) $T_1 < T_2$; $V_1 < V_2$ 4) $T_1 < T_2$; $V_1 > V_2$

KEY :4

82. Among the following find the wrong statement is

1) Law of gravitation is framed using Newton's third law of motion
2) Law of gravitation cannot explain why gravity exists
3) Law of gravitation does not explain the presence of force even when the particles are not in physical contact
4) When the range is long, gravitational force becomes repulsive.

KEY :4

83. The following statement is correct about the motion of earth satellite.

1) It is always accelerating towards the earth
2) There is no force acting on the satellite
3) Move away from the earth normally to the orbit
4) Fall down on to the earth

KEY :1

84. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving around earth in circular orbits of radii r_1 and r_2 ($r_1 > r_2$) respectively. Which of the following statements is true regarding their velocities V_1 and V_2.

1) $V_1 = V_2$ 2) $V_1 < V_2$ 3) $V_1 > V_2$ 4) $\frac{V_1}{r_1} = \frac{V_2}{r_2}$

KEY :2

85. If the area swept by the line joining the sun and the earth from Feb 1 to Feb 7 is 'A', then the area swept by the radius vector from Feb 8 to Feb 28 is

1) A 2) 2A 3) 3A 4) 4A

KEY :3

86. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change

1) gravitational potential energy 2) angular velocity
3) linear orbital velocity 4) centripetal acceleration

KEY :1

87. Average density of the earth (2005A)

1) does not depend on 'g' 2) is a complex function of 'g'
3) is directly proportional to 'g' 4) is inversely proportional to 'g'

KEY :3

88. Two artificial satellites are revolving in the same circular orbit. Then they must have the same

1) Mass 2) Angular momentum
3) Kinetic energy 4) Period of revolution

KEY :4

89. Two identical spherical masses are kept at some distance. Potential energy when a mass 'm' is taken from the surface of one sphere to the other

1) increases continuously 2) decreases continuously
3) first increases, then decreases 4) first decreases, then increases

KEY :3

90. **Six particles each of mass 'm' are placed at the corners of a regular hexagon of edge length 'a'. If a point mass 'm_0' is placed at the centre of the hexagon, then the net gravitational force on the point mass is**

1) $\frac{6Gm^2}{a^2}$ 2) $\frac{6Gmm_0}{a^2}$ 3) zero 4) $\frac{6Gm}{a^4}$

KEY :3

91. An artificial satellite of the earth releases a packet If air resistance is neglected, the point where the packet will hit, will be

1) ahead 2) exactly below 3) behind 4) it will never reach the earth

KEY :4

92. A satellite in vacuum

1) is kept in orbit by solar energy 2) previous energy from gravitational field
3) by remote control 4) No energy is required for revolving

KEY :4

93. The gravitational field is a conservative field. The work done in this field by moving an object from one point to another

1) depends on the end-points only
2) depends on the path along which the object is moved
3) depends on the end-points as well as the path between the points.
4) is not zero when the object is brought back to its initial position.

KEY :1

94. Two heavenly bodies s_1 & s_2 not far off from each other, revolve in orbit

1) around their common centre of mass
2) s_1 is fixed and s_2 revolves around s_1
3) s_2 is fixed and s_1 revolves around s_2
4) cannot say

KEY :1

95. A body of mass 5 kg is taken into space. Its mass becomes.

1) 5 kg 2) 10 kg 3) 2 kg 4) 30 kg

KEY :1

96. If V, T, L, K and r denote speed, time period, angular momentum, kinetic energy and radius of satellite in circular orbit

a) $V \alpha r^{-1}$ **b)** $L \alpha r^{1/2}$ **c)** $T \alpha r^{3/2}$ **d)** $K \alpha r^{-2}$

1) a,b are true 2) b,c are true 3) a,b,d are true 4) a,b,c are true

KEY :2

97. Two satellites are revolving around the earth in circular orbits of same radii. Mass of one satellite is 100 times that of the other. Then their periods of revolution are in the ratio

1) 100:1 2) 1:100 3) 1:1 4) 10:1

KEY :3

98. Two similar satellite s_1 and s_2 of same mass 'm' have completely inelastic collision while orbiting earth in the same circular orbit in opposite direction then

1) total energy of satellites and earth system become zero
2) the satellites stick together and fly into space
3) the combined mass falls vertically down
4) the satellites move in opposite direction

KEY :3

99. For a planet revolving round the sun, when it is nearest to the sun is

1) K.E. is min and P.E. is max. 2) Both K.E. and P.E. are min
3) K.E. is max. and P.E. is min 4) K.E. and P.E. are equal

KEY :3

100. Consider earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a balloon. The gravitational field measured by

1) A goes on decreasing and that of B goes on increasing
2) B goes on decreasing and that of A goes on increasing
3) Each decreases at the same rate
4) Each decreases at different rates.

KEY :4

101. When projectile attains escape velocity, then on the surface of planet , its

1) $KE > PE$ 2) $PE > KE$ 3) $KE = PE$ 4) $KE = 2PE$

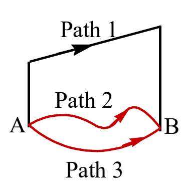
KEY :3

102. The orbit of geo-stationary satellite is circular, the time period of satellite depends on (2008 E)

1) mass of the Earth 2) radius of the orbit
3) height of the satellite from the surface of Earth 4) all the above

KEY :4

103. A gravitation field is present in a region. A point mass is shifted from A to B, along different paths shown in the figure. If W_1, W_2 and W_3 represent the work done by gravitational force for respective paths, then



1) $W_1 = W_2 = W_3$ 2) $W_1 > W_2 > W_3$ 3) $W_1 > W_3 > W_2$ 4) none of these

KEY :1

104. The orbital angular velocity vector of a geo-stationary satellite and the spin angular velocity vector of the earth are

1) always in the same direction 2) always in opposite direction
3) always mutually perpendicular 4) inclined at 23 1/2^0 to each other

KEY :1

105. Repulsive force exist between two protons out side the nucleus this due to

1) Gravitational Forces 2) Electro magnetic Forces
3) Weak Nuclear Forces 4) Strong Nuclear Forces

KEY :2

106. Radio activity decay exist due to

1) Gravitational Forces 2) Electro magnetic Forces
3) Weak Nuclear Forces 4) Strong Nuclear Forces

KEY :3

107. It is not possible to keep a geo-stationary satellite over Delhi. Since Delhi

1) is not present in A.P 2) is capital of India
3) is not in the equatorial plane of the earth 4) is near Agra.

KEY :3

108. If the area swept by the line joining the sun and the earth from Feb 1 to Feb 7 is 'A', then the area swept by the radius vector from Feb 8 to Feb 28 is

1) A 2) 2A 3) 3A 4) 4A

KEY :3

109. The angle between the equatorial plane and the orbital plane of geo-stationary satellite is

1) 45^0 2) 0^0 3) 90^0 4) 60^0

KEY :2

110. The angle between the equatorial plane and the orbital plane of a polar satellite is

1) 45^0 2) 0^0 3) 90^0 4) 60^0

KEY :3

111. Law of gravitation is not applicable if

A) Velocity of moving objects are comparable to velocity of light

B) Gravitational field between objects whose masses are greater than the mass of sun.

1) A is true, B is false 2) A is false, B is true
3) Both A & B are true 4) Both A&B are false

KEY :3

112. Which of the following quantities remain constant in a planetary motion, when seen from the surface of the sun.

1) K.E 2) angular speed
3) speed 4) Angular momentum

KEY :4

::PRACTICE BITS ::

1. A planet revolves round the sun in an elliptical orbit of semi minor and semi major axes *x* and *y* respectively. Then the time period of revolution is proportional to

1) $(x+y)^{\frac{3}{2}}$ 2) $(y-x)^{\frac{3}{2}}$ 3) $x^{\frac{3}{2}}$ 4) $y^{\frac{3}{2}}$

KEY:4

HINT:

From Kepler's 3rd law,

$$T^2 \alpha r^3$$

2. The moon revolves round the earth 13 times in one year. If the ratio of sun-earth distance to earth-moon distance is 392, then the ratio of masses of sun and earth will be

1) 365 2) 365×10^{-12} 3) 3.56×10^{5} 4) 1

KEY:3

HINT

From Kepler's 3rd law, $T^2 \alpha \frac{r^3}{M}$

3. An artificial satellite is revolving around the earth in a circular orbit. Its velocity is one-third of the escape velocity. Its height from the earth's surface is (in Km)

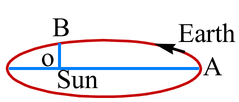
1) 22400 2) 12800 3) 3200 4) 1600

KEY:1

HINT

$$\frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$

4. The Earth moves around the Sun in an elliptical orbit as shown in the figure. The ratio $\frac{OA}{OB} = x$. Then ratio of the speed of the Earth at B and at A is nearly



1) $\sqrt{x}$ 2) x 3) $x\sqrt{x}$ 4) x^2

KEY:2

HINT

From conservation of angular momentum

mvr=constant, $v_1r_1=v_2r_2$

5. The period of moon's rotation around the earth is nearly 29 days. If moon's mass were 2 fold its present value and all other things remain unchanged, the period of moon's rotation would be nearly (in days)

1) $29\sqrt{2}$ 2) $29/\sqrt{2}$ 3) $29\sqrt{3}$ 4) 29

KEY:4

HINT

Time period does not depend upon the mass of the satellite

6. A spaceship is launched into a circular orbit of radius 'R' close to the surface of earth. The additional velocity to be imparted to the spaceship in the orbit to overcome the earth's gravitational pull is (g = acceleration due to gravity)

1) 1.414Rg 2) $1.414\sqrt{Rg}$ 3) 0.414Rg 4) $0.414\sqrt{gR}$

KEY:4

HINT

$$V = V_e - V_0 = \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\left(\sqrt{2} - 1\right)$$

7. If the mass of earth were 2 times the present mass, the mass of the moon were half the present mass and the moon were revolving round the earth at the same present distance, the time period of revolution of the moon would be (in days)

1) 56 2) 28 3) $14\sqrt{2}$ 4) 7

KEY:3

HINT

From Kepler's 3rd law, $T = 2\pi\sqrt{\frac{r^3}{GM}}$

8. If the Earth shrinks such that its density becomes 8 times to the present value, then new duration of the day in hours will be (2008M)

1) 24 2) 12 3) 6 4) 3

KEY:3

HINT

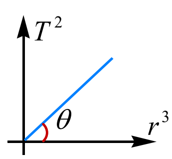
Given $m_1 = m_2 \Rightarrow V_1 d_1 = V_2 d_2$

$\Rightarrow R_1^3 d_1 = R_2^3 d_2 \Rightarrow R_1 = 2R_2$

From law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^2$$

9. If a graph is plotted between T^2 and r^3 for a planet then, its slope will be



1) $\frac{4\pi^2}{GM}$ 2) $\frac{GM}{4\pi^2}$ 3) $4\pi GM$ 4) Zero

KEY:1
HINT

$$\text{Slope} = \frac{T^2}{R^3} = \frac{4\pi^2}{GM} \text{ (From Kepler's 3rd law)}$$

10. The ratio of Earth's orbital momentum (about sun) to its mass is $4.4 \times 10^{15} m^2 s^{-1}$. The area enclosed by the earth's orbit is approximately

1) $1 \times 10^{22} m^2$ 2) $3 \times 10^{22} m^2$ 3) $5 \times 10^{22} m^2$ 4) $7 \times 10^{22} m^2$

KEY:4
HINT

Areal velocity = Area swept/time for one revolution of earth about sun

So, Area = (Areal velocity)(Time period)

$$= \frac{L}{2m} \times 365 \times 86400$$

11. Two identical spheres each of radius R are placed with their centres at a distance nR, where n is integer greater than 2. The gravitational force between them will be proportional to

1) $1/R^4$ 2) $1/R^2$ 3) R^2 4) R^4

KEY:4
HINT

$$F = \frac{Gm_1m_2}{R^2}; \text{Here } m = \frac{4}{3}\pi R^3$$

12. The K.E. of a satellite in an orbit close to the surface of the earth is E. Its max K.E. so as to escape from the gravitational field of the earth is.

1) 2E 2) 4E 3) $2\sqrt{2}$ E 4) $\sqrt{2}$ E

KEY:1
HINT

$$\frac{K_e}{K_0} = \frac{2gR}{gR} \Rightarrow K_e = 2K_0$$

13. A satellite is orbiting round the earth. If both gravitational force and centripetal force on the satellite is F, then, net force acting on the satellite to revolve round the earth is

1) F/2 2) F 3) 2F 4) Zero

KEY:2
HINT

Gravitational force provides centripetal force.

14. Mass M=1 unit is divided into two parts X and (1-X). For a given separation the value of X for which the gravitational force between them becomes maximum is

1) 1/2 2) 3/5 3) 1 4) 2

KEY:1

HINT

$$F = \frac{G \times m(1-x)mx}{R^2} \text{ is maximum when } x = \frac{1}{2}$$

15. Gravitational force between two point masses m and M separated by a distance is F. Now if point mass 3m is placed next to m, the total force on M will be

1) F 2) 2F 3) 3F 4) 4F

KEY:4

HINT

$$F = \frac{GMm}{r^2}; F' = \frac{GM(m+2m)}{r^2}$$

16. If the distance between the sun and the earth is increased by three times, then the gravitational force between the two will

1) remain constant 2) decrease by 63% 3) increase by 63% 4) decrease by 89%

KEY:4

HINT

$$F = \frac{Gm_1m_2}{R^2}$$

17. Energy required to shift a body of mass 'm' from an orbit of radius 2R and 3R is (2002A)

1) $\frac{GMm}{12R}$ 2) $\frac{GMm}{3R^2}$ 3) $\frac{GMm}{8R}$ 4) $\frac{GMm}{6R}$

KEY:1

HINT

$$W = T.E_2 - T.E_1 = \frac{GMm}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

18. The orbital speed of geostationary satellite is

1) 8km/sec from west to east 2) 11.2km/sec from east to west

3) 3.1km/sec from west to east 4) zero

KEY:3

HINT

$$V_0 = \sqrt{g(R+h)}$$

19. Two lead balls of masses m and 5m having radii R and 2R are separated by 12R. If they attract each other by gravitational force, the distance covered by small sphere before they touch each other is

1) 10 R 2) 7.5 R 3) 9 R 4) 2.5 R

KEY:2

HINT

$$x = \frac{m_2 d_{eff}}{m_1 + m_2}; Here\ d_{eff} = 9R$$

20. Two particles each of mass 'm' are placed at A and C are such AB=BC=L. The gravitational force on the third particle placed at D at a distance L on the perpendicular bisector of the line AC is

1) $\frac{Gm^2}{L^2}$ along BD 2) $\frac{Gm^2}{\sqrt{2}L^2}$ along DB 3) $\frac{Gm^2}{L^2}$ along AC 4) $\frac{Gm^2}{E^2}$ along BD

KEY:2

HINT

$$F' = \sqrt{2}F, F = \frac{Gm^2}{2L^2}$$

21. If g on the surface of the earth is 9.8 m/s^2, its value at a height of 6400 km is (Radius of the earth = 6400km).

1) $4.9ms^{-2}$ 2) $9.8ms^{-2}$ 3) $2.45ms^{-2}$ 4) $19.6ms^{-2}$

KEY:3

HINT

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

22. If g on the surface of the earth is $9.8ms^{-2}$, its value at a depth of 3200km (Radius of the earth = 6400km) is

1) $9.8ms^{-2}$ 2) zero 3) $4.9ms^{-2}$ 4) $2.45ms^{-2}$

KEY:3

HINT

$$g' = g\left(1 - \frac{d}{R}\right)$$

23. If mass of the planet is 10% less than that of earth and radius of the planet is 20% greater than that of earth then the weight of 40kg person on that planet is

1) 10 kg wt 2) 25 kg wt 3) 40 kg wt 4) 60 kg wt

KEY:2

HINT

$$g = \frac{GM}{R^2} \Rightarrow g\alpha \frac{M}{R^2}$$

24. The angular velocity of the earth with which it has to rotate so that the acceleration due to gravity on 60^0 latitude becomes zero is

1) $2.5\times10^{-3}\ rad\ s^{-1}$ 2) $1.5\times10^{-3}\ rad\ s^{-1}$ 3) $4.5\times10^{-3}\ rad\ s^{-1}$ 4) $0.5\times10^{-3}\ rad\ s^{-1}$

KEY:1

HINT

$$g - R\omega^2\cos^2\lambda = 0,\ \text{given}\ \lambda = 60^0,\ \text{Find}\ \omega$$

25. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The time period of the second satellite is larger than that of the first one by approximately

1) 0.5% 2) 1.5% 3) 1% 4) 3%

KEY:2

HINT

$$T\alpha R^{3/2} \Rightarrow \frac{\Delta T}{T}\times100 = \frac{3}{2}\frac{\Delta r}{r}\times100$$

26. The value of acceleration due to gravity on the surface of earth is x. At an altitude of 'h' from the surface of earth, its value is y. If R is the radius of earth, then the value of h is

1) $\left(\sqrt{\frac{x}{y}}-1\right)R$ 2) $\left(\sqrt{\frac{y}{x}}-1\right)R$ 3) $\sqrt{\frac{y}{x}}R$ 4) $\sqrt{\frac{x}{y}}R$

KEY:1

HINT

$$x = \frac{GM}{R^2},\ y = \frac{GM}{(R+h)^2}$$

27. The height at which the value of acceleration due to gravity becomes 50% of that at the surface of the earth. (Radius of the earth=6400km) is

1) 2650 2) 2430 3) 2250 4) 2350

KEY:1

HINT

$$g' = \frac{GM}{(R+h)^2} \Rightarrow g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

28. The radius and density of two artificial satellites are R_1, R_2 and ρ_1, ρ_2 respectively. The ratio of acceleration due to gravity on them will be

1) $\frac{R_2\rho_2}{R_1\rho_1}$ 2) $\frac{R_1\rho_2}{R_2\rho_1}$ 3) $\frac{R_1\rho_1}{R_2\rho_2}$ 4) $\frac{R_2\rho_1}{R_1\rho_2}$

KEY:3

HINT

$$g=\frac{4}{3}\pi GR\rho \Rightarrow g \propto R\rho$$

29. The acceleration due to gravity at the latitude 45^0 on the earth becomes zero if the angular velocity of rotation of earth is

1) $\sqrt{\frac{2}{gR}}$ 2) $\sqrt{2gR}$ 3) $\sqrt{\frac{2g}{R}}$ 4) $\sqrt{\frac{5R}{2}}$

KEY:3

HINT

$$0=g-R\omega^2\cos^2 45^0 \Rightarrow \omega=\sqrt{\frac{2g}{R}}$$

30. Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth and moon is 5/3, then radius of moon in terms of radius of earth will be

1) $\frac{5}{18}R_e$ 2) $\frac{1}{6}R_e$ 3) $\frac{3}{18}R_e$ 4) $\frac{1}{2\sqrt{3}}R_e$

KEY:1

HINT

$$g=\frac{4}{3}\pi GR\rho \Rightarrow g \propto R\rho$$

31. The point at which the gravitational force acting on any mass is zero due to the earth and the moon system is. (The mass of the earth is approximately 81 times the mass of the moon and the distance between the earth and the moon is 3,85,000km.)

1) 36,000km from the moon 2) 38,500km from the moon

3) 34500km from the moon 4) 30,000 from the moon

KEY:2

HINT

distance of null point $x=\frac{d}{\sqrt{\frac{m_2}{m_1}}+1}$

32. Two satellite M and N go around the earth in circular orbits at heights of R_M and R_N respectively from the surface of the earth. Assuming the earth to be a uniform sphere of radius R_E, the ratio of velocities of the satellite $\frac{V_M}{V_N}$ is

1) $\left(\frac{R_M}{R_N}\right)^2$ 2) $\sqrt{\frac{R_N+R_E}{R_M+R_E}}$ 3) $\frac{R_N+R_E}{R_M+R_E}$ 4) $\sqrt{\frac{R_N}{R_M}}$

KEY:2

HINT $V_0=\sqrt{\frac{GM}{R+h}} \Rightarrow V_0 \propto \frac{1}{\sqrt{R+h}}$

33. Masses 2 kg and 8 kg are 18 cm apart. The point where the gravitational field due to them is zero is

1) 6 cm from 8 kg mass
2) 6 cm from 2 kg mass
3) 1.8 cm from 8 kg mass
4) 9 cm from each mass

KEY:2

HINT

$$\text{distance of null point } x=\frac{d}{\sqrt{\frac{m_2}{m_1}}+1}$$

34. Particles of masses m_1 and m_2 are at a fixed distance apart. If the gravitational field strength at m_1 and m_2 are $\vec{I}_1$ and $\vec{I}_2$ respectively. Then,

1) $m_1\vec{I_1}+m_2\vec{I_2}=0$
2) $m_1\vec{I_2}+m_2\vec{I_1}=0$
3) $m_1\vec{I_1}-m_2\vec{I_2}=0$
4) $m_1\vec{I_2}-m_2\vec{I_1}=0$

KEY:1

HINT

$$\vec{I_1}=\frac{Gm_2}{d^2} \text{ and } \vec{I_2}=-\frac{Gm_1}{d^2}$$

35. A satellite of mass 'm' revolves round the earth of mass 'M' in a circular orbit of radius'r' with an angular velocity 'ω'. If the angular velocity is ω/8 then the radius of the orbit will be

1) 4r 2) 2r 3) 8r 4) r

KEY:1

HINT

$$\text{From Kepler's 3rd law, } T^2 \propto r^3 \Rightarrow \omega^2 \alpha \frac{1}{r^3}$$

36. A body of mass 'm' is raised from the surface of the earth to a height 'nR' (R -radius of earth). Magnitude of the change in the gravitational potential energy of the body is (g - acceleration due to gravity on the surface of earth) (2007M)

1) $\left(\frac{n}{n+1}\right)mgR$ 2) $\left(\frac{n-1}{n}\right)mgR$ 3) $\frac{mgR}{n}$ 4) $\frac{mgR}{(n-1)}$

KEY:1

HINT

$$\Delta GPE = \frac{mgh}{1+\frac{h}{R}}; \quad 27)\ W = m(\Delta V) + \Delta KE$$

37. A person brings a mass 2 kg from A to B. The increase in kinetic energy of mass is 4J and work done by the person on the mass is -10J. The potential difference between B and A is ....J/kg

1) 4 2) 7 3) -3 4) -7

KEY:4

HINT

38. The work done in shifting a particle of mass 'm' from the centre of earth to the surface of the earth is

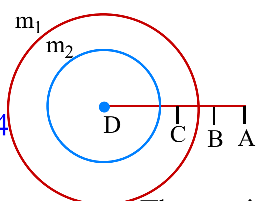
1) –mgR 2) $\frac{1}{2}mgR$ 3) zero 4) mgR

KEY:2

HINT

$$W = GPE_2 - GPE_1; \ \Delta GPE = \frac{mgh}{1+\frac{h}{R}}$$

39. The figure shows two shells of masses m_1 and m_2. The shells are concentric. At which point, a particle of mass m shall experience zero force?



1) A 2) B 3) C 4) D

KEY:4

HINT

The gravitational field intensity at a point inside the spherical shell is zero.

40. Escape velocity of a body of 1kg mass on a planet is 100 m/s. Gravitational potential energy of the body at the plane is

1) -5000J 2) -1000J 3) -2400J 4) 5000J

KEY:1

HINT

$$v_e = \sqrt{\frac{2GM}{R}} = 100 \Rightarrow \frac{GM}{R} = 5000$$

$$\therefore PE(U) = -\frac{GMm}{R} = -5000J$$

41. If three particles, each of mass M are placed at the three corners of an equilateral triangle of side a, the force exerted by this system on another particle of mass M placed (i) at the midpoint of a side and (ii) at the centre of the triangle are, respectively.

1) $0, \frac{4GM^2}{3a^2}$ 2) $\frac{4GM^2}{3a^2}, 0$ 3) $\frac{3GM^2}{a^2}, \frac{GM^2}{a^2}$ 4) 0, 0

KEY:2

HINT

Find individual forces and calculate resultant

Use $F = \frac{Gm_1m_2}{R^2}$

42. **The ratio of escape velocities of two planets if g value on the two planets are $9.9 m/s^2$ and $3.3 m/s^2$ and their radii are 6400km and 3200km respectively is**

1) 2.36 : 1 2) 1.36 : 1 3) 3.36 : 1 4) 4.36 : 1

KEY:1

HINT

$$V_e = \sqrt{2gR} \Rightarrow V_e \propto \sqrt{gR}$$

43. **The distance of Neptune and saturn from the Sun are respectively. 10^{13} and 10^{12} meters and their periodic times are respectively T_n and T_s. If their orbits are circular, the value of T_n/T_s is**

1) 100 2) $10\sqrt{10}$ 3) $\frac{1}{10\sqrt{10}}$ 4) 10

KEY:2

HINT

From Kepler's 3rd law, $T^2 \alpha r^3, \frac{T_1}{T_2} = \left[\frac{r_2}{r_1}\right]^{\frac{3}{2}}$

44. **The escape velocity from the surface of the earth of radius R and density ρ**

1) $2R\sqrt{\frac{2\pi \rho G}{3}}$ 2) $2\sqrt{\frac{2\pi \rho G}{3}}$ 3) $2\pi\sqrt{\frac{R}{g}}$ 4) $\sqrt{\frac{2\pi G\rho}{R^2}}$

KEY:1

HINT

$$V_e = \sqrt{\frac{2GM}{R}} \text{ but } M = \frac{4}{3}\pi R^3 \rho$$

45. **The escape velocity from the earth is 11 km/sec. The escape velocity from a planet having twice the radius and same density as earth is (in km/sec)**

1) 22 km/sec 2) 15.5 km/sec 3) 11 km/sec 4) 5.5 km/sec

KEY:1

HINT

$$V_e \propto R\sqrt{\rho};$$

46. An object of mass 'm' is at rest on earth's surface. Escape speed of this object is V_e . Same object is orbiting earth with $h = R$ then escape speed is V_e^1. Then

1) V_e^1 2) $V_e = 2V_e^1$ 3) $V_e = \sqrt{2}V_e^1$ 4) $V_e^1 = \sqrt{2}V_e$

KEY:3

HINT

$$36)\ \frac{1}{2}mv_0^2 - \frac{GM}{(R+h)} = O$$

47. A satellite revolves in a circular orbit with speed $V = \frac{1}{\sqrt{3}}V_e$. If satellite is suddenly stopped and allowed to fall freely onto earth, the speed with which it hits earth's surface is

1) $\sqrt{gR}$ 2) $\sqrt{\frac{gR}{3}}$ 3) $\sqrt{2gR}$ 4) $\sqrt{\frac{2}{3}gR}$

KEY:4

HINT

$$V = \sqrt{V_e^2 - 2V_0^2}$$

48. A space station is set up in space at a distance equal to earth's radius from the surface of earth. Suppose a satellite can be launched from the space station also. Let v_1 and v_2 be the escape velocities of the satellite on the earth's surface and space station respectively. Then

1) $v_2 = v_1$ 2) $v_2 < v_1$ 3) $v_2 > v_1$

4) 1, 2 and 3 are valid depending on the mass of satellite.

KEY:2

HINT

From the surface of earth

Escape velocity $v_1 = \sqrt{\frac{2GM}{R}}$

$$\frac{1}{2}mv_2^2 - \frac{GMm}{2R} = 0$$

49. A man weight 'W' on the surface of earth and his weight at a height 'R" from surface of earth is (R is Radius of earth)

1) $\frac{W}{4}$ 2) $\frac{W}{2}$ 3) W 4) 4W

KEY:1

HINT

$$W = mg; W' = mg\left(\frac{R}{R+h}\right)^2 = \frac{W}{4}$$

50. A satellite moving in a circular path of radius 'r' around earth has a time period T. If its radius slightly increases by 4%, then percentage change in its time period is

1) 1% 2) 6% 3) 3% 4) 9%

KEY:2

HINT:

$$T^2 \alpha r^3, \frac{\Delta T}{T}\times 100 = \frac{3}{2}\frac{\Delta R}{R}\times 100$$

51. If a rocket is fired with a velocity, $V = 2\sqrt{gR}$ near the earth's surface and goes upwards, its speed in the inter-stellar space is

1) $4\sqrt{gR}$ 2) $\sqrt{2gR}$ 3) $\sqrt{gR}$ 4) $\sqrt{4gR}$

KEY:2

HINT

According to the law of conservation of energy

$$[T.E]_{surface} = [T.E]_{\text{int}er\ stellarspace}$$

$$\Rightarrow \frac{-GMm}{R} + \frac{1}{2}mv^2 = 0 + \frac{1}{2}mv_1^2$$

$$\Rightarrow mgR + \frac{1}{2}m\left(2\sqrt{gR}\right)^2 = \frac{1}{2}mv_1^2$$

$$\Rightarrow \frac{1}{2}mv_1^2 = mgR \Rightarrow v_1^2 = 2\omega R \Rightarrow V_1 = \sqrt{2gR}$$

52. A projectile is fired vertically upwards from the surface of the earth with a velocity Kv_e, where V_e is the escape velocity and K<1. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of the earth will be (neglect air resistance)

1) $\frac{1-K^2}{R}$ 2) $\frac{R}{1-K^2}$ 3) $R\left(1-K^2\right)$ 4) $\frac{R}{1+K^2}$

KEY:2

HINT

According to the law of conservation of energy

$$-\frac{GMm}{R} + \frac{1}{2}mK^2v_e^2 = -\frac{GMm}{R+h} + 0$$

$$v_e^2 = \frac{2GM}{R}$$

$$-\frac{v_e^2}{2}+\frac{K^2v_e^2}{2}=-\frac{v_e^2(R)}{2(R+h)}$$

53. Two different artificial satellites orbiting with same time period around the earth having angular moment 2:1. The ratio of masses of the satellites is

1) 2:1 2) 1:2 3) 1:1 4) 1:3

KEY:1

HINT

$$\frac{dA}{dt}=\frac{L}{2m}$$ =constant. (From Kepler's 2nd law)

54. If the radius of earth shrinks by 0.2% without any change in its mass, escape velocity from the surface of the earth

1) increases by 0.2% 2) decreases by 0.2%

3) increases by 0.1% 4) increases by 0.4%

KEY:3

HINT

$$v_e=\sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v_e\propto\frac{1}{\sqrt{R}}\Rightarrow\frac{\Delta v_e}{v_e}\times 100=-\frac{1}{2}\left(\frac{\Delta R}{R}\times 100\right)$$

55. If 'A' is areal velocity of a planet of mass M, its angular momentum is

1) M/A 2) 2MA 3) A^2M 4) AM^2

KEY:2

HINT:

$$\frac{dA}{dt}=\frac{L}{2M}$$

56. If d the distance between the centres of the earth of mass M_1 and moon of mass M_2, then the velocity with which a body should be projected from the mid point of the line joining the earth and the moon, so that it just escape is

1) $\sqrt{\frac{G(M_1+M_2)}{d}}$ 2) $\sqrt{\frac{G(M_1+M_2)}{2d}}$

3) $\sqrt{\frac{2G(M_1+M_2)}{d}}$ 4) $\sqrt{\frac{4G(M_1+M_2)}{d}}$

KEY:4

HINT

Using law of conservation of energy,

$$\frac{1}{2}mv_e^2=\frac{2GM}{d}(M_1+M_2)$$

57. If 'v_e' is the escape velocity of a body from a planet of mass 'M' and radius 'R'. Then the velocity of the satellite revolving at a height 'h' from the surface of the planet will be

1) $v_e\sqrt{\frac{R}{R+h}}$ 2) $v_e\sqrt{\frac{2R}{R+h}}$ 3) $\sqrt{\frac{R+h}{R}}$ 4) $v_e\sqrt{\frac{R}{2(R+h)}}$

KEY:4

HINT

$$v_e = \sqrt{\frac{2GM}{R}}, v = \sqrt{\frac{GM}{R+h}}$$

$$\Rightarrow \frac{v}{v_e} = \sqrt{\frac{R}{2(R+h)}} \Rightarrow v = v_e\sqrt{\frac{R}{2(R+h)}}$$

58. A particle falls towards earth from infinity. The velocity with which it reaches earth's surface is

1) $v = 2gR$ 2) $v = \sqrt{2gR}$ 3) $v = \sqrt{gR}$ 4) $v = R/g$

KEY:2

HINT

The projecting body having same final velocity to reach projecting place. So, $v = \sqrt{2gR}$

59. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space between the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

1) $\frac{F}{9}$ 2) 3F 3) F 4) $\frac{F}{3}$

KEY:3

HINT

Gravitational force does not depend upon the medium between the masses.

60. Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface, what is the maximum range on the surface of the moon for the same velocity of projection

1) $0.2R_e$ 2) $2R_e$ 3) $0.5R_e$ 4) $5R_e$

KEY:4

HINT

$$R_{max} = \frac{u^2}{g} \Rightarrow R_{max} \propto \frac{1}{g}$$

61. The orbital speed for an earth satellite near the surface of the earth is 7 km/sec. If the radius of the orbit is 4 times the radius of the earth, the orbital speed would be

1) 3.5 km/sec 2) 7 km/sec 3) $7\sqrt{2}$ km/sec 4) 14 km/sec

KEY:1
HINT

$$V_e = \sqrt{\frac{GM}{r}} \Rightarrow V_e \propto \frac{1}{\sqrt{r}}$$

62. Two masses 'M' and '4M' are at a distance 'r' apart on the line joining them, 'P' is point where the resultant gravitational force is zero (such a point is called as null point). The distance of 'P' from the mass 'M' is

1) $\frac{r}{5}$ 2) $\frac{r}{3}$ 3) $\frac{2r}{3}$ 4) $\frac{4r}{5}$

KEY:2
HINT

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1}$$

63. Let 'A' be the area swept by the line joining the earth and the sun during Feb 2012. The area swept by the same line during the first week of that month is

1) A/4 2) 7A/29 3) A 4) 7A/30

KEY:2
HINT:

For 29 days - A, For 1 day - A/29,
For 1 week -7A/29,

64. Three identical particle each of mass "m" are arranged at the corners of an equilateral triangle of side "L". If they are to be in equilibrium, the speed with which they must revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle is

1) $\sqrt{\frac{3Gm}{L}}$ 2) $\sqrt{\frac{Gm}{L}}$ 3) $\sqrt{\frac{Gm}{3L}}$ 4) $\sqrt{\frac{3Gm}{L^2}}$

KEY:2
HINT

$$\sqrt{3}F = \frac{mv^2}{r}; F = \frac{Gm_1m_2}{L^2} \text{ and } r = \frac{L}{\sqrt{3}}$$

65. Two satellites are revolving round the earth at different heights. The ratio of their orbital speeds is 2 : 1. If one of them is at a height of 100km, the height of the other satellite is

1) 19600km 2) 24600km 3) 29600km 4) 14600km

KEY:1
HINT

$$V_e = \sqrt{\frac{GM}{R+h}} \Rightarrow V_e \propto \frac{1}{\sqrt{R+h}}$$

66. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite

is [AIEEE-2004]

1) gx 2) $\left(\frac{gR^2}{R+x}\right)^{1/2}$ 3) $\frac{gR^2}{R+x}$ 4) $\frac{gR}{R-x}$

KEY:2

HINT

$$V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+x}}$$

67. A body is projected vertically up from surface of the earth with a velocity half of escape velocity. The ratio of its maximum height of ascent and radius of earth is

1) 1 : 1 2) 1 : 2 3) 1 : 3 4) 1 : 4

KEY:1

HINT

$$h = \frac{R}{n^2 - 1} \text{ Here n=2}$$

68. The PE of three objects of masses 1kg, 2kg and 3kg placed at the three vertices of an equilateral triangle of side 20cm is

1) 25G 2) 35G 3) 45G 4) 55G

KEY:4

HINT

$$GPE(U) = \frac{Gm_1m_2}{r_{12}} \text{ Use, } U_{net} = U_1 + U_2 + U_3$$

69. If the mass of one particle is increased by 50% and the mass of another particle is decreased by 50% the force between them

1) decreases by 25% 2) decreases by 75% 3) increases by 25% 4) does not change

KEY:1

HINT

$$F_1 = G\frac{m_1m_2}{d^2} \text{ and } F_2 = G\frac{\left(m_1 + \frac{m_1}{2}\right)\left(m_2 - \frac{m_2}{2}\right)}{d^2}$$

70. An astronaut orbiting in a spaceship round the earth has a centripetal acceleration of $2.45m/s^2$. The height of spaceship from earth's surface is (R= radius of earth)

1) 3R 2) 2R 3) R 4) $R/2$

KEY:3

HINT

$$a = \frac{gR^2}{(R+h)^2};$$

71. Two satellites P, Q are revolving around earth in different circular orbits. The velocity of P is twice the velocity of Q. If the height of P from earth's surface is 1600 km. The radius of orbit of Q is (radius of earth R =6400 km).

1) 1600 km 2) 20000 km 3) 32000 km 4) 40000 km

KEY:3

HINT

Given, $v_P = 2v_Q$

$$\sqrt{\frac{GM}{r_P}} = 2\sqrt{\frac{GM}{r_Q}} \rightarrow r_Q = 4r_P$$

72. A planet is revolving around the sun. Its distance from the sun at apogee is r_A and that at perigee is r_p. The masses of planet and sun are 'm' and M respectively, V_A is the velocity of planet at apogee and V_P is at perigee respectively and T is the time period of revolution of planet round the sun, then identify the wrong answer.

1) $T^2 = \frac{\pi^2}{2GM}(r_A + r_P)^3$ 2) $T^2 = \frac{\pi^3}{2GM}(r_A + r_P)^3$

3) $V_A r_A = V_P r_P$ 4) $V_A < V_P; r_A > r_P$

KEY:1

HINT

$$T^2 = \frac{4\pi^2}{GM}\left[\frac{r_A + r_P}{2}\right]^3 \quad \left(\because r = \frac{r_A + r_P}{2}\right)$$

By the law of conservation of angular momentum $V_A r_A = V_P r_P$

73. Suppose the gravitational force varies inversely as the n^{th} power of distance, then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to (2004A)

1) $R^{\left(\frac{n+1}{2}\right)}$ 2) $R^{\left(\frac{n-2}{2}\right)}$ 3) R^n 4) $R^{\left(\frac{n-1}{2}\right)}$

KEY:1

HINT

$$F = \frac{k}{r^n}, F = mr\omega^2$$

74. A satellite moves around the earth in a circular orbit with speed 'V'. If 'm' is mass of the satellite then its total energy is

1) $\frac{1}{2}mv^2$ 2) mv^2 3) $-\frac{1}{2}mv^2$ 4) $\frac{3}{2}mv^2$

KEY:3

HINT

$$TE = -KE = -\frac{1}{2}mv^2$$

75. The mass of a planet is half that of the earth and the radius of the planet is one fourth that of earth. If we plan to send an artificial satellite from the planet, the escape velocity will be, (V_e=11kms^{-1})

1) 11kms^{-1} 2) 5.5kms^{-1} 3) 15.55kms^{-1} 4) 7.78kms^{-1}

KEY:3

HINT

$$v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e \propto \sqrt{\frac{M}{R}}$$

76. Two satellites of masses 400 kg, 500 kg are revolving around earth in different circular orbits of radii r_1, r_2 such that their kinetic energies are equal. The ratio of r_1 to r_2 is

1) 4 : 5 2) 16 : 25 3) 5 : 4 4) 25 : 16

KEY:1

HINT

$$KE = \frac{GMm}{2r} \Rightarrow KE \propto \frac{m}{r} \Rightarrow m \propto r$$

77. The kinetic energy needed to project a body of mass m from earth's surface (radius R) to infinity is

1) $\frac{mgR}{2}$ 2) $2mgR$ 3) mgR 4) $\frac{mgR}{4}$

KEY:3

HINT

$$KE = \frac{1}{2} m V_e^2$$

78. The work done to increase the radius of orbit of a satellite of mass 'm' revolving around a planet of mass M from orbit of radius R in to another orbit of radius 3R is

1) $\frac{2GMm}{3R}$ 2) $\frac{GMm}{3R}$ 3) $\frac{GMm}{6R}$ 4) $\frac{GMm}{24R}$

KEY:2

HINT

$$\text{Workdone} = \text{change in TE}$$

$$\text{Workdone}=\frac{GMm}{2R} - \frac{GMm}{6R}$$

79. A stone is dropped from a height equal to nR, where R is the radius of the earth, from the surface of the earth. The velocity of the stone on reaching the surface of the earth is

1) $\sqrt{\frac{2g(n+1)R}{n}}$ 2) $\sqrt{\frac{2gR}{n+1}}$ 3) $\sqrt{\frac{2gnR}{n+1}}$ 4) $\sqrt{2gnR}$

KEY:3

HINT

$$\frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$

80. Three particles of equal mass 'm' are situated at the vertices of an equilateral triangle of side 'L. The work done in increasing the side of the triangle to 2L is

1) $\frac{2G^2m}{2L}$ 2) $\frac{Gm^2}{2L}$ 3) $\frac{3Gm^2}{2L}$ 4) $\frac{3Gm^2}{L}$

KEY:3

HINT

$$\text{Initial potential energy } U_i = \frac{-3Gm^2}{L}$$

$$\text{Final potential energy } U_f = -\frac{3Gm^2}{2L}$$

$$W = U_f - U_i = -\frac{3Gm^2}{2L} - \left(-\frac{3Gm^2}{L}\right) = \frac{3Gm^2}{2L}$$

81. The time of revolution of planet A round the sun is 8 times that of another planet B. The distance of planet A from the sun is how many times greater than that of the planet B from the sun

1) 2 2) 3 3) 4 4) 5

KEY:3

HINT:

$$\text{From Kepler's 3rd law, } T^2 \alpha r^3, \frac{T_1}{T_2} = \left[\frac{r_2}{r_1}\right]^{\frac{3}{2}}$$

82. A small body is at a distance 'r' from the centre of mercury, where 'r' is greater than the radius of Mercury. The energy required to shift the body from r to 2r measured from the centre is E. The energy required to shift it from 2r or 3r will be

1) E 2) $\frac{E}{2}$ 3) $\frac{E}{3}$ 4) $\frac{E}{4}$

KEY:3

HINT

$$E = U_2 - U_1$$

$$\Rightarrow -\frac{GMm}{2R} + \frac{GMm}{R} = \frac{GMm}{2R}$$

$$E' = -\frac{GMm}{3R} + \frac{GMm}{2R} = \frac{GMm}{6R}$$

83. By what percent the energy of the satellite has to be increased to shift it from an orbit of radius 'r' to $\frac{3r}{2}$

1) 66.7% **2) 33.3%** **3) 15%** **4) 20.3%**

KEY:2

HINT

$$E = \frac{-GMm}{2r} W = E_2 - E_1 = \frac{-GMm}{2r_1}, E_2 = \frac{-GMm}{2r_2}$$

84. The gravitational force between two bodies is 6.67×10^{-7}N when the distance between their centres is 10m. If the mass of first body is 800 kg, then the mass of second body is

1) 1000 kg 2) 1250 kg 3) 1500 kg 4) 2000 kg

KEY:2

HINT

$$F_g = \frac{Gm_1m_2}{R^2} \Rightarrow m_2 = \frac{F_g \times R^2}{Gm_1}$$

85. At what height from the surface of earth, the total energy of satellite is equal to its potential energy at a height 2R from the surface of earth (R=radius of earth)

1) 2R 2) R/2 3) R/4 4) 4R

KEY:2

HINT

$$\frac{-GMm}{2r} = \left(\frac{-GMm}{3R}\right) r = R + h$$

86. A geo-stationary satellite is orbiting the earth at a height 6R above the surface of the earth, where R is the radius of earth. The time period of another satellite revolving around earth at a height 2.5R from earth's surface is

1) $12\sqrt{2}hr$ 2) 12 hr 3) $6\sqrt{2}hr$ 4) 6 hr

KEY:3

HINT

$$\frac{T_1}{T_2} = \sqrt{\frac{R_1^3}{R_2^3}} = \sqrt{\frac{(7R)^3}{\left(\frac{7R}{2}\right)^3}}$$

87. A planet moves around the sun. At a given point P, it is closest from the sun at a distance d_1 and has a speed V_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

1) $\frac{d_1^2V_1}{d_2}$ 2) $\frac{d_2V_1}{d_1}$ 3) $\frac{d_1V_1}{d_2}$ 4) $\frac{d_2^2V_1}{d_1^2}$

KEY:3

HINT

From conservation of angular momentum vr=Constant.

88. A small body is initially at a distance 'r' from the centre of earth. 'r' is greater than the radius of the earth. If it takes W joule of work to move the body from this position to another position at a distance 2r measured from the centre of earth, how many joules would be required to move it from this position to a new position at a distance of 3r from the centre of the earth.

1) W/5 2) W/3 3) W/2 4) W/6

KEY:2

HINT

$$W = GPE_e - GPE_1 \text{ Here, } GPE = \frac{Gm_1m_2}{r_{12}}$$

89. The escape velocity of a planet having mass 6 times and radius 2 times as that of earth is

1) $\sqrt{3}v_e$ 2) $3v_e$ 3) $\sqrt{2}v$ 4) $2v_e$

KEY:1

HINT

$$\frac{v_P}{v_e} = \sqrt{\frac{M_P}{M_e}\frac{R_e}{R_P}}$$

90. The K.E. of a satellite in an orbit close to the surface of the earth is E. Its max K.E. so as to escape from the gravitational field of the earth is.

1) 2E 2) 4E 3) $2\sqrt{2}$ E 4) $\sqrt{2}$ E

KEY:1

HINT

$$\frac{K_e}{K_0} = \frac{2gR}{gR} \Rightarrow K_e = 2K_0$$

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PREVIOUS JEE MAINS QUESTIONS & SOLUTIONS

Keplers Laws of Planetary Motion:

1. Ifthe angular momentum ofa planet ofmass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: [9 Jan. 2019 I]

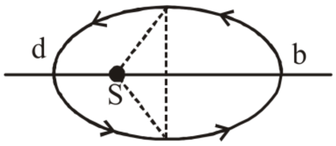
(a) $\frac{L}{m}$ (b) $\frac{4L}{m}$ (c) $\frac{L}{2m}$ ‘ (d) $\frac{2L}{m}$

SOLUTION : (c) Areal velocity; $\frac{dA}{dt}$

$$dA = \frac{1}{2}r^2 d\theta$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$$

2. Figure shows elliptical path abcd of a planet around thesun S such that the area oftriangle csa is $\frac{1}{4}$ the area ofthe ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then: [Online April 9, 2016]



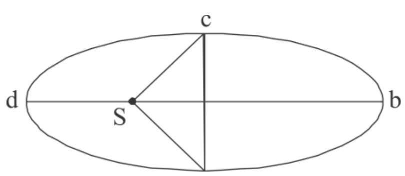
(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$ (c) $t_1 = 3t_2$ (d) $t_1 = t_2$

SOLUTION : . (c)

Let area ofellipse abcd = x

Area of SabcS $= \frac{x}{2} + \frac{x}{4}$ (i. e., arc of abca +SacS)

(Area ofhalfellipse + Area oftriangle)= $\frac{3x}{4}$



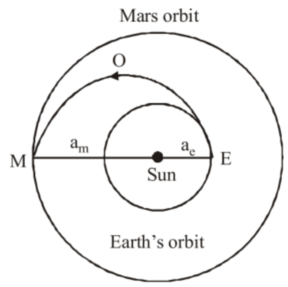
a

$$\text{Area of SadcS} = x - \frac{3x}{4} = \frac{x}{4}$$

$$\frac{\text{AreaofSabcS}}{\text{AreaofSadcS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$$

$$\frac{t_1}{t_2} = 3 \text{ or, } t_1 = 3t_2$$

3. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. Ifthe semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$m, that ofMars orbit $a_m = 2.28 \times 10^{11}$m, taken Kepler's laws give the estimate oftime for Mangalyan to reach Mars from Earth to be close to: [Online April 9, 2014]



(a) 500 days (b) 320 days (c) 260 days (d) 220 days

SOLUTION :' (b)

4. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become [2003]

(a) 10 hours (b) 80 hours (c) 40 hours (d) 20 hours

SOLUTION : '(c)

According to Kepler's law ofperiods $T^{2\propto}R^3$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 = 5 \times 2^3 = 40 \text{ hours}$$

Newton's Universal Law of Gravitation :

5. A straight rod oflength L extends from $x = a$ to$x = L + a$. The gravitational force it exerts on point mass m' at $x = 0$, ifthe mass per unit length ofthe rod is $A + Bx^2$, is givenby:
[12 Jan. 2019 I]

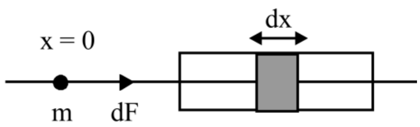
(a) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$ **(b) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$**

(c) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$ **(d) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$**

SOLUTION : ‘. (d)

$$\text{Given} = (A + Bx^2)\ ,$$

Taking small element dm oflength dx at a distance x from$x = 0$



$$\Rightarrow \Gamma = \int_a^{a+L} \frac{Gm}{x^2}(A + Bx^2)dx$$

$$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$

$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

6. Take the mean distance ofthe moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22}kg and 2×10^{30} kg respectively. The radius ofthe earth is 6400 km. Let $\Delta\Gamma_1$ be the difference in the forces exerted bythe moon at the nearest and farthest points on the earth and $\Delta\Gamma_2$ be the difference in the force exerted by the sun at the nearest and farthest points onthe earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is:[Online Apri115, 2018]

(a) 2 **(b) 6** **(c) 10^{-2}** **(d) 0.6**

SOLUTION : (a)

As we know, Gravitational force ofattraction, $\Gamma = \frac{GMm}{R^2}$

$$\Gamma_1 = \frac{GM_e m}{r_1^2} \text{ and } \Gamma_2 = \frac{GM_e M_s}{r_2^2}$$

$$\Delta\Gamma_1 = \frac{2GM_e m}{r_1^3}\Delta r_1 \text{ and } \Delta\Gamma_2 = \frac{GM_e M_s}{r_2^3}\Delta r_2$$

$$\frac{\Delta\Gamma_1}{\Delta\Gamma_2} = \frac{m\Delta r_1 r_2^3}{r_1^3 M_s \Delta r_2}$$

Using $\Delta r_1 = \Delta r_2 = 2R_{earth}$; $m = 8 \times 10^{22}$kg;

$M_s = 2 \times 10^{30}$kg

$r_1 = 0.4 \times 10^6$ km and $r2 = 150 \times 10^6$ km

$$\frac{\Delta\Gamma_1}{\Delta\Gamma_2} = 2$$

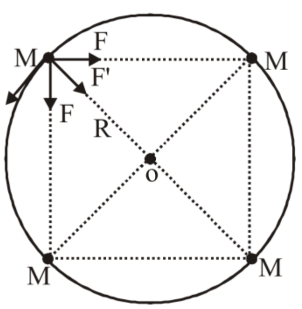
7. Four particles, each ofmass M and equidistant from each other, move along a circle ofradius R under the action of their mutual gravitational attraction. The speed of each particle is: [2014]

(a) $\sqrt{\frac{GM}{R}}$ **(b)** $\sqrt{2\sqrt{2}\frac{GM}{R}}$ **(c)** $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ **(d)** $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

SOLUTION : (d)

$$2F\cos 45° + F' = \frac{Mv^2}{R} \text{ (From figure)}$$

$$\text{Where } F = \frac{GM^2}{(\sqrt{2}R)^2} \text{ and } F' = \frac{GM^2}{4R^2}$$



$$\Rightarrow \frac{2\times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

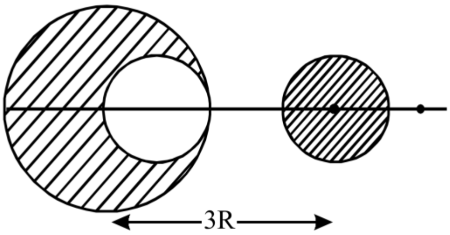
so, $dm = \lambda dx$: $dm = (A + Bx^2)dx$

$$d\Gamma = \frac{\mathrm{Gmdm}}{x^2}$$

$$\Rightarrow \frac{GM^2}{R}\left[\frac{1}{4}+\frac{1}{\sqrt{2}}\right] = Mv^2$$

$$v = \sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

8. From a sphere ofmass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre ofthe original sphere and the removed sphere is 3R, the gravitational force between thetwo sphere is: [Online April 11, 2014]



(a) $\frac{41GM^2}{3600R^2}$ **(b)** $\frac{41GM^2}{450R^2}$ **(c)** $\frac{59GM^2}{450R^2}$ **(d)** $\frac{GM^2}{225R^2}$

SOLUTION : (a)

Volume ofremoved sphere $V_{\text{remo}} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi R^3\left(\frac{1}{8}\right)$

Volume ofthe sphere(remaining) $V_{\text{remain}} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3\left(\frac{1}{8}\right) = \frac{4}{3}\pi R^3\left(\frac{7}{8}\right)$

Therefore mass of sphere carved and remaining sphere are at respectively $\frac{1}{8}$M and $\frac{7}{8}$M.

Therefore, gravitational force between these two sphere,

$$F = \frac{GMm}{r^2} = \frac{G\frac{7M}{8}\times\frac{1}{8}M}{(3R)^2} = \frac{7GM^2}{64\times 9R^2} = \frac{41GM^2}{3600R^2}$$

9. Two particles ofequal mass '*m*' go arounda circle ofradius *R* under the action oftheir mutual gravitational attraction. The speed ofeach particle with respect to their centre of mass is [2011]

(a) $\sqrt{\frac{Gm}{4R}}$ **(b) $\sqrt{\frac{Gm}{3R}}$** **(c) $\sqrt{\frac{Gm}{2R}}$** **(d) $\sqrt{\frac{Gm}{R}}$**

SOLUTION : (a)

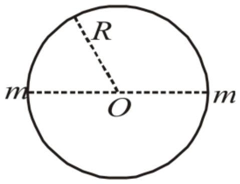
As two masses revolve about the common centre of mass O.

Mutual gravitational attraction = centripetal force

$$\frac{Gm^2}{(2R)^2} = m\omega^2 R$$

$$\Rightarrow \frac{Gm}{4R^3} = \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{Gm}{4R^3}}$$



Ifthe velocity ofthe two particles with respect to the centre of gravity is v then

$v = \text{w } R$

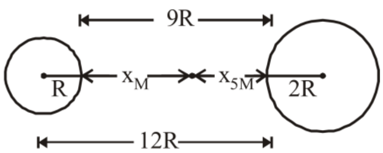
$$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$$

10. Two spherical bodies ofmass *M* and 5*M*& radii *R* & 2*R* respectively are released in free space with initial separation between their centres equal to 12 *R*. If they attract each other due to gravitational force only, then the distance covered bythe smaller body just before collision is [2003]

(a) $2.5R$ **(b) $4.5R$** **(c) $7.5R$** **(d) $1.5R$**

SOLUTION : (c)

We know that Force =mass ×acceleration.



The gravitational force acting on both the masses is the same. $F_1 = F_2$

$$ma_1 = ma_2$$

$$\Rightarrow \frac{9M}{95M} = \frac{5M}{M} = 5 \quad \Rightarrow \frac{9M}{95M} = \frac{1}{5}$$

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass $5M$ $u = 0,$ $S = ut + \frac{1}{2}at^2$; $x_{5M} = \frac{1}{2}a_{5M}t^2$ (ii)

For mass M $u = 0, s = x_M, t = t, a = a_M$

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow x_M = \frac{1}{2}a_M t^2 \text{ ... (iii)}$$

Dividing (ii) by(iii) $\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5}$ [From (i)]

$$5x_{5M} = x_M$$

From the figure it is clear that

$$x_{5M} + x_M = 9R$$

Where O is the point where the two spheres collide.

From(iv) and(v)

$$\frac{x_M}{5} + x_M = 9R$$

$$6x_M = 45R$$

$$x_M = \frac{45}{6}R = 7.5R$$

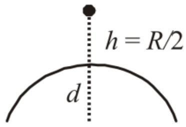
Asseleration due to Gravity :

11. The value ofacceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 and a depth d below the surface ofthe earth. The ratio $\left(\frac{d}{R}\right)$ equals: [5 Sep. 2020 (I)]

(a) $\frac{4}{9}$ **(b) $\frac{5}{9}$** **(c) $\frac{1}{3}$** **(d) $\frac{7}{9}$**

SOLUTION : (b)

According to question, $g_h = g_d = g_1$



·:: (*R*-*d*):

$$g_h = \frac{GM}{\left(R+\frac{R}{2}\right)^2} \text{ and } g_d = \frac{GM(R-d)}{R^3}$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$$

$$\frac{d}{R} = \frac{5}{9}$$

12. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity ofthe earth about the axis passing through the pole is 00. An object is weighed at the equator and at a height h above the poles by using a spring balance. Ifthe weights are found to be same, then h is: ($h << R$, where R is the radius ofthe earth) [5 Sep. 2020 (II)]

(a) $\frac{R^2 c0^2}{2g}$ **(b)** $\frac{R^2 c0^2}{g}$ **(c)** $\frac{R^2 c0^2}{4g}$ **(d)** $\frac{R^2 c0^2}{8g}$

SOLUTION : . (b)

Value of g at equator, $g_A = g \cdot -R\text{o})^2$

Value of g at height h above the pole,

$$g_B = g \cdot \left(1 - \frac{2h}{R}\right)$$

As object is weighed equally at the equator and poles, it means g is same at these places.

$$g_A = g_B$$

$$\Rightarrow g - R\text{oo}^2 = g\left(1 - \frac{2h}{R}\right) \Rightarrow R_{0)}^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2 (0^2}{2g}$$

13. The height ↑ h' at which the weight of a body will be the same as that at the same depth ↑ h'fi$_i$om the surface ofthe earth is (Radius ofthe earth is R and effect ofthe rotation ofthe earth is neglected) : [2 Sep. 2020(II)]

(a) $\frac{\sqrt{5}}{2}R-R$ (b) $\frac{R}{2}$ (c) $\frac{\sqrt{5}R-R}{2}$ (d) $\frac{\sqrt{3}R-R}{2}$

SOLUTION : (c)

The acceleration due to gravity at a height h is given by $g=\frac{GM}{(R+h)^2}$

Here, $G=$ gravitation constant

$M=$ mass of earth

The acceleration due to gravity at depth h is

$$g^{\uparrow}=\frac{GM}{R^2}\left(1-\frac{h}{R}\right)$$

Given, $g=g'$

$$\frac{GM}{(R+h)^2}=\frac{GM}{R^2}\left(1-\frac{h}{R}\right)$$

$$R^3=(R+h)^2(R-h)=\left(R^2+h^2+2hR\right)(R-h)$$

$$\Rightarrow R^3=R^3+h^2R+2hR^2-R^2h-h^3-2h^2R$$

$$\Rightarrow h^3+h^2(2R-R)-R^2h=0$$

$$\Rightarrow h^3+h^2R-R^2h=0$$

$$\Rightarrow h^2+hR-R^2=0$$

$$\Rightarrow h=\frac{-R\pm\sqrt{R^2+4(1)R^2}}{2}=\frac{-R+\sqrt{5}R}{2}=\frac{\left(\sqrt{5}-1\right)}{2}R$$

14. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g=10$ ms -2 at the north pole and the radius ofthe earth $=6400$ km): [7 Jan. 2020 II]

(a) 195.66N (b) 194.32N (c) 194.66N (d) 195.32N

SOLUTION : . (d)

$$\text{Weight at pole, } w = mg = 196N$$

$$\Rightarrow m = 19.6\text{kg}$$

$$\text{Weight at equator, } w' = mg' = m\left(g^{-0)^2}R\right)$$

$$= 19.6\left[10 - \left(\frac{2\pi}{24\times3600}\right)^2 \times 6400 \times 10^3\right]\text{N} \quad \left(\because (i) = \frac{2\pi}{T}\right)$$

$$= 19.6[10 - 0.034] = 195.33N$$

15. The ratio ofthe weights ofa body on the Earth's surface to that on the surface of a planet is 9:4. The mass of theplanet is $\frac{1}{9}$ th ofthat ofthe Earth. If' R' is the radius ofthe Earth, what is the radius ofthe planet? (Take the planets to have the same mass density). [12 April 2019 II]

(a) $\frac{R}{3}$ (b) $\frac{R}{4}$ (c) $\frac{R}{9}$ (d) $\frac{R}{2}$

SOLUTION : (d)

$$\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4} \quad \text{or } \frac{g_e}{g_p} = \frac{9}{4} \quad \text{or } \frac{GM/R^2}{G(M/9)/R_p^2} = \frac{9}{4}$$

$$R_p = \frac{R}{2}$$

16. The value ofacceleration due to gravity at Earth's surface is .8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9ms^{-2}, is close to : (Radius ofearth $= 6.4 \times 10^6$m) [10 April 2019 I]

(a) 2.6×10^6m (b) 6.4×10^6m (c) 9.0×10^6m (d) 1.6×10^6m

SOLUTION : . (a)

Given Acceleration due to gravity at a height h from earth's surface is $g_h = g\left(1 + \frac{h}{Re}\right)^{-2}$

$$4.9 = 9.8\left(1 + \frac{h}{Re}\right)^{-2} \qquad [\text{as } h <<< R_e] h = R_e\left(\sqrt{2} - 1\right)$$

$$h = 6400 \times 0.414\text{km} = 2.6 \times 10^6\text{m}$$

17. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence.

[Online Apri116, 2018]

(a) There will be no change in weight anywhere on the earth

(b) Weight ofthe object, everywhere on the earth, wild decrease

(c) Weight ofthe object, everywhere on the earth, will increase

(d) Except at poles, weight ofthe object on the earth will decrease

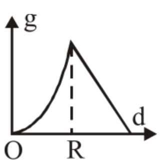
SOLUTION : (d)

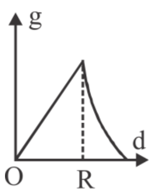
With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - w^2 R\cos^2\varphi$

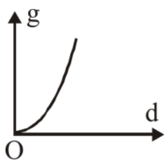
Where φ is latitude, there will be no change in gravity at poles as $\varphi = 90°$

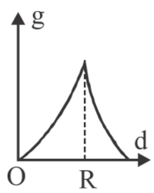
At all other points as w increases g' will decreases hence, weight, $W = mg$ decreases.

18. The variation ofacceleration due to gravity gwith distance d from centre of the earth is best represented by (R =Earth's radius): **[2017, Online May 7, 2012]**

(a) 

(b) 

(c) 

(d) 

SOLUTION : (b)

Variation of acceleration due to gravity, g with distance d' from centre ofthe earth

$$\text{If } d < R, g = \frac{Gm}{R^2}.d$$

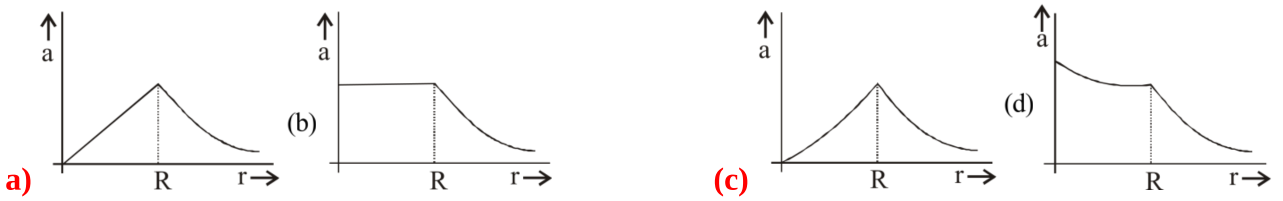
$$i.e., g \propto d \text{(straight line)}$$

$$\text{If } d = R, g_s = \frac{Gm}{R^2}$$

$$\text{If } d > R, g = \frac{Gm}{d^2}$$

$$i.e., g \propto \frac{1}{d^2}$$

19. The mass density of a spherical body is given by $p(r) = \frac{k}{r}$ for $r \leq R$ and $p(r) = 0$ for $r > R$, where r is the distance from the centre.The correct graph that describes qualitatively the acceleration, a, ofa test particle as a function ofr is: [Online April 9, 2017]



SOLUTION : . (b)

Given that, mass density $\left(\frac{\text{mass}}{\text{volume}}\right)$ of a spherical body $p(r) = \frac{k}{r}$

$$\frac{M}{V} = \frac{k}{r} \text{ for inside } r \leq R$$

$$M = \frac{kv}{r} - (i)$$

Inside the surface of sphere Intensity $I = \frac{GMr}{R^3} I = \frac{\Gamma}{m}$

$$g_{inside} = \frac{GMr}{R^3} \text{ or } I = \frac{mg}{m} = g = \frac{G}{R^3} \cdot \frac{kv}{r} \cdot r = \text{constant From eq. (i),}$$

Similarly, $g_{out} = \frac{GM}{r^2}$

Hence, option (2) is correct graph.

20. Ifthe Earth has no rotational motion, the weight ofa person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight $\frac{3}{4}$ W. Radius of the Earth is 6400 km and $g = 10 m/s^2$. [Online April 8, 2017]

(a) $1.1 \times 10^{-3} rad/s$ (b) $0.83 \times 10^{-3} rad/s$ (c) $0.63 \times 10^{-3} rad/s$ (d) $0.28 \times 10^{-3} rad/s$

SOLUTION : . (c)

We know, $g' = g - w^2 R\cos^2\theta$

$$\frac{3g}{4} = g - c0^2 R$$

$$\text{Given, } g' = \frac{3}{4}g$$

$$w^2R = \frac{g}{4}$$

$$w = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}} = \frac{1}{2 \times 8 \times 100} = 0.6 \times 10^{-3}\text{rad/s}$$

21. The change in the value ofacceleration of earth towards sun, when the moon comes fi;om the position of solar eclipse to the position on the other side of earth in line with sun is:(mass ofthe moon $= 7.36 \times 10^{22}$kg, radius ofthe moon's orbit $= 3.8 \times 10^8$m). [Online April 22, 2013]

(a) $6.73 \times 10^{-5} m/s^2$ **(b) $6.73 \times 10^{-3} m/s^2$**

(c) $6.73 \times 10^{-2} m/s^2$ **(d) $6.73 \times 10^{-4} m/s^2$**

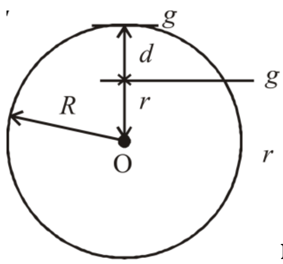
SOLUTION : **(a)**

22. Assuming the earth to be a sphere ofuniform density, the acceleration due to gravity inside the earth at a distance of $r$fi;om the centre is proportional to [Online May 12, 2012]

(a) r **(b) r^{-1}** **(c) r^2** **(d) r^{-2}**

SOLUTION : (a)

Acceleration due to gravity at depth d from the surface ofthe earth or at a distance r from the centre ' O' of the earth $g' = \frac{4}{3}\pi p G v$ Hence $g^{\uparrow} \propto r$



$r = (R - d)$

23. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is [2009]

(a) $\frac{R}{\sqrt{2}}$ **(b) $Rl2$** **(c) $\sqrt{2}R$** **(d) 2R**

SOLUTION : . (d)

$$\text{On earth's surface } g = \frac{GM}{R^2}$$

$$\text{At height above earth's surface } \quad g_h = \frac{GM}{(R+h)^2}$$

$$\frac{g_n}{g} = \frac{R^2}{(R+h)^2} \quad \Rightarrow \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2 \quad \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$$h = 2R$$

24. The change in the value of '*g*' at a height '*h*' above the surface ofthe earth is the same as at a depth *d*' below the surface ofearth. When both *d'* and *h*' are much smaller than the radius ofearth, then which one ofthe following is correct? [2005]

(a) $d = \frac{3h}{2}$ (b) $d = \frac{h}{2}$ (c) $d = h$ (d) $d = 2h$

SOLUTION : . (d)

$$\text{Value of } g \text{ with altitude is, } \; g_h = g\left[1 - \frac{2h}{R}\right];$$

$$\text{Value of } g \text{ at depth d below earth's surface, } \quad g_d = g\left[1 - \frac{d}{R}\right]$$

Equating g_h and g_d, we get $d = 2h$

25. Average density ofthe earth [2005]

(a) is a complex function ofg

(b) does not depend on *g*

(c) is inverselyproportional to *g*

(d) is directly proportional to *g*

SOLUTION : . (d)

$$\text{Value ofg on earth's surface, } \quad g = \frac{GM}{R^2} = \frac{G\text{p}\times V}{R^2} \quad \Rightarrow g = \frac{G\times\text{p}\times\frac{4}{3}\pi R^3}{R^2}$$

$$g = \frac{4}{3}\text{p}\pi G.R$$

where p →average density

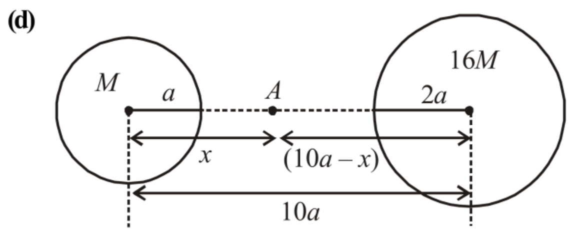
$$p = \left(\frac{3g}{4\pi GR}\right) \quad \Rightarrow \text{p is directly proportional to } g.$$

Gravitational Field and Potential Energy :

26. Two planets have masses M and 16 M and their radii are a and $2a$, respectively. The separation between the centres ofthe planets is $10a$. A body ofmass m is fired from the surface ofthe larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach the surface of smaller planet, the minimum firing speed needed is : [6 Sep. 2020 (II)]

(a) $2\sqrt{\frac{GM}{a}}$ **(b)** $4\sqrt{\frac{GM}{a}}$ **(c)** $\sqrt{\frac{GM^2}{ma}}$ **(d)** $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

SOLUTION : (d)



Let A be the point where gravitation field ofboth planets cancel each other i.e. zero.

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{4}{(10a-x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a \text{ ... (i)}$$

Using conservation of energy, we have $-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$

$$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$$

$$\Rightarrow KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v = \sqrt{\frac{90GM}{8a}} \Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

27. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infmity, is: [4 Sep. 2020 (I)]

(a) $\frac{A}{(x+a)22^1/_2}$ (b) $\frac{A}{(x+a)22^3/_2}$ (c) $A(x+a)22^1/_2$ (d) $A(x+a)22^3/_2$

SOLUTION : . (a)

Given : Gravitational field, $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}, V_\infty = 0$

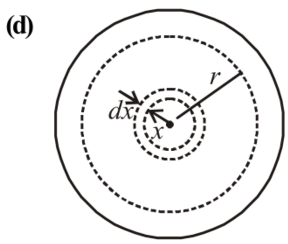
$$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot \vec{d}_x \Rightarrow V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$$

$$V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

28. The mass density of a planet of radius R varies with thedistance $r$fi$_{\mathrm{i}}$om its centre as $\mathrm{p}(r) = \mathrm{p}_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at: [3 Sep. 2020 (II)]

(a) $r = \sqrt{\frac{3}{4}}R$ (b) $r = R$ (c) $r = \frac{1}{\sqrt{3}}R$ (d) $r = \sqrt{\frac{5}{9}}R$

SOLUTION : (d)



Mass ofsmall element ofplanet ofradius x and thickness dx.

$$dm = \mathrm{p} \times 4\pi x^2 dx = \mathrm{p}_0\left(1-\frac{\mathrm{x}^2}{\mathrm{R}^2}\right) \times 4\pi x^2 dx$$

Mass of the planet $M = 4\pi \mathrm{p}_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx$

$$\Rightarrow M = 4\pi \mathrm{p}_0 \left| \frac{r^3}{3} - \frac{r^5}{5R^2} \right|$$

Gravitational field, $E = \frac{GM}{r^2} = \frac{G}{r^2} \times 4\pi p_{0(\ )}(\ \ )$

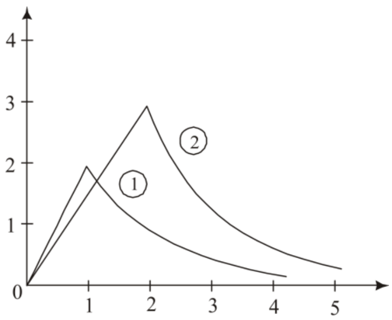
$$\Rightarrow E = 4\pi G p_0 \left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{dE}{dr} = 4\pi G p_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0$$

$$\Rightarrow r = \frac{\sqrt{5}}{3} R$$

29. Consider two solid spheres ofradii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2, respectively. The gra vitational field due to sphere O[i] and O2 are shown. The value of $\frac{m1}{m2}$ is: [8 Jan. 2020 I]



(a) $\frac{2}{3}$ **(b) $\frac{1}{6}$** **(c) $\frac{1}{2}$** **(d) $\frac{1}{3}$**

SOLUTION : (b)

Gravitation field at the surface $E = \frac{Gm}{r^2}$

$$E_1 = \frac{Gm_1}{r_1^2} \text{ and } E_2 = \frac{Gm_2}{r_2^2}$$

From the diagram given in question,

$$\frac{E_1}{E_2} = \frac{2}{3} (r_1 = 1\text{m}, R_2 = 2m \text{ given})$$

$$\Rightarrow \frac{\text{m}_1}{\text{m}_2} = \frac{1}{6}$$

30. **An asteroid is moving directly towards the centre of the earth. When at a distance of10 R(R is the radius of the earth) from the earths centre, it has a speed of12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity fi$_{iom}$ the earth is 11.2km/s)? Give your answer to the nearest integer in kilometer/s** **[NA8 Jan. 2020II]**

SOLUTION : . (16.00)

Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$$-\frac{GM_E m}{10R}+\frac{1}{2}mV_0^2=-\frac{GM_E m}{R}+\frac{1}{2}mV^2$$

[$\because$ Gravitational potential energy $=-\frac{GMm}{r}$]

$$\Rightarrow \frac{GM_E}{R}\left(1-\frac{1}{10}\right)+\frac{V_0^2}{2}=\frac{V^2}{2}\Rightarrow V^2=V_0^2+\frac{9}{5}gR$$

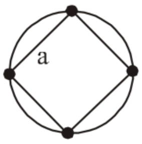
$$\Rightarrow V=\sqrt{V_0^2+\frac{9}{5}gR}\approx 16\ km\ ls \quad [V_0=12\text{km/s given}]$$

31. A solid sphere ofmass M' and radius a' is surrounded by a uniform concentric spherical shell ofthickness 2a and mass 2M. The gravitational field at distance 3a' from thecentre will be: **[9 April 2019 I]**

(a) $\frac{2GM}{9a^2}$ **(b)** $\frac{GM}{9a^2}$ **(c)** $\frac{GM}{3a^2}$ **(d)** $\frac{2GM}{3a^2}$

SOLUTION : (c) $E_g=\frac{GM}{(3a)^2}+\frac{G(2M)}{(3a)^2}=\frac{GM}{3a^2}$

32. **Four identical particles ofmass M are located at the corners of a square of side a'. What should be their speed ifeach of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square?** **[8 April 2019 I]**



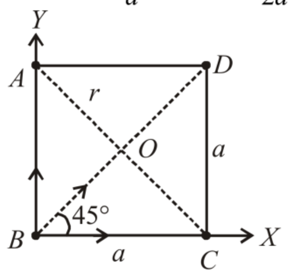
(a) $1.35\sqrt{\frac{GM}{a}}$ **(b)** $1.16\sqrt{\frac{GM}{a}}$ **(c)** $1.21\sqrt{\frac{GM}{a}}$ **(d)** $1.41\sqrt{\frac{GM}{a}}$

SOLUTION : (b)

$$AC = a\sqrt{2}\, r = \frac{AC}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$$

Resultant force on the body$B = \frac{GM^2}{a^2}\hat{\imath} + \frac{GM^2}{a^2}j + \frac{GM^2}{(a\sqrt{2})^2}(\cos 45^o\hat{\imath} + \sin 45^o\hat{\jmath})$

$$\Rightarrow |F| = \frac{GM^2}{a^2}(\sqrt{2}) + \frac{GM^2}{2a^2}$$



$\frac{Mv^2}{r}$ = Resultant force towards centre

$$\frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = \sqrt{\frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

33. Atest particle is moving in circular orbit in the gravitational field produced by a mass density $r(r) = \frac{K}{r^2}$ Identify the correct relation between the radius R ofthe particle's orbit and its period T:

[8 April 2019 II]

(a) T/R is a constant **(b) T^2/R^3 is a constant**

(c) T/R^2 is a constant **(d) TR is a constant**

SOLUTION : . (a)

$$F = \frac{GMm}{r} = \int_0^R a\frac{p(dV)m}{r^2} = mG\int_0^R \frac{k4\pi r^2 dr}{r^2 r^2} = -4\pi kGm\left(\frac{1}{r}\right)_0^R = -\frac{4\pi kGm}{R}$$

Using Newton's second law, we have

$$\frac{mv_0^2}{R} = \frac{4\pi kGm}{R} \quad \text{or} \quad v_0 = \text{C (const.)}$$

$$\text{Time period, } T = \frac{2\pi R}{v_0} = \frac{2\pi R}{C} \quad \text{or} = \frac{T}{R} = \text{constant.}$$

34. Abody ofmass m is moving in a circular orbit ofradius *R* about a planet of mass *M*. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is: [Online Apri115, 2018]

(a) $-\frac{GMm}{2R}$ (b)$+\frac{GMm}{6R}$ (c) $-\frac{GMm}{6R}$ (d) $\frac{GMm}{2R}$

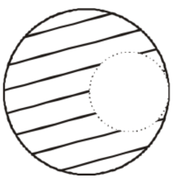
SOLUTION : . (c)

Initial gravitational potential energy, $E_j = -\frac{GMm}{2R}$ Final gravitational potential energy,

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$$

Difference between initial and final energy, $E_f - E_j = \frac{GMm}{R}\left(-\frac{2}{3} + \frac{1}{2}\right) = -\frac{GMm}{6R}$

35. From a solid sphere of mass M and radius R, a spherical portion ofradius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre ofthe cavity thus formed is: (*G = gravitational constant*) [2015]

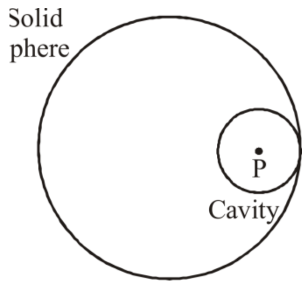


(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$ (c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

SOLUTION : . (d)

Due to complete solid sphere, potential at point $PV_{\text{sphere}} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$



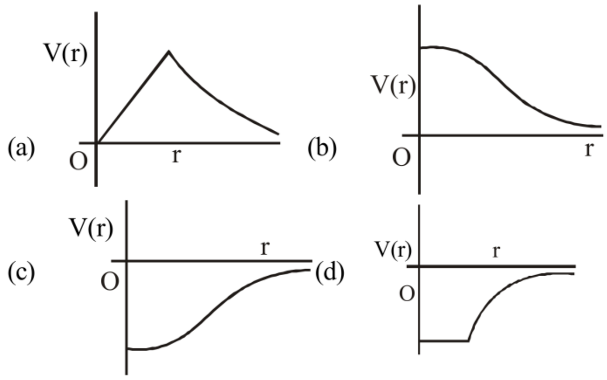
Due to cavity part potential at point P $\quad V_{cavity} = -\frac{3GM}{8R}$

So potential at the centre of cavity

$$= V_{sphere} - V_{cavity} = -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

36. Which of the following most closely depicts the correct variation ofthe gravitational potential V(r) due to a large planet of radius R and uniform mass density? (figures are not drawn to scale)

[Online April 11, 2015]



SOLUTION : . (c)

$$\text{As, } V = -\frac{GM}{2R^3}(3R^2 - r^2)$$

Graph (c) most closely depicts the correct variation of$v(r)$.

37. The gravitational field in a region is given by g = 5N/ kgî+12N/ kgî. The change in the gravitational potential energy of a particle ofmass 1 kg when it is taken from the origin to a point (7m, −3m) is: [Online Apri119, 2014]

(a) 71 J **(b) $13\sqrt{58}$J** **(c) − 71J** **(d) 1 J**

SOLUTION : . (d)

Gravitational field, $I=(5\hat{i}+12\hat{j})$ N/kg

$$I=-\frac{dv}{dr}$$

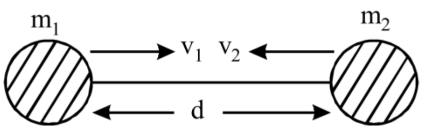
$$v=-\left[\int_0^x I_x\,dx+\int_0^y I_y\,dy\right]\;=-\left[I_x.x+I_y.y\right]$$

$$=-[5(7-0)+12(-3-0)]\;=-[35+(-36)]=1J/kg$$

i. e., change in gravitational potential 1 J/kg.

Hence change in gravitational potential energy 1 J

38. Two hypothetical planets ofmasses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is d'? [Online Apri112, 2014]



(Speed of m_1 is v_1 and that of m_2 is v_2)

(a) $v_1=v_2$

(b) $v_1=m_2\sqrt{\frac{2G}{d(m_1+m_2)}}\,v_2=m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

(c) $v_1=m_1\sqrt{\frac{2G}{d(m_1+m_2)}}\,v_2=m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(d) $v_1=m_2\sqrt{\frac{2G}{m_1}}\,v_2=m_2\sqrt{\frac{2G}{m_2}}$

SOLUTION : (b)

We choose reference point, infinity, where total energy ofthe system is zero.

So, initial energy ofthe system $=0$

$$\text{Final energy}=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2-\frac{Gm_1m_2}{d}$$

From conservation of energy,

Initial energy = Final energy

$$0=\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2-\frac{Gm_1m_2}{d}$$

$$\text{or } \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{d} \quad (1)$$

By conservation oflinear momentum

$$m_1v_1 + m_2v_2 = 0 \text{ or } \frac{v_1}{v_2} = -\frac{m_2}{m_1} \Rightarrow v_2 = -\frac{m_1}{m_2}v_1$$

Putting value ofv_2 in equation (1), we get

$$m_1v_1^2 + m_2\left(-\frac{m_1v_1}{m_2}\right)^2 = \frac{2Gm_1m_2}{d}$$

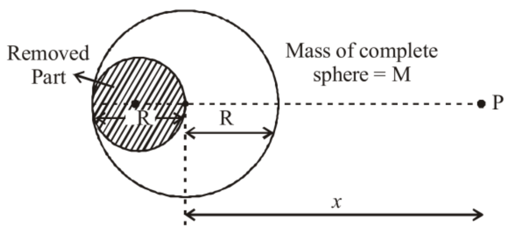
$$\frac{m_1m_2v_1^2 + m_1^2v_1^2}{m_2} = \frac{2Gm_1m_2}{d}$$

$$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}} = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

$$\text{Similarly } v_2 = -m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

39. The gravitational field, due to the ↑left over part' ofa uniform sphere (from which a part as shown, has been 'removed out'), at a very far offpoint, P, located as shown, would be(nearly) :

[Online April 9, 2013]



(a) $\frac{5}{6}\frac{GM}{x^2}$ **(b)** $\frac{8}{9}\frac{GM}{x^2}$ **(c)** $\frac{7}{8}\frac{GM}{x^2}$ **(d)** $\frac{6}{7}\frac{GM}{x^2}$

SOLUTION : (c)

Let mass ofsmaller sphere(which has to k removed) is m Radius $= \frac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \quad \Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere $M' = M - \frac{M}{8} = \frac{7}{8}M$

Therefore gravitational field due to the left over part ofthe sphere $= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$

40. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius ofearth) are 10 nys^2 and 6400 km respectively. The required energy for this work will be [2012]

(a) 6.4×10^{11}Joules (b) 6.4×10^8 Joules (c) 6.4×10^9 Joules (d) 6.4×10^{10} Joules

SOLUTION : (d)

The work done to launch the spaceship $W = -\int_R^\infty \overline{F} \cdot \overline{dr} = -\int_R^\infty \frac{GMm}{r^2} dr$

$$W = +\frac{GMm}{R} \quad \ldots\ldots\ldots \quad .(i)$$

The force ofattraction ofthe earth on the spaceship, when it was on the earth's surface

$$F = \frac{GMm}{R^2}$$

$$\Rightarrow mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2} \ldots (ii)$$

Substituting the value of g in (i) we get

$$W = \frac{gR^2m}{R} \Rightarrow W = mgR$$

$$\Rightarrow W = 1000 \times 10 \times 6400 \times 10^3 = 6.4 \times 10^{10} \text{ Joule}$$

41. Apoint particle is held on the axis ofa ring ofmass m and radius r at a distance $r$fi¡om its centre C. When released, it reaches C under the gravitational attraction ofthe ring. Its speed at C will be [Online May 26, 2012]

(a) $\sqrt{\frac{2Gm}{r}(\sqrt{2}-1)}$ (b) $\sqrt{\frac{Gm}{r}}$ (c) $\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$ (d) $\sqrt{\frac{2Gm}{r}}$

SOLUTION : . (c)

Let M be the mass of the particle

Now, $E_{\text{initia 1}} = E_{f_1 \text{na} 1}$

$$\text{i.e.} \frac{GMm}{\sqrt{2}r} + 0 = \frac{GMm}{r} + \frac{1}{2}MV^2 \quad \text{or} \quad , \quad \frac{1}{2}MV^2 = \frac{GMm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \frac{1}{2}V^2 = \frac{Gm}{r}\left[1 - \frac{1}{\sqrt{2}}\right] \quad \text{or} \quad , \quad V = \sqrt{\frac{2Gm}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

42. Two bodies ofmasses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is: [2011]

(a) $-\frac{4Gm}{r}$ **(b) $-\frac{6Gm}{r}$** **(c) $-\frac{9Gm}{r}$** **(d) zero**

SOLUTION : . (c)

Let P be the point where gravitational field is zero.

$$\frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{2}{r-x} \Rightarrow r - x = 2x \Rightarrow x = \frac{r}{3}$$

Gravitational potential at P, $V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$

43. This question contains Statement-l and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. [2008]

Statement-l : For a mass M kept at the centre ofa cube of side a', the flux ofgravitational field passing through its sides 4 πGM. and

Statement-2: Ifthe direction ofa field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape ofthe surface.

(a) Statement-l is false, Statement-2 is true

(b) Statement-l is true, Statement-2 is true; Statement -2 is a correct explanation for Statement- 1

(c) Statement-l is true, Statement-2 is true; Statement2 is not a correct explanation for Statement-l

(d) Statement-l is true, Statement-2 is false

SOLUTION : (b)

Gravitational field, $E=-\frac{GM}{r^2}$

Flux, $\varphi=\int \overline{E_g}\cdot \vec{d}S=|E\cdot 4\pi r^2|=-4\pi GM$

where, M = mass enclosed in the closed surface

This relationship is valid when $|\vec{E}_g|\propto\frac{1}{r^2}$

44. A particle ofmass 10 g is kept on the surface ofa uniform sphere ofmass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere(you may take $G=6.67\times10^{-11}\mathrm{Nm^2/kg^2}$) [2005]

(a) 3.33×10^{-10}J (b) 13.34×10^{-10}J (c) 6.67×10^{-10}J (d) 6.67×10^{-9}J

SOLUTION : (c)

Initial P.E. $U_i=-\frac{GMm}{R}$

When the particle is far away from the sphere, the P.E. of the system is zero.

$$U_f=0$$

$$W=\Delta U=U_f-U_j=0-\left[\frac{-GMm}{R}\right]$$

$$W=\frac{6.67\times10^{-11}\times100}{0.1}\times\frac{10}{1000}=6.67\times10^{-10}\mathrm{J}$$

45. If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface ofthe earth to a height equal to the radius R' ofthe earth is [2004]

(a) $\frac{1}{4}mgR$ (b) $\frac{1}{2}mgR$ (c) $2\,mgR$ (d) mgR

SOLUTION : . (b)

On earth's surface potential energy, $U = \frac{GmM}{R}$

At a height R from the earth$^{\uparrow}$s surface, $P.E.$ of system $= -\frac{GmM}{2R}$

$$\Delta U = \frac{-GmM}{2R} + \frac{GmM}{R};$$

$$\Rightarrow \Delta U = \frac{GmM}{2R}$$

$$\text{Now } \frac{GM}{R^2} = g; \quad \therefore \frac{GM}{R} = gR$$

$$\Delta U = \frac{1}{2} mgR$$

46. Energyrequired to move abodyofmass $m$fi$_i$om an orbit of radius $2R$ to $3R$ is [2002]

(a) $GMml\ 12R^2$ (b) $GMml3R^2$ (c) $GMml8R$ (d) $GMml6R$.

SOLUTION : . (d)

Gravitational potential energy ofmass m in an orbit ofradius R $u = -\frac{GMm}{R}$

Energy required $=$ potential energy at 3R $-$ potential energy a2R $= \frac{-GMm}{3R} - \left(\frac{-GMm}{2R}\right)$

$$= \frac{-GMm}{3R} + \frac{GMm}{2R} = \frac{-2GMm + 3GMm}{6R} = \frac{GMm}{6R}$$

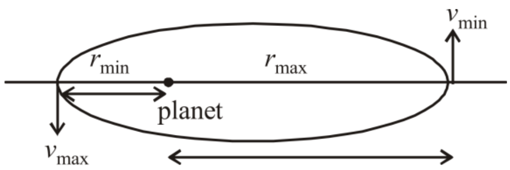
47. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity ofthe satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is:

[NA 6 Sep. 2020 (I)]

(a) 1: 6 (b) 1: 3 (c) 1: 2 (d) 3: 4

SOLUTION : . (a)

By angular momentum conservation $r_{\mathrm{m\ ln}}\, v_{\max} = mr_{\max}\, v_{\min}$



$$\text{Given, } v_{min} = \frac{v_{max}}{6}$$

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

48. Abody is moving inalow circular orbit about aplanet of mass M and radius R. The radius ofthe orbit can be taken to be R itself. Then the ratio ofthe speed of this body in the orbit to the escape velocity from the planet is: [4 Sep. 2020 (II)]

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 1 (d) $\sqrt{2}$

SOLUTION : (a)

Orbital speed ofthe body when it revolves very close to the surface of planet

$$V_0 = \sqrt{\frac{GM}{R}} \text{ (i)}$$

Here, G = gravitational constant

Escape speed from the surface ofplanet

$$V_e = \sqrt{\frac{2GM}{R}} \text{ (ii)}$$

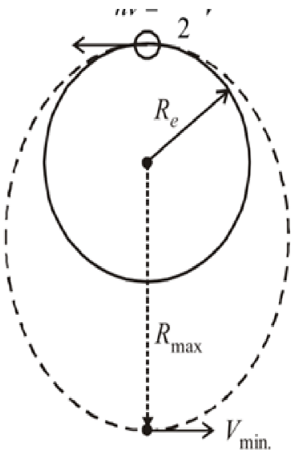
Dividing (i) by(ii), we have

$$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

49. Asatellite is moving inalow nearly circular orbit around the earth. Its radius is roughly equal to that ofthe earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction ofits motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance fi¡om the centre of the earth that the satellite reaches is R. Value of R is: [3 Sep. 2020 (I)]

(a) $4R_e$ (b) $2.5R_e$ (c) $3R_e$ (d) $2R_e$

SOLUTION : . (c)



Orbital velocity, $V_0 = \sqrt{\frac{GM}{R_e}}$

From energy conversation, $-\frac{GMm}{R_e} + \frac{1}{2}m\left(\left(\sqrt{\frac{3}{2}}V\right)\right)^2 = \frac{GMm}{R_{max}} + \frac{1}{2}mV_{min}^2$ (1)

From angular momentum conversation $\sqrt{\frac{3}{2}}VR_e = \bar{V}_{min}\, R_{max}$ (2)

Solving equation (1) and (2) we get, $R_{max} = 3R_e$

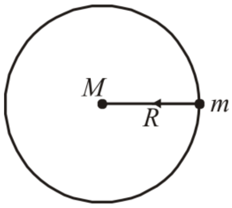
50. The mass density ofa spherical galaxy varies as $\frac{K}{r}$ over a large distance'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as: **[2 Sep. 2020 (I)]**

(a) $T^2 \propto R$ **(b) $T^2 \propto R^3$** **(c) $T^2 \propto \frac{1}{R^3}$** **(d) $T \propto R$**

SOLUTION : (a)

According to question, mass density of a spherical galaxy varies as $\frac{k}{r}$

Mass, $M = \int \rho\, dV \Rightarrow M = \int_0^{r--R_0} \frac{k}{r} 4\pi r^2 dr \Rightarrow M = 4\pi k \int_0^{R_0} r\, dr$



$$\text{or, } M = \frac{4\pi k R_0^2}{2} = 2\pi k R^2$$

$$F_G = \frac{GMm}{R_0^2} = mc0_0^2 R (= F_C) \quad \Rightarrow \frac{G\frac{4\pi k R^2}{2}}{R^2} = 0)^2 0R \Rightarrow 0)0 = \sqrt{\frac{2\pi KG}{R}} \left(\because \text{ti} J = \frac{2\pi}{T}\right)$$

$$T = \frac{2\pi}{0_0}) = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}} \Rightarrow T^2 = \frac{2\pi R}{KG}$$

2π, K and G are constants

$$T^2 \propto R.$$

51. Abody A ofmass m is moving in a circular orbit ofradiusR about a planet. Another body B ofmass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ or A. The collision is completely inelastic. Then, the combined body: [9 Jan. 2020 I]

(a) continues to move in a circular orbit

(b) Escapes fi¡om the Planet's Gravitational field

(c) Falls vertically downwards towards the planet

(d) starts moving in an elliptical orbit around the planet

SOLUTION : . (d)

From law ofconservation ofmomentum, $\vec{P}_i = \vec{P}_f$

$$m_1 u_1 + m_2 u_2 = M V_f$$

$$\Rightarrow v_f = \frac{\left(mv + \frac{mv}{4}\right)}{} \frac{3m}{2} = \frac{5v}{6}$$

Clearly, $v_f < v_j$ Path will be elliptical

52. The energy required to take a satellite to a height h' above Earth surface (radius of Eareth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is: [9 Jan. 2019 II]

(a) 1.6×10^3 km **(b) 3.2×10^3 km** **(c) 6.4×10^3 km** **(d) 28×10^4 km**

SOLUTION : (b)

K.E. ofsatellite is zero at earth surface and at height h from energy conservation

$$U_{su\Gamma} + E = \text{Uface 1h}$$

$$-\frac{GM_em}{R_e} + E_1 = -\frac{GM_em}{(Re + h)}$$

$$\Rightarrow E_1 = GM_{e^m}\,{}_{(R_e - R_e + h)} \Rightarrow E_1 = \frac{GM_em}{(R_e + h)} \times \frac{h}{R_e}$$

Gravitational attraction

$$\Gamma_G = ma_c = \frac{mv^2}{(R_e + h)} = \frac{GM_em}{(R_e + h)^2}$$

$$mv^2 = \frac{GM_em}{(R_e + h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_em}{2(R_e + h)}$$

$$E_1 = E_2$$

$$\text{Clearly, } \frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$$

53. Planet A has mass M and radius R. Planet B has halfthe mass and halfthe radius ofPlanet A. Ifthe escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$ The value of n is: [9 Jan. 2020 II]

(a) 4 **(b) 1** **(c) 2** **(d) 3**

SOLUTION : (a)

Escape velocity of the planet A is $V_A = \sqrt{\frac{2GM_A}{R_A}}$

where M_A and R_A be the mass and radius ofthe planet$A$.

According to given problem$M_B = \frac{M_A}{2}, R_B = \frac{R_A}{2}$

$$V_B = \sqrt{\frac{2G\frac{M_A}{2}}{\frac{R_A}{2}}} \therefore$$

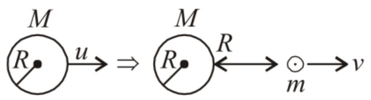
$$\frac{V_A}{V_B} = \sqrt{\frac{\frac{2GM_A}{R_A}}{\frac{2GM_A/2}{R_A/2}}} = \frac{n}{4} = 1$$

$$\Rightarrow n = 4$$

54. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), itejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass ofthe earth): [7 Jan. 2020 I]

(a) $\frac{m}{20}\left(u^2 + \frac{113GM}{200R}\right)$ **(b)** $5m\left(u^2 - \frac{119GM}{200R}\right)$ **(c)** $\frac{3m}{8}\left(\left(\sqrt{\frac{5GM}{6R}}\right)\right)^2$ **(d)** $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

SOLUTION : . (b)

(b)


$$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}m(v^2 - u^2) = \frac{-GMm}{2R} \quad \Rightarrow V = \sqrt{V = u^2 - \frac{GM}{R}} \quad \ldots..\text{(i)}$$

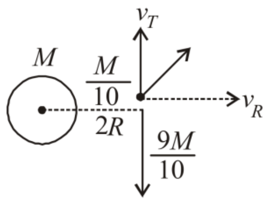
$$v_0 = \sqrt{\frac{GM}{2R}} \quad v_{rad} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10v$$

Ejecting a rocket ofmass $\frac{m}{10}$

$$\frac{9m}{10} \times \sqrt{\frac{GM}{2R}} = \frac{m}{10} \times v_\tau \Rightarrow V_\tau^2 = 81\frac{GM}{2R}$$

Kinetic energy ofrocket, $\quad \text{KE}_{rocket} = \frac{1}{2}\frac{M}{10}(V_T^2 + V_r^2)$

$$= \frac{1}{2} \times \frac{m}{10} \times \left(\left(u^2 - \frac{GM}{R}\right)100 + 81\frac{GM}{R}\right) = \frac{m}{20} \times 100\left(u^2 - \frac{GM}{R} + \frac{81GM}{200R}\right)$$



$$= 5m\left(u^2 - \frac{119GM}{200R}\right)$$

55. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number ofcomplete revolutions made by the spaceship in 24 hours around the planet? [Given : Mass ofPlanet $= 8 \times 10^{22}$kg, Radius of planet $= 2 \times 10^6$m, Gravitational constantG $= 6.67 \times 10^{-11}\text{Nm}^2/\text{kg}^2$]

[10 April 2019 II]

(a) 9 **(b) 17** **(c) 13** **(d) 11**

SOLUTION : (d)

Time period ofrevolution of satellite, $T = \frac{2\pi r}{\text{v}}$

$$v = \sqrt{\frac{GM}{r}}$$

$$T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Substituting the values, we get $\quad T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{667 \times 10^{-11} \times 8 \times 10^{22}}}$ sec

$$\text{T} = 7812.2\text{s}$$

$$T = 2.17hr \Rightarrow 11 \text{ revolutions.}$$

56. A rocket has to be launched from earth in such a waythat it never returns. IfE is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have ifthe same rocket is to be launched fi$_i$om the surface ofthe moon? Assume that the density ofthe earth and the moon are equal and that the earth's volume is 64 times the volume ofthe moon. [8 April 2019 II]

(a) $\frac{E}{64}$ **(b) $\frac{E}{32}$** **(c) $\frac{E}{4}$** **(d) $\frac{E}{16}$**

SOLUTION : . (d)

Escape velocity,

$$v_c = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\text{p}V}{R}} = \sqrt{\frac{2GS\times 4\pi R^3}{R}} = \sqrt{\frac{8}{3}\pi \text{p} G R^2}$$

$$\text{For moon, } v_{\acute{c}} = \sqrt{\frac{8}{3}\pi \text{p} G R_m^2}$$

$$\text{Given, } \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi R_m^3 \text{ or } R_m = \frac{R}{4}$$

$$v_{\acute{e}} = \sqrt{\frac{8}{3}\pi \text{p} G \left(\frac{R}{4}\right)^2} = \frac{v_c}{4}$$

$$\frac{E}{E'} = \frac{\frac{1}{2}mv_e^2}{\frac{1}{2}mv_e^2} = \frac{v_e^2}{v_c^2{}'} = \frac{v_e}{\left(\frac{v_e}{4}\right)} = 16$$

$$\text{or } E' = \frac{E}{16}$$

57. A satellite ofmass M is in a circular orbit ofradius Rabout the centre ofthe earth. A meteorite ofthe same mass, falling towards the earth collides with the satellite completely in elastically. The speeds ofthe satellite and the meteorite are the same, Just before the collision. The subsequent motion ofthe combined bodywill be [12 Jan. 2019 I]

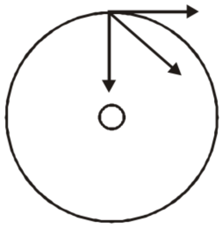
(a) such that it escape to infinity **(b) In an elliptical orbit**

(c) in the same circular orbit ofradius R **(d) in a circular orbit ofa different radius**

SOLUTION : (b)

$$m v\hat{i} + m v \hat{j} = 2m\vec{v} \quad \Rightarrow \vec{v} = \frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}$$

$$\Rightarrow \vec{v} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$



58. Two satellites,AandB, have masses mand2mrespectively. A is in a circular orbit ofradius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is: [12 Jan. 2019 II]

(a) $\frac{1}{2}$ **(b) 1** **(c) 2** **(d) $\sqrt{\frac{1}{2}}$**

SOLUTION : (b)

$$\text{Orbital, velocity, } v = \sqrt{\frac{GM}{r}}$$

$$\text{Kinetic energy of satellite A}, T = m_A V_A^2$$

$$\text{Kinetic energy ofsatellite B}, T = m_B V_B^2$$

$$\Rightarrow \frac{T_A}{T_B} \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}} = 1$$

59. Asatellite is revolving in acircular orbit at aheight hfrom the earth surface, such that h << Rwhere R is the radius of the earth. Assuming that the effect ofearth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field ofearth is: [11 Jan. 2019 I]

(a) $\sqrt{2gR}$ **(b) $\sqrt{gR}$** **(c) $\sqrt{\frac{gR}{2}}$** **(d) $\sqrt{gR}(\sqrt{2}-1)$**

SOLUTION : . (d)

For a satellite orbiting close to the earth, orbital velocity is given by $v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$

Escape velocity (v_e) is $v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}$ [h << R]

$$\Delta v = v_e - v_0 = (\sqrt{2} - 1)\sqrt{gR}$$

60. A satellite is moving with a constant speed v in circular orbit around the earth. An object ofmass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is: [10 Jan. 2019 I]

(a) $2\ mv^2$ **(b) mv^2** **(c) $\frac{1}{2}mv^2$** **(d) $\frac{3}{2}mv^2$**

SOLUTION : (b)

At height rfi¡om center ofearth, orbital velocity $v = \sqrt{\frac{GM}{r}}$

By principle of energy conservation KE of m' $+ \left(-\frac{GMm}{r}\right) = 0 + 0$

(At infinity, PE = KE = 0)

$$\text{or KE of m}' = \frac{GMm}{r} = \left(\left(\sqrt{\frac{GM}{r}}\right)^{m}\right)^2 = mv^2$$

61. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpen-dicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 66 \times 10^{-11} Nm^2kg^{-2}$)

[10 Jan. 2019 II]

(a) $2.4 \times 10^4 m/s$ **(b) $1.4 \times 10^5 m/s$** **(c) $3.8 \times 10^4 m/s$** **(d) $2.8 \times 10^5 m/s$**

SOLUTION : (d)

Let M is mass ofstar m is mass ofmeteroite By energy convervation between 0 and ∞.

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV_{ese}^2 = 0 + 0$$

$$v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4 \times 667 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}} = 2.8 \times 10^5 m/s$$

62. Asatellite is revolving in acircular orbit at aheight 'h'from the earth's surface (radius ofearth R; $h << R$). The minimum increase in its orbital velocityrequired, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect ofatmosphere.) [2016]

(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}(\sqrt{2}-1)$ (c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

SOLUTION : (b)

For h << R , the orbital velocity is $\sqrt{gR}$ Escape velocity $= \sqrt{2gR}$

The minimum increase in its orbital velocity $= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}(\sqrt{2}-1)$

63. An astronaut ofmass m is working on a satellite orbiting the earth at a distance hfi$_i$om the earth$^{\uparrow}$s surface. The radius ofthe earth is R, while its mass is M. The gravitational pull Γ_G on the astronaut is: [Online Apri110, 2016]

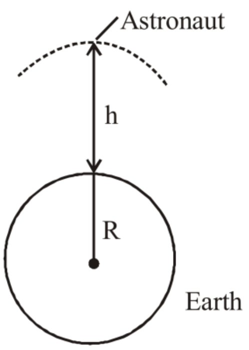
(a) Zero since astronaut feels weightless (b) $\frac{GMm}{(R+h)^2} < \Gamma_G < \frac{GMm}{R^2}$

(c) $\Gamma_G = \frac{GMm}{(R+h)^2}$ (d) $0 < \Gamma_G < \frac{GMm}{R^2}$

SOLUTION : . (c)

According to universal law ofGravitation,

GMm Gravitational force $F = (R+h)^2$



64. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius $R(R << L)$. A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then: [OnlineApri110, 2015]

(a) $T \propto r$ (b) $T \propto \sqrt{r}$ (c) $T \propto r^2$ (d) $T^2 \propto r^3$

SOLUTION : (a)

65. What is the minimum energyrequired to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? [2013]

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

SOLUTION : . (a)

As we know, Gravitational potential energy U $= \frac{-GMm}{r}$

and orbital velocity, $V_0 = \sqrt{GM/R + h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

Therefore minimum required energy, $K = \frac{5GMm}{6R}$

66. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1} , the escape velocity fi$_{i}$om the surface ofthe planet would be [2008]

(a) l.lkm s^{-1} (b) llkm s^{-1} (c) 110kms^{-1} (d) 0.11 km s^{-1}

SOLUTION : **(c)**

67. Suppose the gravitational force varies inversely as the nth power ofdistance. Then the time period ofa planet in circular orbit ofradius 'R' around the sun will be proportional to

(a) R^n (b) $R\left(\frac{n-1}{2}\right)$ [2004] (c) $R\left(\frac{n+1}{2}\right)$ (d) $R\left(\frac{n-2}{2}\right)$

SOLUTION : **(c)**

68. The time period of an earth satellite in circular orbit is independent of [2004]

(a) both the mass and radius ofthe orbit (b) radius of its orbit

(c) the mass ofthe satellite (d) neither the mass ofthe satellite nor the radius of its orbit.

SOLUTION : (c)

Time period of satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

Where $R+h=$ radius of orbit of satellite

$M=$ mass ofearth.

Time period is independent ofmass ofsatellite.

69. A satellite ofmass m revolves around the earth ofradius R at a height $x$fi¡om its surface. Ifg is the acceleration due to gravity on the surface ofthe earth, the orbital speed ofthe satellite is [2004]

(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$ (c) gx (d) $\sqrt{\left(\frac{gR^2}{R+x}\right)}$

SOLUTION : (d)

70. The escape velocity for a body projected vertically upwards fi¡om the surface ofearth is 11 km/s. Ifthe bodyis projected at an angle of $45^{o}$with the vertical, the escape velocitywillbe [2003]

(a) $11\sqrt{2}$km/s (b) 22 km/s (c) 11 km/s (d) $\frac{11}{\sqrt{2}}$km/s

SOLUTION : (c)

71. The kinetic energyneeded to project a bαly ofmass $m$fi¡om the earth surface (radius R) to infinity is [2002]

(a) $mgRl2$ (b) $2mgR$ (c) mgR (d) $mgRl4$.

SOLUTION : (c)

72. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will [2002]

(a) continue to move in its orbit with same velocity

(b) move tangentially to the original orbit in the same velocity

(c) become stationary in its orbit

(d) move towards the earth

SOLUTION : (b)

Due to inertia ofmotion it will move tangentially to the original orbit with the same velocity.

73. The escape velocity of a body depends upon mass as **[2002]**

(a) m^0 **(b)** m^1 **(c)** m^2 **(d)** m^3

SOLUTION : (a) ‘

MECHANICAL PROPERTIES OF SOLIDS

Deforming Force :

External force which try to change in the length, volume or shape of the body is called deforming force.

Elasticity:

Elasticity is that property of the material of a body by virtue of which the body oppose any change in its shape or size when deforming forces are applied to it, and recover its original state as soon as the deforming force are removed.

Perfectly Elastic Body :

The body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Ex. : quartz – Very nearly a perfect elastic body.

Plastic Body:

(a) The body which does not have the property of opposing the deforming force, is known as a plastic body.

(b) The bodies which remain in deformed state even after removed of the deforming force are defined as plastic bodies.

Internal restoring force :

When a external force acts at any substance then due to the intermolecular force there is a internal resistance produced into the substance called internal restoring force.

At equilibrium the numerical value of internal restoring force is equal to the external force.

STRESS:

The internal restoring force acting per unit area of cross-section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area of cross section}} = \frac{F_{internal}}{A} = \frac{F_{external}}{A}$$

SI UNIT : $N\text{-}m^{-2}$ **Dimension :** $M^1L^{-1}T^{-2}$

There are three types of stress –

(I) Longitudinal stress

(II) Volume stress

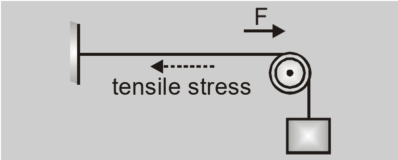
(III) Shear stress

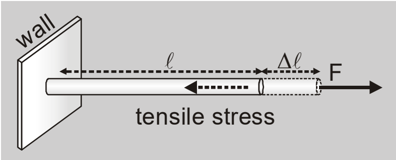
Longitudinal Stress :

When the stress is normal to the surface of body, then it is known as longitudinal stress. there are two types of longitudinal stress

Tensile Stress

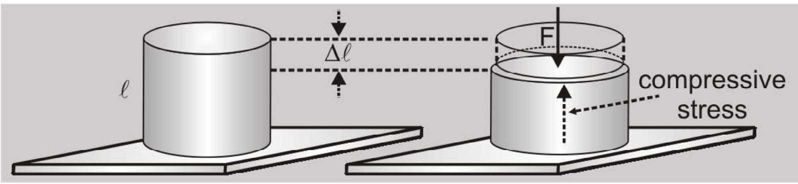
The longitudinal stress, produced due to increase in length of a body, is defined as tensile stress.





Compressive Stress :

The longitudinal stress, produced due to decrease in length of a body, is defined as compressive stress.



Volume Stress :

If equal normal forces are applied every one surface of a body, then it undergoes change in volume. The force opposing this change in volume per unit area is defined as volume stress.

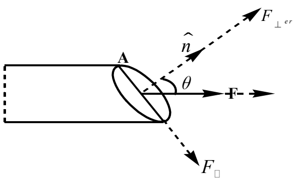
Shear Stress :

When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted.

Difference between pressure and stress

Pressure	Stress
Pressure is always normal to the area.	Stress can be either normal or tangential.
Pressure on a body is always compressive.	Stress can be compressive or tensile or shear.
Pressure is a scalar	Stress is a tensor

If deforming force is applied tangential to the surface, such that tangential stress is developed in a body, then the shape of the body may change.



When a force 'F' acts at an angle 'θ' with outward normal $\hat{n}$ to the area A as shown in figure. In this case, the stress will have the normal and tangential components.

To find the linear (or) longitudinal stress, take the component of the force perpendicular to the plane of a given area A, then divide this component ($F_{\perp er}$) by the area 'A'.

Longitudinal stress = $\frac{F_{\perp er}}{A} = \frac{F\cos\theta}{A}$

To find the shearing stress, take the component of force parallel to the plane of the given area and then devide $F_{\parallel el}$ by the area 'A'.

Shearing stress = $\frac{F_{\parallel el}}{A} = \frac{F\sin\theta}{A}$

The total stress = longitudinal stress + shearing stress

But not F/A.

Breaking Stress :

The stress required to cause actual facture of a material is called the breaking stress or ultimate strength.

Breaking stress = $\frac{F}{A}$

Dependence of breaking stress : (i) Nature of material (ii) Temperature (iii) Impurities.
Independence of breaking stress : (i) Cross sectional area or thickness (i i) Applied force.

Note : The stress required to cause actual fracture of a material is called the breaking stress or the ultimate strength.

Breaking force = Breaking stress x area of cross section

Ø Breaking force is independent of length of the wire, but it depends on the nature of material and area of cross section.

$F \propto A, F \propto r^2$ [in case of cylindrical wire]

Ø The maximum length of the wire that can be hanged without breaking under its own weight

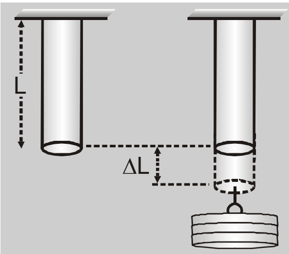
$$\ell = \frac{\text{breaking stress}}{dg}$$

Ø If we cut a wire that can support a maximum load W into two equal parts, then each part of the wire can support a maximum load W.

Punching a hole :

Ø The force required to punch a hole of radius 'r' in a metal plate of thickness 't' is
F = Maximum shearing stress x $2\pi rt$

STRAIN



Strain = $\frac{\text{change in size of the body}}{\text{original size of the body}}$

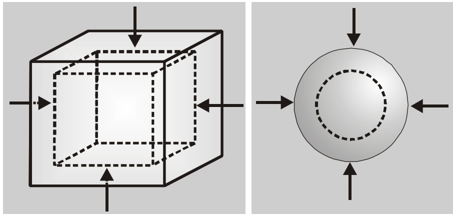
There are three types of strain :

Type of strain depends upon the directions of applied force.

Longitudinal strain =

Longitudinal strain is possible only in solids

Volume strain = $\frac{\text{change in volume of the body}}{\text{original volume of the body}} = \frac{\Delta V}{V}$



Shear strain

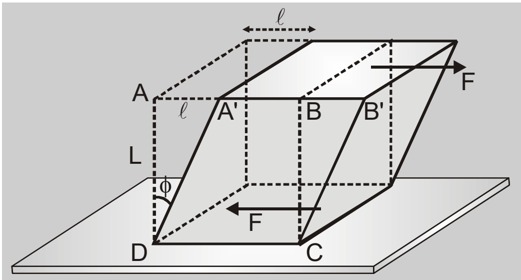
When a deforming force is applied to a body parallel to its surface then its shape (not size) changes.

The strain produced in this way is known as shear strain.

The strain produced due to change of shape of the body is known as shear strain.

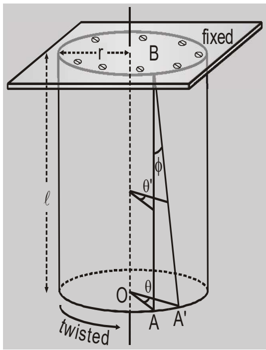
$\tan\phi = \frac{\ell}{L}$

or $\phi = \frac{\ell}{L} = \frac{\text{displacement of upper face}}{\text{distance between two faces}}$



Relation Between angle of twist and Angle of shear

When a cylinder of length 'l' and radius 'r' is fixed at one end



and tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ABA' and angle of twist AOA'.

Arc AA' = r θ and Arc AA' = lϕ

so $r\theta = l\alpha$ r $\phi = \frac{r\theta}{\ell}$

ϕ = angle of twist, θ = angle of shear

Youngs modulus

Longitudinal stress and Longitudinal strain are in constant ratio is called Young's modulus.

$$\text{Young's modulus} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

or $$Y = \frac{\frac{F}{A}}{\frac{e}{\ell}} = \frac{F.\ell}{e \times A} \Rightarrow e = \frac{F.\ell}{A.Y}$$

1. Two wires made of ***same material*** having lengths l_1 & l_2 and radii r_1 and r_2 are subjected to the ***same force***. Then the ratio of their elongations is

$$\frac{e_1}{e_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2} \quad (\text{since } e \propto \frac{1}{r^2})$$

2. Two wires made of ***same material*** having lengths l_1 and l_2 and masses m_1 and m_2 are subjected to the ***same force***. Then the ratio of their elongations is

$$\frac{e_1}{e_2} = \frac{l_1^2}{l_2^2} \times \frac{m_2}{m_1} \quad (\text{since } e \propto \frac{l^2}{m})$$

3. Two wires of ***same material*** and ***same volume*** having areas of cross section A_1 & A_2 are subjected to the ***same force***. Then the ratio of their elogations is

$$\frac{e_1}{e_2} = \frac{A_2^2}{A_1^2} \quad (\text{since } e \propto \frac{1}{A^2})$$

4. Two wires of ***same material*** and ***same volume*** having radii r_1 and r_2 are subjected to the ***same force***. Then the ratio of their elongations is

$$\frac{e_1}{e_2} = \frac{r_2^4}{r_1^4} \quad (\text{since } e \propto \frac{1}{r^4})$$

5. When a body of mass 'm' and density 'd_B' is suspended from a wire its elongation is 'e' when it is in air. If it is completely immersed in a non-viscous liquid of density d_L then its new elongation is

$$e^1 = e(1 - \frac{d_L}{d_B})$$

PROBLEMS

1.A steel wire of 2mm in diameter is stretched by applying a force of 72N. Find the stress in the wire.

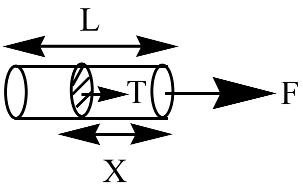
Solution : $r = 1 \times 10^{-3}$ m; F=72N

The stress = $\frac{F}{A} = \frac{F}{\pi r^2} = \frac{72}{\pi(1 \times 10^{-3})^2}$

$= \frac{72}{\pi \times 10^{-6}} = 2.292 \times 10^7 \, Nm^{-2}$

2:A uniform rope of mass M and Length L, on which a force F is applied at one end, then find stress in the rope at a distance x from the end where force is applied?

Solution :



$\frac{M}{L}$ = mass per unit length

From $F = Ma \quad \Rightarrow a = \frac{F}{M}$

Tension, $T = \frac{M}{L}(L - x)a$

$\frac{M}{L}(L-x)\frac{F}{M} = \frac{F}{L}(L-x)$

Stress $= \frac{T}{A} = \frac{F}{A}\left(1 - \frac{x}{L}\right)$

Where tension T and area A must be perpendicular for tensile stress.

3.A force F is required to break a wire of length L and radius r. What is the force required to break a wire of the same material, twice the length and 4 times the radius?

Solution : breaking stress = $\frac{F}{\pi r^2} \Rightarrow F \propto r^2$

$\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2} = \frac{1}{16}$

$\therefore F^1 = 16F$

So the force required is 16 F in the second case

4.The breaking stress of steel is 7.9×10^9 Nm^{-2} and density of steel is 7.9×10^3 kgm^{-3} and g = 10ms^{-2}. The maximum length of steel wire that can hang vertically without breaking is

solution: $Breaking\ Stress = \frac{F}{A} = l\rho g$

$= L = \frac{Breaking\ stress}{\rho g} = \frac{7.9 \times 10^9}{7.9 \times 10^3 \times 10} = 10^5 m$

.

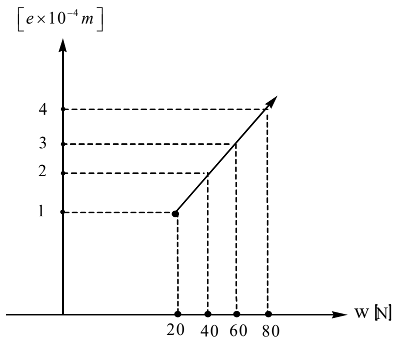
5: A body of mass "m" is connected to an inextensible thread of length "L" is whirled in horizontal circle. Find the maximum angular velocity with which it can be whirled without breaking the thread (Breaking stress of thread =S).

solution: $T = F_{\substack{centripetal \\ force}} = mL\omega^2$

$$S = \frac{T_{max}}{A} = \frac{mL\omega_{max}^2}{A}$$

$$\omega_{max} = \sqrt{\frac{SA}{mL}}$$

6: In the given graph extension (e) of a wire of length 1m is suspended from the top of a roof at one end and with a load "w" connected to other end , if the cross sectional area of the wire is $10^{-6}m^2$, the Young's modulus of the material of wire is



Sol: From graph slope, $\frac{e}{w} = \frac{1}{20} \times 10^{-4}$

$$\frac{w}{e} = \frac{F}{e} = 20 \times 10^4$$

from $Y = \frac{F}{A} \times \frac{\ell}{e} = \left(\frac{w}{e}\right) \times \frac{\ell}{A}$

$$= 20 \times 10^4 \times \frac{1}{10^{-6}} = 2 \times 10^{11} \; N/m^2$$

7. The length of a metal wire is l_1 when the tension in it is F_1 and l_2 when the tension is F_2. find the natural length of wire

Solution: Let l be the natural length of the wire for the force F_1 if elongation is e_1.

$$e_1 = \frac{F_1 l}{AY} \Rightarrow Y = \frac{F_1 l}{Ae_1}$$

Similarly for the force F_2, if elongation is e_2

$$e_2 = \frac{F_1 l}{AY} \qquad \Rightarrow Y = \frac{F_1 l}{Ae_2}$$

$l_1 = l + e_1$ and $l_2 = l + e_1$

$$Y = \frac{F_1 l}{A(l_1 - l)} = \frac{F_2 l}{A(l_2 - l)}$$

$\Rightarrow F_1(l_2 - l) = F_2(l_1 - l)$ and $F_1 l_2 - F_1 l = F_2 l_1 - F_2 l$

$(F_2 - F_1)l = l_1 F_2 - l_2 F_1$ or $l = \frac{l_1 F_2 - l_2 F_1}{(F_2 - F_1)}$.

8 Find the pressure that has to be applied to the ends of a steel wire of length 10 cm, to keep its length constant when its temperature is raised by 100°C is [2014, Jee-main]

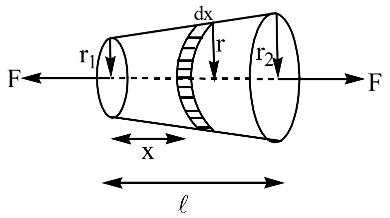
$Y_{steel} = 2\times10^{11} N/m^2 \quad \alpha_{wire} = 1.1\times10^{-5}/K$

Solution: Pressure = $P = \frac{F}{A}$

$$From,\ Y = \frac{F/A}{\frac{\Delta\ell}{\ell}} \Rightarrow \frac{F}{A} = Y\frac{\Delta\ell}{\ell} = Y\alpha\Delta t$$

$\therefore P = Y\alpha\Delta t = 2\times10^{11}\times1.1\times10^{-5}\times10^{2} = 2.2\times10^{+8}\,Pa$

9. A slightly conical wire of length ℓ and radii r_1 and r_2 is stretched by two forces of magnitude F applied parallel to length in opposite directions and normal to end faces. If Y denotes the Youngs's modulus, then find the elongation of the wire ($r_1 > r_2$).



Solution: Consider an element of length dx at distance x as shown in the figure. The radius of the

section $r_x = r_1 + \left(\frac{r_2 - r_1}{\ell}\right)x$

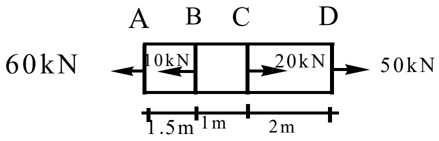
The extension of the element $d\ell = \frac{F(dx)}{A_x Y}$

$$= \frac{Fdx}{\pi r_x{}^2 Y}$$

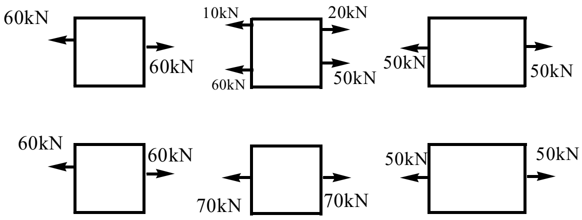
elongation of wire is

$$\ell = \int dl = \int_0^\ell \frac{Fdx}{\pi\left[r_1 + \frac{r_2 - r_1}{\ell}x\right]^2 Y} = \frac{F\ell}{\pi r_1 r_2 Y.}$$

10. A Steel rod of cross-sectional area 1m² is acted upon by forces as shown in the Fig. Determine the total elongation of the bar. Take $Y = 2.0\times10^{11} N/m^2$



Solution: The action of forces on each part of rod is shown in



We know that the extension due to external force F is given by

$$e = \frac{F\ell}{AY}$$

$$\therefore e_{AB} = \frac{\left(60\times10^3\right)\times1.5}{1\times2\times10^{11}} = 4.5\times10^{-7}\,m$$

$$e_{BC} = \frac{\left(70\times10^3\right)\times1.5}{1\times2\times10^{11}} = 3.5\times10^{-7}\,m$$

and $e_{CD} = \dfrac{\left(50\times10^3\right)\times1.5}{1\times2\times10^{11}} = 5.0\times10^{-7}\,m$

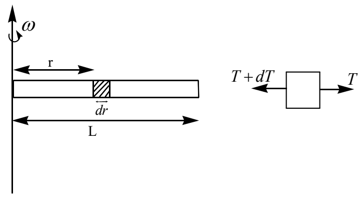
The toal extension $e = e_{AB} + e_{BC} + e_{CD}$

$$= 4.5\times10^{-7} + 3.5\times10^{-7} + 5.0\times10^{-7}$$

$$= 13\times10^{-7}\,m = 1.3\,\mu m$$

11 .A uniform rod of radius "R" and Length "L" is rotated with some angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends, then find tension in the rod?

Solution:



Tension in small element of mass dm is

$$dT = dm\,r\,\omega^2 \Rightarrow dT = -\rho A\,dr\,r\,\omega^2$$

this tension is only due to centripetal force due to all elements between x=L to x=r

$$\therefore T = \int dT = \int_L^r -\rho A\omega^2 r\,dr = \rho A\omega^2 \int_L^r -r dr$$

$$\boxed{\therefore T = \frac{1}{2}\rho A W^2\left[L^2 - r^2\right]}$$

if 'dy' is the elongation in the element 'dr' then $\frac{dy}{dr}=\frac{T}{Ay}$ $\left[\because\ Strain=\frac{stress}{Y}\right]$

$$\int dy=\int\frac{T}{AY}dr$$

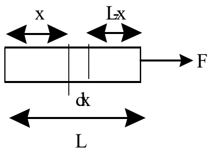
$$y=\int_{O}^{L}\frac{T}{AY}dr=\int_{O}^{L}\frac{1}{2}\frac{\rho A\omega^2}{AY}\left(L^2-r^2\right)dr$$

y = total elongation in the rod

$$\boxed{y=\frac{1}{3}\frac{\rho\,\omega^2 L^3}{Y}}$$

12. A uniform rod of length "L" and mass "M" is pulled horizontally on a smooth surface with a force "F" elongation of the rod of a material of Young's modulus Y is

solution:



Let a small element dx from the free end of the rod, the magnitude of force at this section is

$$F^1=\frac{F}{L}\times(x),$$

elongation on this differential element is $dl=\frac{F}{YAL}\times x\times dx$

$\therefore$ Total elongation,

$$l=\int dl=\int_0^L\frac{F}{YAL}x\,dx=\frac{F}{YAL}\left[\frac{x^2}{2}\right]_0^L=l=\frac{1}{2}\frac{FL}{YA}$$

13. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension when the same tension is applied?

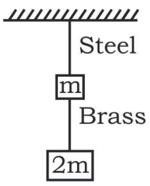
(a) L = 40 cm, r = 0.20 mm (b) L = 100 cm, r = 0.5 mm

(c) L = 200 cm, r = 1 mm (d) L = 300 cm, r = 1.5 mm.

solution: $Y=\frac{FL}{\pi r^2\,\Delta L}$ or $\Delta L\propto\frac{L}{r^2}$.

Here L/r^2 is maximum when L = 40 cm and r = 0.20 mm as compared to other cases.

14.. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the Fig. are a, b, c respectively. Then the corresponding ratio of increase in their lengths would be



(a) $\frac{2ac}{b^2}$ (b) $\frac{3a}{2b^2c}$ (c) $\frac{3c}{2ab^2}$ (d) $\frac{2a^2c}{b}$

solution:

As Dl = $\frac{Fl}{AY}; \frac{\Delta l_s}{\Delta l_B} = \frac{F_s}{F_B} \times \frac{l_s}{l_B} \times \frac{A_B}{A_s} \times \frac{Y_B}{Y_s} = \left(\frac{3M}{2M}\right) \times a \times \frac{1}{b^2} \times \frac{1}{c} = \frac{3a}{2b^2c}$

15.The magnitude of the force developed by raising the temperature from 0ºC to 100ºC of the iron bar 1.00 m long and 1 cm² cross section when it is held so that it is not permitted to expand or bend is $(\alpha = 10^{-50}C^{-1} \text{ and } Y = 10^{11} N m^{-2})$

(a) 10^3 N (b) 10^4 N (c) 10^5 N (d) 10^9 N.

solution: key a $\Delta l = \alpha l \Delta t$ and $F = YA\Delta l/l = YA\alpha\Delta t$

16. A lift is tied with thick iron wires and its mass is 1000 kg. The minimum diameter of wire if the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is 1.4 ´ 10^8 N m^{-2} is (g = 9.8 ms^{-2})

(a) 0.00141 m (b) 0.00282 m (c) 0.005 m (d) 0.01 m.

. solution:

When the lift is accelerated upwards with acceleration a, then tension in the rope is T = m(g + a) = 1000 (9.8 + 1.2) = 11000 N.

Now, stress = $\frac{F}{A} = \frac{T}{\pi r^2}$ or $r^2 = \frac{T}{\pi \times \text{series}}$

$= \frac{11000 \times 7}{22 \times 1.4 \times 10^8} = \frac{1}{4 \times 10^4}$ or $r = \frac{1}{200}; \text{so } D = 2r = \frac{1}{100} = 0.01 m$

SO F = 10^{11} ´ (10^{-4}) ´ 10^{-5} ´ 100 = 10^4 N

17. A wire of length L and of area of cross-section A is stretched through a certain length *l*. If Y is Young's modulus of the material of the wire, then the force constant of the wire is

(a) $\frac{YL}{A}$ (b) $\frac{Yl}{A}$ (c) $\frac{YA}{l}$ (d) $\frac{YA}{L}$.

solution: $Y = \frac{F}{A} \times \frac{L}{l}$ or force constant = $\frac{F}{l} = \frac{YA}{L}$

18. A uniform bar of length L with an elastic modulus Y and thermal coefficient a is held between two rigid planes, one at each end of the bar. In this way the bar is prevented from expansion in these directions when it is heated. When the temperature of the bar is raised by DTºC, the stress developed in the bar is

(a) $Y\alpha\Delta T$ (b) $\frac{Y\alpha\Delta T}{L}$ (c) $\frac{Y\alpha}{\Delta T}$ (d) $\frac{Y\alpha L}{\Delta T}$.

solution: Stress = $\frac{F}{A} - \frac{Y\Delta l}{l} = \frac{Y[l_0(1+\alpha\Delta T) - l_0]}{l_0} = Y\alpha\Delta T$

19. A wire (Y = 2 ´ 10^{11} N/m^2) has length 1 m and area 1 mm^2. The work required to increase its length by 2 mm is

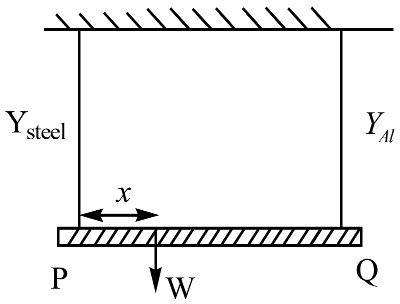
(a) 400 J (b) 40 J (c) 4 J (d) 0.4 J.

solution: $Y = \frac{F}{A} \times \frac{1}{x}$ or $F = \frac{YAx}{1}$

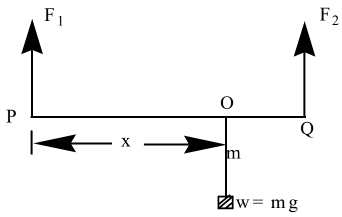
Workdone, $W = \frac{1}{2} F \times x = \frac{1}{2} \frac{YAx^2}{1} = \frac{1 \times 2 \times 10^{11} \times (10^{-6}) \times (2 \times 10^{-3})^2}{2 \times 1} = 0.4\,J$

20. A rod PQ of length 1.05 m having negligible mass is supported at its ends by two wires one of steel (wire A), and the other of aluminium (wire B) of equal lengths as shown in Fg. The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0mm^2 respectively. At what point along the rod a load W be suspended in order to produce (a) equal stress

(b) equal strains in both steel and aluminium.(Y_{steel}=200 GPa, $Y_{aluminium}$=70 GPa)



solution:e end P of the rod PQ, then for rotational equilibrium of the rod about O ,

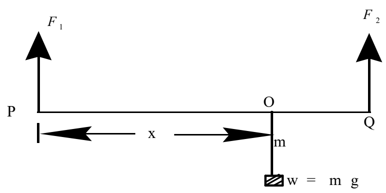


x=0.7 m

b) If F_1 and F_2 are the tensions in the wires A and B respectively to produce equal strain in both wires, i.e, for $(\Delta L)_A = (\Delta L)_B$,

$$= \left(\frac{1mm^2}{2mm^2}\right)\left(\frac{200GPa}{700GPa}\right) = \frac{10}{7}$$

If mass m is now placed at a distance x' from the end P of the rod PQ, for the rotational equilibrium of the rod about O,



$$F_1 x = F_2(1.05 - x)\ or \frac{F_1}{F_2} = \frac{1.05 - x}{x}$$

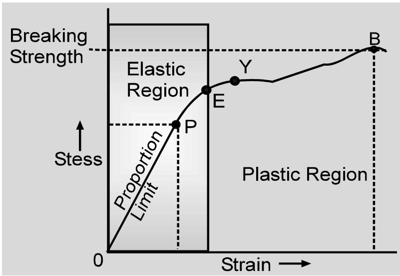
$$\Rightarrow x = \frac{7.35}{17} = 0.43m \Rightarrow 7.35 - 7x = 10x$$

$$\Rightarrow x = \frac{7.35}{17} = 0.43m$$

STRESS - STRAIN GRAPH

Proportion Limit :The limit in which Hook's law is valid and stress is directly proportional to strain is called proportion limit.

Stress α Strain



The limit in which Hook's law is valid and stress is directly proportional to strain is called proportion limit.

Elastic limit

That maximum stress which on removing the deforming force makes the body to recover completely its original state.

Yield Point

The point beyond elastic limit, at which the length of wire starts increasing without increasing stress, is defined as the yield point.

Breaking Point

The position when the strain becomes so large that the wire breaks down at last, is called breaking point. At this position the stress acting in that wire is called breaking stress and strain is called breaking strain.

When the deforming force, applied on a body, is changed rapidly then it temporarily loses its property of elasticity. This is known as elastic fatigue.

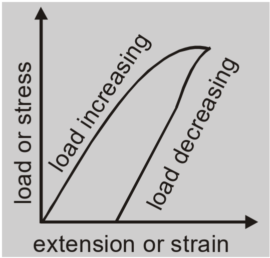
Elastic Fatigue

When the deforming force, applied on a body, is changed rapidly then it temporarily loses its property of elasticity. This is known as elastic fatigue.

Elastic Hysteresis

The strain persists even when the stress is removed.

This lagging behind of strain is called elastic hysteresis.



This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.

Breaking stress also measures the tensile strength.

Metals with small plastic deformation are called brittle.

Metals with large plastic deformation are called ductile.

Elasticity restoring forces are strictly conservative only when the elastic hysteresis is zero. i.e. the loading and unloading stress - strain curves are identical.

The material which have low elastic hysteresis have also low elastic relaxation time.

HOOKE'S LAW

Within elastic limit, the extension of an elastic body is directly proportional to the force that is producing it.

Thomus Young, an English scientist modified the law to a general form.

This modified law is true for all kinds of deformation such as bending, compression, stretching, twisting etc.

This modified form, given below, is now the accepted form of Hooke's Law.

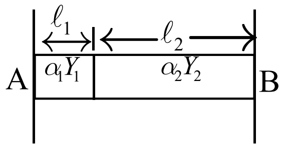
Within elastic limit, stress is proportional to strain.

This constant is known as modulus of elasticity or coefficient of elasticity.

The modulus of elasticity depends only on the type of material used.
It does not depend upon the value of stress and strain.

21. Two rods of different metals, having the same area of cross - section A, are placed end to end between two massive walls as shown in fig. If the temperature of both the rods are now raised by $\Delta t^0 C$ then

a) Find the force with which the rods acts on each other at heigher temperature.



solution:a) Due to heating the increases in length of the composite rod will be

$$(\Delta\ell)_I = \ell_1\alpha_1\Delta t + \ell_2\alpha_2\,\Delta t = (\ell_1\alpha_1 + \ell_2\alpha_2)\Delta t \text{ ... (1)}$$

due to compressive force 'F' from the walls, due to elasticity, the decrease in length will be

$$(\Delta\ell)_D = \frac{F\ell_1}{AY_1} + \frac{F\ell_2}{AY_2} = \frac{F}{A}\left(\frac{\ell_1}{Y_1} + \frac{\ell_2}{Y_2}\right) \text{ .... (2)}$$

As the length of the composite rod remains unchanged the increase in length due to heating must be equal to decrease in length due to compression.

$$(\ell_1\alpha_1 + \ell_2\alpha_2)\Delta t = \frac{F}{A}\left(\frac{\ell_1}{Y_1} + \frac{\ell_2}{Y_2}\right)$$

shift in the joint of a rod

If is $\alpha_1 > \alpha_2$ then final length of first rod

elong ation +/ compression

$$= l + l\alpha_1\Delta\theta - \frac{Fl}{Ay_1}$$

$$l^1 - l = l\alpha_1\Delta\theta - \frac{(\alpha_1 + \alpha_2)\Delta\theta}{\frac{1}{y_1} + \frac{1}{y_2}}$$

Shift in joint = $l^1 - l = x = \dfrac{l\Delta\theta(y_1\alpha_1 - y_2\alpha_2)}{y_1 + y_2}$

22.The edge of an aluminium cube is 10 cm long. one face of cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the displacement of upper face relative to lower face.

solution. Here; $L = 10cm = 10^{-1}m$

$$A = L^2 = 10^{-2}m^2$$

$$F = 100kg\ wt = 100 \times 9.8N = 9.8 \times 10^2\,N.$$

$\eta = 25\,GPa = 25 \times 10^9$ pa $= 25 \times 10^9\ N/m^2$

As

$$\eta = \frac{shear\ stress}{Shear\ strain} = \frac{F/A}{\Delta x/L} = \frac{FL}{A\Delta x}$$

$$\Delta x = \frac{FL}{A\eta} = \frac{(9.8\times10^{2})(10^{-1})}{(10^{-2})(25\times10^{9})}$$

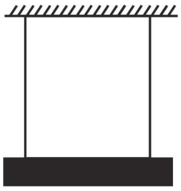
$$= 0.4\times10^{-6}\,m = 4\times10^{-7}\,m.$$

23 The rubber cord catapult has a cross-sectional area 1 mm^2 and total unstretched length 10.0 cm. It is stretched to 12.0 cm and then released to project a missile of mass 5.0 g. Taking Young's modulus for rubber as 5.010^8 N m^{-2}, the tension in the cord is

(a) 1000 N (b) 100 N (c) 10 N (d) 1 N.

solution. $Y = \frac{F}{A}\times\frac{l}{\Delta l}$ or $F = YA\frac{\Delta l}{l}$ = $\frac{(5.0\times10^{8})\times(10^{-6})\times(2\times10^{-2})}{(10\times10^{-2})} = 100\,N$

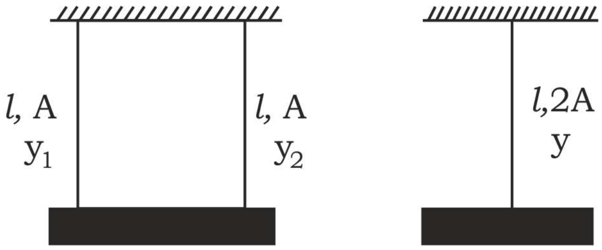
24Two wires of equal length and cross-section area suspended as shown in Fig. in Their Young's modulus are Y_1 and $Y\text{-}_2$ respectively. The equivalent Young's modulus will be



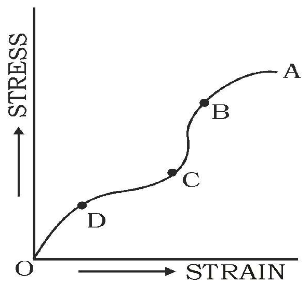
solution. $Y = \frac{F}{A}\times\frac{l}{\Delta l}$

Spring constant of wire, $k = \frac{F}{\Delta l} = \frac{YA}{l}$, $k_{eq} = k_1 + k_2$

or $\frac{Y(2A)}{l} = \frac{Y_1 A}{l} + \frac{Y_2 A}{l}$ or $Y = \frac{Y_1 + Y_2}{2}$



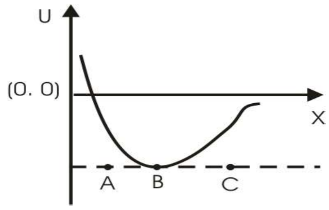
25.fig. shows the stress-strain graph of a certain substance. over which region of the graph is Ilooke's Law obeyed?



(a) BC (b) CD (c) AB (d) OD.

For Hooke's law ; stress μ stain i.e. the graph between stress and strain is a straight line, which is so for portion O to D.

26. The potential energy U between two molecules as a function of the distance X between them has been shown in the adjoining figure. The two molecules are



(a)attracted when X lies between A and B and repelled whenX lies between B and C
(b) attracted when X lies between B and C and are repelled when X lies between A and B
(c) attracted when they reach B. (d) repelled when they reach B.

When there is attraction between molecule, the potential energy decreases with decrease in distance, in case of force of repulsion between the molecules the potential energy increases with the decrease in distance between the molecules

Bulk modulus (B) :

Bulk modulus $= \frac{\text{volume stress}}{\text{bulk strain}}$

V = Constant $B = \frac{\frac{F}{A}}{\frac{-\Delta V}{V}} = -\frac{V}{\Delta V}\frac{F}{A} = \frac{-PV}{\Delta V}$

* negative sign indicates the decrease in volume with increase in pressure

$B_{solids} > B_{liquids} > B_{gases}$

* If a block of coefficient of cubical expansion γ is heated through a rise in temperature of θ, the pressure to be applied on it to prevent its expansion = $K\gamma\theta$, where K is its bulk modulus.
* When a rubber ball of volume V, bulk modulus K is taken to a depth 'h' in water decrease in its volume $\Delta V = \frac{hdgV}{K}$; (d = density of material)
* For an incompressible material, $\Delta V = 0$,
so bulk modulus is infinity.
Solid possesses y, n and k
Liquids and gases possess only K.
* Isothermal bulk modulus of the gas = P(pressure)
Adiabatic bulk modulus of the gas = γp

(where $\gamma = \frac{C_p}{C_V}$)

$$\gamma = \frac{adiabatic\ change\ in\, volume}{isothermal\ change\ in\, volume} = \frac{\Delta V_a}{\Delta V_i}$$

* The reciprocal of bulk modulus is called compressibility. $C = \frac{1}{K}$

Density of compressed liquid :

If a liquid of density 'ρ', volume V and bulk modulus 'K' is compressed, then its density increases

density $\rho = \frac{m}{V}$

$$\rho \propto \frac{1}{V} \Rightarrow \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} \text{ -------- (1)}$$

$$\rho \propto \frac{1}{V} \Rightarrow \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} \text{ -------- (1)}$$

But by definition of bulk modulus

$$B = \frac{-V\,\Delta P}{\Delta V} \Rightarrow -\frac{\Delta V}{V} = \frac{\Delta P}{B} \text{ ----------------(2)}$$

from (1) and (2) $\frac{\Delta\rho}{\rho} = \frac{\Delta P}{K}$

$$\frac{\rho^1 - \rho}{\rho} = \frac{\Delta P}{K} \Rightarrow \rho^1 - \rho = \frac{\Delta P}{K}(\rho) \qquad \boxed{\rho^1 = \rho\left[1 + \frac{\Delta P}{K}\right]}$$

Also $\rho^1 = \rho(1 + C\Delta P)$ where 'C' is the compressibility.

27.A 8m long string of rubber, having density 1.5 x 10^3 kg/m^3 and young's modulus N/m^2 is suspended from the ceiling of a room. The increase in its length due to its own weight will be (g=10m/s^2)

Solution: The increase in its length due to its own weight

$$e = \frac{l^2\rho g}{2Y} = \frac{8^2 \times 1.5 \times 10^3 \times 10}{2 \times 5 \times 10^6} = 9.6 \times 10^{-2} m$$

28.What is the density of water at a depth where the pressure is 80.0 atm, given its density at the surface is $1.03 \times 10^3 kg/m^3$? Compressibility of water $= 45.8 \times 10^{-10} pa^{-1}$.

Solution.

$$B = \frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{B}$$

$$= (79 \times 1.013 \times 10^5) \times (45.8 \times 10^{-10})$$

$$= 36.65 \times 10^{-3}$$

but $\frac{\Delta V}{V} = \frac{(m/\rho) - (m/\rho^1)}{(m/\rho)} = 1 - \frac{\rho}{\rho^1}$

$$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$$

or $\rho^1 = \dfrac{1.03\times10^3}{1-36.65\times10^{-3}} = \dfrac{1.03\times10^3}{0.964}$

$= 1.07\times10^3\ kg/m^3$

29.Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 pa. (Bulk modulus for copper=140 GPa)

solution: Here;

$$L = 10cm = 10^{-1}m,$$

$$V = L^3 = 10^{-3}m^3,$$

$$\Delta P = 7.0\times10^6\,Pa, B = 140\,GPa$$

$$= 140\times10^9\,Pa.$$

As $B = \dfrac{\Delta P\ V}{\Delta V}$,

$$\Delta V = \frac{\Delta P\ V}{B} = \frac{(7.0\times10^6)(10^{-3})}{(140\times10^9)} = 5\times10^{-8}m^3$$

30.A solid sphere of radius 'R' made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area 'A' floats on the surface of the liquid. Find the fractional change in the radius of the sphere , when a mass M is placed on the piston to compress the liquid.

solution: As for a spherical body

$$V = \frac{4}{3}\pi R^3,\ \frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

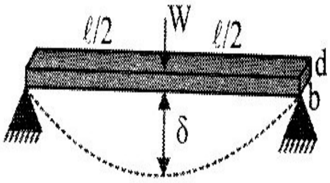
Now by definition of bulk modulus

$$B = -V\frac{\Delta P}{\Delta V}\ i.e \left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} \Rightarrow \frac{Mg}{AB}\left[as\quad \Delta P = \frac{Mg}{A}\right]$$

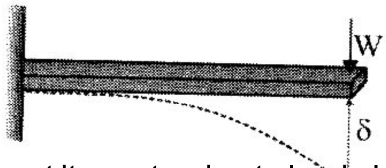
$$\frac{dR}{R} = \frac{1}{3}\frac{\Delta V}{V} \Rightarrow \boxed{\frac{dR}{R} = \frac{Mg}{3AB}}$$

(iii) Bending of Beam :

Beam is the structural member which can carry transverse load. A simply supported beam is supported at its ends. A cantilever beam is fixed at one end.



(a) Simply supported beam

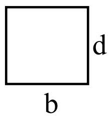


(b) Cantilever beam

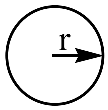
(iv) Deflection of beam : Deflection of beam at its centre due to load placed as shown in Figure.

$\delta = \frac{W\ell^3}{48YI}$ for simply supported beam and $\delta = \frac{W\ell^3}{3YI}$ for cantilever beam where I is called geometric moment of area.

i) For rectangular cross - section $I = \frac{bd^3}{12}$



ii) For circular cross - section $I = \frac{\pi r^4}{4}$



- Ø A force F is applied tangentially on the upper face of a cube of side length L by fixing its lower face. If *l* is the displacement of the upper face and A is the area of the upper face, $A = L^2$.

  $\therefore$ Rigidity modulus, $\eta = \frac{FL}{Al} = \frac{FL}{L^2 l} = \frac{F}{Ll}$.

- Ø One end of the rod is fixed the other free end is twisted through an angle θ by applying a torque τ. The work done on the rod is

  $W = \frac{1}{2}\tau\theta$

wires in series

Two wires of different length ℓ_1, ℓ_2 and of same radii are joined end to end and loaded. Young's modulii Y_1, Y_2 respectively ,and combination behaves as a single wire stress produced in two wires is same

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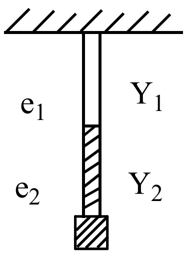


Total elongation is $e = e_1 + e_2$

$$\frac{F}{A}\frac{l_{eff}}{Y_{eff}} = \frac{F}{A}\frac{l_1}{Y_1} + \frac{F}{A}\frac{l_2}{Y_2}$$

$$\boxed{\frac{\ell_1 + \ell_2}{Y_{eff}} = \frac{\ell_1}{Y_1} + \frac{\ell_2}{Y_2}}$$

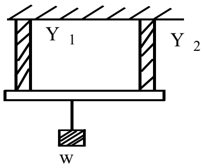
if two wires are having same length, $\ell_1 = \ell_2$ then

$$\frac{2}{Y_{eff}} = \frac{1}{Y_1} + \frac{1}{Y_2} \Rightarrow Y_{eff} = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

TWO WIRES CONNECTED IN PARALLEL

Two wires of same length but different area of cross section A_1, A_2 are joined in parallel and loaded. If the youngs modulii of the materials of the wires are Y_1 & Y_2 and the combination behaves as a single wire

The strain produced in the two wires is same



- Elongation produced in the two wires is same but stress is shared between them

(stress)$_1$+(stress)$_2$=stress

$$F = F_1 + F_2$$

$$\frac{Y_{eff}A_{eff}}{l} = \frac{Y_1A_1}{l} + \frac{Y_2A_2}{l}$$

$$Y_1A_1 + Y_2A_2 = Y_{eff}\left[A_1 + A_2\right]$$

$\frac{Y_1A_1 + Y_2A_2}{A_1 + A_2} = Y_{eff}$, if two wires are of same area of cress section, $A_1 = A_2$

$$Y_{eff} = \frac{Y_1 + Y_2}{2}$$

31. A 40 kg boy whose legs are 4cm² in area 50cm long falls through a height of 2m without breaking his leg bones. If the bones can with stand a stress of $0.9 \times 10^8 N/m^2$. Calculate the Young's modulus of material of the bone.

Sol:

$$mgh = 2\left(\frac{1}{2} \times \frac{stress^2}{Y} \times volume\right),$$

$$Y = \frac{\left(0.9 \times 10^8\right)^2 \times 4 \times 10^{-4} \times 50 \times 10^{-2}}{40 \times 9.8 \times 2}$$

$$= 2.05 \times 10^9 \ N/m^2$$

32. A copper wire 2m long is stretched by 1mm. If the energy stored in the stretched wire is converted into heat, calculate the rise in temperature of the wire.

$(Y = 12.5 \times 10^{10} N/m^2;$

$\rho = 9 \times 10^3 kg/m^3;\ s = 385 J/Kg/K)$

Sol: $m\,s\,\Delta t = \frac{1}{2} Y\,(strain)^2 \times \frac{m}{\rho}$

$$\Delta t = \frac{1}{2} \times \frac{Y}{\rho} \times \left(\frac{e}{\ell}\right)^2$$

$$\Delta t = \frac{1}{2} \times \frac{12.5 \times 10^{10}}{9 \times 10^3} \times \left[\frac{1}{1000 \times 2}\right]^2 = 0.045^0C$$

So the rise in temperature of the wire is 0.0045°C

33. A catapult consists of two parallel rubber cords each of length 20 cm and cross- sectional area 5 cm² when stretched by 8 cm, it can throw a stone of mass 4gm to a vertical height 5 m, the Young's modulus of elasticity of rubber is

$[g = 10\ m/\sec^2]$

Sol: The total elastic potential energy is converted into gravitational potential energy

$$\frac{1}{2} \times \frac{YAe^2}{L} = mgh \quad \Rightarrow Y = \frac{2mghL}{Ae^2}$$

for a single string, $Y = \frac{mghL}{Ae^2}$

$$= \frac{4 \times 10^{-3} \times 10 \times 5 \times 20 \times 10^{-2}}{5 \times 10^{-4} \times \left(8 \times 10^{-2}\right)^2} = \frac{4 \times 10^{-2}}{5 \times 64 \times 10^{-8}}$$

$$= 1.25 \times 10^4 N/m^2$$

Factors effecting Elasticity:

* Annealing decreases elasticity while hammering and rolling increases it.

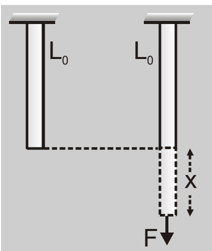
* The impurity having higher elasticity than the sample to which it is added increases the elasticity while the impurity with smaller elasticity decreases the elasticity of the sample.

* Normally, elasticity of the material gets decreased with rise in temperature. However, INVAR STEEL is a material whose elastic behaviour is not affected by rise in temperature.

Work done in stretching a wire (Potential energy of a stretched wire)

For a wire of lenght L_o stretched by a distance x, the restoring elastic force is

:



F = stress × area = $Y\left[\frac{x}{L_o}\right]A$

The work has to be done against the elastic restoring forces.

$dW = F.\,dx = \frac{YA}{L_o}\,x.dx$

The total work done in stretching the wire from x = 0 to x = Dl is, then

$W = \int_0^{\Delta\ell}\frac{YA}{L_o}x.dx = \frac{YA}{L_o}\left[\frac{x^2}{2}\right]_0^{\Delta\ell}$ or $W = \frac{YA(\Delta\ell)^2}{2L_o}$

w = $\frac{1}{2}$ x stretching force x extension.

ii) w = $\frac{1}{2}$ F e $= \frac{1}{2}\frac{YAe^2}{l} = \frac{1}{2}\frac{F^2\ell}{AY} = \frac{1}{2}\frac{F^2\,\ell}{\pi r^2 y}$

iii) w = $\frac{1}{2}$ x stress x strain x volume of the wire

iv) Area under F-e graph gives the work done or the strain energy stored in the

.

previous jeemains questions

Topic:1 Hooke's Law & Young's Modulus

1. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are:

sep 6 mains 2020

(1) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$ (2) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$

(3) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, \frac{A^2}{4B}$ (4) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, \frac{A^2}{2B}$

2. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s⁻¹) with which it can be rotated about its other end in space station is (Breading stress of wire = $4.8\times10^7\,Nm^{-2}$ and area of crosssection of the wire = $10^{-2}cm^2$) is______

9 jan mains 2020

3. A uniform cyclindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to:

(1) $9F/\left(\pi r^2 YT\right)$ (2) $6F/\left(\pi r^2 YT\right)$

12 apr mains2020

(3) $3F/\left(\pi r^2 YT\right)$ (4) $F/\left(3\pi r^2 YT\right)$

4. In an environment, brass and steel wires of length 1 m each with areas of cross section 1 mm² are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to

elongation. The stress required to produce a net elongation of 0.2 mm is, [Given, the Young's modulus for stell and brass are, respectively, $120\times10^9 N/m^2$ and $60\times10^9 N/m^2$]

(1) $1.2\times10^6 N/m^2$ (2) $4.0\times10^6 N/m^2$ 10 apr mains 2020

(3) $1.8\times10^6 N/m^2$ (4) $0.2\times10^6 N/m^2$

5. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(1) 1.00 mm (2) 1.16 mm 10APR MAINS 2019

(3) 0.90 mm (4) 1.36 mm

6. A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given tha g = 3.1 A ms^{-2}, what will be the tensile stress tha would be developed in the wire? 9 APR MAINS 2019

(1) $6.2\times10^6 Nm^{-2}$ (2) $5.2\times10^6 Nm^{-2}$

(3) $3.1\times10^6 Nm^{-2}$ (4) $4.8\times10^6 Nm^{-2}$

7. A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given that g = 3.1 A ms^{-2}, what will be tha tensile stress tha twould be developed in the wire? 9 APR MAINS 2019

(1) $6.2\times10^6 Nm^{-2}$ (2) $5.2\times10^6 Nm^{-2}$

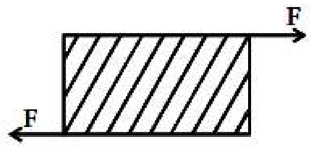
(3) $3.1\times10^6 Nm^{-2}$ (4) $4.8\times10^6 Nm^{-2}$

8. Young's moduli of two wires A and B are in the ratio 7:4. Wire A is 2m long and has radius R. Wire B is 1.5 m long and has radius 2mm. If the two wires stretch by the same length for a given load, then the value of R is close to:8 APR MAINS 2019

(1) 1.5 mm (2) 1.9 mm

(3) 1.7 mm (4) 1.3 mm

9. As shown in the figure, forces of 10^5N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5cm. If the side of another cube of the same material is, 20 cm, then under similar conditions as above, the displacement will be APR 15 MAINS 2018



(1) 1.00 cm (2) 0.25 cm

(3) 0.37 cm (4) 0.75 cm

10. A thin 1m long rod has a radius of 5 mm. A force of 50 πkN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false ? APR MAINS 2016

(1) The maximum value of Y that can be determined is $2\times10^{14} N/m^2$

(2) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length

(3) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain

(4) The figure of merit is the largetst for the length of the rod

11. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. the radii, at the upper and lower ends of

this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal.. APR9 MAINS 2016

(1) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (2) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

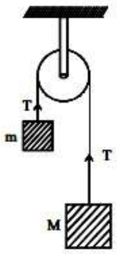
(3) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (4) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

12. The pressure that has to be applied to the ends of a stell wire of elngth 10 cm to keep its length constant when its temperature is raised by 100^{0}C is

(1) $2.2\times10^8\,Pa$ (2) $2.2\times10^9\,Pa$ MAINS 2014

(3) $2.2\times10^7\,Pa$ (4) $2.2\times10^6\,Pa$

13. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If M = 2m, then the stress produced in the wire is



APR MAINS2013

(1) $\frac{2mg}{3A}$ (2) $\frac{4mg}{3A}$ (3) $\frac{mg}{A}$ (4) $\frac{3mg}{4A}$

14. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are respectively $1.0\times10^{11}\,Nm^{-2}$ and $2.0\times10^{11}\,Nm^{-2}$, the total extension of the composite wire is APR23 MAINS 2013

(1) 1.75 mm (2) 2.0 mm

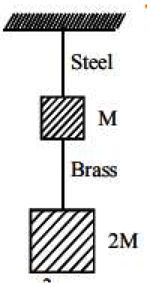
(3) 1.50 mm (4) 1.25 mm

15. A uniform wire (Young's modulus $2\times10^{11}\,Nm^{-2}$) is subjected to longitudinal tensile stress of $5\times10^{11}\,Nm^{-2}$. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to:

(1) 1.0×10^{-4} (2) 1.5×10^{-4} APR22 MAINS 2013

(3) 0.25×10^{-4} (4) 5×10^{-4}

16. If the ratio of lengths, radii and Young's moduli of stell and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is:



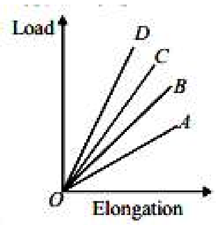
APR9 MAINS 2013

(1) $\frac{3c}{2ab^2}$ (2) $\frac{2a^2c}{b}$ (3) $\frac{3a}{2b^2c}$ (4) $\frac{2ac}{b^2}$

17. A steel wire can sustain 100 kg weight without breaking. If the wire is cut into two equal parts, each part can sustain a weight of MAY19 MAINS 2012
(1) 50 kg (2) 400 kg
(3) 100 kg (4) 200 kg

18. A structural steel rod has a radius of 10 mm and length of 1.0 m. A 100 kN force stretches it along its length. Young's modulus of structural steel is $2\times10^{11}Nm^{-2}$. The percentage strain is about MAY7 2012
(1) 0.16% (2) 0.32% (3) 0.08% (4) 0.24%

19. The load versus elongation graphs for four wires of same length and made of the same material are shown in the figure. The thinnest wire is represented by the line



MAY7 2012

(1) *OA* (2) *OC* (3) *OD* (4) *OB*

20. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? MAY7 2009
(1) 4F (2) 6F (3) 9F (4) F

21. A wire elongates by *l* mm when a load *W* is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(1) *l* (2) 2*l* (3) zero (4) *l*/2 MAY 2006

Topic-2: Bulk and Rigidity Modulus and Work Done in Stretching a Wire

22. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is: 9JAN20120 MAINS
(1) $\sqrt{2}:1$ (2) $1:2$ (3) $2:1$ (4) $1:\sqrt{2}$

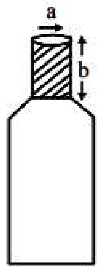
23. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-11}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to: 8APRMAINS2019
(1) 10^6 N/m^{-2} (2) 10^4 N/m^{-2}

(3) 10^8 N/m^{-2} (4) 10^3 N/m^{-2}

24. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mss *m* is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is **2018 MAINS**

(1) $\frac{Ka}{mg}$ (2) $\frac{Ka}{3mg}$ (3) $\frac{mg}{3Ka}$ (4) $\frac{mg}{Ka}$

25. A bottle has an opening of radius a and length b. A cork of length b and radius $(a+\Delta a)$ where $(\Delta a << a)$ is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is



APR12 2014 MAINS

(1) $(\pi\mu Bb)a$ (2) $(2\pi\mu Bb)\Delta a$

(3) $(\pi\mu Bb)\Delta a$ (4) $(4\pi\mu Bb)\Delta a$

26. Steel ruptures when a shear of $3.5\times10^8\,Nm^{-2}$ is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly **APR12 2014 MAINS**

(1) $1.4\times10^4\,N$ (2) $2.7\times10^4\,N$

(3) $3.3\times10^4\,N$ (4) $1.1\times10^4\,N$

27. The bulk moduli of ethanol, mercury and water are given as 0.9, 25 and 2.2 respectively in units of 10^9 Nm^{-2}. For a given value of pressure, the fractional compression in volume is $\frac{\Delta V}{V}$. Which of the following statements about $\frac{\Delta V}{V}$ for these three liquids is correct ? **APR11 2014 MAINS**

(1) Ethanol > Water > Mercury
(2) Water > Ethanol > Mercury
(3) Mercury > Ethanol > Water
(4) Ethanol > Mercury > Water

28. In materials like aluminium and copper, the correct order of magnitude of various elastic modului is **APR9 2014 MAINS**

(1) Young's modulus < shear modulus < bulk modulus
(2) Bulk modulus < Shear modulus <Young's modulus
(3) Shear modulus < Young's modulus < Bulk modulus
(4) Bulk modulus < Young's modulus < Shear modulus

29. If 'S' is stress and 'Y' is young's modulus of material of wire, the energy stored in the wire per unit volume is **APR9 2005 MAINS**

(1) $\frac{S^2}{2Y}$ (2) $2S^2Y$ (3) $\frac{S}{2Y}$ (4) $\frac{2Y}{S^2}$

30. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is

(1) $2F\ell$ (2) $F\ell$ (3) $\frac{F}{2\ell}$ (4) $\frac{F\ell}{2}$ APR2004 MAINS

HINTS & SOLUTIONS

1. (3) Given: $U = \frac{-A}{r^6} + \frac{B}{r^{12}}$

For equilibrium,

$$F = \frac{dU}{dr} = -\left(A\left(-6r^{-7}\right)\right) + B\left(-12r^{-13}\right) = 0$$

$$\Rightarrow 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}} \Rightarrow \frac{6A}{12B} = \frac{1}{r^6}$$

$\therefore$ Separation between molecules, $r = \left(\frac{2B}{A}\right)^{1/6}$

Potential energy,

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

2. (4) Given: wire length, $l = 0.3m$

Mass of the body, $m = 10kg$

Breaking stress, $\sigma = 4.8 \times 10^7 Nm^{-2}$

Area of cross-section, $a = 10^{-2} cm^2$

Maximum angular speed $\omega = ?$

$$T = Ml\omega^2$$

$$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$$

$$\frac{ml\omega^2}{A} \le 48 \times 10^7 \Rightarrow \omega^2 \le \frac{\left(48 \times 10^7\right)A}{ml}$$

$$\Rightarrow \omega^2 \le \frac{\left(48 \times 10^7\right)\left(10^{-6}\right)}{10 \times 3} = 16$$

$$\Rightarrow \omega_{\max} = 4rad/s$$

3. (3) $\Delta_{temp} = \Delta_{force}$

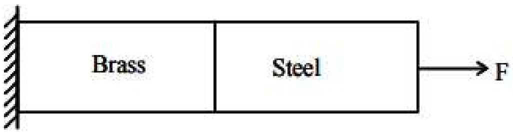
or $L\alpha\left(\Delta T\right) = \frac{FL}{AY}$

$$\therefore \alpha = \frac{FL}{AYT} = \frac{F}{\pi r^2 YT}$$

Coefficient of volume expression

$r = 3\alpha = \frac{3F}{\theta r^2 YT}$

4. (Bonus)



Young modulus, $Y = \frac{Stress}{\left(\frac{\Delta l}{L}\right)}$

Let σ **be the stress**

Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] \quad [\because L_1 = L_2 = 1m]$

$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$

$= 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9 = 8 \times 10^6 \frac{N}{m^2}$

5. (2) Stress $= \frac{F}{A} = \left(\frac{120 \times 60}{180}\right) \times 10^9 = 8 \times 10^6 \frac{N}{m^2}$

$\Rightarrow d^2 = \frac{400 \times 4}{379 \times 10^6 \pi}$

$d = 1.15 mm$

6. (3) Goven,
Radius of wire, r = 2 mm
Mass of the load m = 4 kg

Stress $= \frac{F}{A} = \frac{mg}{\pi(r)^2}$

$= \frac{4 \times 3.1\pi}{\pi \times \left(2 \times 10^{-3}\right)^2} = 3.1 \times 10^6 N/m^2$

7. (3) Given,
Radius of wire, r = 2 mm
Mass of the load m = 4 kg

Stress $= \frac{F}{A} = \frac{mg}{\pi(r)^2} = \frac{4 \times 3.1\pi}{\pi \times \left(2 \times 10^{-3}\right)^2}$

$= 3.1 \times 10^6 N/m^2$

8. (3) $\Delta_1 = \Delta_2$

or $\dfrac{Fl_1}{\pi r_1^2 y_1} = \dfrac{Fl_2}{\pi r_2^2 y_2}$ or $\dfrac{2}{R^2 \times 7} = \dfrac{1.5}{2^2 \times 4}$

$\therefore R = 1.75\, mm$

9. (2) For same material the ratio stress to strain is same
For first cube

$$Stress_1 = \frac{force_1}{area_1} = \frac{10^5}{\left(0.1^2\right)}$$

$$\text{Strain}_1 = \frac{\text{change in length}_1}{\text{original length}_1} = \frac{0.5 \times 10^{-2}}{0.1}$$

For second block,

$$\text{Stress}_2 = \frac{\text{force}_2}{\text{area}_2} = \frac{10^5}{\left(0.2^2\right)}$$

$$\text{strain}_2 = \frac{\text{cahnge in length}_2}{\text{original length}_2} = \frac{\text{x}}{0.2}$$

x is the displacement for second block

For same material, $\dfrac{\text{stress}_1}{\text{strain}_1} = \dfrac{\text{stress}_2}{\text{strain}_2}$

or, $$\frac{\dfrac{10.5}{(0.1)^2}}{\dfrac{0.5 \times 10^{-2}}{0.1}} = \frac{\dfrac{10^5}{(0.2)^2}}{\dfrac{\text{x}}{0.2}}$$

Solving we get, $x = 0.25 cm$

10. (1) Young's modulus $Y = \dfrac{F}{A} / \dfrac{\Delta \ell}{\ell}$

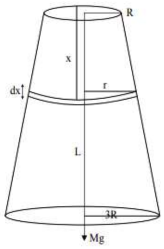
$$Y = \frac{F\ell}{\pi r^2 \Delta \ell}$$

Given, radius r = 5mm, force F = 50 $F = 50\pi kN,$

$$\frac{\ell}{\Delta \ell} = 0.01\ mm$$

$$\therefore Y = \frac{F}{\pi r^2} \frac{\ell}{\Delta \ell} = 2 \times 10^{14} N/m^2$$

11. (3) Consider a small element dx of radius r, $r = \dfrac{2R}{L} x + R$



At equilibrium change in length of the wire

$$\int_0^1 dL = \int \frac{Mgdx}{\pi\left[\frac{2R}{L}x+R\right]^2 y}$$

Taking limit from 0 to L

$$\Delta L = \frac{Mg}{\pi y}\left[-\frac{1}{\left[\frac{2Rx}{L}+R\right]^L}\times\frac{L}{2R}\right] = \frac{MgL}{3\pi R^2 y}$$

The equilibrium extended length of wire $L+\Delta L$

$$= L + \frac{MgL}{3\pi R^2 Y} = L\left(1+\frac{1}{3}\frac{Mg}{\pi Y R^2}\right)$$

12. (1) Young's modulus $\mathrm{Y} = \frac{\text{stress}}{\text{strain}}$

stress = Y x strain
Stress in steel wire = Applied pressure
Pressure = stress = Y x strain

Strain = $\frac{\Delta L}{L} = \alpha\Delta T$

(As length is constant)

$$= 2\times10^{11}\times1.1\times10^{-5}\times100 = 2.2\times10^{8}\,Pa$$

13. (2) Tension in the wire, $T = \left(\frac{2mM}{m+M}\right)g$

$$\text{Stress} = \frac{\text{Force / Tension}}{\text{Area}} = \frac{2\text{mM}}{\text{A}(\text{m}+\text{M})}\text{g}$$

$$= \frac{2(\text{m}\times2\text{m})\text{g}}{\text{A}(\text{m}+2\text{m})}(\text{M}=2\text{m given})$$

$$= \frac{4\text{m}^2}{3\text{mA}}\text{g} = \frac{4\text{mg}}{3\text{A}}$$

14. (4) $Y_c\times(\Delta L_c / L_c) = Y_s\times(\Delta L_s / L_s)$

$$\Rightarrow 1\times10^{11}\times\left(\frac{1\times10^{-3}}{1}\right) = 2\times10^{11}\times\left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore \Delta L_s = \frac{0.5\times10^{-3}}{2} = 0.25m$$

Therefore, total extension of the composite wire

$= \Delta L_c + \Delta L_s$

$= 1mm + 0.25m = 1.25m$

15. (3) Given, $y = 2\times10^{11} Nm^{-2}$

Stress $\left(\frac{F}{A}\right) = 5\times10^{7} Nm^{-2}$

$\Delta V = 0.02\% = 2\times10^{-4} m^3$

$\frac{\Delta r}{r} = ?$

$\gamma = \frac{stress}{strain} \Rightarrow strain\left(\frac{\Delta \ell}{\ell_0}\right) = \frac{\gamma}{stress}$ **...(i)**

$\Delta V = 2\pi x \ell_0 \Delta r - \pi r^2 \Delta \ell$ **....(ii)**

From equation (i) and (ii) putting the value of $\Delta\ell, \ell_0$ **and** ΔV **and solving we get**

$\frac{\Delta r}{r} = 0.25\times10^{-4}$

16. (3) According to questions,

$\frac{\ell_s}{\ell_b} = a, \frac{r_s}{r_b} = b, \frac{y_s}{y_b} = c, \frac{\Delta\ell_s}{\Delta\ell_b} = ?$

As, $y = \frac{F\ell}{A\Delta\ell}\Delta\ell = \frac{F\ell}{Ay}$

$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 . y_s}\left[\because F_s = (M+2M)g\right]$

$\Delta\ell_b = \frac{2mg\ell_b}{\pi r_b^2 . y_b}\left[\because F_b = 2Mg\right]$

$$\therefore \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\frac{3Mg\ell_s}{\pi r_s^2 . y_s}}{\frac{2Mg.\ell_b}{\pi r_b^2 . y_b}} = \frac{3a}{2b^2C}$$

17. (3) Breaking force α **area of cross section of wire Load hold by wire is independent of length of the wire.**

18. (1) Given: $F = 100kN = 10^5 N$

$Y = 2\times10^{11} Nm^{-2}$

$\ell_0 = 1.0m$

radius r = 10 mm = 10^{-2} m

From formula, $Y = \frac{Stress}{Strain}$

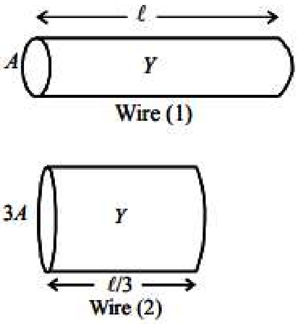
$$\Rightarrow Strain = \frac{Stress}{Y} = \frac{F}{AY}$$

$$= \frac{10^5}{\pi r^2 Y} = \frac{10^5}{3.14 \times 10^{-4} \times 2 \times 10^{11}} = \frac{1}{628}$$

There fore%strain = $\frac{1}{638} \times 100 = 0.16\%$

19. (1) From the graph, it is clear that for the same value of load, elongation is maximum for wire OA. Hence OA is the thinnest wire among the four wires

20. (3)



For wire 1

Length $L_1 = 1$

Area, $A_1 = A$

Fore wire 2

Length, $L_2 = \frac{\ell}{3}$

Area, $A_2 = 3A$

As the wires are made of same material, so they will have same young's modulus

For wire 1,

$$Y = \frac{F / A}{\Delta x / \ell} \quad \text{....(i)}$$

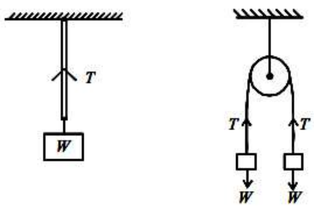
For wire 2,

$$Y = \frac{F' / 3A}{\Delta x / (\ell / 3)} \quad \text{....(ii)}$$

From (i) and (ii) we get,

$$\frac{F}{A} \times \frac{\ell}{\Delta x} = \frac{F'}{3A} \times \frac{\ell}{3\Delta x} \Rightarrow F' = 9F$$

21. (1) Case (i)



At equilibrium, $T = W$

Young's modules, $Y = \frac{W / A}{\ell / L}$ **....(1)**

Elongation, $\ell = \frac{W}{A} \times \frac{L}{Y}$

Case (ii) At equilibrium T = W

$\therefore$ **Young's moduls,** $Y = \dfrac{W/A}{\dfrac{\ell/2}{L/2}}$

$\Rightarrow Y = \dfrac{W/A}{\ell/L} \Rightarrow \ell = \dfrac{W}{A} \times \dfrac{L}{Y}$

$\Rightarrow$ **Elongation is the same**

22. (1) If force F acts a long the length L of the wire of crosssection A, then energy stored in unit volume of wire is given by

Energy density $= \dfrac{1}{2} stress \times strain$

$= \dfrac{1}{2} \times \dfrac{F}{A} \times \dfrac{F}{AY}$

($\therefore$ **stress** $= \dfrac{F}{A}$ **and strain** $= \dfrac{X}{AY}$**)**

$= \dfrac{1}{2}\dfrac{F^2}{A^2 Y} = \dfrac{1}{2}\dfrac{F^2 \times 16}{\left(\pi d^2\right)^2 Y} = \dfrac{1}{2}\dfrac{F^2 \times 16}{\pi d^4 Y}$

If u_1 **and** u_2 **are the densities of two wires, then**

$\dfrac{u_1}{u_2} = \left(\dfrac{d_2}{d_1}\right)^4 \Rightarrow \dfrac{d_1}{d_2} = (4)^{1/4} \Rightarrow \dfrac{d_1}{d_2} = \sqrt{2} : 1$

23. (1) When a catapult is stretched up to length l**, then the stored energy in it** $= \Delta k.E \Rightarrow$

$\dfrac{1}{2} . \left(\dfrac{YA}{L}\right)(\Delta l)^2 = \dfrac{1}{2} mv^2 \Rightarrow y = \dfrac{mv^2 L}{\Delta(\Delta l)^2}$

m = 0.02 kg
v = 20 ms^{-1}
L = 0.42 m

A = $\left(\pi d^2\right)/4$

d = $6 \times 10^{-3} m$

$\Delta l = 0.2\, m$

$y = \dfrac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04} = 2.3 \times 10^6 N/m^2$

So, order is 10^6

24. (3) Bulk modulus, $K = \dfrac{volumetric\ stress}{volumetric\ strain}$

$K = \dfrac{mg}{a\left(\dfrac{dV}{V}\right)}$

$\Rightarrow \dfrac{dV}{V} = \dfrac{mg}{Ka}$

volume of sphere, $V = \frac{4}{3}\pi R^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ **.....(ii)**

Using eq. (i) & (ii) $\frac{3dr}{r} = \frac{mg}{Ka}$

$\therefore \frac{dr}{r} = \frac{mg}{3Ka}$ **(fractional decrement in radius)**

25. (4) $Stress = \frac{Normal\ force}{Area} = \frac{N}{A} = \frac{N}{(2\pi a)b}$

Stress = B x strain

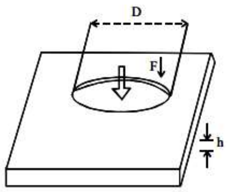
$$\frac{N}{(2\pi a)b} = B\frac{2\pi a\Delta a \times b}{\pi a^2 b}$$

$$\Rightarrow N = B\frac{(2\pi a)^2 \Delta ab^2}{\pi a^2 b}$$

Force needed to push the cork

$$f = \mu N = \mu 4\pi b\Delta aB = (4\pi\mu Bb)\Delta a$$

26. (3)



Shearing strain is created along the side surface of the punched disk. Note that the forces exerted on the disk are exerted along the circumference of the disk, and the total force exerted on its center only
Let us assume that the shearing stress along the side surface of the disk is uniform, then

$$F = \int_{surface} \sigma_{max} dA_{max} = \sigma_{max} \int_{surface} dA$$

$$= \int \sigma_{max}.A = \sigma_{max}.2\pi\left(\frac{D}{2}\right)h$$

$$= 3.5\times10^8 \times \left(\frac{1}{2}\times10^{-2}\right)\times0.3\times10^{-2}\times2\pi$$

$$= 3.297\times10^4 = 3.3\times10^4\,N$$

27. (1) Compressibility $= \frac{1}{Bulk\ modulus}$

As bulk modulus is least for ethanol (0.9) and maximum for mercury (25) among ehtanol, mercury and water. Hence compression in volume $\frac{\Delta V}{V}$.

Ethanol > Water > Mercury

28. **(3) Poisson's ratio,** $\sigma = \dfrac{lateral\ strain(\beta)}{longitudinal\ strain(\alpha)}$

For material like copper, $\sigma = 0.33$

And, $Y = 3k(1-2\sigma)$

Also, $\dfrac{9}{Y} = \dfrac{1}{k} + \dfrac{3}{\eta}$

$Y = 2\eta(1+\sigma)$

Hence, $\eta < Y < k$

29. **(1) Energy stored in the wire per unit volume,**

$E = \dfrac{1}{2} \times stress \times strain$ **....(i)**

We know that,

$Y = \dfrac{stress}{strain}$

$\Rightarrow strain = \dfrac{stress}{Y}$

On substituting the expression of strain in equation (i) we get

$E = \dfrac{1}{2} \times stress \times \dfrac{stress}{Y} = \dfrac{1}{2} . \dfrac{S^2}{Y}$

30. **(4) Let A and L be the area and length of the wire. Work done by constant force in displacing the wire by the distance** ℓ **.**
= change in potential energy

$= \dfrac{1}{2} \times stress \times strain \times volume$

$= \dfrac{1}{2} \times \dfrac{F}{A} \times \dfrac{\ell}{L} \times A \times L = \dfrac{F\ell}{2}$

Fluid Mechanics (Viscosity)

- Fluid is the name given to a substance which begins to flow when external force is applied on it.
- Liquids and gases are fluids.
- Fluids do not have their own shape but take the shape of the containing vessel.
- Fluids flow from one place to other because of pressure difference.
- The branch of physics which deals with the study of fluids at rest is called hydrostatics
- The branch which deals with the study of fluids in motion is called hydrodynamics.

Pressure:

- ¨The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.
- ¨The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it, is called pressure of liquid or hydrostatic pressure.
- If F be the normal force acting on a surface of area A in contact with liquid,

then pressure exerted by liquid on this surface is $P = \frac{F}{A}$

- ¨The average pressure on the surface area ΔA due to a normal force $\Delta F_{\perp}$ is $P_{avg} = \frac{\Delta F_{\perp}}{\Delta A}$
- ¨Pressure at a point is given by $P = \underset{\Delta A \to 0}{Lt} \frac{\Delta F_{\perp}}{\Delta A} = \frac{dF}{dA}$
- ¨ Units : N/m^2 or Pascal (S.I.)
- Dyne/cm^2 (C.G.S.)
- $1 Nm^{-2} = 10 dyne/cm^2$
- ¨ Dimension : $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
- ¨ At a point pressure acts in all directions and a definite direction is not associated with it. So pressure is a tensor quantity.

Atmospheric pressure :

¨The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure.

¨Its value on the surface of the earth at sea level is nearly $1.013 \times 10^5 N/m^2$ or Pascal

¨The practical units of pressure are atmosphere, bar and torr (*mm* of *Hg*)

¨ 1 atmospheric pressure = 1.01325×10^5 pascal ,

1 bar = 760mm of Hg = 76cm of Hg = 0.76m of Hg,

1 torr = 1 mm of Hg.'

- ♦ The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.
- ♦ The pressure at the bottom of the container due to liquid column of height 'h' is $P = h\rho g$,

  where 'ρ' is the density of the liquid
- ♦ If atmospheric pressure(P_0) is considered, then net pressure at the bottom of the container is

$$P = P_0 + h\rho g$$

Gauge pressure :

The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure.

$$P - P_0 = h\rho g$$

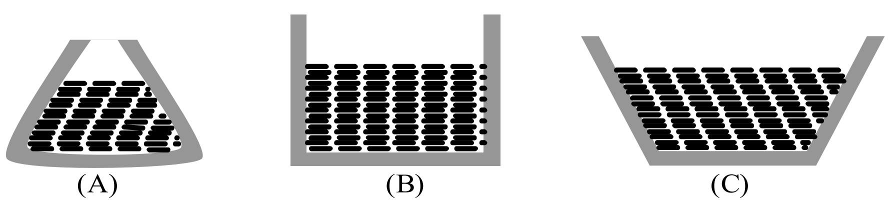
$$P_{absolute} = P_{atm} + P_{gauge}$$

- ♦ Absolute pressure is always positive and is never equal to zero.
- ♦ Gauge pressure may be positive, negative or zero.

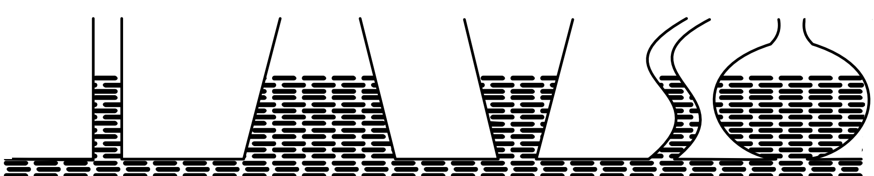
NOTE :

- ♦ Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.

$$P_A = P_B = P_C \text{ but } W_A < W_B < W_C$$

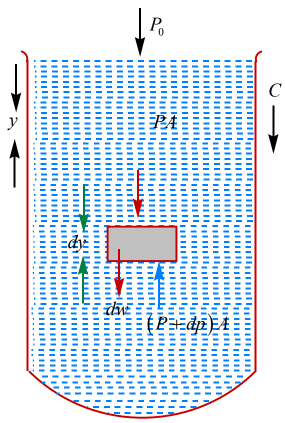


- ♦ In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.



- ♦ Pressure is isotropic i.e., the pressure exerted by a liquid at a point is same in all directions.
- ♦ Pressure is uniform on a horizontal plane for a liquid at rest or moving with uniform velocity or vertical acceleration.'

Pressure varies with depth and height:



up wards force = $(P + dP) = PA + AdP$

downward force = $PA + dw = PA + A\rho gdy$

at equilibrium $PA + AdP = PA + A\rho gdy$

$$\Rightarrow \frac{dP}{dy} = \rho g \Rightarrow \int_{P_0}^{P} dP = \rho g \int_0^y dy$$

$$\Rightarrow P - P_0 = \rho gy \Rightarrow dP = \rho gy$$

♦ Pressure increases with depth linearly

♦ Pressure decreases with height linearly

♦ The average pressure of a liquid on the walls of the container filled up to height 'h' with the liquid is $\frac{1}{2}\rho gh$

Density:

In a fluid, at a point, density ρ is defined as:

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

♦ In case of homogenous isotropic substance, it has no directional properties, so is a scalar.

♦ S.I. unit kg/m^3

C.G.S. unit g/cc

$$1 g/cc = 10^3 kg/m^3$$

Dimensions $[ML^{-3}]$

♦ Density of substance means the ratio of mass of substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. So for a solid body.

Density of body = Density of substance

While for a hollow body, density of body is lesser than that of substance [As $V_{\text{body}} > V_{\text{sub.}}$]

♦ When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

♦ Sometimes instead of density we use the term relative density or specific gravity which is defined as :

$$RD = \frac{\text{Density of body}}{\text{Density of water}}$$

♦ If m_1 mass of liquid of density ρ_1 & m_2 mass of density ρ_2 are mixed, then as $m = m_1 + m_2$ and $V = (m_1/\rho_1) + (m_2/\rho_2)$ [As $V = m/\rho$]

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\Sigma m_i}{\Sigma(m_i/\rho_i)}$$

If $m_1 = m_2$ $\quad \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} =$ Harmonic mean

♦ If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as: $m = \rho_1 V_1 + \rho_2 V_2$ and $V = V_1 + V_2$ [As $\rho = m/V$]

If $V_1 = V_2 = V$ $\quad \rho = (\rho_1 + \rho_2)/2$ = Arithmetic Mean

♦ With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease,

$$i.e., \frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \qquad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

$$\text{or} \quad \rho = \frac{\rho_0}{(1+\gamma\Delta\theta)} \simeq \rho_0(1-\gamma\Delta\theta)$$

♦ With increase in pressure due to decrease in volume, density will increase,

$$i.e., \frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \qquad [\text{As } \rho = \frac{m}{V}]$$

But as by definition of bulk-modulus $B = -V_0 \frac{\Delta p}{\Delta V}$

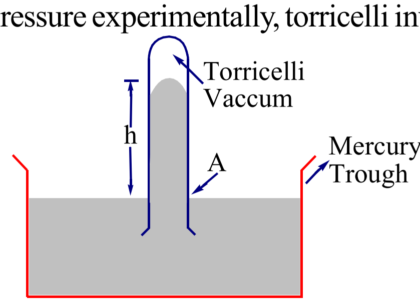
$$i.e., \quad V = V_0\left[1 - \frac{\Delta p}{B}\right]$$

$$\text{So} \quad \rho = \rho_0\left(1 - \frac{\Delta p}{B}\right)^{-1} \simeq \rho_0\left(1 + \frac{\Delta p}{B}\right)$$

MEASUREMENT OF ATMOSPHERIC PRESSURE:

Mercury Barometer :

To measure the atmospheric pressure experimentally, torricelli invented a mercury baromer in 1643.

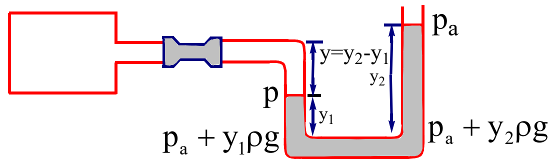


P_a=hrg

The pressure exerted by a mercury column of 1mm high is called 1 Torr.

'Open tube Manometer :

Open-tube manometer is used to measure the pressure gauge. When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.



i.e. $p + y_1 rg = p_a + y_2 rg$

$p - p_a = rg(y_2 - y_1) = rgy$

p = absolute pressure, $p - p_a$ = gauge pressure

Thus, knowing y and r (density of liquid), we can measure the gauge pressure.

Water Barometer:

Let us suppose water is used in the barometer instead of mercury.

$hrg = 1.013 \times 10^5$ or $h = \dfrac{1.013\times10^5}{pg}$

The height of the water column in the tube will be 10.3m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

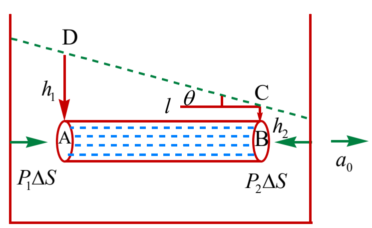
Pressure difference when liquid is accelerating in vertical direction:

i) When liquid column is uniform acceleration upwards, $P = h\rho(g+a)$

ii) When liquid column is in uniform acceleration downwards, $P = h\rho(g-a)$

Pressure difference when liquid is accelerating in horizontal direction:

Consider a liquid placed in beaker which is accelerating horizontally with an acceleration a_0. Let A and B be two points in the liquid at a seperation ℓ in the same horizontal line along the acceleration a_0. We shall first obtain the pressure difference between the points A and B. Construct a small vertical area ΔS around A and an equal area around B. Consider the liquid contained in the horizontal cylinder with two areas as the flat faces. Let the pressure at A be P_1 and the pressure at B be P_2. The forces along the line AB are



- $P_1\,\Delta S$ towards right due to the liquid on the left
- $P_2\,\Delta S$ towards left due to the liquid on the right.

Under the action of these forces, the liquid contained in the cylinder is accelerating towards right. From Newton's second law.

$$P_1\Delta S - P_2\Delta S = ma_0 \text{ or, } (P_1 - P_2)\Delta S = (\Delta S)\,\ell\rho a_0$$

$$\text{or} \quad P_1 - P_2 = \ell\rho a_0 \text{ ---------------(1)}$$

The two points in the same horizontal line do not have equal pressure, if the liquid is accelerated horizontally.

As there is no vertical acceleration, the equation is valid. If the atmospheric pressure is P_0, the pressure at A is $P_1 = P_0 + h_1\rho g$ and the pressure at B is $P_2 = P_0 + h_2\rho g$, where h_1 and h_2 are the depths of A and B from the free surface. Substituting values in (1).

$$h_1\rho g - h_2\rho g = l\rho a_0 \text{ or } \frac{h_1 - h_2}{\ell} = \frac{a_0}{g} \, or, \tan\theta = \frac{a_0}{g}$$

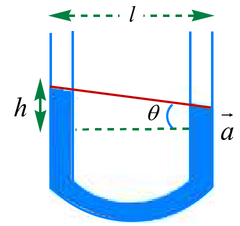
Where θ is the inclination of the free surface with the horizontal.

Here $P_A - P_C = (P_A - P_B) + (P_B - P_C)$

$$= \rho la + \rho g h_2 \; = \rho(la + gh_2)$$

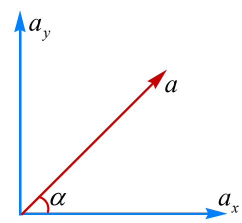
➡ If a U shape tube is moving horizontally with an accleration 'a' as shown in the fig. then

$$\tan\theta = \frac{h}{l} = \frac{a}{g}$$



➡ If container is accelerated 'a' at some angle with the horizontal,

$$\tan\theta = \frac{a_x}{a_y + g}$$



$a_x = a\cos\alpha$ → horizontal component

$a_y = a\sin\alpha$ → vertical component

θ = angle of inclination of free surface of the liquid with horizontal

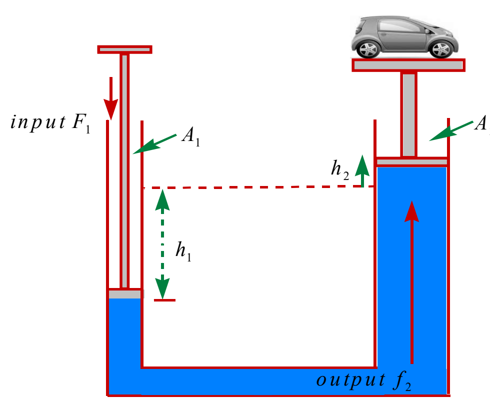
Pascal's Law:

♦ The pressure applied to an enclosed incompressible liquid **is transmitted undiminished** to every point of the liquid and the walls of the container.

♦ The pressure in a liquid at rest is same at all points if we ignore gravity.

Mechanical Gain : It is the ratio of output force to input force (or) Mechanical gain = $\left(\frac{F_2}{F_1}\right)$

Hydraulic lift :



$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = F_1\frac{A_2}{A_1}$$

$$\text{As } A_2 > A_1,\ F_2 > F_1$$

♦ As the same volume of fluid is displaced at both pistons

$$A_1h_1 = A_2h_2$$

$$\Rightarrow h_2 < h_1$$

Pressure Energy:

♦ The energy possessed by a fluid by virtue of its pressure is called the **pressure energy**.

♦ Pressure energy is equal to the work done in keeping an elementary mass of a fluid at a point against the pressure existing at that point.

♦ Pressure energy=Pressure x Volume = P(A x x).

Where P = pressure,
A = Area of cross section,
x = distance through which liquid is moved

♦ Pressure energy per unit volume = $\frac{P\times A\times x}{A\times x} = P$

♦ Pressure energy per unit mass= $\frac{P\times A\times x}{\rho\times A\times x} = \frac{P}{\rho} = \frac{\text{Pressure}}{\text{density}}$

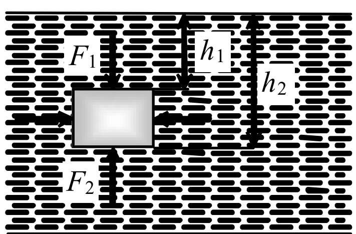
♦ Pressure energy has same units and dimensions as that of energy.

Buoyancy:

♦ When a body is partly or wholly dipped in a fluid, the fluid exerts force on the body due to hydrostatic pressure.

♦ At any small portion of the surface of the body, the force exerted by the fluid is perpendicular to the surface and is equal to the pressure at that point multiplied by the area.

♦ The resultant of all these constant forces is called upthrust or buoyancy.



To determine the magnitude and direction of this force consider a body immersed in a fluid of density σ as shown in figure. The forces on the vertical sides of the body will cancel each other. The top surface of the body will experience a downward force

$F_1 = AP_1 = A(h_1\sigma g + P_0)$ [As $P = h\sigma g + P_0$]

While the lower face of the body will experience an upward force.

$F_2 = AP_2 = A(h_2\sigma g + P_0)$

As $h_2 > h_1, F_2$ will be greater than F_1,
so the body will experience a net upward force

$F = F_2 - F_1 = A\sigma g(h_2 - h_1)$

If L is the vertical height of the body $F = A\sigma gL = V\sigma g$ [As $V = AL = A(h_2 - h_1)$]

i.e., F = Weight of fluid displaced by the body.

♦ This force is called upthrust or buoyancy and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy).

♦ Though we have derived this result for a body fully submerged in a fluid, it can be shown to hold good for partly submerged bodies or a body in more than one fluid also.

♦ When a body is partly or wholly immersed in a fluid the upward force exerted by fluid on thebody is called **buoyancy**.

♦ **Force of buoyancy** is equal to the hydrostatic pressure at the point multiplied by area of cross section of the body.

Laws of floatation:

Let W is weight of a body

W_1 is the the buoyant force .

♦ If $W > W_1$ body sinks

♦ $W = W_1$ body is just submerged (body floats with its volume completely under the liquid)

♦ $W < W_1$ body floats (a part of the body lies outside the liquid)

♦ A body of volume V and density ρ_b is floating with a volume V_{in} inside the fluid of density ρ_l,

then $\boxed{V\rho_b g = V_{in}\rho_1 g}$

weight of the body = weight of the liquid displaced (due to body submerged in the liquid).

♦ A body of mass M and volume V is floating in a liquid of density ρ_l with some volume in air. To make it to just sink, the mass 'm' to be placed on it is given by

$$mg = \Delta V g \rho_l,$$

where ΔV is the volume of body that was initially outside the liquid.

Floatation:

When a body of density ρ_b and volume V immersed in a liquid of density ρ then forces acting are

♦ The weight of body acting vertical downwards through the center of gravity of the body

$$W = mg = V\rho_b g$$

♦ ' The upthrust(force of buoyancy) acting upwards through center of gravity of displaced liquid called center of buoyancy $F_B = V\rho_l g$

1) If $\rho_b > \rho \Rightarrow W > F_B \Rightarrow$ body sinks

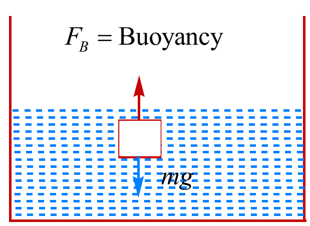
2) If $\rho_b = \rho_l \Rightarrow W = F_B \Rightarrow$ body just floats

3) If $\rho_b < \rho_l \Rightarrow V\rho_b g = V_{in}\rho_1 g \Rightarrow V\rho_b = V_{in}\rho_1$

$\Rightarrow$ body is partially immersed or floats partially.

Archimede's Principle:

When a body is immersed partly or wholly in a fluid it appears that it loses some weight, which is equal to the weight of the liquid displaced (which is equal to the force of buoyancy).



F_B=mg

Apparent loss of weight of a body or weight of fluid displaced = $V_{in}\rho_l g$

V_{in} = Volume of body immersed or volume of fluid displaced

Note :

- ♦ Upthrust or buoyancy is independent of mass, size, density, shape etc. of the body.
- ♦ It depends only on the volume of the body immersed inside the fluid, nature (density) of the fluid and acceleration due to gravity

$$\left(F_B\right)_{eff} = V_{in}\rho_l\left(g \pm a\right)$$

Relative density (Specific gravity) of a solid:

$$\text{RD} = \frac{density\ of\ the\ body}{density\ of\ water\ at\ 4^0 C}$$

‘

$$= \frac{weight\ of\ the\ body}{upthrust\ exerted\ by\ water}$$

$$= \frac{\text{weight of the body in air}}{\text{loss of weight of body in water}}$$

$$\text{R.D.} = \frac{w_1}{w_1 - w_2};$$

w_1 = weight of the body in air

w_2 = weight of the body in water

$w_1 - w_2$ = Loss of weight of body in water

Relative density of a liquid:

If loss of weight of a body in water is ‘a’ and that of in liquid is ‘b’ then,

$$V\rho_w g = a;\ V\rho_L g = b$$

$$\text{RD of liquid} = \frac{\rho_L}{\rho_w} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{\text{b}}{\text{a}} = \frac{W_{air} - W_{liquid}}{W_{air} - W_{water}}$$

Volume of a cavity in a body:

Consider a metal piece of mass M and density ρ_m

The volume of metal $V = \dfrac{M}{\rho_m}$

Weight of the metal in air = W_1

Weight of the metal in water = W_2

Loss of weight of the metal = W_1 - W_2 = F_b

If V' is the geometric volume of the body immersed in the liquid then

$$V'\rho_w g = W_1 - W_2$$

$$V' = \frac{W_1 - W_2}{\rho_w g}$$

Volume of cavity = $V' - V = \left(\frac{W_1 - W_2}{\rho_w g}\right) - \frac{M}{\rho_m}$

Where

V' is the total volume of the metallic body.

V is the volume of the material in the metal piece.

The amount of impurity in a given metal:

Let w_1 be the weight of an alloy in air

w_2 be the weight in water.

Let the alloy consists of two metals having masses m_1 and m_2

Total mass $m = (m_1 + m_2)$. The buoyant force on the alloy is,

$$F_b = w_1 - w_2 = V \rho_w g$$

$$\Rightarrow \frac{w_1 - w_2}{\rho_w g} = V = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

'= Volume of the first metal in the alloy + Volume of the second metal in the alloy

= Volume of the alloy (ρ_1 , ρ_2 are the densities of the metals)

$$\frac{w_1 - w_2}{\rho_w g} = \frac{m_1}{\rho_1} + \frac{(m - m_1)}{\rho_2}$$

$m = m_1 + m_2 =$ total mass

Fraction of Volume of the Body outside the Liquid:

Weight of the body = Weight of the displaced liquid

$V_{total} \rho_b g = V_{in} \rho_l g \left(V_{total} = \text{volume of the body}\right)$

$V_{out} = V_{total} - V_{in} = V\left(1 - \frac{\rho_b}{\rho_l}\right)$

The fraction of volume outside the liquid is $f_{out} = \frac{V_{out}}{V_{total}} = \left(1 - \frac{\rho_b}{\rho}\right)$

Floating of Ice:

- When a block of ice, floats in a liquid of density d_l, melts completely, the level of (liquid + water)

  (i) Rises, if $\rho_1 > \rho_w$

  (ii) Falls, if $\rho_1 < \rho_w$

  (iii) Remains unchanged, if $\rho_1 = \rho_w$

- A piece of solid is embedded inside an ice block which floats in water.

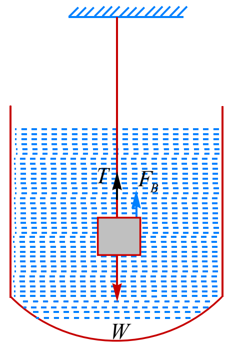
  When ice melts completely, the level of water

  (i) Remains same, if $\rho_s \leq \rho_w$

  (ii) Falls, if $\rho_s > \rho_w$

Tension in the string connected to a submerged body :

When the body hangs by light string and $\rho_b < \rho_l$



Case-I : When the system is at rest or moving with uniform velocity (a=0) in vertical direction

The tension in the string is T = Apparent weight of the body

$$T = W - F_B = V\rho_b g - V\rho_l g = V\left(\rho_b - \rho_l\right)g$$

Case-II: When the system is acclerating vertically upward with acceleration a. Tension

$$T = V\left(\rho_b - \rho_l\right)\left(g - a\right)$$

Case-III : If the system is accelerating vertically downward with an acceleration a (a<g)

$$T = V\left(\rho_b - \rho_l\right)\left(g - a\right)$$

::PROBLEMS::

1. The pressure at the bottom of a lake, due to water, is 4.9x 10^6 N/m^2. What is the depth of lake?

SOLUTION :

$$\text{Pressure } P = h\rho g = 4.9\times10^6 N/m^2$$

ρ density of water = 1000 kg/m^3; g=9.8m/s^2

$$\text{hence } h = \frac{P}{\rho g} = \frac{4.9\times10^6}{1000\times9.8} = 500m$$

2. Equal masses of water and a liquid of density 2 are mixed together, then the mixture has a density of

(a) 2/3 (b) 4/3 (c) 3/2 (d) 3

SOLUTION :

If two liquid of equal masses and different densities are mixed together then density of mixture

$$\rho = \frac{2\rho_1\rho_2}{\rho_1+\rho_2} = \frac{2\times1\times2}{1+2} = \frac{4}{3}$$

3. If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake

(a) 10 m (b) 20 m (c)60 m (d) 30 m

SOLUTION :

Pressure at bottom of the lake = $P_0 + h\rho g$

Pressure at half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

According to given condition

$$P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g)$$

$$\Rightarrow \frac{1}{3}P_0 = \frac{1}{6}h\rho g$$

$$\Rightarrow h = \frac{2P_0}{\rho g} = \frac{2\times10^5}{10^3\times10} = 20m.$$

4. In air, a metallic sphere with an internal cavity weighs 40g and in water it weighs 20g. The volume of cavity if the density of material with cavity be 8g/cm^3 is

1) zero 2) 15 cm^3 3) 5 cm^3 4) 20 cm^3

SOLUTION :

Weight of sphere in air = 40 g

Weight of sphere in water = 20 g

Loss in weight = (40-20)g=20g

Weight of water displaced = loss in weight = 20g

Volume of water displaced = 20 cm^3'

Actual volume of sphere=volume of water

displaced = 20 cm^3

Volume of material in sphere $= \frac{40}{8} = 5cm^3$

Volume of cavity = (20-5) = 15 cm^3

5. **A concrete sphere of radius *R* has a cavity of radius *r* which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be [AIIMS 1995]**

(a) 8 (b) 4 (c)3 (d) Zero

SOLUTION :

Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation

weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$$R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$$

$$R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2)$$

$$\frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1}$$

$$\frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$$

$$\frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1-\rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$$

$$\frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1-0.3}{2.4-1}\right)\times\frac{2.4}{0.3} = 4$$

6. **Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 36 *g* and its density is 9 g/cm^3. If the mass of the other is 48 *g*, its density in g/cm^3 is**

(a) $\frac{4}{3}$ **(b)** $\frac{3}{2}$ **(c)3 (d)5**

SOLUTION :

Apparent weight $= V(\rho - \sigma)g = \frac{m}{\rho}(\rho - \sigma)g$

where $m =$ mass of the body,

$\rho =$ density of the body

$\sigma =$ density of water

If two bodies are in equilibrium then their apparent weight must be equal.

$$\therefore \frac{m_1}{\rho_1}(\rho_1 - \sigma) = \frac{m_2}{\rho_2}(\rho_2 - \sigma)$$

$$\frac{36}{9}(9-1)=\frac{48}{\rho_2}(\rho_2-1)$$

By solving we get $\rho_2 = 3$.

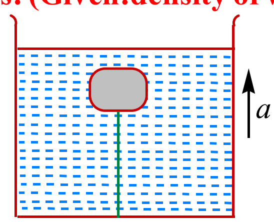
7. What is force on the base of a tank of base area 1.5m^2 when it is filled with water upto a height of 1m $\left(\rho_{water}=10^3 kg/m^3, P_0=10^5 Pa \text{ and } g=10m/s^2\right)$

SOLUTION :

Absolute pressure at the bottom of the container is $P = P_0 + h\rho g = 10^5 + 1\times10^3\times10 = 1.1\times10^5 Pa$

Then force due to water on the base is $F_{base} = PA = \left(1.1\times10^5\right)(1.5) = 1.65\times10^5 N$

8. A tank accelerates upwards with acceleration a=1 m/s^2 contains water. A block of mass 1kg and density 0.8 g/cm^3 is held stationary inside the tank with the help of the string as shown in figure. The tension in the string is: (Given:density of water=1000kg/m^3)



1) T=2.2 N 2) T=2.75 N 3) T=3N 4) T=2.4N

SOLUTION :

$$F\text{=upthrust force} = V\rho_w(g+a)$$

$$=\left(\frac{mass\ of\ block}{density\ of\ block}\right)\rho_w(g+a)$$

$$=\frac{1}{800}(1000)(11) = 13.75N$$

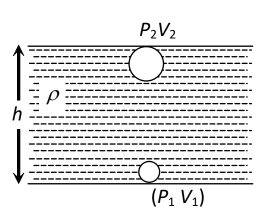
F-T-W=ma; 13.75-T-10=1 (1)

T=2.75N

9. An inverted bell lying at the bottom of a lake 47.6 *m* deep has 50 *cm*^{3} of air trapped in it. The bell is brought to the surface of the lake. The volume of the trapped air will be (atmospheric pressure = 70 *cm* of *Hg* and density of *Hg* = 13.6 *g/cm*^{3})

(a) 350 *cm*^{3} (b) 300 *cm*^{3} (c) 250 *cm*^{3} (d) 22 *cm*^{3}

SOLUTION :



According to Boyle's law, pressure and volume are inversely proportional to each other

$$i.e.\ P \propto \frac{1}{V}$$

$$\Rightarrow P_1V_1 = P_2V_2$$

$$\Rightarrow (P_0 + h\rho_w g)V_1 = P_0V_2$$

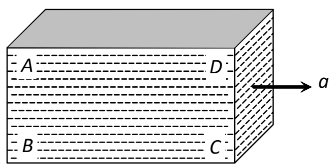
$$\Rightarrow V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$$

$$\Rightarrow V_2 = \left(1 + \frac{47.6 \times 10^2 \times 1 \times 1000}{70 \times 13.6 \times 1000}\right)V_1$$

$$\Rightarrow V_2 = (1+5)50\ cm^3 = 300\ cm^3.$$

[As $P_2 = P_0 = 70\ cm$ of $Hg = 70 \times 13.6 \times 1000$]

10. A closed rectangular tank is completely filled with water and is accelerated horizontally with an acceleration *a* towards right. Pressure is (i) maximum at, and (ii) minimum at



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(a) (i) B (ii) D **(b) (i) C (ii) D** **(c) (i) B (ii) C** **(d) (i) *B* (ii) *A***

SOLUTION :

- Due to acceleration towards right, there will be a pseudo force in a left direction.
- So the pressure will be more on rear side (Points *A* and *B*) in comparison with front side (Point *D* and *C*).
- Also due to height of liquid column pressure will be more at the bottom (points *B* and *C*) in comparison with top (point *A* and *D*).
- So overall maximum pressure will be at point *B* and minimum pressure will be at point *D*.

11. A body of density d_1 is counterpoised by Mg of weights of density d_2 in air of density *d*. Then the true mass of the body is

(a) M **(b)** $M\left(1 - \frac{d}{d_2}\right)$ **(c)** $M\left(1 - \frac{d}{d_1}\right)$ **(d)** $\frac{M(1 - d/d_2)}{(1 - d/d_1)}$

SOLUTION :

Let M_0 = mass of body in vacuum.

Apparent weight of the body in air = Apparent weight of standard weights in air

Actual weight – upthrust due to displaced air = Actual weight – upthrust due to displaced air

$$\Rightarrow M_0 g - \left(\frac{M_0}{d_1}\right)dg = Mg - \left(\frac{M}{d_2}\right)dg$$

$$\Rightarrow M_0 = \frac{M\left[1-\frac{d}{d_2}\right]}{\left[1-\frac{d}{d_1}\right]}$$

12. When equal volumes of two metals are mixed together, the specific gravity of alloy is 4. When equal masses of the same two metals are mixed together, the specific gravity of the alloy now becomes 3. Find specific gravity of each metal?

$$\left(\text{specific gravity}=\frac{\text{density of substance}}{\text{density of water}}\right)$$

SOLUTION :

In case of mixture, $\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2}$

When equal volumes are mixed, $4 = \frac{V\rho_1 + V\rho_2}{V+V} = \frac{\rho_1 + \rho_2}{2} \ldots(i)$

When equal masses are mixed, $3 = \frac{(m+m)}{\frac{m}{\rho_1} + \frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \ldots..(ii)$

Therefore, from (i) and (ii)

specific gravity of the metals are 2 and 6.

13. When a polar bear jumps on an iceberg, its weight 240 kg.wt is just sufficient to sink the iceberg. What is the mass of the iceberg? (specific gravity of ice is 0.9 and that of sea water is 1.02)

SOLUTION :

If M is the mass of iceberg in kg

volume $V = \frac{M}{0.9\times10^3} m^3 \left(\because density\ \rho = specific\ \ gravity \times 10^3 kg/m^3\right)$

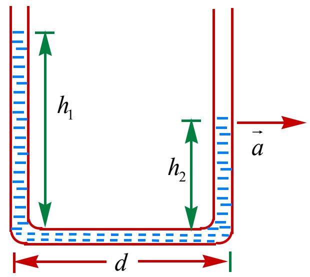
The weight of displaced sea water $= \left(V \times 1.02 \times 10^3 \times g\right)N$.

$$Mg + 240g = \left(\frac{M}{0.9\times10^3}\right)\times\left(1.02\times10^3\right)\times g$$

$$\therefore 240 = M\left(\frac{1.02}{0.9} - 1\right) = \frac{12}{90}M$$

$$\text{or } M = \frac{90}{12}\times 240 = 1800kg.$$

14. Fig. Shows a U-tube of uniform cross-sectional area A accelerated with acceleration 'a' as shown. If d is the separation between the limbs, then the difference in the levels of the liquid in the U-tube is



1) $\frac{ad}{g}$ 2) $\frac{g}{ad}$ 3) adg 4) $ad+g$

SOLUTION :

$$\tan\theta = \frac{h}{d}$$

$$h = \frac{ad}{g}$$

15. The height of a mercury barometer is 75 *cm* at sea level and 50 *cm* at the top of a hill. Ratio of density of mercury to that of air is 10^4. The height of the hill is

(a) 250 *m* **(b) 2.5 *km*** **(c) 1.25 *km*** **(d) 750 *m***

SOLUTION :

Difference of pressure between sea level and the top of hill

$$\Delta P = (h_1 - h_2)\times \rho_{Hg}\times g = (75-50)\times 10^{-2}\times \rho_{Hg}\times g \quad \ldots\text{(i)}$$

pressure difference due to h meter of air

$$\Delta P = h\times \rho_{air}\times g \quad \ldots\text{(ii)}$$

By equating (i) and (ii) we get

$$h\times \rho_{air}\times g = (75-50)\times 10^{-2}\times \rho_{Hg}\times g$$

$$\therefore h = 25\times 10^{-2}\left(\frac{\rho_{Hg}}{\rho_{air}}\right) = 25\times 10^{-2}\times 10^{4} = 2500\ m$$

$\therefore$ Height of the hill = 2.5 *km*.

16. Four-fifths of a cylindrical block of wood, floats in a liquid. Asuming the relative density of wood be 0.8, find the density of the liquid.

SOLUTION :

Let volume of wooden block = V

Volume of liquid displaced = $\frac{4}{5}V$

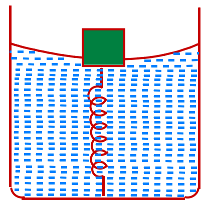
Weight of the block = $V\times 0.8\times 10^3 g$

As the block floats,

weight of the body = weight of the liquid displaced

$$V\times 0.8\times 10^3\times g = \frac{4V}{5}\times \rho_l\times g;\therefore \rho_l = 10^3 kg/m^3$$

17. **A cubical block of wood edge 3 cm floats in water. The lower surface fo the cube just touches the free end of a vertical spring fixed at the bottom of the pot. The maximum weight that can be put on the block without wetting it is (density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g=10 m/s^2)**



1) 1.35 N **2) 1.55 N** **3) 1.65 N** **4) 1.75 N**

SOLUTION :'

In equilibrium total weight $W = Kx + F_b$

$$(M+m)g = Kx + V\rho_w g$$

18. **A hemispherical bowl just floats without sinking in a liquid of density $1.2 \times 10^3 kg/m^3$. If outer diameter and the density of the bowl are 1 m and 2×10^4 kg/m^3 respectively, then the inner diameter of the bowl will be**

(a) 0.94 m **(b) 0.97 m** **(c)0.98 m** **(d) 0.99 m**

SOLUTION :

$$\text{Weight of the bowl} = mg = V\rho g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$$

where D = Outer diameter ,

d = Inner diameter

ρ = Density of bowl

$$\text{Weight of the liquid displaced by the bowl} = V\sigma g = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g$$

where σ is the density of the liquid.

$$\text{For the flotation } \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$$

$$\left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right] 2 \times 10^4$$

By solving we get $d = 0.98$ m.

19. **Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28g and its density is 5.6g/c.c. If the mass of the other body is 36g. Find its density d**

SOLUTION :

$$\text{Apt wt. of 1st body} = m_1 g - F_b = \left(28 - \frac{28}{5.6}\right)g$$

$$\text{Apt wt. of 2nd body} = m_2 g - F_b = \left(36 - \frac{36}{d} \times l\right)g$$

$$\left(28 - \frac{28}{5.6}\right)g = \left(36 - \frac{36}{d}\right)g; (28 - 5) = 36 - \frac{36}{d}$$

$$\Rightarrow \frac{36}{d} = 36 - 23 = 13; d = \frac{36}{13} \approx 2.8 g/cc$$

20. A certain block weighs 15N in air. But it weighs only 12N when completely immersed in water. When immersed completely in another liquid, it weighs 13N. Calculate the relative density of (i) the block and (ii) the liquid.

SOLUTION :

$$\text{i) Relative density of body} = \frac{W_{air}}{W_{air} - W_{water}}$$

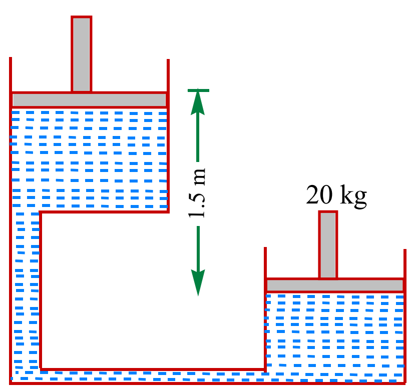
where $W_{air} = 15N$ (weight of the body in air)

and $W_{water} = 12N$ (weight of the body in water)

$$\therefore R.D_{block} = \frac{15N}{15N - 12N} = 5$$

$$\text{ii) } R.D_L = \frac{loss\ of\ weight\ in\ liquid}{loss\ of\ weight\ in\ water} = \frac{15-13}{15-12} = \frac{2}{3}$$

21.. Figure shows a hydraulic press with the larger piston of diameter 35 cm at a height of 1.5 m relative to the smaller piston of diameter 10 cm. The mass on the smaller piston is 20 kg. What isthe force exerted on the load by the larger piston. The density of oil in the process is 750 kg/m³, (Take g=9.8 m/s²).



1) 5 x 10³ N **2) 1.3 x 10³ N** **3) 3.7 x 10³N** **4) 4.8 x 10³N**

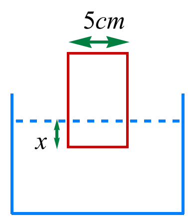
SOLUTION :

$$\frac{F_1}{A_1} - \frac{F_2}{A_2} = h\rho g$$

$$\frac{m_1 g}{\pi r_1^2} - \frac{m_2 g}{\pi r_2^2} = h\rho g$$

$$\rho = \frac{1}{\Pi h}\left[\frac{m_1}{r_1^2} - \frac{m_2}{r_2^2}\right]$$

22. A cubical block of iron of side 5cm is floating in mercury taken in a vessel. What is the height of the block above mercury level. $\left(\rho_{Hg} = 13.6g / cm^3, \rho_{Fe} = 7.2g / cm^3\right)$



SOLUTION :

From the law of flotation, $V_b \rho_b g = V_{in} \rho_L g$

$$\Rightarrow (5)^3 \times (7.2) = \left(5^2 x\right) \times (13.6); \therefore x = 2.65cm$$

Then, the height of the block above mercury level = 5cm - x = 2.35 cm

23. A solid sphere of radius 'R' has a concentric cavity of radius R/3 inside it. The sphere is found to just float in water with the highest point of it touching the water surface. Find the specific gravity of the material of the sphere.

SOLUTION :

$$\frac{V_{cavity}}{V_S} = \frac{V_S - V_{metal}}{V_S} = 1 - \frac{V_{metal}}{V_S} - (1)$$

(V_S = Total volume of the sphere)

According to Archimede's principle Weight of body = Weight of displaced liquid

$$mg = V_S d_w g \Rightarrow d_S V_{metal} g = V_S d_w g$$

(d_s = Density of solid material, d_w = Density of water) $\frac{V_{metal}}{V_{SA}} = \frac{d_w}{d_S} - (2);$

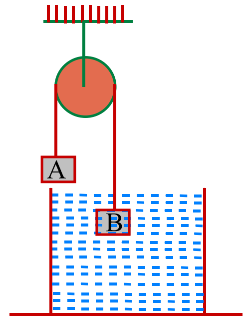
From equation 1 and 2

$$\therefore \frac{V_{cavity}}{V_S} = 1 - \frac{d_w}{d} = 1 - \frac{1}{\frac{d}{d_w}} = 1 - \frac{1}{S.G}$$

$$\therefore \frac{\frac{4}{3}\pi\left(\frac{R}{3}\right)^3}{\frac{4}{3}\pi R^3} = 1 - \frac{1}{S.G} \Rightarrow \frac{1}{27} = 1 - \frac{1}{S.G}$$

$$\Rightarrow \frac{1}{S.G} = 1 - \frac{1}{27} \Rightarrow S.G = \frac{27}{26}$$

24. **In the arrangement shown in the figure $\frac{m_A}{m_B}=\frac{2}{3}$ and the ratio of density of block B and the liquid is 2:1. The system is released from rest. Then**



1) block B will oscillate but not simple harmonically
2) block B will oscillate simple harmonically
3) the system will remain in equilibrium
4) None of the above

SOLUTION :

Let $m_A = 2m, m_B = 3m$

When block-B is inside the liquid $a_1 = \frac{m_A g - (m_B g - \text{upthrust on B})}{m_A + m_B} = \frac{g}{10}$

When block-B is outside the liquid $a_2 = \frac{m_B g - m_A g}{m_A + m_B} = \frac{3mg - 2mg}{5m} = \frac{g}{5}$

Since a_1 & a_2 are constants, motion is periodic, but not simple harmonic

25. **A ball of relative density 0.8 falls into water from a height of 2m. Find the depth to which the ball will sink (neglect viscous focus)**

SOLUTION :

Gravitational potential energy = Apparent weight of the body $\times$ Displacement of the body inside liquid

$$mgh = mg^1h^1$$

(where $g^1 = g\left(\frac{1}{d/\rho_w} - 1\right)$)

Relative density $= \frac{d}{\rho_w} = 0.8$

$$g^1 = g\left(\frac{1}{0.8} - 1\right) = g\left(\frac{0.2}{0.8}\right) = \frac{g}{4},$$

$$h^1 = \frac{g}{g^1}h = \frac{g}{g/4} \times 2 = 8m$$

26. **A log of wood of mass 120 *Kg* floats in water. The weight that can be put on the raft to make it just sink, should be (density of wood = 600 *Kg/m*³)**

(a) 80 *Kg* **(b) 50 *Kg*** **(c) 60 *Kg*** **(d) 30 *Kg***

SOLUTION :

Volume of log of wood $V = \frac{\text{mass}}{\text{density}} = \frac{120}{600} = 0.2\ m^3$

Let x weight that can be put on the log of wood.

So weight of the body = $(120 + x) \times 10\ N$

Weight of displaced liquid = $V\sigma g = 0.2 \times 10^3 \times 10\ N$

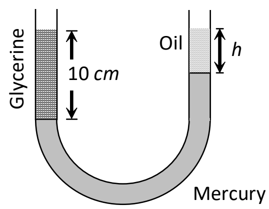
The body will just sink in liquid if the weight of the body will be equal to the weight of displaced liquid.

$$\therefore\ (120 + x) \times 10 = 0.2 \times 10^3 \times 10$$

$$\Rightarrow 120 + x = 200$$

$$\therefore\ x = 80\ kg$$

27. A vertical U-tube of uniform inner cross section contains mercury in both sides of its arms. A glycerin (density = 1.3 g/cm^3) column of length 10 cm is introduced into one of its arms. Oil of density 0.8 gm/cm^3 is poured into the other arm until the upper surfaces of the oil and glycerin are in the same horizontal level. Find the length of the oil column, Density of mercury = 13.6 g/cm^3



(a) 10.4 cm **(b) 8.2 cm** **(c) 7.2 cm** **(d) 9.6 cm**

SOLUTION :

At the condition of equilibrium

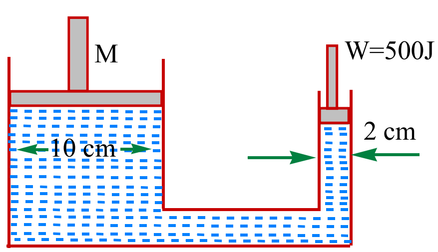
Pressure at point A = Pressure at point B

$$P_A = P_B$$

$$10 \times 1.3 \times g = h \times 0.8 \times g + (10 - h) \times 13.6 \times g$$

By solving we get $h = 9.7\ cm$

28. The hydraulic press shown in the figure is used to raise the mass M through a height of 5.0 mm by performing 500 J of work at the small piston. The diameter of the large piston is 10 cm while that of the smaller one is 2 cm. The mass M is



1) 10^4 kg **2) 10^3 kg** **3) 100 kg** **4) 10^5kg**

SOLUTION :

Since, M moves 5.0 mm and volume of liquid remains constant

$$\therefore \pi\times\left(5\times10^{-2}\right)^2\left(5\times10^{-3}\right)=\pi\times\left(1\times10^{-2}\right)^2 x$$

$$\therefore x=125\times10^{-3}m$$

Suppose force F does 500 J work on small piston.

Fx=500

$$F=\frac{500}{125\times10^{-3}}N=4\times10^3 N$$

Using Pascal's law, $Mg=\frac{F\left(5\times10^{-2}\right)^2}{\left(1\times10^{-2}\right)^2}$

$$Mg=100\times10^3=10^5 N\Rightarrow M=10^4 kg$$

29. An ice berg of density 900 *Kg/m*³ is floating in water of density 1000 *Kg/m*³. The percentage of volume of ice-cube outside the water is

(a) 20% **(b) 35%** **(c) 10%** **(d) 25%**

SOLUTION :

Let the total volume of ice-berg is V and its density is *r*.

If this ice-berg floats in water with volume V_{in} inside it

then $V_{in}\sigma g = V\rho g$

$$V_{in}=\left(\frac{\rho}{\sigma}\right)V \quad [\sigma = \text{density of water}]$$

$$\text{or } V_{out}=V-V_{in}=\left(\frac{\sigma-\rho}{\sigma}\right)V$$

$$\frac{V_{out}}{V}=\left(\frac{\sigma-\rho}{\sigma}\right)=\frac{1000-900}{1000}=\frac{1}{10}$$

\ $V_{out}=10\%$ of V

30. A rubber ball of mass m and density ρ is immersed in a liquid of density 3ρ to a depth h and released. To what height will the ball jump up above the surface due to buoyancy force of liquid on the ball? (neglect the resistance of water and air).

SOLUTION :

Volume of ball $V=\frac{m}{\rho}$

Acceleration of ball moving in upward direction inside the liquid

$$a=\frac{F_{net}}{m}=\frac{upthrust-weight}{m}=\frac{V_{total}\rho_l g-mg}{m}$$

$$a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g\,(upwards)$$

$\therefore$ velocity of ball while crossing the surface $v = \sqrt{2ah} = \sqrt{4gh}$

$\therefore$ The ball will jump to a height

$$H = \frac{v^2}{2g} = \frac{4gh}{2g} = 2h$$

31. A small ball of density ρ is immersed in a liquid of density $\sigma\,(\sigma > \rho)$ to a depth h and released. The height above the surface of water upto which the ball will jump is

1) $\left(\frac{\rho}{\sigma} - 1\right)h$ **2)** $\left(\frac{\rho}{\sigma} + 1\right)h$ **3)** $\left(\frac{\sigma}{\rho} - 1\right)h$ **4)** $\left(\frac{\sigma}{\rho} + 1\right)h$

SOLUTION :

$$H = \frac{\upsilon^2}{2g}$$

$$\upsilon = \sqrt{2ah} \textbf{ and } a = \left(\frac{\sigma - \rho}{\rho}\right)g$$

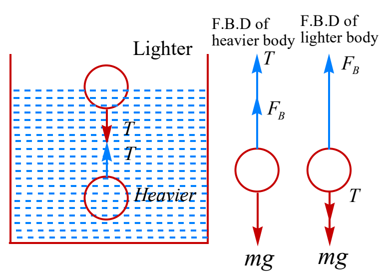
$$\upsilon = \sqrt{\frac{2(\sigma - \rho)gh}{\rho}}$$

$$\therefore H = \frac{\upsilon^2}{2g} = \left(\frac{\sigma - \rho}{\rho}\right)h$$

32. Two spheres of volume 250cc each but of relative densities 0.8 and 1.2 are connected by a string and the combination is immersed in a liquid. Find the tension in the string. ($g = 1 m/s^2$).

SOLUTION :

The tension on denser sphere is upwards and on lighter sphere is downards.



$$V_{b_1}\rho_1 g + T = V_{b_1}\rho_1 g$$

$$\therefore \left(250 \times 10^{-6} \times 800g\right) + T = 250 \times 10^{-6} \times \rho_{liquid}\, g - (i)$$

$$V_{b_2}\rho_2 g - T = V_{b_2}\rho_l g$$

$$\left(250\times10^{-6}\times1200g\right)-T=250\times10^{-6}\times\rho_{liquid}g-(ii)$$

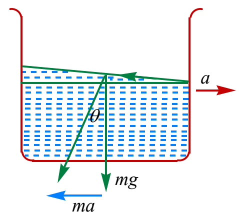
subtract Eqs. (ii) from (i), we get

$$2T=250\times10^{-6}\times400g\Rightarrow T=0.5N$$

33. A bowl of soap water is at rest on a table in the dining compartment of a train, if the acceleration of the train is g/4 in forward direction, the angle made by its surface with horizontal is

1) $\tan^{-1}\left(\frac{1}{2}\right)$ **2)** $\tan^{-1}\left(\frac{1}{4}\right)$ **3)** $\tan^{-1}\left(\frac{1}{5}\right)$ **4)** $\tan^{-1}\left(\frac{1}{3}\right)$

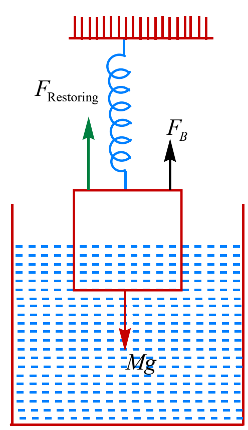
SOLUTION :



$$\therefore \tan\theta=\frac{ma}{mg}=\frac{a}{g}$$

34. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, when its lengths vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is (AIEEE-13)

SOLUTION :



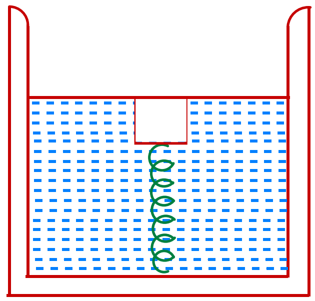
In equilibrium, upward force = downward force

$$F_{spring}+F_B=mg\Rightarrow kx_0+\left(\frac{L}{2}A\right)\sigma g=mg$$

$$\Rightarrow x_0=\frac{mg-\frac{\sigma LAg}{2}}{k}=\frac{mg}{k}\left(1-\frac{\sigma LA}{2m}\right)$$

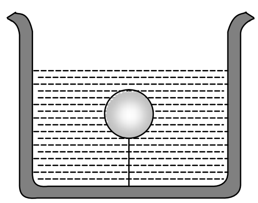
35. A block is fully submerged in a vessel filled with water by a spring attached to the bottom of the vessel. In equilibrium position spring is compressed. If the vessel now moves downwards with an acceleration a(<g). What happens to the length of the spring?

SOLUTION :



When the vessel moves downwards, with acceleration, block experiences net pseudo force upwards. Hence apparent weight of the block decreases and the block moves upwards. Hence length of the spring increases. Additional force on the block in upward direction=pseudo force on the block-decrease in buoyance force

36. A solid sphere of density $\eta\,(>1)$ times lighter than water is suspended in a water tank by a string tied to its base as shown in fig. If the mass of the sphere is m then the tension in the string is given by



(a) $\left(\frac{\eta-1}{\eta}\right)mg$ **(b)** ηmg **(c)** $\frac{mg}{\eta-1}$ **(d)** $(\eta-1)mg$

SOLUTION :

Tension in spring $T =$ upthrust $-$ weight of sphere

$$= V\sigma g - V\rho g$$

$$= V\eta\rho g - V\rho g \ \ (\text{As } \sigma = \eta\rho)$$

$$= (\eta-1)V\rho g$$

$$= (\eta-1)mg.$$

37. A hollow sphere of volume V is floating on water surface with *half* immersed in it. What should be the minimum volume of water poured inside the sphere so that the sphere now sinks into the water

(a) $V/2$ **(b)** $V/3$ **(c)** $V/4$ **(d)** V

SOLUTION :

When body (sphere) is half immersed, then

upthrust = weight of sphere

$$\frac{V}{2}\times\rho_{\text{liq}}\times g = V\times\rho\times g \quad \therefore \quad \rho=\frac{\rho_{\text{liq}}}{2}$$

When body (sphere) is fully immersed then,
Upthrust = wt. of sphere + wt. of water poured in sphere

$$V\times\rho_{\text{liq}}\times g = V\times\rho\times g + V'\times\rho_{\text{liq}}\times g$$

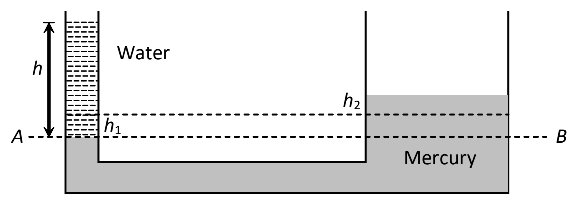
$$V\times\rho_{\text{liq}} = \frac{V\times\rho_{\text{liq}}}{2} + V'\times\rho_{\text{liq}}$$

$$V'=\frac{V}{2}$$

38. **Two communicating vessels contain mercury. The diameter of one vessel is n times larger than the diameter of the other. A column of water of height h is poured into the left vessel. The mercury level will rise in the right-hand vessel (s = relative density of mercury and ρ = density of water) by**

(a) $\frac{n^2h}{(n+1)^2s}$ **(b)** $\frac{h}{(n^2+1)s}$ **(c)** $\frac{h}{(n+1)^2s}$ **(d)** $\frac{h}{n^2s}$

SOLUTION :



If the level in narrow tube goes down by h_1 then in wider tube goes up to h_2,

$$\text{Now, } \pi r^2 h_1 = \pi(nr)^2 h_2$$

$$\Rightarrow h_1 = n^2 h_2$$

Now, pressure at point A = pressure at point B

$$h\rho g = (h_1+h_2)\rho' g$$

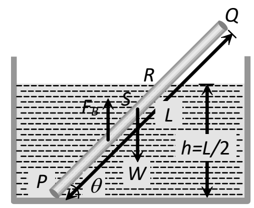
$$\Rightarrow h = (n^2h_2+h_2)sg \quad \left(\text{As } s=\frac{\rho'}{\rho}\right)$$

$$\Rightarrow h_2 = \frac{h}{(n^2+1)s}$$

39. **A uniform rod of density ρ is placed in a wide tank containing a liquid of density $\rho_0(\rho_0>\rho)$. The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position the rod makes an angle θ with the horizontal**

(a) $\sin\theta=\frac{1}{2}\sqrt{\rho_0/\rho}$ **(b)** $\sin\theta=\frac{1}{2}\cdot\frac{\rho_0}{\rho}$ **(c)** $\sin\theta=\sqrt{\rho/\rho_0}$ **(d)** $\sin\theta=\rho_0/\rho$

SOLUTION :



Let $L = PQ =$ length of rod $\therefore\ SP = SQ = \frac{L}{2}$

Weight of rod, $W = Al\rho g$, acting At point S

And force of buoyancy, $F_B = Al\rho_0 g$, $[l = PR]$ which acts at mid-point of PR.

For rotational equilibrium,

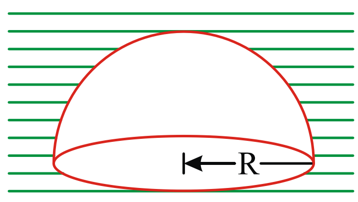
$$Al\rho_0 g \times \frac{l}{2}\cos\theta = AL\rho g \times \frac{L}{2}\cos\theta$$

$$\Rightarrow \frac{l^2}{L^2} = \frac{\rho}{\rho_0} \Rightarrow \frac{l}{L} = \sqrt{\frac{\rho}{\rho_0}}$$

From figure, $\sin\theta = \frac{h}{l} = \frac{L}{2l} = \frac{1}{2}\sqrt{\frac{\rho_0}{\rho}}$

Comprehension :

A solid hemisphere of radius R is made to just sink in a liquid of density ρ . Find the



40. vertical thrust on the curved surface

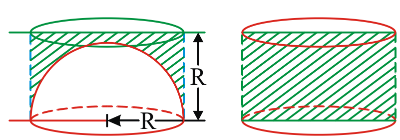
A) $\frac{\pi R^3 \rho g}{3}$ B) $\frac{\pi R^3 \rho g}{2}$ C) 0 D) $\pi R^3 \rho g$

SOLUTION :

Vertical thrust of the liquid is equal to weight of the liquid column above the curve (spherical) surface.

$$F_V = v\rho g .$$

Volume of the shaded portion $= \left(\pi R^3\right) - \frac{2}{3}\pi R^3 = \frac{\pi R^3}{3}$



(I) (II)

Substituting V in the equation $F_v = V\rho g$, we have

$$F_V = \frac{\pi R^3}{3}\rho g\left(down\right)$$

41. **Vertical thrust on the flat surface**

A) $\frac{\pi R^3 \rho g}{3}$ B) $\frac{\pi R^3 \rho g}{2}$ C) 0 D) $\pi R^3 \rho g$

SOLUTION :

The upward thrust on the base of the hemisphere is $F_v' = V\rho g$,

where, V = volume of the liquid column above the base $= (\pi R^2)R = \pi R^3$

Then, we have $F_v' = \pi R^3 \rho g (up)$

42. **Side thrust on the hemisphere**

A) $\frac{\pi R^3 \rho g}{3}$ B) $\frac{\pi R^3 \rho g}{2}$ C) 0 D) $\pi R^3 \rho g$

SOLUTION :

Let the horizontal and vertical thrusts on the tortoise be F_h and F_v, respectively.

we know that $F_h = \rho g y_C A_v$

where $y_C = R$

$A_v = \pi R^2$.

This gives $F_h = \rho g \pi R^3$ towards right

43. **A block of ice floats on a liquid of density 1.2in a beaker then level of liquid when ice completely melt**

(a) Remains same **(b) Rises** **(c) Lowers** **(d)(a), (b) or (c)**

SOLUTION :

The volume of liquid displaced by floating ice $V_D = \frac{M}{\sigma_L}$

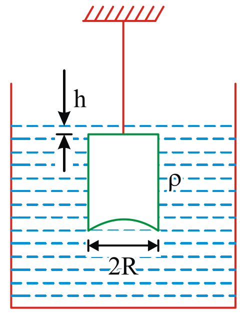
Volume of water formed by melting ice, $V_F = \frac{M}{\sigma_W}$

If $\sigma_1 > \sigma_W$, then, $\frac{M}{\sigma_L} < \frac{M}{\sigma_W}$

i.e. $V_D < V_F$

i.e. volume of liquid displaced by floating ice will be lesser than water formed and so the level if liquid will rise.

44. **A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is**

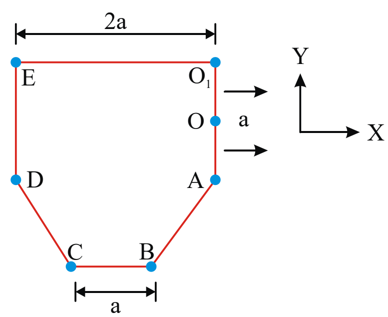


A) $\rho g\left(V+\pi R^2 h\right)$ B) Mg C) $Mg-V\rho g$ D) $Mg+\pi R^2 h\rho g$

SOLUTION :

$$\text{Upthrust}=F_2-F_1;$$
$$F_2=F_1+\text{Upthrust}$$
$$=\rho g h\left(\pi R^2\right)+V\rho g;$$
$$=\rho g\left(V+\pi R^2 h\right)$$

45. The top view of closed compartment containing liquid is moving with an acceleration along x - axis as shown. Find the incorrect statement.



A) The pressure at A and O is same B) The pressure at O and O_1 is same

C) The pressure at B and C is same D) The pressure at D and E is same

SOLUTION :

$$\frac{dP}{dx}=-\rho x \text{ acceleration}$$

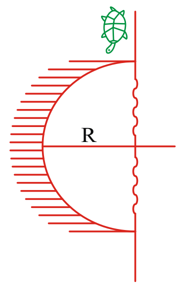
♦ The pressure decreases in positive x - direction.

♦ The pressure is lower in front side.

♦ The pressure at B and C can not be same.

Comprehension :

A tortoise is just sinking in water of density ρ. The tortoise is assumed to be a hemisphere of radius R.



46. Find Vertical thrust

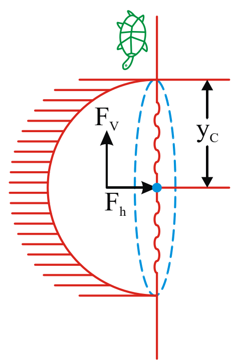
A) $\rho g\pi R^3$ B) $\frac{1}{3}\rho g\pi R^3$ C) $\frac{2}{3}\rho g\pi R^3$ D) 0

SOLUTION :

Similarly using the formula $F_v = \rho Vg$

where V = volume of the tortoise $= \frac{2}{3}\pi R^3$

we have $F_v = \frac{2}{3}\rho g\pi R^3$ (up)



47. Fint the total hydrostatic force

A) $\rho g\pi R^3$ B) $\sqrt{\frac{13}{3}}\rho g\pi R^3$ C) $\frac{2}{3}\rho g\pi R^3$ D) $\sqrt{\frac{16}{3}}\rho g\pi R^3$

SOLUTION :

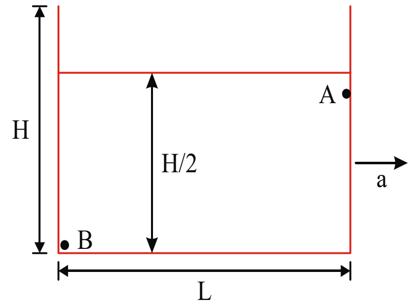
Hence the net hydrostatic force on the tortoise is $F = \sqrt{F_h^2 + F_v^2}$

$$= \sqrt{\left(\rho g\pi R^3\right)^2 + \left(\frac{2}{3}\rho g\pi R^3\right)^2}$$

$$= \sqrt{\frac{13}{3}}\rho g\pi R^3$$

48. A vessel of height H and length L contains a liquid of density ρ upto height H/2. The vessel starts accelerating horizontally with acceleration 'a' towards right. If A is the point at the surface

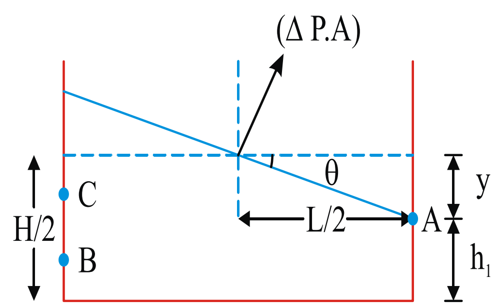
of the liquid at right end while the vessel is accelerating and B is the point at bottom of the vessel on the other end, the difference of pressures at B and A will be



A) $\frac{\rho}{2}(gH+aL)$ B) $\frac{\rho}{2}(gH-aL)$ C) $2\rho(gH-aL)$ D) $\frac{3\rho}{2}(gH+aL)$

SOLUTION :

Let force $(\Delta P)A$ acts on the surface of the liquid while vessel is accelerating.



The surface can not sustain tangential force.

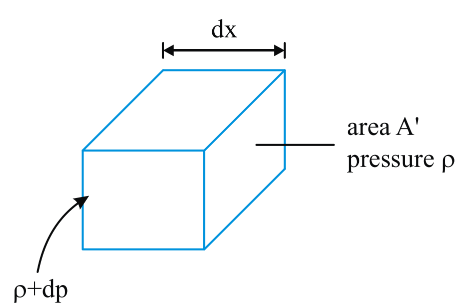
$(\Delta P.A)\cos\theta = mg$;

$$(\Delta P.A)\sin\theta = ma$$

$$\tan\theta = a/g \text{ and } y = \frac{L}{2}\tan\theta = \frac{aL}{2g}$$

$$h_1 = \frac{H}{2} - \frac{aL}{2g}$$

Considering a fluid element at distance x from left side of the vessel, then



$$(p+dp)A' - pA' = (A'dx\rho)a$$

$$\int_A^C dp = \int_A^C \rho a dx \quad ; \quad p_C - p_A = a\rho L$$

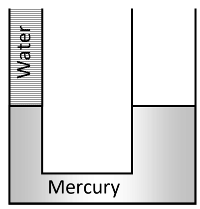
If points A and C are at same level

$$p_B - p_C = \rho g h_1 = \rho g\left[\frac{H}{2} - \frac{aL}{2g}\right]$$

$$p_B - (\rho g L + p_A) = \rho g\left[\frac{H}{2} - \frac{aL}{2g}\right]$$

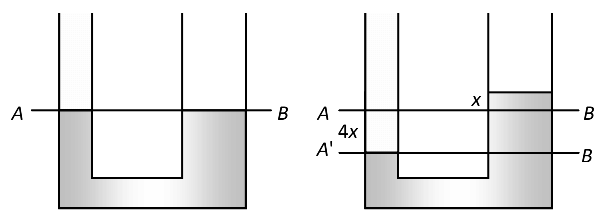
$$p_B - p_A = \frac{\rho}{2}(gH + aL)$$

49. **A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 *cm* from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water**



(a) 1.2 *cm* **(b) 2.35 *cm*** **(c) 0.56 *cm*** **(d) 0.8 *cm***

SOLUTION :



If the rise of level in the right limb be x *cm*.

the fall of level of mercury in left limb be $4x$ *cm*

(because the area of cross section of right limb is 4 times as that of left limb.}

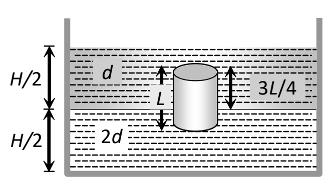
$\therefore$ Level of water in left limb is $(36 + 4x)$ *cm*.

Now equating pressure at interface of *Hg* and water (at *A'B'*)

$$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$$

By solving we get $x = 0.56$ *cm*.

50. **A homogeneous solid cylinder of length $L\,(L < H/2)$. Cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by**



(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$ (c) d (d) $\frac{d}{5}$

SOLUTION :

Weight of cylinder = upthrust due to both liquids

$$V \times D \times g = \left(\frac{A}{5} \times \frac{3}{4}L\right) \times d \times g + \left(\frac{A}{5} \times \frac{L}{4}\right) \times 2d \times g$$

$$\left(\frac{A}{5} \times L\right) \times D \times g = \frac{A \times L \times d \times g}{4}$$

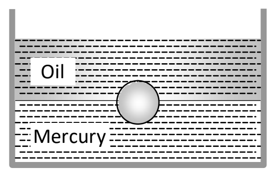
$$\frac{D}{5} = \frac{d}{4}$$

$$\therefore D = \frac{5}{4}d$$

51. **A vessel contains oil (density = 0.8 *gm/cm*³) over mercury (density = 13.6 *gm/cm*³). A homogeneous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material of the sphere in *gm/cm*³ is**

(a) 3.3 **(b) 6.4** **(c) 7.2** **(d) 12.8**

SOLUTION :



As the sphere floats in the liquid. Therefore its weight will be equal to the upthrust force on it

$$\text{Weight of sphere} = \frac{4}{3}\pi R^3 \rho g \quad \ldots\text{(i)}$$

$$\text{Upthrust due to oil and mercury} = \frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g \quad \ldots\text{(ii)}$$

Equating (i) and (ii)

$$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8g + \frac{2}{3}\pi R^3 \times 13.6g$$

$$\Rightarrow 2\rho = 0.8 + 13.6 = 14.4 \Rightarrow \rho = 7.2$$

Fluid Dynamics:

It is the study of behaviour of fluids in motion. Fluid dynamics is caused by the difference in pressures of a fluid between two points.

The Rate Of Flow Of A Liquid:

The rate of flow of a liquid means the volume of a liquid that flows across any cross section in unit time and is given by

$$Q = \frac{Volume}{time} = \frac{V}{t} = A\left(\frac{l}{t}\right) = Av\left(v = vel.\text{of the fluid}\right)$$

SI unit : $\left[\frac{m^3}{\sec}\right]$;

D.F: L^3T^{-1}

Mass of the liquid that flows per unit time $\frac{M}{t} = \frac{Volume}{time} \times density = A\left(\frac{l}{t}\right)\rho = Av\rho$

Where

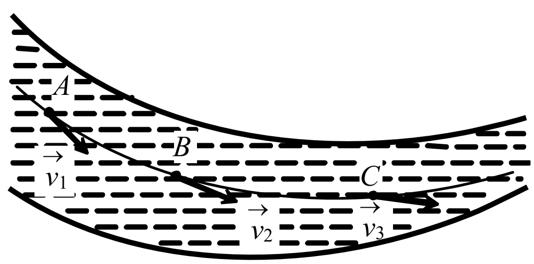
A is the area of cross section of the tube,

V is the velocity of the liquid

ρ is the density of the liquid.

Stream line flow :

A streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point.



The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

Path *ABC* is streamline as shown in the figure and v_1 , v_2 and v_3 are the velocities of the liquid particles at *A*, *B* and *C* point respectively.

Characteristics of fluid flow:

- ♦ A stream line may be a straight line or a curve.
- ♦ The tangent drawn at any point of curved stream line gives the direction of velocity of the particle at that point.
- ♦ Two stream lines never intersect, if they intersect, at the point of intersection the fluid may have two directions of velocity which is impossible.
- ♦ An imaginary tube consisting of a number of stream lines is called tube of flow.
- ♦ There is no radial flow of liquid.

Types of flow of liquid:

There are two types of liquid flow.

1) Stream line flow
2) Turbulent flow.

Stream Line Flow:

If the velocity of all the fluid particles crossing a point remains constant both in magnitude and direction then the flow of the fluid is known as **Stream Line Flow**

A **Turbulent Flow:**

If the velocity of the different fluid particles crossing a point doest not remain constant in magnitude and direction then the flow of the fluid is known as **Turbulent Flow**

Eddies and whirl pools are formed in **turbulent flow.**

Critical Velocity:

Critical Velocity is the velocity beyond which stream line flow is gradually changed to turbulent flow.

$$\text{Critical Velocity } V_c = \frac{R\eta}{D\rho}$$

$$R = \frac{D\rho V_c}{\eta}$$

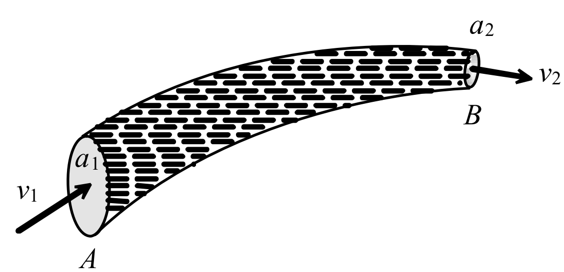
where η =coefficient of viscosity,
R=Reynolds number,
D=diameter of the tube,
ρ =density of the liquid

♦ Reynold's number depending upon the diameter of the pipe, the density and coefficient of viscosity of the liquid.

♦ If $0<R<1000$, the fluid flow is said to be stream line.
♦ If the value of $R > 2000$ then the liquid flow becomes turbulent.
♦ If $1000 < R < 2000$ then the flow in unsteady

Equation of Continuity:

When an incompressible fluid flows steadily through a tube of non-uniform cross section, the rate of mass of fluid entering the tube is equal to rate of mass of the fluid leaving the tube.



$$\frac{m}{l} = \text{constant}$$

$$\frac{m_1}{t_1} = \frac{m_2}{t_2}$$

$$A_1\left(\frac{l_1}{t_1}\right)\rho_1 = A_2\left(\frac{l_2}{t_2}\right)\rho_2$$

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

$$\text{as} \quad \rho_1 = \rho_2$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

$$V \propto \frac{1}{A}$$

$$v \propto \frac{1}{r^2}$$

Equation of continuity represents the law of conservation of mass in case of moving fluids.

Types of Energies in fluid flow:

A fluid in motion prossesses three types of energies namely

(i) $KE = \frac{1}{2}mv^2$ kinetic energy,

(ii) PE=mghpotential energy

(iii) Pressure energy = P x V

Bernoulli's theorem:

♦ Bernoulli's Theorem states that the sum of the pressure energy, kinetic energy and potential energy at any point in steady flow calculated per unit mass or per unit volume is constant.

(or)

♦ Bernoulli's Theorem can also be stated as follows: "In a stream line flow of fluid, the sum of gravitational head, pressure head and velocity head at any point in the path of the flow is constant"

♦ $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$ (per unit volume)

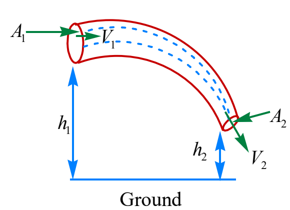
♦ $\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{Constant}$ (per unit mass)

♦ $\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{Constant}$ (per unit weight)

Here ⇉ $\frac{P}{\rho g}$ is called pressure head,

⇉ $\frac{v^2}{2g}$ is called **Velocity head**

⇉ h is called **gravitational head.**



Here ♦ $\frac{1}{2}\rho v^2$ is called dynamic pressure.

♦ $(P + \rho gh)$ is called static pressure

♦ Bernoulli's theorem represents Law of conservation of energy.

♦ When the flow is horizontal, h is same and hence sum of pressure head and velocity head is constant.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

where P_1 and P_2 are pressures at two points,

v_1, v_2 are velocities at two points

ρ is the density of the liquid.

♦ For horizontal flow of liquid, maximum pressure corresponds to minimum velocity and vice versa

$$\left(P + \frac{1}{2}\rho v^2 = \text{constant}\right)$$

Torricelli's theorem:

♦ The velocity of efflux of a liquid through an orifice is equal to that of the velocity acquired by a freely falling body from a height which is equal to that of the liquid level from the orifice.

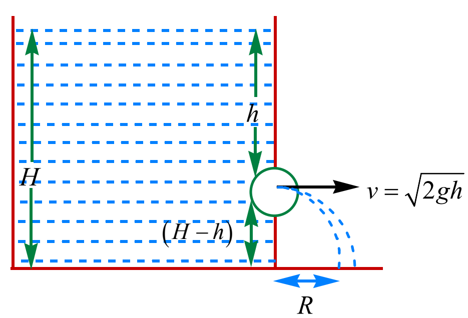
$$\boxed{v = \sqrt{2gh}}$$

♦ Time taken by the efflux liquid to reach the ground is given by

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Where H = height of the liquid in the container

h = the distance between the free surface of the liquid and centre of the hole



♦ Horizontal range of the liquid is given by R = $V \times t$

$$R = \sqrt{2gh.\frac{2(H-h)}{g}} \qquad \text{R} = 2\sqrt{h(H-h)}$$

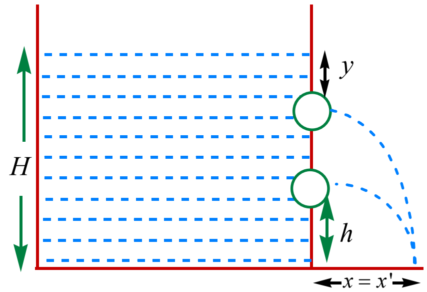
♦ The volume of the liquid coming out of the orifice per second is

$$Q = Av = A\sqrt{2gh} = \pi r^2\sqrt{2gh}$$

♦ Horizontal range is maximum when the orifice is at the middle of liquid level and bottom.

i.e., if $h = \frac{H}{2}$ then $R_{Max} = H = 2h$

♦ If the level of free surface, in a container is at height H from the base. There are two holes at height 'h' above the bottom and other at depth 'y' below the free surface, then



$$x = 2\sqrt{h(H-h)} \text{ and } x^1 = 2\sqrt{y(H-y)}$$

now if $x = x^1$

i.e. $h(H-h) = y(H-y)$

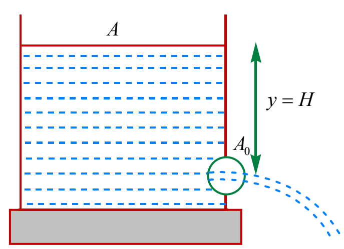
from above equation $y = \frac{1}{2}[H \pm (H-2h)]$

i.e. $y = h$ or $(H-h)$

so the range is same for liquid coming out of holes at same distance below the top and above the bottom.

♦ A tank having an area of cross-section A is filled with water upto height 'H' and 'A_0' is the area of cross-section of hole at the bottom of tank. If A_0 is the area of orifice at a depth 'y' below the free surface and A is that of container, then the volume of liquid coming out of the orifice per second will be

$$(dV/dt) = vA_0 = A_0\sqrt{2gy} \quad \left[as\ v = \sqrt{2gy}\right]$$



Due to this, the level of liquid in the container will decrease and so, if the level of liquid in the container above the hole changes from y to y-dy in time t to t+dt then -dV=Ady.
Substituting the value of dV in the above equation,

$$-A\frac{dy}{dt} = A_0\sqrt{2gy}; \int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$$

So the time taken for the level to fall from H to H^1

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H^1} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H^1}\right]$$

♦ Time after which level of water falls from H to $\frac{H}{2}$ is

$$t_1 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{\frac{H}{2}}\right)$$

♦ Time after which water level falls from $\frac{H}{2}$ to '0' is

$$t_2 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{2}} - 0\right]$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2}-1}{1}$$

♦ **In a cylindrical vessel containing liquid of density $'\rho'$, there are two holes in the side wall at heights h_1 and h_2 respectively such that the range of efflux at the bottom of vessel is same. If v_1, v_2 are the velocities of efflux and t_1, t_2 are the times taken by the efflux liquid to reach the floor respectively from holes at heights h_1, h_2 then**

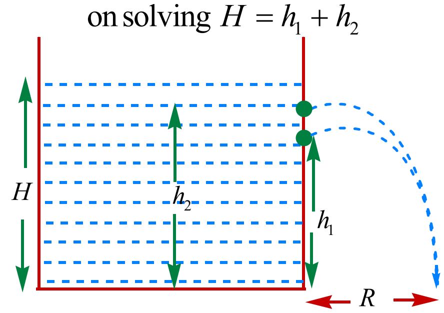
$$v_1 = \sqrt{2g(H-h_1)};\quad t_1 = \sqrt{\frac{2h_1}{g}}$$

$$v_2 = \sqrt{2g(H-h_2)};\quad t_2 = \sqrt{\frac{2h_2}{g}}$$

since $x_1 = x_2 \Rightarrow v_1 t_1 = v_2 t_2$

$$\sqrt{2g(H-h_1)}\sqrt{\frac{2h_1}{g}} = \sqrt{2g(H-h_2)}\sqrt{\frac{2h_2}{g}}$$

on solving $H = h_1 + h_2$

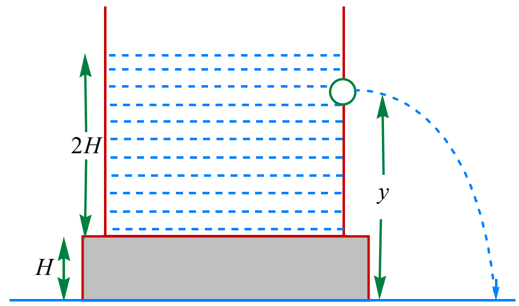


Note :

In the above case, the height of a hole, in terms of h_1 and h_2 for which the range of efflux would be maximum will be

$$h = \frac{H}{2} = \left(\frac{h_1 + h_2}{2}\right)$$

♦ **A tank is filled upto a height, 2H with a liquid and is placed on a platform of height H from the ground. The density 'y' from the ground where a small hole is made in the tank, to get the maximum horizontal range R**



Horizontal range will be maximum when the hole in the tank lies at the middle of total height of water surface from the ground.

$$\text{For maximum range } h = \frac{H}{2} = \left(\frac{h_1 + h_2}{2}\right)$$

$$\text{i.e } y = \left(\frac{2H + H}{2}\right) = \frac{3H}{2}$$

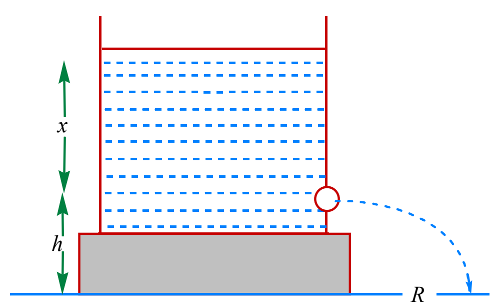
♦ **A water tank is kept on the top of a table of height h. If a small hole is punched in the side of the tank at its base it is found that the resultant stream of water strikes the ground at a horizontal distance R from the tank then the depth of water in the tank** $x = \frac{R^2}{4h}$

$$R = ut = \sqrt{2gx}\sqrt{\frac{2h}{g}}$$

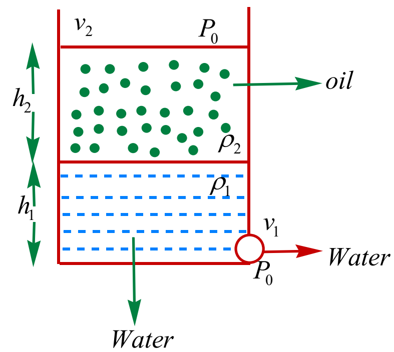
$$R = 2\sqrt{xh}$$

$$R^2 = 4xh$$

$$x = \frac{R^2}{4h}$$



♦ A tank is filled with water of density ρ_1 and oil of density ρ_2. The height of water column is h_1 and that of the oil is h_2. The velocity of efflux through a hole at the bottom of the tank is obtained as follows According to Bernoulli's theorem



$$P_1 + \frac{1}{2}\rho_1 v_1^2 + 0 = P_2 + \frac{1}{2}\rho_2 v_2^2 + (\rho_1 g h_1 + \rho_2 g h_2)$$

$$\text{But } P_1 = P_2 = P_0$$

$$v_2 = 0 (\because v_2 << v_1)$$

$$\frac{1}{2}\rho_1 v_1^2 = \rho_1 g h_1 + \rho_2 g h_2$$

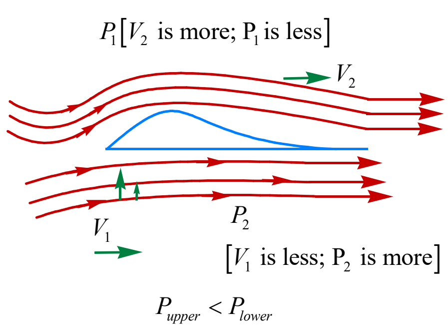
$$v_1 = \sqrt{\frac{2g(\rho_1 h_1 + \rho_2 h_2)}{\rho_1}}$$

Applications of Bernoulli's theorem:

Dynamic lift:

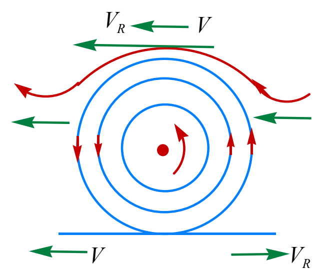
The upward lift experienced by a body in **motion in a fluid** is called **dynamic lift.**

- ♦ The dynamic lift experienced by a body when it is in motion in air is called aerodynamic lift.
- ♦ Aeroplanes get the dynamic lift because of the shape of their wings.
- ♦ The upper surface of the wing is more curved than the lower surface. Air flows with greater speed above the wing and so pressure above the wing will be less than that at the bottom. This difference in pressures produces the aerodynamic lift and allows aeroplane to fly.



Dynamic lift $= (P_2 - P_1)A = \frac{1}{2}\rho.(V_1^2 - V_2^2) \times A$

Spinning ball :



- ♦ The plane of motion of a spinning ball gets changed due to an effect called **Magnus effect.**
- ♦ Resultant velocity **on the top** $= V + V_R = V + R\omega$
- ♦ Resultant velocity **on the bottom** $= V - V_R = V - R\omega$
- ♦ Pressure at the top of the ball will be less than that of at the bottom.

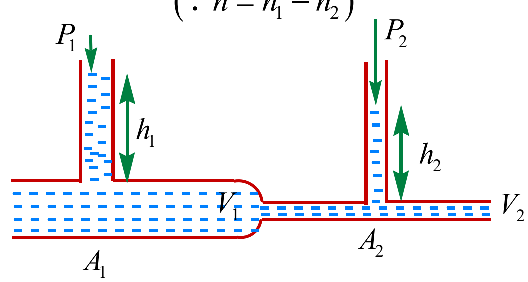
NOTE :Atomiser, paintgun and Bunsen burner, work onthe principle of Bernoulli's Theorem.

Venturimeter:

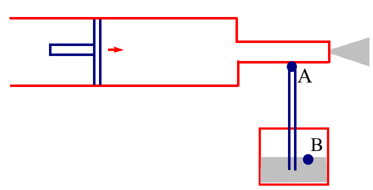
- ♦ It is an ideal device of measuring rate of flow of a liquid through pipe.It is also known as **venturi tube** or **flow meter**.
- ♦ The decrease in cross-sectional area of the flow passage causes increase in pressure
- ♦ The measurement of the pressure difference enables the determination of the rate of flow through the pipe.

$$Q = A_1A_2\sqrt{\frac{2(P_1 - P_2)}{(A_1^2 - A_2^2)\rho}} = A_1A_2\sqrt{\frac{2(h_1 - h_2)\rho g}{(A_1^2 - A_2^2)\rho}} = A_1A_2\sqrt{\frac{2hg}{(A_1^2 - A_2^2)}}$$

$$(\because h = h_1 - h_2)$$

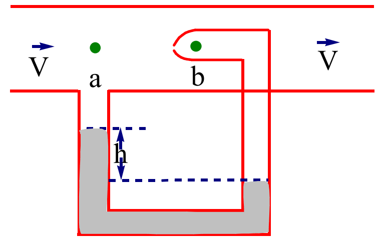


Aspirator pump:



In aspirator pump when air is pressed inside the tube, it comes out rapidly so that pressure at A reduces whereas pressure at B is more. For this pressure difference liquid rises till the barrel and sprayed with the expelled air.

Pitot tube:



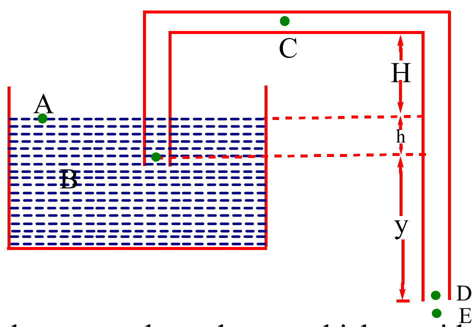
It is used to measure the speed of fluid flowing through a pipe. Here, the pressure in the left arm of the manometer whose opening is parallel to the direction of flow, is equal to the pressure in the fluid stream while pressure in right arm can be computed by using Bernoulli's theorem. It is obvious that velocity of fluid is zero at point b.

$$P_a + \frac{\rho v_2^2}{2} = P_0$$, where ρ is the density of fluid flowing through tube.

$$P_b - P_a = \rho_0 gh$$ where is the density of liquid in the manometer tube.

$$\text{So, } \frac{\rho v^2}{2} = \rho_0 gh \Rightarrow v = \sqrt{\frac{2\rho_0 gh}{\rho}}$$

Siphon Tube:



Siphon tube is used to empty the tanks etc, which are either very heavy or can't be lifted. In short, we can say siphon is used to remove liquid from containers without using pumps etc.
The siphon tune is of uniform cross-section and to operate it successfully, it must be intially filled with liquid. For the situation show in figure.

$$v_A = 0 \text{ and } v_B = v_C = v_D = v$$

Because siphon tube is of uniform cross section so from equation of continuity, flow speed is same at all points within the siphon tube.

Applying Bernoulli's equation at A, B, C, D and E (consider water level in the tank as reference for gravitational potential energy)

$$P_0 + 0 + 0 = P_B + \frac{\rho v^2}{2} - \rho gh$$

$$= P_C + \frac{\rho v^2}{2} + \rho gH = P_D + \frac{\rho v^2}{2} + \rho g(h+y)$$

$$= P_0 + \frac{\rho v_E^2}{2} - \rho g(h+y)$$

From above equation, we have $P_A = P_E = P_0$

$$V_E = \sqrt{2g(h+y)}$$

so for liquid to come out $\quad h + y > 0$

::PROBLEMS::

1. What are the dimensions of Reynolds number?

SOLUTION :

$$R = \frac{\rho v_0 D}{\eta}$$

ρ is density of the fluid,

v_0 is the critical velocity,

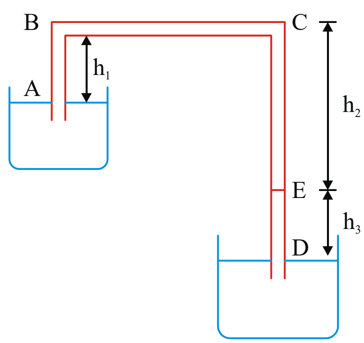
D is diameter through which the fluid is flowing

η is the coefficient of viscosity of the fluid)

$$R = \frac{[ML^{-3}][LT^{-1}][L]}{[ML^{-1}T^{-1}]}\left[\frac{ML^{-1}t^{-1}}{ML^{-1}T^{-1}}\right] = [M^0L^0T^0]$$

$\therefore$ R is dimensionless.

2. In the syphon as shown, which of the option is not correct, if $h_2 > h_1$ and $h_3 < h_1$?



A) $p_E < p_D$ B) $p_E > p_C$ C) $p_B > p_C$ D) $p_B < p_E$

SOLUTION :

If p_0 = atmospheric pressure $P_A = P_0 = P_B + \rho h_1 g$;

$$P_D = P_0 = P_E + \rho h_3 g$$

Since, $h_3 < h_1, So, P_E > P_B$

3. What should be the average velocity of water in a tube of diameter 2cm so that the flow is (i) laminar (ii) turbulent? The viscosity of water is 0.001 Pa-s. (for water pipe R<2000 steam line flow; R>3000 turbulent flow)

SOLUTION :

Here, d=2cm=0.02m;

$$\eta = 0.001Pa-s; \rho = 10^3 kg/m^3$$

i) For stream line flow; Reynolds no. R=2000, V=?

Now, $v = \frac{R\eta}{\rho d} = \frac{2000\times 0.001}{10^3\times 0.02} = 0.1ms^{-1}$

ii) For turbulent flow, Reynolds number,

$$R = 3000; v = ?\, v = \frac{3000\times 0.001}{10^3\times 0.02} = 0.15ms^{-1}$$

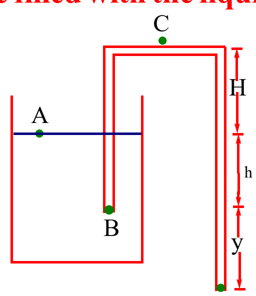
4. **A cylindrical tank has a hole of 2 cm^2 at its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 100 cm^3/s, then find the maximum height upto which water can rise in the tank (Take g=10 ms^{-2})**

1) 2.5×10^{-2} m 2) 1.25×10^{-2}m 3) 5.5×10^{-2} m 4) 3.5×10^{-2}m

SOLUTION :

$$Q_{in} = Q_{out} \qquad Q = a\sqrt{2gh}$$

5. **A siphon tube is used to remove liquid from a container as shown in fig. In order to operated the siphon tube, it must initially be filled with the liquid.**



(i) Determine the speed of the liquid through the siphon

(ii) Determine the pressure at the point C.

SOLUTION :

(i) Applying Bernoulli's equation at points A and D,

we get $P_A + \frac{1}{2}rv_A^2 + rgy_A = P_D + \frac{1}{2}rv_D^2 + rgy_D$

Assuming datum for potential energy at the free surface,

we have $y_A = 0; y_D = -(h+y);$

$$P_A = P_D = P_{atm} v_A^2 \approx 0; v_D = v$$

$$P_{atm} + 0 + 0 = P_{atm} + \frac{1}{2}rv^2 + rg[-(h+y)] \quad \text{or} \quad v = \sqrt{2g(h+y)}$$

(ii) Applying Bernoulli's equation at A and C,

we get $P_A + \frac{1}{2}rv_A^2 + rgy_A = P_C + \frac{1}{2}rv_C^2 + rgy_C$

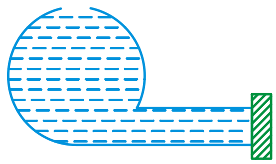
Here, $Y_C = +H; v_C = v$ (according to the continuity equation)

$$P_{atm} + 0 + 0 = P_C + \frac{1}{2}rv^2 + rgh$$

$$\text{or,}\ P_{atm} = P_C + rg(h+y) + rgh$$

$$\text{or,}\ P_C = P_{atm} - rg(h+H+y)$$

6. **In a vessel as shown, the opening has a cross - sectional area A. F_1 is the net force applied on the plate by liquid and air, which is kept to close the opening. The plate is now displaced a short distance away from the opening in which case liquid strikes the plate inelastically with a force F_2. Find F_2 / F_1**



SOLUTION :

$$F_1 = \Delta P(A) = \rho g h A$$

$$F_2 = \rho A V^2 = \rho A(2gh);$$

$$\frac{F_2}{F_1} = 2$$

7. A vessel has water to a height of 50 cm. It has three horizontal tubes of same diameter each of length 15cm coming out at heights 10 cm, 15 cm, 20 cm. The length of a single tube of same diameter as that of the three tubes which can replace them when placed horizontally at the bottom of the vessel is:

1) 45 cm 2) 5 cm 3) 8 cm 4) 16 cm

SOLUTION :

h is total height of water, h_1, h_2, h_3 are the heights at which tubes are fitted

$$Q_1 = \frac{\pi(h-h_1)dgr^4}{8\eta l}; Q_2 = \frac{\pi(h-h_2)dgr^4}{8\eta l}$$

$$Q_3 = \frac{\pi(h-h_3)dgr^4}{8\eta l} \text{ and } Q = Q_1 + Q_2 + Q_3$$

with $Q = \frac{\pi h d g r^4}{8\eta l}; \frac{h}{l} = \frac{h_1}{l_1} + \frac{h_2}{l_2} + \frac{h_3}{l_3}$

8. Air is streaming past a horizontal aeroplane wing such that the speed of air is 120 m/s over the upper surface and 90 m/s at the lower surface, with respect to the plane. If the density of air is 1.3 kg/m^3, find the difference in pressure between the top and bottom of the wing. If the wing is 10m long and has an average width of 2m, calculate the gross lift of the wing.

SOLUTION :

According to Bernoulli's equation, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

i.e., $P_1 = P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \ = \frac{1}{2}(1.3)\left(120^2 - 90^0\right) = 4.1\times10^3 N/m^2$

The net uplift due to the difference in pressure is

$$F_{up} = \Delta P \times A = \left(4.1\times10^{-}\right)(10\times2) = 8.2\times10^4 N$$

9. A sphere falls from rest into water from a height of 2m. The relative density of the sphere is 0.80. Find the depth to which ball will sink (in m)

SOLUTION :

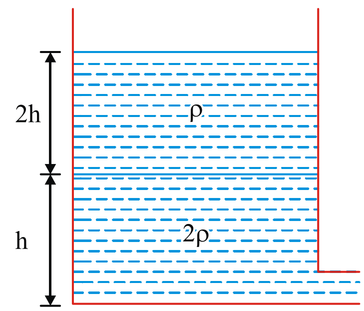
$$a = \frac{upthrust - weight}{mass} = \frac{V(1)g - V(0.8)g}{V(0.8)} = g/4$$

$$V_2^2 = V_1^2 - 2(a)h_0$$

$$0 = 2gh - 2ah_0$$

$$h_0 = \frac{gh}{a} = \frac{(g)(2)}{(g/4)} = 8m$$

10. The velocity of the liquid coming out of a small hole of a vessel containing two different liquids of densities 2ρ and ρ as shown in fig is



A) $\sqrt{gh}$ B) $2\sqrt{gh}$ C) $2\sqrt{2gh}$ D) $\sqrt{6gh}$

SOLUTION :

$$P_0 + \rho g(2h) + (2\rho)gh = P_0 + \frac{1}{2} \times 2\rho \times v^2$$

liquid of density 2ρ is coming out

11. A cylindrical tank of height H is open at the top end and it has a radius R. water is filled in it up to a height of h. The time taken to empty the tank through a hole of radius r at its bottom is

A) $\sqrt{\frac{2h}{g}}\frac{R^2}{r^2}$ B) $\sqrt{\frac{2H}{g}}\frac{R^2}{r^2}$ C) $\sqrt{hH}$ D) $\sqrt{\frac{2H}{g}}\frac{R}{r}$

SOLUTION :

$$-\pi R^2 \frac{dh}{dt} = \pi r^2 \sqrt{2gh} \quad ; \quad \frac{dh}{dt} = -\frac{r^2}{R^2}\sqrt{2gh}$$

$$dt = -\frac{R^2}{r^2}\frac{1}{\sqrt{2gh}}\,dh$$

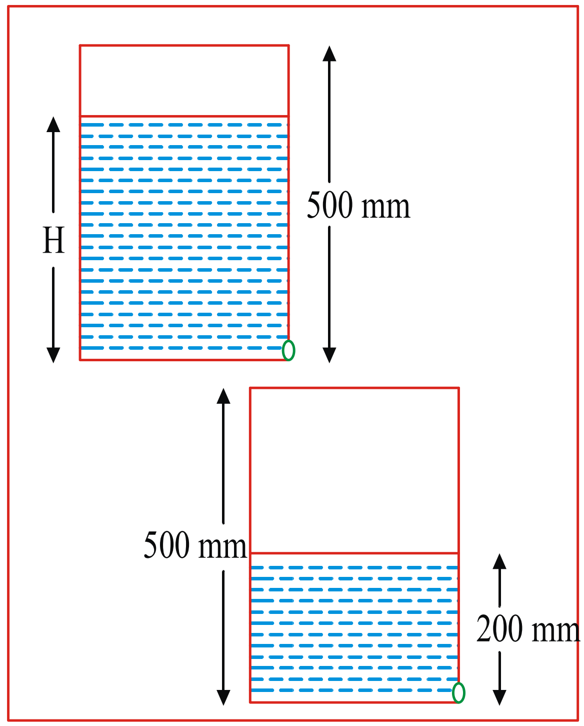
$$\therefore \int_0^t dt = \frac{R^2}{r^2\sqrt{2g}}\int_h^0 h^{-1/2}dh$$

$$t = -\frac{R^2}{r^2\sqrt{2g}} \times 2\left[\sqrt{h}\right]_h^0$$

$$t = \sqrt{\frac{2h}{g}}\left[\frac{R^2}{r^2}\right]$$

12. A cylindrical vessel of height 500 mm has an orifice at its bottom. The orifice is initially closed and water is filled in it upto a height H. Now the top is sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. (Take atmospheric pressure $=1.0\times10^5 N/m^2$. denisty of water $1000kg/m^3$ and $g=10m/s^2$. Neglect any effect of surface tension.)

SOLUTION :



$$P = P_0 - \rho gh = 98\times10^3 N/m^2 ;$$

$$P_0V_0 = PV$$

$$10^5\left[A(500-H)\right] = 98\times10^3\left[A(500-200)\right]$$

$$H = 206mm ;$$

level fall $= 206 - 200 = 6mm$

13. A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10cm², the velocity of water is 1 m/s and the pressure is 2000 Pa. What is the pressure at another point where the cross-sectional area is 5cm².

SOLUTION :

According to equation of continuity $A_1v_1 = A_2v_2; v_2 = \frac{10}{5}\times1 = 2m/s$

Now, according to Bernouli's equation, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ ($\because$ horizontal pipe)

$$2000 + \frac{1}{2}10^3(1)^2 = P_2 + \frac{1}{2}(10^3)(2^2); P_2 = 500 Pa$$

14. A sphere of solid material of specific gravity 8 has a concentric spherical cavity and just sinks in water. The ratio of radius of cavity to that of outer radius of the sphere must be

A) $\frac{7^{1/3}}{2}$ B) $\frac{5^{1/3}}{2}$ C) $\frac{9^{1/3}}{2}$ D) $\frac{3^{1/3}}{2}$

SOLUTION :

Let ρ be the density of the material, ρ_0 be the density of water.

When the spehere has just started sinking,

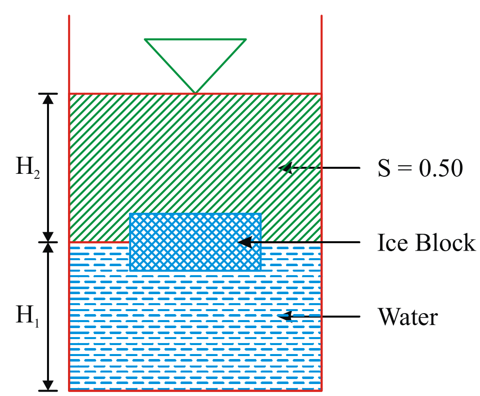
the weight of the spehre= weight of water displaced (approx)

$$\Rightarrow \frac{4}{3}\pi(R^3 - r^3)\rho g = \frac{4}{3}\pi R^3 \rho_0 g$$

$$\Rightarrow (R^3 - r^3)\rho = R^3 \rho_0 \Rightarrow \frac{(R^3 - r^3)}{R^3} = \frac{\rho_0}{\rho}$$

$$\Rightarrow \frac{r}{R} = \frac{(7)^{1/3}}{2}$$

15. A block of ice (specific gravity S_{i-} = 0.90) is floating in a container having two immiscible liquids (one of specific gravity S_1 = 0.50 and other is water) as shown in the figure. (H_1, H_2 are heights of water, other liquid columns respectively.) Now the ice block melts completely, then



1. H_2 will decrease
2. H_1 will increase
3. H_1+H_2 will remains unchanged
4. H_1+H_2 decreases

SOLUTION :

Density of ice is in between that of water and liquid.

16. **Two unequal blocks of densities σ_1 and σ_2 placed over each other are immersed in fluid of density σ. The block of density σ_1 is fully submerged and the block of density σ_2 is partly submerged so that ratio of their masses is 1/2 and $\sigma/\sigma_1 = 2$ and $\sigma/\sigma_2 = 0.5$. Find the degree of submergence of the upper block of density σ_2.**

A) 50% submerged B) 25% submerged C) 75% submerged D) Fully submerged

SOLUTION :

$$\left(m_1 + \frac{m_2}{K}\right)g = \left[\frac{m_1}{\sigma_1} + \frac{m_2}{K\sigma_2}\right]\sigma g$$

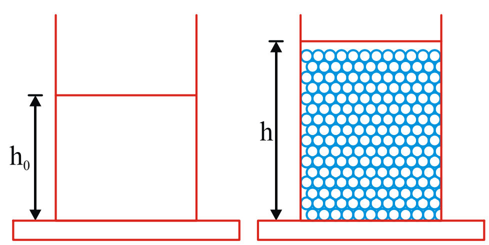
dividing by $m_2 g$,

$$\frac{m_1}{m_2} + \frac{1}{K} = \frac{m_1\sigma}{m_2\sigma_1} + \frac{\sigma}{K\sigma_2}$$

$$= \frac{m_1}{m_2}.2 + \frac{1}{K}.0.5;$$

$$\frac{m_1}{m_2} = \frac{0.5}{K}; K = 1.0, \text{ fully submerged}$$

17. **In a cylindrical container water is filled up to a height of $h_0 = 1.0m$. Now a large number of small iron balls are gently dropped one by one into the container till the upper layer of the balls touches the water surface. If average density of the contents is $\rho = 4070 kg/m^3$, density of iron is $\rho_i = 7140 kg/m^3$ and density of water is $\rho_0 = 1000 kg/m^3$, find the height h of the water level (in S.I units) in the container with the iron balls.**



SOLUTION :

$$\rho = \frac{total\ mass}{total\ volume} = \frac{m_1 + m_2}{Ah}$$

$$\rho_0 = \frac{m_1}{Ah_0}; \rho_i = \frac{m_2}{A(h - h_0)}$$

$$\Rightarrow h = \frac{\rho_0 h_0 + (h + h_0)\rho_i}{\rho} = \frac{(\rho_i - \rho_0)}{(\rho_i - \rho)} = 2m$$

18. **A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 (\rho_2 < \rho_1)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = kv^2 (k > 0)$. The terminal speed of the ball is   (AIEEE-2008)**

1) $\sqrt{\frac{Vg\rho_1}{k}}$ 2) $\frac{Vg(\rho_1-\rho_2)}{k}$ 3) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$ 4) $\frac{Vg\rho_1}{k}$

SOLUTION :

$$mg = F_b + F_v$$

19. Calculate rate of flow of glycerin of density 1.25x10³kg/m³ through the conical section of a horizontal pipe, if the radii of its ends are 0.1m and 0.04m and pressure drop across its length is 10 N/m². "

SOLUTION :

According to equation of continuity $\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{(0.1)^2}{(0.04)^2} = \frac{25}{4}$

and, according to Bernoulli's equation for a horizontal tube, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$v_2^2 - v_1^2 = 2\frac{(P_1 - P_2)}{\rho} = 16\times10^{-3} m^2/s^2$$

$$\text{but } v_2 = \frac{25}{4}v_1 = 6.25v_1$$

$$\therefore \left[(6.25)^2 - 1^2\right]v_1^2 = 16\times10^{-3} m^2/s^2$$

$$\text{or } v_1 \approx 0.0205 m/s.$$

the rate of volume flow $= A_1 v_1 = \pi(0.1)^2 \times (0.02) = 6.28\times10^{-4} m^3/s$

And the rate of mass flow is $\frac{dm}{dt} = \rho A v$. $= (1.25\times10^3)\times(6.28\times10^{-4}) = 0.785 kg/s$

20. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

A) more than half-filled if ρ_c is less than 0.5 B) more than half-filled if ρ_c is more than 1.0

C) half-filled if ρ_c is more than 0.5 D) less than half-filled if ρ_c is less than 0.5

SOLUTION :

$$\frac{V_m + V_a + V_w}{2}\rho_w g = V_m \rho_c \rho_w g + V_w \rho_w g$$

$$V_w = V_m(1 - 2\rho_c) + V_a$$

$$\text{If } \rho_c > \frac{1}{2} \Rightarrow V_w < V_a$$

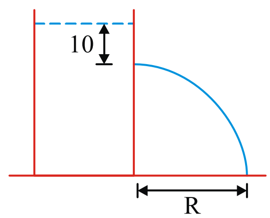
$$\text{if } \rho_c < \frac{1}{2} \Rightarrow V_w < V_a$$

where V_w = volume occupied by water in the shell

V_a = volume occupied by air in the shell

V_m = volume of the material in the shell

21. The range of water flowing out of a small hole made at a depth 10m below water surface in a large tank is R. Find the extra force applied on water surface so that range becomes 2R (in atm, an approximate value)



SOLUTION :

$$V = (2gh)^{1/2}$$

Range will be twice, if efflux velocity becomes twice

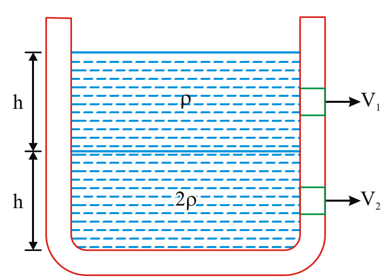
or h becomes four times or 40m.

Extra pressure = 30m of water head

But 1 atm = $0.7 \times 13.6m$ of water = 10.336m of water,

30m of water=3.0 atm

22. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth $\frac{h}{2}$ and $\frac{3h}{2}$ from the surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then $\frac{v_1}{v_2}$ is



A) $\frac{1}{2\sqrt{2}}$ B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{\sqrt{2}}$

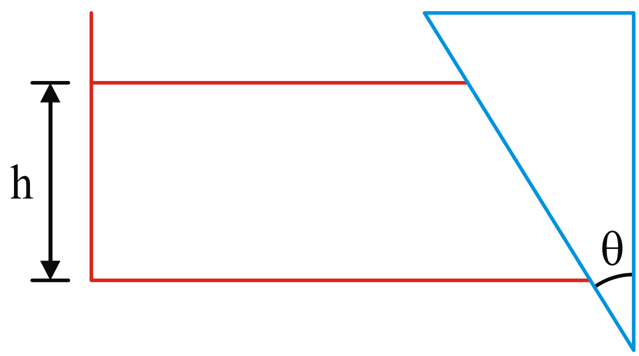
SOLUTION :

$$P_0 + \rho g \frac{h}{2} = P_0 + \frac{1}{2}\rho v_1^2$$

$$P_0 + \rho g \frac{h}{2} + (2\rho) g \frac{h}{2} = P_0 + \frac{1}{2}(2\rho) v_2^2$$

$$\frac{v_1}{v_2} = \frac{\sqrt{gh}}{\sqrt{2gh}} = \frac{1}{\sqrt{2}}$$

23. A slit is cut at the bottom, along the right bottom edge of a rectangular tank. The slit is closed by a wooden wedge of mass m and apex angle θ as shown in figure. The vertical plane surface of the wedge is in contact with the right vertical wall of the container. Coefficient of static friction between these two surfaces is μ. To what maximum height, can water be filled in the tank without leakage from the slit? The width of tank is b and density of water is ρ.



A) $\sqrt{\frac{2m}{\rho b(\tan\theta - \mu)}}$ B) $\sqrt{\frac{4m}{\rho b(\tan\theta - \mu)}}$ C) $\sqrt{\frac{2m}{\rho b(\sin\theta - \mu\cos\theta)}}$ D) $\sqrt{\frac{2m\cos\theta}{\rho b(\tan\theta - \mu\cos\theta)}}$

SOLUTION :

$$mg + \mu N = F_{pressure}$$

$$\Rightarrow mg + \mu\left[\rho g \frac{h}{2}(bh)\right] = \rho g\left[\frac{1}{2} \times h\tan\theta \times h\right]$$

$$\Rightarrow mg + \frac{\mu\rho g h^2 b}{2} = \frac{\rho g h^2 \tan\theta}{2}$$

$$\Rightarrow m = \frac{bh^2\rho}{2}[\tan\theta - \mu]$$

$$\therefore h = \sqrt{\frac{2m}{(\tan\theta - \mu)b\rho}}$$

24. A vessel with a symmetrical hole in its bottom is fastened on a cart. The mass of the vessel and the cart is 1.5kg. With what force F(in $\times 10^2$ N) should the cart be pulled so that the maximum amount of water remains in the vessel. The dimensions of the vessel are as shown in figure. Given that b=50cm, c=10cm, area of base A=40 cm^2, L=20cm, g =10 m/s^2.

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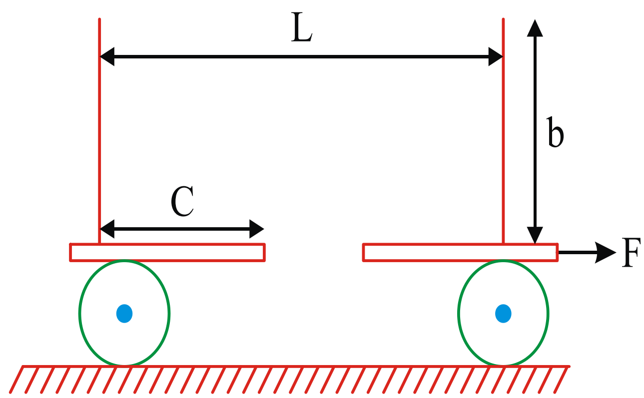
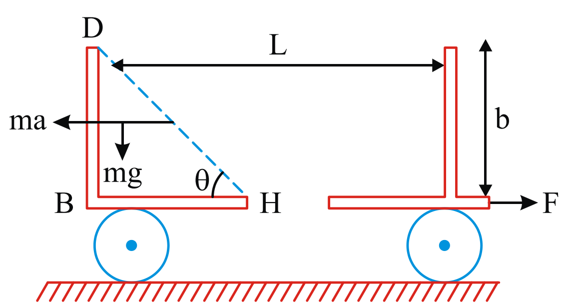


SOLUTION :

As the cart is drawn by a force F, the water in the vessel takes up a slant position rising upward at the back of the vessel.

To prevent water from flowing out of the hole H, the acceleration of the vessel should have such a value that it occupies a face area DBH and a width of vessel given by A/L.



$$\text{Area of } \Delta DBH = \frac{1}{2}bc$$

$$\text{volume of the liquid retained} = \frac{1}{2}bc \times \frac{A}{2}$$

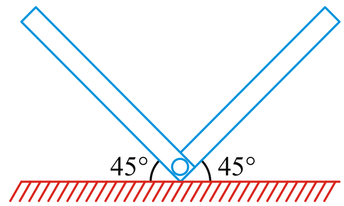
$$\text{Mass of the cart and water} = M + \frac{bcA\rho}{2L}$$

$$\tan\theta = \frac{ma}{mg};$$

$$a = g\tan\theta = g \times \frac{b}{c}$$

$$\text{Required force is } \left(M + \frac{bcA\rho}{2L}\right)\frac{gb}{c} = [1.5 + 0.5] \times 50 = 2.0 \times 50 = 100N$$

25. A thin V-shaped glass tube is fixed in the vertical plane as shown. Initially, the left part of the tube contains a column of water of length d = $\sqrt{2}$ m. A valve at the bottom of the tube prevents the water from moving to right part. At some time, the valve is quickly opened. Neglecting friction, find the time (in seconds) it takes for the water to move completely into the right part of the tube. (Take g= π^2 m/s^2)



Fixed horizontal surface

SOLUTION :

The water of density ρ will execute SHM inside the fixed v-shape glass tube with time period

$$T = 2\pi\sqrt{\frac{m}{2\rho Ag\cos 45°}}$$

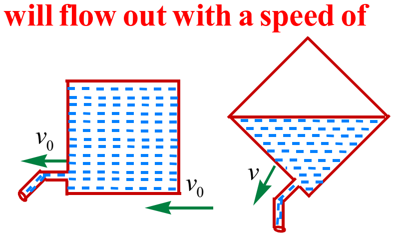
where m = mass of water

A = cross-sectional area of tube.

$$T = 2\pi\sqrt{\frac{d}{\sqrt{2}g}}$$

Hence the required time is half the time period of oscillation.

26. A square box of water has a small hole located in one of the bottom corners. When the box is full and sitting on a level surface, complete opening of the hole results in a flow of water with a speed v_0, as shown. When the box is still half empty, it is tiled by 45^0 so that the hole is ta the lowest point. Now the water will flow out with a speed of

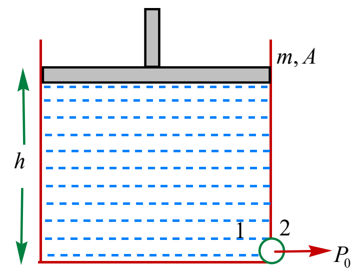


1) v_0 2) $\frac{v_0}{2}$ 3) $\frac{v_0}{\sqrt{2}}$ 4) $\frac{v_0}{\sqrt[4]{2}}$

SOLUTION :

$$V_0 = \sqrt{2gh} \Rightarrow V' = \sqrt{2g\frac{h}{\sqrt{2}}} = \frac{V_0}{\sqrt[4]{2}}$$

27. A cylindrical vessel contains a liquid of density ρ upto a height h. The liquid is closed by a piston of mass m and area of cross-section A. There is a small hole at the bottom of the vessel. Find the speed v with which the liquid comes out of the hole.



SOLUTION :

Applying Bernoulli's theorem at 1 and 2;

difference in pressure energy between 1 and 2 = difference in kinetic energy between 1 and 2

$$\left(P_0 + \rho gh + \frac{mg}{A}\right) + 0 = (p_0) + \frac{1}{2}\rho v^2$$

$$p_0 + \rho gh + \frac{mg}{A} - p_0 = \frac{1}{2}\rho v^2$$

$$\rho gh = \frac{1}{2}\rho v^2 \Rightarrow v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$$

28. Two balls of same size but different masses m_1 and m_2 $(m_2 > m_1)$ are attached to the two ends of a thin light thread and dropped from a certain height. It is known that the viscous drag of air depends on the size and velocities of the balls. Other than the gravitational pull from the earth and the viscous drag, the buoyant force from air also act on the balls. The buoyant force on a ball equals to the weight of air displaced by the ball. After sufficiently long time from the instant the balls were dropped both of them acquire uniform velocity known as terminal velocity. When the balls have acquired terminal velocity, the tension in the thread is

A) Zero   B) $(m_2 - m_1)g$ C) $0.5(m_2 + m_1)g$ * D) $0.5(m_2 - m_1)g$

SOLUTION :

For vertical translational equilibrium, use freebody diagram for single and two body system seperately.

29. A gas flows with a velocity v along a pipe of cross sectional area 's' and bent an angle of 90^0 at point A. What force does the gas exert on the pipe at 'A'. If it's density is ρ ?

1) $\frac{\sqrt{2}SV}{\rho}$ 2) $\sqrt{2}SV^2\rho$ 3) $\sqrt{3}\frac{SV^2\rho}{2}$ 4) $\sqrt{3}SV^2\rho$

SOLUTION :

Take x-axis along the flow and y-axis perpendicular to it

$$V_{initial} = V\vec{i}, V_{final} = V\vec{j}; \left|\Delta\vec{V}\right| = \sqrt{V^2 + V^2} = \sqrt{2}V$$

$$F = m\frac{\Delta v}{t} = m\frac{\Delta v}{t} = \rho \times S\frac{l}{t} \times \sqrt{2}V = \sqrt{2}SV^2\rho$$

30. A pump draws water from a reservoir and sends it through a horizontal pipe with speed v. Find the relation between power of the pump and velocity of liquid.

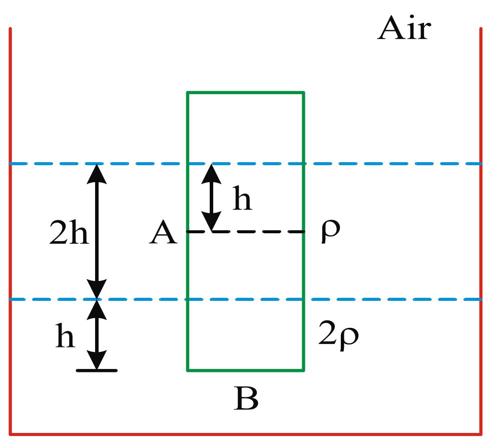
SOLUTION :

From work - energy theorem

$$P = \frac{KE\left(imparted\ to\ water\right)}{time} = \frac{KE}{volume\ of\ water} \times \frac{volume\ of\ water}{time}$$

$$= \left(\frac{1}{2}\rho v^2\right)\left(Av\right) \text{ or } P \alpha v^3$$

31. A cylinder stands vertical in two immiscible liquids of densities ρ and 2ρ as shown. Find the difference in pressure at point A and B:



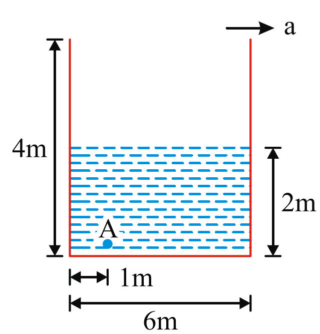
A) $2\rho gh$ B) $3\rho gh$ C) $4\rho gh$ D) None

SOLUTION :

Difference in pressure at A and B = $4\rho gh - \rho gh = 3\rho gh$

Comprehension :

An open rectangular tank of dimensions 6m x 5m x 4m contains water upto a height of 2m. The vessel is accelerated horizontally with an acceleration of $a \ \ m/s^2$ as shown. Take $\rho_{water} = 10^3 kg/m^3$, $g = 10m/s^2$, atmospheric pressure = $10^5 N/m^2$. Base on above information answer the following questions :

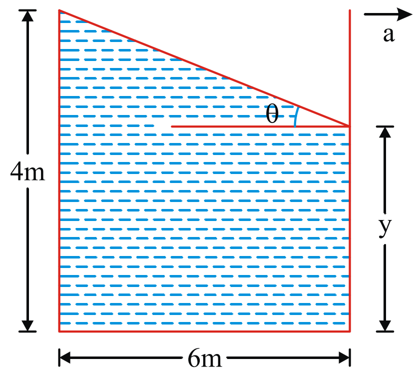


32. Determine the maximum value of a so that no water comes out from tank.

A) g　　B) $\frac{2g}{3}$　　C) $\frac{g}{3}$　　D) $2g$

SOLUTION :

The angle made by free surface of liquid with horizontal is $\theta = \tan^{-1}\left(\frac{a}{g}\right)$



For no water to spill out, limiting case is shown in the fig.

As volume of water inside the beaker remains the same,

$$\text{so} \quad 2\times6\times5 = \frac{1}{2}(4+y)\times6\times5$$

$$\Rightarrow y = 0$$

$$\tan\theta = \frac{4-y}{6} = \frac{2}{3} = \frac{a}{g}$$

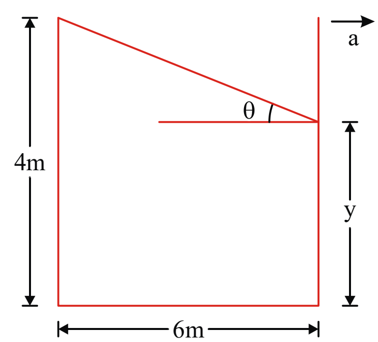
$$\Rightarrow \quad a = \frac{2g}{3}$$

33. Determine the height to which the water should be filled in the tank so that when $a = 5m/s^2$, no water comes out from the tank

A) 2m　　B) 3m　　C) 2.5m　　D) 3.5m

SOLUTION :

For no water to come out, the situation is as shown in fig



$$\tan\theta = \frac{4-y}{6} = \frac{a}{g}$$

$$\Rightarrow \frac{4-y}{6}=\frac{5}{10}$$

$$\Rightarrow y=1m$$

So, volume of water inside the tank is, $\frac{1}{2}(4+y)\times6\times5$

Let h be the initial height, then

$$5\times6\times h=\frac{1}{2}\times(4+1)\times6\times5;$$

$$h=5/2=2.5m$$

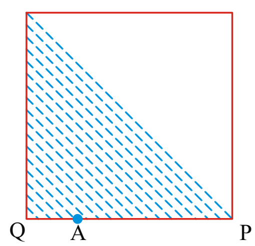
34. Instead of open top if the vessel is closed, then absolute pressure at point A would be

[Take $a=\frac{20}{3}m/s^2$ and initially height of water in tank is 2m]

A) $1.33\times10^5 N/m^2$ B) $1.0\times10^5 N/m^2$ C) $3.33\times10^{45} N/m^2$ D) None

SOLUTION :

For $a=\frac{20}{3}=\frac{2g}{3}$ the situation would be as shown in fig.



pressure $P=p_0$

Pressure at $A, P_A=p_0+\rho a\times5$

$$=10^5+10^3\times\frac{20}{3}\times5$$

$$=1.33\times10^5 N/m^2$$

35. A liquid of density $\rho=\rho_0[1+\alpha y]$ is stored in a container where y is the distance from the the liquid surface and $\alpha=\frac{2}{3}m^{-1}$. A small hole is made at the bottom of the container. Find nearest integer of velocity of efflux (in m/s) when the liquid height is 1m. Assume flow is laminar . $(g=10m/s^2)$

SOLUTION :

$$\frac{dp}{dy}=-\rho g=-\rho_0(1+\alpha y)g$$

$$\Rightarrow\int dp=-\int\rho_0(1+\alpha y)g$$

after applying Bernoulli's principle,

the velocity is 4 m/s.

36. A light cylindrical vessel is kept on a horizontal surface. It's base are is A. A hole of cross-sectional area 'a' is made just at it's bottom side. The minimum coefficient of friction necessary for sliding of vessel due to the impact force of the emerging liquid

1) Varying **2)** $\frac{a}{A}$ **3)** $\frac{2a}{A}$ **4) None**

SOLUTION :

$$F = v\frac{dm}{dt} = va\rho v = v^2 a\rho = \left(\sqrt{2gy}\right)^2 a\rho = 2a\rho gy$$

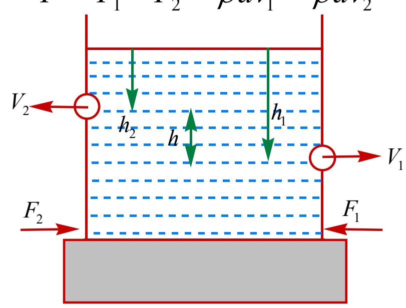
Force of friction f=F=2a ρ gy

$$\left[\mu N = \mu\left(A\rho gy\right)\right] = 2a\rho gy; \mu = \frac{2a}{A}$$

37. There are two identical small holes, each of area of cross-section 'a' on the opposite sides of a tank containing a liquid of density ρ. The difference in height between the holes is 'h'. Tank is resting on a smooth horizontal surface. Find the horizontal force which will have to be applied on the tank to keep it in equilibrium.

SOLUTION :

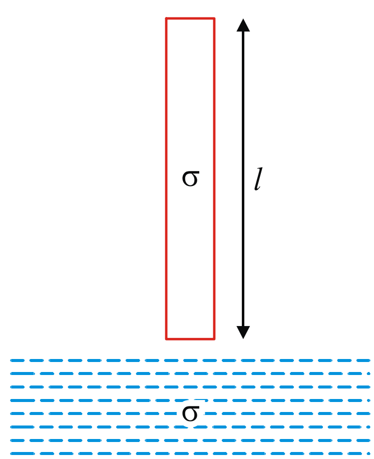
$$F = F_1 - F_2 = \rho a v_1^2 - \rho a v_2^2$$



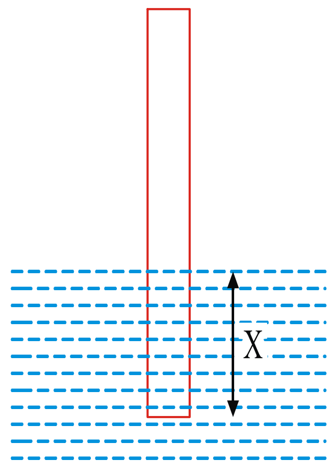
$$= \rho a\left(2gh_1\right) - \rho a\left(2gh_2\right) = 2\rho ag\left(h_1 - h_2\right) = 2\rho agh$$

38. A uniform vertical cylinder is released from rest with its lower end just touching the liquid surface of a deep lake. Calculate the maximum displacement of the cylinder in meters. Take

$\ell = 4m \ and \ \frac{\sigma}{\rho} = \frac{1}{2}$



SOLUTION :



$$Al\,\sigma g - Ax\,\rho g = Al\,\sigma a$$

$$a = g - \frac{\rho g x}{\sigma l};$$

$$\int_0^v v.dv = \int_0^x g - \frac{\rho g x}{\sigma \ell} dx$$

$$\Rightarrow \frac{v^2}{2} = gx - \frac{\rho g}{\sigma \ell}\frac{x^2}{2}$$

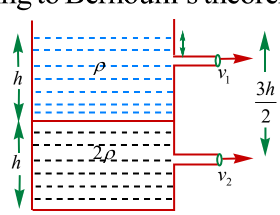
At maximum displacement,

$$\Rightarrow x = \frac{2\sigma\ell}{\rho} = 2 \times \frac{1}{2} \times 4 = 4m$$

39. A tank is filled with two immiscible liquids of densities 2ρ and ρ each of height h. Two holes are made to the side wall at $\frac{h}{2}$ and $\frac{3h}{2}$ from upper surface of the liquid, then find the ratio of velocity of efflux of the liquids through the holes

SOLUTION :

According to Bernoulli's theorem,



For $v_1, p + \rho g\left(\frac{h}{2}\right) = P\frac{1}{2}(\rho)\left(v_1^2\right) \Rightarrow v_1 = \sqrt{gh}$

For $v_2, P + (\rho g h) + (2\rho)g\left(\frac{h}{2}\right) = P + \frac{1}{2}(2\rho)v_2^2$

$$\Rightarrow v_2 = \sqrt{2gh} \therefore \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

40. A hose shoots water straight up to a height of 2.5 m. The opening end of the hose has an area of 0.75cm^2. What is the speed of the water as it leaves the hose? How much water will come out in one minute?

SOLUTION :

Kinetic energy at bottom = Potential energy at the top

$$\frac{1}{2}v^2 = gh \Rightarrow v = \sqrt{2gh} \Rightarrow v = \sqrt{2\times9.8\times2.5}$$

$$= \sqrt{5\times9.8} = \sqrt{49} = 7m/s; v = 7m/s \text{ or } 700\text{cm/s}.$$

The rate of flow of water = Av. So in one minute the volume of water that flows out

$$= Av\times60 = (0.75)\times700\times60 = 0.75\times42\times10^3$$

$$= 3.15\times10^4 cm^3 = 31.5 \text{ litre.}$$

41. A wooden block is floating in a water tank. The block is pressed to its bottom. During this process work done is equal to

A) Work done against upthrust exerted by the water

B) Work done against upthrust plus loss of gravitational potential energy of the block

C) Work done against upthrust minus loss of gravitational potential energy of the block

D) all the above

SOLUTION :

Initially the wooden block floats with partially immersed in water. Initially, upthrust exerted by water is equal to weight of the block.

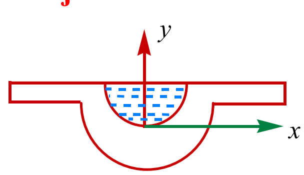
But when it is pressed down more water is displaced. Hence upthrust exerted by water increases.

The force required to press down the block is equal to $F = upthrust - mg$.

Hence, work done by the force F will be equal to work done against (upthrust - mg).

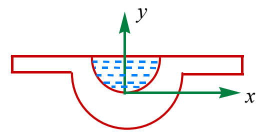
It means, work done by the external force is equal to work done against upthrust loss of gravitational potential energy of the block.

42. A small hole is made at the bottom of a symmetrical jar as shown in figure. A liquid is filled into the jar upto a certain height. The rate of dissension of liquid is independent of level of the liquid in the jar. Then the surface of jar is a surface of revolution of curve



1) $y=kx^4$ **2) $y=kx^2$** **3) $y=kx^3$** **4) $y=kx^5$**

SOLUTION :



Let y be the height of liquid at same instant then

(a=area of hole $\frac{-dy}{dt}$ =constant given)

$$a\sqrt{2gy} = \pi r^2\left(-\frac{dy}{dt}\right)$$

$\pi, \left(\frac{-dy}{dt}\right)$ and g are constant

squaring the equation, we get $y = kx^4$

43. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, find the value of R. (EAMCET-2011)

SOLUTION :

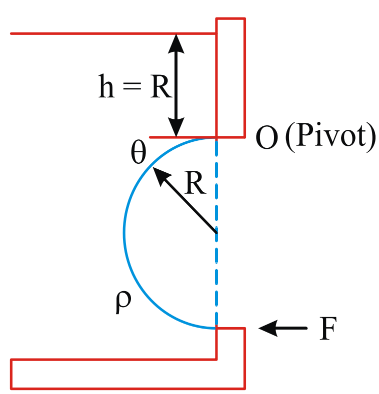
Velocity of efflux at a depth h is given by $v = \sqrt{2gh}$.

Volume of water flowing out per second from both the holes are equal

$$\therefore a_1v_1 = a_2v_2\left(\because \frac{V}{t} = av\right)$$

$$\left(L^2\right)\sqrt{2g(y)} = \pi R^2\sqrt{2g(4y)} \Rightarrow R = \frac{L}{\sqrt{2\pi}}$$

44. The fig shows a semi-cylindrical massless gate of unit length perpendicular to the plane of the page and is pivoted at the point O holding a stationary liquid of density ρ. A horizontal force F is applied at its lowest position to keep it stationary. The magnitude of the force is :



A) $\frac{3}{2}\rho gR^2$ B) $\frac{9}{2}\rho gR^2$ C) ρgR^2 D) $2\rho gR^2$

SOLUTION :

Torque about 'O' due to water pressure is given by $\tau = \int d\tau$

where $d\tau = dF.R\sin\theta$

after integrating from 0 to 180^0

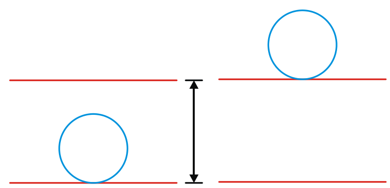
we will get $\tau = 4R^3\rho g$

this should be balanced by applied torque.

$$\therefore \tau = 4R^3\rho g = F(2R)$$

$$\Rightarrow F = 2\rho g R^2$$

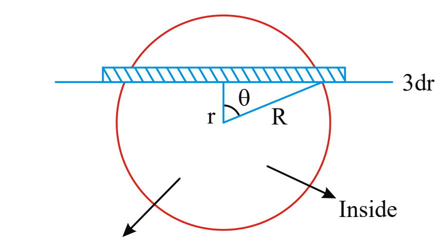
45. In figure-I is shown a sphere of mass m and radius r resting at the bottom of a large container filled with water. Depth of the container is h. Density of material of the sphere is the same as that of water. Now the whole sphere is slowly pulled out of water as shown in figure-II



Work done by the agent in pulling the sphere is equal to

A) mgr B) 0.5mgr C) mg(0.5r+h) D) mg(r+h)

SOLUTION :



$$\cos\theta = \frac{r}{R} \Rightarrow r = R\cos\theta \Rightarrow dr = -R\sin\theta . d\theta$$

net force on immersed body's part=0 $\left(\because \rho_{body} = \rho_{water}\right)$

$\therefore dw =$ workdone in shifting above part through dr height $= V_{above}\rho g(dr) \rightarrow (1)$

determination of $\underline{V_{above}}$:- $\Omega = 2\pi(1-\cos\theta)$

$\Rightarrow$ volume related to surface area above water level is $= \frac{R^3}{3}\Omega$ $\left[\begin{array}{l} \because 4\pi \rightarrow \frac{1}{3}\pi R^3 \\ \Omega \rightarrow \frac{4}{3}\pi R^3\left(\frac{\Omega}{4\pi}\right) \end{array}\right]$

$$\therefore V_{above} = \frac{R^3}{3} 2\pi(1-\cos\theta) \to (2)$$

$$\therefore dw = \frac{R^3}{3}[2\pi - 2\pi\cos\theta]\rho g(-R)\sin\theta\, d\theta$$

$$\therefore dw = -\frac{R^4 \rho g}{3} 2\pi \sin\theta d\theta + \frac{R^4\rho g}{3} 2\pi\cos\theta.\sin\theta.d\theta$$

$$= \frac{R^4 \rho g \pi}{3}[\sin 2\theta d\theta - 2\sin\theta d\theta]$$

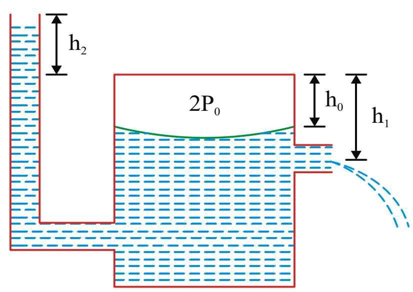
$$\Rightarrow w = \frac{R^4\rho g\pi}{3}(4)$$

$[\because$ after integration limits of θ : 0 to 180^0]

$$\text{w} = \frac{4\pi}{3} R^3 \rho g R \; = mgR$$

Comprehension :

Figure shows a large closed cylindrical tank containing water. Initially, the air trapped above the water surface has a height h_0 and pressure $2p_0$ where p_0 is the atmospheric pressure. There is a hole in the wall of the tank at a depth h_1 below the top from which water comes out. A long vertical tube is connected as shown.



46. Find the height h_2 of the water in the long tube above the top initially

A) $\frac{3p_0}{\rho g} - \frac{h_0}{3}$ B) $\frac{2p_0}{\rho g} - \frac{h_0}{2}$ C) $\frac{p_0}{\rho g} - h_0$ D) $\frac{p_0}{2\rho g} - 2h_0$

SOLUTION :

$$2P_0 = (h_2 + h_0)\rho g + p_0$$

(since liquids at the same level have the same pressure)

$P_0 = h_2\rho g + h_0 \rho g$;

$h_2 \rho g = P_0 - h_0 \rho g$

$$h_2 = \frac{P_0}{\rho g} - \frac{h_0 \rho g}{\rho g} = \frac{P_0}{\rho g} - h_0$$

47. Find the speed with which water comes out of the hole.

A) $\frac{1}{\rho}\left[p_0-\rho g\left(h_1-2h_0\right)\right]^{1/2}$ B) $\left[\frac{2}{\rho}\left[p_0+\rho g\left(h_1-h_0\right)\right]\right]^{1/2}$ C) $\left[\frac{3}{\rho}\left[p_0+\rho g\left(h_1+h_0\right)\right]\right]^{1/2}$ D) $\left[\frac{4}{\rho}\left[p_0-\rho g\left(h_1-h_0\right)\right]\right]^{1/2}$

SOLUTION :

KE of the water = Pressure energy of the water at that layer $\frac{1}{2}mV^2 = m\times\frac{P}{\rho}$

$$V^2=\frac{2P}{\rho}=\frac{2}{\rho}\left[P_0+\rho g\left(h_1-h_0\right)\right]$$

$$V=\left[\frac{2}{\rho}\left\{P_0+\rho g\left(h_1-h_0\right)\right\}\right]^{1/2}$$

48. Find the height of the water in the long tube abvoe the top when the water stops coming out of the hole.

A) $-2h_0$ B) h_0 C) h_2 D) $-h_1$

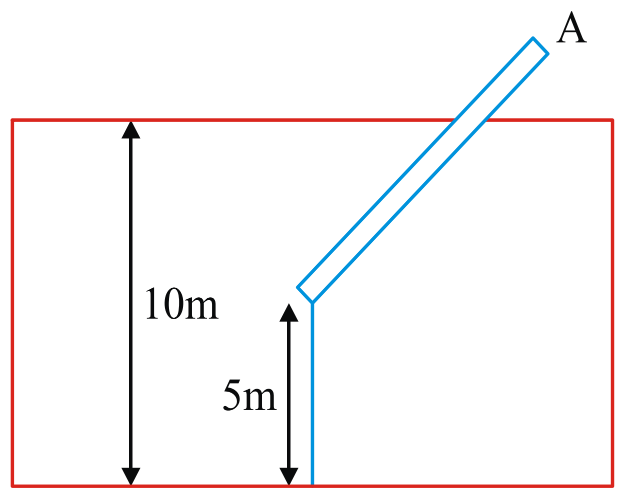
SOLUTION :

we know $2P_0+\rho g\left(h_1-h_0\right)=P_0+\rho gX$

$$\Rightarrow X=\frac{P_0}{\rho g}+\left(h_1-h_0\right)=h_2+h_1$$

i.e, X is h_1 meter below the top or X is - h_1 above the top.

49. A rod is of length 6 m and of specific gravity $\rho = 25/36$. One end of the rod is tied to a 5 m long light rope which in turn is tied to the floor of a pool 10 m deep as shown. Find the length of the part of rod in metres which is out of water.



SOLUTION :

Taking moments about A

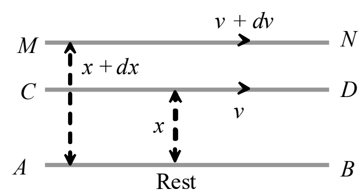
$$\left(6-x\right)^2 A\rho_w g/2 = 18A\rho_r g$$

Solving, we get $x = 1\,m$

VISCOSITY :

- In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layers in contact, the two layer exert tangential force on each other which tries to destroy the relative motion between them.
- The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called viscous force.
- Viscosity in gases due to collisions between gas molecules and transfer of momentum.
- Viscosity in liquids is due to cohesive force between molecules in succesive layers
- Waves in sea water subside due to viscosity.
- Rain drops fall to the ground with terminal velocity due to viscosity.
- It is only due to viscosity, a liquid flow becomes orderly.

Viscous Force (Newton's Formula):



Consider the two layers *CD* and *MN* of the liquid at distances x and $x + dx$ from the fixed surface *AB*, having the velocities v and $v + dv$ respectively. Then $\frac{dv}{dx}$ denotes the rate of change of velocity with distance and is known as velocity gradient.

According to Newton's hypothesis, the tangential force F acting on a plane parallel layer is proportional to the area of the plane A and the velocity gradient $\frac{dv}{dx}$ in a direction normal to the layer, *i.e.*,

$$F \propto A \quad \text{and} \quad F \propto \frac{dv}{dx}$$

$$\therefore \quad F \propto A\frac{dv}{dx}$$

$$\text{or} \qquad F = -\eta A\frac{dv}{dx}$$

Where η is a constant called the coefficient of viscosity.

The viscous force acting between two adjacent layers of a liquid is given by $F = -\eta A\frac{dv}{dx}$

- The viscous force acts tangential to the liquid layer
- Negative sign indicates the force is opposite in direction to the relative velocity of flow of the liquid.

- velocity gradient = $\frac{dv}{dx}$. It is a vector
- η is coefficient of viscosity. It may be defined as the tangential force per unit area required to maintain unit velocity gradient (or) it is the ratio between tangential stress and velocity gradient.

  It is also called **coefficient of dynamic viscosity.**
- Dimensional formula of $\eta = M^1L^{-1}T^{-1}$
- The SI unit of η is Pa-s. Its CGS unit is poise.
- 1pa-s=10 poise.

Note : The value of η changes from liquid to liquid and for ideal liquid η=0

Coefficient of kinematic viscosity:

- The ratio between the coefficient of viscosity and density of the liquid is called Coefficient of Kinematic viscosity.
- Coefficient of Kinematic viscosity = $\frac{\eta}{\rho}$
- Its S.I. unit is m^2s^{-1}
- Its practical unit is stoke. 1 stoke = $10^{-4}m^2s^{-1}$.
- Its dimensional formula is $\left[L^2T^{-1}\right]$, same as that of areal velocity.

Effect of temperature:

- In the case of liquids, coefficient of viscosity decreases with increase of temperature as the cohesive forces decrease with increase of temperature.
- In the case of gases, coefficient of viscosity increases with increase of temperature because number of collisions between the molecules of the gas increases.

Effect of pressure:

- For liquids the value of η increases with increase of pressure.
- Above 33^0C the viscosity of water increases with pressure, and that below this temperature, initially the pressure effect is negative
- For gases, value of η increases with increase of pressure at low pressure. But at high pressure, η is independent of pressure.
- The machine parts get jammed in winter, because the viscosity oil (used as lubricants in machine parts) increases due to fall in temperature
- A viscous fluid tends to cling to a solid in contact with it. That is why dust particles cling to a fan blade even when it is rotating rapidly.

Poiseuille's equation:

According to Poiseuille's Formula the rate of flow of a liquid through a horizontal capillary tube is

$$V = \frac{\pi \Pr^4}{8\eta l}$$

Where V is volume of the liquid flowing out per second,

'p' is the pressure difference across the capillary pipe,

'r' is the radius of the pipe,

'l' is the length of the capillary pipe,

'η' is the coefficient of viscosity of the liquid,

$\pi / 8$ is a proportionality constant.

Poiseuille's equation is valid for

a) flow through a horizontal capillary tube.
b) steady and laminar flow
c) the liquid the contact with the walls of the capillary tube must be at rest.
d) the pressure at any cross section must be same

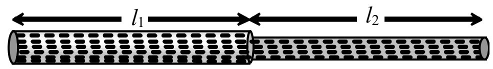
Arrangement of capillary tubes:

a) Capillary tubes in series :

♦ **From Poiseuille's equation,** $V = \frac{\pi P r^4}{8\eta l} (or) V = \frac{P}{\left(\frac{8\eta l}{\pi r^4}\right)} = \frac{P}{R}$

♦ **Total volume of the liquid flowing out per sec remains constant**

Where $R = \frac{8\eta l}{\pi r^4}$ **is called fluid resistance.**



When two capillaries are connected in series across a constant pressure difference P, the fluid resistance $R = R_1 + R_2$.

$$R = \frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4} = \frac{8\eta}{\pi}\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)$$

♦ Volume of liquid flowing per second is same through both capillaries

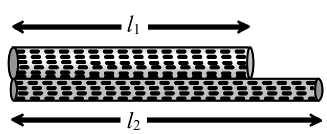
$$\Rightarrow V = \frac{P}{R_1 + R_2} = \frac{P}{R} = \frac{\pi P}{8\eta\left(\frac{\ell_1}{r_1^4} + \frac{\ell_2}{r_2^4}\right)}$$

Here $P = P_1 - P_2 = (P_1 - P_0) + (P_0 - P_2)$

$(P_1 - P_0)$ and $(P_0 - P_2)$ are pressure differences across individual capillaries.

$$\frac{P_1 - P_0}{P_0 - P_2} = \frac{R_1}{R_2} = \frac{\ell_1}{\ell_2} \times \frac{r_2^4}{r_1^4}$$

b) Capillary tubes in parallel :



♦ When two capillary tubes are connected in parallel across constant pressure difference P, then fluid resistance R is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

♦ $R_1 = \frac{8\eta l_1}{\pi r_1^4}$ and $R_2 = \frac{8\eta l_2}{\pi r_2^4}$

$$\frac{1}{R} = \frac{\pi}{8\eta}\left(\frac{r_1^4}{l_1} + \frac{r_2^4}{l_2}\right)$$

♦ Here volume of the fluid flowing per second in each tube is different but pressure difference

$P = (P_1 - P_2)$ is same.

♦ Total volume flowing per second is $V=V_1+V_2$

$$\Rightarrow V = \frac{P}{R_1} + \frac{P}{R_2} \text{ or } V = \frac{\pi P}{8\eta}\left(\frac{r_1^4}{l_1} + \frac{r_2^4}{l_2}\right)$$

$$\frac{V_1}{V_2} = \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \left(\frac{r_1}{r_2}\right)^4$$

Stoke's law - Terminal velocity:

When a spherical body is dropped in a fluid, the fluid layer in contact with the body is dragged with the body. The remaining layers of the fluid excerts viscous force on the body to oppose its motion.

When the sum of the viscous force and upthrust of the liquid on the body is equal to its weight, the body then begins to fall with a constant velocity, known as **terminal velocity**

According to stoke,

i) the viscous force (F) $= 6\pi\eta r\ v_t$.

Where

η=coeff. of viscosity of the fluid,

r=radius of the body

v_t terminal velocity.

ii) Upward of the body(W) = mg=(volume of sphere x density) $\times g = \frac{4}{3}\pi r^3 \rho g$

iii) Upward thrust(T) = weight of the fluid displaced

=(volume of the fluid displaced x density of the fluid) g

$= \frac{4}{3}\pi r^3 \rho g$

When the body attains terminal velocity the net force acting on the body is zero

W-R-F=0;

F=W-T

$$\Rightarrow v_t = \frac{2r^2 g(\rho - \sigma)}{9\eta}$$

♦ $v_t\ \alpha\ (\rho - \sigma)$

If $\rho < \sigma$, body will rise instead of falling.

For example **air bubble in water**

♦ $v_t\ \alpha\ \frac{1}{\eta}$,

greater the viscosity, smaller is the terminal velocity.

For example a lead shot attains less terminal velocity in glycerine than in water.

♦ $v_t \alpha r^2 \Rightarrow \frac{v_{t_1}}{v_{t_2}} = \left(\frac{r_1}{r_2}\right)^2$

♦ When 'n' identical droplets are falling down with their terminal velocity 'v' are combined to form a big drop, then terminal velocity of big drop is

$$\frac{v_{big}}{v} = \frac{R^2}{r^2} \Rightarrow v_{big} = n^{2/3} v \left(\because R = n^{1/3} r\right)$$

♦ Two spherical bodies having masses m_1, m_2 respectively falling in viscous medium the ratio of terminal velocities is

$$\frac{v_1}{v_2} = \left(\frac{m_1}{m_2}\right)^{2/3}$$

:: PROBLEMS ::

1. **A boat of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2m/s by applying a tangential force. If the river is 1m deep and the water in contact with the bed is stationary, find the tangential force need to keep the boat moving with constant speed. (coefficient of viscosity of water = 10^{-2} poise)**

SOLUTION :

$$\text{Velocity gradient} = \frac{dv}{dx} = \frac{2-0}{1-0} = 2s^{-1}$$

From, Newton's law of viscous force,

$$F_v = \eta A \frac{dv}{dx} = \left(10^{-2} \times 10^{-1}\right)(10)(2) = 0.02N$$

2. **A spherical ball falls through viscous medium with terminal velocity v. If this ball is replaced by another ball of the same mass but half the radius, then the terminal velocity will be (neglect the effect of buoyancy.)**

A) v B) $2v$ C) $4v$ D) $8v$

SOLUTION :

$$m = \frac{4}{3}\pi r^3 \rho g$$

$$v_0 \alpha r^2 \rho \;\; ;$$

$$v_0' \alpha \frac{r^2}{4}.(8\rho)\alpha 2r^2\rho$$

3. **A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 \left(\rho_2 < \rho_1\right)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2 \left(k > 0\right)$. The terminal speed of the ball is:**

A) $\sqrt{\frac{Vg\left(\rho_1 - \rho_2\right)}{k}}$ B) $\frac{Vg\rho_1}{k}$ C) $\sqrt{\frac{Vg\rho_1}{k}}$ D) $\frac{Vg\left(\rho_1 - \rho_2\right)}{k}$

SOLUTION :

The force acting on the ball are gravity force, buoyancy force and viscous force.

When ball acquires terminal speed, it is in dynamic equilibrium,

let terminal speed of ball is v_T.

$$\text{So, } v\rho_2 g + kv_T^{\,2} = V\rho_1 g\,;$$

$$v_T = \sqrt{\frac{V\left(\rho_1 - \rho_2\right)g}{k}}$$

4. **Water flows through a capillary tube of radius 'r' and length at a rate of 40 ml per second, when connected to a pressure difference of 'h' cm of water. Another tube of the some length but radiud . r/2 is connected in series with this tube and the combination is connected to the same pressure heaD) [density of water is ρ]**

A) The pressure difference accross each tube is $p_1 = \frac{\rho gh}{17}$ and $P_2 \frac{16}{17}\rho gh$

B) The pressure difference accross each tube is $p_1 = \frac{\rho gh}{16}$ and $P_2 \frac{17}{16}\rho gh$

C) The rate of flow of water through the combination is $\frac{40}{17} c.c/\text{sec}$.

D) The rate of flow of water through the combination is $\frac{17}{40} c.c/\text{sec}$.

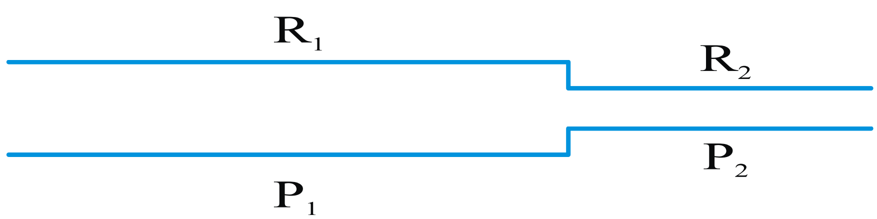
KEY : A, C

SOLUTION :

According to Pissiulle's equation.

$$\text{Rate of flow} = \frac{P}{\left(\frac{8\eta L}{\pi r^4}\right)}$$

Making an analogy with current flow,



$$\text{Let } R_1 = \frac{8\eta L}{\pi r^4};$$

$$R_2 = \frac{8\eta L}{\pi\left(\frac{g}{2}\right)^4} = 16\,R$$

$$P_1 + P_2 = P = \rho gh \quad \text{and} \quad P_1/P_2 = \frac{R_1}{R_2} = \frac{1}{16}$$

$$\text{Also Rate of flow} = \frac{P}{P_1 + P_2} = \frac{P}{17R_1} = \frac{40}{17} = \text{mL/s}$$

5. **A 16cm^3 of water flows per second through a capillary tube of radius r cm and of length 1cm, when connected to a pressure head of h cm of water. If a tube of the same length and radius r/s is connected to the same pressure head, find the mass of water flowing per minute through the tube.**

SOLUTION :

$$V \propto \frac{\text{Pr}^4}{l} \Rightarrow \frac{V_2}{V_1} = \frac{P_2}{P_1} \times \frac{r_2^4}{r_1^4} \times \frac{l_1}{l_2}$$

$$= \frac{(r/2)^4}{r^4} \times \frac{l}{l} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}; V_2 = \frac{16}{16} = 1cm^3/s$$

Volume of water flowing per minute = $1 \times 60 = 60cm^3/\min$

$\therefore$ **Mass of water flowing per minute** $= V \times d = 60 \times 1 = 60 gram/\min$.

6. **Water is flowing in a river. If the velocity of a layer at a distance of 10 cm from the bottom is 20 cm/s. Find the velocity of layer at a height of 40 cm from the bottom**

A) $10\,m/s$ B) $20\,m/s$ C) $30\,m/s$ D) $80\,m/s$

SOLUTION :

$$F = -\eta A\frac{dv}{dx}\ ;$$

$$F\alpha\frac{dv}{dx}\ or\ F\alpha\frac{\Delta v}{\Delta x}$$

$$F\alpha\frac{v}{x}\ (or)\ x\alpha\frac{v}{F}$$

but F is constant

$$\therefore\ x\alpha v$$

7. **Water flows in a streamline manner through a capillary tube of radius a. The pressure difference being P and the rate of flow is Q. If the radius is reduced to a/2 and the pressure difference is increased to 2P, then find the rate of flow**

SOLUTION :

$$\text{Rate of flow } Q = \frac{P}{\left(8\eta l/\pi r^4\right)}$$

$$\Rightarrow Q \propto \left(\mathrm{P} r^4\right) \Rightarrow \frac{Q_1}{Q_2} = \frac{P_1}{P_2}\left(\frac{r_1}{r_2}\right)^4$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{P}{2P}\left(\frac{a}{a/2}\right)^4 \Rightarrow Q_2 = \frac{Q}{8}$$

Rate of flow will become 1/8 times

8. **If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid.**

A) 0.2 m/s B) 0.4 m/s C) 0.133 m/s D) 0.1m/s

SOLUTION :

Terminal velocity of spherical body in a viscous liquid is given by: $v_T = \frac{2r^2}{9\eta}(\rho-\sigma)g$

Where $r =$ radius of the sphere,

ρ =density of the sphere,

η =coefficient of viscosity,

σ = density of liquid.

$$\therefore\ v \propto (\rho-\sigma)$$

$$\frac{v_g}{v_s} = \frac{\rho_g-\sigma}{\rho_s-\sigma} = \frac{19.5-1.5}{10.5-1.5} = 2$$

$$\therefore\ v_s = \frac{v_g}{2} = \frac{0.2}{2} = 0.1\,m/s$$

9. Capillary tubes of lengths *l* and *2l* are connected in series. Their radii are r and 2r respectively. If stream line flow is maintained and pressure difference across first and second capillary tubes are P_1 and P_2 respectively, then find the ratio $\frac{P_1}{P_2}$

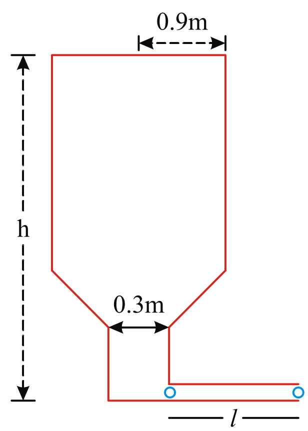
SOLUTION :

Equating the rate of flow of liquid

Here $Q_1 = Q_2$ and $Q = \frac{\pi r^4 P}{8\eta l}$

$$\therefore \frac{\pi (r)^4 P_1}{8\eta l} = \frac{\pi (2r)^4 P_2}{8\eta (2l)} \Rightarrow \frac{P_1}{P_2} = 8$$

10. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper radius 0.9m and lower radius 0.3m A capillary tube of length l is attached at the bottom of the tank as shown in fig. The capillary has outer radius 0.002m and inner radius A) When pressure P is applied at the top of the tank volume flow rate of liquid is $8\times10^{-6} m^3/s$ and if capillary tube is detached, the liquid comes out from the tank with a velocity $10 m/s$. Then the coefficient viscosity of liquid is ($\pi a^2 = 10^{-6} m^2$ $a^2/l = 2\times10^{-6} m$.)



A) $\eta = 1.25\times10^{-3} N-s/m^2$.
B) $\eta = 2.50\times10^{-3} N-s/m^2$
C) $\eta = 5.00\times10^{-3} N-s/m^2$
D) $\eta = 7.25\times10^{-3} N-s/m^2$

SOLUTION :

Applying Bemoullie's equation $P + \rho gh = \frac{1}{2}\rho\left(v_2^{\,2} - v_1^{\,2}\right)$;

but $v_1 = \left(A_2 v_2 / A_1\right)$

$$\Delta P = P + \rho gh; \quad \therefore \frac{dV}{dt} = \frac{\pi a^4 \Delta P}{8\eta l}$$

11. Three capillary tubes of same radius 1 cm but of lengths 1m, 2m and 3m are fitted horizontally to the bottom of a long vessel containing a liquid at constant pressure and flowing through these. What is the length of a single tube which can replace the three capillaries.

SOLUTION :

$$V_1 = \frac{\pi \Pr^4}{8\eta l_1}, V_2 = \frac{\pi \Pr^4}{8\eta l_2}, V_3 = \frac{\pi \Pr^4}{8\eta l_3}$$

and $V = \frac{\pi \Pr^4}{8\eta l}$

Now $V = V_1 + V_2 + V_3 \Rightarrow \frac{1}{\ell} = \frac{1}{\ell_1} + \frac{1}{\ell_2} + \frac{1}{\ell_3}$

Substituting the values, we get

$$l = \frac{l_1 l_2 l_3}{l_1 l_2 + l_1 l_3 + l_2 l_3} = \frac{1 \times 2 \times 3}{1 \times 2 + 1 \times 3 + 2 \times 3} = \frac{6}{11} m$$

12. A cylindrical vessel of area of cross-section A and filled with liquid to a height of h_1 has a capillary tube of length *l* and radius r protuding horizontally at its bottom. If the viscosity of liquid is η and density ρ. Find the time in which the level of water in the vessel falls to h_2.

A) $\frac{8\eta l A}{\pi \rho g r^4} \ln \frac{h_1}{h_2}$ B) $\frac{8\eta l A}{\pi \rho g r^4}$ C) $\frac{\eta A}{g}\left(\sqrt{h_1} - \sqrt{h_2}\right)$ D) $\frac{8\eta l A}{\pi \rho g r^4} \ln \frac{h_2}{h_1}$

SOLUTION :

Let h be the height of water level in the vessel at instant t which decreases by dh in time dt.

$\therefore$ Rate of flow of water through the capillary tube, $V = -A\left(\frac{dh}{dt}\right)$ .... (1)

Further, the rate of flow from Poiseuille formula $V = \frac{\pi \Pr^4}{8\eta l}$ .... (2)

The hydrostatic pressure at depth h is $P = \rho g h$

From eqns (1) and (2), we have

$$-A\frac{dh}{dt} = \frac{\pi \rho h r^4}{8\eta l}$$

$$dt = -\frac{8\eta l A}{\pi \rho r^4} \frac{dh}{h} \quad ;$$

$$t = \frac{-8\eta l A}{\pi \rho g r^4} \int_{h_1}^{h_2} \frac{dh}{h}$$

13. Two identical drops of water are falling through air with a steady speed of 'v' each. If the drops coalesce to form a single drop, what is the new terminal velocity? (EAMCET-2013)

SOLUTION :

From conservation of mass $\frac{4}{3}\pi R^3 \times \rho = \frac{4}{3}\pi r^3 \times \rho + \frac{4}{3}\pi r^3 \times \rho$

or $R = \left(2^{\frac{1}{3}}\right) r$ and $v_t \alpha r^2$ (stokes law)

$$\frac{v^1}{v}=\frac{R^2}{r^2}=2^{2/3}$$

$$\therefore v^1=2^{2/3}v\cdot$$

14. A spherical steel ball stretched at the top of a long column of glycerin of length l falls through a distacne $l/2$ with accelerated motion and the remaining distance $l/2$ with uniform velocity. Let t_1 and t_2 denote the times taken to cover the first and second half and w_1 and w_2 are the work done against gravity in the two halves, then compare times and work done.

SOLUTION :

Average velocity in first half of the distance <v,
while in the second half the average velocity is v.
Therefore, $t_1>t_2$.
The work done against gravity in both halves is mg $l/2$

$$\therefore t_1>t_2 \therefore w_1=w_2$$

15. **A volume V of a viscous liquid flows per unit time due to a pressure head ΔP along a pipe of diameter d and length l. Instead of this pipe, a set of four pipes each of diameter $d/2$ and length $2l$ is connected to the same pressure head ΔP. Now the volume of liquid flowing per unit time is:**

A) $V/16$ B) $V/8$ C) $V/4$ D) V

SOLUTION :

$$V=\frac{\pi \Pr^4}{8l\eta} \quad ;$$

$$V\propto\frac{r^4}{l}$$

$$\frac{V_1}{V_2}=\left(\frac{r_1}{r_2}\right)^4\times\frac{l_2}{l_1}=32 \quad ;$$

$$V_2=\frac{V}{32}$$

For 4 pipes,

$$V_2^1=4V_2=4\times\frac{V}{32}=\frac{V}{8}$$

16. A small steel ball falls through a syrup at a constant speed of 1.0 m/s. If the steel ball is pulled upwards with a force equal to twice its effective weight, how fast will it move upward?

SOLUTION :

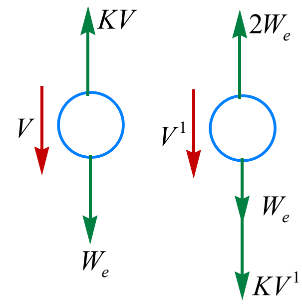
$6\pi\eta rv=mg^1$ (effective weight)

Let W_e = effective weight and $6\pi\eta r=K$

In equilibrium, $w_e=kv$...(1)

Again, $2w_e-w_e=kv^1$.....(2)

From Eqs (1) and (2) $v^1=v=1.0m/s$



17. **A marble of mass x and diameter $2r$ is gently released in a tall cylinder containing honey. If the marble displaces mass. y (<x) of the liquid, the terminal velocity is proportional to**

A) $x+y$ b. $x-y$ C) $\frac{x+y}{r}$ D) $\frac{x-y}{r}$

SOLUTION :

$$v_0 = \frac{2}{9} r^2 \frac{(\rho - \rho^1) g}{\eta} \ ;$$

$$x = \frac{4}{3} \pi r^3 \rho \ (or)$$

$$\rho \alpha \frac{x}{r^3} \ ; \ \rho^1 \alpha \frac{y}{r^3} \ ;$$

$$v_0 \alpha \frac{x-y}{r}$$

18. **Between a plate of area 100 cm^2 and another plate of area 100 m^2 there is a 1 mm, thick layer of water, if the coefficient of viscosity of water is 0.01 poise, then the force required to move the smaller plate with a velocity 10 cm/s with reference to large plate is.**

A) $100\ dyn$ B) $10^4\ dyn$ C) $10^6\ dyn$ D) $10^9\ dyn$

SOLUTION :

$$F = \eta A \frac{dv}{dx}$$

where A=100cm^2;

$$\frac{dv}{dx} = \frac{10cm/s}{1m/m} = 100\, s^{-1}$$

19. **A tube of length l and radius R carries a steady flow of fluid whose density is ρ and viscos ity η. The velocity v of flow is given by $v = v_0(1 - r^2/R^2)$, Where r is the distance of flowing fluid from the axis.**

A) The volume of fluid, flowing across the section of the tube, in unit time is $2\pi v_0 (R^2/4)$

B) The kinetic energy of the fluid within the volume of the tube is $K.E. = \pi \rho l v_0^{\,2} (R^2/6)$

C) The frictional force exerted on the tube by the fluid is $F = 4\pi\eta k v_0$

D) The pressure difference at the ends of tube is $P = \frac{4\eta l v_0}{R^2}$

KEY : ALL

SOLUTION :

The volume of fluid flowing through this section per second $dv = (2\pi r dr) v_0 (1 - r^2/R^2)$

total volume $V = \int_0^R (2\pi r\, dr) v_0 (1 - r^2/R^2)$

$= 2\pi v_0 (R^2/4)$

(ii). The kinetic energy of the fluid within the volume element of thickness dr
K.E of fluid within the tube is

$$= \frac{1}{2}(2\pi l)\rho v_0^{\ 2} \int_0^R (1 - r^2/R^2) r\, dr$$

we get K.E $\pi \rho l v_0^{\ 2} (R^2/6)$

(iii). The viscous drag exerts a force on the

tube $F = -\eta A \left(\frac{dv}{dr}\right)_{r=R}$

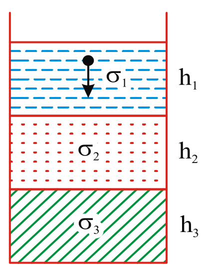
Here $\left(\frac{dv}{dr}\right)_{r=R} = v_0 (-2r/R^2)_{r-R} = -2v_0/R \quad \therefore \quad F = 4\pi\eta l v_0$

(iv) $\Delta P = P_2 - P_1 = P$

where $P_1 = 0$ and $P_2 = P$

$$P = \frac{force(F)}{area(\pi R^2)} = \frac{4\eta l v_0}{R^2}$$

20. A ball moves successively through three liquids, at rest as shown, of densities σ_1, σ_2 and σ_3 nd viscosity coefficient η_1, η_2 and η_3 and respectively with the same (terminal) velocity. Then



A) $\eta_3 > \eta_2 > \eta_1$ B) $\frac{\sigma_1}{\eta_1} = \frac{\sigma_2}{\eta_2} = \frac{\sigma_3}{\eta_3}$ C) $\frac{\eta_1}{\eta_3} > \frac{\eta_3}{\eta_2}$ D) $\frac{\eta_2\sigma_1 - \eta_1\sigma_2}{\eta_3\sigma_1 - \eta_1\sigma_3} = \frac{\eta_2 - \eta_1}{\eta_3 - \eta_1}$

KEY : C, D

SOLUTION :

$$\frac{\rho - \sigma_1}{\eta_1} = \frac{\rho - \sigma_2}{\eta_2} = \frac{\rho - \sigma_3}{\eta_3}$$

$$\sigma_1 < \sigma_2 < \sigma_3 \Rightarrow \eta_1 > \eta_2 > \eta_3$$

Also, $\frac{\eta_1}{\eta_3} = \frac{\rho - \sigma_1}{\rho - \sigma 3} > 1 \frac{\eta_3}{\eta_2} = \frac{\rho - \sigma_3}{\rho - \sigma_2} < 1 \Rightarrow (\mathrm{C})$

Eliminating ρ gives (D)

21. **Two capillary tubes of same radius r but of lengths l_1 and l_2 are fitted in parallel to the bottom of a vessel. The pressure head is P. What should be the length of a single tube of same radius that can replace the two tubes so that the rate of flow is same as before ?**

A) $l_1 + l_2$ B) $\frac{1}{l_1}+\frac{1}{l_2}$ C) $\frac{l_1 l_2}{l_1+l_2}$ D) $\frac{1}{l_1+l_2}$

SOLUTION :

Fluid resistance $R=\frac{8\eta L}{\pi r^4}$

in parallel, $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

or $\frac{\pi r^4}{8\eta l_{eq}}=\frac{\pi r^4}{8\eta l_1}+\frac{\pi r^4}{8\eta l_2}$

$l_{eq}=\frac{l_1 l_2}{l_1+l_2}$

22. **When water flows at a rate Q through a tube of radius r placed horizontally, a pressure difference p develops across the ends of the tube. If the radius of the tube is doubled and the rate of flow halved, the pressure difference will be**

A) $8p$ B) p C) $p/8$ D) $p/32$

SOLUTION :

Poiseuille's formula gives the quantity of liquid flowing through a ckapillary, $Q=\frac{\pi}{8}\frac{pr^4}{\eta l}$

i.e., $p-\frac{8}{\pi}Q.\frac{nl}{r^4}$;

if $Q'=\frac{Q}{2}, r'=2r$

$\frac{p}{n}=\frac{8}{\pi}\frac{Q}{2}.\frac{nl}{(2r)^4}=\frac{8}{p}\frac{Q.nl}{r^4}\times\frac{1}{32}$ $(\because n=32)$

i.e., pressure $p'=\frac{p}{32}$

23. $L, L/2$ **and** $L/3$ **are connected in series. Their radii are** $r, r/2$ **and** $r/3$ **respectively. Then, if stream-line flow is to be maintained and the pressure across the first capillary is** P**, then:**

A) the pressure difference across the ends of second capillary is $8P$

B) the pressure difference across the third capillary is $43P$

C) the pressure difference across the ends of second capillary is $16P$

D) the pressure difference across the third capillary is $59P$

SOLUTION :

$$\frac{dQ}{dt} = \frac{\pi \Pr^4}{8\eta L}$$

As capillaries are joined in series, so (dQ/dt) will be same for each capillary.

Hence, $$\frac{\pi \Pr^4}{8\mu L} = \frac{\pi P'(r/2)^4}{8\eta(L/2)} = \frac{\pi P''(r/3)^4}{8\eta(L/3)}$$

So pressure difference across the ends of 2nd capillary $P' = 8p$

across the ends of 3rd capillary $P'' = 27p$

‘

Surface Tension

♦ The attractive forces between molecules of same substance are called cohesive forces.
♦ The attractive forces between molecules of different substances are called adhesive forces.
♦ The maximum distance upto which the cohesive force between two molecules exists is called the molecular range and is of the order of 10^{-9}m.
♦ An imaginary sphere drawn around a molecule with molecular range as radius is called the sphere of influence of that molecule.
♦ The force per unit length normal to any imaginary line drawn on the free surface of a liquid is known as **surface Tension.**

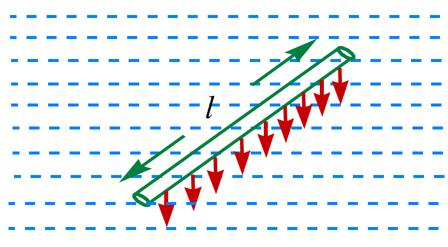
$$T = \frac{F}{l};$$

SI Unit : N/m,
CGS Unit : dyne/cm.
Dimensional formula : ML^0T^{-2}
♦ Surface tension is due to cohesive force between the molecules of a liquid.
♦ Surface tension is a molecular phenomenon.
♦ Surface tension is independent of surface area.
♦ It decreases with increase of temperature.

Applications of force due to surface tension :

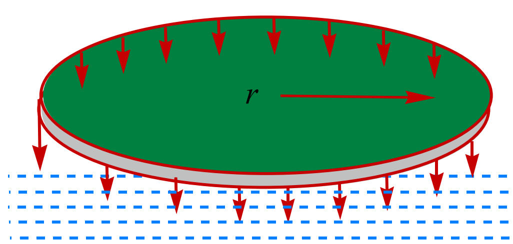
♦ Force required to pull a wire of length 'l' from the surface of water of surface tension T is

$$F = 2\ell T$$



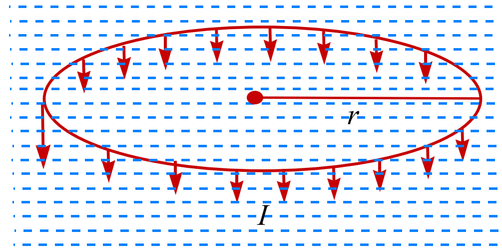
♦ Force required to pull a circular ring of radius R from the surface of water of surface tension T is

$$F = 2\pi rT$$



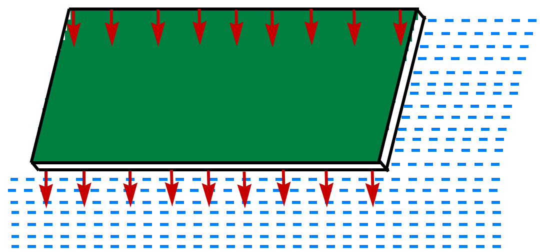
♦ Force required to pull a thin circular ring of radius r from the surface of water of surface tension T is

$$F = 4\pi rT$$



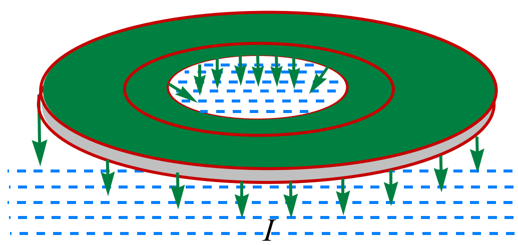
♦ Force required to pull a rectangular plate of length 'l' and breadth 'b' from the surface of water of surface tension T is

$$F = 2(l + b)T$$



♦ Force required to pull a circular disc of radius R with hole of radius r from the surface of water of surface tension T is

$$F = 2\pi(R + r)T$$



♦ Between two glass plates a water drop is squeezed to form a thin film of thickness (d) and surface area (A). The force required to seperate the two plates is,

F = 2TA/d.

Surface Energy:

♦ Work done to increase surface area of a film by one unit is known as surface energy. It is numerically equal to surface tension. (or) The additional potential energy possessed due to increase in surface area by one unit is called **surface energy**

$$S = \frac{W}{\Delta A};$$

SI Unit : J/m^2;

D.F=MT^{-2}

Applications of Surface Energy :

- Work done in forming a liquid drop is,

  W = Change in surface Area x Surface tension

  W $= \Delta A T - 4\pi r^2 T$.
- Work done in increasing the size of liquid drop from radius r_1 to r_2 is

  $W = 4\pi T(r_2^2 - r_1^2)$
- Work done in blowing a soap bubble of radius r is

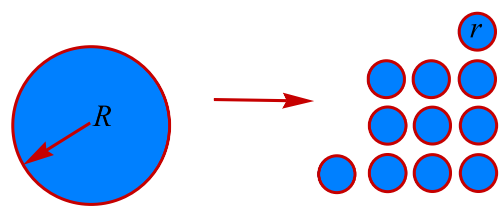
  $W = 8\pi r^2 T$. (Soap bubble has two free surfaces)
- Work done in increasing the size of a soap bubble from radius r_1 to r_2 is,

  $W = 8\pi T(r_2^2 - r_1^2)$
- Work done in forming a circular liquid film of radius 'r' is,

  $w = 2\pi r^2 T$.
- Work done in increasing the area of circular soap film from radius r_1 to r_2 is

  $W = 2\pi T(r_2^2 - r_1^2)$.

When a big liquid drop splits into 'n' identical droplets, then



a) Surface area increases,

work is done on the system against surface tension and potential energy increase.

b) Energy is absorbed by the system.

c) Temperature of each droplet decreases.

d) As mass is constant

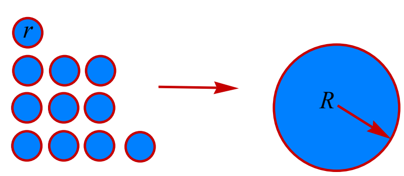
$$M = nm \Rightarrow \frac{4}{3}\pi R^3 \rho = n\left(\frac{4}{3}\pi r^3\right)\rho \Rightarrow R = n^{1/3} r$$

Increase in surface area $\Delta FA = n4\pi r^2 - 4\pi R^2$

Workdone $= T\Delta A = T4\pi\left[nr^2 - R^2\right]$

$$W = T4\pi R^2\left[n^{\frac{1}{3}} - 1\right]$$

When 'n' small droplets each of radius 'r' are merged to form a big drop of radius 'R', then



a) Work is done by surface tension and total surface area decreases,

b) Energy is released by the system.

c) Temperature of big droplet increases.

d) In this case workdone by the system is

$$W = 4\pi r^2 T\left[n^{1/3} - 1\right]$$

$$W = 4\pi r^2 T\left[n - n^{2/3}\right]$$

♦ **If** $V = \frac{4}{3}\pi R^3$ **= volume of big drop**

$$W = \frac{3VT}{R}\left(n^{1/3} - 1\right) = 3VT\left[\frac{n^{1/3}}{R} - \frac{1}{R}\right]$$

$$W = 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$$

::PROBLEMS::

1. **A small piece of wire of length 4 cm is floating on the surface of water. If a force of 560 dynes in excess of its apparent weight is required to pull it up from the surface, find the surface tension of water.**

SOLUTION :

Length of wire $l = 4cm$. Contact length of solid with liquid surface $L = 2l = 8cm$.

$$\text{Surface tension } T = \frac{F}{L} \Rightarrow T = \frac{560}{8} = 70 dyne / cm$$

$$T_{water} = 70 dyne\ cm^{-1} = 0.07 Nm^{-1}$$

2. **An annular metal ring of inner radius 7cm and outer radius 14cm and negligible weight is floating on the surface of a liquid. If surface tension of liquid is 0.08Nm^{-1}, calculate the force required to detach it from liquid surface.**

SOLUTION :

The contact length of annular ring with liquid surface

$$L = 2\pi r_1 + 2\pi r_2 \Rightarrow L = 2 \times \frac{22}{7} \times (14 + 7) cm$$

$$L = 132 cm = 1.32 m.$$

The force required.

F=T(L)=0.08x1.32=0.1056N

3. **A wire is bent in the form of a 'U'-shape and a slider of negligible mass is connecting the two vertical sides of the U-shape. This arrangement is dipped in a soap solution and lifted, a thin soap film is formed in the frame. It supports a weight of 2.0 x 10^{-2}N. If the length of the slider is 40cm, what is the surface tension of the film?**

SOLUTION :

W=2.0 x 10^{-2}N

l=40cm = 0.4 m

Upward force due to surface tension = $T \times 2l$

In equilibrium, W = $T \times 2l$

$$T = \frac{W}{2l} = \frac{2 \times 10^{-2}}{2 \times 0.4} = 2.5 \times 10^{-2} Nm^{-1}.$$

4. **When a wire of length $l(l >> r)$ and cross sectional radius r is kept floating on surface of a liquid. Maximum radius of wire such that it may not sink is**

SOLUTION :

Weight = maximum force of Surface Tension

$$Mg = T[2l] \qquad \Rightarrow \rho(\pi r^2) lg = T(2l)$$

$$r_{max} = \sqrt{\frac{2T}{\pi \rho g}}$$ (ρ is the density of the wire)

5. **If the surface tension of soap solution is 35 dynes/cm, calculate the work done to form an air bubble of diameter 14mm with that solution.**

SOLUTION :

Surface tension T=35 dynes cm^{-1}=0.035Nm^{-1}

$$\text{Radius of the bubble } r = \frac{14mm}{2} = 7mm = 710^{-3}m$$

$$W = \Delta A \times T = 8\pi r^2 T = 8 \times \frac{22}{7} \times 49 \times 10^{-6} \times 0.035$$

$$W = 4.312 \times 10^{-5} J$$

6. A soap bubble is blown to a radius of 3cm. If it is to be further blown to a radius of 4cm what is the work done? (Surface tension of soap solution = 3.06 x 10^{-2} Nm^{-1})

SOLUTION :

Initial radius of soap bubble R_1=3cm=3x10^{-2}m

Final radius of sopa bubble R_2=4cm=4x10^{-2}m

Work done in blowing soap bubble from radius R_1 to R_2 is

$$\Rightarrow W = 8\pi\left(R_2^2 - R_1^2\right)T$$

$$= 8 \times \frac{22}{7} \times 3.06 \times 10^{-2}(16-9) \times 10^{-4}$$

$$= 176 \times 3.06 \times 10^{-6} J = 539.6 \times 10^{-6} J$$

7. A water drop of diameter 2mm is split up into 10^9 identical water drops. Calculate the work done in this process. (The surface tension of water is 7.3x$10^{-2}$$Nm^{-1}$).

SOLUTION :

Let a water drop of radius R be split up into 10^9 identical water drops each of radius r.

$$R = \frac{D}{2} = \frac{2}{2} = 1mm = 1 \times 10^{-3}m$$

No. droplets n=10^9; T=7.3x$10^{-2}$$Nm^{-1}$

$$W = 4\pi R^2 T\left[n^{1/3} - 1\right]$$

$$= 4\pi\left(10^{-3}\right)^2 \times 7.3 \times 10^{-2}\left[\left(10^9\right)^{1/3} - 1\right] = 9.17 \times 10^{-4} J$$

8. 1000 drops of a liquid each of diameter 4mm coalesce to form a single large drop. If surface tension of liquid is 35 dyne cm^{-1}. Calculate the energy evolved by the system in the process.

SOLUTION :

No. of drops n=1000 $\Rightarrow n^{1/3} = 10; n^{2/3} = 100$

Surface tension of liquid T=35 dyne cm^{-1}

Radius of each small drop r=2mm=0.2cm

$$\text{Energy evolved in merging } W = 4\pi r^2 T\left[n - n^{2/3}\right]$$

$$\Rightarrow W = 4 \times \frac{22}{7} \times \left(2 \times 10^{-1}\right)^2 35[1000 - 100]$$

$$\Rightarrow W = \frac{88 \times 35 \times 4 \times 10^{-2}}{7}[900]$$

$$= 15840 ergs. \approx 1.58 \times 10^{-3} J$$

9. If n drops of a liquid, each with surface energy E, join to from a single drop, then

A) Some energy will be released in the process

B) Some energy wil be released in the process

C) the energy released or ansorbed wil be $E\left(n-n^{2/3}\right)$

D) the energy released or absorbed will be $nE\left(2^{2/3}-1\right)$

KEY : A , C

SOLUTION :

Let S = surface tension
= surface energy per unit drop
r = radius of each small drop
R radius of a single drop

$$n\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$$

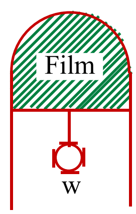
$$R=rn^{1/3}$$

Initial surface energy $E_i=n\times 4\pi r^2\times S=nE$

Final surface energy, $E_f=4\pi R^2 S=4\pi r^2 n^{2/3}S=n^{2/3}E$

Therefore, energy released $=E_i-E_f=E\left(n-n^{2/3}\right)$

10. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N. The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is (AIEEE-2012)



1) $0.0125Nm^{-1}$ 2) $0.1Nm^{-1}$ 3) $0.05Nm^{-1}$ 4) $0.025Nm^{-1}$

SOLUTION :

At equilibrium, weight of the given block is balanced by force due to surface tension,
i.e., 2L.T=W

$$T=\frac{W}{2L}=\frac{1.5\times10^{-2}N}{2\times0.3m}=0.025Nm^{-1}$$

11. Work W is required to from a bubble of volume V from a given solution. What amount of work is required to be done to form a bubble of volume $2V$?

A) W B) $2W$ C) $2^{1/3}W$ D) $4^{1/3}W$

SOLUTION :

.
$$V\times\frac{4}{3}\pi R^3;2V=\frac{4}{3}\pi R^3;2\times\frac{4}{3}\pi R^3=\frac{4}{3}\pi R^3$$

$$R'=2^{1/}R$$

$$W'\pi\times\left[2^{1/3}R\right]^2\sigma=2^{2/3}\times2\times4\pi R^2\sigma=4^{1/3}W$$

12. A large number of liquid drops each of radius "r" merge to form a single spherical drop of radius "R". If the energy released in the process is converted into the kinetic energy of the big drop formed. Find the speed of the big drop (d is density of the liquid)?

SOLUTION :

$$\text{Energy released is } W = 3VT\left[\frac{1}{r}-\frac{1}{R}\right].....(1)$$

If V is volume of big drop, M the mass of the drop and ρ the density then

$$\text{Kinetic energy} = \frac{1}{2}Mv^2 = \frac{1}{2}(\rho V)v^2...(2)$$

As (1) = (2)

$$3VT\left[\frac{1}{r}-\frac{1}{R}\right] = \frac{1}{2}(\rho V)v^2$$

$$\text{or } v = \sqrt{\frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$$

13. A drop of liquid of density ρ is floating half immersed in a liquid of density d. If σ is the surface tension, then what is the diameter of the drop of the liquid.

1) $\sqrt{\frac{3\sigma}{g(2\rho-d)}}$ **2)** $\sqrt{\frac{6\sigma}{g(2\rho-d)}}$ **3)** $\sqrt{\frac{4\sigma}{g(2\rho-d)}}$ **4)** $\sqrt{\frac{12\sigma}{g(2\rho-d)}}$

SOLUTION :

In equilibrium

force due to surface tension + Force of buoyancy = Weight of the spherical liquid drop

$$2\pi rT + \frac{2}{3}\pi r^3 d_2 g = \frac{4}{3}\pi r^3 d_1 g$$

$$T = \sigma, d_1 = \rho, d_2 = d$$

$$2\pi r\sigma + \frac{2}{3}\pi r^3 dg = \frac{4}{3}\pi r^3 \rho g$$

$$r^2 = \frac{3\sigma}{g(2\rho-d)} \text{ or } r = \sqrt{\frac{3\sigma}{g(2\rho-d)}}$$

$$\text{Diameter} = 2r = \sqrt{\frac{12\sigma}{g(2\rho-d)}}$$

14. A large number of droplets, each of radius a, Coalesce to from a bigger drop of radius b. Assume that the energy released in the process is converted into the kinetic energy of the drop. The velocity of the drop is (σ =surface tension, ρ = density)

A) $\left[\frac{\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$ **B)** $\left[\frac{2\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$ **C)** $\left[\frac{3\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$ **D)** $\left[\frac{6\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$

SOLUTION :

$$\text{Energy released}\left[n\times 4\pi a^2-4\pi b^2\right]\sigma$$

$$\text{Now, } 4\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi b^3 \text{ or } n=\frac{b^3}{a^3}$$

Therefore, enery released is

$$\left[\frac{b^3}{a^3}\times 4\pi a^2-4\pi b^2\right]\sigma=4\pi b^2\left(\frac{b}{a}-1\right)\sigma$$

$$\text{Now, } \frac{1}{2}\left(\frac{4}{3}\pi b^2\right)\rho v^2=4\pi b^2\left[\frac{b}{a}-1\right]\sigma$$

$$\text{or } v=\left[\frac{6\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$$

15. A drop of radius R is split under isothermal conditions into 'n' droplets, each of radius 'r', the ratio of surface energies of big and each small drop is

SOLUTION :

$$\frac{U_{big}}{U_{small}}=\frac{\left(4\pi R^2\right)T}{\left(4\pi r^2\right)T}=\frac{R^2}{r^2}=\frac{\left(n^{1/3}r\right)^2}{r^2}=n^{2/3}:1$$

16. Eight spherical droplets, each of radius 'r' of a liquid of density 'ρ' and surface tension 'r' coalesce to form one big drop. If 's' in the specific heat of the liquid. Then the rise in the temeprature of the liquid in this process is

1) $\frac{2T}{3r\rho s}$ **2)** $\frac{3T}{r\rho s}$ **3)** $\frac{3T}{2r\rho s}$ **4)** $\frac{T}{r\rho s}$

SOLUTION :

$$Q=ms\Delta t\Rightarrow T\Delta A=ms\Delta t$$

$$T\left(16\pi r^2\right)=\frac{4}{3}\pi R^3\rho S\Delta T$$

$$T\left(16\pi r^2\right)=\frac{4}{3}(2\pi)^3\rho S\Delta T\Rightarrow\Delta t=\frac{3T}{2r\rho s}$$

17. Number of droplets are combined isothermally to form a big drop, the ratio of initial and final surface energies of the system is

SOLUTION :

$$\frac{u_i}{u_f}=\frac{n\left(4\pi r^2\right)T}{\left(4\pi R^2\right)T}=\frac{nr^2}{R^2}=\frac{nr^2}{\left(n^{\frac{1}{3}}r\right)}=n^{\frac{1}{3}}:1$$

18. Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is (AIEEE-2011)

1) $2^{5/3}\pi r^2T$ **2)** $4\pi r^2T$ **3)** $2\pi r^2T$ **4)** $2^{8/3}\pi r^2T$

SOLUTION :

Let R be the radius of the bigger drop,
then volume of bigger drop = 2 x volume of small drop

$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3;\ R = 2^{1/3} r$$

Surface energy of bigger drop,

$$E = 4\pi R^2 T = 4\times 2^{2/3}\pi r^2 T = 2^{8/3}\pi r^2 T$$

19. When a big drop of water is formed from n small drops of water, the energy loss is 3E, where, E is the energy of the bigger drop. If R is the radius of the bigger drop and r is the radius of the smaller drop, then number of smaller drops (n) is (EAMCET-2014)

SOLUTION :

The energy of n small drops-the energy of the bigger drop=Energy loss by bigger drop

$$n\times 4\pi r^2\times T - 4\pi R^2\times T = 3\times 4\pi R^2\times T$$

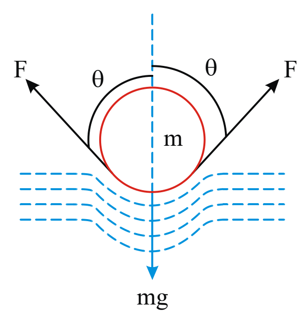
$$n\times 4\pi r^2 = 12\pi R^2 + 4\pi R^2 \Rightarrow n = 4\frac{R^2}{r^2}$$

20. Find the maximum possible mass of a greased needle floating on water surface. T is the surface tension of water, l is the length of the needle

A) $m_{\max} = \frac{2Tl}{g}$ **B)** $m_{\max} = \frac{g}{2Tl}$ **C)** $m_{\max} = \frac{2Tg}{l}$ **D)** $m_{\max} = \frac{Tl}{g}$

SOLUTION :

Let the mass of the needle be m.Asthe liquid surface is distorted,the surface tensionforces acing on both sides of the needle make an angle θ,say,with vertical.Since the forces acting on the needle are F,F and mg, resolving the forces vertically for its equilibrium, we have



$$\sum F_y = F\cos\theta + F\cos\theta - mg = 0$$

This gives $m = \frac{2F\cos\theta}{g}$

where $F = Tl$

Then $m = \frac{2Tl\cos\theta}{g}$

For m to be maximum, $\cos\theta = 1$

Hence, $m_{\max} = \frac{2Tl}{g}$

21. **A straw 6 cm long floats on water. The water film on one side has surface tension of 50 dyne/cm. On the other side, camphor reduces the surface tension to 40 dyne/cm. The resultant force acting on the straw is**

1) 60 dyne **2) 10 dyne** **3) 30 dyne** **4) 0 dyne**

SOLUTION :

$\vec{F} = \vec{F_1} + \vec{F_2} = (T_1 - T_2)l$

22. **A drop of water of mass m and density ρ is placed between two well cleaned glass plates, the distance between which is d. What is the force of attraction between the plates? (T=surface Tension)**

1) $\frac{Tm}{2\rho d^2}$ **2)** $\frac{4Tm}{\rho d^2}$ **3)** $\frac{2Tm}{\rho d^2}$ **4)** $\frac{Tm}{\rho d^2}$

SOLUTION :

$$F = 2TA/d$$
$$m = \rho V = \rho Ad$$
$$A = \frac{m}{\rho d}$$
$$F = \frac{2Tm}{\rho d^2}$$

23. **Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03Nm^{-1})** **(AIEEE-2011)**

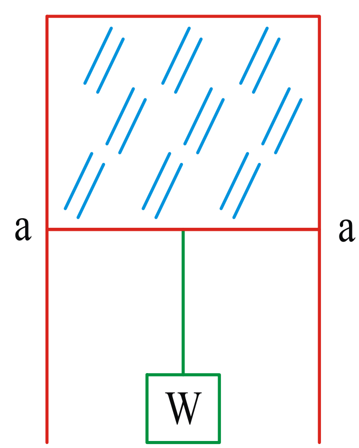
1) $0.2\pi mJ$ **2)** $2\pi mJ$ **3)** $0.4\pi mJ$ **4)** $4\pi mJ$

SOLUTION :

Work done = change in surface energy

$$\Rightarrow W = 2T \times 4\pi\left(R_2^2 - R_1^2\right)$$
$$= 2 \times 0.03 \times 4\pi\left[(5)^2 - (3)^2\right] \times 10^{-4} J = 0.4\pi mJ$$

24. **A film of soap solution is trapped between a vertical frame and a light wire ab of length $0.1m$. If $g = 10m/s^2$. Then the load W that should be suspended from the wire to keep it in equilibrium is**



A) 0.2 g **B) 0.3g** **C) 0.4 g** **D) 0.5g**

SOLUTION :

$$25\times10^{-3}\times2\times0.1=m\times10$$

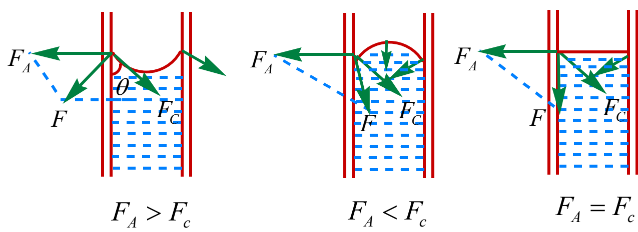
$$m=\frac{5\times10^{-3}}{10}kg=0.5g$$

Angle of contact:

- ♦ Angle of contact of a liquid with respect to a solid is the angle between the tangent drawn to the liquid surface at the point of contact and the surface of the solid, measured inside the liquid.
- ♦ The angle of contact depends on solid-liquid pair, temperature and impurities.
- ♦ The angle of contact may assume any value between 0^0 to 180^0.
- ♦ If the angle of contact is less than 90^0 then the liquid wets the solid
- ♦ If the angle of contact is greater than 90^0 then the liquid does not wet the solid.
- ♦ The angle of contact between pure water and glass = 0^0
- ♦ The angle of contact between Hg and glass=140^0.
- ♦ The angle of contact is not changed by the angle of inclination of solid object in the liquid.
- ♦ Water proofing agents increase the angle of contact.
- ♦ Wetting agents decrease the angle of contact.

Shape of a liquid surface in a tube:

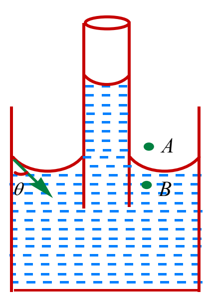
- ♦ When a glass capillary tube is dipped in water, water rises into the tube.
- ♦ When a glass capillary tube is dipped in mercury, mercury depresses into the tube.
- ♦ When a silver capillary tube is dipped in water, water neither rises nor falls.



Case(i) Case(ii) Case(iii)

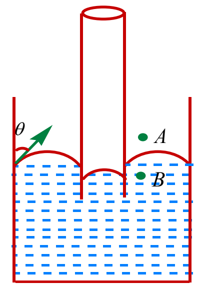
(F_A:adhesive forces, F_C=Cohesive forces, P_A:Pressure at A, P_B : Pressure at B)

Case (i) : - When glass capillary tube is dipped in water, observations are



1) Capillary rise
2) $F_A > F_C$ (adhesive > Cohesive)
3) Concave meniscus
4) Water wets the glass
5) $\theta < 90^o$
6) $P_A > P_B$

Case (ii) : - When glass capillary tube is dipped in Mercury, observations are



1) Capillary fall

2) $F_A < F_C$ (adhesive < Cohesive)

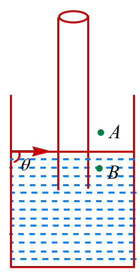
3) Convex meniscus

4) $\theta > 90^o$

5) $P_B > P_A$

6) Mercury does not wet the glass

Case (iii) : - When silver capillary tube is dipped in water, observations are



1) water neither rises nor falls

2) $F_A = F_C$ (adhesive = Cohesive)

3) Flat surface (not curved)

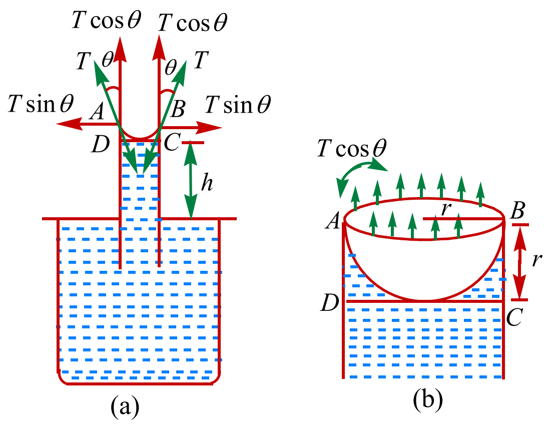
4) Critical wetting

5) $\theta = 90^o$

6) $P_A = P_B$

Capillarity:

- The rise or fall of a liquid column in a capillary tube dipped in a liquid is known as capillarity.
- Capillarity is due to relative strengths of cohesive and adhesive forces.
- In a gravity free space, a liquid in a capillary tube will rise to full length of the tube but not over flow.
- The weight of the liquid column in the capillary tube is balanced by the force due to surface Tension.



$$2\pi r T \cos\theta = Mg$$

$$U = weight \times \frac{h}{2} = \frac{mgh}{2} = \frac{2\pi T^2 \cos^2\theta}{dg}$$

$$T = \frac{r(h + r/3)dg}{2\cos\theta}$$

If r is very small compared to h,

then $T = \dfrac{hrdg}{2\cos\theta}$

where

r = radius of the capillary tube

h = height of liquid column,

d = density of the liquid,

g = acceleration due to gravity,

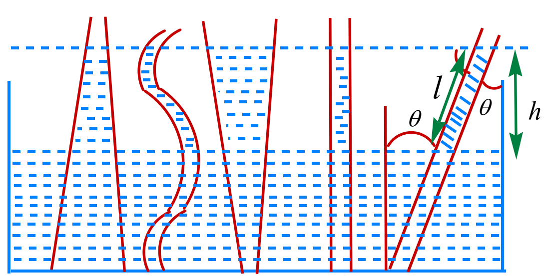
θ = angle of contact

♦ Gravitational Potential energy of a liquid rises in a tube is

$$U = weight \times \frac{h}{2} = \frac{mgh}{2} = \frac{2\pi T^2 \cos^2\theta}{dg} \text{ (when r is very less than h)}$$

♦ When diameter of capillary tube increases twice, the height of liquid column falls down to half. $(r_1h_1 = r_2h_2)$.

♦ Since $h \propto \frac{1}{r}$, the graph between h and r is a rectangular hyperbola.



♦ For a given radius, the capillary rise in a capillary tube does not depend either on the angle of inclination or on the shape of the tube.

$$\cos\theta = \frac{h}{l} \Rightarrow \boxed{h = l\cos\theta}$$

$$l_1 \cos\theta_1 = l_2 \cos\theta_2$$

h = height of water in the tube

l = length of water in the tube

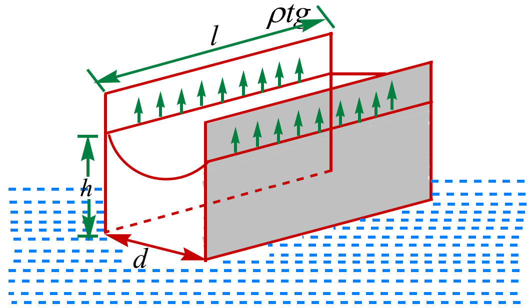
♦ If the radii of the two limbs of a U-tube are r_1 and r_2, the difference between the levels of a liquid in 'u' tube is

$$\Delta h = \frac{2T}{dg}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\left(if\ r_1 < r_2 << h\right)$$

♦ If two parallel plates with the spacing 't' are placed in water reservoir, then height of rise

$$2T\ell - mg = V\rho g \approx \ell ht\rho g \ \ (\rho \text{ density of the liquid})$$

$$h = \frac{2T}{\rho tg}$$

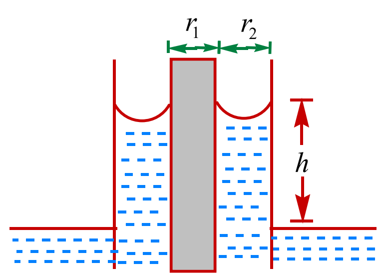


♦ If two concentric cylinders of radii r_1 & r_2 (inner one is solid are placed in water reservoir,

$$T\cos\theta\left(l_1 + l_2\right) = mg$$

$$T\left[2\pi r_1 + 2\pi r_2\right] = \left[\pi r_2^2 h - \pi r_1^2 h\right]\rho g\left(\because \theta = 0^0\right)$$

$$h = \frac{2T}{\left(r_2 - r_1\right)\rho g}\left(if\ r_1 < r_2 << h\right)$$



♦ A drop of liquid of density d_1 are floating half immersed in a liquid of density d_2. If T is the surface tension of the liquid, then the radius of the drop is (if $\theta = 0^0$);

$$F_{surface\ tension} + F_{buoyancy} = mg$$

$$2\pi rT\cos\theta + \frac{2}{3}\pi r^3 d_2 g = \frac{4}{3}\pi r^3 d_1 g$$

$$r = \sqrt{\frac{3T}{\left(2d_1 - d_2\right)g}}$$

■**A capillary tube is vertically dipped in a liquid. The height of the liquid in the tube is 'h' and the total set up is kept in a lift.**

- If the lift is moving up with an acceleration 'a' then the height of the liquid in the tube is given by

$$h' = h\left[\frac{g}{g+a}\right]$$

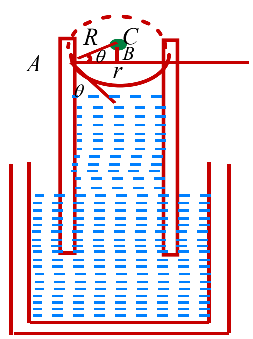
- If the lift is moving down with an acceleration 'a' then the height of the liquid in the tube is given by

$$h' = h\left[\frac{g}{g-a}\right]$$

- If the lift is falling freely the height of the liquid raised in the tube is equal to full length of the tube available, but not over flow.

$$h' = h\left[\frac{g}{g-g}\right] = \infty \text{ (is not true in such situations)}$$

The relation between radius of tube and radius of meniscus is:



AB = radius of tube (r)

AC = radius of meniscus (R)

In $\Delta ABC \quad \cos\theta = \frac{AB}{AC} = \frac{r}{R}$

$\Rightarrow \boxed{r = R\cos\theta}$

If $\theta = 0^o$ then r = R;

If $\theta = 90^o$, $R = \frac{r}{\cos 90^0} = \infty$

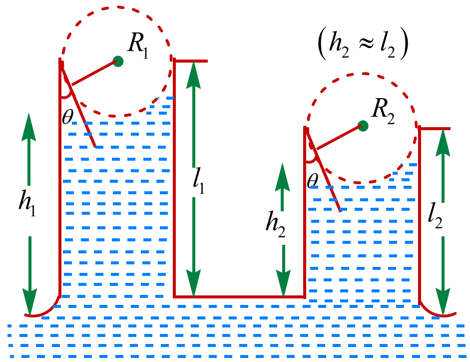
i.e., The liquid meniscus is plane.

Capillary tube of insufficient length:

Theoretically the rise of liquid in the tube is $h = \frac{2T}{Rpg}$.

If the length of the tube above liquid is ℓ, if less than h, then the liquid will rise to full length of the tube and the free surface of the liquid will acquire larger radius of curvature in such a way, that the product

$$h_1R_1 = h_2R_2 \approx l_2R_2$$



We have, $h = \frac{2T}{R\rho g}$

Thus, When a capillary tube of insufficient length is dipped in the liquid, the liquid will not overflow but stays at the top with adjustable meniscus.

Effect of temperature on surface tension:

Over small ranges of temperature, the surface tension of a liquid decreases linearly with the rise of temperature due to increase in inter molecular distances according to the relation

$$S_t = S_0 (1 - \alpha\Delta t)$$

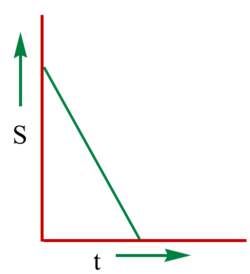
where s_t = surface tension at t^0C

S_0 = surface tension at 0^0C

Δt = Change in temperature.

α = temperature coefficient of surface tension.

Dependence of Surface Tension -On temperature :



- ♦ In the case of molten copper and molten cadmium,surface tension increases with increase of temperature.
- ♦ The surface tension of any liquid at its critical temperature is zero.

Effect of impurities on Surface Tension:

With the addition of impurities surface tension may increase or decrease, depending on the type of impurity.

- ♦ There are two types of impurities.

  They are
  (i) weakly soluble impurities,
  (ii) highly soluble impurities.
- ♦ If the added impurities are weakly soluble in liquid its surface tension decreases.

  Eg: When soap is mixed with water surface tension decreases.
- ♦ If the added impurities are highly soluble in liquid, its surface tension increases.

  Eg: When highly soluble salts like NaCl, $ZnSO_4$, etc. are mixed with water, its surface tension increases.

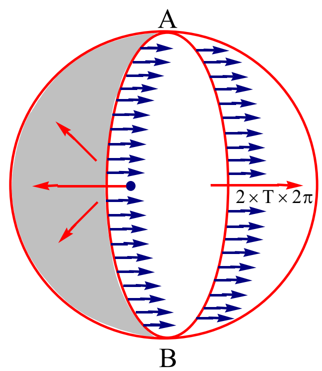
Excess Pressure inside a curved liquid surface :

The pressure on the concave side of curved liquid surface is greater than that on the convex side. This is the reason why pressure difference exists across two sides of a curved surface.

Case : A

Excess pressure inside a soap bubble

= Pressure inside the bubble - outside pressure soap bubble has two free surface



(A) one is exposed to outside air

(B) another to inside air.

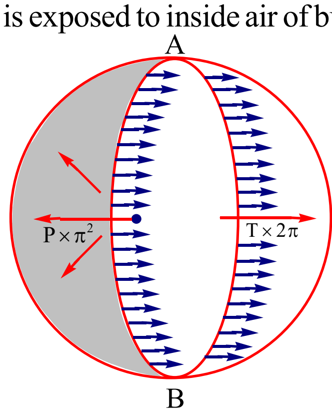
Balancing atmospheric pressure = surface tension at 2 free surface soap surface has [2free surfaces]

$$P\times pr^2 = 2\times T\times 2pr \Rightarrow P = \frac{4T}{r}$$

Case : B

Excess pressure in air bubble inside a liquid.

Here only one free surface which is exposed to inside air of bubble.



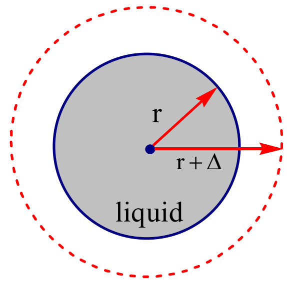
Balancing liquid pressure = surface tension at one free surface

$$P\times pr^2 = T\times 2pr[\text{one free surface}]$$

$$P = \frac{2T}{r}$$

Case : C

Excess pressure in a liquid drop



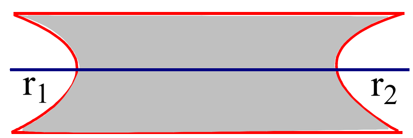
here, one free surface exposed to air

Balancing atm pressure = surface tension at one free surface

$$P \times pr^2 = T \times 2pr \Rightarrow P = \frac{2T}{r}$$

Case :D

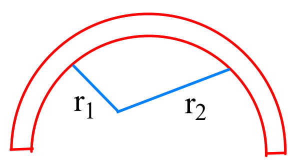
If curvature are in same direction



$$P = T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Case : E

If the curvature are in same direction



$$P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

From above

For cylindrical surface $P = \frac{T}{r} \backslash r_1 = r, r_2 = \infty$

For spherical surface $P = \frac{2T}{r} \backslash r_1 = r_2 = r$

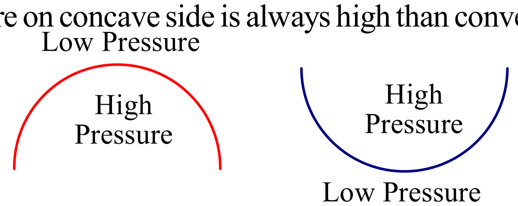
Note: 1

(A) Always for liquid surface one free surface is taken

(B) Always for liquid film two free surface is taken.

Note : 2

For liquid surface, pressure on concave side is always high than convex side.



When the soap bubble coalesce, then the radius of curvature of common surface

Excess pressure in first bubble $P_1 = \frac{4T}{r_1}$

Excess pressure in second bubble $P_2 = \frac{4T}{r_2}$

Excess pressure on common surface $P = P_1 - P_2$
if radius of curvature of common surface is r,

$$P = \frac{4T}{r}$$

$$P = P_1 - P_2$$

$$\frac{4T}{r} = \frac{4T}{r_1} - \frac{4T}{r_2}$$

$$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$r = \frac{r_1 r_2}{r_2 - r_1}, \text{ where } r_1 < r_2$$

When two soap bubble of radii r_1 & r_2, combine to form a new bubble in vaccum under isothermal condition,

At isothermal condition SPV=0

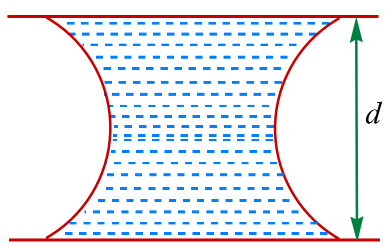
$$P_1V_1 + P_2V_2 = PV$$

$$\frac{4T}{r_1}.\frac{4}{3}pr_1^3 + \frac{4T}{r_2}.\frac{4}{3}pr_2^3 = \frac{4T}{r}.\frac{4}{3}pr^3$$

$$r = \sqrt{r_1^2 + r_2^2}$$

Liquid betwen two plates:

When a small drop of water is placed between two glass plates put face to face. it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount P given by

$$P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\because R_1 = R = \frac{d}{2}; R_2 = \infty\right)$$

$$= \frac{T}{R} = \frac{T}{d/2} = \frac{2T}{d}$$

Force required to separate glass plates is

$$F = \frac{2T}{d} \times A$$

$$\Rightarrow \frac{2T \times A}{V \times A} = \frac{2TA^2}{V} \left(\because V = A \times d \right)$$

Detergents And Surface Tension:

♦ We clean dirty clothes containing grease and oil stains sticking to cotton (or) other fabrics by adding detergents (or) soap to water.

Adding detergent or soap to water makes the angle of contact less than 90^0 and there by wets the clothes.

♦ The kind of process using surface active detergents or surfactants is important not only for cleaning, but also in recovering oil, mineral ores, etc.

Wetting Agents:

Wetting agent is a material, mixed with liquid, to decrease the angle of contact with the given solid.

Eg: Soaps and detergents

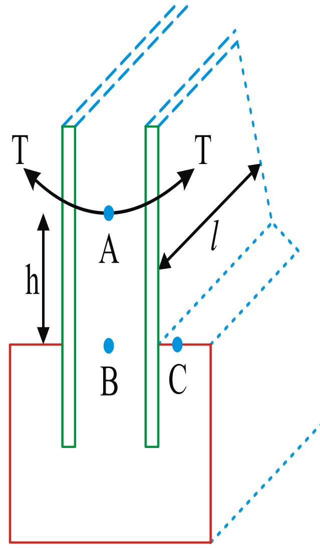
Water Proofing Agent:

Water proofing agent is a material applied on the surface of solid to increase the angle of contact with water.

Eg : Wax

::PROBLEMS ::

1. Expression for the height of capillary rise between two parallel plates dipping in a liquid of density σ separated by a distance D) The surface tension of the liquid is T. [Take angle of contact to be zero]



A) $h = \frac{2T}{\sigma dg}$ **B)** $h = \frac{2d}{\sigma T}$ **C)** $h = \frac{\sigma T}{d}$ **D)** $h = \frac{2T^2}{\sigma d}$

SOLUTION :

The meniscus between two plates is cylindrical in shape. Pressure at A(the lowestpoint of the meniscus)

$$p_A = p_0 - T/r$$

Pressure at B = Pressure at $C = p_0$ = Pressure at $A + \sigma gh$

$$\therefore P_B = p_0 = p_0 - \frac{T}{r} + \sigma gh,\ h = \frac{T}{\sigma gr} = \frac{2T}{\sigma gd}$$

Alternative method:

Force upward $= 2lT\cos\theta = 2lT \left(\because \theta = 0^0\right)$

Gravitational pull = (Volume × Density)g $= lhd\sigma g$

$$\therefore 2lT = lhd\rho g \Rightarrow h = \frac{2T}{d\sigma g}$$

2. Find the weight of water supported by surface tension in a capillary tube with a radius of 0.2mm. Surface tension of water is $0.072\,Nm^{-1}$ and angle of contact of water is 0^0.

SOLUTION :

Assume the weight of water to be ‘F’

Weight of water in capillary tube = upward force due to surface tension

i.e., $F = 2\pi r\left(T\cos\theta\right)$

Surface tension of water $T = 0.072\,Nm^{-1}$

Angle of contact $\theta = 0^0$, Radius of capillary tube (r) $= \frac{0.2}{1000} m = 0.2 \times 10^{-3} m$

$$F = 2\pi r(T\cos\theta) = 2 \times \frac{22}{7} \times 0.2 \times 10^{-3} \times 0.072 \times 1$$

$$= 90.51 \times 10^{-6} N \Rightarrow F = 90.51 \times 10^{-6} N$$

3. A capillary tube of radius 'r' is immersed in water and water rises to a height of 'h'. Mass of water in the capillary tube is $5x10^{-3}$ kg. The same capillary tube is now immersed in a liquid whose surface tension in $\sqrt{2}$ times the surface tension of water. The angle of contact between the capillary tube and this liquid is 45^0. The mass of liquid which rises into the capillary tube now is, (in kg) (EAM-13)

SOLUTION :

Height of water rise in a capillary tube

$$h = \frac{2T\cos\theta}{rdg}; h_1 = \frac{2T_1\cos\theta_1}{rdg}; h_2 = \frac{2T_2\cos\theta_2}{rdg}$$

Given, $T_2 = \sqrt{2}T, \theta = 45^0, \cos 45^0 = \frac{1}{\sqrt{2}}$

$$\therefore h_2 = \frac{2\sqrt{2}T \times \frac{1}{\sqrt{2}}}{rdg}$$

From Eqs(i) and (ii), we observe $h_2=h$

Hence, same mass of liquid rises into the capillary as before 5×10^{-3} kg.

4. A U-tube is supported with its limbs vertical and is partly filled with water. If internal diameters of the limbs are $1x10^{-2}$m and $1x10^{-4}$m respectively, what will be the difference in heights of water columns in the two limbs (Surface tension of water is $0.07Nm^{-1}$)

SOLUTION :

Surface tension, $T=0.07Nm^{-1}$;

Density, $d=1000kgm^{-3}$;

$g=9.8ms^{-2}$

Angle of contact $\theta = 0^0$;

Radius, $r_1=0.5x10^{-2}$m;

Radius, $r_2=0.5x10^{-4}$m

Then, $h = \frac{2T}{dg}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$$= \frac{2 \times 0.07}{10^3 \times 9.8}\left(\frac{1}{0.5 \times 10^{-2}} - \frac{1}{0.5 \times 10^{-4}}\right) = 0.283m$$

5. A glass capillary sealed at the upper end is of length 0.11m and internal diameter 2×10^{-5} m. The tube is immersed vertically into a liquid of surface tension $5.06 \times 10^{-2} N/m$. To what length the capillary has to be immersed so that liquid level inside and outside the capillary becomes the same.?

A) 5cm **B) 3cm** **C) 1cm** **D) 7cm**

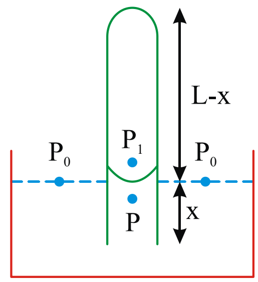
SOLUTION :

Let P_0 be atmospheric pressure and P_1 the pressure of air within the sealed tube.

Therefore equilibrium pressure just below the meniscus should be equal to atmospheric pressure because levels of water inside and outside the tube is same.

$$\text{i.e.,}\quad \left(P_1 - \frac{2T}{r}\right) = p_0 or P_1 = p_0 + \frac{2T}{r}$$

If L=0.11m is the length of tube and x the



length of immersed part, then from Boyle's law $p_1V_1 = p_2V_2$; $P_0La = \left(P_0 + \frac{2T}{r}\right)(L-x)a$

where a is the cross-sectional area of tube,

$$\text{i.e.,} P_0 \times L = \left(P_0 + \frac{2T}{r}\right)(L-x)$$

$$P_0 \times L = P_0(L-x) + \frac{2T}{r}(L-x)$$

$$x = \frac{4TL}{P_o d + 4T}$$

6. What would be the pressure inside a small air bubble of 1.0mm radius situated just below the surface of water. T=72 x 10^{-3}N/m
Atm. pr. = 1.013 x 10^5N/m²

SOLUTION :

$$\text{Excess pressure } P_{ex} = \frac{2T}{r}$$

$$P_{ex} = \frac{2 \times 72 \times 10^{-3}}{1 \times 10^{-3}} = 1440 N/m^2$$

For air bubble in water as on free surface since the bubble is just below the water surface, the external pressure on it is equal to the atm. pressure P, hence the pressure inside the bubble.

$P + P_{ex} = 1.013 \times 10^5 + 1440$
$= 1.0274 \times 10^5$ N/m²

7. Water rises to a height of 10 cm in a capillary tube and mercury falls to a depth of 3.42 cm in the same capillary tube. If the denisty of mercury is 13.6 g/c.C) and the angles of contact for mercury and water are 135^0 and 0^0, respectively, the ratio of surface tension for water and mercury is

A) $1:0.15$ **B)** $1:3$ **C)** $1:6.5$ **D)** $1.5:1$

SOLUTION :

$$\text{Heoight}, \; h = \frac{2T\cos\theta}{r\rho g}$$

$$\therefore \text{ For water, } h_w = \frac{2\times T_w \times \cos 0^0}{r\times 1\times g}$$

$$\text{And, for mercury, } h_m = \frac{2\times T_m \times \cos 135^0}{r\times 13.6\times g}$$

$$\therefore \frac{h_w}{h_m} = \frac{2\times T_w\times 1}{r\times 1\times g}\times\frac{r\times 13.6\times g\times\sqrt{2}}{2\times T_m\times 1}$$

$$\left[\because \cos 135^0 = -1/\sqrt{2}\right] \Rightarrow \frac{10}{3.42} = \frac{T_w}{T_m}\times 13.6\times\sqrt{2}$$

$$\therefore \frac{T_w}{T_m} = \frac{10}{3.42\times 13.6\times 1.414} = \frac{1}{6.5}$$

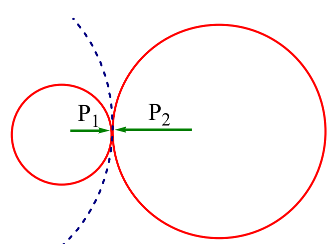
$$\therefore T_w : T_m = 1 : 6.5$$

8. Two separate air bubbles (r_1=0.02cm, r_2 = 0.004cm) formed of same liquid T = 0.07N/m come together to form a double bubble. Find the radius and sense of curvature of the internal film surface common to both the bubbles

SOLUTION :

$$r = \frac{r_1 r_2}{r_2 - r_1}$$

$$r = \frac{0.003\times 0.004}{0.004 - 0.002} = 0.004\text{m}$$



As the excess pressure is always towards convace surface & pressure in smaller bubble is greater than larger bubble, the common surface is concave towards the centre of the smaller bubble.

9. Two soap bubbles of radii R_1 and R_2 are kept in vacuum at constant temperature, the ratio of masses of air inside them, is

SOLUTION :

$$\textbf{From, } PV = \frac{m}{M}RT$$

$$\text{i.e., } m \propto PV$$

$$P_1 = P_0 + \frac{4T}{R_1} = 0 + \frac{4T}{R_1}; P_2 = P_0 + \frac{4T}{R_2} = 0 + \frac{4T}{R_2}$$

$$\frac{m_1}{m_2} = \frac{P_1V_1}{P_2V_2} = \frac{\left(\frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3}{\left(\frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3} \qquad \therefore \frac{m_1}{m_2} = \frac{R_1^2}{R_2^2}$$

10. Two soap bubble of raddi R_1 and R_2 are in atmosphere of pressur eP_0 at constant temperature. Ratio of masses of air inside them is

SOLUTION :

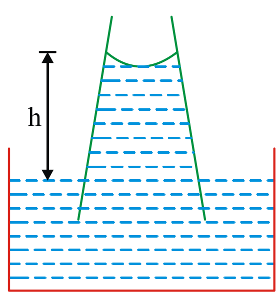
$$PV = \frac{m}{M}RT$$

$$\therefore m \propto PV = \frac{P_1V_1}{P_2V_2}$$

$$p_1 = p_0 + \frac{4T}{R_1}, p_2 = p_0 + \frac{4T}{R_2}$$

$$= \frac{\left(P_0 + \frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3}{\left(P_0 + \frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3} = \frac{\left(P_0 + \frac{4T}{R_1}\right)R_1^3}{\left(P_0 + \frac{4T}{R_2}\right)R_2^3}$$

11. A capillary of the shape as shown is dipped in a liquiD) Contact angle between the liquid and the capillary is 0^0 and mass of liquid inside the meniscus is to be neglecteD) T is surface tension of the liquid, r is raidus of the meniscus, g is acceleration due to gravity and ρ is density of the liquid then height h in equilibrium is



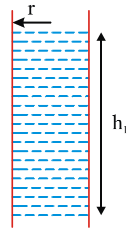
A) Greater than $\frac{2T}{r\rho g}$ **B) Equal to** $\frac{2T}{r\rho g}$ **C) less than** $\frac{2T}{r\rho g}$

D) of any value depending upon actual values

SOLUTION :

. As weight of liquid in capillary is balanced by surface tension,

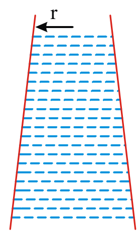
then $T = 2\pi r = \pi r^2 h, pg$ (for uniform r radius tube)



$; \quad h_1 = \frac{2T}{rpg}$

But weight of liquid in tapered tube is more than uniform tube of radius r, then in order to balance is

$< h_1$



$; \quad h_1 < \frac{2T}{rpg}$

12. Two soap bubbles are combined isothermally to form a big bubble of radius R. If ΔV is change in volume, ΔS change in surface area and P_1 is atmospheric pressure then show that

$3P_0(\Delta V) + 4T(\Delta S) = 0$

SOLUTION :

PV=constant.

After comibining the two bubbles $\left(P_0 + \frac{4T}{R}\right)V$ =constant;

$$P_0 V + \frac{4T}{R} \cdot \frac{4}{3}\pi R^3 = C$$

Before combining the two bubbles $P_0(V_1 + V_2) + \frac{4T}{3}(S_1 + S_2) = C....(2)$

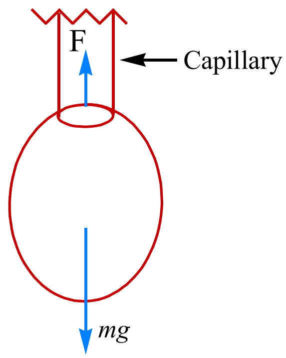
According to Boyle's law $P_1V_1 + P_2V_2 = PV$

from equations (1) and (2)

$$P_0(V_1 + V_2) + \frac{4T}{3}(S_1 + S_2) = P_0V + \frac{4T}{3}S$$

$$P_0\Delta V + \frac{4T}{3}\Delta S = 0; 3P_0(\Delta V) + 4T(\Delta S) = 0$$

13. **Soapy water drips from a capillary tube. When the drop breaks away, the diameter of its neck is D. The mass of the drop is m. Find the surface tension of soapy water?**



1) $\frac{mg}{\pi^2 D}$ 2) $\frac{mg}{\pi D^2}$ 3) $\frac{mg}{\pi D}$ 4) $\frac{mg}{2\pi D}$

SOLUTION :

When the drop breaks away from the capillary, weight of the drop = force of surface tension.

$$\text{or } mg = \pi D \times T \text{ or } T = \frac{mg}{\pi D}$$

14. **When air bubble comes from bottom to the top of a lake its radius becomes n times. If temperature remains constant through out the lake the depth of the lake will be,**

SOLUTION :

From Boyle's law PV=constant

$$\Rightarrow P_1V_1 = P_2V_2$$

$$\left(P_0 + h\rho g + \frac{2T}{R}\right)\frac{4}{3}\pi R^3 = \left(P_0 + \frac{2T}{nR}\right)\frac{4}{3}\pi (nR)^3$$

$$h\rho g = P_0\left(n^3 - 1\right) + \frac{2T}{nR}n^3 - \frac{2T}{R}$$

$$h = \frac{P_0\left(n^3 - 1\right) + \frac{2T}{R}\left[n^2 - 1\right]}{\rho g}$$

15. **A capillary tube is immersed vertically in water such that the height of liquid column is found to be 'x' on the surface of the earth. When it is taken to minute the capillary rise is 'y' if 'R' is the radius of the earth. Then the depth of mine is**

1) $d = R\frac{(y-x)}{x}$ 2) $d = R\frac{(y-x)}{y}$ 3) $d = R\left(\frac{x}{y-x}\right)$ 4) $d = R\left(\frac{y}{y-x}\right)$

SOLUTION :

$$g^1 = g(l - d/R)$$

16. A vertical glass capillary with inside diameter 0.50mm is submerged into water so that the length of its part emerging outside the water surface is equal to 25 mm.Find the radius of curvature of the meniscus.Surface tension of water is $73\times10^{-3}N/m$.

A) $R=0.6m$ **B)** $R=6mm$ **C)** $R=0.6mm$ **D)** $R=0.6Km$

SOLUTION :

In the capillary tube, the water should rise to a height $h=\frac{2T}{r\rho g}$

Here $T=73\times10^{-3}N/m$

$$r=\frac{0.50mm}{2}=0.25\times10^{-3}m$$

$$\therefore h=\frac{2\times73\times10^{-3}}{0.25\times10^{-3}\times10^{3}\times9.8}=59\times10^{-3}m=59mm$$

Now $h>h'$;

i.e., length is outside water surface.

Therefore, radius of meniscus >radius of capillary r.

If R is the radius of meniscus,

then we have

$$\frac{2T}{R}=h'\rho g$$

$$R=\frac{2T}{h'\rho g}$$

Here $h'=25mm=25\times10^{-3}m$

$$\therefore R=\frac{2\times73\times10^{-3}}{25\times10^{-3}\times9.8}=0.6\times10^{-3}m$$

17. The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water. [The surface tension of water at temperature of the experiment is 7.30×10^{-2} Nm^{-1}. 1 atmospheric pressure=1.01×10^{5} Pa, density of water=1000kgm^{-3}, g=9.80ms^{-2}].

SOLUTION :

The pressure required to blow the bubble is

$P=P_{atm}+hdg+2T/r$

$=1.01784\times10^{5}+(2\times7.3\times10^{-2}/10^{-3})$

$=(1.01784+0.00146)\times10^{5}=1.02\times10^{5}$Pa.

18. A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07N/m. Assume that the angle of contact between water and glass is 0^{0}.

SOLUTION :

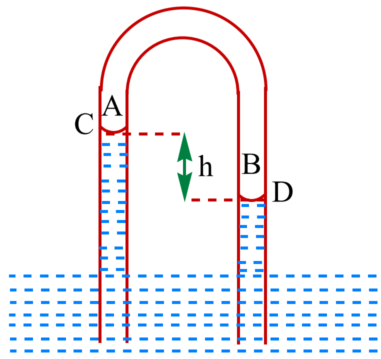
Let P_A and P_B are the pressure at points A and B respectively.

The pressure at point C. $P_C = P_A - \frac{2T}{R_1}$

. Where $R_1 = \frac{r_1}{\cos 0^0} = r_1$

The pressure at point D, $P_D = P_B - \frac{2T}{R_2}$

Where, $R_2 = \frac{r_2}{\cos 0^0} = r_2$



If 'h' is the difference in levels of liquid in two limbs, then

$$P_D - P_C = h\rho g \Rightarrow \left(P_B - \frac{2T}{R_2}\right) - \left(P_A - \frac{2T}{R_1}\right) = h\rho g$$

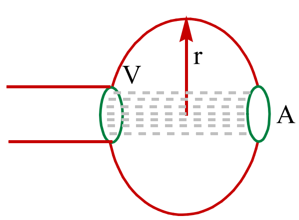
As $P_A = P_B$ and $R_1 = r_1 = 1.5mm$

$$R_2 = r_2 = 3.0mm, so\ 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = h\rho g$$

$$0.2 \times 0.07\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right) = h \times 1000 \times 9.8$$

After solving, we get $h = 4.76 \times 10^{-3}$ m

19. **A soap bubble is being blown at the end of a very narrow tube of radius b. Air (density ρ) moves with a velocity 'v' inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After some time the bubble, having grown to a radius 'r', separates from the tube. Find the value of 'r'. Assume that r>>b so that you can consider the air to be falling normally on the bubble's surface.**



1) $\frac{4T}{\rho v^2}$ 2) $\frac{4T}{\rho v}$ 3) $\frac{2T}{\rho v}$ 4) $\frac{4T}{\rho v^2}$

SOLUTION :

The bubble sill separate from the tube when thrust force exerted by the air is equal to the force due to excess pressure.

$$\text{i.e, } \rho A v^2 = \left(\frac{4T}{r}\right) A$$

$$r = \frac{4T}{\rho v^2}$$

20. Two verical parallel glass plates are particly submerged in water. The distance between the plates is d and the length is l. Assume that the water between the plates does not reach the upper edges of the plates and that the wetting is complete. the water will rise to height (ρ - density of water and σ = surface tension of water)

A) $\frac{2\sigma}{\rho g d}$ B) $\frac{\sigma}{2\rho g d}$ C) $\frac{4\sigma}{\rho g d}$ D) $\frac{5\sigma}{\rho g d}$

SOLUTION :

Total upward force dur to surface tension = $2\sigma l$

Weight of lifted liquid = (hlD) ρ g

Equating, we get $h = \frac{2\sigma}{\rho g d}$

21. A capillary tube of radius' r' is lowered into water whose surface tension is 'α' and density 'd'.The liquid rises to a height. Assume that the contact angle is Zero. Choose the correct statement(s):

A) Magnitude of work done by force of surface tension is $\frac{4\pi\alpha^2}{dg}$

B) Magnitude of work done by force ofsurface tension is $\frac{2\pi\alpha^2}{dg}$

C) Potential energy acquired by the water is $\frac{2\pi\alpha^2}{dg}$

D) The amount of heat developed is $\frac{2\pi\alpha^2}{dg}$

KEY : A , C, D

SOLUTION :

$$h = \frac{2\alpha}{dgr}$$

hence work done by force of surface tension is

$$W_s = \alpha \times 2\pi r \times h = \frac{4\pi\alpha^2}{dg}$$

But centre of mass of liquid in the capillary tube is at a hight h/2.

$$\text{Hence potencial energy gained} = \frac{Mgh}{2} = \pi r^2 \times h \times d \times g \times \frac{h}{2} = \frac{2\pi\alpha^2}{dg}$$

Hence work done by gravity $=\left(-\dfrac{2\pi\alpha^2}{dg}\right)$

Amount of heat developed $=\dfrac{2\pi\alpha^2}{dg}$

22. When a capillary tube is dipped in a liqiud, the liquid rises to a height h in the tube. the free liquid surface inside the tube is hemispherical in shape. The tube is now pushed down so that the height of the tube outside the liquid is less than h. Then

A) The liquid will come out of the tube like in a small fountain

B) The liquid will ooze out of the tube slowly

C) The liquid will fill the tube but not come out of its upper end

D) The free liquid surface inside the tube will not be hemispherical

KEY : C, D

SOLUTION :

The nagle of contact at the free liquid surface inside the capillary tube will change such that the certical component of the surface tension forces just balances the weight of the liquid column.

23. A glass rod of radius r_1 is inserted symmetrically into a vertical capillary tube of radius r_2 such that their lower ends are at the same lebel. The arrangement is now dipped in water. The height to which water will rise into the tube will be σ = surface tension of warer , ρ = density of water)

A) $\dfrac{2\sigma}{(r_2-r_1)\rho g}$ **B)** $\dfrac{\sigma}{(r_2-r_1)\rho g}$ **C)** $\dfrac{2\sigma}{(r_2+r_1)\rho g}$ **D)** $\dfrac{2\sigma}{(r_2^2+r_1^2)\rho g}$

SOLUTION :

Total upward force due to surface tension = $\sigma(2\pi r_1+2\pi r_2)$.

This supports the weight of the liquid column of heighrt h.

Weight of liquid column = $h=\left[\pi r_2^2-\pi r_1^2\right]\rho g$ Equating,

we get $h\pi(r_2+r_1)(r_2-r_1)\rho g=2\pi\sigma(r_1+r_2)$

or $h(r_2-r_1)\rho g=2\sigma$

$$h=\frac{2\sigma}{(r_2-r_1)\rho g}$$

::THEORY BITS ::

1. The weight of the body is maximum in

1) air 2) hydrogen 3) water 4) vacuum

KEY:4

2. If a big drop of liquid at 27^0C is broken into number of small drops, then the temperature of the droplets is

1) $=27^0C$ 2) $>27^0C$ 3) $<27^0C$ 4) $=54^0C$

KEY:3

3 When a body is full immersed in a liquid, the loss of weight of the body is equal to

1) Apparent weight of the body 2) Force of buoyance

3) Half the force of buoyancy 4)Twice the force of buoyancy

KEY:2

4. A boat full of scrap iron is floating on water in a lake. If all the iron is dropped into the water,the level of water will

1) go up 2) fall down 3) remain the same 4) Can not be decided

KEY:2

5. A large block of ice floats in a liquid.When ice melts the liquid level rises.The density of liquid is

1) Greater than that of water 2) Less than that of water

3) Equal to that of water 4) Half of that of water

KEY:1

6. Identify the correct choice.

A)When a abody floats in a liquid, it displaces the liquid whose weight is equal to its own weight.

B)When a body sinks in a liquid, it displaces the liquid whose volume is equal to its own volume.

1) A is true but B is false 2) A is false but B is true

3) Both A and B are true 4) Both A and B are false

KEY:3

7. A swimmer goes from the surface of water to a depth of 20m, the change in the pressure on his body is nearly

1) 3 atmospheres 2) 1 atmosphere 3) 2 atmospheres 4) zero

KEY:3

8. A bucket of water contain a wooden block floating in water with(4/5)th of its volume sub merged in the water.The bucket is placed on the floor of a lift and the lift now starts moving down with uniform accleration. The block of wood now

1) moves upward 2) moves downward

3) remains at same place 4)moves horizontally

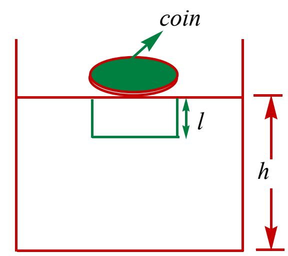
KEY:3

9. Surface tension of water is T_1 . When oil spreads on water surface tension becomes T_2, then

1) $T_1 > T_2$ 2) $T_1 = T_2$ 3) $T_1 < T_2$ 4) $T_1 = \frac{T_2}{2}$

KEY:1

10. A wooden block with a coin placed on its top floats in water as shown. After some time the coin falls into water. Then



1) l decreases and h increases
2) l increases and h decreases
3) both l and h increase
4) both l and h decrease

KEY:4

11. Which factor controls the better flow rate of a liquid through the syringe?

1) the pressure exerted by the thumb
2) the length of the needle
3) the nature of the liquid
4) the radius of the syringe bore.

KEY:4

12. In order that a floating object be in a stable equillibrium, its centre of bouyancy should be

1) vertical below its centre of gravity
2) horizontally inline with its centre of gravity
3) vertically above its centre of gravity
4) may be anywhere

KEY:3

13. When there are no external forces, shape of the liquid is determined by

1) Density of liquid
2) Temperature only
3) Surface tension
4) Viscosity

KEY:3

14. A piece of ice floats in a liquid denser than water. The liquid fills the vessel upto the edge.If ice melts completely then

1) water level remains unchanged
2) water level decreases
3) water overflows
4) data is insufficient.

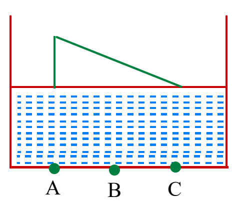
KEY:3

15. With the increase in temperature the anlge of contact between glass and water

1) decreases
2) increases
3) remains cont.
4) some times increases and some times decreases

KEY:1

16. An object of uniform density is allowed to float in water kept in a beaker. The object has triangular cross-section as shown in the figure. If the water pressure measured at the three point A,B and C below the object are P_A, P_B and P_C respectively then:



1) $P_A > P_B > P_C$
2) $P_A > P_B < P_C$
3) $P_A = P_B = P_C$
4) $P_A = P_C < P_B$

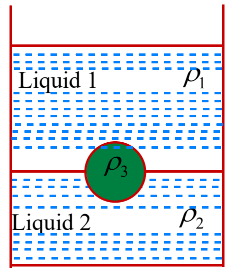
KEY:3

17. In a gravity free space, shape of a large drop of liquid is

1) Spherical
2) Cylindrical
3) Neither Spherical nor cylindrical
4) May be Spherical or cylindrical

KEY:1

18. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3 is dropped in the jar. It come to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 *and* ρ_3? (AIEEE-2008)



1) $\rho_1 < \rho_2 < \rho_3$ 2) $\rho_1 < \rho_3 < \rho_2$ 3) $\rho_3 < \rho_1 < \rho_2$ 4) $\rho_1 > \rho_3 > \rho_2$

KEY:2

19. When a capillary tube is immersed into a liquid the liquid neither rises nor falls in the capaillary, then the angle of contact is

1) 20^0 2) 90^0 3) 30^0 4) 70^0

KEY:2

Equation of continuity, Bernoulli's Theorem

20. Stream line motion becomes turbulent motion when the velocity of the liquid is

1) beyond critical velocity 2) critical velocity
3) below critical velocity 4) variable velocity

KEY:1

21. A liquid does not wet the solid surface if the angle of contact is

1) 0^0 2) $=45^0$ 3) $= 90^0$ 4) $>90^0$

KEY:4

22. In a laminar flow at a given point the magnitude and direction of the velocity of the fluid

1) both are constant 2) magnitude is only constant
3) direction is only constant. 4) both are not constant.

KEY:1

23. The liquid flow is most stream lined when

1) liquid of high viscosity and high density flowing through a tube of small radius.
2) liquid of high viscosity and low density flowing through a tube of small radius
3) liquid of low viscosity and low density flowing through a tube of large radius
4) liquid of low viscosity and high density flowing through a tube of large radius

KEY:2

24. The rate of flow of the liquid is the product of

1) Area of cross section of the liquid and velocityof the liquid.
2) Length of the tube of the flow and velocity of the liquid.
3) Volume of the tube of the flow and velocity of the liquid.
4) Viscous force acting on the liquid layer and velocity of the liquid

KEY:1

25. The fundemantal quantity which has the same power in the dimensional formula of surface tension and coefficient of viscosity is

1) Mass 2) Length 3) Time 4) None

KEY:1

26. The volume of liquid flowing per second out of an orifice at the bottom of the tank does not depend upon

1) the density of the liquid 2) acceleration due to gravity

3) the height of the liquid above orifice 4) the area of the orifice

KEY:1

27. Water flows through a horizontal pipe of radius 'r' at a speed V. If the radius of the pipe is doubled, the speed of flow of water under similar conditions is

1) 2V 2) V/2 3) V/4 4) 4V

KEY:3

28. A liquid is under stream lined motion through a horizontal pipe of nonuniform cross section. If the volume rate of flow at cross section 'a' is V, the volume rate of flow at cross section a/2 is

1) V/2 2) V 3) V/4 4) V

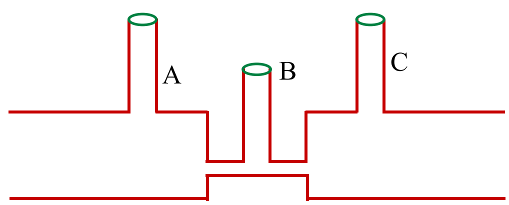
KEY:4

29 100 kg of iron and cotton are weighed by using a spring balance on the surface of the earth. If R_1 and R_2 are the reading shown by the balance,then

1) $R_1 < R_2$ 2) $R_1 = R_2$ 3) $R_1 > R_2$ 4) $R_1 = R_2 = 0$

KEY:3

30. Three tubes A, B, C are connected to a horizontal pipe in which liquid is flowing. The radii of the pipes at the joints of A, B and C are 2 cm, 1 cm and 2 cm respectively. The height of the



1) in A is maximum 2) in A and C is equal

3) is same in all the three 4) in A and B is same

KEY:2

31. If air is blown with a straw under of the pans of a physical balance present in equilibrium position, then that pan.

1) rises up 2) remains in the same position

3) lowers down 4) rises or lowers depending upon the velocity of air blown.

KEY:3

32. The liquid meniscus in a capillary tube will be convex, if the angle of contact is

1) greater than 90^0 2) less than 90^0 3) equal to 90^0 4) equal to zero

KEY:1

33. A train goes past a person standing at the edge of a platform at high speed. Then the person will be

1) attracted towards the train 2) unaffected by the train

3) reflected by the train 4) affected only if its speed is greater than critical velocity.

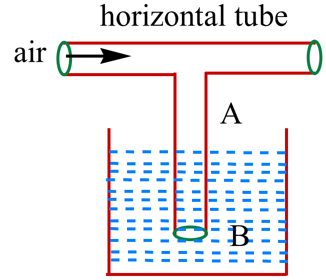
KEY:1

34. The velocity distribution curve of the stream line flow of a liquid advancing through a capillary tube is

1) Circular 2) elliptical 3) parabolic 4) a straight line

KEY:3

35. **Water stands at level A in the arrangement shown in figure . If a jet of air is gently blown into the horizontal tube in the direction shown in figure, then**



1) Water will fall below A in the capillary tube
2) Water will rise above A in the capillary tube
3) There will be no effect on the level of water in the capillary tube
4) Air will emerge from end B in the form of bubbles.

KEY:2

36. **A car moving on a road when overtaken by a bus**
1) is pulled towards the bus
2) is pulled away from the bus
3) is not affected by the bus
4) information is insufficient.

KEY:1

37. **The rise of liquid into capillary tube is h_1. If the apparatus is taken in a lift moving up with acceleration, the height is h_2, then**
1) $h_1 = h_2$
2) $h_1 > h_2$
3) $h_2 > h_1$
4) $h_2 = 0$

KEY:2

38. **A capillary tube, made of glass is dipped into mercury. Then**
1) mercury rises in the capillary tube
2) mercury descends in capillary tube
3) mercury rises and flows out of capillary tube
4) mercury neither rises nor descends in the capillary tube.

KEY:2

39. **A water barrel having water up to depth 'd' is placed on a table of height 'h'. A small hole is made on the wall of the barrel at its bottom. If the stream of water coming out of the hole falls on the ground at a horizontal distance 'R' from the barrel, then the value of 'd' is**
1) $\frac{4h}{R^2}$
2) $4hR^2$
3) $\frac{R^2}{4h}$
4) $\frac{h}{4R^2}$

KEY:3

40. **The end of a glass tube becomes round on heating due to**
1) friction
2) Viscosity
3) Gravity
4) Surface tension

KEY:4

41. **The main cause of viscosity is**
1) Force of repulsion between molecules
2) Cohesive forces
3) adhesive forces
4) both cohesive and adhesive forces.

KEY:2

42. **Viscosity is the property by virtue of which a liquid**
1) occupies minimum surface area
2) offers resistance for the relative motion between its layers.
3) becomes spherical in shape
4) tends to gain its deformed position.

KEY:2

43. **Viscosity is most closely related to**

1) density 2) velocity 3) friction 4) energy

KEY:3

44. Rain drops fall with terminal velocity due to

1) Buoyancy 2) Viscosity 3) Low weight 4) Surface tension

KEY:2

45. Two identical lead shots are dropped at the same time in two glass jars containing water and glycerine. The lead shot dropped in glycerine descends slowly because.

1) Viscous force is more in water than in glycerine
2) Viscous force is more in glycerine than in water
3) Surface tension is more in water
4) Surface tension is more in glycerine

KEY:2

46. After the storm, the sea water waves subside due to

1) Surface tension of sea-water 2) Disapperance of heavy currents
3) The viscosity of sea water 4) Gravitational pull of the storm

KEY:3

47. When a metallic sphere is dropped in a long column of a liquid, the motion of the sphere is opposed by the viscous force of the liquid. If the apparent weight of the sphere equals to the retarding forces on it, the sphere moves down with a velocity called.

1) Critical velocity 2) Terminal velocity 3) Velocity gradient 4) Constant velocity

KEY:2

48. The tangential forces per unit area of the liquid layer required to maintain unit velocity gradient is known as

1) Coefficient of gravitation of liquid layer 2) Coefficient of friction between layers
3) Coefficient of viscosity of the liquid 4) Temperature coefficient of viscosity

KEY:3

49. The quality of fountain-pen ink depends largely on

1) Surface tension of the liquid 2) Viscosity of ink
3) Impurities in ink 4) Density of ink

KEY:2

50. The tangential force (or) viscous force on any layer of the liquid is directly proportional to velocity gradient $\left(\frac{dv}{dx}\right)$. Then the direction of velocity gradient is :

1) Perpendicular to the direction of flow of liquid
2) Parallel to the direction of flow of liquid
3) Opposite to the direction of flow of the liquid
4) Independent of the direction of flow of liquid.

KEY: 1

51. The equation of continuity leads to

1) Law of conservation of moments of liquid flow. 2) law of conservation of energy
3) law of equipartition of energy 4) law of conservation of mass.

KEY:4

52. Viscosity of the fluids is analogous to

1) Random motion of the gas molecules 2) Friction between the solid surfaces
3) integral motion 4) Non uniform motion of solids

KEY:2

53. The surface tension of a liquid __with rise of temperature.

1) Increases 2) Decreases 3) Remains same

4) First decreased and then increases

KEY:2

54. The viscous drag is

1) inversely proportional to the velocity gradient
2) directly proportional to the surface area of layers in contact
3) independent of nature of liquid
4) perpendicular to the direction of liquid flow

KEY:2

55. A capillary is dipped in water vessel kept on a freely falling lift, then

1) water will not rise in the tube
2) water will rise to the maximum available height of the tube
3) water will rise to the height observed under normal condition
4) water will rise to the height below that observed under normal condition.

KEY:2

56. Viscosity is exhibited by

1) Solids, liquids, and gases.
2) liquids and gases
3) Solids and gases
4) Solids and liquids

KEY:2

57. A spinning ball is moving in a direction opposite to the direction of the wind. The ball moves in a curved path as

1) The pressure at the top and the bottom of the ball are equal.
2) The pressure at the top > the pressure at the bottom
3) The pressure at the top < the pressure at the bottom
4) There is no relation between the pressures.

KEY:2

58. A good Lubricant must have

1) high viscosity
2) low viscosity
3) high density
4) low surface tension

KEY:1

59. Coefficient of viscosity of a gas

1) increases with increase of temperature
2) decreases with increase of temperature
3) remains constant with increase of temperature
4) may increase or decrease with increase of temperature

KEY:.1

60. Viscosity of water at constant temperature is

1) more in deep water
2) more in shallow waters
3) less in deep water
4) same in both deep water and shallow waters

KEY:1

61. Hot syrup flows faster because

1) Surface tension increases with temperature
2) Viscosity decreases with temperature
3) Viscosity increases with temperature
4) Surface tension decreases with temperature

KEY:2

62. The pressure at a depth 'h' in a liquid of density " ρ " is plotted on the Y-axis and the value of 'h' on the X-axis , the graph is a straight line. The slope of the straight line is (g = acceleration due to gravity)

1) ρ g
2) 1/ ρ g
3) ρ /g
4) g/ ρ

KEY:1

63. If the flow is stream lined then Reynolds number is less than

1) 2000
2) 3000
3) 1000
4) 4000

KEY:3

64. A drop of water of radius 'r' is falling through the air of coefficient of viscosity 'η' with a constant velocity of 'v'. The resultant force on the drop is

1) $\frac{1}{6\pi\eta rv}$ 2) $6\pi\eta rv$ 3) $\sqrt{6\pi\eta rv}$ 4) zero

KEY:4

65. The paint -gun works on the principle of

1) Boyle's law 2) Bernoulli's principle
3) Archimedi's principle 4) Newton's laws of motion

KEY:

66. The rate of flow of a liquid through a capillary tube is

1) directly proportional to the length of the tube
2) inversely proportional to the difference of pressure between the ends of the tube.
3) directly proportional to the 4^{th} power of the radius of the tube.
4) independent of the nature of the liquid

KEY:3

67. Poiseuili's equation holds good when

1) the flow is steady and stream line 2) the pressure is constant at every cross section
3) The liquid in contact with the walls is stationary 4) All the above

KEY:4

68. If l is length of the tube and r is the radius of the tube, then the rate of volume flow of a liquid is maximum for the following measurements, under the same pressure difference.

1) l, r 2) $\frac{l}{2}, 2r$ 3) $2l, \frac{r}{2}$ 4) $2l, 2r$

KEY:2

69. When a boat in a river enters the sea water, then it

1) sinks a little 2) rises a little 3) remains same 4) will drawn

KEY:2

70. After terminal velocity is reached the acceleration of a body falling through a viscous fluid is

1) zero 2) g 3) less than g 4) greater than g

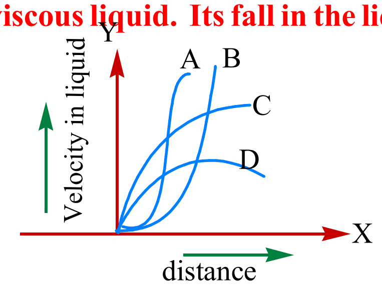
KEY:1

71. Clouds appear to float in air due to

1) low density 2) Air current 3) Viscosity of air 4) Buoyancy

KEY:4

72. A small ball is dropped in a viscous liquid. Its fall in the liquid is best described by the figure



1) curve A 2) curve B 3) curve C 4) curve D

KEY:3

73. A solid rubber ball of density 'd' and radius 'R' falls vertically through air. Assume that the air resistance acting on the ball is F = KRV where K is constant and V is its velocity. Because of this air resistance the ball attains a constant velocity called terminal velocity V_T after some

time. Then V_T

1) $\frac{4\pi R^2 dg}{3K}$ 2) $\frac{3K}{4\pi R^2 dg}$ 3) $\frac{4}{3}\frac{\pi r^3 dg}{K}$ 4) $\pi rdgk$

KEY:1

74. Bernoulli's theorem is applicable in the case of

1) Compressible liquid in stream lined flow
2) Compressible liquid in turbulent flow
3) incompressible liquid in stream lined flow
4) incompressible liquid in turbulent flow.

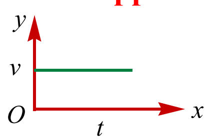
KEY:3

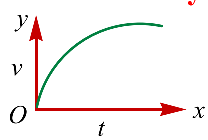
75. The terminal velocity of a small ball falling in a viscous liquid depends upon
i) its mass m ii) its radius r
iii) the coefficient of viscosity of the liquid η and
iv) acceleration due to gravity. Which of the following relations is dimensionally true for the terminal velocity.

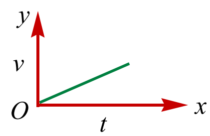
1) $V = \frac{Kmg}{\eta r}$ 2) $V = \frac{Kmgr}{\eta}$ 3) $V = \frac{Kmg\eta}{r}$ 4) $V = \frac{Kr\eta}{mg}$

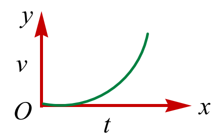
KEY:1

76. A ball is dropped into coaltar. Its velocity-time curve will be

1) 

2) 

3) 

4) 

KEY:2

77. Two needles are floating on the surface of water. A hot needle when touches water surface between the needles, then they move

1) Closer 2) Away 3) Out of the liquid 4) Into the liquid

KEY:2

78 Which of the following is a characteristic of turbulen flow?

1) velocity more than critical velocity
2) irregular flow
3) molecules crossing from one layer to the other
4) 1, 2, 3.

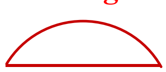
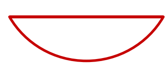
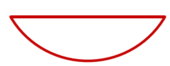
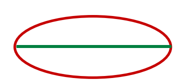
KEY:4

79. Liquid drops acquire spherical shape due to

1) gravity
2) surface tension
3) viscosity
4) intermolecular separation

KEY:2

80. Vertical sections of a wing of a fan are shown in the following figures. The maximum up thrust will be in figure.

1)  2)  3)  4) 

KEY:1

81. The height upto which water will rise in a capillary tube will be:

1) maximum when water temperature is 4^0C
2) minimum when water temperature is 4^0C
3) minimum when water temperature is 0^0C
4) same at all temperatures

KEY:2

82. At critical temperature surface tension becomes

1) 0 2) 1 3) Infinite 4) Negative

KEY:1

83. Droplets of a liquid are generally more spherical in shape than large drops of the same liquid because

1) Force of surface tension is equal and opposite to the force of gravity

2) Force of surface tension predominates the force of gravity
3) Force of gravity predominates the surface tension
4) Force of surface tension and force of gravity act in the same direction and are equal.

KEY:2

84. Mercury does not wet glass, wood or iron because

1) cohesive force is less than adhesive force
2) cohesive force is greater than adhesive force
3) angle of contact is less than 90^0
4) cohesive force is equal to adhesive force

KEY:2

85. With the increase of temperature,

1) The viscosity of a liquid increases
2) The viscosity of a liquid decreases
3) The viscosity of a gas decreases
4) The viscosity of a gas remains unchanged.

KEY:2

86. The surface tension of a liquid at its boiling point is

1) Maximum
2) Zero
3) Same as at room temperature
4) Minimum but more than zero

KEY:2

87. In turbulent flow, the velocity of the liquid molecules in contact with the walls of the tube.

1) is zero
2) is maximum
3) is equal to critical velocity
4) may have any value

KEY:1

88. The addition of soap changes the surface tension of water to T_1 and that of salt solution changes to T_2. Then

1) $T_1 = T_2$
2) $T_1 > T_2$
3) $T_1 < T_2$
4) $T_1 \geq T_2$

KEY:3

89. Machine parts are jammed in winter due to

1) increase in viscosity of lubricant
2) decrease in viscosity of lubricant
3) increase in surface tension of lubricant
4) decrease in surface tension of lubricant

KEY:1

90. Two pieces of glass plate one upon the other with a little water between them cannot be separated easily because of

1) inertia
2) pressure
3) viscosity
4) surface tension

KEY:4

91. When stirring of a liquid is stopped, the liquid comes to rest due to

1) surface tension
2) gravity
3) viscosity
4) buoyancy

KEY:3

92. The quantity on which the rise of liquid in a capillary tube does not depend is

1) density of liquid
2) radius of capillary tube
3) angle of contact
4) atmospheric pressure

KEY:4

93. The force which tends to destroy the relative motion between liquid layers is known as

1) Force due to surface tension
2) Viscous force
3) Gravitational force
4) Force of Cohesion

KEY:2

94. The potential energy of molecule on the surface of a liquid as compared to inside the liquid is

1) zero
2) smaller
3) the same
4) Greater

KEY:4

95. A drop of water breaks into two droplets of equal size. In this process which of the following statements is correct ?

1) the sum of temperature of the two droplets together is equal to the original temperature of

the drop.

2) the sum of masses of the two droplets is equal to the original mass of the drop.

3) the sum of the radii of the two droplets is equal to the radius of the original drop.

4) the sum of the surface areas of the two droplets is equal to the surface area of the original drop.

1) 1 is correct 2) 2 is correct 3) 3 is correct 4) 4 is correct

KEY:2

96. The dynamic lift of an aeroplane is based on

1) Torricelli theorem 2) Bernoulli's theorem

3) Conservation of angular Momentum 4) Principle of continuity

KEY:2

97. It is difficult to fill a capillary tube with mercury than with water since

1) Angle of contact between glass & mercury is more than 90^0 and the angle of contact between glass and water is less than 90^0.

2) Angle of contact is between glass and mercury is less than 90^0 and the angle of contact between glass and water is more than 90^0.

3) Angle of contact is same for both water and mercury.

4) Mercury is less dense than water.

KEY:1

98. When temperature is increased: (2004M)

a) viscosity of the gas increases

b) viscosity of the gas decreases

c) viscosity of the liquid decreases **d) viscosity of the liquid increases**

1) a and c are true 2) b and c are true 3) b and d are true 4) a and d are true

KEY:1

99. A water proofing agent changes the angle of contact from

1) Acute to $\pi/2$ 2) $\pi/2$ to obtuse

3) Acute to obtuse value 4) Obtuse to acute value

KEY:3

100. The nature of r-h graph ('r' is radius of capillary tube and 'h' is capillary rise) is

1) Straight Line 2) Parabola

3) Ellipse 4) Rectangular hyperbola

KEY:4

101. If 'L' is the capillary rise or dip and 'A' the cross sectional area of the tube, other conditions being the same, then

1) LA = Constant 2) $L\sqrt{A}$ = Constant

3) L/A = Constant 4) $L/\sqrt{A}$ = Constant

KEY:2

102. Water rises in a capillary tube to a height H, when the capillary tube is vertical. If the same capillary is now inclined to the vertical the length of water column in it will

1) Increase 2) Decrease 3) will not change

4) May increase or decrease depending on the angle of inclination.

KEY:1

103. The excess pressure inside a soap bubble is

1) Inversely proportional to the surface tension 2) Inversely proportional to its radius

3) Directly proportional to square of its radius 4) Directly proportional to its radius

KEY:2

104. The force of buoyancy is equal to

1) Weight of the body 2) Weight of the liquid displaced by the body

3) Apparent weight of the body 4) Divorce force

KEY:2

105. Which of the following substances has the greatest viscosity?

1) Mercury 2) Water 3) Kerosine 4) Glycerine

KEY:4

106. If two soap bubbles of different radii are connected by a tube.

1) air flows from the bigger bubbles to the smaller bubble till the sizes become equal.
2) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
3) air flows from the smaller bubble to the bigger.
4) there is no flow of air.

KEY:2

107. When the value of Reynold's number is less, the predominant forces are

1) Viscous forces 2) inertial forces
3) Surface tension forces 4) gravitational forces

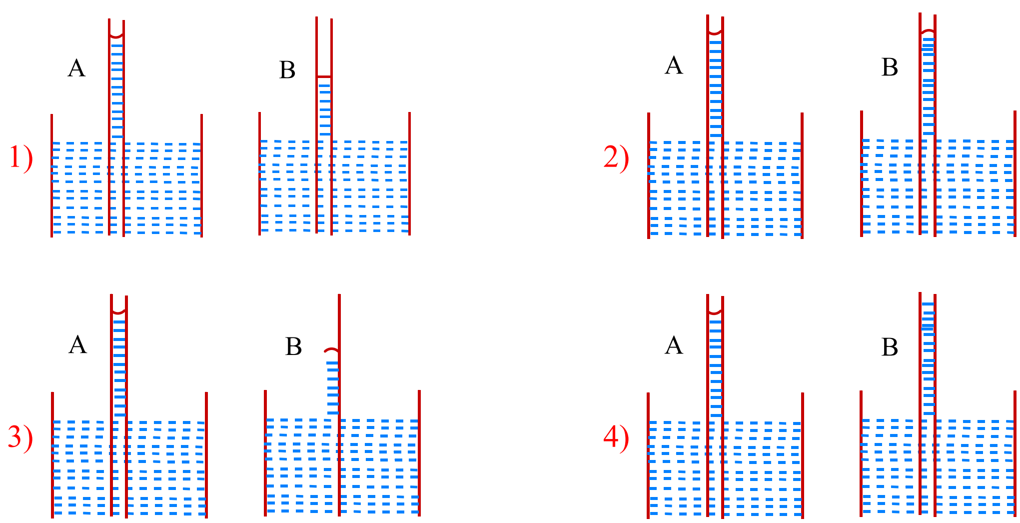
KEY:1

108. A gale is on a house. The force on the roof due to the gale is

1) directed downward 2) zero
3) directed upward 4) information insufficient

KEY:3

109. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid column in the two tubes?



KEY:1

110 For an ideal fluid, viscosity is

1) zero 2) infinity 3) finite but small 4) unity

KEY:1

111. The water proofing agents:

1) increase the surface tension T and decrease the angle of contact θ 2) increase both T and θ
3) decrease both T and θ 4) decrease T and increase θ

KEY:2

112. Water is flowing in a pipe of uniform cross section under constant pressure. At some place the

pipe becomes narrow. The pressure of water at this place.

1) remains same 2) may increase or decrease
3) increases 4) decreases.

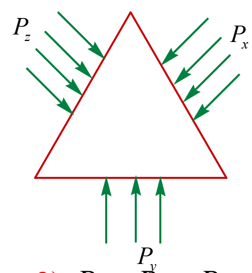
KEY:4

113. As the depth of the river increases, the velocity of flow

1) increases 2) decreases
3) remains unchanged 4) may increase or decrease

KEY:2

114. A triangular element of the liquid is shown in the fig., P_x, P_y and P_z represent the pressures on the element of the liquid. Then:



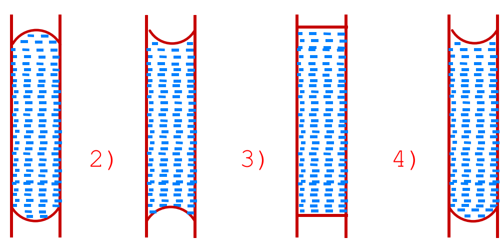
1) $P_x = P_y \neq P_z$ 2) $P_x = P_y = P_z$

3) $P_x \neq P_y \neq P_z$ 4) $P_x^2 + P_y^2 + P_z^2$ = constant

KEY:2

115. A vertical glass capillary tube, open at both ends, contains some water. Which of the following shapes may be taken by the water in the tube ?

1) 2) 3) 4)



KEY:2

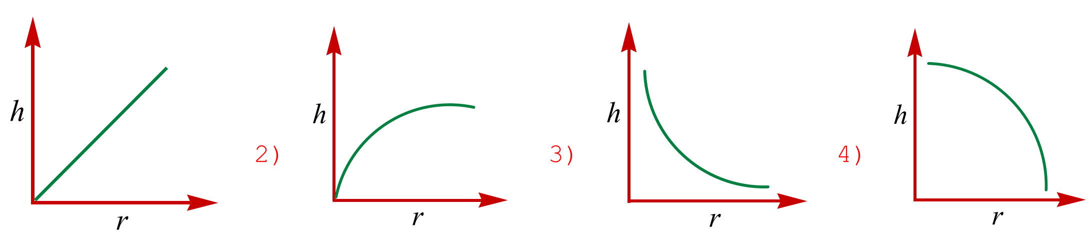
116 .If air is blown through the space between a calendar suspended from a nail on wall and the wall, then

1) The calendar moves close to the wall. 2) The calendar moves farther from the wall.
3) The position of the calendar does not change.
4) The position of the calendar may or may not change.

KEY: 1

117. Which of the following graphs may represent the relation between capillary rise h and the radius r of the capillary.

1) 2) 3) 4)



KEY:3

: :PRACTICE BITS : :

1. A bucket containing water of depth 15cm is kept in a lift which is moving vertically upward with an acceleration 2g. Then the pressure on the bottom of the bucket in kgwt/cm² is

1) 0.45 2) 0.045 3) 0.015 4) 0.15

KEY : 2

HINT :

$$P = hd\left(g + a\right)$$

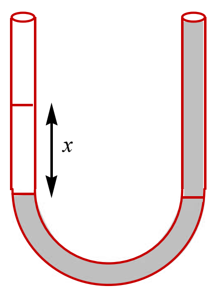
2. A bird of mass 1.23kg is able to hover by imparting a down ward velocity of 10m/s uniformly to air of density ' ρ ' kg/m³ over on effective area 0.1m². The acceleration due to gravity is 10m/s². Then the magnitude of ' ρ '. in kg/m³.

1) 0.34 2) 0.89 3) 1.23 4) 4.8

KEY : 2

HINT :

3. One end of a U-tube of uniform bore (area A) containing mercury is connected to a suction pump. Because of it the level of liquid of density ρ falls in one limb. When the pump is removed, the restoring force in the other limb is:



1) $2x\rho Ag$ 2) $x\rho g$ 3) $A\rho g$ 4) $x\rho Ag$

KEY:1

HINT:

Force due to excess pressure

=Restoring force= $\left(\rho gh\right)A = \rho g\left(2x\right)A$

4. A boat having length 2m and width 1m is floating in a lake. When a man stands on the boat, it is depressed by 3 cm. The mass of the man is

1) 50kg 2) 55kg 3) 60 kg 4) 70 kg

KEY:3

HINT:

Total weight = Force of buoyancy,

Mg=Vdg

$$\left(M + m\right)g = \Delta V\rho g \Rightarrow \left(M + m\right) = \Delta xA\rho$$

5. A cube of wood supporting 200g mass just floats in water. When the mass is removed, the cube rises by 1cm, the linear dimension of cube is

1) 10cm 2) 20cm 3) $10\sqrt{2}$ cm 4) $5\sqrt{2}$ cm

KEY:3
HINT:

Let l be the side of the cube.
Volume of the cube outside=volume of water displaced due to mass.
Water displaced is 200 gm and its volume is 200 cm^3.

$m = A \times \Delta y \times \rho ; 1l^2 = 200 \Rightarrow l = 10\sqrt{2}$

6. A large block of ice 4 m thick has a vertical hole drilled through it and is floating in the middle of water in a lake. The minimum length of rope required to scoop up a bucket full of water through the hole is (density of ice = 0.9 CGS unit, density of water = 1 CGS unit)

1) 40 cm 2) 24 cm 3) 20 cm 4)360 cm

KEY:1
HINT:

$$Mg = V_{in} d_1 g, Ald_{ice} = Al_{in} d_1$$

Length of rope = side of the block - $l_{immersed}$

7. A hollow metal sphere is found to float in water with the highest point just touching the free surface of water. If 'd' is the density of the metal in cgs units, the fraction that represents the volume of the hollow in terms of the volume of the sphere is

1) $\frac{1}{d}$ 2) $\left(1-\frac{1}{d}\right)$ 3) $\frac{d}{(d-1)}$ 4) $\left(1+\frac{1}{d}\right)$

KEY:2
HINT:

$$\frac{V_{cavity}}{V_S} = \frac{V_S - V_{metal}}{V_S} = 1 - \frac{V_{metal}}{V_S} = 1 - \frac{d_w}{d}$$

8. Excess pressure of one soap bubble is four times that of other. Then the ratio of volume of first bubble to second one is

1) 1:64 2) 64:1 3) 4:1 4) 1:2

KEY : 1
HINT :

$$\Delta = \frac{4T}{R}; \left(V = \frac{4}{3}\pi R^3, R \propto V^{\frac{1}{3}}\right)$$

9. A solid body is found floating in water with $\left(\frac{\alpha}{\beta}\right)^{th}$ of its volume submerged. The same solid is found floating in a liquid with $\left(\frac{\alpha}{\beta}\right)^{th}$ of its volume above the liquid surface. The specific gravity of the liquid is

1) $\frac{\beta-\alpha}{\alpha}$ 2) $\frac{\alpha-\beta}{\beta}$ 3) $\frac{\alpha}{\beta-\alpha}$ 4) $\frac{\beta}{\alpha-\beta}$

KEY:3
HINT:

$$\frac{\alpha}{\beta} = \frac{d_w}{d_w} ....(1) \quad 1 - \frac{\alpha}{\beta} = \frac{d_b}{d_l} ....(2)$$

from 1 and 2 $\frac{d_l}{d_b} = \frac{\alpha}{\beta - \alpha}$

10. A solid sphere of radius 'R' has a concentric cavity of radius 'R/2' inside it. The sphere is found to just float in water with the highest point of it touching the water surface. The specific gravity of the material of the sphere is

1) 1 2) 7/8 3) 8/7 4) 8/9

KEY:3

HINT:

$$\frac{V_{cavity}}{V_A} = \frac{V_S - V_{metal}}{V_S} = 1 - \frac{V_{metal}}{V_S} = 1 - \frac{d_w}{d} = 1 - \frac{1}{S.G}$$

11. An inverted u-tube has its two limbs in water and kerosene contained in two beakers. If water rises to a height of 10cm, to what height does kerosene (density=0.8gm/cc) rise in the other limb?

1)10 cm 2) 12.5 cm 3)15 cm 4)20 cm

KEY : 2

HINT :

$$P_1 = P_2 \Rightarrow h_1\rho_1 g = h_2\rho_2 g \Rightarrow g_1\rho_1 = h_2\rho_2$$

12. A vessel contains oil of density 0.8gm/cc. over mercury of density 13.6gm/c.c. A sphere floats with half of it's volume immersed in mercury and the other half in the oil. The density of material of sphere. (in gm/c.c)

1) 14.4 2) 7.2 3) 3.6 4) 12.2

KEY : 2

HINT :

$$vdg = v_1 d_1 g + v_2 d_2 g$$

13. An air tight container having a lid with negligible mass and an area of 8cm^2 is partially evacuated. If a 48N force is required to pull the lid off the container and the atmospheric pressure is 1.0 x 10^5 Ps, the pressure in the container before it is opened must be

1) 0.6atm 2) 0.5 atm 3) 0.4 atm 4) 0.2 atm

KEY : 3

HINT :

$$P_1 = P_2 = \frac{F}{A}; P_{atm} - P_{in} = \frac{F}{A}$$

14. A brass sphere weighs 100 gm.wt in air. It is suspended by a thread in a liquid of specific gravity =0.8. If the specific gravity of brass is 8, the tension in the thread in the in newtons is

1) 0.882 2)8.82 3)0.882 4)0.00882

KEY : 3

HINT :

$$T = mg\left(1 - \frac{d_{liquid}}{d_{body}}\right)$$

15. A cube of side 20cm is floating on a liquid with 5cm of the cube outside the liquid. If the density of liquid is 0.8 gm/c.c then the mass of the cube is

1) 4.2 kg 2) 4.8 kg 3) 5kg 4) 5.2 kg

KEY : 2

HINT :

$$mg = V_{in}\rho g$$

16. A soap film in formed on a frame of area 4x10^{-3}m^2. If the area of the film in reduced t o half, then the change in the Potential energy of the film is (surface tension of soap solution $= 40\times10^{-3}\,N/m$ **)**

1) 32x10^{-5}J 2)16x10^{-5}J 3)8x10^{-5}J 4) 16x10^{5}J

KEY:2

HINT:

$$PE_1 = W_1 = 2TA$$

$$PE_2 = W_2 = 2T\left(\frac{A}{2}\right).$$

17. If a body floats with (m/n)th of its volume above the surface of water, then the relative density of the material of the body is

1) (n-m) / n 2) m/n 3) n/m 4) (n-m) / m

KEY : 1

HINT :

$$\text{Specific gravity} = \frac{\text{weight of the body}}{\text{force of buoancy}} = \frac{W}{F_B} = \frac{d_B}{d_w}$$

$$V_{in} = V - V_{out} = V - = \frac{m}{n}V = V\left(\frac{n-m}{n}\right)$$

$$\Rightarrow \frac{V_m}{V} = \frac{d_B}{d_w} = S.G$$

18. Water from a top emerges vertically downwards with initial velocity 4 ms^{-1}. The cross-sectional area of the tap is A. The flow is steady and pressure is constant throughout the stream of water. The distance h vertically below the tap, where the cross-sectional area of the stream becomes $\left(\frac{2}{3}\right)A$ **is (g=10m/s^2) (EAMCET-2010)**

1) 0.5 m 2) 1 m 3) 1.5 m 4) 2.2 m

KEY:2

HINT:

$$\text{The equation of continuity } A_1v_1 = A_2v_2, A\times 4 = \frac{2}{3}A\times v_2,$$

$$\text{From Bernoulli's theorem } P + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\text{or } g\left(h_1 - h_2\right) = \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

$$g\times h = \frac{1}{2}\left[(6)^2 - (4)^2\right]\left[\because h_1 - h_2 = h\right] \Rightarrow h = 1m$$

19. A pipe having an internal diameter D is connected to another pipe of same size. Water flows into the second pipe through 'n' holes, each of diameter d. If the water in the first pipe has speed v, then speed of water leaving the second pipe is (EAMCET-2012)

1) $\frac{D^2v}{nd^2}$ 2) $\frac{nD^2v}{d^2}$ 3) $\frac{nd^2v}{D^2}$ 4) $\frac{d^2v}{nd^2}$

KEY : 1

HINT :

$$A_1v_1 = A_2v_2 \quad \pi\left(\frac{D}{2}\right)^2 v = \pi\left(\frac{d}{2}\right)^2 v' \Rightarrow v' = \frac{D^2v}{nd^2}$$

20.. The velocity of the wind over the surface of the wing of an aeroplane is 80 ms^{-1} and under the wing 60 ms^{-1}. If the area of the wing is $4m^2$, the dynamic lift experienced by the wing is [density of air = 1.3 kg. m^{-3}]

1) 3640 N 2) 7280 N 3)14560N 4)72800 N

KEY : 2

HINT :

$$F = \frac{d}{2}\left(V_2^2 - V_1^2\right)A$$

21. A vessel has a small hole at its bottom. If water can be poured into it upto a height of 7 cm without leakage (g=10 ms^{-2}), the radius of the hole is (surface tension of water is 0.07 Nm^{-1}).

1) 2 mm 2) 0.2 mm 3) 0.1 mm 4) 0.4 mm

KEY : 2

HINT :

$$T = \frac{rhdg}{2}$$

22. An aeroplane of mass 5000 kg in flying at an altitude of 3 km. If the area of the wings is $50m^2$ and pressure at the lower surface of wings is $0.6x10^5$ pa, the pressure on the upper surface of wings is (in Pascal) (g=10 ms^{-2})

1) $59x10^3$ 2) $2x10^4$ 3) $6x10^3$ 4) 59

KEY : 1

HINT :

$$\left(p_1 - p_2\right)A = mg$$

23. Water flows through a non-uniform tube of area of cross sections A, B and C whose values are 25, 15 and 35 cm^2 respectively. The ratio of the velocities of water at the sections A, B and C is

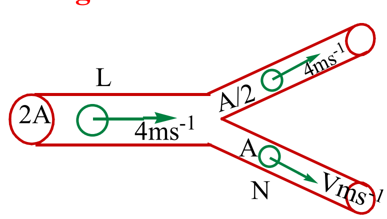
1) 5 : 3 : 7 2) 7 : 3 : 5 3) 21 : 35 : 15 4) 1 : 1 : 1

KEY :3

HINT :

$$Av = \text{const}, A_1v_1 = A_2v_2$$

24. An incompressible liquid flows through a horizontally tube L M N as shown in the figure. Then the velocity 'V' of the liquid through the tube N is



1) 1 ms^{-1} 2) 2 ms^{-1} 3) 4.5 ms^{-1} 4) 6 ms^{-1}

KEY : 4

HINT :

$$A_1v_1 + A_2v_2 = A_3v_3$$

25. A liquid is kept in a cylindrical jar, which is rotated about the cylindrical axis. The liquid rises at its sides. The radius of the jar is 'r', and speed of rotation is 'ω'. The difference in height at the center and the sides of the jar is

1) $\frac{r^2\omega^2}{g}$ 2) $\frac{r^2\omega^2}{2g}$ 3) $\frac{g}{r^2\omega^2}$ 4) $\frac{2g}{r^2\omega^2}$

KEY : 2

HINT :

Difference in potential energy = Rotational

$$\text{Kinetic energy} \therefore mg\left(h_1 - h_2\right) = \frac{1}{2}I\omega^2$$

26. At the mouth of the tap, the area of cross-section is 2.0 cm^2 and the speed of water is 3m/s. The area of cross-section of the water column 80cm below the tap is (use g=10m/s^2)

1) 0.6 cm^2 2)1.2 cm^2 3)1.5 cm^2 4) 2.0 cm^2

KEY : 2

HINT :

$$v_2^2 = v_1^2 + 2gh;\ A_1v_1 = A_2v_2$$

27. Capillary tubes of diameters 1, 1.5, 2 mm are dipped vertically in the same liquid. The capillary ascents of the liquid in the tube are in the ratio

1) 2 : 3 : 4 2) 6 : 4 : 3 3) 3 : 4 : 6 4) 4 : 3 : 2

KEY : 2

HINT :

$$T = \frac{rhdg}{2\cos\theta}; h \propto \frac{1}{r}$$

28. When two capillary tubes A and B are immersed in water, the heights of water columns are found to be in the ratio 2 : 3. The ratio of the radii of tubes A and B is

1) 2 : 3 2) 4 : 9 3) 9 : 4 4) 3 : 2

KEY : 4

HINT :

$$T = \frac{rhdg}{2\cos\theta}; r \propto \frac{1}{h}$$

29. A cylindrical tank of 1 meter radius rests on a platform 5m high. Initially the tank i filled with water to a height of 5m. A plug whose of area $10^{-4}m^2$ is removed from an orifice on the side of the tank at the bottom. The initial speed with which water flows out from the orifice in ms^{-1} is (g=10ms^{-2})

1) 10 2) 5 3) $5\sqrt{2}$ 4) $10\sqrt{2}$

KEY : 1

HINT :

$$v = \sqrt{2gh}$$

30. In the above problem, the initial speed with which water strikes the ground in ms^{-1} is

1) 10 2) 5 3) $5\sqrt{2}$ 4) $10\sqrt{2}$

KEY : 4

HINT : $$v_x = \sqrt{2gh_1}, v_y = \sqrt{2gh_2}; v = \sqrt{v_x^2 + v_y^2}$$

31. There is a hole at the side-bottom of a big water tank. The area of the hole is 4mm². **Through out it a pipe is connected. The upper surface of water is 5 m above the hole. The rate of flow of water through the pipe is (in m^3s^{-1}) (g= $10ms^{-2}$)**

1) 4×10^{-5} 2) 4×10^{5} 3) 4×10^{-6} 4) 28×10^{-5}

KEY : 1

HINT :

$$Q = A\sqrt{2gh}$$

32. The flow rate from a tap of diameter 1.25 cm is 3 lit/min. The coefficient of viscosity of water is 10^{-3} Pas.The nature of flow is

1) Turbulent 2) Laminar

3) Neither laminar (or) turbulent 4) Data inadequate

KEY : 1

HINT :

$$R = \frac{\rho v d}{\eta}, \left(Q = \frac{Al}{t} = Av \Rightarrow v = \frac{Q}{A}\right)$$

33. A force of 10N is required to draw rectangular glass plate on the surface of a liquid with some velocity. Force needed to draw another glass plate of 3 times length and 2 times width is

1) 5/3N 2) 10N 3) 60N 4) 30N

KEY : 3

HINT :

$$F = \eta A \frac{dv}{dx}; F \alpha A$$

34. Water is flowing through a capillary tube at the rate of 20 x 10^{-6} m^3/s. Another tube of same radius and double the length is connected in series to the first tube. Now the rate of flow of water in m^3s^{-1} is

1) 10×10^{-6} 2) 3.33×10^{-6} 3) 6.67×10^{-6} 4) 20×10^{-6}

KEY : 3

HINT :

$$Q = \frac{\pi \text{P} r^4}{8\eta l}; Q \alpha \frac{1}{l}$$

35. An artery in a certain person has been widened $1\frac{1}{2}$ times the original diameter. If the pressure difference across the artery is maintained constant, the blood flow through the artery will be increased

1) 3/2 times 2) 9/4 times 3) no change 4) 81/16 times

KEY : 4

HINT :

$$) \; Q = \frac{\pi \text{P} r^4}{8\eta l} : Q \alpha r^4$$

36. The potential energy of the liquid of surface tension "T" and density ρ that rises into the capillary tube is

1) $\pi^2 T^2 \rho^2 g$ 2) $4\pi T^2 \rho^2 g$ 3) $\frac{2\pi T^2}{\rho g}$ 4) $\frac{\pi T^2}{\rho g}$

KEY:3

HINT:

$$P.E = mg\left(h/2\right) = \pi r^2 h\rho gh/2$$

37. Water flowing from a hose pipe fills a 15 litre container in one minute. The speed of water from the free opening of radius 1 cm is (in ms^{-1})

1) 2.5 2) $\frac{\pi}{2.5}$ 3) $\frac{2.5}{\pi}$ 4) 5 π

KEY : 3

HINT :

$$Q = \frac{\pi \Pr^4}{8\eta l}; Q = \pi r^2 v$$

38. A small air bubble of 0.1 mm diameter is formed just below the surface of water. If surface tension of water is 0.072 Nm^{-1}, the pressure inside the air bubble in kilo pascal is (Atmospheric pressure = 1.01 x 10^5 pa)

1) 28.9 2) 0.289 3) 0.0289 4) 103.88

KEY:4

HINT:

$$P_{in} = P_0 + \frac{2T}{R}$$

39. If a soap bubble of radius 3 cm coalesce with another soap bubble of radius 4 cm under isothermal conditions, the radius of the resultant bubble formed is in cm

1) 7 2) 1 3) 5 4) 12

KEY : 4

HINT :

$$r = \sqrt{r_1^2 + r_2^2}$$

40. Two liquids are allowed to flow through two capillary tubes of lengths in the ratio 1 : 2 and radii in the ratio 2 :3 under the same pressure difference. If the volume rates of flow of the liquids are in the ratio 8 : 9, the ratio of their coefficients of viscosity is

1) 1 : 3 2) 3 :1 3) 4 : 9 4) 9 : 4

KEY : 3

HINT :

$$Q = \frac{\pi \Pr^4}{8\eta l}; \eta \alpha \frac{r^4}{Ql}$$

41. The viscous resistance of a tube to liquid flow is R. Its resistance for a narrow tube of same length and 1/3 times radius is

1) R/3 2) 3R 3) 27R 4) 81R

KEY : 4

HINT :

$$R = \frac{8\eta l}{\pi r^4}; R\alpha \frac{1}{r^4}$$

42. **Two identical tall jars are filled with water to the brim. The first jar has a small hole on the side wall at a depth h/3 and the second jar has a small hole on the side wall at a depth of 2h/3, where 'h' is the height of the jar. The water issuing out from the first jar falls at a distance R_1 from the base and the water issuing out from the second jar falls at a distance R_2 from the base. The correct relation between R_1 and R_2 is**

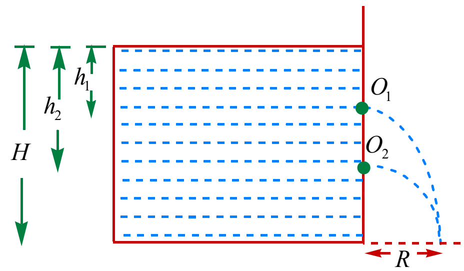
1) $R_1 > R_2$ 2) $R_1 < R_2$ 3) $R_2 = 2 \times R_1$ 4) $R_1 = R_2$

KEY:4

HINT:

$$R_1 = v_1 t_1; R_2 = v_2 t_2$$

43. **There are two holes O_1 and O_2 in a tank of height H. The water emerging from O_1 and O_2 strikes the ground at the same points, as shown in fig. Then:**



1) $H = h_1 + h_2$ 2) $H = h_2 - h_1$ 3) $H = h_1 h_2$ 4) $H = h_2/h_1$

KEY:1

HINT:

$$R = V_{efflux} \times time; R_1 = R_2$$

44. **A tank full of water has a small hole at the bottom. If one-fourth of the tank is emptied in t_1 seconds and the remaining three-fourths of the tank is emptied in t_2 seconds. Then the ratio $\frac{t_1}{t_2}$ is**

1) $\sqrt{3}$ 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{2}{\sqrt{3}} - 1$

KEY:4

HINT:

$$t_1 \alpha \left(\sqrt{h} - \sqrt{3h/4}\right); t_2 \alpha \left(\sqrt{3h/4}\right)$$

45. **There are two holes one each along the opposite sides of a wide rectangular tank. The cross section of each hole is $0.01 m^2$ and the vertical distance between the holes is one meter. The tank is filled with water. The net force on the tank in Newton when water flows out of the holes is: (Density of water $1000 kg/m^3$)**

1) 100 2) 200 3) 300 4) 400

KEY:2

HINT:

$$F_{net} = F_1 - F_2 = \rho a V_1^2 - \rho a V_2^2 = \rho a \left(2gh_1 - 2gh_2\right)$$

$$= 2\rho ga(h_1 - h_2) = 2\rho gah$$

46. A tank with vertical walls is mounted so that its base is at a height H above the horizontal ground. The tank is filled with water to a depth 'h' . A hole is punched in the side wall of the tank at a depth ' x ' below the water surface. To have maximum range of the emerging stream, the value of x is

1) $\frac{H+h}{4}$ 2) $\frac{H+h}{2}$ 3) $\frac{H+h}{3}$ 4) $\frac{3(H+h)}{4}$

KEY:2

HINT:

$$R = \sqrt{2gx}\sqrt{\frac{2(H+(h-x))}{g}}$$

47. A hole is made at the bottom of tank filled with water (density = 10^3 kg/m^3). If the total pressure at the bottom of the tank is 3 atm (1 atm = 10^5 N/m^2), then the velocity of efflux is

1) $\sqrt{400}$ m/s 2) $\sqrt{200}$ m/s 3) $\sqrt{600}$ m/s 4) $\sqrt{500}$ m/s

KEY:1

HINT:

Pressure due to water in the tank=3atm-1 atm=2atm=20m of water column

height of the water in teh tank is h=20m

Velocity of efflux=$\sqrt{2gh} = \sqrt{2\times 10\times 20} = \sqrt{400}m/\sec$

48. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cm-s^{-1}. If they coalesce to form one big drop, what will be the terminal speed of bigger drop? (Neglect the buoyancy of the air) (EAMCET-2009)

1) 1.5 cms^{-1} 2) 6 cms^{-1} 3) 24 cms^{-1} 4) 32 cms^{-1}

KEY : 3

HINT :

$$V_B = n^{2/3}V_S$$

49. The velocity of a ball of mass 'm' density 'd_1' when dropped in a container filled with glycerin of density 'd_2' becomes constant after some time. The viscous force acting on the ball will be

1) $mg\left(\frac{d_1}{d_2}\right)$ 2) $mg\left(1-\frac{d_2}{d_1}\right)$ 3) $mg\left(\frac{d_1+d_2}{d_1}\right)$ 4) $mg\left(\frac{d_1+d_2}{d_2}\right)$

KEY : 2

HINT :

Viscous force=Apparent weight-Force of buoyancy

$$\Rightarrow F_Y = mg - F_B$$

50. One spherical ball of radius R, density d released in a liquid of density d/2 attains a terminal velocity V. Another ball of radius 2R and density 1.5d, released in the liquid will attain a terminal velocity

1) 2V 2) 4V 3) 6V 4) 8V

KEY:4

HINT:

$$v = \frac{2}{9}\frac{r^2(\rho - \rho_0)g}{\eta}\; \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2} \times \left(\frac{\rho_1 - \rho_0}{\rho_2 - \rho_0}\right)$$

51. When a solid ball of volume V is falling through a viscous liquid, a viscous force F acts on it. If another ball of volume 2V of the same material is falling through the same liquid then the viscous force experienced by it will be (when both fall with terminal velocities).

1) F 2) F/2 3) 2F 4) F/4

KEY:3

HINT:

$$F = 6\pi\eta r v \left(\because v \propto r^2, F \propto r^3, F \propto V\right)$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{V_1}{V_2}$$

52. The level of a liquid in a vessel kept constant at 50cm. It has three identical horizontal tubes each of length 60cm coming out at heights 5, 10 and 15 cm respectively. If a single tube of the same radius as that of the three tubes can replace the three tubes when placed horizontally at the bottom of the vessel length of that tube is

1) 25 cm 2) 40 cm 3) 12.5 cm 4) 50 cm

KEY:1

HINT:

$$\frac{\pi P r^4}{8\eta l} = \frac{\pi P_1 r^4}{8\eta l_1} + \frac{\pi P_2 r^4}{8\eta l_2} + \frac{\pi P_3 r^4}{8\eta l_3}$$

$$\frac{H}{L} = \frac{H - h_1}{l} + \frac{H - h_2}{l} + \frac{H - h_3}{l}$$

53. A tube of radius R and length L is connected in series with another tube of radius R/2 and length L/8. If the pressure across the tubes taken together is P, the pressure across the two tubes seperately are :

1) and 2) and 3) and 4) and

KEY:4

HINT:

54. A capillary tube is attached horizontally at a constant head arrangement. If the radius of the capillary tube is increased by 10%, the rate of flow of liquid changes by about

1) -40% 2) +40% 3) +21% 4) +46%

KEY:4

HINT:

55. **Three horizontal capillary tubes of same radii and lengths L_1, L_2 and L_3 are filled side by side a little above the bottom, to the wall of a tank that filled with water. The length of a single capillary tube of same radius that can replace the three tubes such that the rate of flow of water through the single tube equals the combined rate of flow through the three tubes is**

1) 2) 3) 4)

KEY:2

HINT:

56. **A metallic wire of diameter "d" is lying horizontally on the surface of water. The maximum length of wire so that is may not sink will be**

1) $\sqrt{\dfrac{2T}{\pi dg}}$ 2) $\sqrt{\dfrac{2Tg}{\pi d}}$ 3) $\sqrt{\dfrac{2\pi d}{Tg}}$ 4) any length

KEY:4

HINT:

the expression is independent of length

57. **A liquid is filled into a semi elliptical crossection with a as semi major axis and b as semi minor axis. The ratio of surface tension forces on the curved part and the plane part of the tube in vertical position will be**

1) 2) 3) 4)

KEY:1

HINT:

58. **The length of a rubber cord floating on water is 5 cm. The force needed to pull the cord out of water is ..... N (surface tension of water is 7.2×10^{-2} Nm^{-1}) .**

1) 7.2×10^{-3} 2) 7.2×10^{-4} 3) $7.2 \times 10^{-}$ ^{5}4) 7.2×10^{-2}

KEY : 3

HINT :

59. **Calculate the force required to separate the glass plates of area 10^{-2} m^2 with a film of water 0.05 mm thickness between them (surface tension of water = 70×10^{-3} N/m)**

1) 28 N 2) 112 N 3) 5.6 N 4) 11.2 N

KEY : 1

HINT :

60. **A thin wire ring of 3 cm radius float on the surface of a liquid. The pull required to raise the ring**

before the film breaks is 30.14 x 10^{-3}N more than its weight. The surface tension of the liquid (in Nm^{-1}) is

1) 80x10^{-3} 2) 87x10^{-3} 3) 90x10^{-3} 4) 98x10^{-3}

KEY : 1

HINT :

61. When a 'U' shaped slider of negligible mass is dipped in a soap solution and lifted, a thin film of soap is formed in the frame. It supports a weight of 2.0c10^{-2}N. If the length of the slider is 40 cm, the surface tension of the film of soap is

1) 25 Nm^{-1} 2) 2.5 Nm^{-1} 3) 2.5 x 10^{-2} Nm^{-1} 4) 2.5 x 10^{-3} Nm^{-1}

KEY : 3

HINT :

62. A ring of inner and outer radii 8 and 9 cm is pulled out of water surface with a force of [S.T of water(T)=70 dyne/cm]

1) 26 x 10^{-2} N 2) 12.6 x 10^{-2} N 3) 7.48 x 10^{-2} N 4) 3.08 x 10^{-2} N

KEY : 3

HINT :

63. In Fig(i) a thin film supports a small weight  **. The weight suppported by a film of the same liquid at the same tempertaure in fig.(ii) is ___**

1)3.5x10^{-2}N 2)3.5x10^{-3}N 3)3.5x10^{-1}N 4)3.5x10^{-4} N

KEY : 1

HINT :

Force due to S.T.=Weight

same liquid, same temperature, same length of the film, supports same weight.

64. A film of water is formed between two straight parallel wires of length 10cm each separated by 0.5 cm. If their separation is increased by 1 mm while still maintaining their parallelism how much work will have to be done of water (Surface tension of water  **)**

1) 2) 3)

KEY:2

HINT:

Find out

65. The work done is blowing a soap bubble of volume "V" is W. The work done in blowing a soap bubble of volume "2V" is

1) W 2) $2^{\frac{2}{3}}W$ 3) $3^{\frac{2}{3}}W$ 4) 2 W

KEY:2

HINT:

66. Work of joule is required to be done in increasing the size of a soap film from 10cm x 6 cm to 10cm x 11 cm. The surface tension of the film is(in N/m)

1) 5×10^{-2} 2) 6×10^{-2} 3) 1.5×10^{-2} 4) 1.2×10^{-2}

KEY : 2

HINT :

67. The work done in increasing the raduis of a soap bubble from 4 cm to 5 cm is ......Joule(given surface tension of soap water to be 25×10^{-3} N/m)

1) 0.5657×10^{-3} 2) 5.657×10^{-3} 3) 56.5×10^{-3} 4) 565×10^{-3}

KEY : 1

HINT :

68. The reading of a pressure meter attached with a closed water pipe is 3.5×10^5 N m^{-2}. On opening the valve of the pipe, the reading of pressure meter is reduced to 3 x 10^5 N m^{-2}. Calculate the speed of water flowing in the pipe.

1)10 cm/s 2)10 m/s 3) 0.1 m/s 4) 0.1 cm/s

KEY : 2

HINT :

69. A mercury drop of radius 1 cm is sprayed into 10^6 drops of equal size. The energy expended in joule is (surface tension of mercury is 460×10^{-3}N/m)

1) 0.057 2) 5.7 3) 4)

KEY : 1

HINT :

70. **A wooden cube is found to float in water with ½ cm of its vertical side above the water. On keeping a weight of 50gm over its top, it is just submerged in the water. The specific gravity of wood is**

1) 0.8 2) 0.9 3) 0.85 4) 0.95

KEY:4

HINT:

weight of the body=weight of displaced liquid

71. **8000 identical water drops combine together to form a big drop. Then the ratio of the final surface energy to the initial surface energy of all the drops together is.**

1) 1 : 10 2) 1 : 15 3) 1 : 20 4) 1 : 25

KEY : 3

HINT :

72. **A capillary tube of radius 0.25 mm is dipped vertically in a liquid of density 800 kg m^{-3} and of surface tension $3x10^{-2}$ Nm^{-2}. The angle of contact of liquid-glass is and given $g = 10ms^{-2}$ the rise of liquid in the capillary tube is ..... Cm**

1) 9 2) 0.9 3) 9×10^{-3} 4) 0.09

KEY : 2

HINT :

73. **When a clean lengthy capillary tube is dipped vertically in a beaker containing water, the water rises to a height of 8 cm. What will happen if another capillary tube of length 4 cm and same radius is dipped vertically in the same beaker containing water. (Angle of contact of water is 0^0.)**

1) Water will flow out like a fountain.
2) Water will rise to a height of 4 cm only and the angle of contact will be zero.
3) Water will rise to a height of 4 cm only and the angle of contact will be 60^0.
4) Water will not rise at all

KEY : 3

HINT :

74. **A capillary tube is taken from the Earth to the surface of the Moon. The rise of the liquid column on the Moon (acceleration due to gravity on the Earth is 6 times that of the Moon) is**

1) six times that on the Earth surface 2) 1/6 that on the Earth's surface
3) equal to that on the Earth's surface 4) zero

KEY : 1

HINT :

75. **When a capillary tube is lowered into water, the mass of the water raised above the outside level is 5 gm. If the radius of the tube is doubled the mass of water that raises in the tube above**

the outside level is

1)1.25 gm 2) 5 gm 3)10 gm 4) 20 gm

KEY : 3

HINT :

76. When a body lighter than water is completely submerged in water, the buoyant force acting on it is found to be 'n' times its weight. The specific gravity of the material of the body is

1) $\frac{1}{1+n}$ 2) $\frac{1}{n}$ 3) n 4) $n+\frac{1}{n}$

KEY : 2

HINT :

77. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator the length of water coloumn in the capillary tube will be

1) 4 cm 2) 20 cm 3) 8 cm 4) 10 cm

KEY : 2

HINT :

In a freely falling lift capillary height=full length of the capillary tube.

78. When a cylindrical tube is dipped vertically into a liquid the angle of contact is 140^0. when the tube is dipped with an inclination of 40^0 the angle of contact is

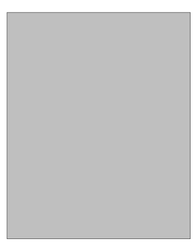
1)100^0 2) 140^0 3) 180^0 4) 60^0

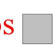
KEY : 2

HINT :

Angle of contact is independent of tilting angle

79. Water rises in a striaght capillary tube upto a height of 5 cm when held vertical in water. If the tube is bent as shown in figure then the height of water column in it will be



1) 5 cm 2) less than 5 cm 3) more than 5 cm 4) 5 cos 

KEY :1

HINT :

Capillary rise independent of shape of capillary pipe if radius of the pipe does not changes.

80. A tube is mounted so that it's base is at height 'h' above the horizontal ground. The tank is filled with water to a depth 'h'. A hole is punched in the side of the tank at depth 'y' below water surface. Then the value of 'y' so that the range of emerging stream would be maximum is

1) h 2) h/2 3) h/4 4) 3h/4

KEY:1

HINT:

R to be maximum, $\frac{dR}{dy} = 0$, which gives y=h

81. Twoliquiddropshavetheirdiameters as 1 mm and 2 mm. The ratio of excess pressures in them is

1) 1 : 2 2) 2 : 1 3) 4 : 1 4) 1 : 4

KEY : 2

HINT :

$$\Delta P = \frac{2T}{r}$$

82. The pressure inside soap bubble is 1.01 and 1.02 atmosphere respectively the ratio of their volume

1) 102:101 2) $(102)^3:(101)^3$ 3) 8 : 1 4) 2 : 1

KEY : 3

HINT :

$$\Delta P = \frac{4T}{R}; \left(V = \frac{4}{3}\pi R^3, R \propto V^{\frac{1}{3}} \right)$$

83. A spherical soap bubble of radius 1 cm is formed inside another of radius 3 cm. The radius of single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is ....cm.

1) 1 2) 0.8 3) 0.5 4) 0.25

KEY:2

HINT:

$$r = \frac{r_1 r_2}{r_1 + r_2}$$

84. If the shearing stress between the horizontal layers of water in a river is 1.5 milli newton/ m^2 and $\eta_{water} = 1 \times 10^{-3}\, pa.s$, The velocity gradient is ... s^{-1}

1) 1.5 2) 3 3) 0.7 4) 1

KEY : 1

HINT :

$$\frac{dv}{dx} = \frac{F}{A\eta}$$

85. The depth of water at which air bubble of radius 0.4mm remains in equilibrium is $\left(T_{water} = 72 \times 10^{-3} N/m\right)$

1) $3.67cm$ 2)3.67 m 3)6.37 cm 4)5.32 cm

KEY:1

HINT:

$$hdg = \frac{2T}{r}$$

86. Two separate air bubbles having radii ($r_1 = .002cm, r_2 = .004cm$) formed of same liquid $T = 0.07$ N/m come together to form a double bubble. Find the radius of curvature of the internal film surface common to both the bubbles.

1) 0.001cm 2) 0.002 cm 3) 0.004 cm 4) 0.003 cm

KEY:3

HINT:

$$r = \frac{r_1 r_2}{r_2 - r_1}$$

as the excess pressure is always towards concave surface & pressure in smaller bubble is greater than larger bubble, the common surface is concave towards the centre of the small bubble.

87. The excess pressure inside a spherical soap bubble of radius 1 cm is balanced by a column of oil (Specific gravity = 0.8), 2 mm high, the surface tension of the bubble is (EAM-10)

1) 3.92 N/m 2) 0.0392 N/m 3) 0.392 N/m 4) 0.00392 N/m

KEY:2

HINT:

The excess pressure of soap bubble

$$\Delta p = \frac{4T}{R}, h\rho g = \frac{4T}{R}; T = \frac{Rh\rho g}{4} = 0.0392 N/m$$

88. In a car lift compressed air experts a force F_1 on a small piston having a radius of 5cm. This pressure is transmitted to a second piston of radius 15cm. If the mass of the car to be lifted is 1350 kg. What is F_1?

1) 14.7 x10^3 N 2)1.47x10^3N 3) 2.47x10^3 N 4) 24.7x10^3 N

KEY : 2

HINT :

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

89. The lower end of a capillary tube of radius r is placed vertically in water then with the rise of water in the capillary heat evolved is

1) $+\frac{\pi r^2 h^2 dg}{2J}$ 2) $+\frac{\pi r^2 h^2 dg}{J}$ 3) $-\frac{\pi r^2 h^2 dg}{2J}$ 4) $-\frac{\pi r^2 h^2 dg}{J}$

KEY:1

HINT:

$$\text{Work done} = \text{heat evolved};\ W = JQ; Q = \frac{mgh}{2J}$$

90. Four identical capillary tubes a, b, c and d are dipped in four beakers containing water with tube 'a' vertically, tube 'b' at 30^0, tube 'c' at 45^0 and tube 'd' at 60^0 inclination with the vertical. Arrange the lengths of water column in the tubes in descending order.

1) d, c, b, a 2) d, a, b, c 3) a,c,d,b 4) a,b,c,d

KEY:1

HINT:

$h \propto$ angle of inclination

91. A vessel whose bottom has round holes with diameter of 1mm is filled with water Assuming that surface tension acts only at holes, then the maximum height to which the water can be

filled in vessel without leakage is (Given surface tension of water is 75 x 10 $^{-3}$ N/m and g = 10m / s^2)

1) 3 cm 2) 0.3 cm 3) 3 mm 4) 3 m

KEY:1

HINT:

Gauge pressure=excess pressure above the meniscus

$$\Rightarrow hdg = \frac{2T}{r} \Rightarrow h = \frac{2T}{rdg}$$

92. Water rises to a height h_1 in a capillary tube in a stationary lift. If the lift moves up with uniform acceleration it rises to a height h_2, then acceleration of the lift is

1) $\left[\frac{h_2 - h_1}{h_2}\right]g$ 2) $\left[\frac{h_2 - h_1}{h_1}\right]g$ 3) $\left[\frac{h_1 - h_2}{h_1}\right]g$ 4) $\left[\frac{h_1 - h_2}{h_2}\right]g$

KEY:4

HINT:

$$h_2\left(g + a\right) = h_1 g$$

93. A liquid drop of diameter D breaks up into 27 drops. Find the resultant change in energy.

1) $2\pi TD^2$ 2) πTD^2 3) $\frac{\pi TD^2}{2}$ 4) $4\pi TD^2$

KEY:1

HINT:

$$W = T \times 4\pi R^2\left(n^{1/3} - 1\right)$$

94. The radii of the two columns in a 'U' tube are 'r_1' & 'r_2', when a liquid of density (angle of contact is 0^o } is filled in it, the level difference of the liquid in the two arms in 'h'. The surface tension of the liquid is {g = acceleration due to gravity } (2004-M)

1) $\frac{\rho g h r_1 r_2}{2\left(r_2 - r_1\right)}$ 2) $\frac{\rho g h\left(r_2 - r_1\right)}{2 r_2 r_1}$ 3) $\frac{2\left(r_1 - r_2\right)}{\rho g h r_2 r_1}$ 4) $\frac{2\left(r_1 - r_2\right)}{\rho g h}$

KEY:1

HINT:

$$h = \frac{2T}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Previous JEE Mains Questions and Solutions

VISCOSITY

Pressure , Density , Pascal's Law and Archimedis Principle :

1. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. Ifthe specific gravity ofthe shell material is $\frac{27}{8}$ w.r.t water, the value of r is: [5 Sep. 2020 (I)]

(a) $\frac{8}{9}R$ (b) $\frac{4}{9}R$ (c) $\frac{2}{3}R$ (d) $\frac{1}{3}R$

SOLUTION : (a)

In equilibrium, $mg = F_B$

$F_B = V\rho_0 g$ and mass =volume × density

$$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$$

Given, relative density, $\frac{\rho_0}{\rho_w} = \frac{27}{8}$

$$\Rightarrow \left[1 - \left(\frac{r}{R}\right)^3\right]\frac{27}{8}\rho_w = \rho_w$$

$$\Rightarrow 1 - \frac{r^3}{R^3} = \frac{9}{27} \Rightarrow 1 - \frac{1}{3} = \frac{r^3}{R^3} \Rightarrow \frac{2}{3} = \frac{r^3}{R^3}$$

$$\Rightarrow \frac{r}{R} = \left(\frac{2}{3}\right)^{1/3} \Rightarrow 1 - \frac{r^3}{R^3} = \frac{8}{27} \Rightarrow \frac{r^3}{R^3} = 1 - \frac{8}{27} = \frac{19}{27}$$

$$r = 0.89R = \frac{8}{9}R.$$

2. An air bubble of radius 1 cm in water has an upward acceleration $9.8\ \mathrm{cm\ s^{-2}}$. The density of water is $1\ \mathrm{gm\ cm^{-3}}$ and water offers negligible drag force on the bubble. Themass ofthe bubble is $(\mathrm{g} = 980\mathrm{cm/s^2})$ [4 Sep. 2020 (I)]

(a) $4.5\mathrm{lgm}$ (b) $3.15yn$ (c) $4.15\mathfrak{n}$ (d) $1.52\mathfrak{n}$

SOLUTION : (c)

Given: Radius of air bubble = 1 cm,

Upward acceleration ofbubble, $a = 9.8\mathrm{cm/s^2}$,

$\rho_{\mathrm{water}} = 1\mathrm{gcm^{-3}}$

$$\text{Volume } V=\frac{4\pi}{3}r^3=\frac{4\pi}{3}\times(1)^3=4.19\text{cm}^3$$

$\uparrow a$

$$\text{Fbuoyant} - mg = ma \Rightarrow m = \frac{F_{\text{buoyant}}}{g+a}$$

$$m=\frac{(V\text{p}_{0J}g)}{g+a}=\frac{V\text{p}_{0J}}{1+\frac{a}{g}}=\frac{(4.19).\times 1}{1+\frac{98}{980}}=\frac{4..19}{101}=4.15\text{g}$$

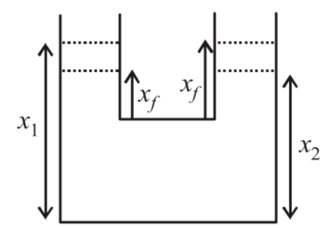
3. Two identical cylindrical vessels are kept on the ground and each contain the same liquid ofdensity d. The area of the base ofboth vessels is S but the height ofliquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe ofnegligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy ofthe system in the process is:

[4 Sep. 2020 (II)]

(a) $gdS(x_2^2+x_1^2)$ **(b)** $gdS(x_2+x_1)^2$ **(c)** $\frac{3}{4}gdS(x_2-x_1)^2$ **(d)** $\frac{1}{4}gdS(x_2-x_1)^2$

SOLUTION : **(d)**

(d)



Initial potential energy, $U_1=(\text{p}Sx_1)g\cdot\frac{x_1}{2}+(\text{p}Sx_2)g\cdot\frac{x_2}{2}$

Final potential energy, $U_f=(\text{p}Sx_f)g\cdot\frac{x_f}{2}\times 2$

By volume conservation, $Sx_1+Sx_2=S(2x_f)$

$$x_f=\frac{x_1+x_2}{2}$$

When valve is opened loss in potentail energy occur till water level become same.

$$\Delta U=U_i-U_f$$

$$\Delta U=\rho Sg\left[\left(\frac{x_1^2}{2}+\frac{x_2^2}{2}\right)-x_f^2\right]$$

$$= \rho S g\left[\frac{x_1^2}{2}+\frac{x_2^2}{2}-\left(\frac{x_1+x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2}\left[\frac{x_1^2}{2}+\frac{x_2^2}{2}-x_1x_2\right] = \frac{\rho S g}{4}(x_1-x_2)^2$$

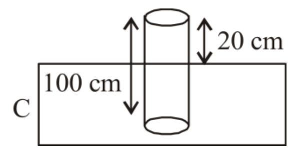
4. A leak proofcylinder oflengLh 1 m, made ofa metal which has very low coefficient ofexpansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature ofwater is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density ofwater at $T = 4°C$, relative to the density at $T = 0°C$ is close to: **[8Jan2020(D]**

(a) 1.26 **(b) 1.M** **(c) 1.01** **(d) 1.03**

SOLUTION : **(c)**

When cylinder is floating in water at 0°C

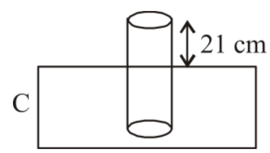
Net thrust $= A(h_2 - h_1)\rho_{0°c}g = A(100-80)\rho_{0°c}g$



0°

When cylinder is floating in water at 4°C

Net thrust $= A(h_2 - h_1)\rho_{4°c}g = A(100-21)\rho_{4°c}g$



4°

$$\frac{\rho_{4°c}}{\rho_{0°c}} = \frac{80}{79} = 1.01$$

5. Consider a solid sphere of radius R and mass density $(r) = \rho_0\left(\left(1-\frac{r^2}{R^2}\right)\right)$, $0 < r \le R$. The minimum density ofa liquid in which it will float is: **[8 Jan 2020 (D]**

(a) $\frac{\rho_0}{3}$ **(b)** $\frac{\rho_0}{5}$ **(c)** $\frac{2\rho_0}{5}$ **(d)** $\frac{2\rho_0}{3}$

SOLUTION : **(c)**

For minimum density ofliquid, solid sphere has to float (completely immersed) in the liquid.

$$mg = F_B \text{ (also } V_{\text{irersed}} = V_{\text{tota1}})$$

$$\text{or} \int \text{p}\, dV = \frac{4}{3}\pi R^3 \rho_\ell$$

$$\left[\text{p}(r) = \text{p}_0\left(1 - \frac{r^2}{R^2}\right) 0 < r \le R \text{ given}\right]$$

$$\Rightarrow \int_0^R \text{p}_0\, 4\pi\left(1 - \frac{r^2}{R^2}\right).r^2 dr = \frac{4}{3}\pi R^3 \text{p}_l$$

$$\Rightarrow 4\pi \text{p}_0\left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]_0^R = \frac{4}{3}\pi R^3 \text{p}_\ell$$

$$\frac{4\pi \text{p}_0 R^3}{3} \times \frac{2}{5} = \frac{4}{3}\pi R^3 \text{p}_p$$

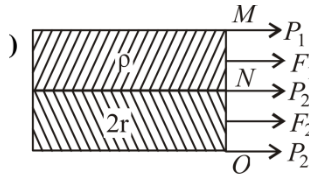
$$\text{p}_\ell = \frac{2\text{p}_0}{5}$$

$$= 0$$

6. **Two liquids of densities p_1 and $\text{p}_2(\text{p}_2 = 2\text{p}_1)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of5 m. The ratio ofthe forces due to these liquids exerted on upper part *MN*to that at the lower part NO is(Assume that the liquids are not mixing): [8 Jan 2020 (II)]**

(a) 1/3 **(b) 2/3** **(c) ½** **(d) ¼**

SOLUTION : (d)



Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.

Pressure is given by $P = \text{p}gh$

$$\text{Now, } P_1 = 0\ (h = 0)$$

$$P_2 = \text{p}g(5)$$

$$P_3 = \text{p}g(15)$$

$$= 15\text{p}g$$

$$\text{Force on upper part, } F_1 = \frac{(P_1+P_2)}{2}A$$

Force on lower part, $F_2 = \frac{(P_2+P_3)}{2} A$

$$\frac{F_1}{F_2} = \frac{5\text{p}g}{20\text{p}g} = \frac{5}{20} = \frac{1}{4}$$

7. A cubical block ofside 0.5m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density ofwater $= 10^3 \text{kg/m}^3$] [10 April 2019 (II)]

(a) 46.3 kg (b) 87.5 kg (c) 65.4kg (d) 30.lkg

SOLUTION : (b)

When a ιx)dy floats then the weight ofthe body = upthrust $(50)3 \times \frac{30}{100} \times (1) \times g = M_{\text{cube}} g$ (i)

Let m mass should be placed, then $(50)\ 3 \times (1) \times g = (M_{\text{cube}} + m) g$ (ii)

Subtracting equation (i) fiom equation (ii), we get

$$\Rightarrow \text{mg} = (50)^3 \times \text{g}(1 - 0.3) = 125 \times 0.7 \times 10^3 \text{g}$$

$$\Rightarrow \text{m} = 87.5\text{kg}$$

8. A submarine experiences apressure of 5.05×10^6 Pa at depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_1 - d_1$ is approximately (density ofwater= 10^3kym^3 and acceleration due to gravity = 10 ms- 2):

[10 April 2019 (II)]

(a) 300 m (b) 400 m (c) 600m (d) 500 m

SOLUTION : (a)

$$\text{P}_1 = \text{P}_0 + \text{pgd}_1$$

$$\text{P}_2 = \text{P}_0 + \text{pgd}_2$$

$$\Delta\text{P} = \text{P}_2 - \text{P}_1 = \text{pg}\Delta\text{d}$$

$$3.03 \times 10^6 = 10^3 \times 10 \times \Delta\text{d}$$

$$\Rightarrow \Delta\text{d} = 300\text{m}$$

9. A wooden block floating in a bucket ofwater has $\frac{4}{5}$ ofits volume submerged. When certain amount ofan oil poured into the bucket, it is found that the block is just under the oil surface with halfofits volume under water and halfin oil. The density ofoil relative to that ofwater is: [9 April 2019 (II)]

(a) 0.5 (b) 0.8 (c) 0.6 (c) 0.7

SOLUTION : (c)

$$Mg = \left(\frac{4V}{5}\right) p\omega g$$

$$\text{or} \left(\frac{M}{V}\right) = \frac{4p\omega}{5} \text{ or } p = \frac{4po)}{5}$$

When block floats fully in water and oil, then

$$Mg = F_{b_J} + F_{b_2}$$

$$(pV)g = \left(\frac{V}{2}\right) \rho_{oi1} g + \frac{V}{2} \rho \omega g$$

$$\text{or } p_{oi1} = \frac{3}{5} pc0 = 0.6 \rho c0$$

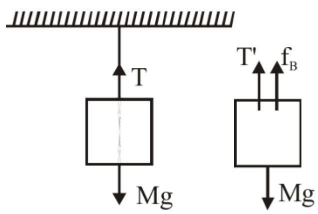
10. A load of mass M kg is suspended fi: om a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid ofrelative density2. The relative density ofthe material ofload is 8. The new value of increase in length ofthe steel wire is:

[12 Jan. 2019 (II)]

(a) 3.0mm (b) 4.0mm (c) 5.0mm (d) Zero

SOLUTION : (a)

$$\text{Using } \frac{\Gamma}{A} = Y \cdot \frac{\Delta \ell}{\ell}$$



$$\Rightarrow \Delta\ell \propto \Gamma \quad \text{........ (i)}$$

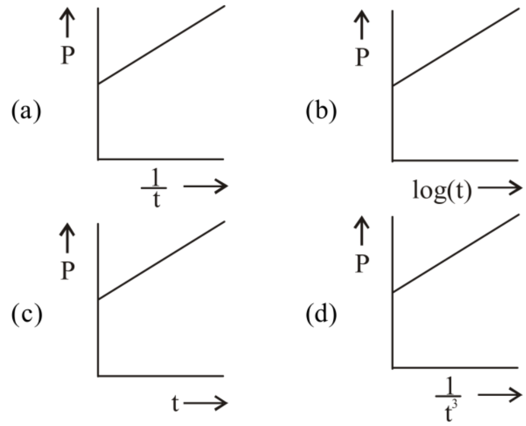
$$T = Mg - f_B = Mg - Pb = \left(1 - \frac{Pp}{Pb}\right) Mg = \left(1 - \frac{2}{8}\right) Mg$$

$$T = \frac{3}{4} Mg$$

From eqn (i)

$$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4} \text{ [Given: } \Delta\ell = 4 \text{ mm] } \Delta\ell' = \frac{3}{4} \cdot \Delta\ell = \frac{3}{4} \times 4 = 3 \text{ mm}$$

11. A soap bubble, blown by a mechanical pump at the mouth ofa tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by: [12 Jan. 2019 (II)]



SOLUTION : **(d)**

$$V = ctor, \frac{4}{3}\pi r^3 = ct$$

$$\Rightarrow r = kt^{\frac{1}{3}}$$

$$P = P_{0^+}\frac{4T}{kt^{1/3}}$$

$$P = P_0 + c\left(\frac{1}{t^{1,3}}\right)$$

12. Aliquid ofdensity p is coming out ofa hose pipe ofradius a with horizontal speed vand hits a mesh. 50% ofthe liquid passes through the mesh unaffected. 25% looses all ofits momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be: [11 Jan. 2019 (I)]

(a) $\frac{1}{4}pv^2$ (b) $\frac{3}{4}pv^2$ (c) $\frac{1}{2}pv^2$ (d) pv^2

SOLUTION : **(b)**

$$\text{Mass per unit time ofthe liquid} = pav$$

$$\text{Momentum per second carried by liquid} = pav \times v$$

$$\text{Net force due to bounced back liquid,} \Gamma_1 = 2 \times \left[\frac{1}{4}pav^2\right]$$

$$\text{Net force due to stopped liquid, } \Gamma_2 = \frac{1}{4}pav2$$

$$\text{Total force, } \Gamma = \Gamma_1 + \Gamma_2 = \frac{1}{2}pav^2 + \frac{1}{4}pav^2 = \frac{3}{4}pav2$$

$$\text{Net pressure} = \frac{3}{4}pv^2$$

13. A thin uniform tube is bent into a circle ofradius r in the verticalplane. Equal volumes oftwo immiscible liquids, whose densities are p_1 and $p_1 (p_1 > p_2)$ fill half the circle. The angle θ between the radius vector passing through the common interface and the vertical is

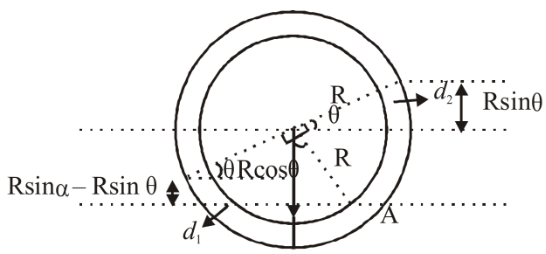
[Online April 15, 2018]

(a) $\theta = \tan^{-1}\left[\frac{\pi}{2}\left(\frac{p_1-\rho_2}{p_1+p_2}\right)\right]$ (b) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{p_1-p_2}{p_1+p_2}\right)$

(c) $\theta = \tan^{-1}\pi\left(\frac{p_1}{p_2}\right)$ (d) None of above

SOLUTION : (d)

Pressure at interface A must be same from both the sides to be in equilibrium.



$$(R\cos\theta + R\sin\theta)p_2 g = (R\cos\theta - R\sin\theta)p_1 g$$

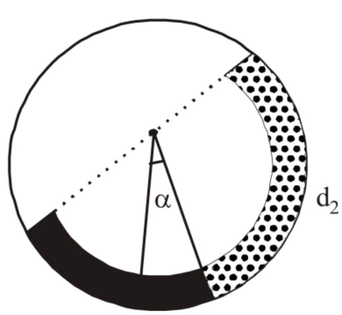
$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta}$$

$$\Rightarrow p_1 - p_1\tan\theta = p_2 + p_2\tan\theta$$

$$\Rightarrow (p_1 + p_2)\tan\theta = p_1 - p_2$$

$$\theta = \tan^{-1}\left(\frac{p_1 - p_2}{p_1 + p_2}\right)$$

14. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90^o angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[2014]**

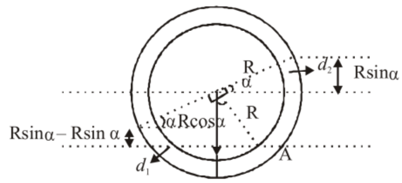


d_1

(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

SOLUTION : **(c)**

Pressure at interface A must be same fi$_{io}$m both the sides to be in equilibrium.



$$(R\cos\alpha + R\sin\alpha)d_2 g = (R\cos\alpha - R\sin\alpha)d_1 g$$

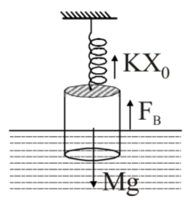
$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

15. **A uniform cylinder oflength L and mass M having crosssectional areaA is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid ofdensity 0 at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:** **[2013]**

(a) $\frac{Mg}{k}$ **(b)** $\frac{Mg}{k}\left(1-\frac{LAo}{M}\right)$ **(c)** $\frac{Mg}{k}\left(1-\frac{LAo}{2M}\right)$ **(d)** $\frac{Mg}{k}\left(1+\frac{LAo}{M}\right)$

SOLUTION : **(c)**

From figure, $kx_0 + \Gamma_B = Mg$



$$kx_0 + o\frac{L}{2}Ag = Mg \qquad \text{[mass = density × volume]}$$

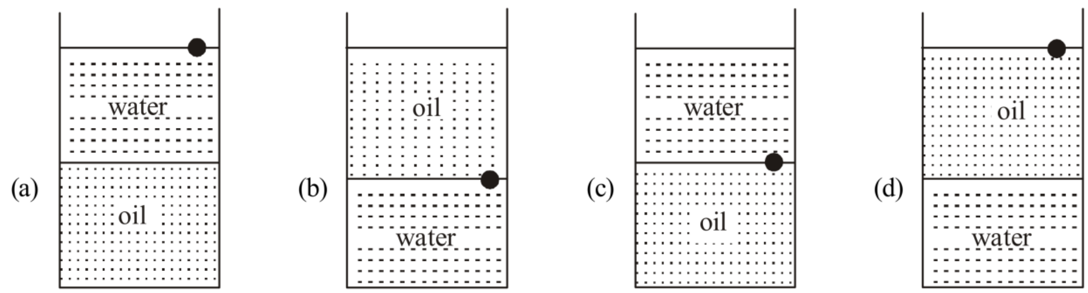
$$\Rightarrow kx_0 = Mg - 0\frac{L}{2}Ag$$

$$\Rightarrow x_0 = \frac{Mg - \frac{oLAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LAo}{2M}\right)$$

Hence, extension ofthe spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LAo}{2M}\right)$$

16. Aball is made ofa material ofdensity $\rho$where $\rho_{oi1} < \rho < \rho_{water}$ with ρ_{oi1} and ρ_{water} represe-nting the densities of oil and water, respectively. The oil and water are immiscible. Ifthe above ball is in equilibrium in a mixture ofthis oil and water, which ofthe following pictures represents its equilibrium position? [2010]



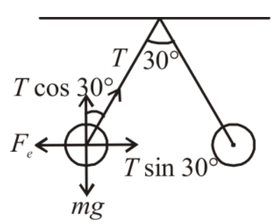
SOLUTION : (b)

Oil will float on water so, (b) or (d) is the correct option. But density ofball is more than that of oil,, hence it will sink in oil.

17. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8 gcm^{-3}$, the angle remains the same. If density of the material ofthe sphere is $.6 gcm^{-3}$, the dielectric constant of the liquid is [2010]

(a) 4 (b) 3 (c) 2 (d) 1

SOLUTION : (c)



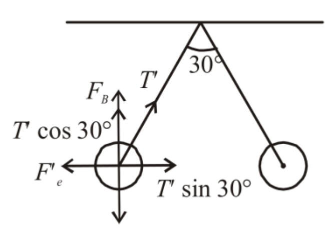
$$F_e = T \sin 30° \text{ and } mg = T \cos 30°$$

$$\Rightarrow \tan 30° = \frac{F_e}{mg} \text{ .... (1)}$$

$$\text{In liquid, } F_e' = T' \sin 30° \text{ (A)}$$

$$mg = F_B + T' \cos 30°$$

$$\text{But } F_B = \text{Buoyant force}$$



$$mg$$

$$= V(d - p)g = V(1.6 - 0.8)g = 0.8Vg$$

$$= 0.8\frac{m}{d}g = \frac{0.8.mg}{16} = \frac{mg}{2}$$

$$mg = \frac{mg}{2} + T' \cos 30^\circ$$

$$\Rightarrow \frac{mg}{2} = T' \cos 30^\circ \text{ (B)}$$

From (A) and (B), $\tan 30^\circ = \frac{2F_e}{mg}$

From (1) and (2)

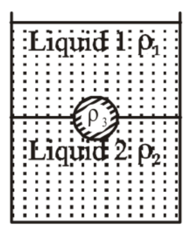
$$\Rightarrow F_e = 2F_{\acute{e}}$$

If K be the dielectric constant, then

$$F_{\acute{e}} = \frac{F_e}{K}$$

$$F_e = \frac{2F_e}{K} \Rightarrow K = 2$$

18. Ajar is filled with two non-mixing liquids 1 and 2 having densities p_1 and, p_2 respectively. A solid ball, made of a material of density p_3 , is dropped in the jar. It comes to equilibrium in the position shown in the figure.Which of the following is true for p_1, $p_2$andp_3? [2008]



(a) $p_3 < p_1 < p_2$ (b) $p_1 > p_3 > p_2$ (c) $p_1 < p_2 < p_3$ (d) $p_1 < p_3 < p_2$

SOLUTION : (d)

As liquid 1 floats over liquid 2. The lighter liquid floats over heavier liquid. So, $p_1 < p_2$

Also $p_3 < p_2$ because the ball of density p_3 does not sink to the bottom ofthejar.

Also $p_3 > p_1$ otherwise the ball would have floated in liquid 1.

we conclude that $p_1 < p_3 < p_2$.

Fluid Flow , Reynold's Number and Bernouli's Principle :

19. Afluid is flowing through ahorizontal pipe ofvarying crosssection, with speed v ms 1 at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms 1. Ifthe density ofthe fluid is p kg m^3 and the flow is streamline, then V is equal to:

[6 Sep. 2020 (ID]

(a) $\sqrt{\frac{P}{p}+v}$ (b) $\sqrt{\frac{2P}{p}+v^2}$ (c) $\sqrt{\frac{P}{2p}+v^2}$ (d) $\sqrt{\frac{P}{p}+v^2}$

SOLUTION : (d)

Using Bernoulli's equation

$$P_1+\frac{1}{2}pu_1^2+\rho gh_1=P_2+\frac{1}{2}pu_2^2+pgh_2$$

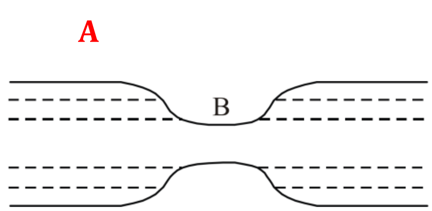
For horizontal pipe, $h_1=0$ and $h_2=0$ and taking $P_1=P, P_2=\frac{P}{2}$, we get

$$\Rightarrow P+\frac{1}{2}p\iota)^2=\frac{P}{2}+\frac{1}{2}pV^2$$

$$\Rightarrow \frac{P}{2}+\frac{1}{2}p\iota)^2=\frac{1}{2}pV^2$$

$$\Rightarrow V=\sqrt{0^2+\frac{P}{p}}$$

20. Water flows in a horizontal tube (see figure). The pressure ofwater changes by 700 Nm 2 between A and B where the area of cross section are 40 cm^2 and20 cm^2, respectively. Find the rate of flow ofwater through the tube. (density ofwater = 1000 kgm 3) [9 Jan. 2020 (D]



(a) 3020 cm^3/s (b) 2720 cm^3/s (c) 2420 cm^3/s (d) 1810 cm^3/s

SOLUTION : (b)

According to question, area of cross-section at $A, aA=40cm^2$ and at $B, aB=20cm^2$

Let velocity ofliquid flow at $A,=V_A$ and at $B,=V_B$

Using equation of continuity $a_AV_A=a_BV_B$

$$40V_A=20V_B \Rightarrow 2V_A=V_B$$

Now, using Bernoulli's equation

$$P_A+\frac{1}{2}\text{p}V_A^2=P_B+\frac{1}{2}\text{p}V_B^2 \Rightarrow P_A-P_B=\frac{1}{2}\text{p}\left(V_B^2-V_A^2\right)$$

$$\Rightarrow \Delta P=\frac{1}{2}1000\left(V_B^2-\frac{V_B^2}{4}\right) \Rightarrow \Delta P=500\times\frac{3V_B^2}{4}$$

$$\Rightarrow V_B=\sqrt{\frac{(\Delta P)\times 4}{1500}}=\sqrt{\frac{(700)\times 4}{1500}}\text{m/s}=1.37\times 10^2\text{cm/s Volume flow rate Q}=a_B\times v_B$$

$$=20\times 100\times V_B=2732\text{cm}^3/\text{s}\approx 2720\text{cm}^3/\text{s}$$

21. **An ideal fluid flows (laminar flow) through apipe ofnonuniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities offluid in this pipe is: [7 Jan. 2020 0D]**

(a) $\frac{9}{16}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{4}$ (d) $\frac{81}{256}$

SOLUTION : **(a)**

From the equation ofcontinuity $A_1v_1=A_2v_2$

Here, v_1 and v_2 are the velocities at two ends ofpipe. A_1 and A_2 are the area ofpipe at two ends

$$\Rightarrow \frac{v_1}{v_2}=\frac{A_2}{A_1}=\frac{\pi(4.8)^2}{\pi(6.4)^2}=\frac{9}{16}$$

22. **Water from a tap emerges vertically downwards with an initial speed of 1.0 ms 1. The cross-sectional area of the tap is $10^{◅}\text{m}^2$. Assume that the pressure is constant throughout the stream ofwater and that the flow is streamlined. The cross-sectional area ofthe stream, 0.15m below the tap would be: [Take g = 10 ms 2) [10 April 2019 (II)]**

(a) $2\times 10^5\text{m}^2$ (b) $5\times 10^5\text{m}^2$ (c) $5\times 10^{◅}\text{m}^2$ (d) $1\times 10^5\text{m}^2$

SOLUTION : **(b)**

Using Bernoullie's equation $P+\frac{1}{2}\left(v_1^2-v_2^2\right)+\rho gh=P$

$$\Rightarrow v_2^2=v_1^2+2gh \quad \Rightarrow v_2=\sqrt{v_1^2+2gh}$$

Equation ofcontinuity $\text{A}_1\text{v}_1=\text{A}_2\text{v}_2$

$$(1\text{cm}^2)(1\text{m/s})=(\text{A}_2)\left(\sqrt{(1)^2+2\times 10\times\frac{15}{100}}\right)$$

$$10^{◅}\times 1=\text{A}_2\times 2\,;\quad \text{A}_2=\frac{10^{-4}}{2}=5\times 10^5\text{m}^2$$

23. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is ofthe order of: (density ofwater $= 1000\ \text{kg/m}^3$, coefficient ofviscosity ofwater $= 1\text{mPa}s$) [8April 2019 I]

(a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

SOLUTION : (b)

$$\text{Rate offlow ofwater}(V) = 100\text{lit/ min} = \frac{100\times10^{-3}}{60} \times \frac{5}{3} \times 10^{-3}\text{m}^3$$

$$\text{Velocity of flow ofwater } (v) = \frac{V}{A} = \frac{5\times10^{-3}}{3\times\times(5\times10^{-2})^2} = \frac{10}{15\pi} = \frac{2}{3\pi}\text{m/s} = 0.2\text{m/s}$$

$$\text{Reynold number } (N_R) = \frac{Dvp}{\eta} = (10\times10^{-2}) \times \underline{2} \times 1000 = \frac{3\pi}{1} = 2\times10^4$$

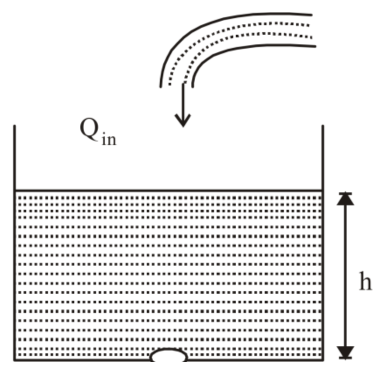
$$\text{Order of} N_R = 10^4$$

24. Water flows into a large tank with flat bottom at the rate of $10^{-4}\text{m}^3\text{s}^{-(1)}$ Water is also leaking out ofa hole of area $1\ \text{cm}^2$ at its bottom. If the height ofthe water in the tank remains steady, then this height is: [10 Jan. 2019 I]

(a) 5.1 cm (b) 7 cm (c) 4 cm (d) 9 cm

SOLUTION : . (a)

(a)



$-\tau_{Q_{out}}$

Since height ofwater column is constant therefore, water inflowrate $(Q_{\dot{m}})$

$$= \text{water outflow rate}$$

$$Q_{in} = 10^{\to\downarrow}\text{m}^3\text{s}^1$$

$$Q_{out} = Au = 10^{\to l} \times \sqrt{2gh}$$

$$10^{\triangleleft} = 10^{\triangleleft} \times \sqrt{20\times h}$$

$$h = \frac{1}{20}\text{m} = 5\ \text{cm}$$

25. The top of a water tank is open to air and its water lavel is maintained. It is giving out $0.74 m^3$ water per minute through a circular opening of2 cm radius in its wall. The depth ofthe centre ofthe opening from the level ofwater in the tank is close to: [9Jan. 2019 (II)]

(a) $6.0 m$ (b) $4.8 m$ (c) $9.6 m$ (d) $2.9 m$

SOLUTION : . (b)

$$\text{Here, volume tric flow rate} = \frac{0.74}{60} = \pi r^2 v = (\pi \times 4 \times 10^{\to t}) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10} (\because \pi^2 = 10)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24} \approx 4.8 m$$

i. e., The depth ofthe centre ofthe opening from the level ofwater in the tank is close to $4.8 m$

26. When an air bubble ofradius r rises from the bottom to thesurface of a lake, its radius becomes $\frac{5r}{4}$ Taking the atmospheric pressure to be equal to 10m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect oftemperature): [Online Apri115, 2018]

(a) $10.5 m$ (b) $8.7 m$ (c) $112 m$ (d) $9.5 m$

SOLUTION : . (d)

$$\text{Using } P_1V_1 = P_2V_2$$

$$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\pi\frac{125r^3}{64}$$

$$\frac{pg(10) + pgh}{pg(10)} = \frac{125}{64}$$

$$640+64\,h = 1250$$

$$\text{On solving we get } h = 9.5 m$$

27. Two tubes ofradii r_1 and r_2, and lengths l_1 and l_2, respectively, are connected in series and a liquid flows through each ofthem in streamline conditions. P_1 and P_2 are pressure differences across the two tubes. If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will oe equal to:

[Online April 9, 2017]

(a) r_1 (b) $2r_1$ (c) $4r_1$ (d) $\frac{r_1}{2}$

SOLUTION : (d)

The volume ofliquid flowing through both the tubes i. e., rate offlow ofliquid is same.

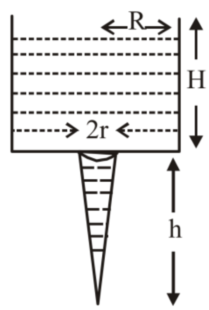
$$\text{Therefore, } V = V_1 = V_2 \quad \text{i.e., } \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2} \quad \text{or } \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

$$P_2 = 4P_1 \text{ and } l_2 = l_1/4$$

$$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{l_1/4} \Rightarrow r_2^4 = \frac{r_1^4}{16}$$

$$r_2 = r_1/2$$

28. Consider a waterjar ofradius R that has water filled up to height H and is kept on a stand ofheight h (see figure). Through a hole ofradius $r(r << R)$ at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. Ifthe radius ofthe cross-section ofwater stream when it hits the ground is x. Then : [Online April 9, 2016]



(a) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$ (b) $x = r\left(\frac{H}{H+h}\right)$ (c) $x = r\left(\frac{H}{H+h}\right)^{2}$ (d) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

SOLUTION : (a)

$$\text{According to Bernoulli's Principle, } \frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2$$

$$v_1^2 + 2gh = v_2^2$$

$$2gH + 2gh = v_2^2 \text{ (i)}$$

$$a_1 v_1 = a_2 v_2$$

$$\pi r^2\sqrt{2gh} = \pi x^2 v_2 \quad ; \quad \frac{r^2}{x^2}\sqrt{2gh} = v_2$$

Substituting the value of v_2 in equation (i) $2gH + 2gh = \frac{r^4}{x^4}2gh$

$$\text{or, } x = r\left|\frac{H}{H+h}\right|^{\frac{1}{4}}$$

29. Ifit takes 5 minutes to fill a 15 litre bucket fi$_i$om a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water $= 10^3 kg/m^3$) and viscosity of water $= 10^{-3}$ Pa. s) close to : [Online April 10, 2015]

(a) 1100 (b) 11,000 (c) 550 (d) 5500

SOLUTION : . (d)

Given: Diameter of water tap $= \frac{2}{\sqrt{\pi}}$ cm Radius, $r = \frac{1}{\sqrt{\pi}} \times 10^{-2}$m

$$\frac{dm}{dt} = \text{pAV}$$

$$\frac{15}{5\times60} = 10^3 \times \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4}\text{V} \Rightarrow \text{V} = 0.05\text{m/s}$$

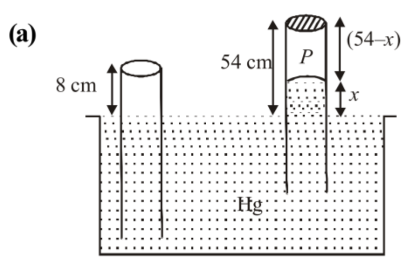
$$\text{Reynold's number, } R_e = \frac{\text{p}Vr}{n}$$

$$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}} \cong 5500$$

30. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end ofthe tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length ofthe air column above mercury in the tube now? (Atmospheric pressure = 76 cm ofHg) [2014]

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

SOLUTION : (a)



Length ofthe air column above mercury in the tube is, $P + x = P_0$

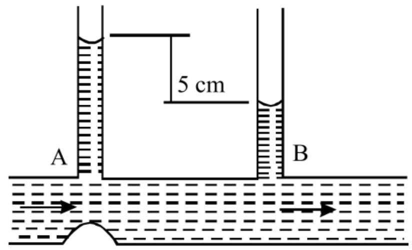
$$\Rightarrow P = (76 - x)$$

$$\Rightarrow 8 \times A \times 76 = (76 - x) \times \text{A} \times (54 - x)$$

$$x = 38$$

Thus, length ofair column $= 54 - 38 = 16$ cm.

31. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section ofthe tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is $(g = 10ms^{-2})$ [Online Apri119, 2014]



(a) $7.\frac{5cc}{s}$ (b) $8.\frac{0cc}{s}$ (c) $10.0cc/s$ (d) $12.5cc/s$

SOLUTION : (a)

According to Bernoulli's theorem, $P_1 + \frac{1}{2}pv_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$v_2^2 - v_1^2 = 2gh \text{ .... } (1)$$

According to the equation of continuity $A_1v_1 = A_2v_2$ .... (2)

$$\frac{A_1}{A_2} = \frac{6mm^2}{10mm^2}$$

From equation (2), $\frac{A_1}{A_2} = \frac{v_2}{v_1} = \frac{6}{10}$

or, $v_2 = \frac{6}{10}v_1$

Putting this value of v_2 in equation (1)

$$\left(\frac{6}{10}v_1\right)^2 - (v_1)^2 = 2 \times 10^3 \times 5$$

$$[g = 10m/s^2 = 10^3 cm/s^2]$$

Solving we get $v_1 = \frac{10}{8}$

Therefore the rate at which water flows through the tube $= A_1v_1 = A_2v_2 = \frac{6\times10}{8} = 7.5cc/s$

32. A cylindrical vessel ofcross-section A contains water to a height h. There is a hole in the bottom ofradius 'a'. The time in which it will be emptied is: [Online Apri112, 2014]

(a) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$ (b) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$ (c) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$ (d) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

SOLUTION : **(b)**

Let the rate offalling water level be $-\frac{dh}{dt}$

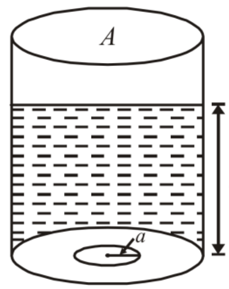
Initially at $t=0\,;\,h=h$

$t=t\,;\,h=0$

Then, $A\left(-\frac{dh}{dt}\right)=\pi a^2.v$

$dt=-\frac{A}{\pi a^2\sqrt{2gh}}dh$

[$\because$ velocity of efflux of liquid $v=\sqrt{2gh}$]



Integrating both sides

$$\int_0^t d\,t=-\frac{A}{\sqrt{2g}\pi a^2}\int_h^0 h^{-1/2}\,dh$$

$$[t]_0^t=-\frac{A}{\sqrt{2g}\pi a^2}.\left[\frac{h^{1/2}}{1/2}\right]_h^0$$

$$t=\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$

33. Water is flowing at a speed of 1.5ms^{-1} through horizontal tube ofcross-sectional area 10^{-2}m^2 and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density ofwater $=10^3\text{kgm}^{-3}$) [Online April 9, 2014]

(a) 15N **(b) 22.5N** **(c) 33.7N** **(d) 45N**

SOLUTION : **(a)**

For 1 m length ofhorizontal tube

Mass ofwater M =density $\times$ volume $=10^3$ $\times$area $\times$ lengLh $=10^3\times10^{-2}\times1=10$ kg

Therefore minimum force $=\frac{\Delta p}{\Delta t}$(rate of change of momentum) $=10\times1.5=15N$

34. Air ofdensity 1.2 kg m^{-3} is blowing across the horizontal wings ofan aeroplane in such a way that its speeds above and below the wings are 150 ms^{-1} and 100 ms^{-1}, respectively. The pressure difference between the upper and lower sides ofthe wings, is:

[OnlineApril 22, 2013]

(a) 60Nm^{-2} **(b) 180Nm^{-2}** **(c) 7500Nm^{-2}** **(d) 12500Nm^{-2}**

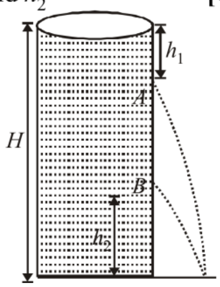
SOLUTION : **(c)**

$$\text{Pressure difference} P_2 - P_1 = \frac{1}{2}p(v_2^2 - v_1^2) = \frac{1}{2} \times 1.2((150)^2 - (100)^2)$$

$$= \frac{1}{2} \times 1.2(22500 - 10000) = 7500\text{Nm}^{-2}$$

35. In a cylindrical water tank, there are two small holes A and B on the wall at a depth of h_1, from the surface ofwater and at a height of h_2 fi¡om the bottom ofwater tank. Surface ofwater is at heigh H from the bottom ofwater tank. Water coming out from both holes strikes the ground at the same point S. Find the ratio os h_1 and h_2

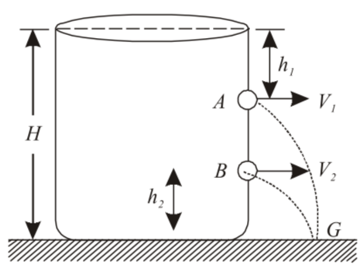
the ratio of h_1 and h_2 **[Online May 26, 2012]**



(a) Depends on H **(b) 1 : 1** **(c) 2: 2** **(d) 1: 2**

SOLUTION : **. (b)**

(b)



i.e. $R_1 = R_2 = R$ or, $v_1 t_1 = v_2 t_2$ (i)

Where v_1 = velocity of efflux at $A = \sqrt{(2gh_1)}$ and

v_2 = velocity of efflux at $B = \sqrt{(2g(H - h_2)}$

t_1 = time of fall water stream through $A = \sqrt{\left(\frac{2(H - h_1}{g}\right)}$

t_2 = time of fall ofthe water stream through $B = \sqrt{\frac{2h_2}{g}}$ Putting these values is eqn (i) we get

$$(H - h_1)h_1 = (H - h_2)h_2 \text{ or } [H - (h_1 + h_2)][h_1 - h_2] = 0$$

Here, $H = h_1 + h_2$ is irrelevant because the holes are at $\underline{l\eta} = 1$

two different heights. Hence $h_1 = h_2$

Viscosity and Terminal Velocity :

36. **Water is flowing through a horizontal tube having crosssectional areas ofits two ends being A and A' such that the ratio A/A' is 5. Ifthe pressure difference ofwater between the two ends is $3 \times 10^5 Nm^{-2}$, the velocity of water with which it enters the tube will be (neglect gravity effects) [Online May 12, 2012]**

(a) $5ms^{-1}$ (b) $10ms^{-1}$ (c) $25\ ms^{-1}$ (d) $50\sqrt{10}ms^{-1}$

SOLUTION : (a)

According to Bernoulli' s theorem $P_1 + \frac{1}{2} \mathrm{p} v_1^2 = P_2 + \frac{1}{2} \mathrm{p} v_2^2$ (i) From question,

$$P_1 - P_2 = 3 \times 10^5, \frac{A_1}{A_2} = 5$$

According to equation of continuity $A_1 v_1 = A_2 v_2 \quad \Rightarrow v_2 = 5 v_1$

From equation (i) $P_1 - P_2 = \frac{1}{2} \mathrm{p} \left(v_2^2 - v_1^2\right)$

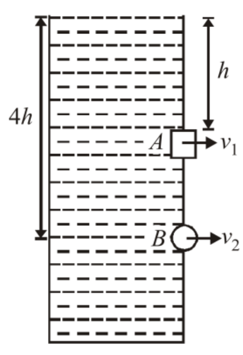
$$\text{or } 3 \times 10^5 = \frac{1}{2} \times 1000 \left(5 v_1^2 - v_1^2\right)$$

$$\Rightarrow 600 = 6 v_1 \times 4 v_1$$

$$\Rightarrow v_1^2 = 25$$

$$v_1 = 5 \mathrm{m/s}$$

37. **A square hole of side length ℓ is made at a depth ofh and a circular hole ofradius r is made at a depth of4hfi$_i$om the surface ofwater in a water tank kept on a horizontal surface. If $\ell << h, r << h$ and the rate ofwater flow from the holes is the same, then r is equal to [May7, 2012]**



(a) $\frac{\ell}{\sqrt{2\pi}}$ (b) $\frac{\ell}{\sqrt{3\pi}}$ (c) $\frac{\ell}{3\pi}$ (d) $\frac{\ell}{2\pi}$

SOLUTION : . (a)

$$\text{As } A_1v_1 = A_2v_2 \text{ (Principle of continuity)}$$

$$\text{or, } \ell^2\sqrt{2gh} = \pi r^2\sqrt{2g \times 4h}$$

$$\text{(Efflux velocity} = \sqrt{2\text{gh}})$$

$$r^2 = \frac{\ell^2}{2\pi} \text{ or } r = \sqrt{\frac{\ell^2}{2\pi}} = \frac{\ell}{\sqrt{2\pi}}$$

38. Water is flowing continuouslyfiom atap having an internal diameter 8×10^{-3}m. The water velocity as it leaves the tap is 0.4ms^{-1}. The diameter ofthe water stream at a distance 2 $\times 10^{-1}$m below the tap is close to: [2011]

(a) 7.5×10^{-3}m (b) 9.6×10^{-3}m (c) 3.6×10^{-3}m (d) 5.0×10^{-3}m

SOLUTION : (c)

Using Bernoulli's theorem, for horizontal flow $P_0 + \frac{1}{2}\text{p}v_1^2 + \text{p}gh = P_0 + \frac{1}{2}\text{p}v_2^2 + 0$

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{016 + 2 \times 10 \times 0.2} = 2.03\text{n}\sqrt{}\text{ s}$$

According to equation of continuity $A_2v_2 = A_1v_1$

$$\pi\frac{D_2^2}{4} \times v_2 = \pi\frac{D_1^2}{4}v_1$$

$$\Rightarrow D_2 = D_1\sqrt{\frac{v_1}{v_2}} = 3.55 \times 10^{-3}\text{m}$$

39. A cylinder ofheight 20 m is completely filled with water. The velocity ofefflux ofwater (in ms^{-1}) through a small hole on the side wall ofthe cylinder near its bottom is [2002]

(a) 10 (b) 20 (c) 25.5 (d) 5

SOLUTION : (b)

Given, Height ofcylinder, h $= 20$ cm Acceleration due to gravity, g $= 10\text{ms}^{-2}$

$$\text{Velocity ofefflux } v = \sqrt{2gh}$$

Where h is the height ofthe free surface ofliquid from th e hole

$$\Rightarrow v = \sqrt{2 \times 10 \times 20} = 20\text{m/s}$$

40. In an experiment to Veriђ$r$stokeS laW, asmall spherical ball of radius r and density p falls under gravity through a distance h in air before entering a tank ofwater. Ifthe terminal velocityof the ball inside water is same as its velociLyjust before enterin g the water surface, then the value ofh is proportional to: (ignore viscosity of air) [5 Sep. 2020 (II)]

(a) r^4 (b) r (c) r^3 (d) r^2

SOLUTION : (a)

$$\text{Using, } v^2 - u^2 = 2gh \Rightarrow v^2 - 0^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

$$\text{Terminal velocity, } V_T = \frac{2}{9}\frac{r^2(\text{p}-0)g}{\eta}$$

After falling through h the velocity should be equal to terminal velocity

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(\text{p}-0)g}{\eta} \quad \Rightarrow \quad 2gh = \frac{4r^4g^2(\text{p}-0)^2}{81\eta^2}$$

$$\Rightarrow h = \frac{2r^4g(\text{p}-0)^2}{81\eta^2} \Rightarrow h\alpha r^4$$

41. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient ofviscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the Same fluid, the ratio $(v_1 l v_2)$ equals: [12 April 2019 (II)]

(a) 9 (b) 1/27 (c) 1/9 (d) 27

SOLUTION : (a)

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3 \quad \text{or } r = \frac{R}{3}$$

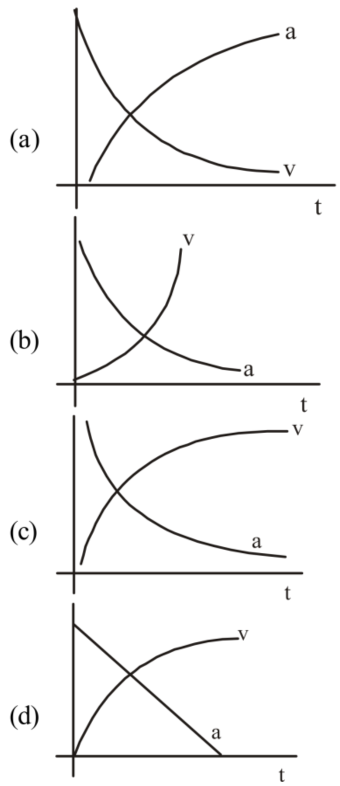
$$\text{Terminal velocity, } v \propto r^2$$

$$\frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$$

$$\text{or } v_2 = \left(\frac{r_2^2}{r_1^2}\right)^{v_1}$$

$$v_1 = \frac{1}{9}$$

42. **Which of the following option correctly describes the variation ofthe speed v and acceleration'a' ofa point mass falling vertically in a viscous medium that applies a force $\Gamma = -kv$, where ↑ k′ is a constant, on the tx)dy? (Graphs are schematic and not drawn to scale)** **[Online April 9, 2016]**



SOLUTION : **(c)**

When a point mass is falling vertically in a viscous medium, the medium or viscous fluid exerts drag force on the body to oppose its motion and at one stage body falling with constant terminal velocity.

43. **The velocity ofwater in a river is 18 km/hr near the surface. Ifthe river is 5 m deep, find the shearing stress between the horizontal layers ofwater. The co-efficient ofviscosity of water $= 10^{-2}$ poise.** **[Online April 19, 2014]**

(a) $10^{-1} N/m^2$ **(b) $10^{-2} N/m^2$** **(c) $10^{-3} N/m^2$** **(d) $10^{-4} N/m^2$**

SOLUTION : **(b)**

$$\eta = 10^{-2} \text{ poise}$$

$$v = 18\text{km/h} = \frac{18000}{3600} = 5\text{m/s} \quad l = 5\text{m}$$

$$\text{Strain rate} = \frac{v}{l}$$

$$\text{Coefficient of viscosity, } \eta = \frac{\text{shearingstress}}{\text{strainrate}}$$

$$\text{Shearing stress} = \eta \times \text{strain rate} = 10^{-2} \times \frac{5}{5} = 10^{-2}\text{Nm}^{-2}$$

44. The average mass ofrain drops is 3.0×10^{-5} kg and their avarage terminal velocity is 9 m/s. Calculate the energy transferred by rain to each square metre ofthe surface at a place which receives 100 cm ofrain in a year. [Online April 11, 2014]

(a) 3.5×10^5J (b) 4.05×10^4J (c) 3.0×10^5J (d) 9.0×10^4J

SOLUTION : (b)

Total volume ofrain drops, received 100 cm in a year byarea 1 m^2

$$= 1m^2 \times \frac{100}{100} m = 1m^3$$

As we know, density ofwater, $d = 10^3 kg/m^3$

Therefore, mass ofthis volume ofwater $M = d \times v = 10^3 \times 1 = 10^3$ kg

Average terminal velocity ofrain drop $v = 9m/s$ (given)

Therefore, energy transferred by rain, $E = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times (9)^2$

$$= \frac{1}{2} \times 10^3 \times 81 = 4.05 \times 10^4 J$$

45. A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is 3m and that of kerosene 2m. When the hole is opened the velocity of fluid coming out from it is nearly: (take $g = 10ms^{-2}$ and density ofwater $= 10^3$ kg m^{-3}) [Online April 11, 2014]

(a) $10.7ms^{-1}$ (b) $9.6ms^{-1}$ (c) $8.5ms^{-1}$ (d) $7.6ms^{-1}$

SOLUTION : (b)

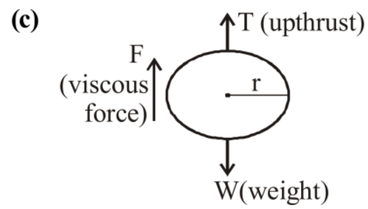
According to Toricelli's theorem, Velocity ofefflex,

$$V_{eff} = \sqrt{2gh} = \sqrt{2 \times 98 \times 5} \cong 9.8ms^{-1}$$

46. In an experiment, a small steel ball falls through a liquid at a constant speed of10 cm/s. Ifthe steel ball is pulled upward with a force equal to twice its effective weight, how fast will it move upward? [Online April 25, 2013]

(a) 5 cm/s (b) Zero (c) 10 cm/s (d) 20 cm/s

SOLUTION : **(c)**



$$\text{Weight of the body} W = mg = \frac{4}{3}\pi r^3 pg$$

$$T = \frac{4}{3}\pi r^3 og \quad \text{and } \Gamma = 6\pi\eta vr$$

When the body attains terminal velocity net force acting on the body is zero.

$$\text{i. e., W-T} - \Gamma = 0$$

$$\text{And terminal velocity } v = \frac{2}{9}\frac{r^2(p-0)g}{\eta}$$

As in case of upward motion upward force is twice its effective weight, therefore,

it will move with same speed 10 cm/s

47. The terminal velocity of a small sphere of radius a in a viscous liquid is proportional to

[Online May 26, 2012]

(a) a^2 **(b) a^3** **(c) a** **(d) a^{-1}**

SOLUTION : **. (a)**

Terminal velocity in a viscous medium is given by:

$$V_T = \frac{2a^2(p-0)g}{9\eta}$$

$$V_T \propto a^2$$

48. Ifa ball of steel (density $p = 7.8 gcm^{-3}$) attains a terminal velocity of 10 cm s^{-1} when falling in water (Coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s), then, its terminal velocity in glycerine ($p = 1.2 gcm^{-3}$, $\eta = 13.2$ Pa. s) would be, nearly **[2011 RS]**

(a) 6.25×10^{-4} cm s^{-1} **(b) 6.45×10^{-4} cm s^{-1}**

(c) 1.5×10^{-5} cm s^{-1} **(d) 1.6×10^{-5} cm s^{-1}**

SOLUTION : **(a)**

When the ball attains terminal velocity

Weight ofthe ball = viscous force + buoyant force

$$Vpg = 6\pi\eta rv + Vp_p g$$

$$\Rightarrow Vg(p - p_\ell) = 6\pi\eta rv$$

$$\text{Also } Vg(p - p_i^{'}) = 6\pi\eta^{'} rv^{'}$$

$$v^{'}\eta^{'} = \frac{(p - p_l)}{(p - p_l)} \times v\eta$$

$$\Rightarrow v^{'} = \frac{(p - p_p)^{'}}{(p - p_p)} \times \frac{v\eta}{\eta^{'}}$$

$$= \frac{(7.8 - 1.2)}{(7.8 - 1)} \times \frac{10 \times 8.5 \times 10^{-4}}{13.2}$$

$$v^{'} = 6.25 \times 10^{-4} \text{cm/s}$$

49. A spherical solid ball ofvolume V is made of a material of density p_1. It is falling through a liquid of density $p_1 (p_2 < p_1)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $\Gamma_{\text{viscous}} = -ky^2 (k > 0)$. The terminal speed ofthe ball is [2008]

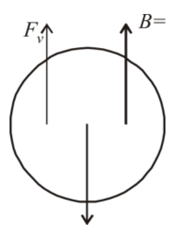
(a) $\sqrt{\frac{Vg(p_1 - p_2)}{k}}$ (b) $\frac{Vgp_1}{k}$ (c) $\sqrt{\frac{Vgp_1}{k}}$ (d) $\frac{Vg(p_1 - p_2)}{k}$

SOLUTION : (a)

When the ball attains terminal velocity

Weight ofthe ball = Buoyant force + Viscous force

$$Vp_2 g$$



$$W = VP_1 g$$

$$Vp_1 g = Vp_2 g + kv_t^2 \Rightarrow y_{g(p_1 - p_2)g = kv_t^2}$$

$$\Rightarrow v_t = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$$

50. If the terminal speed of a sphere of gold (density $= 19.5 kg/m^3$) is $0.2 m/s$ in a viscous liquid (density $= 1.5 kg/m^3$), fmd the terminal speed of a sphere of silver (density $= 10.5 kg/m^3$) ofthe same size in the same liquid [2006]

(a) 0.4 m/s (b) 0.133 m/s (c) 0.1 m/s (d) 0.2 m/s

SOLUTION : (c)

Given,

Density ofgold, $p_G = 19.5 kg/m^3$

Density of silver, $p_5 = 10.5 kg/m^3$

Density ofliquid, $0 = 1.5 kg/m^3$

Terminal velocity, $v_T = \frac{2r^2(p-0)g}{9\eta}$

$$\frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)} \Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$$

$$v_{T_2} = 0.1 m/s$$

51. Spherical balls ofradius 'R' are falling in a viscous fluid ofviscosity ' with a velocity v'. The retarding viscous force acting on the spherical ball is [2004]

(a) inversely proportional to both radius 'R' and velocity'v'

(b) directly proportional to both radius 'R' and velocity'v'

(c) directlyproportional to 'R' but inverselyproportional to 'v'

(d) inverselyproportional to 'R' but directly proportional to velocity'v'

SOLUTION : (b)

From Stoke's law, force of viscosity acting on a spherical body is $F = 6\pi\eta r v$

hence F is directly proportional to radius & velocity.

SURFACETENSION

Surface Tension , Surface Energy and Capillarity :

1. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. Ifthe contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is $\left[p_{(1\text{liquid})} = 900 kgm^{-3}, g = 10ms^{-2}\right]$ (Give answer in closest integer) [NA 3 Sep. 2020 (I)]

SOLUTION : (101)

Given: Radius ofcapillary tube, $r = 0.015$ cm $= 15 \times 10^{-5}$ mm $h = 15$ cm $= 15 \times 10^{-2}$ mm

Using, $h = \underline{2T\cos\theta}$ $[\cos\theta = \cos 0^{\circ} = 1]$

pgr

Surface tension, $T = \frac{rhpg}{2} = \frac{15\times10^{-5}\times15\times10^{-2}\times900\times10}{2} = 101$ milli newton m^{-1}

2. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is: [3 Sep. 2020 (I)]

(a) 4: 1 (b) 0.8: 1 (c) 8: 1 (d) 2: 1

SOLUTION : (c)

According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \text{(i)}$$

And $\Delta P_2 = P_2 - P_0 = 0.02 = \underline{4T}$ (ii)

R_2

Dividing, equation(ii) by(i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Volume $V = \frac{4}{3}\pi R^3$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$$

3. A capillarytube made ofglass ofradius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension $= 0.05Nm^{-1}$, density $= 667$ kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn fiiom liquid-glass interfaces (from opp. sides of the capillary) make an angle of60° with one another. Then h is close to $(g = 10ms^{-2})$. [2 Sep. 2020 (II)]

(a) 0.049 m (b) 0.087 m (c) 0.137 m (d) 0.172 m

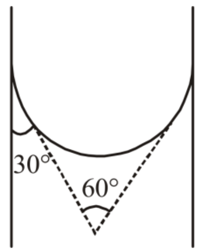
SOLUTION : (b)

Given,Angle ofcontact $\theta = 30^\circ$

Surface tension, $T = 0.05 \text{Nm}^{-1}$

Radius ofcapillarytube, $r = 0.15$ mm $= 0.15 \times 10^{-3}$m

Density ofmethylene iodide, $\text{p} = 667 \text{ kg m}^{-3}$



$$\text{Capillaryrise, } h = \frac{2T\cos\theta}{\text{p}gr} = \frac{2\times 0.05\times\frac{\sqrt{3}}{2}}{667\times 10\times 0.15\times 10^{-3}} = 0.087\text{m}$$

4. A small spherical droplet ofdensity d is floating exactly halfimmersed in a liquid ofdensity p and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet): [9 Jan. 2020 (II)]

(a) $r = \sqrt{\frac{2\text{T}}{3(d+\text{p})g}}$ (b) $r = \sqrt{\frac{\text{T}}{(d-\text{p})g}}$ (c) $r = \sqrt{\frac{\text{T}}{(d+\text{p})g}}$ (d) $r = \sqrt{\frac{3\text{T}}{(2d-\text{p})g}}$

SOLUTION : (d)

For the drops to be in equilibrium upward force on drop = downward force on drop

$$T.2\pi R = \frac{4}{3}\pi R^3 dg - \frac{2}{3}\pi R^3 \rho g$$

$$\Rightarrow T(2\pi R) = \frac{2}{3}\pi R^3(2d - \text{p})g$$

$$\Rightarrow T = \frac{R^2}{3}(2d - \text{p})g \Rightarrow R = \sqrt{\frac{3T}{(2d-\text{p})g}}$$

5. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio oftheir densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube ofradius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to: [10 April 2019 (I)]

(a) $4/5$ (b) $2/5$ (c) $3/5$ (d) $2/3$

SOLUTION : (b)

As we know that

$$\underline{2T\cos\theta} = R_h$$

rpg

$$\frac{T_{Hg}}{T_{Water}} = 7.5$$

$$\frac{p_{Hg}}{p_W} = 13.6 \& \frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135^o}{\cos 0^o} = \frac{1}{\sqrt{2}}$$

$$\frac{R_{Hg}}{R_{Water}} = (\quad)(\quad)(\quad)(\quad)(\quad)(\quad)$$

$$= 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

6. If M’ is the mass ofwater that rises in a capillary tube of radius r’, then mass ofwater which will rise in a capillary tube of radius 2r ’ is: [9 April 2019 I]

(a) M (b) $\frac{M}{2}$ (c) 4 M (d) 2 M

SOLUTION : (d)

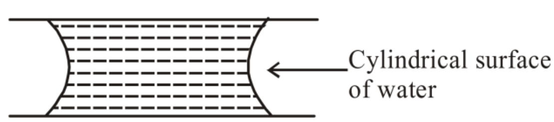
We have, $h = \frac{2T\cos\theta}{rpg}$

Mass of the water in the capillary

$$m = pV = p \times \pi r^2 h = p \times \pi r^2 \times \frac{2T\cos\theta}{rpg}$$

$$\Rightarrow m \alpha r$$

7. If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension ofwater is T then the pressure in water between the plates is lower by: [Online Apri110, 2015]

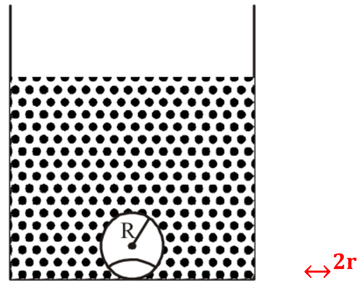


(a) $\frac{2T}{R}$ (b) $\frac{4T}{R}$ (c) $\frac{T}{4R}$ (d) $\frac{T}{R}$

SOLUTION : (d)

8. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres ofradius R and making a circular contact ofradius r with the

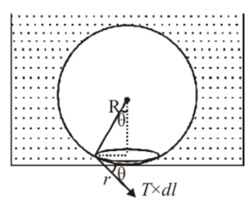
bottom ofthe vessel. If $r \ll R$ and the surface tension of water is T, value of r just before bubbles detach is: (density of water is p_w) [2014]



(a) $R^2\sqrt{\frac{2p_w g}{3T}}$ (b) $R^2\sqrt{\frac{p_w g}{6T}}$ (c) $R^2\sqrt{\frac{p_w g}{T}}$ (d) $R^2\sqrt{\frac{3p_w g}{T}}$

SOLUTION : (a)

When the bubble gets detached, Buoyant force = force due to surface tension



Force due to excess pressure = upthrust Access pressure in air bubble $= \frac{2T}{R}$

$$\frac{2T}{R}(\pi r^2) = \frac{4\pi R^3}{3T}\rho_w g$$

$$\Rightarrow r^2 = \frac{2R^4\rho_w g}{3T} \Rightarrow r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

9. A large number ofliquid drops each ofradius r coalesce to from a single drop ofradius R. The energyreleased in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given, surface tension ofliquid T, density p)

[Online Apri119, 2014, 2012]

(a) $\sqrt{\frac{T}{p}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (b) $\sqrt{\frac{2T}{p}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (c) $\sqrt{\frac{4T}{p}\left(\frac{1}{r}-\frac{1}{R}\right)}$ (d) $\sqrt{\frac{6T}{p}\left(\frac{1}{r}-\frac{1}{R}\right)}$

SOLUTION : (d)

When drops combine to form a single drop ofradius R.

Then energy released, $E = 4\pi TR^3\left[\frac{1}{r}-\frac{1}{R}\right]$

If this energy is converted into kinetic energy then

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\frac{1}{2} \times \left[\frac{4}{3}\pi R^3 p\right] v^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$v^2 = \frac{6T}{p}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$v = \sqrt{\frac{6T}{p}\left[\frac{1}{r} - \frac{1}{R}\right]}$$

10. Two soap bubbles coalesce to form a single bubble. IfV is the subsequent change in volume ofcontained air and S change in total surface area, T is the surface tension and P atmospheric pressure, then which ofthe following relation is correct?

[Online Apri112, 2014]

(a) $4PV + 3ST = 0$ (b) $3PV + 4ST = 0$

(c) $2PV + 3ST = 0$ (d) $3PV + 2ST = 0$

SOLUTION : (b)

11. An air bubble ofradius 0.1 cm is in a liquid having surface tension $0.06 N/m$ and density $10^3 kg/m^3$. The pressure inside the bubble is $1100\ Nm^{-2}$ greater than the atmospheric pressure. At what depth is the bubble below the surface of the liquid? $(g = 9.8 ms^{-2})$ [Online April 11, 2014]

(a) $0.1m$ (b) $0.15m$ (c) $0.20m$ (d) $0.25m$

SOLUTION : (a)

Given: Radius of air bubble, $r = 0.1\ cm = 10^{-3}m$

Surface tension of liquid, $S = 0.06 N/m = 6 \times 10^{-2} N/m$

Density ofliquid, $p = 10^3 kg/m^3$

Excess pressure inside the bubble, $p_{exe} = 1100 Nm^{-2}$

Depth of bubble below the liquid surface, $h = ?$

As we know, pExcess $= hpg + \frac{2s}{r}$

$$\Rightarrow 1100 = h \times 10^3 \times 9.8 + \frac{2 \times 6 \times 10^{-2}}{10^{-3}}$$

$$\Rightarrow 1100 = 9800h + 120$$

$$\Rightarrow 9800h = 1100 - 120$$

$$\Rightarrow h = \frac{980}{9800} = 0.1\text{m}$$

12. Acapillarytube is immersed verticallyin water and the height ofthe water column is x. When this arrangement is taken into a mine ofdepth d, the height ofthe water colunm is y. IfR is the radius ofearth, the ratio $\frac{x}{y}$ is: [Online April 9, 2014]

(a) $\left(1 - \frac{d}{R}\right)$ (b) $\left(1 - \frac{2d}{R}\right)$ (c) $\left(\frac{R-d}{R+d}\right)$ (d) $\left(\frac{R+d}{R-d}\right)$

SOLUTION : (a)

Acceleration due to gravity changes with the depth, $g' = g\left(1 - \frac{d}{R}\right)$

Pressure, $P = pgh$

Hence ratio, $\frac{x}{y}$ is $\left(1 - \frac{d}{R}\right)$

13. Wax is coated on the inner wall ofa capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle ofcontact θ and the height h upto which water rises change. These changes are: [Online April 23, 2013]

(a) θ increases and h also increases
(b) θ decreases and h also decreases
(c) θ increases and h decreases
(d) θ decreases and h increases

SOLUTION : (c)

Angle of contact θ $\cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$

when water is on a waxy or oily surface

$T_{SA} < T_{SL}$ $\cos\theta$ is negative i.e.,

$90^\circ < \theta < 180^\circ$

i.e., angle of contact θ increases

And for $\theta > 90^\circ$ liquid level in capillary tube fall.

i.e., h decreases

14. A thin tube sealed at both ends is 100 cm long. It lies horizontally, the middle 20 cm containing mercury and two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, bywhat amount will the mercurybe displaced?

(Given : cross-section of the tube can be assumed to be unifoim) [Online April 23, 2013]

(a) 2.95 cm (b) 5.18 cm (c) 8.65 cm (d) 0.0 cm

SOLUTION : (b)

15. This question has Statement-l and Statement-2. Ofthe four choices given after the Statements, choose the one that best describes the two Statetnents.

Statement-l: A capillary is dipped in a liquid and liquid rises to a height h in it. As the temperature ofthe liquid is raised, the height h increases (ifthe density of the liquid and the angle ofcontact remain the same).

Statement-2: Surface tension of a liquid decreases with the rise in its temperature.

[Online April 9, 2013]

(a) Statement-l is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-l.

(b) Statement- 1 is false, Statement-2 is true.

(c) Statement- 1 is true, Statement-2 is false.

(d) Statement-l is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-l.

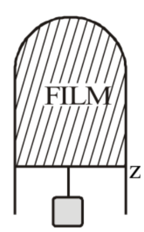
SOLUTION : (b)

Surface tension of a liquid decreases with the rise in temperture. At the boiling point ofliquid, surface tension is zero.

$$\text{Capillary rise h} = \frac{2T\cos\theta}{rdg}$$

As surface tension T decreases with rise in temperature hence capillary rise also decreases.

16. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length ofthe slider is 30 cm and its weight is negligible. The surface tension ofthe liquid film is [2012]



W

(a) $0.0125\ Nm^{-1}$ (b) $0.1\ Nm^{-1}$ (c) $0.05\ Nm^{-1}$ (d) $0.025\ Nm^{-1}$

SOLUTION : (d)

Let T is the force due to surface tension per unit length, then $\Gamma = 2lT$

l = length of the slider.

At equilibrium, $F = W$

$$2Tl = mg$$

$$\Rightarrow T = \frac{mg}{2l} = \frac{1.5\times10^{-2}}{2\times30\times10^{-2}} = \frac{1.5}{60} = 0.025\text{Nm}^{-1}$$

17. Work done in increasing the size ofa soap bubble fiom a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03Nm^{-1}) [2011]

(a) 0.2πmJ (b) 2πmJ (c) 0.4πmJ (d) 4πmJ

SOLUTION : (c)

Work done = increase in surface area × surface tension

$$\Rightarrow W = 2T4\pi[(5^2) - (3)^2] \times 10^{-4}$$

$$= 2 \times 0.03 \times 4\pi[25 - 9] \times 10^{-4}\text{J}$$

$$= 0.4\pi \times 10^{-3}\text{J} = 0.4\pi\text{mJ}$$

18. Two mercury drops (each of radius 'r') merge to form bigger drop. The surface energy ofthe bigger drop, if T is the surface tension, is : [2011 RS]

(a) $4\pi r^2 T$ (b) $2\pi r^2 T$ (c) $2^{8/3}\pi r^2 T$ (d) $2^{5/3}\pi r^2 T$

SOLUTION : (c)

As volume remains constant

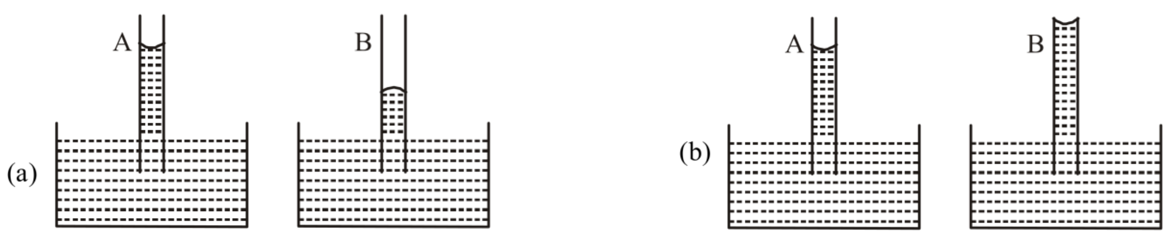
Sum of volumes of 2 smaller drops = Volume of the bigger drop

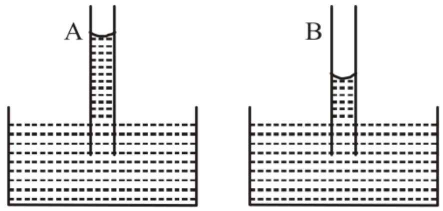
$$2.\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2^{1/3} r$$

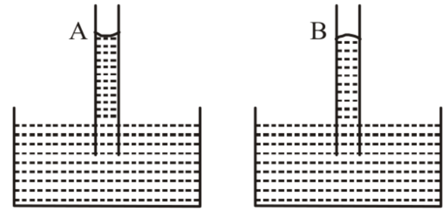
Surface energy = Surface tension × Surface area = $T.4\pi R^2$

$$= T4\pi 2^{2/3} r^2 = T.2^{8/3}\pi r^2$$

19. Acapillary tube(A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Wmich of the following shows the relative nature ofthe liquid columns in the two tubes? [2008]



(c) 

(d) 

SOLUTION : (c)

In case of water, the meniscus shape is concave upwards.

From ascent formula $h = \frac{2o\cos\theta}{rpg}$

The surface tension (σ) of soap solution is less than water.

Therefore height ofcapillaryrise for soap solution should be less as compared to water.

As in the case ofwater, the meniscus shape of soap solution is also concave upwards.

20. A20 cm long capillary tube is dipped in water. The water rises up to 8 cm. Ifthe entire arrangement is put in a freely falling elevator the length ofwater column in the capillary tube will be [2005]

(a) 10 cm (b) 8 cm (c) 20 cm (d) 4 cm

SOLUTION : (c)

Water fills the tube entirely in gravityless condition i.e., 20 cm.

21. If two soap bubbles of different radii are connected by a tube [2004]

(a) air flows fi$_i$om the smaller bubble to the bigger

(b) air flows fi$_{i0}$m bigger bubble to the smaller bubble till the sizes are interchanged

(c) air flows fi$_i$om the bigger bubble to the smaller bubble till the sizes become equal

(d) there is no flow ofair.

SOLUTION : (a)

Let pressure outside be P_0 and r and R be the radius of smaller bubble and bigger bubble respectively.

Pressure P_1 For smaller bubble $= P_0 + \frac{2T}{r}$

P_2 **For bigger bubble** $= P_0 + \frac{2T}{R}\,(R > r)$

$$P_1 > P_2$$

hence air moves from smaller bubble to bigger bubble.

THERMOMETRY

Heat

The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as heat.

- We can change the temperature of a body by giving heat (temperature rises) or by removing heat (temperature falls) from body.
- The amount of heat (Q) is given to a body depends upon it's mass (m), change in it's temperature ($\Delta\theta° = \Delta\theta$) and nature of material *i.e.* $Q = m.c.\Delta\theta$; where c = specific heat of material.
- Heat is a scalar quantity. It's units are *joule*, *erg*, *cal*, *kcal etc*.
- The calorie (*cal*) is defined as the amount of heat required to raise the temperature of 1 *gm* of water from $14.5°C$ to $15.5°C$.

  Also 1 $kcal = 1000\ cal = 4186\ J$ and 1 $cal = 4.18\ J$

- **British Thermal Unit (BTU) :** One BTU is the quantity of heat required to raise the temperature of one pound ($1\ lb$) of water from $63°F$ to $64°F$

  $$1\ BTU = 778\ ft.\ lb = 252\ cal = 1055\ J$$

- In solids thermal energy is present in the form of kinetic energy, in liquids, in the form of translatory energy of molecules. In gas it is due to the random motion of molecules.
- Heat always flows from a body of higher temperature to lower temperature till their temperature becomes equal (Thermal equilibrium).
- The heat required for a given temperature increase depends only on how many atoms the sample contains, not on the mass of an individual atom.

Temperature

Temperature is defined as the degree of hotness or coldness of a body. The natural flow of heat is from higher temperature to lower temperature.

Two bodies are said to be in thermal equilibrium with each other, when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

- Temperature is one of the seven fundamental quantities with dimension $[\theta]$. It is a scalar physical quantity with S.I. unit kelvin.
- When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls *i.e.* temperature can be regarded as the effect of cause "heat".

- ➢According to kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).
- ➢Although the temperature of a body can to be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.
- ➢Highest possible temperature achieved in laboratory is about $10^8 K$ while lowest possible temperature attained is $10^{-8} K$.
- ➢Temperature of the core of the sun is $10^7 K$ while that of its surface is $6000 K$.
- ➢Normal temperature of human body is $310.15 K$ ($37°C = 98.6°F$).
- ➢NTP or STP implies $273.15K$ ($0°C = 32°F$)

Scales of Temperature

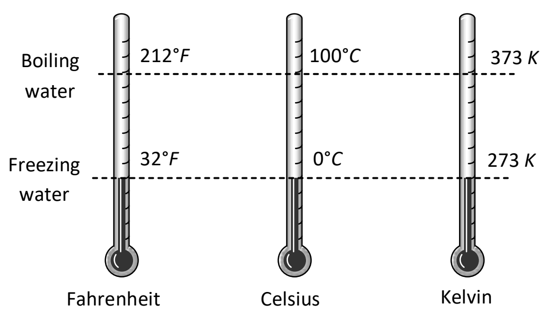


Fig. 12.1

The centigrade (°C), Farenheite (°F), Kelvin (K), Reaumer (R), Rankine (Ra) are commonly used temperature scales.

- ➢To construct a scale of temperature, two fixed points are taken. First fixed point is the freezing point (ice point) of water, it is called lower fixed point (LFP). The second fixed point is the boiling point (steam point) of water, it is called upper fixed point (UFP).
- ➢**Celsius scale :** In this scale LFP (ice point) is taken 0° and UFP (steam point) is taken 100°. The temperature measured on this scale all in degree Celsius (°C).
- ➢**Farenheite scale :** This scale of temperature has LFP as 32°F and UFP as 212°F. The change in temperature of 1°F corresponds to a change of less than 1° on Celsius scale.
- ➢**Kelvin scale :** The Kelvin temperature scale is also known as thermodynamic scale. The triple point of water is also selected to be the zero of scale of temperature. The temperature measured on this scale are in Kelvin (K).

The triple point of water is that point on a P-T diagram where the three phases of water, the solid, the liquid and the gas, can coexist in equilibrium.

Different measuring scales

Scale	Symbol	LFP	UFP	Number of divisions
Celsius	$°C$	$0°C$	$100°C$	100
Fahrenheit	$°F$	$32°F$	$212°F$	180
Reaumer	$°R$	$0°R$	$80°R$	80
Rankine	$°Ra$	$460\ Ra$	$672\ Ra$	212
Kelvin	K	$273.15\ K$	$373.15\ K$	100

- Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales}$$

- All these temperatures are related to each other by the following relationship

$$\frac{C-0}{100} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15} = \frac{R-0}{80-0} = \frac{Ra-460}{672-460}$$

$$\text{or } \frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4} = \frac{Ra-460}{10.6}$$

- The Celsius and Kelvin scales have different zero points but the same size degrees. Therefore any temperature difference is the same on the Celsius and Kelvin scales $(T_2 - T_1)°C = (T_2 - T_1)\ K$.

Thermometry

A branch of science which deals with the measurement of temperature of a substance is known as thermometry.

- The linear variation in some physical properties of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance.

- Thermometric properties (x) may be as follows
  - Length of liquid in capillary
  - Pressure of gas at constant volume.
  - Volume of gas at constant pressure.
  - Resistance of a given platinum wire.

- In old thermometry, freezing point ($0°C$) and steam point ($100°C$) are taken to define the temperature scale.

So if the thermometric property at temperature $0°C$, $100°C$ and $t°C$ are x_0, x_{100} and x respectively then

$$\frac{t-0}{100-0} = \frac{x-x_0}{x_{100}-x_0}$$

$$t°C = \frac{x-x_0}{x_{100}-x_0} \times 100°C$$

- General equation used to measure temperature t. $X_t = X_0(1+\alpha t)$

- In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water $273.16\ K$) the other is itself $0\ K$ where the value of thermometric property is assumed to be zero.

So if the value of thermometric property at $0\ K$, $273.16\ K$ and TK are 0, x_{Tr} and x respectively then

$$\frac{T}{273.16} = \frac{x}{x_{Tr}}$$

$$T = 273.16\left[\frac{x}{x_{Tr}}\right]K$$

Faulty Thermometer:

- If the reading on a faulty thermometer is 'x' and its lower and upper fixed points are L and U respectively then
- correct reading on Celsius scale is

$$\frac{C}{100} = \frac{X - L}{U - L}$$

- Correct reading on Fahrenheit scale is $\frac{F-32}{180} = \frac{X-L}{U-L}$
- Correct reading on Kelvin sclae is $\frac{K-273}{100} = \frac{X-L}{U-L}$
- Error in measurement by faulty thermometer = measures value - true value

Correction = -Error

Types of Thermometers:

Types of thermometer and its range	Thermometric property	Advantages	Disadvantages	Where is it used
Mercury-in-glass -39^0C to 45^0C	Length of column of mercury in capillary tube	(i) Quick and easy to use (direct reading) (ii) Easily portable	(i) Fragile (ii) Small size limits precision (iii) Limited range	Laboratory use where high accuracy is not required
Constant volume gas thermometer -27^0C to 1500^0C	Pressure of a fixed mass of gas at constant volume	(i) Very acurate (ii) Very sensitive (iii)Wide range (iv) Easily reproducible	(i) Very large volume of bulb (ii) Slow to use and convenient	(a) Standard when compared to other thermometer
Platinum resistance -180^0C to 1150^0C	Electrical resistance of a platinum coil	(i) Accurate (ii) Wide range	Not suitable for varying temperature (i.e, is slow to respond to changes	Best thermometer for small steady temperature difference
Thermocouple -250^0C to 1150^0C	Emf produced between junctions of dissimilar metals at different temperature for measurement of emfs	(i) Fast response because of low heat capacity (ii) Wide range (iii) Can be employed for remote readings	Accuracy is lost if emf is measured using a moving coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable)	Best thermometer for varying temperature
Radiation pyrometer Above 3000^0C	Colour of radiation emitted by a hot body	Does not come into contact when temperature is measured	(i) cumbersome (ii) some direct reading (needs a trained observer)	Only thermometer possible for very high thermometers

PROBLEMS

1 On the Celsius scale the absolute zero of temperature is at

(a) $0°C$ (b) $-32°C$ (c) $100°C$ (d) $-273.15°C$

SOLUTION:

$$T = 273.15 + t°C$$
$$\Rightarrow 0 = 273.15 + t°C$$
$$\Rightarrow t = -273.15°C$$

2 Oxygen boils at $-183°C$. This temperature is approximately

(a) $215°F$ (b) $-297°F$ (c) $329°F$ (d) $361°F$

SOLUTION:

$$\frac{C}{5} = \frac{F-32}{9}$$
$$\Rightarrow \frac{-183}{5} = \frac{F-32}{9}$$
$$\Rightarrow F = -297°F$$

3 The temperature of a body on Kelvin scale is found to be $x\,K$. When it is measured by Fahrenheit thermometer, it is found to be $x°F$, then the value of x is

(a) 40 (b) 313 (c) 574.25 (d) 301.25

SOLUTION:

$$\frac{F-32}{9} = \frac{K-273}{5}$$
$$\Rightarrow \frac{x-32}{9} = \frac{x-273}{5}$$
$$\Rightarrow x = 574.25$$

4 A centigrade and a Fahrenheit thermometer are dipped in boiling water. The water temperature is lowered until the Fahrenheit thermometer registers 140°. What is the fall in temperature as registered by the Centigrade thermometer

(a) 30° (b) 40° (c) 60° (d) 80°

SOLUTION:

$$\frac{C}{5} = \frac{F-32}{9}$$
$$\Rightarrow \frac{C}{5} = \frac{(140-32)}{9}$$
$$\Rightarrow C = 60°$$

5 At what temperature the centigrade (Celsius) and Fahrenheit, readings are the same

(a) $-40°$ (b) $+40°$ (c) 36.6° (d) $-37°$

SOLUTION:

$$\frac{C}{5} = \frac{F-32}{9}$$

$$\Rightarrow \frac{t}{5} = \frac{t-32}{9}$$

$$\Rightarrow t = -40°$$

6 If a thermometer reads freezing point of water as 20°C and boiling point as 150°C, how much thermometer read when the actual temperature is 60°C

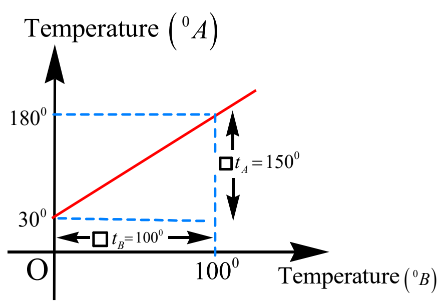
(a) 98°C (b) 110°C (c) 40°C (d) 60°C

SOLUTION:

Temperature on any scale can be converted into other scale by $\frac{x-LFP}{UFP-LFP} = \text{Constant}$

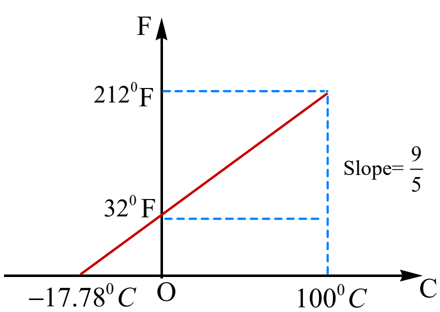
for all scales $\frac{x-20}{150-20} = \frac{60}{100} \Rightarrow x = 98°C$

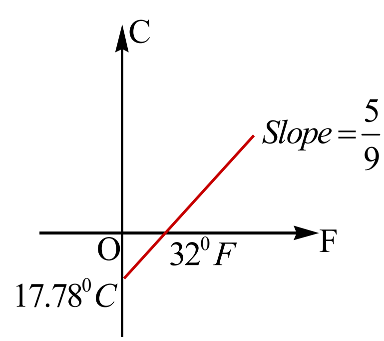
7. **The graph between two temperature scales A and B is shown in Fig. Between upper fixed point and lower fixed point there are 150 equal divisions on scale A and 100 on scale B. The relation between the temperatures in two scales is given by__**



SOLUTION: When $t_B = 0, t_A = 30^0$ $\therefore \frac{t_A - 30}{150} = \frac{t_B - 0}{100}$

8. **Graph shows the relation between Centigrade and Fahrenheit scales of temperature. Find slope in each graph?**





SOLUTION:

Case (i): A plot of Fahrenheit temperature (F) versus Celsius temperature (C)

$$F = \frac{9}{5}C + 32 \quad (\because y = mx + c)$$

Slope of the graph m=9/5

Case (ii):

$$\frac{C}{5} = \frac{F-32}{9}; C = \frac{5}{9}F - \frac{160}{9} (\because y = mx + c)$$

Slope of the graph, m=5/9

A plot of Celsius temperature (C) versus Fahrenheit temperature (F)

9. An accurate Celsius thermometer and a faulty Fahrenheit thermometer register 60^0 and 141^0 respectively when placed in the same constant temperature enclosure. What is the error in the Fahrenheit thermometer?

SOLUTION:

From $\frac{C}{5} = \frac{F-32}{9} \Rightarrow \frac{60}{5} = \frac{F-32}{9} \Rightarrow F = 140^0 F$

Error=141-140=1^0F; Correction=-1^0F

10. Two absolute scales X and Y have triple points of water defined to be 300 X and 450 Y. How are TX and T_Y related to each other?

SOLUTION:

Here, temperature 300 on absolute scale X=273.16K (Triple point of water)

$\therefore$ Value of temperature T_X on absolute

scale $X = \frac{273.16}{300}T_X$

Similarly, value of temperature T_Y on absolute scale

$$Y = \frac{273.16}{450}T_Y$$

Since both these values are equal,

$$\frac{273.16}{300}T_X = \frac{273.16}{450}T_Y \qquad \therefore T_X = \frac{2}{3}T_Y$$

11. The readings corresponding to the ice point and steam point for a constant pressure gas thermometer are 500cc, and 545 cc. If the reading corresponding to room temperature be 510 cc, find the room temperature?

SOLUTION:

Given: $V_0 = 500cc; V_{100} = 545cc.\ and\ V_t = 510cc.$

$$\text{Using, } t = \left[\frac{V_t - V_0}{V_{100} - V_0}\right]\times 100 = \left[\frac{510-500}{545-500}\right]100$$

$$= 22.22^0 C$$

12.. **The resistance of a platinum wire is 15Ω at 20°C. This wire is put in a hot furnace and the resistance of the wire is found to be 40Ω. Find the temperature of the hot furnace if temperature coefficient of resistance of platinum is** $3.6\times10^{-3}\ {}^0C^{-1}$

SOLUTION:

$$R_t = R_0\left(1+\alpha t\right) \Rightarrow \frac{R_2}{R_1} = \frac{\left(1+\alpha t_2\right)}{(1+\alpha t_1)}$$

$$\frac{40}{15} = \frac{\left(1+\alpha t_2\right)}{\left(1+\alpha t_1\right)} \Rightarrow 40-15 = \alpha\left(15t_2 - 40t_1\right)$$

$$15t_2 = \frac{25}{3.6\times10^{-3}} + 40\times20 \approx 7745$$

$$\Rightarrow t_2 = \frac{7745}{15} \approx 516^0 C$$

13. The resistance of a platinum resistance thermometer is found to be 11.0 ohm when dipped in a triple point cell. When it is dipped in a bath, resistance is found to be 28.887 ohm. Find the temperature of the bath in ^0C on platinum scale.

SOLUTION: In terms of triple point of water,

$$T_K = \left[273.16\frac{R}{R_{Tr}}\right]K$$

$$\text{so } T_K = 273.16\times\frac{28.887}{11.0} = 717.32K$$

$$\text{Now as } T_C = 717.32 - 273.15 = 444.17^0 C$$

14. What is the temperature for which the reading on Kelvin and Fahrenheit scales are same?

SOLUTION:

On the Kelvin and Fahrenheit scales

$$\frac{K-273.15}{100}=\frac{F-32}{180} \quad (\text{if } X=K=F)$$

$$\frac{X-273.15}{100}=\frac{X-32}{180}$$

$$X=\frac{9}{4}(255.38)=574.6$$

$$\therefore 574.6K=574.6^0F.$$

15. At what temperature is the Fahrenheit scale reading equal to half that on the Celsius scale?

SOLUTION:

$$As t_F=\frac{9}{5}t_c+32 \ and \ t_F=\frac{1}{2}t_c,$$

$$\frac{1}{2}t_c=\frac{9}{5}t_c+32$$

$$or \ t_c=-\frac{320}{13}=-24.6^0C$$

16. A constant volume gas thermometer shows pressure readings of 50 cm and 90 cm of mercury at 0^0C and 100^0C respectively. What is the temperature on gas scale when the pressure reading is 60 cm of mercury?

SOLUTION:

Given that $P_0=50cm$ of Hg,

$$P_{100}=90cm \text{ of Hg}$$

$$P_t=60cm \text{ of Hg}$$

$$t=\frac{P_t-P_0}{P_{100}-P_0}\times 100=\frac{60-50}{90-50}\times 100=25^0C$$

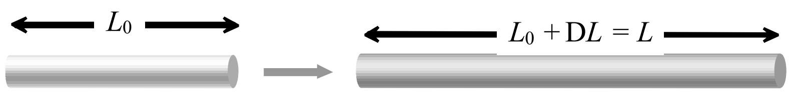
Thermal Expansion:

When matter is heated without any change in it's state, it usually expands. According to atomic theory of matter, a symmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.

Linear expansion :

When a solid is heated and it's length increases, then the expansion is called linear expansion.

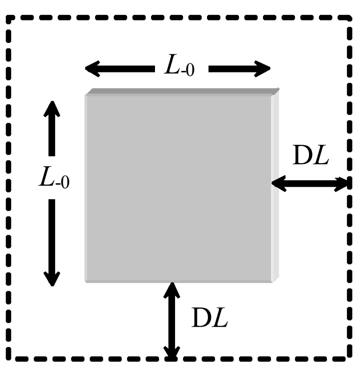


(A) Before heating   **(B)** After heating

- Change in length $\Delta L = L_0 \alpha \Delta T$
  (L_0 = Original length, T = Temperature change)
- Final length $L = L_0 (1 + \alpha \Delta T)$
- Co-efficient of linear expansion $\alpha = \dfrac{\Delta L}{L_0 \Delta T}$
- Unit of α is $^\circ C^{-1}$ or K^{-1}. It's dimension is $[\theta^{-1}]$

Superficial (areal) expansion :

When the temperature of a $2D$ object is changed, it's area changes, then the expansion is called superficial expansion.



(i) Change in area is $\Delta A = A_0 \beta \Delta T$

(A_0 = Original area, T = Temperature change)

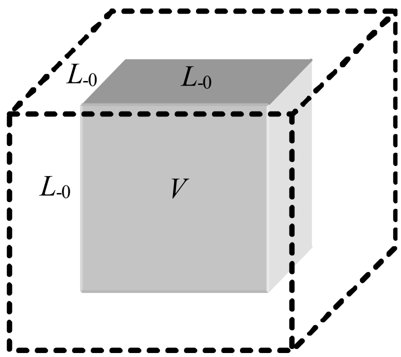
(ii) Final area $A = A_0(1 + \beta\Delta T)$

(iii) Co-efficient of superficial expansion $\beta = \frac{\Delta A}{A_0 \Delta T}$

(iv) Unit of β is $°C^{-1}$ or K^{-1}.

Volume or cubical expansion :

When a solid is heated and it's volume increases, then the expansion is called volume or cubical expansion.



(i) Change in volume is $\Delta V = V_0 \gamma \Delta T$

(V_0 = Original volume, T = change in temperature)

(ii) Final volume $V = V_0(1 + \gamma\Delta T)$

(iii) Volume co-efficient of expansion $\gamma = \frac{\Delta V}{V_0 \Delta T}$

iv) Unit of γ is $°C^{-1}$ or K^{-1}.

Contraction on heating : Some rubber like substances contract with rising temperature, because transverse vibration of atoms of substance dominate over longitudinal vibration which is responsible for expansion.

Coefficients of Linear Expansion

The ratio of increase in length of a solid per degree rise in temperature to its original length is called coefficient of linear expansion (α)

$$\alpha = \frac{l_2 - l_1}{l_1 \times (t_2 - t_1)} /^0 C$$

α in differential form $\alpha = \frac{1}{l_0}\left(\frac{dl}{dt}\right) /^0 C$

length of the solid after heating

$\ell_2 = \ell_1\left[1+\alpha\left(t_2 - t_1\right)\right]$

The coefficient of linear expansion of a solid depends on the nature of the material and scale of temperature. (it is independent on dimension of material)

↳ The linear expansion of a solid $l_2 - l_1 = e = l_1\alpha(t_2 - t_1)$

↳ It depends on three factors.

a) Its original length (l_1)

b) The nature of the material (α)

c) Change in temperature $(t_2 - t_1)$

↳ Increase in length $\Delta l = l\alpha\Delta t$

↳ Fractional change in length $\frac{\Delta l}{l} = \alpha\Delta t$

Percentage change in length $\frac{\Delta l}{l}\times 100 = \alpha\Delta t\times 100$

↳ For anisotropic solids, if α_x, α_y *and* α_z are coefficients of linear expansions along x, y and z directions respectively then the average coefficient of linear expansion is $\boxed{\alpha = \frac{\alpha_x + \alpha_y + \alpha_z}{3}}$

↳ Numerical value of coefficient of linear expansion of a solid is α_C when the temperature is measured in Celsius scale and its value is α_F when the temperature is measured in Fahrenheit scale then

a) $\boxed{\alpha_F = \left(\frac{5}{9}\right)\alpha_C}$ (or) $\boxed{\alpha_C = \left(\frac{9}{5}\right)\alpha_F}$

b) $\alpha_F < \alpha_C$

↳ A composite rod is made by joining two rods of different materials and of same cross section. If l_1, l_2 are their initial lengths at $t_1\,^0C$, then

(a) the increase in length of composite rod at $t_2\,^0C$ is given by $\Delta l = \left(l_1\alpha_1 + l_2\alpha_2\right)\left(t_2 - t_1\right)$

b) The effective coefficient of linear expansion of the composite rod is given by $\boxed{\alpha = \frac{l_1\alpha_1 + l_2\alpha_2}{l_1 + l_2}}$

↳ If two metal rods of coefficients of linear expansions α_1 *and* α_2 have same length at $t_1\,^0C$ *and* $t_2\,^0C$ respectively, then the common temperature at which they have again the same length is

$$\boxed{t = \frac{\alpha_1 t_1 - \alpha_2 t_2}{\alpha_1 - \alpha_2}}$$

↳ If two rods of same length l having different coefficients of linear expansion α_1 *and* α_2 $(\alpha_1 > \alpha_2)$ are at the same temperature $t_1\,^0C$ then a) difference in their lengths at higher temperature $t_2\,^0C$ is given by

$\Delta l = x = (\alpha_1 - \alpha_2) l (t_2 - t_1)$

↳ The diameter of a metal ring is 'D' and the coefficient of linear expansion is α. If the temperature of the ring is increased by Δt then the increase in circumference of the ring

$$\Delta C = C\alpha\Delta t = 2\pi r\alpha\Delta t = 2\pi\left(\frac{D}{2}\right)\alpha\Delta t = \pi D\alpha\Delta t$$

Increase in circumference $= \pi D \alpha \Delta t$

Coefficient of Areal (or) Superficial expansion:

The ratio of increase in its area per degree rise in temperature to its original area is called coefficient of areal expansion (β).

$$\beta = \frac{A_2 - A_1}{A_1 \times (t_2 - t_1)} /^0 C$$

Final area $A_2 = A_1[1 + \beta(t_2 - t_1)]$

↳ Change in area $\Delta A = A\beta\Delta t$.

↳ Fractional change in area $\frac{\Delta A}{A} = \beta\Delta t$

Percentage change in area $\frac{\Delta A}{A} \times 100 = \beta\Delta t \times 100$

↳ The diameter of a metal ring is 'D' and the coefficient of areal expansion is β. If the temperature of the ring is increased by Δt then

a) The increase in area of the ring

$$\Delta A = A\beta\Delta t = \pi R^2 \beta\Delta t = \frac{\pi D^2 \beta\Delta t}{4}$$

$$= \frac{\pi D^2 \alpha}{2}\Delta t \left(\because \beta = 2\alpha\right)$$

Coefficient of volume expansion:

↳ The ration of increase in its volume per degree rise in temperature to its original volume is called coefficient of volume expansion γ.

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} /^0 C; V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

↳ Final temperature $t_2 = \frac{V_2 - V_1}{V_1\gamma} + t_1$

↳ Change in volume $\Delta V = \gamma V_1 (t_2 - t_1)$.

↳ Fractional change in volume $\frac{\Delta V}{V} = \gamma\Delta t$

↳ Percentage change in volume

$$\frac{\Delta V}{V} \times 100 = \gamma \Delta t \times 100$$

↳ Volume expansion of a body is independent of its cavities.

Relation among α, β, γ :

↳ $\boxed{\beta = 2a}$, $\boxed{\gamma = 3a}$

↳ $\alpha : \beta : \gamma = \alpha : 2\alpha : 3\alpha = 1 : 2 : 3$

↳ $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

Variation of density of substance with temperature

↳ When a solid is heated its volume increases and hence its density decreases, as mass remains constant.

If ρ_1 *and* ρ_2 are densities of a solid at $t_1\,^0C$ and $t_2\,^0C$, and as $m_1 = m_2$; $\rho_1 V_1 = \rho_2 V_2$

$$\rho_1 V_1 = \rho_2 V_1 \left[1 + \gamma\left(t_2 - t_1\right)\right]$$

$$\rho_1 = \rho_2 \left[1 + \gamma\left(t_2 - t_1\right)\right]$$

If ρ_1 and ρ_0 are densities at t^0C *and* 0^0C.

$$\boxed{\rho_1 = \frac{\rho_0}{(1 + \gamma t)} \, (or) \, \rho_1 = \rho_0 (1 + \gamma t)^{-1}}$$

$$\rho_1 \approx \rho_0 \left(1 - \gamma t\right)$$

↳ For anisotropic materials γ is the sum of linear coefficients in three mutually perpendicular directions.

$$\gamma = \alpha_x + \alpha_y + \alpha_z$$

For isotropic solids $\gamma = 3\alpha$

Applications:

Same Expansion In Different Rods:

If two rods of different materials have the same difference between their lengths at all temperatures only when their linear expansions are equal.

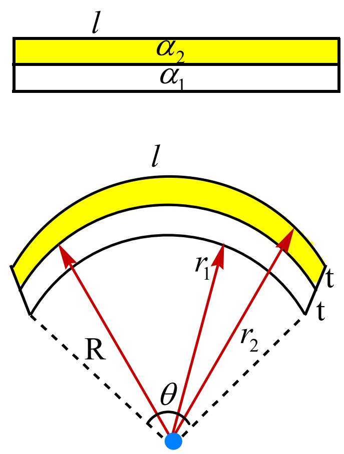
$$\Delta l_1 = \Delta l_2 ; l_1 \alpha_1 \Delta t = l_2 \alpha_2 \Delta t$$

Then $l_1 \alpha_1 = l_2 \alpha_2, \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$

if the constant difference in their lengths is x then

$$l_1 = \frac{x\alpha_2}{\alpha_1 \square \alpha_2}, l_2 = \frac{x\alpha_1}{\alpha_1 \square \alpha_2}, x = l_2 - l_1$$

Bimetallic Strip:



Where t is thickness of each strip

↳ Bimetallic strip works on the principle that different metals expand differently for the same rise in temperature.

↳ If a bimetallic strip made of brass and iron is heated brass bends on convex side $(\alpha_b - \alpha_i)$

↳ If it is cooled brass bends on concave side.

↳ Radius of curvature of a bimetallic strip.

$$\theta = \frac{1}{R}(or)\ \theta = \frac{dl}{dr} = \frac{l_2 - l_1}{r_2 - r_1}; \frac{l}{R} = \frac{l(\alpha_2 - \alpha_1)\Delta T}{2t}$$

$$\boxed{\therefore R = \frac{2t}{(\alpha_2 - \alpha_1)\Delta T}} \qquad (\because \alpha_2 > \alpha_1)$$

t = thickness of each metal strip used.

↳ Bimetallic strip can be used as temperature sensor in thermometers and fire alarms.

↳ As an automatic switch or circuit breaker in electric iron, refrigerators, incubators, thermostats, flash lights etc.

↳ As a balance wheel in wrist watches.

Pendulum Clocks:

Variation of Time Period of Pendulum Clocks:

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$$

If temperature is increased by Δt,

$$T = 2\pi\sqrt{\frac{l_o(1 + \alpha\Delta t)}{g}}$$

(by using Binomial expansion)

$$T = T_0\left(1 + \frac{\alpha}{2}\Delta t\right)$$

$\Rightarrow \Delta T = T - T_0 = T_0 \dfrac{\alpha}{2} \Delta t$

ΔT = increase in time period.

↳ Pendulum clocks looses time in summer and gains time in winter

The loss or gain per day $= \dfrac{1}{2} \alpha \Delta t \times 86400$ Sec.

Compensated pendulum length is always constant at all temperatures, so it shows correct time at all temperatures.

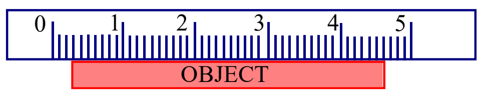
Grid Iron Pendulum:

The total expansion of brass rods should be equal to that of steel rods. $\Delta l_1 = \Delta l_2$

$$\boxed{n_1 l_1 \alpha_1 = n_2 l_2 \alpha_2}$$

Measuring Tapes:

Expansion of a Measuring Scale



A scale made of material having coefficient of linear expansion (α_s) is caliberated at $t_1{}^0$C. The scale gives correct measurement only at calibrated temperature ($t_1{}^0$C). If the length of an object is measured at higher temperature $t_2{}^0$C($t_2{}^0$>$t_1{}^0$), there will be an error in the measurement at that temperature ($t_2{}^0$C) due to expansion of sacle i.e. due to increase in length of each division. The error in measurement of scale is $\Delta L = L\alpha_s \Delta t$

Where L measured length of object at $t_2^0 C$ and Δt is raise in temperature

a) When temperature increases the length of object oberved by the scale is less than correct length. Hence true value at measured temperature $t_2^0 C$ is $L_{TV} = L + L\alpha_s \Delta T$; $L_{TV} = L(1+\alpha_s \Delta T)$

b) When temperature decrease below the caliberation temperature, length of object observed by the scale is more than correct length. Hence true value at measured temperature $t_2^0 C$ is $L_{TV} = L - L\alpha_s \Delta T$; $L_{TV} = L(1-\alpha_s \Delta T)$

c) At room temperature (t^0C) the length of a material rod is measured using a metal centimeter scale. The measured length is lcm. If the scale is calibrated to read accurately at temperature)^{0}C, then actual length of metal rod at 0^0C is .....

The length of the metal rod at t^0C is $l_{TV}(1+\alpha_M t)$

The one cm on the metal scale at t^0C is $1(1+\alpha_s t)$

The length of the metal rod at t^0C

$$\frac{l_{TV}(1+\alpha_M t)}{(1+\alpha_s t)} = l_{TV}(1+\alpha_M t)(1-\alpha_s t)$$

$$l = l_{TV}(1+(\alpha_M - \alpha_s)t)$$

$$l_{TV} = l(1-(\alpha_M - \alpha_s)t)$$

when l_{TV} = correct length at calibration temperature.

α_M = coefficient of linear expansion of metal rod.

α_s = coefficient of linear expansion of metal scale.

Note (1) : If $\alpha_M > \alpha_s \quad l_{TV} < 1$

(2) If $\alpha_M < \alpha_s \quad l_{TV} > 1$

Thermal Stress:

- It is developed due to prevention of expansion of a solid when it is heated.
- A rod of length l_0 clamped between two fixed walls.

For Δt Change in temperature

Young's modulus

$$Y = \frac{F/A}{\Delta l / l_0} = \frac{Fl_0}{A\Delta l} = \frac{F}{A\alpha\Delta t}\left(\because \Delta l = l_0\alpha\Delta t\right)$$

or $\dfrac{F}{A} = Y\alpha\Delta t$

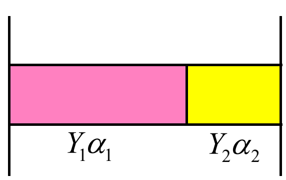
- Thermal force $\boxed{F = YA\alpha\Delta t}$.

Thermal force is independent of length of rod.

- Thermal stress= $Y\alpha\Delta t$
- For same thermal stress in two different rods heated through the same rise in temperature,

$$\boxed{Y_1\alpha_1 = Y_2\alpha_2}$$

- Two rods of different metals having the same area of cross section A are placed between the two massive walls as shown in the fig. The first rod has a length l_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for second rod are l_2, α_2 & Y_2. The temperature of both rods is now raised by t^0C.



Total length prevented from expansion

$$\Delta l_1 + \Delta l_2 = \frac{F \times l_1}{Y_1 \times A} + \frac{F \times l_2}{Y_2 \times A}$$

$$\boxed{Thermal\ \ force = F\frac{At\left[\left(l_1\alpha_1\right) + \left(l_2\alpha_2\right)\right]}{\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)}}$$

$\left(\because \Delta l = l\alpha t = \frac{Fl}{YA}\right)$

$$Thermal\ stress = \frac{F}{A} = \frac{t\left[(l_1\alpha_1)+(l_2\alpha_2)\right]}{\left(\frac{l_1}{Y_1}+\frac{l_2}{Y_2}\right)}$$

↳ Lengths of individual rods due to thermal stress:

Length of the first rod=Original length + increase in length due to rise in temperature-decrease in length due to thermal force

$$l_1^1 = l_1 + l_1\alpha_1 t - \left(\frac{Fl_1}{AY_1}\right)$$

Length of the second rod

$$l_2^1 = l_2 + l_2\alpha_2 t - \left(\frac{Fl_2}{AY_2}\right)$$

↳ Junction displacement=difference in lengths of any one of the rods after heating and before heating.

$$\boxed{\therefore x = l_1\alpha_1 t - \frac{Fl_1}{AY_1}} \text{ (or) } \boxed{\therefore x = l_2\alpha_2 t - \frac{Fl_2}{AY_2}}$$

Effect of Moment of inertia of a Rigid body due to thermal expansion

Moment of inertia of a rigid body about an axis of rotation is given by $I=MK^2$.......(1)

where M is mass of the body and K is its radius of gyration. On heating, due to thermal expansion radius of gyration of the about same axis increases and hence its moment of inertia increases.

As mass remains constant on heating $I \propto K^2$

Differentiating and then dividing the same equation we have

$$\frac{\Delta I}{I} = 2\frac{\Delta K}{K} \text{.......(2)}$$

Here, $\frac{\Delta K}{K}$ is fractional change in radius of gyration

If α is coefficient of linear expansion of the material

Then $\Delta K = K\alpha\Delta t$

Sunstituting in equation (2)

fractional change in moment of inertia $\frac{\Delta I}{I} = 2\alpha\Delta t$

Change in moment of inertia $\Delta I = 2\alpha\Delta t$

Effect of angular velocity of a rotating rigid body due to thermal expansion

On heating since no external torque acts on the body its angular momentum remains constant i.e, when I increases ω decreases.

Angular momentum of an object is $L = I\omega$

Where I is moment of inertia and ω angular velocity. On heating I and ω changes but L remains constant, since no external torque is acitng on object, then $I\omega$ = constant.

$$I_1\omega_1 = I_2\omega_2 \qquad I\omega = 1(1+2\alpha\Delta t)\omega_2$$

$$\omega_2 = \frac{\omega}{1+2\alpha\Delta t} \approx \omega(1-2\alpha\Delta t)$$

Then $-\frac{\Delta\omega}{\omega} = 2\alpha\Delta t$

↳ If a cube of coefficient of cubical expansion γ is heated, then the pressure to be applied on it to prevent its expansion is P then $\Delta V = V\gamma\Delta t$

$$K = \frac{P}{\frac{\Delta V}{V}} = \frac{P}{\gamma\Delta t} \quad \Rightarrow P = K\gamma\Delta t$$

$$\Rightarrow P = 3K\alpha\left(t_2 - t_1\right)$$

where K is bulk modulus

Barometer With Brass Scale:

↳ Relation between faulty and actual barometric heights is given by

$$\boxed{h_2 = h_1\left[1 + (\gamma_{Hg} - \alpha_s)\left(t_2 - t_1\right)\right]}$$

h_1 =height of barometer at t_1^0C where the scale is marked.

h_2 =height of barometer at t_2^0C where the measurement is made.

γ_{Hg} =real coefficient of expansion of mercury

α_s =Coefficient of linear expansion of scale

EXAMPLES:

- ➢Between the rails a gap is left to allow for their expansion in summer. If l is the length of the rail and Δt is the change in temperature then the gap is given by $\Delta l = l\alpha\Delta t$
- ➢A wire of length l is bent in the form of a ring with a small gap of length x_1 at $t_1\,^0C$. On heating the ring to $t_2\,^0C$ the gap increases to x_2 in length. The $\alpha = \frac{x_2 - x_1}{x_1\left(t_2 - t_1\right)}$
- ➢Gap behaves like the material for all thermal expansions.
- ➢Telephone wires are loosely connected between the poles in summer, to allow for their contraction in winter.
- ➢Concrete roads are laid in sections and gaps are provided between them to allow for expansion.
- ➢Pipes used to convey steam from boiler must have loops to prevent cracking of pipes due to thermal expansion.
- ➢Huge iron girders used in the construction of bridges and buildings are allowed to rest on rollers on either side providing scope for expansion.
  Hence the damage to the structure can be avoided.
- ➢When a drop of water falls on a hot glass chimney, the portion of the spot where the water falls, contracts and the remaining portion expands. So, the glass chimney breaks (brittle nature of the glass also)
- ➢Pyrex glass is used to prepare test tubes for heating purpose because its linear expansion coefficient is small. $\left(\alpha = 3\times10^{-6}\,{}^0C^{-1}\right)$

➢Silica glass (quartz) is used for making bulbs of thermometer because of low linear expansion coefficient. $\left(\alpha = 0.5\times10^{-6}\,{}^{0}C^{-1}\right)$

➢Invar is an alloy of Iron, Nickel and Carbon. Invar has very low linear expansion coefficient, so used in wrist watches, pendulum clocks and standard scales.

➢A hole is drilled at the centre of a metallic plate. When plate is heated, the diameter of hole increases.

➢When two holes are drilled on a metal plate and heated the distance between the holes increases.

➢When a solid and hollow sphere with same outer radius made up of same metal are heated to same temperature then both expand equally.

➢Platinum is used to seal glass because their coefficients of expansion are almost same.

PROBLEMS

1. A bimetallic strip is made of aluminium and steel $\left(\alpha_{Al} > \alpha_{steel}\right)$. On heating, the strip will

1) remain straight
2) get twisted
3) will bend with aluminium on concave side
4) will bend with steel on concave side

SOLUTION:

Due to unequal expansions the one having more α bend towards convexside.

2 What length of brass and iron at $0^{0}C$ must be used if the difference between their lengths is always 0.2m? The value of α for brass and iron are $18\times10^{-6}/{}^{0}C$ and $12\times10^{-6}/{}^{0}C$ respectively.

(2014E) (2013 M).

SOLUTION:

$$l_1\alpha_1 = l_2\alpha_2 \; and \; l_2 - l_1 = x; \; l_1 = \frac{l_2\alpha_2}{\alpha_1}$$

$$l_2 - l_1 = x = l_2 - \frac{l_2\alpha_2}{\alpha_1} = l_2\left(\frac{\alpha_1 - \alpha_2}{\alpha_1}\right)$$

$$l_2 = \frac{x\alpha_1}{\alpha_1 - \alpha_2} = \frac{0.2\times12\times10^{-6}}{(18-12)\times10^{-6}} = 0.40m$$

$$l_1 = \frac{x\alpha_2}{\alpha_1 - \alpha_2} = \frac{0.2\times18\times10^{-6}}{(18-12)\times10^{-6}} = 0.60m$$

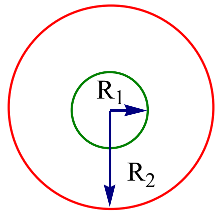
3. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly

1) its speed of rotation increases.
2) its speed of rotation decreases.
3) its speed of rotation remains same.
4) its speed increases because its moment of inertia increases.

SOLUTION:

If the rod heated its length increases and moment of inertia increases, so that its angular velocity decreases.

4. In the given figure, when temperature is increased then which of the following increases ?



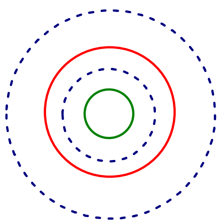
1) R_1 2) R_2 3) $R_2 - R_1$ 4) all

SOLUTION:

All of the above

----- represents expanded Boundary

______ represents original Boundary



As the intermolecular distance between atoms increases on heating nence the inner and outer perimeter increases. Also if teh atomic arrangement in radial direction is observed then we can say that it also increases hence all A,B,C are true.

5. As the temperature is increased, the time period of a pendulum

1) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
2) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.
3) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.
4) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.

SOLUTION:

If the temperature of a pendulum increases length of the pendulum increases $\left(T \propto l\right)$.

So that time period increases. $\Delta V = V\gamma\Delta t$

6. The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature ΔT so that its new temperature is T + ΔT. The increase in the volume of the sphere is approximately

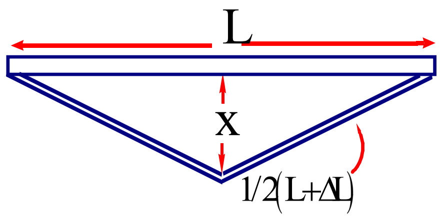
1) $2\pi R\alpha\Delta T$ 2) $\pi R^2\alpha\Delta T$ 3) $4\pi R^3\alpha\Delta T/3$ 4) $4\pi R^3\alpha\Delta T$

SOLUTION:

The increase in the volume of the sphere $= \frac{4}{3}\pi r^3 \times 3\alpha.\Delta t$

$$\Delta V = 4\pi r^3\alpha\Delta t$$

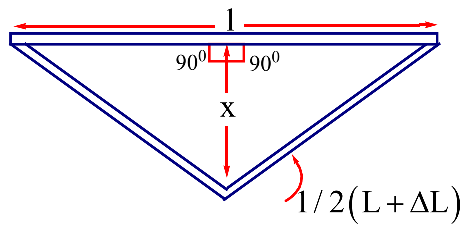
7. A rail track made of steel having length 10m is clamped on a railway line at its two ends (figure). On a summer day due to rise in temperature by 20^0C. It is deformed as shown in figure. Find x (displacement of the centre) if α_{steel}=1.2 x 10^{-5} /$^0C^{-1}$



1) 5cm 2) 20cm 3) 15cm 4) 11cm

SOLUTION:

Diagram show the deformmation of a railway track due to rise in temperature



Applying Pythagoras theorem in right angled triangle ,

$$x^2 = \left(\frac{L+\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2$$

$$\Rightarrow x = \sqrt{\left(\frac{L+\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$\Rightarrow x = \sqrt{\left(\frac{L}{2}\right)^2 + \frac{2L\Delta L}{4} + \left(\frac{\Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(L^2 + \Delta L^2 + 2L\Delta L\right) - L^2} = \frac{1}{2}\sqrt{\left(\Delta L^2 + 2L\Delta L\right)}$$

As increase in length ΔL is very small, therefore neglecting $(\Delta L)^2$,

$$\text{we get } x = \frac{\sqrt{2L\Delta L}}{2} \qquad ....(i)$$

But $\Delta L = L\alpha\Delta t$ ...(ii)

According to the problem, $L = 10m$

$\alpha = 1.2 \times 10^{-5}\ /^0C^{-1}$,

$\Delta T = 20^0C$

Substituting value ΔL in Eq. (i) from Eq (ii)

$$x = \frac{1}{2}\sqrt{2L \times L\alpha\Delta t} = \frac{1}{2}L\sqrt{2\alpha\Delta t}$$

$$= 11cm$$

8 A pendulum clock loses 12s a day if the temperature is 400C and gains 4s day if the temperature is 200C. The temperature at which the clock will show correct time and the coefficient of linear expansion (α) of the metal of the pendulum shaft are respectively **(JEE Mains-2016)**

1) 25^0C; $\alpha = 1.85 \times 10^{-5}/^0C$
2) 60^0C; $\alpha = 1.85 \times 10^{-4}/^0C$
3) 30^0C; $\alpha = 1.85 \times 10^{-3}/^0C$
4) 55^0C; $\alpha = 1.85 \times 10^{-2}/^0C$

SOLUTION:

$\therefore$ Let at temperature θ, clock gives correct time

$$\Delta T = \left(\frac{1}{2}\alpha\Delta\theta\right)T, T = 1 \text{ day} = 86400s$$

$$12 = \frac{1}{2}\alpha(40-\theta)T \ldots (i)$$

$$4 = \frac{1}{2}\alpha(\theta-20)T \ldots (ii)$$

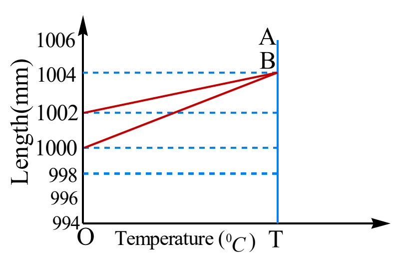
$$i/ii \Rightarrow \theta = 25^0C$$

substituting θ in equation (ii), we get

$$4 = \frac{1}{2}\alpha(25-20)\times 86400$$

$$\alpha = \frac{1}{5\times 86400} = 1.85x10^{-5}/^0 C$$

9 The variation of lengths of two metal rods A and B with change in temperature are shown in figure. The coefficient of linear expansion α_A for the metal A and the temperature T will be: (Given $\alpha_B = 9\times10^{-6}/^0 C$)



1) $\alpha_A = 3\times10^{-6}/^0 C, 500^0 C$
2) $\alpha_A = 3\times10^{-6}/^0 C, 222.22^0 C$
3) $\alpha_A = 27\times10^{-6}/^0 C, 500^0 C$
4) $\alpha_A = 27\times10^{-6}/^0 C, 222.22^0 C$

SOLUTION:

$$\text{Slope of the line } A = \frac{1006-1000}{T}\frac{\Delta L}{\Delta T}$$

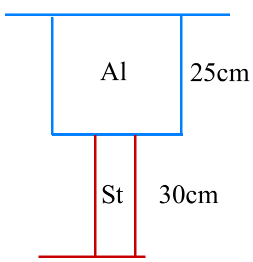
$$\frac{6}{T} = 1000mm\ \alpha_A ----(1)$$

similarly for B line

$$\frac{2}{T} = 1002mm\ \alpha_B ----(2)$$

From (1) & (2) $\alpha_A = 3\alpha_B$

10. Two bars are unstressed and have lengths of 25 cm and 30 cm at 20^{0}C as shown in Figure. Bar (1) is of aluminium and bar (2) is of steel. The cross-sectional area of bars are 20 cm^2 for aluminium and 10cm^2 for steel. Assuming that the top and bottom supports are rigid, stress in Al steel bars in $\frac{N}{mm^2}$ when the temperature is 70^{0}C. (Nearly) $(Y_a = 0.70\times10^5 N/mm^2 . Y_s = 2.1\times10^5 N/mm^2 .$ $\alpha_a = 24\times10^{-6} /{}^0C$ and $\alpha_a = 12\times10^{-6} /{}^0C)$



1) 75, 150 2) 25, 50 3) 50, 100 4) 100, 200

SOLUTION:

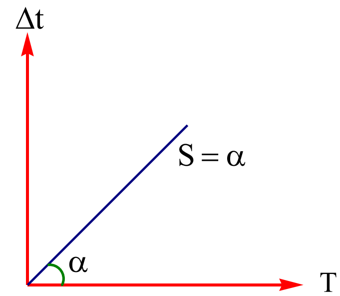
Contraction of the two bars due to compressive stress = Elongation of the two bars due to rise of temperature

$$\left(\frac{Sl}{Y}\right)_{Al} + \left(\frac{Sl}{Y}\right)_{St} = (\alpha L\Delta t)_{Al} + (\alpha L\Delta t)_{St}$$

Force in steel = for in aluminium

$$S_{Al} + A_{Al} = S_{St} \times A_{St}$$

11. A simple pendulum made of a bob of mass m and a metallic wire of negligible mass has time period 2s at T=0^{0}C. If the temperature of the wire is increased and the corresponding change in its time period is plotted against it temperature, the resulting graph is a line of slope S. If the coefficient of linear expansion of metal is α then the value of S is **(JEE Mains-2016 online)**



1) α 2) $\alpha/2$ 3) 2α 4) $1/\alpha$

SOLUTION:

$$t_0 = 2 = 2\pi\sqrt{\frac{l_0}{g}}$$

$$t_0 = 2\pi\sqrt{\frac{l_0(1+\alpha T)}{g}} = 2(1+\alpha T)^{1/2} = 2 + \alpha T$$

$$t - t_0 = \alpha T \Rightarrow \Delta t = \alpha T$$

12. A wire of length L_0 is supplied heat to raise its temperature by T. if γ is the coefficient of volume expansion of the wire and Y is Young's modulus of the wire then the energy density stored in the wire is

1) $\frac{1}{2}\gamma^2 T^2 Y$ 2) $\frac{1}{3}\gamma^2 T^2 Y^3$ 3) $\frac{1}{18}\frac{\gamma^2 T^2}{Y}$ 4) $\frac{1}{18}\gamma^2 T^2 Y$

SOLUTION:

Elastic potential energy per unit volume

$$E = \frac{1}{2} \times Stress \times Strain = \frac{1}{2} \times Y \times (Strain)^2$$

$$\Rightarrow E = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 = \frac{1}{2} \times Y \times \alpha^2 \times \Delta T^2$$

13. The coefficient of linear expansion of an in homogeneous rod changes linearly form α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of rod is

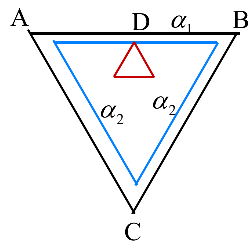
1) $\alpha_1 + \alpha_2$ 2) $\frac{\alpha_1 + \alpha_2}{2}$ 3) $\sqrt{\alpha_1 \alpha_2}$ 4) $\alpha_1 - \alpha_2$

SOLUTION:

$$\alpha_n = \alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{l}\right)x; \quad \Delta L = \int_0^L \alpha_n dx \Delta t$$

$$L = \left(\frac{\alpha_1 + \alpha_2}{2}\right) L\Delta T; \alpha_{eff} = \left(\frac{\alpha_1 + \alpha_2}{2}\right)$$

14. An equilateral triangle ABC is formed by joining three rods of equal length and D is the mid-point of AB. The coefficient of linear expansion for AB is α_1 and for AC and BC is α_2. The relation between α_1 and α_2, if distance DC remains constant for small changes in temperature is **(2010 E)**



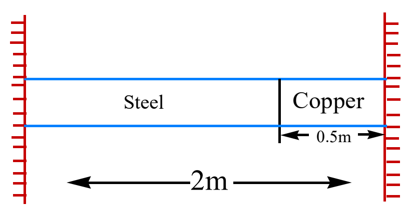
1) $\alpha_1 = \alpha_2$ 2) $\alpha_1 = 4\alpha_2$ 3) $\alpha_2 = 4\alpha_1$ 4) $\alpha_1 = \frac{1}{2}\alpha_2$

SOLUTION: Before and after changing the temperature,

$$\sqrt{l_2^2 - \frac{l_1^2}{4}} = \sqrt{\left[l_2\left(l+\alpha t\right)\right]^2 - \frac{1}{4}\left[l_1\left(1+\alpha t\right)\right]^2}$$

$$and\ l_1 = l_2$$

15. **When composite rod is free, composite length increases to 2.002m from temperature 20^{0}C to 120^{0}C. When composite rod is fixed between the support, there is no change in component length. Find Y and α of steel if $Y_{cu} = 1.5\times10^{13} N/m^2$ $\alpha_{cu} = 1.6\times10^{-5}\ /^0C$**



SOLUTION:

$$\Delta l = l_s\alpha_s\Delta T + l_c\alpha_c\Delta T$$

$$0.002 = \left[1.5\alpha_s + 0.5\times1.6\times10^{-5}\right]100$$

$$\alpha_s = \frac{1.2\times10^{-5}}{1.5} = 8\times10^{-6}\ {}^0C$$

If there is no change in composite length, thermal force of steel and copper rod should be equal

$$F_{st} = F_{cu};\ Y_s A\alpha_s\Delta t = Y_{cu}A\alpha_{Cu}\Delta t$$

$$\frac{Y_s}{Y_c} = \frac{\alpha_c}{\alpha_s};\ Y_s = Y_c\times\frac{\alpha_c}{\alpha_s} = \frac{1.5\times10^{13}\times1.6\times10^{-5}}{8\times10^{-6}}\quad Y_s = 3\times10^{13} N/m^2$$

16. A blacksmith fixes iron ring on the rim of the wooden wheel of bullock cart. The diameter of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27^0 C. The temperature to which the ring should be heated so as to fit the rim on the wheel $\left(\alpha_{iron} = 1.20\times10^{-5}\ /^0\ C\right)$?

SOLUTION:

$$\text{Given } t_1 = 27^0 C; l_1 = 5.231m; l_2 = 5.243m$$

$$l_2 = l_1\left[1+\alpha_1\left(t_2 - t_1\right)\right]$$

$$5.243=5.231\left[1+1.20\times10^{-5}\left(t_2 - 27\right)\right]$$

$$\textbf{or } t_2 = 218^0 C$$

17. An aluminium sphere of 20 cm diameter is heated from $0^0 C$ to $100^0 C$. Its volume changes by (given that the coefficient of linear expansion for aluminium $\left(\alpha_{Al} = 23\times10^{-6}\ /^0\ C\right)$ (AIEEE 2011)

SOLUTION:

Given d=20cm

$$V = V_0\left(1+\gamma t\right) = V_0\left(1+3\alpha t\right)\left(\text{since } \gamma = 3\alpha\right)$$

$$\text{change in volume} = V - V_0 = 3V_0\alpha t$$

$$= 3\times\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times 23\times10^{-6}\times100$$

$$= 3\times\frac{4}{3}\pi\left(\frac{0.2}{2}\right)^3\times 23\times10^{-6}\times100$$

$$= 28.9cc\left(1cc = 10^{-6}m^3\right)$$

18. An iron rod of length 50 cm is joined at an end to copper rod of length 100 cm at 20^{0}C. Find the length of the system at100^{0}Cand average coefficient of linear expansion of the system.

$\left(\alpha_{iron} = 12\times10^{-6}\ /^0\ C \text{ and } \alpha_{copper} = 17\times10^{-6}\ /^0\ C.\right)$

SOLUTION:

Increase in length of composite rod is

$$\Delta l = \Delta l_1 + \Delta l_2 = \left(\alpha_1 l_1 + \alpha_2 l_2\right)\Delta t$$

$$= \left(12\times10^{-6}\times50 + 17\times10^{-6}\times100\right)\times\left(100-20\right)\text{=0.192cm}$$

Length of the composite rod at $100^0 C$ is $l + \Delta l = 150.192cm$

Average linear expansion co-efficient

$$\alpha_{avg} = \frac{\Delta l}{l\Delta t} = \frac{0.192}{150\times80} = 16\times10^{-6}\ /^0\ C$$

19. Density of gold is 19.30 g/cm^3 at 20^0C. Compute the density of gold at 90^0C by adding steam to it. $(\alpha = 14.2\times10^{-6}/^0C)$

SOLUTION:

$$\frac{\rho_2}{\rho_1}=\frac{V_1}{V_2}=\frac{V_1}{V_1(1+3\alpha\Delta T)}=(1+3\alpha\Delta T)^{-1}$$

$$\text{or } \frac{\rho_2}{\rho_1}=(1-3\alpha\Delta T) \text{ or}$$

$$\rho_2=\rho_1(1-3\alpha\Delta T)$$

$$=(19.30)\left[1-3(14.2\times10^{-6})(70)\right]=19.24g/cm^3$$

20: A steel bar of cross sectional area 1 cm^2 and 50 cm long at 30^0C fits into the space between two fixed supports. If the bars now heated to 280^0C, what force will it exert against the supports?

(α for steel = $11\times10^{-6}/^0C$ and

Young's modulus for steel = $2\times10^{11}N/m^2$)

SOLUTION:

Force exerted on the supports

$$=\text{Stress}\times\text{Area of cross section}=Y\alpha A(t_2-t_1).$$

$$=2\times10^{11}\times11\times10^{-6}\times10^{-4}\times250=55000N$$

21. Uniform pressure P is exerted on all sides of a solid cube of bulk modulus, B and volume coefficient of expansion γ, at temperature t^{0}C. By what amount should the temperature of cube be raised in order to bring its volume back to the value it had before the pressure was applied?

SOLUTION:

$$\text{As } B=\frac{P}{\Delta V/V},\Delta V=\frac{VP}{B}......(i)$$

If ΔT is the required increase in temperature,

$$\Delta V=\gamma V\Delta T.....(ii)$$

From eqns. (i) and (ii),

$$\gamma V\Delta T=\frac{VP}{B}\ or\ \Delta T=\frac{P}{\gamma B}$$

22. The balance wheel of a mechanical wrist watch has a frequency of oscillation given by $f=\frac{1}{2\pi}\sqrt{C/I}$, where I is the moment of inertia of the wheel and C is the torsional rigidity of its spring. The wrist watch keeps accurate time at 25^{0}C. How many seconds would it gain a day at-25^{0}C if the balance wheel made of Aluminium?

(Given, $\alpha_{Al}=25.5\times10^{-6}/^0C$)

SOLUTION:

$$f=\frac{1}{2\pi}\sqrt{\frac{C}{1}}=\frac{1}{2\pi k}\sqrt{\frac{C}{M}}\qquad\left[\because I=MK^2\right]$$

$$f\alpha\frac{1}{T}; f\alpha\frac{1}{k} \qquad \Rightarrow \frac{df}{f}=\frac{-dT}{T}=\frac{-dk}{k}$$

$$\text{As } \frac{dk}{k}=\alpha dt \Rightarrow \frac{dT}{T}=+\alpha dt$$

Number of seconds gained/day

$$dT=\left(8.64\times10^{4}\right)\left(\alpha dT\right)=110.2s/day$$

23. An aluminium measuring rod, which is correct at 5⁰C measures the length of a line as 80 cm at 45⁰C. If thermal coefficient of linear expansion of aluminium is $2.50\times10^{-5}/^{0}C$. The correct length of the line is

SOLUTION:

$$L_2=L_1+L_1\alpha\Delta t$$

$$L_2=80+\left(2.50\times10^{-5}\right)\left(80\right)\left(40\right)=80.08cm$$

24. A mass of 2kg is suspended from a fixed pointed by a wire of length 3m and diameter 0.5 mm. Initially the wire is just unstretched, the mass resting on a fixed support. By how much must the temperature fall if the mass is to be entirely supported by the wire (Given Y for wire = 206 G Pa, $\alpha=11\times10-6/^{0}C$)

SOLUTION:

Contraction due to cooling is equal to the stretching produced by the weight 'mg'.

$$\therefore \Delta L=\frac{mgL}{AY}=\frac{2\times9.8\times3}{\pi\left(0.25\right)^{2}\times10^{-6}\times206\times10^{9}}$$

Now the contraction due to cooling

$$=L\alpha\Delta t=3\times11\times10^{-6}\times\Delta t$$

solving $\Delta t=44^{0}C$

25. A metallic rod of length 1 cm and cross-sectional area A cm^2 is heated through $t^{0}C$. After expansion if a mechanical force is applied normal to its length on both sides of the rod and restore its original length, what is the value of force? The young's modulus of elasticity of the metal is E and mean coefficient of linear expansion is α per degree Celsius.

SOLUTION:

Change in the length $=\Delta l=l\alpha t$

Length of rod at $t^{0}C\ is\ l+l\alpha t$

Decrease in length due to stress= Δl

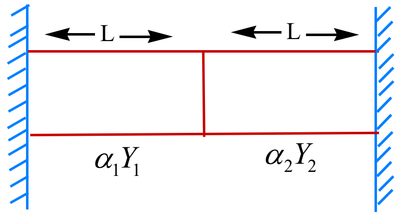
But length of rod remains constant $\therefore \Delta l_t-\Delta l=0$

$$\therefore \Delta l=\Delta l_t=l\alpha t \Rightarrow E=\frac{stress}{strain}=\frac{F}{A}\times\frac{l+\Delta l_t}{-\Delta l_t}$$

$$\therefore F=\frac{EA\Delta l_t}{l+\Delta l_t}=\frac{-EAl\alpha t}{l+l\alpha t}=\frac{EA\alpha t}{\left(1+\alpha t\right)}$$

Here, negative sign indicates that forces are compressive in nature.

26. Two metal rods are fixed end to end between two rigid supports as shown in figure. Each rod is length l and area of cross-section is A. When the system is heated up, determine the condition when the junction between rods does not shift? (Y_1 and Y_2 are Young's modulus of materials of rods, α_1 and α_2 are coefficients of linear expansion)



SOLUTION:

Since, each rod is prevented from expansion so, they are under compression and mechanical strain.

The strain in each rod is zero.

$$\frac{e_1}{l} = \alpha_1 \Delta T - \frac{F}{AY_1};\ \frac{e_2}{l} = \alpha_2 \Delta T - \frac{F}{AY_2};\ \frac{e_1}{l} = \frac{e_2}{l} = 0$$

$$\alpha_1 \Delta T - \frac{F}{AY_1} = 0 \ and \ \alpha_2 \Delta T - \frac{F}{AY_2} = 0$$

$$\alpha_1 \Delta T - \frac{F}{AY_1} ....(1) \quad and \quad \alpha_2 \Delta T - \frac{F}{AY_2} ....(2)$$

Dividing (1) by (2), we get $\alpha_1 Y_1 = \alpha_2 Y_2$

27. A bimetallic strip of thickness 2 cm consists of zinc and silver rivetted together. The approximate radius of curvature of the strip when heated through $50^0 C$ will be: (linear expansivity of zinc and silver are $32 \times 10^{-6} /^0 C$ and $19 \times 10^{-6} /^0 C$ respectively)

SOLUTION:

$$\text{Radius of curvature } R = \frac{2t}{(\alpha_2 - \alpha_1)\Delta T}$$

$$R = \frac{2 \times 1}{(32 - 19) \times 10^{-6} \times 50} = 30.77m$$

28. A clock with a metallic pendulum is 5 seconds fast each day at a temperature of 15^0C and 10 seconds slow each day at a temperature of 30^0C. Find coefficient of linear expansion for the metal.

SOLUTION:

The time lost or gained per day is

$$\Delta t = \frac{1}{2} \alpha \Delta T \times 86400 \quad [as\ 1\ day = 86400\ s.]$$

If graduation temperature of clock is T_0 then gain in time at $15^0 C$ is

$$5 = \frac{1}{2}(\alpha)(T_0 - 15) \times 86400 ....(i)$$

At $30^0 C$ clock is loosing time thus

$$10 = \frac{1}{2}\alpha(30 - T_0)86400 ....(ii)$$

Dividing equation (ii) by (i), we get

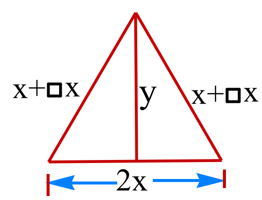
$$2(T_0 - 15) = (30 - T_0) \; or \; T_0 = 20^0 C$$

Thus from equation (i)

$$5 = \frac{1}{2}\alpha[20 - 15]86400$$

$$\alpha = 2.31 \times 10^{-5} /^0 C$$

29. A steel rail 30 m long is firmly attached to the road bed only at its ends. The sun raises the temperature of the rail by $50^0 C$, causing the rail to buckle. Assuming that the buckled rail consists of two straight parts meeting in the centre, calculate how much centre of the rail rise? Given, $\alpha_{steel} = 12 \times 10^{-6} /^0 C$.



SOLUTION:

Let the initial length be 2x and the final total length be $2(x + \Delta x)$ as shown.

Let y be the height of the centre of the buckled rail.

Clearly, $\Delta x = \alpha x \, \Delta T$ and

$$y = \sqrt{(x + \Delta x)^2 - x^2} = \sqrt{2x(\Delta x)} = \sqrt{2x^2 \alpha \Delta t}$$

$$y = x\sqrt{2\alpha\Delta t} \quad \left[neglecting \; (\Delta x)^2\right]$$

$$\text{Thus, } y = \left[15\sqrt{2(12 \times 10^{-6})50}\right] cm = 0.52 m$$

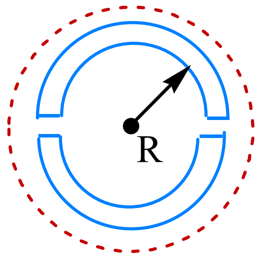
30. A metal rod of Young's modulus F and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by $t^0 C$, then the linear stress developed in it is (AIE-2011)

SOLUTION:

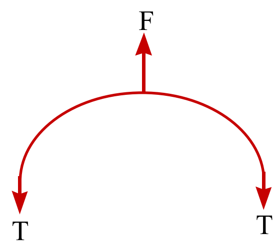
$$\Delta L = \alpha L \Delta T = \frac{FL}{AY} \Rightarrow Stress = \frac{F}{A} = Y\alpha\Delta T = Y\alpha t$$

31. A wooden wheel of radius R is made of two semi circular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature. It presses the semi-circular

parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y, then the force that one part of the wheel applies on the other part is (AIEEE 2012)



SOLUTION:



Increase in length

$$\Delta L = \alpha L \Delta T \Rightarrow \qquad \frac{\Delta L}{L} = \alpha \Delta T$$

the thermal stress developed is

$$\frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T; \;\; T = SY\alpha\Delta T$$

From the FBD of one part of the wheel, F = 2T

Where F is the force applied by one part of the wheel on other part, F=2SY $\alpha\Delta T$

32. **Two rods of different materials and identical cross sectional area, are joined face to face at one end and their free ends are fixed to the rigid walls. If the temperature of the surroundings is increased by 30°C, the magnitude of the displacement of the joint of the rod is (length of rods $l_1 = l_2 = 1\,unit$, ratio of their young's moduli, $Y_1/Y_2 = 2$, coefficients of linear expansion are α_1 and α_2)**

1) $5(\alpha_2 - \alpha_1)$ 2) $10(\alpha_1 - \alpha_2)$ 3) $10(\alpha_2 - 2\alpha_1)$ 4) $5(2\alpha_1 - \alpha_2)$

SOLUTION:

$$Y_1\left(Strain\right)_1 = Y_2\left(Strain\right)_2$$

$$Y_1\left[\frac{\alpha_1 l \Delta T_1 - x}{l}\right] = Y_{21}\left[\frac{\alpha_2 l \Delta T_2 + x}{l}\right]$$

Displacement of the rod

$$x = \frac{(Y_2\alpha_2 - Y_1\alpha_1)}{Y_1 + Y_2} \times l \times \Delta T$$

33. Calculate the compressional force required to prevent the metallic rod of length l cm and cross-sectional area A cm^2 when heated through t^0C, from expanding along length wise. The Young's modulus of elasticity of the metal is E and mean coefficient of linear expansion is α per degree Celsius

1) $EA\alpha t$ 2) $\frac{EA\alpha t}{1+\alpha t}$ 3) $\frac{EA\alpha t}{1-\alpha t}$ 4) $El\alpha t$

SOLUTION:

Natural length of the rod at $t^0C = l + l\alpha t$

decrease in natural length due to stress $= \Delta l$

$$\Delta l_1 - \Delta l = 0; E = \frac{-F(l+\Delta l)}{A\Delta l}; F = \frac{-EA\alpha t}{1+\alpha t}$$

34. The coefficient of linear expansion for a certain metal varies with temperature as $\alpha(T)$. If L_0 is the initial length of the metal and the temperature of metal changed from T_0 to $T\ (T_0 > T)$ then

1) $L = L_0 \int_{T_0}^{T} \alpha(T) dT$ 2) $L = L_0\left[1 + \int_{T_0}^{T} \alpha(T) dT\right]$ 3) $L = L_0\left[1 - \int_{T_0}^{T} \alpha(T) dT\right]$ 4) $L > L_0$

SOLUTION: $\frac{dL}{L_0} = -\alpha(T)\,dt;$ Integrate the equation

35. A solid sphere of radius r and mass m is spinning about a diameter as axis with a speed ω_0. The temperature of the sphere increases by 100^0C without any other disturbance. If the coefficient of linear expansion of material of sphere is $2\times10^{-4}/^0C$, the ratio of angular speed at 100^0C and ω_0 is

1) 1:1 2) 1:1.04 3) 1.04:1 4) 1:1.02

SOLUTION:

$$I_1\omega_1 = I_2\omega_2; R_1^2\omega_1 = R_2^2\omega_2$$

$$\frac{\omega_2}{\omega_1} = \left(\frac{R_2}{R_1}\right)^2 = \frac{1}{(1+\alpha\Delta t)^2}$$

Thermal Expansion in Liquids:

- (1) Liquids do not have linear and superficial expansion but these only have volume expansion.
- Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.

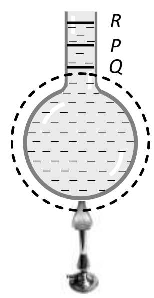


Fig. 12.11

$PQ \to$ represents expansion of vessel

$QR \to$ represents the real expansion of liquid

$PR \to$ Represent the apparent expansion of liquid

- The actual increase in the volume of the liquid = The apparent increase in the volume of liquid + the increase in the volume of the vessel.
- Liquids have two coefficients of volume expansion.

Co-efficient of apparent expansion (γ_a) : It is due to apparent (that appears to be, but is not) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)_a}{V \times \Delta\theta}$$

Co-efficient of real expansion (γ_r) : It is due to the actual increase in volume of liquid due to heating.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)_r}{V \times \Delta\theta}$$

- Also coefficient of expansion of flask $\gamma_{Vessel} = \frac{(\Delta V)_{Vessel}}{V \times \Delta\theta}$
- $\gamma_{Real} = \gamma_{Apparent} + \gamma_{Vessel}$
- Change (apparent change) in volume in liquid relative to vessel is

$$\Delta V_{app} = V\,\gamma_{app}\,\Delta\theta = V(\gamma_{Real} - \gamma_{Vessel})\Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$$

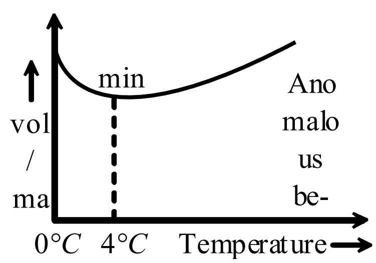
α = Coefficient of linear expansion of the vessel.

Anomalous expansion of water :

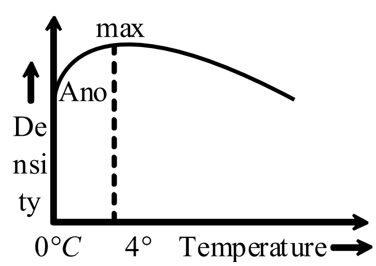
Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than $4°C$. In the range $0°C$ to $4°C$, water contracts on heating and expands on cooling, *i.e.* g is negative. This behaviour of water in the range from $0°C$ to $4°C$ is called anomalous expansion.

This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature increases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches $0°C$ first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about $4°C$.

At $4°C$, density of water is maximum while its specific volume is minimum.



(A)



(B)

Variation of Density with Temperature

Most substances (solid and liquid) expand when they are heated,

i.e., volume of a given mass of a substance increases on heating, so the density should decrease $\left(\text{as } \rho \propto \frac{1}{V}\right)$.

For a given mass $\rho \propto \frac{1}{V} \Rightarrow \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{V}{V + \gamma V \Delta\theta} = \frac{1}{1 + \gamma\,\Delta\theta}$

$$\Rightarrow \rho' = \frac{\rho}{1 + \gamma\,\Delta\theta} = \rho(1 + \gamma\,\Delta\theta)^{-1} = \rho(1 - \gamma\,\Delta\theta)$$

CALORIMETRY

Specific Heat

When a body is heated it's temperature rises (except during a change in phase).

Gram specific heat : The amount of heat energy required to raise the temperature of unit mass of a body through $1°C$ (or K) is called specific heat of the material of the body.

If Q heat changes the temperature of mass m by $\Delta\theta$ then specific heat $c = \frac{Q}{m\Delta\theta}$

(i) Units : *Calorie/gm* $°C$ (practical), *J/kg K* (S.I.)

Dimension : $[L^2 T^{-2} \theta^{-1}]$

(ii) For an infinitesimal temperature change $d\theta$ and corresponding quantity of heat dQ.

pecific heat $c = \frac{1}{m} \cdot \frac{dQ}{d\theta}$

Molar specific heat : Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree it is represented by (capital) C.

Molar specific heat $(C) = M \times \text{Gram specific heat } (c)$

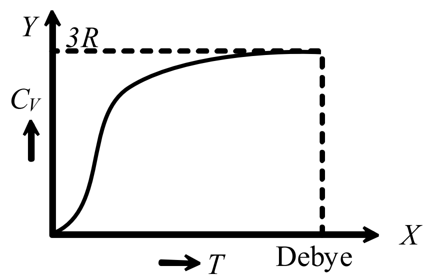
(M = Molecular mass of substance)

$$C = M\frac{Q}{m\Delta\theta} = \frac{1}{\mu}\frac{Q}{\Delta\theta} \quad \left(\text{where, Number of moles } \mu = \frac{m}{M}\right)$$

Units : *calorie/mole* × °*C* (practical); *J/mole* × *kelvin* (S.I.)

Dimension : $[ML^2T^{-2}\theta^{-1}]$

Specific Heat of Solids



When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume C_V.

(1) From the graph it is clear that at $T = 0$, C_V tends to zero

(2) With rise in temperature, C_V increases and at a particular temperature (called Debey's temperature) it becomes constant = $3R = 6$ *cal*/ *mole* × *kelvin* = 25 *J/mole* × *kelvin*

(3) For most of the solids, Debye temperature is close to room temperature.

Dulong and Petit law :

Average molar specific heat of all metals at room temperature is constant, being nearly equal to $3R = 6$ *cal*. $mole^{-1}$ K^{-1} = 25 $J\,mole^{-1}$ K^{-1}, where R is gas constant for one mole of the gas. This statement is known as Dulong and Petit law.

Debey's law :

It was observed that at very low temperature molar specific heat $\propto T^3$ (exception are Sn, Pb and Pt)

Specific heat of ice :

In C.G.S. $c_{\text{ice}} = 0.5\frac{cal}{gm \times °C}$

In S.I. $c_{ice} = 500\frac{cal}{kg \times °C} = 2100\frac{Joule}{kg \times °C}$.

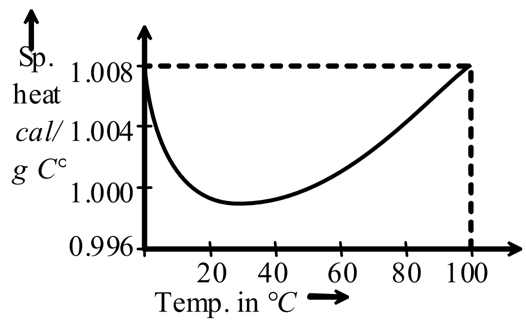
Specific Heat of Liquid (Water)

(1) Among all known solids and liquids specific heat of water is maximum *i.e.* water takes more time to heat and more time to cool *w.r.t.* other solids and liquids.

(2) It is observed that by increasing temperature, initially specific heat of water goes on decreasing, becomes minimum at 37°*C* and then it start increasing.

Specific heat of water is – $\frac{1\,cal}{gm \times °C} = 1000\,\frac{cal}{kg \times °C} = 4200\,\frac{J}{kg \times °C}$

(This value is obtained between the temperature 14.5°*C* to 15.5°*C*)



(3) The variation of specific heat with temperature for water is shown in the figure. Usually this temperature dependence of specific heat is neglected.

(4) As specific heat of water is very large; by absorbing or releasing large amount of heat its temperature changes by small amount. This is why, it is used in hot water bottles or as coolant in radiators.

Specific Heat of Gases

(1) In case of gases, heat energy supplied to a gas is spent not only in raising the temperature of the gas but also in expansion of gas against atmospheric pressure.

(2) Hence specific heat of a gas, which is the amount of heat energy required to raise the temperature of one gram of gas through a unit degree shall not have a single or unique value.

(3) If the gas is compressed suddenly and no heat is supplied from outside *i.e.* $\Delta Q = 0$, but the temperature of the gas raises on the account of compression.

$$\Rightarrow \quad c = \frac{Q}{m(\Delta\theta)} = \frac{0}{m\Delta\theta} = 0$$

(4) If the gas is heated and allowed to expand at such a rate that rise in temperature due to heat supplied is exactly equal to fall in temperature due to expansion of the gas. *i.e.* $\Delta\theta = 0$

$$\Rightarrow \quad c = \frac{Q}{m(\Delta\theta)} = \frac{Q}{0} = \infty$$

(5) If rate of expansion of the gas were slow, the fall in temperature of the gas due to expansion would be smaller than the rise in temperature of the gas due to heat supplied. Therefore, there will be some net rise in temperature of the gas *i.e.* ΔT will be positive.

$$\Rightarrow c = \frac{Q}{m(\Delta\theta)} = \text{Positive}$$

(6) If the gas were to expand very fast, fall of temperature of gas due to expansion would be greater than rise in temperature due to heat supplied. Therefore, there will be some net fall in temperature of the gas *i.e.* Δ will be negative.

$$\Rightarrow \quad c = \frac{Q}{m(-\Delta\theta)} = \text{Negative}$$

Hence the specific heat of gas can have any positive value ranging from zero to infinity. Further it can even be negative. The exact value depends upon the mode of heating the gas. Out of many values of specific heat of a gas, two are of special significance, namely C_p and C_V, in the chapter "Kinetic theory of gases" we will discussed this topic in detail.

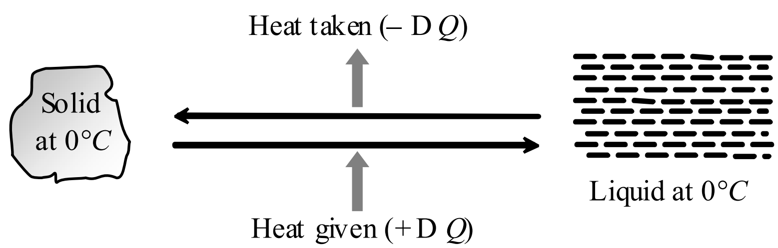
Specific heat of steam : $c_{\text{steam}} = 0.47\,cal/gm \times °C$

Phase Change and Latent Heat

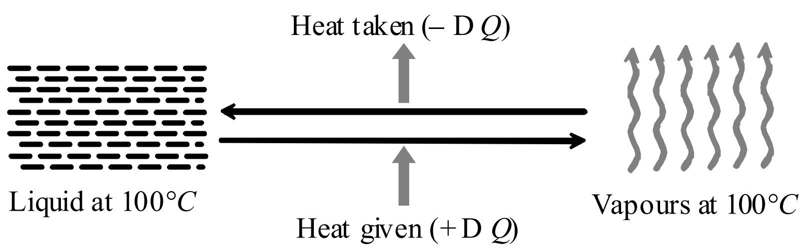
(1) **Phase :** We use the term phase to describe a specific state of matter, such as solid, liquid or gas. A transition from one phase to another is called a phase change.

(i) For any given pressure a phase change takes place at a definite temperature, usually accompanied by absorption or emission of heat and a change of volume and density.

(ii) In phase change ice at $0°C$ melts into water at $0°C$. Water at $100°C$ boils to form steam at $100°C$.



(A)



(B)

(iii) In solids, the forces between the molecules are large and the molecules are almost fixed in their positions inside the solid. In a liquid, the forces between the molecules are weaker and the molecules may move freely inside the volume of the liquid. However, they are not able to come out of the surface. In vapours or gases, the intermolecular forces are almost negligible and the molecules may move freely anywhere in the container. When a solid melts, its molecules move apart against the strong molecular attraction. This needs energy which must be supplied from outside. Thus, the internal energy of a given body is larger in liquid phase than in solid phase. Similarly, the internal energy of a given body in vapour phase is larger than that in liquid phase.

(iv) In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.

Latent heat : The amount of heat required to change the state of the mass *m* of the substance is written as : $Q = mL$, where L is the latent heat. Latent heat is also called as Heat of Transformation. It's unit is *cal/gm* or *J/kg* and Dimension: $[L^2T^{-2}]$

(i) **Latent heat of fusion :** The latent heat of fusion is the heat energy required to change 1 *kg* of the material in its solid state at its melting point to 1 *kg* of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 *kg* of liquid changes to 1 *kg* of solid. For water at its normal freezing temperature or melting point ($0°C$), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{\text{ice}} \approx 80\ cal/gm \approx 60\ kJ/mol \approx 336\ kilo\ joule/kg$$

Latent heat of vaporisation : The latent heat of vaporisation is the heat energy required to change 1 *kg* of the material in its liquid state at its boiling point to 1 *kg* of the material in its gaseous state. It is also the amount of heat energy released when 1 *kg* of vapour changes into 1 *kg* of liquid. For water at its normal boiling point or condensation temperature ($100°C$), the latent heat of vaporisation (latent heat of steam) is

$$L_V = L_{\text{steam}} \approx 540\ cal/gm \approx 40.8\ kJ/mol \approx 2260\ kilo\ joule/kg$$

(iii) Latent heat of vaporisation is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence very less amount of heat is required. So, latent heat of vaporisation is more than the latent heat of fusion.

Thermal Capacity and Water Equivalent

Thermal capacity : It is defined as the amount of heat required to raise the temperature of the whole body (mass *m*) through $0°C$ or $1K$.

$$\text{Thermal capacity} = mc = \mu C = \frac{Q}{\Delta\theta}$$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension : $[ML^2T^{-2}\theta^{-1}]$,

Unit : *cal/°C* (practical) *Joule/k* (S.I.)

Water Equivalent : Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by *W*.

If m = Mass of the body, c = Specific heat of body, $\Delta\theta$ = Rise in temperature.

Then heat given to body $\Delta Q = mc\Delta\theta$ ….. (i)

If same amount of heat is given to *W gm* of water and its temperature also rises by $\Delta\theta$.

Thenheat given to water $Q = W \times 1 \times \Delta\theta$… (ii) [As $c_{\text{water}} = 1$]

From equation (i) and (ii) $\Delta Q = mc\Delta\theta = W \times 1 \times \Delta\theta$

$\Rightarrow$ Water equivalent (*W*) = *mc gm*

(i) Unit : *Kg* (S.I.) Dimension : $[ML^0T^0]$

(ii) Unit of thermal capacity is *J/kg* while unit of water equivalent is *kg*.

(iii) Thermal capacity of the body and its water equivalent are numerically equal.

(iv) If thermal capacity of a body is expressed in terms of mass of water it is called water-equivalent of the body.

Some Important Terms

Evaporation : Vaporisation occurring from the free surface of a liquid is called evaporation. Evaporation is the escape of molecules from the surface of a liquid. This process takes place at all temperatures and increases with the increase of temperature. Evaporation leads to cooling because the faster molecules escape and, therefore, the average kinetic energy of the molecules of the liquid (and hence the temperature) decreases.

Melting (or fusion)/freezing (or solidification) : The phase change of solid to liquid is called melting or fusion. The reverse phenomenon is called freezing or solidification.

When pressure is applied on ice, it melts. As soon as the pressure is removed, it freezes again. This phenomenon is called **regelation**.

Vaporisation/liquefication (condensation) : The phase change from liquid to vapour is called vaporisation. The reverse transition is called liquefication or condensation.

Sublimation : Sublimation is the conversion of a solid directly into vapours. Sublimation takes place when boiling point is less than the melting point. A block of ice sublimates into vapours on the surface of moon because of very very low pressure on its surface. Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.

Hoar frost : Direct conversion of vapours into solid is called hoar frost. This process is just reverse of the process of sublimation, *e.g.*, formation of snow by freezing of clouds.

Vapour pressure : When the space above a liquid is closed, it soon becomes saturated with vapour and a dynamic equilibrium is established. The pressure exerted by this vapour is called Saturated Vapour Pressure (S.V.P.) whose value depends only on the temperature – it is independent of any external pressure. If the volume of the space is reduced, some vapour liquefies, but the pressure is unchanged.

A saturated vapour does not obey the gas law whereas the unsaturated vapour obeys them fairly well. However, a vapour differs from a gas in that the former can be liquefied by pressure alone, whereas the latter cannot be liquefied unless it is first cooled.

Boiling : As the temperature of a liquid is increased, the rate of evaporation also increases. A stage is reached when bubbles of vapour start forming in the body of the liquid which rise to the surface and escape. A liquid boils at a temperature at which the S.V.P. is equal to the external pressure.

It is a fast process. The boiling point changes on mixing impurities.

Dew point : It is that temperature at which the mass of water vapour present in a given volume of air is just sufficient to saturate it, *i.e.* the temperature at which the actual vapour pressure becomes equal to the saturated vapuor pressure.

Humidity : Atmospheric air always contains some water vapour. The mass of water vapour per unit volume is called absolute humidity.

The ratio of the mass of water vapour (m) actually present in a given volume of air to the mass of water vapour (M) required to saturate the same volume at the same temperature is called the relative humidity (R.H.). Generally, it is expressed as a percentage,

i.e., $\text{R.H.}(\%) = \frac{m}{M} \times 100(\%)$

R.H. May also be defined as the ratio of the actual vapour pressure (p) of water at the same temperature,

i.e. $\text{R.H.}(\%) = \frac{p}{P} \times 100(\%)$

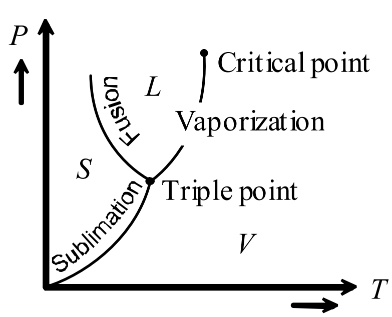
Thus R.H. may also be defined as

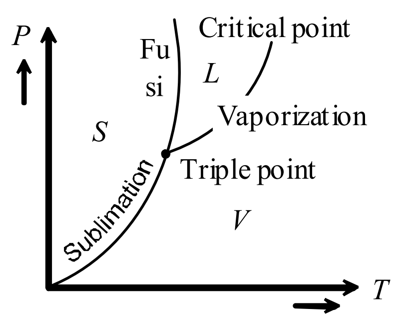
$$\text{R.H.}(\%) = \frac{\text{S.V.P. at dew point}}{\text{S.V.P. at given temperature}} \times 100$$

Variation of melting point with pressure : For those substances with contract on melting (*e.g.* water and rubber), the melting point decreases with pressure. The reason is the pressure helps shrinking and hence melting. Most substances expand on melting. (*e.g.* max, sulpher *etc.*)

An increase of pressure opposes the melting of such substances and their melting point is raised.

Variation of latent heat with temperature and pressure : The latent heat of vapourization of a substance varies with temperature and hence pressure because the boiling point depends on pressure. It increases as the temperature is decreased. For example, water at 1 *atm* boils at $100°C$ and has latent heat 2259 Jg^{-1} but at 0.5 *atm* it boils at $82°C$ and has latent heat 2310 Jg^{-1}





The latent heat of fusion shows similar but less pronounced pressure dependence.

The figures show the *P–T* graphs for (*a*) a substance (*e.g.*, water) which contracts on melting an (*b*) a substance (*e.g.* wax) which expands on melting. The *P–T* graph consists of three curves.

(i) Sublimation curve which connects points at which vapour (*V*) and solid (*S*) exist in equilibrium.

(ii) Vapourization curve which shows vapour and liquid (*L*) existing in equilibrium.

(iii) Fusion curve which shows liquid and solid existing in equilibrium.

The three curves meet at a single point which is called the triple point. It is that unique temperature-pressure point for a substance at which all the three phases exist in equilibrium.

Freezing mixture : If salt is added to ice, then the temperature of mixture drops down to less than $0°C$. This is so because, some ice melts down to cool the salt to $0°C$. As a result, salt gets dissolved in the water formed and saturated solution of salt is obtained; but the ice point (freeing point) of the solution formed is always less than that of pure water. So, ice cannot be in the solid state with the salt solution at $0°C$. The ice which is in contact with the solution, starts melting and it absorbs the required latent heat from the mixture, so the temperature of mixture falls down.

Principle of Calorimetry

Calorimetry means 'measuring heat'.

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat lost = Heat gained

i.e. principle of calorimetry represents the law of conservation of heat energy.

(1) Temperature of mixture (θ_{mix}) is always e" lower temperature (θ_L) and d" higher temperature (θ_H), *i.e.*, $\theta_L \le \theta_{mix} \le \theta_H$.

It means the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

(2) **Mixing of two substances when temperature changes only :** It means no phase change. Suppose two substances having masses m_1 and m_2, gram specific heat c_1 and c_2, temperatures θ_1 and θ_2 $(\theta_1 > \theta_2)$ are mixed together such that temperature of mixture at equilibrium is θ_{mix}

Hence, Heat lost = Heat gained

$$\Rightarrow m_1c_1(\theta_1 - \theta_{mix}) = m_2c_2(\theta_{mix} - \theta_2) \Rightarrow \theta_{mix} = \frac{m_1c_1\theta_1 + m_2c_2\theta_2}{m_1c_1 + m_2c_2}$$

Mixing of two substances when temperature and phase both changes or only phase changes: A very common example for this category is ice-water mixing.

Suppose water at temperature $\theta_W °C$ is mixed with ice at $0_i °C$, first ice will melt and then it's temperature rises to attain thermal equilibrium. Hence; Heat given = Heat taken

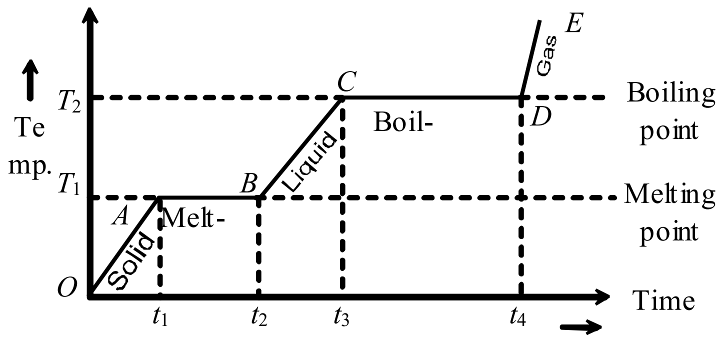
$$\Rightarrow m_W C_W(\theta_W - \theta_{mix}) = m_i L_i + m_i C_W(\theta_{mix} - 0°)$$

$$\Rightarrow \theta_{mix} = \frac{m_W\theta_W - \dfrac{m_i L_i}{C_W}}{m_W + m_i}$$

(i) If $m_W = m_i$ then $\theta_{mix} = \dfrac{\theta_W - \dfrac{L_i}{C_W}}{2}$

(ii) By using this formulae if $\theta_{mix} < \theta_i$ then take $\theta_{mix} = 0°C$

Heating Curve:



If to a given mass (m) of a solid, heat is supplied at constant rate P and a graph is plotted between temperature and time, the graph is as shown in figure and is called heating curve. From this curve it is clear that

(1) In the region OA temperature of solid is changing with time so, $Q = mc_S \Delta T \Rightarrow P\,\Delta t = mc_S \Delta T$ [as $Q = P\Delta t$]

But as $(\Delta T/\Delta t)$ is the slope of temperature-time curve

$$c_s \propto \frac{1}{\text{Slope of line } OA}$$

i.e. specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

(2) In the region AB temperature is constant, so it represents change of state, *i.e.*, melting of solid with melting point T_1. At A melting starts and at B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion. $Q = mL_F$ or $L_F = \frac{P(t_2 - t_1)}{m}$ [as $Q = P(t_2 - t_1)$]

or $L_F \propto$ length of line AB

i.e. Latent heat of fusion is proportional to the length of line of zero slope. [In this region specific heat $\propto \frac{1}{\tan 0} = \infty$]

(3) In the region BC temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC

ie, $$c_L \propto \frac{1}{\text{Slope of line } BC}$$

(4) In the region CD temperature is constant, so it represents the change of state, *i.e.*, boiling with boiling point T_2. At C all substance is in liquid state while at D in vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation

i.e., $L_V \propto$ Length of line CD [In this region specific heat $\propto$]

(5) The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

Note:

The temperature of mixture can never be lesser than lower temperature and can never be greater than higher temperature

$$\theta_L \le \theta_{mix} \le \theta_H$$

If 'm' g of steam at $100^0 C$ is mixed with 'm' g of ice at $0^0 C$ then

a) Resultant temperature of mixture is $100^0 C$

b) Mass of steam condensed $= \frac{m}{3} g$

c) Mass of steam left uncondensed $= \frac{2m}{3} g$

d) The final mixture contains $\frac{4m}{3} g$ of water and $\frac{2m}{3} g$ of steam both at $100^0 C$

Super cooling :

- Most liquids, if cooled in a pure state in a perfectly clean vessel, with least disturbance, can be lowered to a temperature much below the normal freezing point, without solidifying. This is known as super cooling or super fusion.
- In super cooling, water can be cooled upto $-10^0 C$ without becoming solid.

Saturated and Unsaturated Vapours :

(a) When the pressure exerted by a vapour is maximum it is called saturated vapour, when pressure exerted is not maximum, it is called unsaturated vapour.

(b) Saturated vapours do not obey the gas laws and saturated vapour pressure of liquid is independent of volume occupied. But unsaturated vapour obey the gas laws.

(c) At boiling point of a liquid saturated vapour pressure is equal to atmospheric pressure at that place.

NOTE:

Physical Quantity	Units	
	SI	CGS (Practical)
Heat	Joule	Calories
Specific Heat	Joule/Kg-K	Cal/g-^{0}C
Molar specific Heat	Joule/mol-K	Cal/mol-^{0}C
Thermal capacity	Joule/Kg	Cal/^{0}C
Water Equivalent	Kg	g

PROBLEMS

1. Find the water equivalent of copper block of mass 200g. The specific heat of copper is $0.09\, cal / g^0 C$.

SOLUTION:

Water equivalent w= mS $= 200 \times 0.09 = 18g$

2. A lead piece of mass 25g gives out 1200 calories of heat when it is cooled from $90^0 C$ to $10^0 C$. What is its (i) specific heat

(ii) thermal capacity

(iii) water equivalent.

SOLUTION:

Mass of lead piece (m) = 25 g = 0.025 kg

Heat energy given out $(dQ) = 1200 \times 4.2 J$

(i) specific heat $S = \frac{1}{m}\frac{dQ}{d\theta} = \frac{1}{0.025} \times \frac{1200 \times 4.2}{80} = 2520 JKg^{-1}K^{-1}$

(ii) Thermal capacity = mS = 0.025 × 2520 = 63 J/K

(iii) Water equivalent $\frac{63}{4200} Kg = 0.015 Kg$

3. Two spheres of radii in the ratio 1:2, have specific heats in the ratio 2:3. The densities are in the ratio 3:4. Find the ratio of their thermal capacities.

SOLUTION:

Thermal capacity of a body = mS.

The ratio of thermal capacities

$$\frac{m_1 S_1}{m_2 S_2} = \frac{V_1 \rho_1 S_1}{V_2 \rho_2 S_2} = \frac{\frac{4}{3}\pi r_1^3 \rho_1 S_1}{\frac{4}{3}\pi r_2^3 \rho_2 S_2} = \left(\frac{r_1}{r_2}\right)^3 \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{S_1}{S_2}\right).$$

Here, $\frac{r_1}{r_2} = \frac{1}{2}; \frac{S_1}{S_2} = \frac{2}{3}; \frac{\rho_1}{\rho_2} = \frac{3}{4}$

The ratio of thermal capacities $= \left(\frac{1}{2}\right)^3 \left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{16}$

4. A sphere of aluminium of 0.047 kg is placed for sufficient time in a vessel containing boiling water, so that the sphere is at $100^0 C$. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg of water at $20^0 C$. The temperature of water rises and attains a steady state at $23^0 C$. Calculate the specific heat capacity of aluminium.

($S_{cu} = 386 J/Kg - K$, $S_w = 4180 J/Kg - K$)

SOLUTION:

Heat lost by aluminium sphere =

(heat gained by water) + (heat gained by calorimeter)

$$0.047 \times S_{Al} \times \left(100^0 - 23^0\right) = 0.25 \times 4180\left(23^0 - 20^0\right)$$

$$+0.14 \times 386\left(23^0 - 20^0\right)$$

$\therefore S_{Al} = 911 J/Kg - K$

5: The temperature of equal masses of three different liquids A, B and C are 12ºC, 19ºC and 28ºC respectively. The common temperature when A and B are mixed is 16ºC and when B and C are mixed is 23ºC. What should be the common temperature when A and C are mixed?

SOLUTION:

Given $\theta_A = 12^o C, \theta_B = 19^o C$ and $\theta_C = 28^o C$. Let S_A, S_B and S_C are the specific heats of respective liquids.

When liquid A and B are mixed

Heat gain = Heat lost

$$mS_A(16-12) = mS_B(19-16)$$

$$\text{or } S_B = \frac{4}{3}S_A........(i)$$

When liquid B and C are mixed

Heat gain = Heat lost

$$mS_B(23-19) = mS_C(28-23) \text{ or}$$

$$S_B = \frac{5}{4}S_C.....(ii)$$

From (i) and (ii), we get $S_A = \frac{15}{16}S_C$

When A and C are mixed, let equilibrium temperature of mixture is θ, then

Heat gain = Heat lost

$$mS_A(\theta-12) = mS_C(28-\theta) \Rightarrow \theta = 20.26^o C$$

6: A piece of ice of mass 100 g and at temperature $0^0 C$ is put in 200 g of water at $25^0 C$. How much ice will melt as the temperature of the water reaches $0^0 C$? (specific heat capacity of water $= 4200 J\,kg^{-1}K^{-1}$ and latent heat of fusion of ice $= 3.4\times10^5 J\,Kg^{-1}$)

SOLUTION:

The heat released as the water cools down from $25^0 C$ to $0^0 C$ is

$$Q = mS\Delta\theta = (0.2)(4200)(25) = 21000J$$

The amount of ice melted by this heat is

$$m = \frac{Q}{L} = \frac{21000}{3.4\times10^5} = 62g$$

7: 10 litres of hot water at $70^o C$ is mixed with an equal volume of cold water at $20^o C$. Find the resultant temperature of the water. (Specific heat of water = 4200 J/kg -K)

SOLUTION:

Resultant temperature, $\theta = \frac{m_1S_1\theta_1 + m_2S_2\theta_2}{m_1S_1 + m_2S_2}$

Here, $m_1 = m_2 = 10kg$,

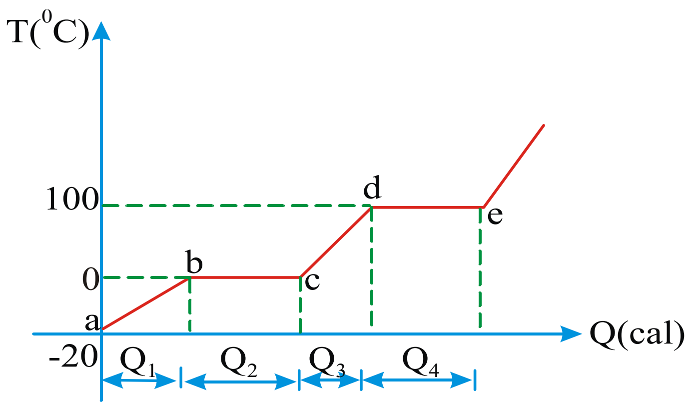
(since mass of 1 litre of water is 1 kg).

$\theta_1 = 70^o C; \theta_2 = 20^o C$

and $S_1 = S_2 = 4200J/kg - K$

$$\theta = \frac{10 \times 4200 \times 70 + 10 \times 4200 \times 20}{10 \times 4200 + 10 \times 4200} = 45^{o} C$$

8: The following graph represents change of state of 1 gram of ice at $-20^{0} C$. Find the net heat required to convert ice into steam at $100^{0} C$



SOLUTION:

$S_{ice} = 0.53 cal / g - ^{0}C$

In the figure :

a to b: Temperature of ice increases until it reaches its melting point $0^{0} C$.

$$Q_1 = mS_{ice}\left[0-(-20)\right] = (1)(0.53)(20) = 10.6 cal$$

b to c: Temperature remains constant until all the ice has melted

$$Q_2 = mL_f = (1)(80) = 80 cal$$

c to d : Temperature of water again rises until it reaches its boiling point $100^{0} C$

$$Q_3 = mS_{water}\left[100-0\right] = (1)(1.0)(100) = 100 cal$$

d to e : Temperature is again constant until all the water is transformed into the vapour phase

$$Q_4 == mL_v = (1)(539) = 539 cal$$

Thus, the net heat required to convert 1g of ice at $-20^{0} C$ into steam at $100^{0} C$ is

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 729.6 cal$$

9 : A calorimeter of water equivalent 83.72 Kg contains 0.48 Kg of water at 35°C. How much mass of ice at 0°C should be added to decrease the temperature of the calorimeter to 20°C. (S_W = 4186J / Kg-K and $L_{ice} = 335000 J / Kg$)

SOLUTION:

Heat capacity of the calorimeter = 83.72J K^{-1}

From law of method of mixtures,

$$\left.\begin{matrix} \text{Heat lost by calorimeter} \\ + \\ \text{Heat lost by water} \end{matrix}\right\} = \text{Heat gained by the ice}$$

$$83.72\times15+0.48\times4186\times15=m\times(335000+83720)$$

$$\therefore m = 0.07498\,Kg$$

10. Steam is passed into a calorimeter with water having total thermal capacity 110 cal/gm and initial temperature 30°C. If the resultant temperature is 90°C, the increase in the mass of the water is

1) 12 gm 2) 1.2 gm 3) 5 gm 4) 12.4 gm

SOLUTION:

From principle of calorimetry

$$(mS)_{water}\,\Delta\theta_1 = m\left(L_{steam} + S\Delta\theta_2\right)$$

$(mS)_{water} = 110$, $S_w = 1$, $L_{ice} = 540$,

$$\Delta\theta_1 = 60 \;\&\; \Delta\theta_2 = 10$$

11: A steam at 100°C is passed into 1 kg of water contained in a calorimeter of water equivalent 0.2 kg at 9°C till the temperature of the calorimeter and water in it is increased to 90°C. Find the mass of steam condensed in kg (S_W=1 cal/g °C, & L_{steam} = 540 cal/g)(EAM-14E)

SOLUTION:

Let, m be the mass of the steam condensed. mass of the steam passed into calorimeter,

$m_2 = 1\text{kg} = 1000$ g.

Water equivalent of calorimeter,

$m_1S_1 = 0.2$ kg $= 200$g

$\theta_1 =$ temperature of the steam $= 100$°C

$\theta_2 =$ temperature of the water $= 9$°C

$\theta_3 =$ resultant temperature $= 90$°C

From law of method of mixtures,

Heat lost = heat gained (calorimeter + water)

$$m\left[L_{steam} + S_W\left(\theta_1 - \theta_3\right)\right] = \left[m_1S_1 + m_2S_W\right]\left(\theta_3 - \theta_2\right)$$

$$m\left[540 + 1(100 - 90)\right] = \left[200 + 1000\times1\right](90 - 9)$$

$$\Rightarrow m = 176g = 0.176kg \approx 0.18kg$$

12: 1g steam at 100°C is passed in an insulating vessel having 1g ice at 0°C. Find the equilibrium composition of the mixture. (Neglecting heat capacity of the vessel).

SOLUTION:

Available heat from steam $mL = 1\times540 = 540\,cal$

Heat required for melting of ice and to rise its temperature to 100° C $= m_{ice}L_{ice} + m_{water}S_{water}\Delta\theta$

$$=(1\times 80)+[1\times 1\times(100-0)]=180\,cal$$

Let m be the mass of steam condensed, then

$$m\times 540=180 \;\Rightarrow\; m=\frac{180}{540}=\frac{1}{3}g$$

Final contents : Water $=1+\frac{1}{3}=\frac{4}{3}g$,

steam $=1-\frac{1}{3}=\frac{2}{3}g$

13: 20g of steam at 100°C is passed into 100g of ice at 0°C. Find the resultant temperature if latent heat of steam is 540 cal/g, latent heat of ice is 80 cal/ g and specific heat of water is 1 cal/g°C.

SOLUTION:

For steam

Heat lost by the steam in condensation

$Q_1=m_sL_s=20\times 540=10800\,cal$ ....(1)

For ice

Heat gained by the ice in melting and to rise its temperature from 0^oC to 100^0C is $Q_2=m_{ice}L_{ice}+m_{ice}S_w\Delta t$

$=100\times 80+100\times 1\times 100=18000\,cal$ ......(2)

From eq. (1) and (2) ; $Q_2>Q_1$

Let θ = resultant temperature of the mixture

According to law of method of mixtures

Heat lost by steam = Heat gained by ice

$$m_sL_s+m_sS_{water}(100-\theta)=m_{ice}L_{ice}+m_{ice}S_{water}(\theta-0)$$

$$(20\times 540)+20\times 1(100-\theta)=(100\times 80)+(100\times 1\times\theta)$$

$$\Rightarrow\; \theta=40^oC$$

14. A thermally insulated vessel contains some water at 0°C. The vessel is connected to a vacuum pump to pump out water vapour. This results in some water getting frozen. The maximum percentage amount of water that will be solidified in this manner will be ($L_{steam}=21\times 10^5 J/kg$ and $L_{ice}=3.36\times 10^5 J/kg$).

1) 86.2% 2) 33.6% 3) 21% 4) 24.36%

SOLUTION:

Let m_1 mass is vaporised and m_2 mass gets solidified

Then heat taken in vaporisation = heat given during

or $m_1(21\times 10^5)=(m_2)(3.36\times 10^5)$

$$m_2 = 6.25m_1;$$

$$\% = \frac{m_2}{m_1 + m_2} \times 100$$

15 : 6 gm of steam at $100^0 C$ is mixed with 6 gm of ice at $0^0 C$. Find the mass of steam left uncondensed

($L_f = 80cal/g$ **,** $L_v = 540cal/g$ **,**

$S_{Water} = 1cal/g -^0 C$ **)**

SOLUTION:

For steam

Heat lost by the steam in condensation

$Q_1 = m_s L_s = 6 \times 540 = 3240cal$ ....(1)

For ice

Heat gained by the ice in melting and to rise its temperature from $0^o C$ $to 100^0 C$ is

$Q_2 = m_{ice} L_{ice} + m_{ice} S_w \Delta t$

$= 6 \times 80 + 6 \times 1 \times 100 = 1080cal$ ......(2)

From eq (1) and (2) $Q_1 > Q_2$

i.e , the total steam should not condensed in to water.

Let 'm' gm of steam is condensed into water by giving 1080cal. of heat .

$$mL_s = 1080 \,; m = \frac{1080}{540} = 2gm$$

$\therefore$ mass of the steam left uncondensed $= 6 - 2 = 4g$

16. 2kg of water contained in a vessel of negligible heat capacity is heated with a coil of 1kw. The loss of energy form the vessel is at the rate of 160J/s. How much time temperature will raise from

1) 8 min 20 sec 2) 6 min 2 sec 3) 7 min 4) 14 min

SOLUTION:

$$Net = \frac{Q}{t} = 100 - 160$$

$$\frac{ms\Delta T}{t} = 840$$

17. 30 gms of water at 30ºC is in a beaker. Which of the following, when added to water, will have greatest cooling effect? (Specific heat of copper = 0.1 cal/gmºC)

1) 100gm of water at10ºC 2) 15gm of water at 0ºC]

3) 3gm of ice at 0ºC 4) 18gm of copper at 0ºC

SOLUTION:

From principle of calorimetry

(i) $mS_w(30-\theta_1)=m_1S_w(\theta_1-10)$

(ii) $mS_w(30-\theta_2)=m_2S_w(\theta_2-0)$

(iii) $mS_w(30-\theta_3)=m_3L_{ice}+m_3S_w(\theta_3-0)$

(iv) $mS_w(30-\theta_4)=m_4S_{cu}(\theta_4-0)$

here $m_1=100g, m_2=15g, m_3=3g, m_4=18g$

above calculations will be show that θ_1 is least.

18: A piece of ice(heat capacity =2100J/Kg ^{0}C and latent heat $=3.36\times10^5 J/Kg$) of mass m grams is at $-5^o C$ at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1gm of ice has melted. Assuming there is no other heat exchange in the process. Find the value of m. (JEE-2010)

SOLUTION:

Here, heat given is used to increase the temperature of the ice to $0^o C$ and to melt 1gm of ice.
Given m is mass of ice in gm.

$\therefore 420=(m\times2100\times5+1\times3.36\times10^5)\times10^{-3}$

$\Rightarrow m=8\,gm$.

19. In an industrial process 10 kg of water per hour is to be heated from 20^{0}C to 80^{0}C. To do this steam at 150^{0}C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90^{0}C . How many kilograms of steam is required per hour (specific heat of steam = 1 cal/gm , latent heat of vapourisation = 540 cal/gm)?

1) 1gm 2) 1 kg 3) 10 gm 4) 10 kg

SOLUTION:

Let m kg of steam is required for this process

$$m_w\times S_w\times(80-20)$$

$$=m\times S_{steam}\times(150-100)+mL_{steam}+mS_w(100-\theta)$$

20.When a small ice crystal is placed into super cooled water, it begins to freeze instantaneously. What amount of ice is formed from 1kg of water super cooled to $-8^o C$.

SOLUTION: $mL=m^1S\Delta\theta$; $m\times80=1000\times1\times8$; m = 100g

21 : The specific heat of a substance varies as $(3\theta^2+\theta)\times10^{-3}\,cal/g-^oC$. What is the amount of heat required to raise the temperature of 1kg of substance from 10^{o}C to 20^{o}C?

SOLUTION:

For small change in temperature dθ, heat required, $dQ = mSd\theta$.

$$\therefore Q = \int_{\theta_1}^{\theta_2} mSd\theta$$

$$\therefore Q = \int_{10}^{20} 1000\left(3\theta^2 + \theta\right)\times 10^{-3} d\theta = \left|\theta^3 + \frac{\theta^2}{2}\right|_{10}^{20}$$

$$= \left(20^3 + \frac{20^2}{2}\right) - \left(10^3 + \frac{10^2}{2}\right) = 8200 - 1050 = 7150 cal$$

22: The melting point of ice is 0°C at 1 atm. At what pressure will it be -1°C?

(Given, $V_2 - V_1 = \left(1 - \frac{1}{0.9}\right)\times 10^{-3} m^3$ **)**

SOLUTION:

Here $\Delta T = (-1-0) = -1, T = 273 + 0 = 273K$ and $V_2 - V_1 = \left(1 - \frac{1}{0.9}\right)\times 10^{-3} m^3$ (given)

$L = 80 cal / g$

we have, $\frac{\Delta P}{\Delta T} = \frac{L}{T\left(V_2 - V_1\right)}$

$$\frac{\Delta P}{(-1)} = \frac{80\times 4.2\times 10^3}{273\left(1 - \frac{1}{0.9}\right)\times 10^{-3}}$$

$\therefore \Delta P = 110.8\times 10^5 N / m^2 = 110.8 atm$

$P_2 - P_1 = 110.8 atm \Rightarrow P_2 = 110.8 + P_1 = 111.8$ atm

23. 30g of ice 0°c and 20 g of steam at 100°c are mixed. The composition of the resultant mixture is

1) 40g of water and 10g steam at 100°c
2) 10g of ice and 40g of water at 0°c
3) 50g of water at 100°c
4) 35g of water and 15g of steam at 100°c

SOLUTION:

$$m_s L_s = (m_{ice} L_{ice} + m_{ice} s\Delta\theta)$$

Where m_s=mass of steam condensed to rise temperature of ice to 100°c water.

24. Ice at 0°C is added to 200gm of water initially at 70°C in a vacuum flask. When 50gm of ice has been added and has all melted, the temperature of flask and contents is 40°C. When a further 80gm of ice is added and has all melted, the temperature of whole become 10°C. Neglecting heat lost to surroundings the latent heat of fusion of ice is

1) 80 cal/gm 2) 90 cal/gm 3) 70 cal/gm 4) 540 cal/gm

SOLUTION:

According to principle of calorimetry

$ML_f + MS\Delta\theta = (mS\Delta\theta)_{water} + (mS\Delta\theta)_{flask}$

$5L_f = 400 + 3w....(i),$ *here* $w = (mS)_{flask}$

Now the system contains (200+50) gm of water at 40°C,

so when further 80gm of ice is added

$8L_f = 670 + 3w.....(ii)$

from (i) & (ii) we get L_f.

25. 'n' number of liquids of masses m,2m,3m,4m, ... having specific heats S, 2S,3S, 4S, ... are at temperatures t, 2t, 3t, 4t . . . . are mixed. The resultant temperature of mixture is

1) $\frac{3n}{2n+1}t$ 2) $\frac{2n(n+1)}{3(2n+1)}t$ 3) $\frac{3n(n+1)}{2(2n+1)}t$ 4) $\frac{3n(n+1)}{(2n+1)}t$

SOLUTION:

From principle of calorimetry

$$\theta = \frac{m_1S_1\theta_1 + m_2S_2\theta_2 + ...}{m_1S_1 + m_2S_2 + ...}$$

$$\theta = \frac{(1^3 + 2^3 + 3^3 + ... + n^3)t}{(1^2 + 2^2 + 3^2 + ... + n^2)}$$

$$= \frac{3n(n+1)}{2(2n+1)}t$$

26. 2 kg of ice at -20⁰C is mixed with 5 kg of water at 20⁰C in an insulating vessel having a negligible heat capacity. The final mass of water in the vessel. (The specific heat of water and ice are 1k cal/ kg⁰C and 0.5 k cal/kg/⁰C respectively and the latent heat of fusion of ice is 80 k cal/kg) is

1) 7 kg 2) 6 kg 3) 4 kg 4) 2 kg

SOLUTION:

Let m be mass of ice melted into water

$$m_{ice} \times S_{ice} \times 20 + m \times L_{ice} = m_{water} \times S_w \times 20$$

final mass of water in vessel = m + 5kg.

27. The specific heat of a substance varies with temperature as s=0.20+0.14 θ+0.023 θ^2 (cal/gm°C) .Heat required to raise the temperature of 2 gm of the substance from 5^0C to 15^0C is (θ is in ^oC)

1) 24 cal 2) 56 cal 3) 82 cal 4) 100 cal

SOLUTION:

$$Q=\int_{\theta_1}^{\theta_2} m\times S\times d\theta$$

$$Q=2\int_{5}^{15}\left(0.2+0.14\theta+0.023\theta^2\right)d\theta$$

28. A heater melts 0°C ice in a bucket completely into water in 6 minutes and then evaporates all that water into steam in 47 minutes 30 sec. If latent heat of fusion of ice is 80 cal/gram, latent heat of steam will be (specific heat of water is 1 cal /gam-°C)

1) 536 Cal/gram 2) 533.3 Cal/gram 3) 540 Cal/gram 4) 2.268×10^6 J/Kg

SOLUTION:

Let m be the mass of ice in bucket

Heat given out by heater in 6min is 80m

Heat given out in 47.5min is $100m+mL_v$

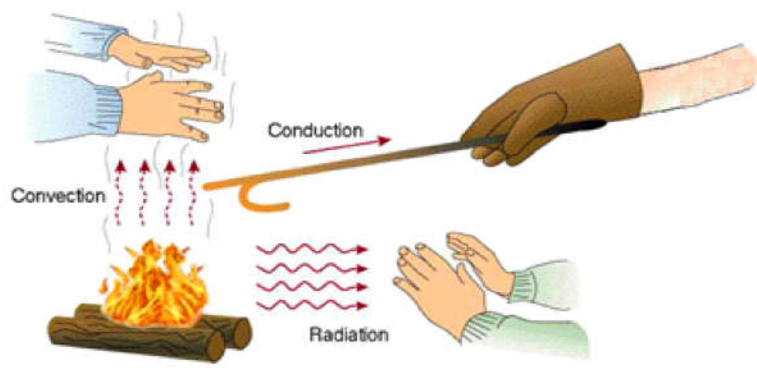
$m\times 80 \longrightarrow 6$ minutes

$(m\times1\times100)+(m\times L_v)\rightarrow 47.5$ minutes

$\therefore\ 80\times47.5=6(100+L_v)$

Transmission of Heat

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by one of the following modes.
Conduction



Conduction :

The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called conduction.

(1) Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.

(2) Medium is necessary for conduction

(3) It is a slow process

(4) The temperature of the medium increases through which heat flows

(5) Conduction is a process which is possible in all states of matter.

(6) When liquid and gases are heated from the top, they conduct heat from top to bottom.

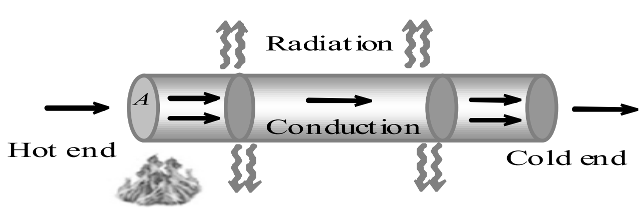
(7) In solids only conduction takes place

(8) In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules, therefore they are poor conductors.

(9) In metallic solids free electrons carry the heat energy, therefore they are good conductor of heat.

Conduction in Metallic Rod :

When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end.



,

Temperature gradient (T.G.) :

The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

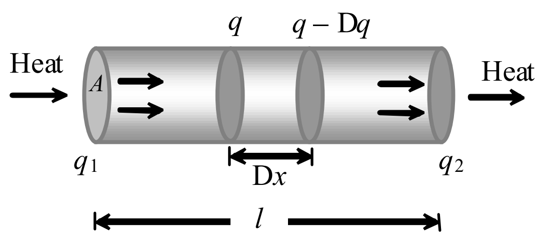
Hence

(i) Temperature gradient = $\frac{-\Delta\theta}{\Delta x}$

(ii) The negative sign show that temperature θ decreases as the distance x increases in the direction of heat flow.

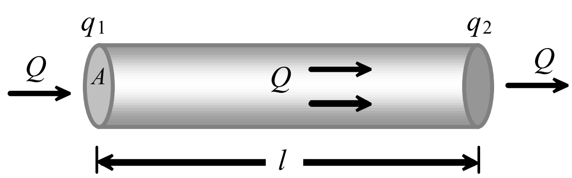
(iii) For uniform temperature fall $\frac{\theta_1 - \theta_2}{l} = \frac{\Delta\theta}{\Delta x}$

(iv) Unit : *K*/*m* or °*C/m* (S.I.) and Dimensions $[L^{-1}\theta]$



Law of thermal conductivity :

Consider a rod of length *l* and area of cross-section *A* whose faces are maintained at temperature q_1 and q_2 respectively. The curved surface of rod is kept insulated from surrounding to avoid leakage of heat



(i) In steady state the amount of heat flowing from one face to the other face in time *t* is given by

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

where *K* is coefficient of thermal conductivity of material of rod.

(ii) Rate of flow of heat *i.e.* heat current $\frac{Q}{t} = H = \frac{KA(\theta_1 - \theta_2)}{l}$

(iii) In case of non-steady state or variable cross-section, a more general equation can be used to solve problems.

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$$

More about *K* :

It is the measure of the ability of a substance to conduct heat through it.

(i) Units : *Cal*/*cm*-*sec* °*C* (in C.G.S.), *kcal*/*m*-*sec*-*K* (in M.K.S.) and *W*/*m*- *K* (in S.I.).

Dimension : $[MLT^{-3}\theta^{-1}]$

(ii) The magnitude of *K* depends only on nature of the material.

(iii) Substances in which heat flows quickly and easily are known as good conductor of heat. They possesses large thermal conductivity due to large number of free electrons

e.g. Silver, brass *etc.* For perfect conductors, $K = \infty$.

(iv) Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons *e.g.* Glass, wood *etc.* and for perfect insulators, $K = 0$.

(v) The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

(vi) Human body is a bad conductor of heat (but it is a good conductor of electricity).

Decreasing order of conductivity :

For some special cases it is as follows

(a) $K_{Ag} > K_{Cu} > K_{Al}$

(b) $K_{Solid} > K_{Liquid} > K_{Gas}$

(c) $K_{Metals} > K_{Non-metals}$

Relation between temperature gradient and thermal conductivity :

In steady state, rate of flow of heat $\frac{dQ}{dt} = -KA\frac{d\theta}{dx} = -KA\,'$ (T.G.)

(T.G.) $\propto \frac{1}{K}$ ($\frac{dQ}{dt}$ = constant)

Temperature difference between the hot end and the cold end in steady state is inversely proportional to K,

i.e. in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where K = ¥, temperature difference in steady state will be zero.

Thermal resistance (*R*) :

The thermal resistance of a body is a measure of its opposition to the flow of heat through it. It is defined as the ratio of temperature difference to the heat current (= Rate of flow of heat)

(i) Hence $R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$

(ii) **Unit :** $^oC \times sec / cal$ or $K \times sec / kcal$ and

Dimension : $[M^{-1}L^{-2}T^3\theta]$

Wiedmann-Franz law :

At a given temperature T, the ratio of thermal conductivity to electrical conductivity is constant

i.e., $(K / \sigma T)$ = constant,

i.e., a substance which is a good conductor of heat (*e.g.,* silver) is also a good conductor of electricity. Mica is an exception to above law.

Thermometric conductivity or diffusivity :

It is a measure of rate of change of temperature (with time) when the body is not in steady state (*i.e.,* in variable state)

It is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material. Thermal capacity per unit volume = $\frac{mc}{V} = \rho c$

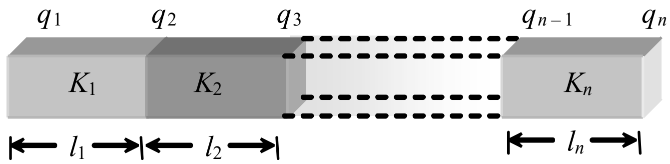
(ρ = density of substance) Þ Diffusivity (D) = $\frac{K}{\rho c}$

Unit : m^2/sec

Combination of Metallic Rods

Series combination :

Let n slabs each of cross-sectional area A, lengths $l_1, l_2, l_3, ... l_n$ and conductivities $K_1, K_2, K_3 ...... K_n$ respectively be connected in the series



Heat current :

Heat current is the same in all the conductors.

i.e., $\frac{Q}{t} = H_1 = H_2 = H_3 .... = H_n$

$$\frac{K_1 A(\theta_1 - \theta_2)}{l_1} = \frac{K_2 A(\theta_2 - \theta_3)}{l_2} = \frac{K_n A(\theta_{n-1} - \theta_n)}{l_n}$$

Equivalent thermal resistance :

$$R = R_1 + R_2 + ... R_n$$

Equivalent thermal conductivity :

It can be calculated as follows

From $R_S = R_1 + R_2 + R_3 + ...$

$$\frac{l_1 + l_2 + ... l_n}{K_s} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + ... + \frac{l_n}{K_n A}$$

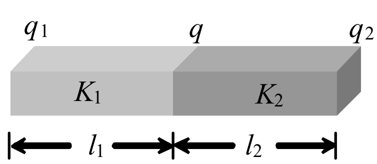
$$K_s = \frac{l_1 + l_2 + ... l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + ... \frac{l_n}{K_n}}$$

(a) For n slabs of equal length $K_s = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + ... \frac{1}{K_n}}$

(b) For two slabs of equal length, $K_s = \frac{2K_1 K_2}{K_1 + K_2}$

Temperature of interface of composite bar :

Let the two bars are arranged in series as shown in the figure.



Then heat current is same in the two conductors.

i.e., $\frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$

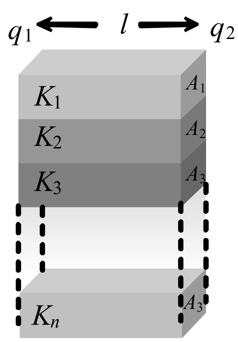
By solving we get $\theta = \dfrac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$

(a) If $l_1 = l_2$ then $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

(b) If $K_1 = K_2$ and $l_1 = l_2$ then $\theta = \dfrac{\theta_1 + \theta_2}{2}$

'Parallel Combination :

Let n slabs each of length l, areas $A_1, A_2, A_3, ... A_n$ and thermal conductivities $K_1, K_2, K_3, ... K_n$ are connected in parallel then



Equivalent resistance :

$$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \frac{1}{R_n}$$

For two slabs $R_s = \dfrac{R_1 R_2}{R_1 + R_2}$

Temperature gradient :

Same across each slab.

Heat current :

in each slab will be different. Net heat current will be the sum of heat currents through individual slabs.

i.e., $H = H_1 + H_2 + H_3 + ...H_n$

$$\frac{K(A_1 + A_2 + ... + A_n)(\theta_1 - \theta_2)}{l} = \frac{K_1 A_1(\theta_1 - \theta_2)}{l} + \frac{K_2 A_2(\theta_1 - \theta_2)}{l} + ... + \frac{K_n A_n(\theta_1 - \theta_2)}{l}$$

$$K = \frac{K_1A_1 + K_2A_2 + K_3A_3 + ... K_nA_n}{A_1 + A_2 + A_3 + ... A_n}$$

(a) For n slabs of equal area $K = \dfrac{K_1 + K_2 + K_3 + ... K_n}{n}$

(b) For two slabs of equal area $K = \dfrac{K_1 + K_2}{2}$.

Growth of Ice on Lake

(1) Water in a lake starts freezing if the atmospheric temperature drops below $0^o C$. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature is $-\theta^o C$.

(2) The temperature of water in contact with lower surface of ice will be zero.

(3) If A is the area of lake, heat escaping through ice in time dt is $dQ_1 = \frac{KA[0-(-\theta)]\,dt}{y}$

(4) Suppose the thickness of ice layer increases by dy in time dt, due to escaping of above heat.

Then $dQ_2 = mL = \rho(dy\ A)L$

(5) As $dQ_1 = dQ_2$,

hence, rate of growth of ice will be

$(dy/dt) = (K\theta/\rho L y)$

So, the time taken by ice to grow to a thickness y is

$$t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\rho L}{2K\theta}y^2$$

(6) If the thickness is increased from y_1 to y_2 then time taken

$$t = \frac{\rho L}{K\theta}\int_{y_1}^{y_2} y dy = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$$

(7) Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

(8) Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.

(9) It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2,\ i.e.,\ t_1 : t_2 : t_3 :: 1 : 4 : 9$$

(10) The time intervals to change the thickness from 0 to y, from y to $2y$ and so on will be in the ratio

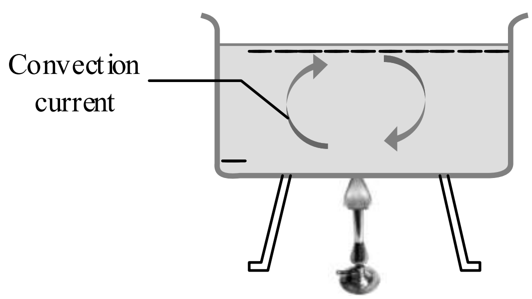
$$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2) : (2^2 - 1^2) : (3^2 : 2^2)$$

$$\Rightarrow\ \Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$$

Convection :

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.

(1) **Natural convection :**



This arise due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try setting down. It mostly occurs on heating a liquid/fluid.

Forced convection :

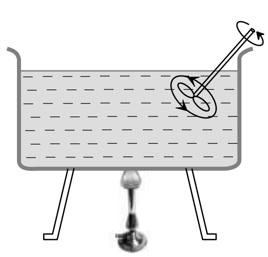


Fig. 15.17

If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection. In this case Newton's law of cooling holds good. According to which rate of loss of heat from a hot body due to moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings

ie $\frac{Q}{t} \propto A(T - T_0)$

$$\frac{Q}{t} = h\,A(T - T_0)$$

where h = Constant of proportionality called convection coefficient, T = Temperature of body and T_0 = Temperature of surrounding

Convection coefficient (h) depends on properties of fluid such as density, viscosity, specific heat and thermal conductivity.

- Natural convection takes place from bottom to top while forced convection in any direction.
- In case of natural convection, convection currents move warm air upwards and cool air downwards. That is why heating is done from base, while cooling from the top.
- Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.
- Natural convection is not possible in a gravity free region such as a free falling lift or an orbiting satellite.
- The force of blood in our body by heart helps in keeping the temperature of body constant.
- If liquids and gases are heated from the top (so that convection is not possible) they transfer heat (from top to bottom) by conduction.
- Mercury though a liquid is heated by conduction and not by convection.

Radiation

- The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.
- Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum *e.g.* the heat from the sun reaches the earth through radiation.
- The wavelength of thermal radiations ranges from $7.8 \times 10^{-7}\,m$ to $4 \times 10^{-4}\,m$. They belong to *infra-red* region of the electromagnetic spectrum. That is why thermal radiations are also called *infra-red* radiations.
- Medium is not required for the propagation of these radiations.
- They produce sensation of warmth in us but we can't see them.
- Every body whose temperature is above zero Kelvin emits thermal radiation.
- Their speed is equal to that of light *i.e.* $(= 3 \times 10^8\,m/s)$.
- Their intensity is inversely proportional to the square of distance of point of observation from the source (*i.e.* $I \propto 1/d^2$).
- Just as light waves, they follow laws of reflection, refraction, interference, diffraction and polarisation.

- When these radiations fall on a surface then exert pressure on that surface which is known as radiation pressure.
- While travelling these radiations travel just like photons of other electromagnetic waves. They manifest themselves as heat only when they are absorbed by a substance.
- Spectrum of these radiations can not be obtained with the help of glass prism because it absorbs heat radiations. It is obtained by quartz or rock salt prism because these materials do not have free electrons and interatomic vibrational frequency is greater than the radiation frequency, hence they do not absorb heat radiations.

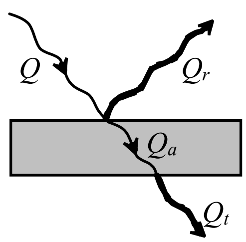
Colour of Heated Object

When a body is heated, all radiations having wavelengths from zero to infinity are emitted.

- Radiations of longer wavelengths are predominant at lower temperature.
- The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing.
- A blue flame is at a higher temperature than a yellow flame

Interaction of Radiation with Matter :

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.



(1) $Q = Q_a + Q_r + Q_t$

(2) $\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = a + r + t = 1$

(3) $a = \frac{Q_a}{Q}$ = Absorptance or absorbing power

$r = \frac{Q_r}{Q}$ = Reflectance or reflecting power

$t = \frac{Q_t}{Q}$ = Transmittance or transmitting power

(4) r, a and t all are the pure ratios so they have no unit and dimension.

(5) **Different bodies**

(i) If $a = t = 0$ and $r = 1$ ® body is perfect reflector

(ii) If $r = t = 0$ and $a = 1$ ® body is perfectly black body

(iii) If, $a = r = 0$ and $t = 1$ ® body is perfect transmitter

(iv) If $t = 0$ Þ $r + a = 1$ or $a = 1 - r$ *i.e.* good reflectors are bad absorbers.

Emissive Power, Absorptive Power and Emissivity

If temperature of a body is more than it's surrounding then body emits thermal radiation

Monochromatic Emittance or Spectral emissive power (e_l) :

For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface

with in a unit wavelength around l

i.e. lying between $\left(\lambda - \frac{1}{2}\right)$ to $\left(\lambda + \frac{1}{2}\right)$.

Spectral emissive power $(e_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$

Unit : $\frac{Joule}{m^2 \times sec \times \AA}$

Dimension : $[ML^{-1}T^{-3}]$

Total emittance or total emissive power (e) :

It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$e = \int_0^\infty e_\lambda d\lambda$$

Unit : $\frac{Joule}{m^2 \times \text{sec}}$ or $\frac{Watt}{m^2}$

Dimension : $[MT^{-3}]$

Monochromatic absorptance or spectral absorptive power (a_l) :

It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unit less quantity. It is represented by a_l.

Total absorptance or total absorpting power (a):

It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

Emissivity (*e*) :

Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at that temperature

i.e. $\varepsilon = \frac{e}{E}$ (*e* ® read as epsilon)

- ♦ For perfectly black body $e = 1$
- ♦ For highly polished body $e = 0$
- ♦ But for practical bodies emissivity (e) lies between zero and one ($0 < e < 1$).

Perfectly Black Body

(1) A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it.

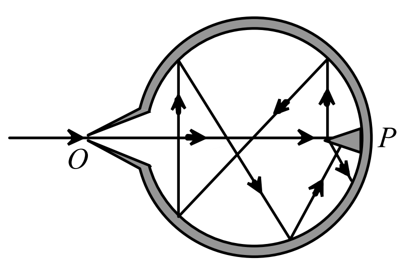
(2) As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity *i.e.* $t = 0$ and $r = 0$ Þ $a = 1$.

(3) We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.

(4) When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.) it emits all possible radiation so it is an example of black body.

Ferry's black body :

A perfectly black body can't be realised in practice. The nearest example of an ideal black body is the Ferry's black body. It is a doubled walled evacuated spherical cavity whose inner wall is blackened. The space between the wall is evacuated to prevent the loss of heat by conduction and radiation. There is a fine hole in it. All the radiations incident upon this hole are absorbed by this black body. If this black body is heated to high temperature then it emits radiations of all wavelengths. It is the hole which is to be regarded as a black body and not the total enclosure



A perfectly black body can not be realised in practice but materials like Platinum black or Lamp black come close to being ideal black bodies. Such materials absorbs 96% to 85% of the incident radiations.

Prevost Theory of Heat Exchange :

(1) Every body emits heat radiations at all finite temperature (Except 0 K) as well as it absorbs radiations from the surroundings.

(2) Exchange of energy along various bodies takes place via radiation.

(3) The process of heat exchange among various bodies is a continuous phenomenon.

(4) At absolute zero temperature (0 K or $-273°C$) this law is not applicable because at this temperature the heat exchange among various bodies ceases.

- If $Q_{emission}$ - > $Q_{absorbed}$ ® temperature of body decreases and consequently the body appears colder.
- If $Q_{emission}$ - < $Q_{absorbed}$ ® temperature of body increases and it appears hotter.
- If $Q_{emission}$ - = $Q_{absorbed}$ ® temperature of body remains constant (thermal equilibrium)

Kirchoff's Law:

According to this law the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Hence $\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots \left(\frac{E}{A}\right)_{\text{Perfectly black bod y}}$

But for perfectly black body $A = 1$

$i.e.\ \frac{e}{a} = E$

If emissive and absorptive powers are considered for a particular wavelength l,

$\left(\frac{e_\lambda}{a_\lambda}\right) = (E_\lambda)_{\text{black}}$

Now since $(E_l)_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

Stefan's Law:

According to it the radiant energy emitted by a perfectly black body per unit area per sec (*i.e.* emissive power of black body) is directly proportional to the fourth power of its absolute temperature,

$i.e\ \ \cdot\ E \propto T^4$

$$E = sT^4$$

where s is a constant called Stefan's constant having dimension $[MT^{-3}\theta^{-4}]$

value $5.67 \times 10^{-8}\ W/m^2K^4$.

(i) For ordinary body : $e = eE = \varepsilon\sigma T^4$

(ii) Radiant energy : If Q is the total energy radiated by the ordinary body then

$$e = \frac{Q}{A \times t} = \varepsilon \sigma T^4$$

$$Q = A\, \varepsilon \sigma T^4 t$$

(iii) Radiant power (P) : It is defined as energy radiated per unit area

i.e. $P = \frac{Q}{t} = A \varepsilon \sigma T^4$.

(iv) If an ordinary body at temperature T is surrounded by a body at temperature T_0, then Stefan's law may be put as

$$e = \varepsilon\, \sigma\, (T^4 - T_0^4)$$

Rate of Loss of Heat (R_H) and Rate of Cooling (R_C):

Rate of loss of heat (or initial rate of loss of heat) :

If an ordinary body at temperature T is placed in an environment of temperature T_0 ($T_0 < T$) then

heat loss by radiation is given by $\Delta Q = Q_{\text{emission}} - Q_{\text{absorption}} = A \varepsilon\, \sigma (T^4 - T_0^4)$

$$\text{Rate of loss of heat } (R_H) = \frac{dQ}{dt} = A \varepsilon\, \sigma (T^4 - T_0^4)$$

If two bodies are made of same material, have same surface finish and are at the same initial temperature then

$$\frac{dQ}{dt} \propto A \Rightarrow \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{A_1}{A_2}$$

Initial rate of fall in temperature (Rate of cooling) :

If m is the body and c is the specific heat then

$$\frac{dQ}{dt} = mc.\frac{dT}{dt} = mc\frac{d\theta}{dt} \qquad (\because Q = mc\,\Delta T \text{ and } dT = d\theta)$$

$$\text{Rate of cooling } (R_c) = \frac{d\theta}{dt} = \frac{(dQ/dt)}{mc}$$

$$= \frac{A \varepsilon\, \sigma}{mc}(T^4 - T_0^4)$$

$$= \frac{A \varepsilon\, \sigma}{V \rho\, c}(T^4 - T_0^4)$$

where m = density (r) ′ volume (V)

for two bodies of the same material under identical environments,

the ratio of their rate of cooling is $\frac{(R_c)_1}{(R_c)_2} = \frac{A_1}{A_2} \cdot \frac{V_2}{V_1}$

Dependence of rate of cooling :

When a body cools by radiation the rate of cooling depends on

- Nature of radiating surface *i.e.* greater the emissivity, faster will be the cooling.
- Area of radiating surface, *i.e.* greater the area of radiating surface, faster will be the cooling.
- Mass of radiating body *i.e.* greater the mass of radiating body slower will be the cooling.
- Specific heat of radiating body *i.e.* greater the specific heat of radiating body slower will be cooling.
- Temperature of radiating body *i.e.* greater the temperature of body faster will be cooling.
- Temperature of surrounding *i.e.* greater the temperature of surrounding slower will be cooling.

Newton's Law of Cooling:

When the temperature difference between the body and its surrounding is not very large

i.e. $T - T_0 = \mathrm{D}T$

then $T^4 - T_0^4$ may be approximated as $4T_0^3 \Delta T$

By Stefan's law, $\frac{dT}{dt} = \frac{A\varepsilon\sigma}{mc}[T^4 - T_0^4]$

Hence $\frac{dT}{dt} = \frac{A\varepsilon\sigma}{mc} 4T_0^3 \Delta T$

$$\frac{dT}{dt} \propto \Delta T$$

$$\frac{d\theta}{dt} \propto \theta - \theta_0$$

i.e., if the temperature of body is not very different from surrounding,

rate of cooling is proportional to temperature difference between the body and its surrounding.

This law is called Newton's law of cooling.

- Greater the temperature difference between body and its surrounding greater will be the rate of cooling.
- If $\theta = \theta_0$, $\frac{d\theta}{dt} = 0$ *i.e.* a body can never be cooled to a temperature lesser than its surrounding by radiation.
- If a body cools by radiation from $\theta_1^o C$ to $\theta_2^o C$ in time *t*,

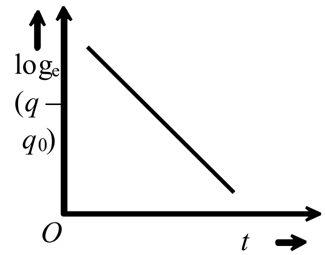
  then $\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$ and $\theta = \theta_{av} = \frac{\theta_1 + \theta_2}{2}$.

  The Newton's law of cooling becomes $\left[\frac{\theta_1 - \theta_2}{t}\right] = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$.

  This form of law helps in solving numericals.
- Practical examples
  - Hot water loses heat in smaller duration as compared to moderate warm water.
  - Adding milk in hot tea reduces the rate of cooling.

Cooling Curves

Curve between $\log(\theta - \theta_0)$ and time :



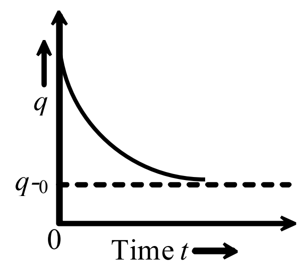
$$\text{As } \frac{d\theta}{dt} \propto -(\theta - \theta_0) \Rightarrow \frac{d\theta}{(\theta - \theta_0)} = -Kdt$$

$$\text{Integrating } \log_e(\theta - \theta_0) = -Kt + C$$

$$\log_e(\theta - \theta_0) = -Kt + \log_e A$$

This is a straight line with negative slope

Curve between temperature of body and time:



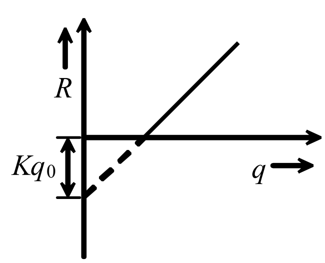
$$\text{As } \log_e(\theta - \theta_0) = -Kt + \log_e A$$

$$\Rightarrow \log_e \frac{\theta - \theta_0}{A} = -Kt$$

$$\Rightarrow \theta - \theta_0 = Ae^{-kt}$$

which indicates temperature decreases exponentially with increasing time.

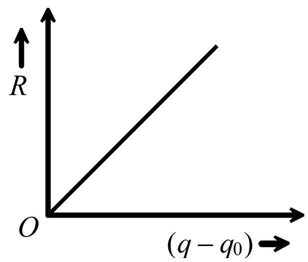
Curve between the rate of cooling:



(R) and body temperature (θ). $R = K(\theta - \theta_0) = K\theta - K\theta_0$

This is a straight line intercept R-axis at $-K\theta_0$

Curve between rate of cooling (R) :



R and temperature difference between body (θ) and surrounding (θ_0)

$R \propto (\theta - \theta_0)$.

This is a straight line passing through origin.'

Wien's Displacement Law:

According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant,

i.e. $\lambda_m T = b = \text{constant}$

where b is Wien's constant and has value $2.89 \times 10^{-3}\, m\text{-}K$.

As the temperature of the body increases, the wavelength at which the spectral intensity (E_λ) is maximum shifts towards left. Therefore it is also called Wien's displacement law.

This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m the temperature of the star $T (= b/\lambda_m)$ is determined.

Law of Distribution of Energy (Plank's Hypothesis):

(1) The theoretical explanation of black body radiation was done by Planck.

(2) According to Plank's atoms of the walls of a uniform temperature enclosure behave as oscillators, each with a characteristic frequency of oscillation.

(3) These oscillations emits electromagnetic radiations in the form of photons (The radiation coming out from a small hole in the enclosure are called black body radiation). The energy of each photon is *hn.* Where *n* is the frequency of oscillator and *h* is the Plank's constant. Thus emitted energies may be *hn*, 2*hn*, 3*hn* ... *nhn* but not in between.

According to Planck's law $E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{[e^{hc/\lambda KT} - 1]} d\lambda$

where c = speed of light

k = Boltzmann's constant.

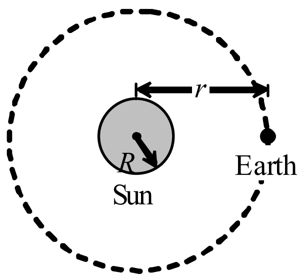
This equation is known as Plank's radiation law. It is correct and complete law of radiation

(4) This law is valid for radiations of all wavelengths ranging from zero to infinite.

(5) For radiations of short wavelength $\left(\lambda << \frac{hc}{KT}\right)$ Planck's law reduces to *Wien's energy distribution law* $E_\lambda d\lambda = \frac{A}{\lambda^5} e^{-B/\lambda T} d\lambda$

(6) For radiations of long wavelength $\left(\lambda >> \frac{hc}{KT}\right)$ Planck's law reduces to *Rayleigh-Jeans energy distribution law* $E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$

Temperature of the Sun and Solar Constant:



If R is the radius of the sun and T its temperature, then the energy emitted by the sun per sec through radiation in accordance with Stefan's law will be given by

$$P = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

In reaching earth this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$i.e.\ T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5\times10^8}{7\times10^5}\right)^2 \times \frac{1.4\times10^3}{5.67\times10^{-8}}\right]^{1/4} \simeq 5800\ K$$

As $r = 1.5\times10^8\ km$, $R = 7\times10^5\ km$,

$$S = 2\frac{cal}{cm^2 min} = 1.4\frac{kW}{m^2}$$

$$\sigma = 5.67\times10^{-8}\frac{W}{m^2K^4}$$

This result is in good agreement with the experimental value of temperature of sun, *i.e.*, 6000 K.

PROBLEMS

1 The heat is flowing through two cylindrical rods of same material. The diameters of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, the ratio of rate of flow of heat through them will be

(a) 1:1 (b) 2:1 (c) 1:4 (d) 1:8

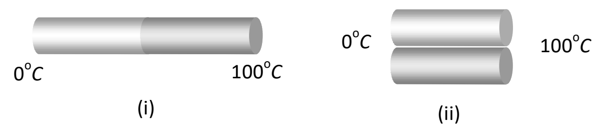
‘

$$\frac{Q}{t} = \frac{KA\,\Delta\theta}{l}$$

$$\frac{Q}{t} \propto \frac{A}{l} \propto \frac{d^2}{l} \quad (d = \text{Diameter of rod})$$

$$\frac{(Q/t)_1}{(Q/t)_2} = \left(\frac{d_1}{d_2}\right)^2 \times \frac{l_2}{l_1} = \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right) = \frac{1}{8}$$

2. Two identical square rods of metal are welded end to end as shown in figure (i), 20 calories of heat flows through it in 4 minutes. If the rods are welded as shown in figure (ii), the same amount of heat will flow through the rods in



(a) 1 minute (b) 2 minutes (c) 4 minutes (d) 16 minutes

SOLUTION:

$$\frac{Q}{t} = \frac{KA\,\Delta\theta}{l} = \frac{\Delta\theta}{(l/KA)} = \frac{\Delta\theta}{R} \quad (R = \text{Thermal resistance})$$

$$t \propto R \quad (\because Q \text{ and } \Delta\theta \text{ are same})$$

$$\frac{t_P}{t_S} = \frac{R_P}{R_S} = \frac{R/2}{2R} = \frac{1}{4}$$

$$\text{Series resistance } R_S = R_1 + R_2$$

$$\text{parallel resistance } R_P = \frac{R_1 R_2}{R_1 + R_2})$$

$$t_P = \frac{t_S}{4} = \frac{4}{4} = 1 \text{ min}.$$

3. Two rods *A* and *B* are of equal lengths. Their ends are kept between the same temperature and their area of cross-sections are A_1 and A_2 and thermal conductivities K_1 and K_2. The rate of heat transmission in the two rods will be equal, if

(a) $K_1 A_2 = K_2 A_1$ (b) $K_1 A_1 = K_2 A_2$ (c) $K_1 = K_2$ (d) $K_1 A_1^2 = K_2 A_2^2$

SOLUTION:

$$\left(\frac{Q}{t}\right)_1 = \frac{K_1 A_1 (\theta_1 - \theta_2)}{l}$$

$$\left(\frac{Q}{t}\right)_2 = \frac{K_2 A_2 (\theta_1 - \theta_2)}{l}$$

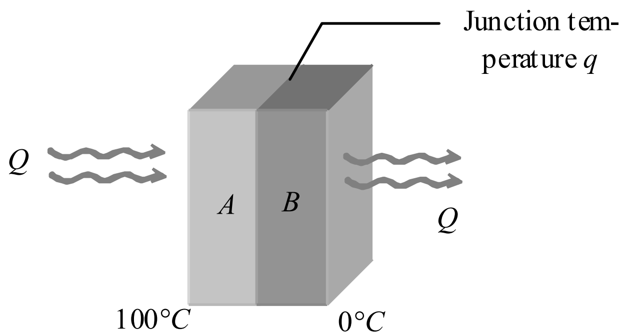
$$\text{given } \left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$K_1 A_1 = K_2 A_2$$

4. Two rectangular blocks A and B of different metals have same length and same area of cross-section. They are kept in such a way that their cross-sectional area touch each other. The temperature at one end of A is $100°C$ and that of B at the other end is $0°C$. If the ratio of their thermal conductivity is 1 : 3, then under steady state, the temperature of the junction in contact will be

(a) $25°C$ (b) $50°C$ (c) $75°C$ (d) $100°C$

SOLUTION:



It is given that $\frac{K_1}{K_2} = \frac{1}{3}$

$K_1 = K$ then $K_2 = 3K$

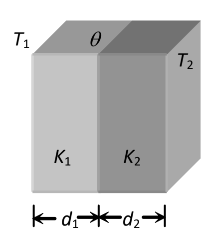
the temperature of the junction in contact $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

$$= \frac{1 \times 100 + 3 \times 0}{1 + 3} = \frac{100}{4} = 25°C$$

5. Two walls of thicknesses d_1 and d_2 and thermal conductivities k_1 and k_{-2} are in contact. In the steady state, if the temperatures at the outer surfaces are T_1 and T_2, the temperature at the common wall is

(a) $\frac{k_1T_1d_2 + k_2T_2d_1}{k_1d_2 + k_2d_1}$ (b) $\frac{k_1T_1 + k_2d_2}{d_1 + d_2}$ (c) $\left(\frac{k_1d_1 + k_2d_2}{T_1 + T_2}\right)T_1T_2$ (d) $\frac{k_1d_1T_1 + k_2d_2T_2}{k_1d_1 + k_2d_2}$

SOLUTION:



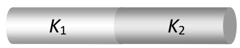
In series both walls have same rate of heat flow. Therefore

$$\frac{dQ}{dt}=\frac{K_1A(T_1-\theta)}{d_1}=\frac{K_2A(\theta-T_2)}{d_2},$$

$$\Rightarrow K_1d_2(T_1-\theta)=K_2d_1(\theta-T_2)$$

$$\Rightarrow \theta=\frac{K_1d_2T_1+K_2d_1T_2}{K_1d_2+K_2d_1}$$

6. If two metallic plates of equal thicknesses and thermal conductivities K_1 and K_2 are put together face to face and a common plate is constructed, then the equivalent thermal conductivity of this plate will be



(a) $\frac{K_1K_2}{K_1+K_2}$ (b) $\frac{2K_1K_2}{K_1+K_2}$ (c) $\frac{(K_1^2+K_2^2)^{3/2}}{K_1K_2}$ (d) $\frac{(K_1^2+K_2^2)^{3/2}}{2K_1K_2}$

SOLUTION:

In series $R_{eq}=R_1+R_2$

$$\Rightarrow \frac{2l}{K_{eq}A}=\frac{l}{K_1A}+\frac{l}{K_2A}$$

$$\frac{2}{K_{eq}}=\frac{1}{K_1}+\frac{1}{K_2}\Rightarrow \frac{2K_1K_2}{K_1+K_2}$$

7. Two spheres of different materials one with double the radius and one-fourth wall thickness of the other, are filled with ice. If the time taken for complete melting ice in the large radius one is 25 *minutes* and that for smaller one is 16 *minutes*, the ratio of thermal conductivities of the materials of larger sphere to the smaller sphere is

(a) 4 : 5 (b) 5 : 4 (c) 25 : 1 (d) 1 : 25

SOLUTION:

$$Q=\frac{KA(\Delta\theta)t}{l}$$

$\because$ Q and $\Delta\theta$ are same for both spheres hence

$$K\propto\frac{l}{At}\propto\frac{l}{r^2t}$$

$$\Rightarrow \frac{K_{\text{larger}}}{K_{\text{smaller}}}=\frac{l_l}{l_s}\times\left(\frac{r_s}{r_l}\right)^2\times\frac{t_s}{t_l}.$$

It is given that $r_l=2r_s$, $l_l=\frac{1}{4}l_s$ and $t_1=25\,min$, $t_s=16\,min$.

$$\Rightarrow \frac{K_{\text{larger}}}{K_{\text{smaller}}}=\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^2\times\frac{16}{25}=\frac{1}{25}$$

8. The ratio of the diameters of two metallic rods of the same material is 2 : 1 and their lengths are in the ratio 1 : 4. If the temperature difference between their ends are equal, the rate of flow of heat in them will be in the ratio

(a) 2 : 1 (b) 4 : 1 (c) 8 : 1 (d) 16 : 1

SOLUTION:

$$\frac{Q}{t} = \frac{KA(\Delta\theta)}{l} \Rightarrow \frac{Q}{t} \propto \frac{A}{l} \propto \frac{r^2}{l}$$

$$\Rightarrow \frac{(Q/t)_1}{(Q/t)_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{l_2}{l_1} = \left(\frac{2}{1}\right)^2 \times \left(\frac{4}{1}\right) = \frac{16}{1}$$

9. Two cylinders P and Q have the same length and diameter and are made of different materials having thermal conductivities in the ratio 2 : 3. These two cylinders are combined to make a cylinder. One end of P is kept at $100°C$ and another end of Q at $0°C$. The temperature at the interface of P and Q is

(a) $30°C$ (b) $40°C$ ‘ (c) $50°C$ (d) $60°C$

SOLUTION:

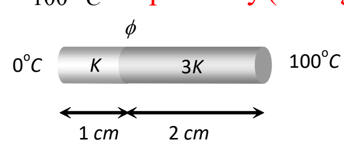
Temperature of interface $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

where $K_1 = 2K$ and $K_2 = 3K$ $\left(\because \frac{K_1}{K_2} = \frac{2}{3}\right)$

$$\theta = \frac{2K \times 100 + 3K \times 0}{2K + 3K}$$

$$= \frac{200K}{5K} = 40°C$$

10. Two bars of thermal conductivities K and $3K$ and lengths $1 cm$ and $2 cm$ respectively have equal cross-sectional area, they are joined lengths wise as shown in the figure. If the temperature at the ends of this composite bar is $0°C$ and $100°C$ respectively (see figure), then the temperature ϕ of the interface is



(a) $50°C$ (b) $\frac{100}{3}°C$ (c) $60°C$ (d) $\frac{200}{3}°C$

SOLUTION:

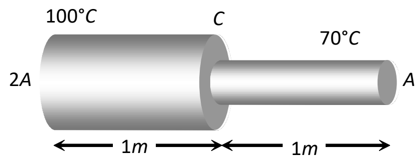
Temperature of interface

$$\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1}$$

$$= \frac{K \times 0 \times 2 + 3K \times 100 \times 1}{K \times 2 + 3K \times 1}$$

$$= \frac{300K}{5K} = 60°C$$

11. A metal rod of length $2m$ has cross sectional areas $2A$ and A as shown in figure. The ends are maintained at temperatures $100°C$ and $70°C$. The temperature at middle point C is



(a) $80°C$ (b) $85°C$ (c) $90°C$ (d) $95°C$

‘

Let θ be temperature middle point C and in series rate of heat flow is same

$$\Rightarrow K(2A)(100-\theta) = KA(\theta - 70)$$

$$\Rightarrow 200 - 2\theta = \theta - 70$$

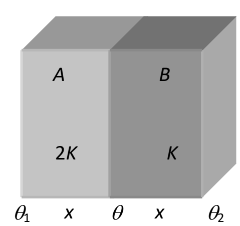
$$\Rightarrow 3\theta = 270$$

$$\Rightarrow \theta = 90°C$$

12. A wall is made up of two layers A and B. The thickness of the two layers is the same, but materials are different. The thermal conductivity of A is double than that of B. In thermal equilibrium the temperature difference between the two ends is $36^o C$. Then the difference of temperature at the two surfaces of A will be

(a) $6^o C$ (b) $12^o C$ (c) $18^o C$ (d) $24^o C$

SOLUTION:



Suppose thickness of each wall is x then $\left(\frac{Q}{t}\right)_{combination} = \left(\frac{Q}{t}\right)_A$

$$\Rightarrow \frac{K_S A(\theta_1 - \theta_2)}{2x} = \frac{2KA(\theta_1 - \theta)}{x}$$

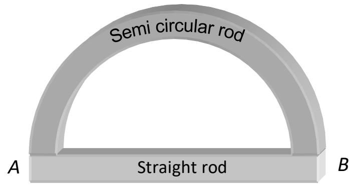
$$\because K_S = \frac{2 \times 2K \times K}{(2K+K)} = \frac{4}{3}K$$

$$(\theta_1 - \theta_2) = 36°$$

$$\Rightarrow \frac{\frac{4}{3}KA \times 36}{2x} = \frac{2KA(\theta_1 - \theta)}{x}$$

Hence temperature difference across wall A is $(\theta_1 - \theta) = 12^o C$

13. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in the figure. The points *A* and *B* are maintained at different temperature. The ratio of the heat transferred through a cross-section of a semi-circular rod to the heat transferred through a cross section of the straight rod in a given time is



(a) $2 : p$ (b) $1 : 2$ (c) $p : 2$ (d) $3 : 2$

'

$$\frac{dQ}{dt} = \frac{KA\Delta\theta}{l},$$

For both rods *K*, *A* and $\Delta\theta$ are same $\Rightarrow \frac{dQ}{dt} \propto \frac{1}{l}$

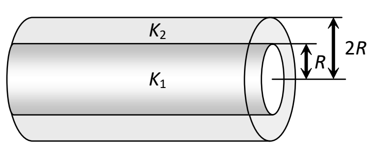
$$\frac{(dQ/dt)_{semi\ circular}}{(dQ/dt)_{straight}} = \frac{l_{straight}}{l_{semicircular}} = \frac{2r}{\pi r} = \frac{2}{\pi}.$$

14. A cylinder of radius *R* made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius *R* and outer radius 2*R* made of material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(a) $K_1 + K_2$ (b) $\frac{K_1K_2}{K_1 + K_2}$ (c) $\frac{K_1 + 3K_2}{4}$ (d) $\frac{3K_1 + K_2}{4}$

SOLUTION:

Both the cylinders are in parallel, for the heat flow from one end as shown.



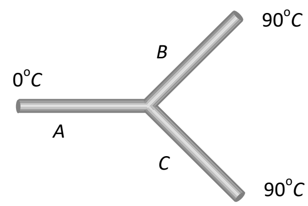
Hence $K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$;

where A_1 = Area of cross-section of inner cylinder = πR^2

A_2 = Area of cross-section of cylindrical shell $= \pi\{(2R)^2 - (R)^2\} = 3\pi R^2$

$$\Rightarrow K_{eq} = \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{\pi R^2 + 3\pi R^2} = \frac{K_1 + 3K_2}{4}$$

15. Three rods made of the same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at $0^o C$ and $90^o C$ respectively. The temperature of the junction of the three rods will be

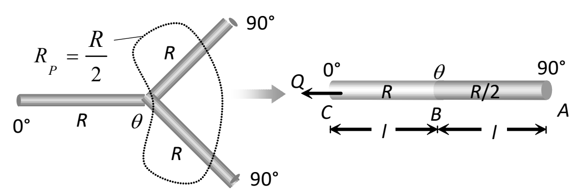


(a) $45^o C$ (b) $60^o C$ (c) $30^o C$ (d) $20^o C$

SOLUTION:

Let the temperature of junction be θ. Since roads B and C are parallel to each other (because both having the same temperature difference).

Hence given figure can be redrawn as follows



$$\because \frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R}$$

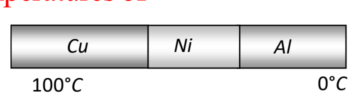
$$\left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{BC}$$

$$\Rightarrow \frac{(90-\theta)}{R/2} = \frac{(\theta - 0)}{R}$$

$$\Rightarrow 180 - 2\theta = \theta$$

$$\Rightarrow \theta = 60°C$$

16. A composite metal bar of uniform section is made up of length 25 *cm* of copper, 10 *cm* of nickel and 15 *cm* of aluminium. Each part being in perfect thermal contact with the adjoining part. The copper end of the composite rod is maintained at $100^o C$ and the aluminium end at $0^o C$. The whole rod is covered with belt so that there is no heat loss occurs at the sides. If $K_{Cu} = 2K_{Al}$ and $K_{Al} = 3K_{Ni}$, then what will be the temperatures of



(a) $23.33^o C$ and $78.8^o C$ (b) $83.33^o C$ and $20^o C$ (c) $50^o C$ and $30^o C$ (d) $30^o C$ and $50^o C$

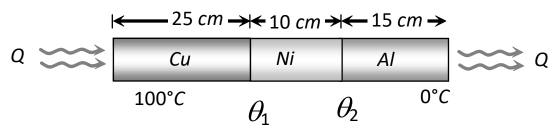
SOLUTION:

If suppose $K_{Ni} = K \Rightarrow K_{Al} = 3K$ and $K_{Cu} = 6K$.

Since all metal bars are connected in series

So $\left(\frac{Q}{t}\right)_{Combination} = \left(\frac{Q}{t}\right)_{Cu} = \left(\frac{Q}{t}\right)_{Al} = \left(\frac{Q}{t}\right)_{Ni}$

and $\frac{3}{K_{eq}} = \frac{1}{K_{Cu}} + \frac{1}{K_{Al}} + \frac{1}{K_{Ni}} = \frac{1}{6K} + \frac{1}{3K} + \frac{1}{K} = \frac{9}{6K}$



Hence, if $\left(\frac{Q}{t}\right)_{Combination} = \left(\frac{Q}{t}\right)_{Cu}$

$$\Rightarrow \frac{K_{eq}\,A(100-0)}{l_{Combination}} = \frac{K_{Cu}\,A(100-\theta_1)}{l_{Cu}}$$

$$\Rightarrow \frac{2K\,A\,(100-0)}{(25+10+15)} = \frac{6K\,A\,(100-\theta_1)}{25} \Rightarrow \theta_1 = 83.33\,°C$$

Similar if $\left(\frac{Q}{t}\right)_{Combination} = \left(\frac{Q}{t}\right)_{Al}$

$$\Rightarrow \frac{2K\,A(100-0)}{50} = \frac{3K\,A(\theta_2-0)}{15}$$

$$\Rightarrow \theta_2 = 20^{o}\,C$$

17. Three rods of identical area of cross-section and made from the same metal form the sides of an isosceles triangle ABC, right angled at B. The points A and are maintained at temperatures and respectively. In the steady state the temperature of the point C is . Assuming that only heat conduction takes place, is equal to

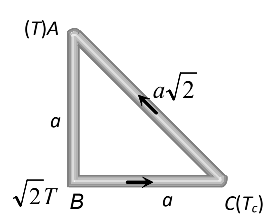
(a) $\frac{1}{(\sqrt{2}+1)}$ (b) $\frac{3}{(\sqrt{2}+1)}$ (c) $\frac{1}{2(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$

SOLUTION:

$\because\ T_B > T_A$

$\Rightarrow$ Heat will flow B to A via two paths (i) B to A (ii) and along BCA as shown.

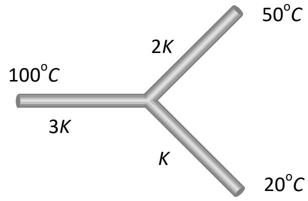
Rate of flow of heat in path BCA will be same



i.e. $\left(\frac{Q}{t}\right)_{BC} = \left(\frac{Q}{t}\right)_{CA} \Rightarrow \frac{k(\sqrt{2}T - T_C)A}{a} = \frac{k(T_C - T)A}{\sqrt{2}a}$

$$\Rightarrow \frac{T_C}{T} = \frac{3}{1+\sqrt{2}}$$

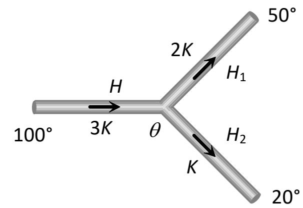
18. Three rods of the same dimension have thermal conductivities $3K$, $2K$ and K. They are arranged as shown in fig. Given below, with their ends at $100^\circ C$, $50^\circ C$ and $20^\circ C$. The temperature of their junction is



(a) $60^o\ C$ (b) $70^o\ C$ (c) $50^o\ C$ (d) $35^o\ C$

SOLUTION:

Let the temperature of junction be θ then according to following figure.



$$H = H_1 + H_2$$

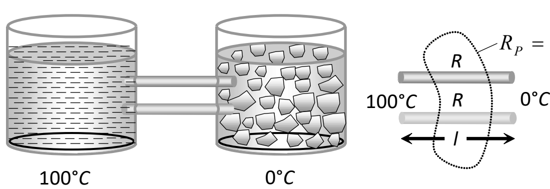
$$\Rightarrow \frac{3K \times A \times (100-\theta)}{l} = \frac{2KA(\theta-50)}{l} + \frac{KA(\theta-20)}{l}$$

$$\Rightarrow 300 - 3\theta = 3\theta - 120 \Rightarrow \theta = 70^\circ C$$

19. Two identical conducting rods are first connected independently to two vessels, one containing water at $100^o\ C$ and the other containing ice at $0^o C$. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g / s be the rate of melting of ice in two cases respectively. The ratio of q_1 / q_2 is

(a) $\frac{1}{2}$ ‘ (b) $\frac{2}{1}$ (c) $\frac{4}{1}$ (d) $\frac{1}{4}$

SOLUTION:



$R_P = \frac{R}{2}$

Initially the rods are placed in vessels as shown below

$$\frac{Q}{t} = \frac{(\theta_1-\theta_2)}{R} \Rightarrow \left(\frac{Q}{t}\right)_1 = \frac{mL}{t} = q_1 L = \frac{(100-0)}{\frac{R}{2}} \quad \text{... (i)}$$

Finally when rods are joined end to end as shown

$$\Rightarrow \left(\frac{Q}{t}\right)_2 = \frac{mL}{t} = q_2 L = \frac{(100-0)}{2R} \quad \text{... (ii)}$$

From equation (i) and (ii), $\frac{q_1}{q_2} = \frac{4}{1}$

20. A solid cube and a solid sphere of the same material have equal surface area. Both are at the same temperature $120^{o}C$, then

(a) Both the cube and the sphere cool down at the same rate

(b) The cube cools down faster than the sphere

(c) The sphere cools down faster than the cube

(d) Whichever is having more mass will cool down faster

SOLUTION:

$$\text{Rate of cooling of a body } R = \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$$

$$\Rightarrow R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{Volume}}$$

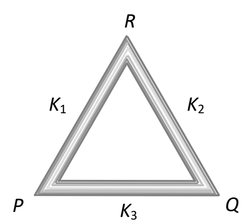
$$\Rightarrow \text{For the same surface area. } R \propto \frac{1}{\text{Volume}}$$

$$\because \text{Volume of cube} < \text{Volume of sphere}$$

$$\Rightarrow R_{Cube} > R_{Sphere}$$

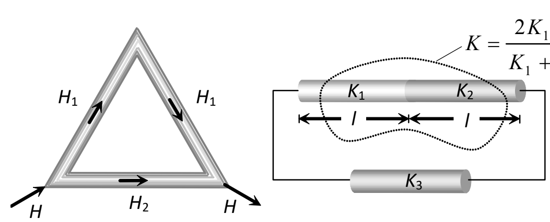
i.e. cube, cools down with faster rate.

21. Three rods of same dimensions are arranged as shown in figure they have thermal conductivities K_1, K_2 and K_3 The points P and Q are maintained at different temperatures for the heat to flow at the same rate along PRQ and PQ then which of the following option is correct



(a) $K_3 = \frac{1}{2}(K_1 + K_2)$ (b) $K_3 = K_1 + K_2$ (c) $K_3 = \frac{K_1 K_2}{K_1 + K_2}$ (d) $K_3 = 2(K_1 + K_2)$

SOLUTION:



The given arrangement of rods can be redrawn as follows

It is given that $H_1 = H_2$

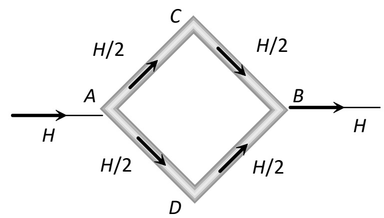
$$\Rightarrow \frac{KA(\theta_1 - \theta_2)}{2l} = \frac{K_3 A(\theta_1 - \theta_2)}{l}$$

$$\Rightarrow K_3 = \frac{K}{2} = \frac{K_1 K_2}{K_1 + K_2}$$

22. Four identical rods of same material are joined end to end to form a square. If the temperature difference between the ends of a diagonal is $100^o C$, then the temperature difference between the ends of other diagonal will be

(a) $0^o C$ (b) $\frac{100}{l}\,^o C$; where l is the length of each rod (c) $\frac{100}{2l}\,^o C$ (d) $100^o C$

SOLUTION:



Suppose temperature difference between A and B is $100°C$ and $\theta_A > \theta_B$

Heat current will flow from A to B via path ACB and ADB. Since all the rod are identical so $(\Delta\theta)_{AC} = (\Delta\theta)_{AD}$

(Because heat current $H = \frac{\Delta\theta}{R}$; here R = same for all.)

$$\Rightarrow \theta_A - \theta_C = \theta_A - \theta_D$$

$$\Rightarrow \theta_C = \theta_D$$

i.e. temperature difference between C and D will be zero.

23. A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of $0.1gm$ of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is that of first, the rate at which ice melts in will be ‘

(a) 3.2 (b) 1.6’ (c) 0.2 ‘ (d) 0.1

SOLUTION:

$$\frac{Q}{t} = \frac{KA\Delta\theta}{l} \Rightarrow \frac{mL}{t} = \frac{K(\pi r^2)\Delta\theta}{l}$$

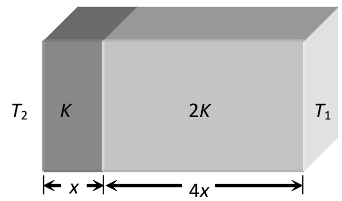
$\Rightarrow$ Rate of melting of ice $\left(\frac{m}{t}\right) \propto \frac{Kr^2}{l}$

Since for second rod K becomes $\frac{1}{4}th$ r becomes double and length becomes half, so rate of melting will be twice

i.e. $\left(\frac{m}{t}\right)_2 = 2\left(\frac{m}{t}\right)_1 = 2 \times 0.1 = 0.2\ gm/sec.$

24. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 ($T_2 >$

T_1). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with ¦ which equal to



(a) 1 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

SOLUTION:

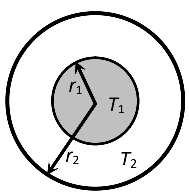
Equation of thermal conductivity of the given combination $K_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} = \frac{x + 4x}{\frac{x}{K} + \frac{4x}{2K}} = \frac{5}{3}K$.

Hence rate of flow of heat through the given combination is $\frac{Q}{t} = \frac{K_{eq}.A(T_2 - T_1)}{(x + 4x)} = \frac{\frac{5}{3}K\,A\,(T_2 - T_1)}{5x}$

$$= \frac{\frac{1}{3}K\,A\,(T_2 - T_1)}{x}$$

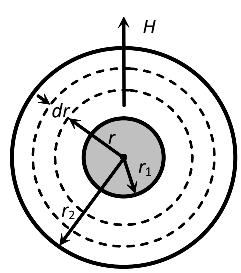
On comparing it with given equation we get $f = \frac{1}{3}$

25. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(a) $\frac{r_1 r_2}{(r_1 - r_2)}$ (b) $(r_2 - r_1)$' (c) $(r_2 - r_1)(r_1 r_2)$ (d) In $\left(\frac{r_2}{r_1}\right)$

SOLUTION:



Consider a concentric spherical shell of radius r and thickness dr as shown in fig.
The radial rate of flow of heat through this shell in steady state will be

$$H = \frac{dQ}{dt} = -KA\frac{dT}{dr} = -K(4\pi r^2)\frac{dT}{dr}$$

$$\Rightarrow \int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{T_1}^{T_1} dT$$

Which on integration and simplification gives

$$H = \frac{dQ}{dt} = \frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$

$$\Rightarrow \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

26. Ice starts forming in lake with water at $0^o C$ and when the atmospheric temperature is $-10^o C$. If the time taken for 1 *cm* of ice be 7 *hours*, then the time taken for the thickness of ice to change from 1 *cm* to 2 *cm* is

(a) 7 *hours* (b) 14 *hours* (c)Less than 7 *hours* (d)More than 7 *hours*

SOLUTION:

$$t = \frac{\rho L}{2K\theta}(x_2^2 - x_1^2)$$

$$\Rightarrow t \propto (x_2^2 - x_1^2)$$

$$\Rightarrow \frac{t}{t'} = \frac{(x_2^2 - x_1^2)}{(x'^2_2 - x'^2_1)} \Rightarrow \frac{9}{t'} = \frac{(1^2 - 0^2)}{(2^2 - 1^2)}$$

$$\Rightarrow t' = 21\ hours$$

27. There is formation of layer of snow $x\ cm$ thick on water, when the temperature of air is $-\theta^o C$ (less than freezing point). The thickness of layer increases from x to y in the time t, then the value of is given by

(a) $\frac{(x+y)(x-y)\rho L}{2k\theta}$ (b) $\frac{(x-y)\rho L}{2k\theta}$ (c) $\frac{(x+y)(x-y)\rho L}{k\theta}$ (d) $\frac{(x-y)\rho Lk}{2\theta}$

SOLUTION:

Since $t = \frac{\rho L}{2k\theta}(x_2^2 - x_1^2)$

$$\therefore t = \frac{\rho L}{2k\theta}(x^2 - y^2) = \frac{\rho L(x+y)(x-y)}{2K\theta}$$

28. A 5*cm* thick ice block is there on the surface of water in a lake. The temperature of air is –10°*C*; how much time it will take to double the thickness of the block ($L = 80\ cal/g$, $K_{icc} = 0.004\ Erg/s\text{-}k$, $d_{ice} = 0.92\ g\ cm^{-3}$)

(a) 1 *hour* (b) 191 *hours* (c) 19.1 *hours* (d) 1.91 *hours*

SOLUTION:

$$t = \frac{Ql}{KA(\theta_1 - \theta_2)} = \frac{mLl}{KA(\theta_1 - \theta_2)} = \frac{V\rho Ll}{KA(\theta_1 - \theta_2)}$$

$$= \frac{5 \times A \times 0.92 \times 80 \times \frac{5+10}{2}}{0.004 \times A \times 10 \times 3600} = 19.1\ hours.$$

29. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from , by . If the temperature of is

(a)The temperature of is (b) $\lambda_B = 1.5\,\mu m$

(c)The temperature of is (d)The temperature of is

SOLUTION:

(a,b) According to Stefan's law

$$E = eA\sigma T^4 \Rightarrow E_1 = e_1 A\sigma T_1^4$$

$$E_2 = e_2 A\sigma T_2^4$$

$$\because E_1 = E_2$$

$$\therefore e_1 T_1^4 = e_2 T_2^4$$

$$\Rightarrow T_2 = \left(\frac{e_1}{e_2} T_1^4\right)^{\frac{1}{4}} = \left(\frac{1}{81} \times (5802)^4\right)^{\frac{1}{4}}$$

And, from Wein's law $\lambda_A \times T_A = \lambda_B \times T_B$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A}$$

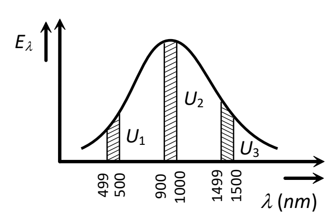
$$\Rightarrow \frac{\lambda_B - \lambda_A}{\lambda_B} = \frac{T_A - T_B}{T_A}$$

$$\Rightarrow \frac{1}{\lambda_B} = \frac{5802 - 1934}{5802} = \frac{3968}{5802} \Rightarrow \lambda_B = 1.5\ \mu m$$

30. A black body is at a temperature of . The energy of radiation emitted by this object with wavelength between and is , between and is and between and is . The Wein's constant . Then

(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

SOLUTION:'



.

Wein's displacement law is $\lambda_m T = b$

$$\Rightarrow \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6}{2880} = 1000\ nm.$$

Energy distribution with wavelength will be as follows

From the graph it is clear that $U_2 > U_1$.

31. A black metal foil is warmed by radiation from a small sphere at temperature T and at a distance . It is found that the power received by the foil is 'P'. If both the temperature and the distance are doubled, the power received by the foil will be

(a) $16P$ (b) $4P$

(c) $2P$ (d) P

SOLUTION:

$$\text{Energy received per second } i.e., \text{ power } P \propto (T^4 - T_0^4)$$

$$\Rightarrow P \propto T^4 \ (\because T_0 << T)$$

$$\text{Also energy received per sec } (p) \propto \frac{1}{d^2}$$

(inverse square law)

$$\Rightarrow P \propto \frac{T^4}{d^2} \Rightarrow \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{d_2}{d_1}\right)^2$$

$$\Rightarrow \frac{P}{P_2} = \left(\frac{T}{2T}\right)^2 \times \left(\frac{2d}{d}\right)^2 = \frac{1}{4} \Rightarrow P_2 = 4P.$$

32. Two metallic spheres S_1 and S_2 are made of the same material and have identical surface finish. The mass of is three times that of . Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of to that of is

(a) $1/3$ (b) $(1/3)^{1/3}$ (c) $1/\sqrt{3}$ (d) $\sqrt{3}/1$

SOLUTION:

$$\text{Rate of cooling } (R) = \frac{\Delta\theta}{t} = \frac{A \in \sigma(T^4 - T_0^4)}{mc}$$

$$\Rightarrow R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{volume}} \propto \frac{r^2}{r^3} \propto \frac{1}{r}$$

$$\Rightarrow \text{Rate } (R) \propto \frac{1}{r} \propto \frac{1}{m^{1/3}} \left[\because m = \rho \times \frac{4}{3}\pi r^3 \Rightarrow r \propto m^{1/3}\right]$$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

33. Three discs A, B and C having radii $2m$, $4m$, and $6m$ respectively are coated with carbon black on their other surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_a, Q_b, and Q_c respectively

(a) Q_a is maximum (b) Q_b is maximum (c) Q_c is maximum (d) $Q_a = Q_b = Q_c$

SOLUTION:

$$\text{Radiated power } P = A\varepsilon\sigma T^4 \Rightarrow P \propto AT^4$$

$$\text{From Wein's law, } \lambda_m T = \text{constant} \Rightarrow T \propto \frac{1}{\lambda_m}$$

$$\therefore\ P \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{(\lambda_m)^4}$$

$$\Rightarrow Q_A : Q_B : Q_C = \frac{2^2}{(300)^4} : \frac{4^2}{(400)^4} : \frac{6^2}{(500)^4}$$

$\therefore\ Q_B$ will be maximum

34. The total energy radiated from a black body source is collected for one minute and is used to heat a quantity of water. The temperature of water is found to increase form $20^\circ C$ to $20.5^\circ C$. If the absolute temperature of the black body is doubled and the experiment is repeated with the same quantity of water at , the temperature of water will be

(a) $21^\circ C$ (b) $22^\circ C$ (c) $24^\circ C$ (d) $28^\circ C$

SOLUTION:

The total energy radiated from a black body per minute.

$$Q \propto T^4 \Rightarrow \frac{Q_2}{Q_1} = \left(\frac{2T}{T}\right)^4 = 16$$

$$\Rightarrow Q_2 = 16 Q_1$$

If m be mass of water taken and S be its specific heat capacity,

then $Q_1 = ms(20.5 - 20)$

$Q_2 = ms(\theta - 20)$

$\theta^\circ C =$ Final temperature of water

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{\theta - 20}{0.5}\ \ \frac{16}{1} = \frac{\theta - 20}{0.5} \Rightarrow \theta = 28^\circ C$$

35. A solid sphere and a hollow sphere of the same material and size are heated to the same temperature and allowed to cool in the same surroundings. If the temperature difference between each sphere and its surroundings is , then

(a) The hollow sphere will cool at a faster rate for all values of

(b) The solid sphere will cool at a faster rate for all values of

(c) Both spheres will cool at the same rate for all values of

(d) Both spheres will cool at the same rate only for small values of

SOLUTION:

$$\text{Rate of cooling } \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$$

As surface area, material and temperature difference are same, so rate of loss of heat is same in both the spheres.
Now in this case rate of cooling depends on mass.

$$\Rightarrow \text{Rate of cooling } \frac{\Delta\theta}{t} \propto \frac{1}{m}$$

$\because\ m_{solid} > m_{hollow}$. Hence hollow sphere will cool fast.

36. A solid copper cube of edges is suspended in an evacuated enclosure. Its temperature is found to fall from to in . Another solid copper cube of edges , with similar surface nature, is suspended in a similar manner. The time required for this cube to cool from to will be approximately

(a) $25\ s$ (b) $50\ s$ (c) $200\ s$ (d) $400\ s$

SOLUTION:

$$\text{Rate of cooling } \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$$

$$\Rightarrow t \propto \frac{m}{A} \quad [\because \ \Delta\theta, t, \sigma, \ (T^4 - T_0^4) \text{ are constant}]$$

$$\Rightarrow t \propto \frac{m}{A} \propto \frac{\text{Volume}}{\text{Area}} \propto \frac{a^3}{a^2}$$

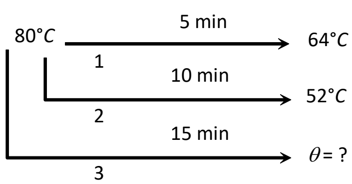
$$\Rightarrow t \propto a \Rightarrow \frac{t_1}{t_2} = \frac{a_1}{a_2}$$

$$\Rightarrow \frac{100}{t_2} = \frac{1}{2} \Rightarrow t_2 = 200\, sec.$$

37. A body initially at $80^\circ\, C$ cools to $64^\circ\, C$ in 5 *minutes* and to $52^\circ\, C$ in 10 *minutes*. The temperature of the body after 15 *minutes* will be

(a) $42.7^\circ\, C$ (b) $35^\circ\, C$ (c) $47^\circ\, C$ (d) $40^\circ\, C$

SOLUTION:



According to Newton law of cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

For first process : $\frac{(80-64)}{5} = K\left[\frac{80+64}{2} - \theta_0\right]$ ...(i)'

For second process : $\frac{(80-52)}{10} = K\left[\frac{80+52}{2} - \theta_0\right]$ ...(ii)

For third process : $\frac{(80-\theta)}{15} = K\left[\frac{80+\theta}{2} - \theta_0\right]$ ...(iii)

On solving equation (i) and (ii) we get $K = \frac{1}{15}$ and $\theta_0 = 24^\circ C$.

Putting these values in equation (iii) we get $\theta = 42.7^\circ C$

38. A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 0.1 *gm* of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is $\frac{1}{4}$ that of first, the rate at which ice melts in $gm/\sec$ will be

(a) 3.2 (b) 1.6 (c) 0.2 (d) 0.1

SOLUTION:

$$\frac{Q}{t}=\frac{KA\Delta\theta}{l}\Rightarrow\frac{mL}{t}=\frac{K(\pi r^2)\Delta\theta}{l}$$

$$\Rightarrow \text{Rate of melting of ice} \left(\frac{m}{t}\right)\propto\frac{Kr^2}{l}$$

Since for second rod K becomes $\frac{1}{4}th$ r becomes double and length becomes half, so rate of melting will be twice

$$\text{i.e.} \left(\frac{m}{t}\right)_2=2\left(\frac{m}{t}\right)_1=2\times0.1=0.2\ gm/sec.$$

39. One end of a copper rod of length $1.0\ m$ and area of cross-section 10^{-3} is immersed in boiling water and the other end in ice. If the coefficient of thermal conductivity of copper is $92\ cal/m\text{-}s\text{-}^oC$ and the latent heat of ice is $8\times10^4\ cal/kg$, then the amount of ice which will melt in one minute is

(a) $9.2\times10^{-3}\ kg$ (b) $8\times10^{-3}\ kg$ (c) $6.9\times10^{-3}\ kg$ (d) $5.4\times10^{-3}\ kg$

SOLUTION:

Heat transferred in one minute is utilised in melting the ice

$$\text{so, } \frac{KA(\theta_1-\theta_2)t}{l}=m\times L$$

$$\Rightarrow m=\frac{10^{-3}\times92\times(100-0)\times60}{1\times8\times10^4}=6.9\times10^{-3}\ kg$$

40. An ice box used for keeping eatable cold has a total wall area of $1\ metre^2$ and a wall thickness of $5.0cm$. The thermal conductivity of the ice box is $K=0.01\ joule/metre-^oC$. It is filled with ice at 0^oC along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is $334\times10^3\ joules/kg$. The amount of ice melted in one day is($1 day = 86,400\ seconds$)

(a) $776\ gms$ (b) $7760\ gms$ (c) $11520\ gms$ (d) $1552\ gms$

SOLUTION:

$$\frac{dQ}{dt}=\frac{KA}{l}d\theta=\frac{0.01\times1}{0.05}\times30=6J/sec$$

Heat transferred in on day (86400 sec)

$$\theta=6\times86400=518400\ J$$

$$\text{Now } Q=mL$$

$$\Rightarrow m=\frac{Q}{L}=\frac{518400}{334\times10^3}$$

$$=1.552\ kg=1552g.$$

41. A solid copper sphere (density and specific heat capacity c) of radius r at an initial temperature 200K is suspended inside a chamber whose walls are at almost 0K. The time required (in s) for the temperature of the sphere to drop to 100 K is

(a) $\frac{72}{7}\frac{r\rho c}{\sigma}$ (b) $\frac{7}{72}\frac{r\rho c}{\sigma}$ (c) $\frac{27}{7}\frac{r\rho c}{\sigma}$ (d) $\frac{7}{27}\frac{r\rho c}{\sigma}$

SOLUTION:

$$\frac{dT}{dt} = \frac{\sigma A}{mcJ}(T^4 - T_0^4)$$

In the given problem fall in temperature of body $dT = (200 - 100) = 100\,K$,
temp. of surrounding $T_0 = 0K$,
Initial temperature of body $T = 200\,K$].

$$\frac{100}{dt} = \frac{\sigma 4\pi r^2}{\frac{4}{3}\pi r^3 \rho c J}(200^4 - 0^4)$$

$$\Rightarrow dt = \frac{r\rho c J}{48\sigma}\times 10^{-6} s = \frac{r\rho c}{\sigma}.\frac{4.2}{48}\times 10^{-6}$$

$$= \frac{7}{80}\frac{r\rho c}{\sigma}\mu s \simeq \frac{7}{72}\frac{r\rho c}{\sigma}\mu s \qquad [\text{As } J = 4.2]$$

42. A sphere and a cube of same material and same volume are heated upto same temperature and allowed to cool in the same surroundings. The ratio of the amounts of radiations emitted will be

(a) 1 : 1 (b) $\frac{4\pi}{3} : 1$ (c) $\left(\frac{\pi}{6}\right)^{1/3} : 1$ (d) $\frac{1}{2}\left(\frac{4\pi}{3}\right)^{2/3} : 1$

SOLUTION:

$$Q = \sigma A t\,(T^4 - T_0^4)$$

If T, T_0, σ and t are same for both bodies

then $\frac{Q_{sphere}}{Q_{cube}} = \frac{A_{sphere}}{A_{cube}} = \frac{4\pi r^2}{6a^2}$ ..... (i)

But according to problem, volume of sphere = Volume of cube

$$\Rightarrow \frac{4}{3}\pi r^3 = a^3$$

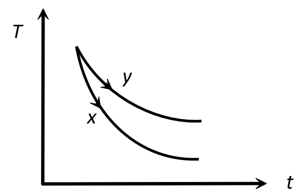
$$\Rightarrow a = \left(\frac{4}{3}\pi\right)^{1/3} r$$

Substituting the value of a in equation (i) we get

$$\frac{Q_{sphere}}{Q_{cube}} = \frac{4\pi r^2}{6a^2} = \frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3} r\right\}^2}$$

$$= \frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3} r^2} = \left(\frac{\pi}{6}\right)^{1/3} : 1$$

43. The graph. Shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity (e) and absorptivity (a) of the two bodies



(a) $e_x > e_y$ & $a_x < a_y$ (b) $e_x < e_y$ & $a_x > a_y$ (c) $e_x > e_y$ & $a_x > a_y$ (d) $e_x < e_y$ & $a_x < a_y$

SOLUTION:

$$\text{Rate of cooling}\left(-\frac{dT}{dt}\right) \propto \text{emissivity } (e)$$

$$\text{From graph, } \left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$$

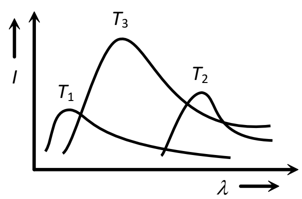
$$\Rightarrow e_x > e_y$$

Further emissivity (e) ∝ Absorptive power (a)

$$\Rightarrow a_x > a_y$$

(∵ good absorbers are good emitters).

44. The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown. Their temperature are such that



(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$ (c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

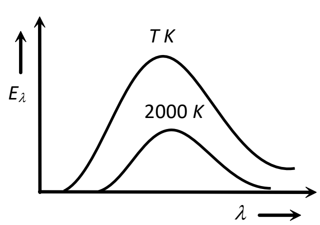
SOLUTION:

According to Wien's law $\lambda_m \propto \frac{1}{T}$

from the figure $(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

therefore $T_1 > T_3 > T_2$.

45. The adjoining diagram shows the spectral energy density distribution E_λ of a black body at two different temperatures. If the areas under the curves are in the ratio 16 : 1, the value of temperature T is



(a) 32,000 K (b) 16,000 K (c) 8,000 K (d) 4,000 K

SOLUTION:

$$\frac{A_T}{A_{2000}} = \frac{16}{1} \quad \text{(given)}$$

Area under $e_\lambda - \lambda$ curve represents the emissive power of body

emissive power $\propto T^4$

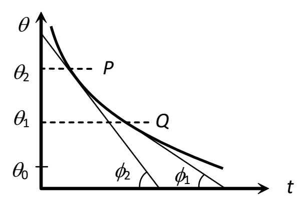
(Hence area under curve)

$$\Rightarrow \frac{AT}{A_{2000}} = \left(\frac{T}{2000}\right)^4$$

$$\Rightarrow \frac{16}{1} = \left(\frac{T}{2000}\right)^4$$

$$\Rightarrow T = 4000\,K.$$

46. A body cools in a surrounding which is at a constant temperature of θ_0. Assume that it obeys Newton's law of cooling. Its temperature θ is plotted against time t. Tangents are drawn to the curve at the points $P(\theta = \theta_1)$ and $Q(\theta = \theta_2)$. These tangents meet the time axis at angles of ϕ_2 and ϕ_1, as shown



(a) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (b) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$ (c) $\frac{\tan\phi_1}{\tan\phi_2} = \frac{\theta_1}{\theta_2}$ (d) $\frac{\tan\phi_1}{\tan\phi_2} = \frac{\theta_2}{\theta_1}$

SOLUTION:

For θ–t plot, rate of cooling $= \frac{d\theta}{dt}$ = slope of the curve.

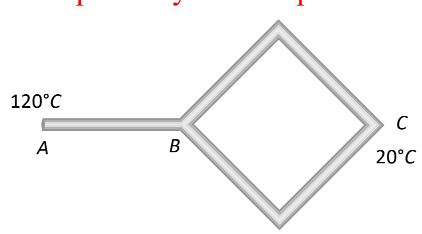
At P, $\frac{d\theta}{dt} = \tan\phi_2 = k(\theta_2 - \theta_0)$,

where k = constant.

At Q $\frac{d\theta}{dt} = \tan\phi_1 = k(\theta_1 - \theta_0)$

$$\Rightarrow \frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$$

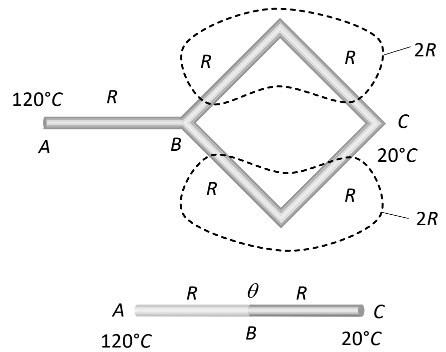
47. Five identical rods are joined as shown in figure. Point A and C are maintained at temperature $120°C$ and $20°C$ respectively. The temperature of junction B will be



(a) $100°C$ (b) $80°C$ (c) $70°C$ (d) $0°C$

SOLUTION:

If thermal resistance of each rod is considered R then, the given combination can be redrawn as follows



$$(\text{Heat current})_{AC} = (\text{Heat current})_{AB}$$

$$\frac{(120-20)}{R} = \frac{(120-\theta)}{R} \Rightarrow \theta = 70°C$$

48. On a clear sunny day, an object at temperature T is placed on the top of a high mountain. An identical object at the same temperature is placed at the foot of mountain. If both the objects are exposed to sun-rays for two hours in an identical manner, the object at the top of the mountain will register a temperature

(a) Higher than the object at the foot

(b) Lower than the object at the foot

(c) Equal to the object at the foot

(d) None of the above

SOLUTION:

According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t} = K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

In the first case, $\frac{(60-50)}{10} = K\left[\frac{60+50}{2}-\theta_0\right]$

$$1 = K(55-\theta)\text{.... (i)}$$

In the second case, $\frac{(50-42)}{10} = K\left[\frac{50+42}{2}-\theta_0\right]$

$$0.8 = K(46-\theta_0)\text{.... (ii)}$$

Dividing (i) by (ii), we get $\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$

or $46-\theta_0 = 44-0.8\theta_0$

$$\Rightarrow \theta_0 = 10^{o}C$$

THEORY BITS

1. Temperature of gas is a measure of

1) The average translational kinetic energy of the gas molecules
2) The average potential energy of the gas molecules
3) The average distance of the gas molecules
4) The size of the molecules of the gas

KEY: 1

2. The coefficient of linear expansion for a cetain metal varies with temperature as $\alpha(T)$. If L_0 is the intial length of the metal and the temperature of the metal is changed from T_0 to T (T_0>T).Then

1) $L = L_0 \int_{T_0}^{T} \alpha(T) dt$ 2) $L = L_0 \left[1 + \int_{T_0}^{T} \alpha(T) dT\right]$

3) $L = L_0 \left[L - \int_{T_0}^{T} \alpha(T) dT\right]$ 4) $L > L_0$

KEY:2

3. On the Celsius scale the absolute zero of temperature is at

1) 0^0C 2) -32^0C 3) 100^0C 4)-273.15^0C

KEY:3

4. The correct value of 0^0C on the Kelvin scale is

1) 273.15^0C 2) 273.16^0C 3) 273^0C 4) 273.2^0C

KEY:1

5. Which of the following is the largest rise in temperature?

1) $1^o F$ 2) $1^o R$ 3) 1K 4) $1^o C$

KEY:2

6. Melting and Boiling point of water on Fahrenheit scale of temperature respectively

1) $212^0 F, 32^0 F$ 2) $32^0 F, 212^0 F$ 3) $0^0 F, 100^0 F$ 4) $32^0 F, 132^0 F$

KEY:2

7. The substance which has negative coefficient of linear expansion is

1) Lead 2) Aluminium 3) Iron 4) Invar steel

KEY:1

8. Mercury boils at $356^0 C$. However, mercury thermometers are made such that they can measure temperatures upto $500^0 C$. This is done by

1) maintaining vacuum above the mercury column
2) filling Nitrogen gas at high pressure above the mercury column
3) filling Nitrogen gas at low pressure above the mercury column
4) filling oxygen gas at high pressure above the mercury column

KEY:2

9. For measuring temperature near absolute zero, the thermometer used is

1) thermoelectric thermometer 2) radiation thermometer
3) magnetic thermometer 4) resistance thermometer

KEY:3

10. Which of the following is the smallest rise in temperature?

1) $1^0 F$ 2) $1^0 R$ 3) 1K 4) $1^0 C$

KEY:1

11. For measurements of very high temperature say around $5000^0 C$ (of sun), one can use:

1) Gas thermometer 2) Platinum resistance thermometer
3) Vapour pressure thermometer 4) Pyrometer (Radiation thermometer)

KEY:4

12. The temperature at which two bodies appear equally hot or cold when touched by a person is

1) $0^0 C$ 2) $37^0 C$ 3) $25^0 C$ 4) $4^0 C$

KEY:2

13. Celsius is the Unit of

1) Temperature 2) Heat 3) Specific heat 4) Latent heat

KEY:1

14. The range of clinical thermometer is

1) $37^0 C\ to\ 42^0 C$ 2) $95^0 F\ to\ 110^0 F$ 3) $90^0 F\ to\ 112^0 F$ 4) $95^0 C\ to\ 104^0 C$

KEY:2

15. Solids expand on heating because

1) the K.E. of the atoms increases 2) the P.E. of the atoms increases
3) total energy of the atoms increases 4) the K.E. of the atoms decrease

KEY:1

16. A ring shaped piece of a metal is heated, If the material expands, the hole will

1) Contract 2) Expand 3) Remain same
4) Expand or Contract depending on the width

KEY:2

17. Expansion during heating

1) occurs only in solids 2) decreases the density of the material
3) occurs at same rate for all liquids and gases 4) increases the weight of the material

KEY:2

18. When a metal bar is heated, the increase in length is greater, if

1) the bar has large diameter 2) the bar is long
3) the temperature rise small 4) small diameter

KEY: 2

19. Which of the following scales of temperature has only positive degrees of temperature?

1) Centigrade 2) Fahrenheit scale 3) Reaumur scale 4) Kelvin scale

KEY:4

20. A solid ball of metal has a spherical cavity inside it. The ball is cooled.The Volume of the cavity will

1) decrease 2) increase 3) remain same 4) have its shape changed

KEY:1

21. The standard scale of temperature is

1) The mercury scale 2) The gas scale 3) The platinum resistance scale 4) liquid scale

KEY: 2

22. Two spheres of same size are made of same material but one is hollow and the other is solid. They are heated to same temperature, then

1) Both spheres will expand equally. 2) Hollow sphere will expand more than solid one.
3) Solid sphere will expand more than hollow one. 4) Hollow sphere will expand double that of solid one

KEY:1

23. When a metal bar is cooled, then which one of these statements is correct.

1) Length, density and mass remain same

2) Length decreases, density increases but mass remains same
3) Length and mass decrease but density remains the same
4) Length and density decrease but mass remains the same

KEY:2

24. If temperature of two spheres of same size but made of different materials changes by ΔT then

1) Both expands equally
2) Sphere with greater α expands or contracts more than other.
3) Sphere with greater α expands or contracts less than other.
4) Both contracts equally.

KEY:2

25. The coefficient of linear expansion of a solid depends upon

1) the unit of pressure
2) the nature of the material only
3) the nature of the material and temperature
4) unit of mass

KEY:2

26. When hot water is poured on a glass plate, it breaks because of

1) unequal expansion of glass
2) equal contraction of glass
3) unequal contraction of glass
4) glass is delicate

KEY:1

27. If α_c and α_k denote the numerical values of coefficient of linear expansions of the solid, expressed per ^{0}C and per Kelvin respectively, then.

1) $\alpha_c > \alpha_k$ 2) $\alpha_c < \alpha_k$ 3) $\alpha_c = \alpha_k$ 4) $\alpha_c = 2\alpha_k$

KEY:3

28. If α_c and α_f denote the numerical values of coefficient of linear expansion of a solid, expressed per ^{0}C and per ^{0}F respectively, then

1) $\alpha_c > \alpha_f$ 2) $\alpha_f > \alpha_c$ 3) $\alpha_f = \alpha_c$ 4) $\alpha_f + \alpha_c = 0$

KEY:1

29. The coefficient of linear expansion of a metal rod is 12×10^{-6} / ^{0}C, its value in per $_0$C, its value in per ^{0}F

1) $\frac{20}{3}\times10^{-6}/^0F$ 2) $\frac{15}{4}\times10^{-6}/^0F$ 3) $21.6\times10^{-6}/^0F$ 4) $12\times10^{-6}/^0F$

KEY:1

30. A brass disc fits into a hole in an iron plate. To remove the disc.

1) the system must be cooled
2) the system must be heated
3) the plate may be heated (or) cooled
4) the disc must be heated

KEY:1

31. The coefficient of volume expansion is

1) equal to the coefficient of linear expansion.
2) Twice the coefficient of linear expansion
3)Equal to the sum of coefficients of linear and superficial expansions.
4) Twice the coefficient of areal expansion.

KEY:3

32. Two metal strips that constitute a bimetallic strip must necessarily differ in their.

1) length
2) mass
3) coefficient of linear expansion
4) resistivity

KEY:3

33. Thermostat is based on the principle of

1) equal expansion of two rods of different lengths.

2) Different expansion of two rods of different lengths.
3) Different expansion of two rods of same length
4) Equal expansion of two rods of same length.

KEY:3

34. A pendulum clock shows correct time at 0^0 C. At a higher temperature the clock.
1) loses time 2) gains time
3) neither loses nor gains time 4) will not operate

KEY:1

35. To keep the correct time modern day watches are fitted with balance wheel made of
1) Steel 2) Platinum 3) Invar 4) tungsten

KEY:3

36. When the temperature of a body increases
1) density and moment of inertia increase
2) density and moment of inertia decrease
3) density decreases and moment of inertia increases.
4) density increases and moment of inertia decreases.

KEY:3

37. The coefficient of linear expansion of crystal in one direction is α_1 and that in every direction perpendicular to it α_2. The coefficient of cubical expansion is
1) $\alpha_1 + \alpha_2$ 2) $2\alpha_1 + \alpha_2$ 3) $\alpha_1 + 2\alpha_2$ 4) None

KEY:3

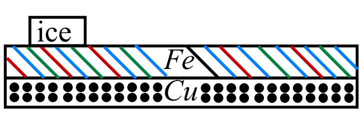
38. In balance wheel of watch, the factors that make its oscillations uniform are
1) tension in string 2) moment of inertia of balance wheel
3) temperature 4) pressure

KEY:2

39. Always platinum is fused into glass, because
1) platinum is good conductor of heat 2) melting point of platinum is very high
3) they have equal specific heats 4) their coefficients of linear expansion are equal

KEY:4

40. A cube of ice is placed on a bimetallic strip at room temperature as shown in the figure. What will happen if the upper strip of iron and the lower strip is of copper?



1) Ice moves downward 2) Ice moves upward 3) Ice remains in rest 4) None of the above

KEY:1

41. To withstand the shapes of concave mirrors against temperature variations used in high resolution telescope, they are made of
1) Quartz 2) Flint glass 3) Crown glass 4) Combination of Flint and Silica

KEY:1

42. The holes through which the fish plates are fitted to join the rails are oval in shape because
1) Bolts are in oval shape

2) To allow the movement of rails in the direction of length due to change in temperature.

3) To make the fitting easy and tight	4) Only oval shape holes are possible

KEY:2

43. The diameter of a metal ring is D and the coefficient of linear expansion is α. If the temperature of the ring is increased by 1^0C, the circumference and the area of the ring will increase by

1) $\pi D\alpha$, $2\pi D\alpha$	2) $2\pi D\alpha$, $\pi D^2\alpha$	3) $\pi D\alpha$, $\frac{\pi D\alpha}{2}$	4) $\pi D\alpha$, $\frac{\pi D^2\alpha}{2}$

KEY:4

44. When a metal ring is heated

1) The inner radius decreases and outer radius increases

2) The outer radius decreases and inner radius increases

3) Both inner and outer radii increase	4) Both inner and outer radii decrease

KEY:3

45. The linear expansion of a solid depends on

1) Its original mass

2) Nature of the material and temperature difference.

3) The nature of the material only

4) pressures

KEY:2

46. The moment of inertia of a uniform thin rod about its perpendicular bisector is I. If the temperature of the rod is increased by Δt, the moment of inertia about perpendicular bisector increases by (coefficient of linear expansion of material of the rod is α).

1) Zero	2) Iα Δt	3) 2 Iα Δt	4) 3 Iα Δt

KEY:3

47. A bimetal made of copper and iron strips welded together is straight at room temperature. It is held vertically so that the iron strip is towards the left hand and copper strip is towards right hand. The bimetal strip is then heated. The bimetal strip will

1) remain straight	2) bend towards right	3) bend towards left	4) have no change

KEY:3

48. If L_1 and L_2 are the lengths of two rods of coefficients of linear expansion α_1 and α_2 respectively the condition for the difference in lengths to be constant at all temperatures is

1) $L_1\alpha_1 = L_2\alpha_2$	2) $L_1\alpha_2 = L_2\alpha_1$	3) $L_1\alpha_1^2 = L_2\alpha_2^2$	4) $L_1\alpha_2^2 = L_2\alpha_1^2$

KEY:1

49. A semicircular metal ring subtends an angle of 180^0 at the center of the circle. When it is heated, this angle

1) remains constant	2) increases slightly	3) decreases slightly	4) becomes 360^0

KEY:1

50. The coefficients of linear expansion of P and Q are α_1 and α_2 respectively. If the coefficient of cubical expansion of 'Q' is three times the coefficient of superficial expansion of P, then which of the following is true ?

1) $\alpha_2 = 2\alpha_1$	2) $\alpha_1 = 2\alpha_2$	3) $\alpha_2 = 3\alpha_1$	4) $\alpha_1 = 3\alpha_2$

KEY:1

51. **When a rod is heated, its linear expansion depends on**

a) initial length b) area of cross section c) mass d) temperature rise

1) only a is correct 2) a & d are correct 3) b & c are correct 4) a & c are correct

KEY:2

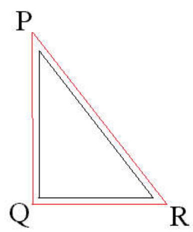
52. **PQR is a right angled triangle made of brass rod bent as shown. If it is heated to a high temperature the angle PQR.**

1) increases

2) decreases

3) remains same

4) becomes 135^0



KEY:3

53. **A brass scale gives correct length at 0^0C. If the temperature be 25^0C and the length read by the scale is 10 cm. Then the actual length will be**

1) more than 10 cm 2) less than 10 cm 3) equal to 10 cm 4) we can not say

KEY:1

54. **The coefficient of volume expansion is**

1) twice the coefficient of linear expansion. 2) twice the coefficient of real expansion.

3) thrice the coefficient of real expansion. 4) thrice the coefficient of linear expansion

KEY:4

55. **For a constant volume gas thermometer, one should fill the gas at**

1) Low temperature and low pressure 2) Low temperature and high pressure

3) High temperature and low pressure 4) High temperature and high pressure

KEY:3

56. **When a metal sphere is heated maximum percentage increase occurs in its**

1) Density 2) Surface area 3) Radius 4) Volume

KEY:4

57. **When a copper ball is cooled the largest percentage increase will occur in its**

1) diameter 2) area 3) volume 4) density

KEY:4

58. **A solid sphere and a hollow sphere of same material have same mass. When they are heated by 50^0C, increase in volume of solid sphere is 5 c.c. The expansion of hollow sphere is**

1) 5 c.c. 2) more than 5 c.c. 3) Less than 5 c.c. 4) None

KEY:2

59. **The numerical value of coefficient of linear expansion is independent of units of**

a) length **b) temperature**

c) area **d) mass**

1) only (a) is correct 2) (a) & (b) are correct

3) (a), (b) & (c) are correct 4) (a), (b) & (c) are correct

KEY:4

60. **Expansion during heating**

a) occurs in solids only **b) causes decrease in weight**

c) is due to increase of interatomic spacing

1) only (a) is wrong 2) (a), (b) & (c) are wrong

3) (a) & (b) are wrong 4) (a), (b) & (c) are correct

KEY:3

61. Due to thermal expansion with rise in temperature

a) Metallic scale reading becomes lesser than true value

b) Pendulum clock goes fast

c) A floating body sinks a little more

d) The weight of a body in a liquid increases

1) only (a) is correct 2) (a) & (b) are correct

3) (a), (b) & (d) are correct 4) (a), (c) & (d) are correct

KEY:4

62. Which of the following statements are true

a) Rubber contracts on heating

b) Water expands on freezing

c) Water contracts on heating from 0^0C to 4^0C

d) Water expands on heating from 4^0C to 40^0C

1) only (a) is correct 2) (b) & (c) are correct

3) (c) & (d) are correct 4) all are correct

KEY:4

63. The substance which contracts on heating is

1) Silica glass 2) Iron 3) Invar steel 4) Aluminum

KEY:1

64. When a metal ring having some gap is heated

a) length of gap increases

b) radius of the ring decreases

c) the angle subtended by the gap at the centre remains same

d) length of gap decreases

1) only (d) is correct 2) (a) & (b) are correct

3) (a) & (c) are correct 4) all are correct

KEY:3

65. At what temperature, does the celsius and fahrenheit scales show the same reading but with opposite sign ?

1) 44 2) 20 3) 40 4) 10

KEY:3

66. Gas thermometers are more sensitive than liquid thermometers because

1) gases expand more than liquids 2) gases do not easily changed their state

3) gases are much lighter 4) gases are easy to obtain

KEY:1

67. When a copper solid sphere is heated, its

a) moment of inertia increases

b) Elasticity decreases

c) density decreases

d) mass increases

1) only (b) is true 2) (a) & (b) are true

3) (a), (b) & (c) are true 4) all are true

KEY:3

68. A platinum resistance thermometer is constructed which reads 0^0C at ice point and 100^0C at steam point. The resistance of platinum coil varies with t as $R_t = R_0\left(1+\alpha t+\beta t^2\right)$. If t_p denotes temperature on the platinum resistance thermometer and t denotes temperature on mercury scale. Then resistance as the function of t_p will be

1) $R_0\left[1+\alpha t_p+\beta\times(100)t_p\right]$ 2) $R_0\left[1+\alpha t_p+\alpha\times(100)t_p\right]$

3) $R_0\left[1+\beta t_p+\alpha t_p\right]$ 4) $R_0\left[\beta t_p+\alpha t_p\right]$

KEY:1

69 Which of the following statements is correct for a thermometer ?

1) The bulb of the thermometer is made of a good conducting material
2) The bulb of the thermometer is made of a poor conducting material
3) Sole purpose of making the walls of the bulb thin is to proveide maximum possible space for liquid
4) None of these

KEY:1

70. Of the following thermometers the one which is most useful for the measurement of a rapidly varying temperature is a

1) platinum resistance thermometer 2) gas thermometer
3) thermoelectric thermometer 4) saturation vapour pressure thermometer

KEY:3

71. Two thermometers are constructed in the same way except that one has a spherical bulb and the other a cylindrical bulb; which one will respond quickly to temperature changes ?

1) Spherical bulb thermometer 2) Cylindrical bulb thermometer
3) Both equally 4) either

KEY:2

72. Which of the following statements is true for a thermometer ?

1) Coefficient of cubical expansion of liquid must be greater than that of bulb material
2) Coefficient of cubical expansion of liquid may be equal to that to bulb material
3) Coefficient of cubical expansion of liquid must be less than that of bulb material
4) None of the above

KEY:1

73. The coefficient of linear expansion of an in homogeneous rod changes linearly from α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of the rod is

$$\alpha_{eq} = \frac{l_1\alpha_1 + l_2\alpha_2}{l_1 + l_2}$$

1) $\alpha_1+\alpha_2$ 2) $\frac{1}{2}(\alpha_1+\alpha_2)$ 3) $\sqrt{\alpha_1\alpha_2}$ 4) $\alpha_1-\alpha_2$

KEY:2

74. By what temperature a rod should be heated so that its length becomes $2l_0$ of its initial length l_0. Assume that α (coefficient of linear expansion) in that range of temperature remains constant and melting point is very high.

1) $\frac{2}{\alpha}$ 2) $\frac{1}{\alpha}$ 3) $\frac{\ln 2}{\alpha}$ 4) $\frac{\ln 1}{\alpha}$

KEY:3

75. Column I gives some devices and Column II gives some processes on which the functioning of these devices depend. Match the devices in Column I with the processes in Column II.

Column I	**Coumn II**
A. Bimetallic strip	p. Radiation from a hot body
B. Steam engine	q. Energy conversion
C. Incandescent lamp	r. Melting
D. Electric fuse	s. Thermal expansion of solids

KEY:A-s, B-q, C-p,q, D-q,r

76.

List-I	**List-II**
a) Isotorpic solids	e) Expands on melting
b) Ice	f) Equal expansion in all directions
c) Aniostropic solids	g) Contracts on heating
d) Copper	h) unequal expansion in different directions

KEY:a-f, b-g, c-h, d-e

77.

List-I	**List-II**
a) Thermal expansion	e) Pendulum clock
b) α, β, γ	f) Depends on dimension, material, temperature
c) Bimetallic strip	g) Depends on nature of material only
d) Invar steel	h) Balance wheel of a watch

KEY:a-f, b-g, c-h, d-e

78.

List-I	**List-II**
a) Bimetal thermostat	e) Pendulum clock
b) Compensated pendulum	f) Invar steel
c) Metal tape	g) Differential expansion of metals
d) Alvinar	h) Hair spring

KEY:a-g, b-e, c-f, d-h

79.

List-I	**List-II**
a) Thermal stress	e) $3\alpha\Delta t 100$
b) Loss in time of a pendulum clock per sec	f) $\dfrac{d}{(\alpha_2 - \alpha_1)\Delta t}$
c) Percentage increase in volume of a solid	g) $Y\alpha\Delta t$
d) Radius of circular arc of bimetallic strip	h) $(1/2)\alpha\Delta t$

KEY:a-g, b-h, c-e, d-f

80. Cooking is difficult on mountains because

1) water boils at low temperature
2) water boils at high temperature
3) water does not boil
4) it is cool there

KEY:1

81. Heat capacity of a substance is infinite. It means

1) heat is given out 2) heat is taken in

3) no change in temperature whether heat is taken in (or) given out

4) all of the above

KEY:3

82. Heat required to raise the temperature of one gram of water through 1^0C is

1) 0.001 Kcal 2) 0.01 Kcal

3) 0.1 Kcal 4) 1.0 Kcal

KEY:1

83. A large block of ice is placed on a table when the surroundings are at 0^0C

1) ice melts at the sides 2) ice melts at the top

3) ice melts at the bottom 4) ice does not melt at all

KEY:3

84. In defining the specific heat, temperature is represented in ^0F instead of ^0C. Then the value of specific heat will

1) decreases 2) increases

3) remain constant 4) be converted to heat capacity

KEY:1

85. Why the specific heat at a constant pressure is more than that at constant volume

1) there is greater inter molecular attraction at constant pressure

2) at constant pressure molecular oscillation are more violent

3) external work need to be done for allowing expansion of gas at constant pressure

4) due to more reasons other than those mentioned in the above

KEY:3

86. The ratio [C_p / C_v] of the specific heats at a constant pressure and at a constant volume of any perfect gas

1) can't be greater than 5/4 2) can't be greater than 3/2

3) can't be greater than 5/3 4) can have any value

KEY:3

87. During melting process, the heat given to a solid is used in (generally)

1) Increasing the temperature 2) Increasing the density of material

3) Increasing the average distance between the molecules 4) Increasing the average K.E. of the molecules

KEY:3

88. When two blocks of ice are pressed against each other then they stick together (coalesce) because

1) cooling is produced 2) heat is produced

3) increase in pressure, increase in melting point 4) increase in pressure, decrease in melting point

KEY:4

89. Ice is found to be slippery when a man walks on it This is so because

1) increase in pressure causes ice to melt faster

2) increase in pressure causes ice to melt slower

3) its surface is smooth and cold

4) ice is colder

KEY:1

90. A piece of ice at 0^0C is dropped into water at 0^0C. Then ice will

1) melt 2) be converted to water 3) not melt 4) partially melt

KEY:3

91. Paraffin wax expands on melting. The melting point of wax with increasing pressure is

1) increases 2) decreases 3) remains same 4)we can't say

KEY:1

92. In a pressure cooker cooking is done quickly because

1) the cooker does not absorb any heat 2) it has a safety valve

3) boiling point of water rises due to increased pressure

4) it is a prestige to cook in a cooker

KEY:3

93. Which of the following at 100^0C produces most severe burns ?

1) hot air 2) water 3) steam 4) oil

KEY:3

94. The latent heat of vaporisation of a substance is always

1) greater than its latent of fusion 2) greater than its latent heat of sublimation

3) equal to its latent heat of sublimation 4) less than its latent heat of fusion

KEY:1

95. The latent heat of vaporisation of water is more than latent heat of fusion of ice, why

1) on vaporisation much larger increase in volume takes place

2) increase in kinetic energy is much larger on boiling

3) kinetic energy decreases on boiling

4) volume decreases when the ice melts

KEY:1

96. The heat capacity of material depends upon

1) the structure of a matter 2) temperature of matter

3) density of matter 4) specific heat of matter

KEY:4

97. Which of the following states of matter have two specific heats ?

1) solid 2) gas 3) liquid 4) vapour

KEY:2

98. The specific heat of a gas in an isothermal process is

1) infinity 2) zero

3) negative 4) remains constant

PRACTICE BITS

1. Specific heat of aluminium is 0.25 cal/g-^{0}c. The water equivalent of an aluminium vessel of mass one kilogram is

1) 40 cal/^{0}c 2) 250 g 3) 250 cal/^{0}c 4) 40 g

KEY :2

SOLUTION :

water equivalent = mS gram.

2. A metal block absorbs 4500 cal of heat when heated from 30^{0}C to 80^{0}C. Its thermal capacity is

1) 90 gm 2) $90\ cal/^0C$ 3) 9 gm 4) $9\ cal/^0C$

KEY :2

SOLUTION :

$$H = \frac{\Delta Q}{\Delta \theta}$$

3. Two beakers A and B contain liquids of masses 300 g and 420 g respectively and specific heats 0.8cal/g-^{0}C and 0.6cal/g-^{0}C. The amount of heat on them is equal. If they are joined by a metal rod

1) heat flows from the beaker B to A 2) heat flows from A to B

3) no heat flows 4) heat flows neither from A to B nor B to A

KEY :2

SOLUTION :

Quantity of heat on A = Quantity of heat on B

$$m_A \times S_A \times \theta_1 = m_B \times S_B \times \theta_2 \Rightarrow \theta_1 > \theta_2$$

4. The ratio of densities of two substances is 2:3 and that of specific heats is 1 : 2. The ratio of thermal capacities per unit volume is

1) 1 : 2 2) 2 : 1 3) 1 : 3 4) 3 : 1

KEY :3

SOLUTION :

$$H = mS = \rho VS \Rightarrow \frac{H_1}{H_2} = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{S_1}{S_2}\right)$$

5. Two spheres of copper of diameters 10cm and 20 cm will have thermal capacities in the ratio

1) $\frac{1}{8}$ 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{1}{6}$

KEY :1

SOLUTION :

$$H = mS = \rho \frac{4}{3}\pi r^3 S \Rightarrow \frac{H_1}{H_2} = \left(\frac{r_1}{r_2}\right)^3$$

6. Two liquids A and B of equal volumes have their specific heats in the ratio 2 : 3. If they have same thermal capacity, then the ratio of their densities is

1) 1 : 1 2) 2 : 3 3) 3 : 2 4) 5 : 6

KEY :3

SOLUTION :

$$m_1S_1 = m_2S_2 \Rightarrow v_1\rho_1S_1 = v_2\rho_2S_2 \,;\, \frac{\rho_1}{\rho_2} = \frac{S_2}{S_1}$$

7. Three liquids A,B and C of masses 400gm, 600 gm and 800 gm are at 30^0c, 40^0c and 50^0c respectively. When A and B are mixed resultant temperature is 36^0c when B and C are mixed resultant temperature is 44^0c Then ratio of their specific heats are

1) 2:1:1 2) 3:2:1 3) 2:2:1 4) 1:4:9

KEY :3

SOLUTION :

When A, B are mixed

$$m_AS_A(\Delta\theta)_A = m_BS_B(\Delta\theta)_B \;......\; \text{(i)}$$

When B, C are mixed

$$m_BS_B(\Delta\theta)_B = m_CS_C(\Delta\theta)_C \;......\; \text{(ii)}$$

From (i) and (ii) we get relation between S_A and S_C.

When A and C are mixed

$$m_AS_A(\Delta\theta)_A = m_CS_C(\Delta\theta)_C$$

8. 50g of copper is heated to increase its temperature by 10^0C. If the same quantity of heat is given to 10 g of water, the rise in its temperature is (S_{cu}= 420J/kg/^{0}C and S_w= 4200J/kg/^{0}C)

1) 5^0C 2) 6^0C 3) 7^0C 4) 8^0C

KEY :1

SOLUTION :

$$Q_1 = Q_2 \Rightarrow mS_c \times \Delta\theta_1 = mS_w \times \Delta\theta_2$$

9. 1gm of ice at 0^0c is converted to steam at 100^0c the amount of heat required will be (L_{steam}=536 cal/g)

1) 756 cal 2)12000 cal 3)716 cal 4)450 cal

KEY :3

SOLUTION :

$Q = mL_{ice} + mS_w(100-0) + mL_s$

10. Boiling water at 100^0C and cold water at t^0C are mixed in the ratio 1:3 and the resultant maximum temperature was 37^0C. Assuming no heat losses, the value of 't' is

1) 4^0C 2) 9^0C 3) 12^0C 4) 16^0C

KEY :4

SOLUTION :

Heat lost by hot water = Heat gained by cold water.

$$m_1S_1 \times \Delta\theta_1 = m_2S_2\Delta\theta_2 \left(Given, \frac{m_1}{m_2} = \frac{1}{3} \right)$$

11. The fraction of ice that melts by mixing equal masses of ice at -10°C and water at 60°C is

1)6/11 2) 11/16 3) 5/16 4) 11/15

KEY :2

SOLUTION :

Here a part of ice is melted because heat given by water when it comes to 0ºC is less than the heat required for ice to melt completely.

Let m| is the mass of the ice melted.

$$m'S_{ice}(10)+m'L_{ice}=m_{water}S_w(60)$$

12. Power of a man who can chew 0.3 kg ice in one minute is (in cal/s)

1) 400 2) 4 3) 24 4) 240

KEY :1

SOLUTION :

$$P=\frac{mL_f}{t}$$

13. A beaker contains 200g of water. The heat capacity of the beaker is equal to that of 20g water. The initial temperature of water in the beaker is 20^0c. If 440g of hot water at 92^0c is poured in it, the final temperature (neglecting radiation loss) will be nearly

1) 58^0c 2) 68^0c 3) 73^0c 4) 78^0c

KEY :2

SOLUTION :

From principle of calorimetry

$$m_{water}\times S_w\times(\theta-20)+(mS)\times(\theta-20)$$

$$=m_{hot\,water}\times S_w\times(92-\theta)$$

14. If 10g of the ice at 0^0c is mixed with 10g of water at 100^0c, then the final temperature of the mixture will be

1) 5^0c 2) 10^0c 3) 100 K 4) 0^0c

KEY :2

SOLUTION :

From principle of calorimetry

$$(m\times L)+(m\times S\times\Delta\theta_1)=m\times S\times\Delta\theta_2$$

15. The final temperature, when 10 g of steam at $100^0\,C$ is passed into an ice block of mass 100g $\left(L_{steam}=540\,cal/g, L_{ice}=80\,cal/g; S_{water}=1\,cal/g^0C\right)$ **is**

1) 21.8^0C 2) 15.7^0C 3) 16.9^0C 4) 20.4^0C

KEY :1

SOLUTION :

Heat lost by steam = Heat gained by ice

$$m_{steam}\times L_v+m_{steam}S_w\left(100^0-\theta^0\right)=m_{ice}L_f+m_{ice}S_w\left(\theta^0-0\right)$$

16. The quantity of heat which can rise the temperature of x gm of a substance through t1°C can rise the temperature of y gm of water through t2°C is same. The ratio of specific heats of the substances is

1) yt1/xt2 2) xt2/yt1 3) yt2/xt1 4) xt1/yt2

KEY :3

SOLUTION : $Q_1 = Q_2 \Rightarrow m_1 S_1 \theta_1 = m_2 S_2 \theta_2$

17. Two liquids A and B are at 30^0c and 20^0c respectively. When they are mixed in equal masses the temperature of the mixture is found to be 26^0c. The ratio of specific heats is

1) 4 : 3 2) 3 : 4 3) 2 : 3 4) 3 : 2

KEY :4

SOLUTION : Heat lost by A = Heat gain by B $mS_A(\Delta\theta)_A = mS_B(\Delta\theta)_B \Rightarrow \frac{S_A}{S_B} = \frac{(\Delta\theta)_B}{(\Delta\theta)_A}$

18. M g of ice at 0^0c is mixed with M g of water at 10^0c. The final temperature is

1) 8^0c 2) 6^0c 3) 4^0c 4) 0^0c

KEY :4

SOLUTION :

$(M \times 80) > (M \times 10)$ $\therefore$ Final Temp. is 0^0c

19. 10 grams of steam at $100^0 C$ is mixed with 50 gm of ice at $0^0 C$ then final temperature is

1) $20^0 C$ 2) $50^0 C$ 3) $40^0 C$ 4) $100^0 C$

KEY :3

SOLUTION :

Heat lost = Heat gained

$m_{ice} L_{ice} + m_{ice} S_w (\theta) =$

$m_{steam} L_{steam} + m_{steam} S_w (100 - \theta)$

20. The heat energy required to vapourise 5kg of water at 373 K is

1) 2700 K.cal 2) 1000 K.cal 3) 27 K.cal 4) 270 K.cal

KEY :1

SOLUTION : $Q = m \times L_{steam}$

21. Two liquids A and B are at temperatures of 75^0c and 150^0c respectively. Their masses are in the ratio of 2 : 3 and specific heats are in the ratio 3 : 4. The resultant temperature of the mixture, when the above liquids, are mixed (Neglect the water equivalent of container) is

1) 125^0c 2) 100^0c 3) 50^0c 4) 150^0c

KEY :1

SOLUTION :

$m_A S_A (\theta - 75) = m_B S_B (150 - \theta)$

22. 1g of ice at $0^0 C$ is mixed 1g of steam at $100^0 C$. The mass of water formed is

1) 1.33g 2) 13.3 g 3) 0.133 g 4) 13.3g

KEY :1

SOLUTION :

Here the resultant temperature is $100^0 C$

$m^|$ is mass of the steam condensed

$m^| L_v = m_{ice} L_f + m_{ice} S_{water} \times \Delta\theta$

$\therefore$ water formed = 1g + m$^|$

23. A liquid of mass 'm' and specific heat 'S' is at a temperature '2t'. If another liquid of thermal capacity 1.5 times, at a temperature of t/3 is added to it, the resultant temperature will be

1) $\frac{4}{3}t$ 2) t 3) $\frac{t}{2}$ 4) $\frac{2}{3}t$

KEY :2

SOLUTION :

From principle of calorimetry $\theta = \frac{m_1 S_1 \theta_1 + m_2 S_2 \theta_2}{m_1 S_1 + m_2 S_2}$

$(Given, m_2 S_2 = 1.5 \times m_1 S_1)$

24. Three liquids with masses m_1, m_2, m_3 are thoroughly mixed. If their specific heats are S_1, S_2, S_3 and their temperatures $\theta_1, \theta_2, \theta_3$ respectively, the temperature of the mixture is

1) $\frac{S_1\theta_1 + S_2\theta_2 + S_3\theta_3}{m_1 S_1 + m_2 S_2 + m_3 S_3}$ 2) $\frac{m_1 S_1\theta_1 + m_2 S_2\theta_2 + m_3 S_3\theta_3}{m_1 S_1 + m_2 S_2 + m_3 S_3}$

3) $\frac{m_1 S_1\theta_1 + m_2 S_2\theta_2 + m_3 S_3\theta_3}{m_1\theta_1 + m_2\theta_2 + m_3\theta_3}$ 4) $\frac{m_1\theta_1 + m_2\theta_2 + m_3\theta_3}{S_1\theta_1 + S_2\theta_2 + S_3\theta_3}$

KEY :2

SOLUTION :

Let $\theta_1 > \theta_2 > \theta_3$ and θ= resultant temperature.

From principle of colorimetry $m_3 S_3(\theta_3 - \theta) = m_1 S_1(\theta - \theta_1) + m_2 S_2(\theta - \theta_2)$

25. A piece of metal of mass 112g is heated to $100^0 C$ and dropped into a copper calorimeter of mass 40g containing 200g of water at $16^0 C$. Neglecting heat loss, the specific heat of the metal is nearly, if the equilibrium temperature reached is $24^0 C$

($S_{cu} = 0.1 cal/g{-}^o C$ **)**

1) $0.292 cal/gm{-}^0 C$ 2) $0.392 cal/gm{-}^0 C$ 3) $0.192 cal/gm{-}^0 C$ 4) $0.492 cal/gm{-}^0 C$

KEY :3

SOLUTION :

Heat lost by metal = heat gained by calorimeter and water

$m_{metal} S_{metal}(100 - 24) =$

$(m_{cu} S_{cu} + m_{water} S_w)(24 - 16)$

PREVIOUS JEE MAINS QUESTIONS AND SOLUTOINS

1. Two different wires having lengths L_1 and L_2, and respective temperature coefficient oflinear expansion α_1 and α_2 are joined end-to-end. Then the effective temperature coefficient of linear expansion is : [Sep. 05, 2020 (II)]

(a) $\frac{\alpha_1 L_1+\alpha_2 L_2}{L_1+L_2}$ (b) $2\sqrt{\alpha_1\alpha_2}$ (c) $\frac{\alpha_1+\alpha_2}{2}$ (d) $4\frac{\alpha_1\alpha_2 L_2 L_1}{\alpha_1+\alpha_2(L_2+L_1)^2}$

Sol : (a) Let L_1' and L_2' be the lengths of the wire when temperature is changed by ΔT°C.

At T°C,

$$L_{eq}=L_1+L_2$$

At $T+\Delta$°C

$$L_{eq}'=L_1'+L_2'$$

$$L_{eq}(1+\alpha_{eq}\Delta T)=L_1(1+\alpha_1\Delta T)+L_2(1+\alpha_2\Delta T)$$

$$[L^{\prime}=L(1+\alpha\Delta T)]$$

$$\Rightarrow (L_1+L_2)(1+\alpha_{eq}\Delta T)=L_1+L_2+L_1\alpha_1\Delta T+L_2\alpha_2\Delta T$$

$$\Rightarrow \alpha_{eq}=\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$$

2. A bakelite beaker has volume capacity of500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume ofthe beaker remains constant as temperature is varied. If $\gamma_{(beaker)}=6\times10^{-6}{}^\circ C^{-1}$ and $\gamma_{umeexpansion}^{(mercury)}=1.5\times10^{-4}{}^\circ C^{-1}$, Coefficient of$_{VO}$, then V_m (in cc) is close to where γ is the

[NA Sep. 03, 2020 (D]

Sol : Volume capacity ofbeaker, $V_0=500$ cc

$$V_b=V_0+V_0\gamma_{beaker}\Delta T$$

When beaker is partially filled with V_m volume ofmercury,

$$V_b^1=V_m+V_m\gamma_m\Delta T$$

Unfilled volume $(V_0-V_m)=(V_b-V_m^1)$

$$\Rightarrow V_0\gamma\text{ beaker}=V_m\gamma_M$$

$$V_m=\frac{V_0\gamma_{beaker}}{\gamma_M}$$

or, $V_m=\frac{500\times6\times10^{-6}}{15\times10^{-5}}=20$ cc.

3. When the temperature of a metal wire is increased from 0°C to 10°C, its length increased by 0.02%. The percentage change in its mass density will be closest to:[Sep. 02, 2020 (II)]

(a) 0.06 (b) 2.3 (c) 0.008 (d) 0.8

Sol : . (a) Change in length of the metal wire (Δl) when its temperature is changed by ΔT is given by

$$\Delta l=l\alpha\Delta T$$

Here, α = Coefficient oflinear expansion

Here, Δl = 0.02%, $\Delta T = 10^{o}C$

$$\alpha = \frac{\Delta l}{l\Delta T} = \frac{0.02}{100 \times 10}$$

$$\Rightarrow \alpha = 2 \times 10^{-5}$$

Volume coefficient of expansion, \[= $3\alpha = 6 \times 10^{-5}$

$$\rho = \frac{M}{V}$$

$$\frac{\Delta V}{V} \times 100 = \gamma \Delta T = (6 \times 10^{-5} \times 10 \times 100) = 6 \times 10^{-2}$$

Volume increase by 0.06% therefore density decrease by 0.06%.

4. A non-isotropic solid metal cube has coefficients of linear expansion as: $5 \times 10^{5}/^{\circ}C$ along the x-axis and $5 \times 10^{6}/^{\circ}C$ along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}/^{\circ}C$ then the value of C is [NA7 Jan. 2020 I]

Sol : (60.00) Volume, $V = lbh$

$$\gamma = \frac{\Delta V}{V} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

(γ = coefficient ofvolume expansion)

$$\Rightarrow \gamma = 5 \times 10^{5} + 5 \times 10^{-6} + 5 \times 10^{-6}$$

$$= 60 \times 10^{-6}/\circ c$$

Value of $C = 60.00$

5. At $40^{o}C$, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end ofthe wire. When the wire is cooled down from $40^{o}C$ to $20^{o}C$ it regains its original length of 0.2m. The value of M is close to: [12 Apri12019 I]

(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/oC and $10^{11} N/m^{2}$, respectively; $g = 10 ms^{-2}$)

(a) 9 kg (b) 0.5 kg (c) 1.5kg (d) 0.9kg

Sol : 5. (Bonus) $\Delta_{temp} = \Delta_{load}$ and $A = \pi r^{2} = \pi(10^{3})^{2} = \pi \times 10^{-6}$

$$L\alpha\Delta T = \frac{FL}{AY}$$

$$\text{or } 0.2 \times 10^{5} \times 20 = \frac{F \times 0.2}{(\pi \times 10^{-6}) \times 10^{11}}$$

$$F = 20\pi N \therefore m = \frac{f}{g} = 2\pi = 6.28 \text{ kg}$$

6. Two rods A and B of identical dimensions are at temperature 30°C. IfA is heated upto 180°C and B upto $T^{o}C$, then the new lengths are the same. If the ratio of the coefficients of linear

expansion ofA and B is 4: 3, then the value ofT is: [11 Jan. 2019 II]

(a) 230°C (b) 270°C (c) 200°C (d) 250°C

Sol : (a) Change in length in both rods are same i.e.

$$\Delta\ell_1 = \Delta\ell_2$$

$$\ell\alpha_1\Delta\theta_1 = \ell\alpha_2\Delta\theta_2$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1}\left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$

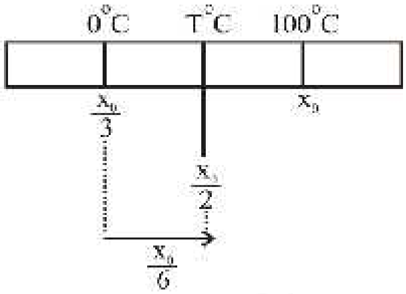
$$\frac{4}{3} = \frac{\theta - 30}{180 - 30}$$

7. Athermometer graduated according to alinear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in oC, ifthis thermometer in the contact with the object reads $x_0/2$? [11 Jan. 2019 II]

(a) 25 (b) 60 (c) 40 (d) 35

Sol: a) Let required temperature = T°C

M.P. B.P.



$$\Rightarrow T°C = - = x_0 x_0 x_0$$

2 3 6

$$\& \left(x_0 - \frac{x_0}{3}\right) = (100 - 0°C)$$

$$\Rightarrow \frac{2x_0}{3} = 100 \Rightarrow x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{x_0}{6} = \frac{150}{6} = 25°C$$

8. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is: [9 Jan. 2019 I]

(a) $\frac{F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$ (c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{2F}{A\alpha\Delta T}$

Sol : (a) Young' s modulus Y = $\frac{\text{stress}}{\text{strain}} = \frac{F/A}{A(\Delta\ell/\ell)}$ Using, coefficient oflinear expansion,

$$\alpha = \frac{\Delta\ell}{\ell\Delta T} \Rightarrow \frac{\Delta\ell}{\ell} = \alpha\Delta T$$

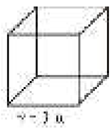
$$Y = \frac{F}{A(\alpha\Delta T)}$$

9. An external pressure P is applied on a cube at 0^{o}C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient oflinear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by: [2017]

(a) $\frac{3\alpha}{PK}$ (b) $3PK\alpha$ (c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

Sol : (c) As we know, Bulk modulus

$$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$



$$V = V_0(1 + \gamma\Delta t)$$

$$\frac{\Delta V}{V_0} = \gamma\Delta t$$

$$\frac{P}{K} = \gamma\Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

10. A steel rail of length 5 m and area of cross-section 40 cm^2 is prevented from expanding along its length while the temperature rises by 10°C. If coefficient of linear expansion and Young's modulus ofsteel are $1.2 \times 10^5 K^1$ and $2 \times 10^{11} Nm^2$ respectively, the force developed in the rail is approximately: [Online April 9, 2017]

(a) 2×10^7N (b) 1×10^5N (c) 2×10^9N (d) 3×10^5N

Sol : (b) Young' s modulus = $\frac{\text{Thermalstress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$

$$Y = \frac{F}{A\alpha\Delta\theta}\left(Q\frac{\Delta L}{L} = \alpha\Delta\theta\right)$$

Force developed in the rail $F = YA\alpha\Delta t$

$$= 2 \times 10^{11} \times 40 \times 10^4 \times 1.2 \times 10^5 \times 10$$

$$= 9.6 \times 10^4 = 1 \times 10^5 N$$

11. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let l be the length ofthe rod, A its area of cross-section, Y its Young's modulus, and α

its coefficient oflinear expansion. Then, F is equal to:[Online April 8, 2017]

(a) $l^2 Y\alpha\Delta T$ (b) $lAY\alpha\Delta T$ (c) $AY\alpha\Delta T$ (d) $\frac{AY}{\alpha\Delta T}$

Sol: (c) Due to thermal exp., change in length $(\Delta f) = l\alpha\Delta T$ (i)

Young' s modulus (Y) $= \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

$$Y = \frac{F/A}{\Delta l/l} \Rightarrow \frac{\Delta l}{l} = \frac{F}{AY}$$

$$\Delta 1 = \frac{Fl}{AY}$$

From eq^n(i), $\frac{Fl}{AY} = l\alpha\Delta T$

$F = AY\alpha\Delta T$

12. The ratio of the coefficient of volume expansion of a glass container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperatures? [Online April 23, 2013]

(a) 2:5 (b) 1:4 (c) 1:64 (d) 1:8

Sol: (b) When there is no change in liquid level in vessel then $\gamma'_{real} = \gamma'_{vessel}$

Change in volume in liquid relative to vessel $\Delta V_a P = V\gamma'\Delta\theta = V(\quad)$

13. On a linear temperature scale Y, water freezes at $-160°Y$ and boils at $-50°$ Y. On this Y scale, a temperature of340 K wouldbe read as: (water fieezes at273 K andboils at 373 K) [Online April 9, 2013]

(a) $-73.7°Y$ (b) $-233.7°Y$ (c) $-86.3°Y$ (d) $-106.3°Y$

Sol: (c) $\frac{\text{Reading on any scale} - LFP}{UFP - LFP}$ = constant for all scales

$$\frac{340-273}{373-273} = \frac{\circ Y - (-160)}{-50-(-160)}$$

$$\Rightarrow \frac{67}{100} = \frac{y+160}{110}$$

$$Y = -86.3°Y$$

14. A wooden wheel ofradius R is made oftwo semicircular part (see figure). The two parts are held together by a ring made ofa metal strip ofcross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part ofthe wheel applies on the other part is: r2012]

(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$ (c) $\pi SY\alpha\Delta$ (d) $2SY\alpha\Delta$

Sol: (d) The Young modulus is given as

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/S}{\Delta L/L}$$

Here, $\Delta L = 2\pi\Delta RL = 2\pi R$

$$Y = \frac{F}{S2\pi\Delta R} \times 2\pi R$$

$$\Rightarrow Y = \frac{FR}{S.\Delta R} \dots (i)$$

The coefficient oflinear expansion $\alpha = \frac{\Delta R}{R\Delta T}$

$$\Rightarrow \frac{\Delta R}{R} = \alpha.\Delta T \Rightarrow \frac{R}{\Delta R} = \frac{1}{\alpha\Delta T} \dots (ii)$$

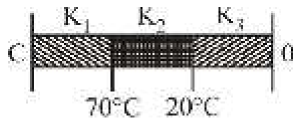
From equation (i) and(ii)

$$Y = \frac{F}{S.\alpha\Delta T} \Rightarrow F = Y.S.\alpha\Delta T$$

The ring is pressing the wheel from both sides, Thus

$$F_{net} = 2F = 2YS\alpha\Delta T$$

15. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They arejoined together at their ends to make a long rod(see figure). One end ofthe long rod is maintained at $100^\circ C$ and the other at $0^\circ C$ (see figure). If the joints of the rod are at $70^\circ C$ and $20^\circ C$ in steady state and there is no loss of energy fiom the surface of the rod, the correct relationship between K_1, K_2 and K_3 is: [Sep. 06, 2020 (II)]



(a) $K_1 : K_3 = 2 : 3$, $K_1 < K_3 = 2 : 5$ (b) $K_1 < K_2 < K_3$

(c) $K_1 : K_2 = 5 : 2$, $K_1 : K_3 = 3 : 5$ (d) $K_1 > K_2 > K_3$

Sol: (a) As the rods are identical, so theyhave same length (l) and area ofcross - section (A) . They are connected in series. So, heat current will be same for all rods.

Heat current $= \left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD}$

$$\Rightarrow \frac{(100-70)K_1A}{l} = \frac{(70-20)K_2A}{l} = \frac{(20-0)K_3A}{l}$$

$$\Rightarrow K_1(100-70) = K_2(70-20) = K_3(20-0)$$

$$\Rightarrow K_1(30) = K_2(50) = K_3(20)$$

$$\Rightarrow \frac{K_1}{10} = \frac{K_2}{6} = \frac{K_3}{15}$$

$$\Rightarrow K_1 : K_2 : K_3 = 10 : 6 : 15$$

$$\Rightarrow K_1 : K_3 = 2 : 3.$$

16. A bullet of mass 5 g, travelling with a speed of210 m/s, strikes a fixed wooden target. One halfofits kinetics energy is converted into heat in the bullet while the other half is converted

into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g–°C) (1 cal = 4.2×10^7 ergs) close to: [Sep. 05, 2020 (D)]

(a) $87.5^{\circ}C$ (b) 83.3° (c) $119.2^{\circ}C$ (d) $38.4^{\circ}C$

Sol: (a) According to question, one half ofits kinetic energy is converted into heat in the wood.

$$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

$$\Rightarrow \Delta T = \frac{v^2}{4 \times s} = \frac{210 \times 210}{4 \times 4.2 \times 0.3 \times 1000} = 87.5^{\circ}C$$

17. The specific heat ofwater = $4200 Jkg^{-1}K^{-1}$ and the latent heat of ice = $3.4 \times 10^5 Jkg^{-1}$. 100 grams of ice at 0°C is placed in 200 g ofwater at 25°C. The amount Of ice that will melt as the temperature ofwater reaches 0°C is close to (in grams): [Sep. 04, 2020 (I)]

(a) 61.7 (b) 63.8 (c) 69.3 (d) 64.6

Sol: (a) Here ice melts due to water.

Let the amount ofice melts = m_{ice}

$$m_w s_w \Delta\theta = m_{ice} L_{ice}$$

$$m_{ice} = \frac{m_w s_w \Delta\theta}{L_{ice}}$$

$$= \frac{0.2 \times .4200 \times 25}{34 \times 10^5} = 0.0617kg = 61.7g$$

18. A calorimter ofwater equivalent 20 g contains 180 g of water at 25°C. m' grams of steam at 100°C is mixed in it till the temperature ofthe mixture is 31°C. The value of'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = $1 cal g^{-1} {}^{\circ}C^{-1}$) [Sep. 03, 2020 (II)]

(a) 2 (b) 4 (c) 32 (d) 2.6

Sol: (a) Heat given by water = $m_w C_w (T_{mix} - T_w)$

$$= 200 \times 1 \times (31 - 25)$$

Heat taken by steam = $mL_{stem} + mC_w(T_s - T_{mix})$

$$= m \times 540 + m(1) \times (100 - 31)$$

$$= m \times 540 + m(1) \times (69)$$

From the principal ofcalorimeter,

Heat lost = Heat gained

$$(200)(31 - 25) = m \times 540 + m(1)(69)$$

$$\Rightarrow 1200 = m(609) \Rightarrow m \approx 2.$$

19. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature Twhen different amounts ofwater (given in liters) are taken f : om each container and mixed (assume no loss of heat during the process) [8 Jan. 2020 II]

C_1 C_2 C_3 T

$1l$ $2l$ $60^\circ C$

$1l$ $2l$ $30^\circ C$

$2l$ $1l$ $60^\circ C$

$1l$ $1l$ $1l$ θ

The value of θ (inC to the nearest integer) is________ .

Sol: (50.00)

Let Q_1, Q_2, Q_3 be the temperatures ofcontainer C_1, C_2 and C_3 respectively.

Using principle ofcalorimetry in container C_1, we have

$$(\theta_1 - 60) = 2ms(60 - \theta)$$

$$\Rightarrow \theta_1 - 60 = 120 - 2\theta$$

$$\Rightarrow \theta_1 = 180 - 2\theta \text{ (i)}$$

For container C_2

$$ms\ (\theta_2 - 30) = 2ms(30 - \theta)$$

$$\Rightarrow \theta_2 = 90 - 2\theta 3 \text{ (ii)}$$

For container C_3

$$2ms(\theta_1 - 60) = ms(60 - \theta)$$

$$\Rightarrow 2\theta_1 - 120 = 60 - \theta$$

$$\Rightarrow 2\theta_1 + \theta = 180 \text{ (iReject)}$$

Also, $\theta_1 + \theta_2 + \theta_3 = 3\theta$ (iv)

Adding (i), (ii) and(iii)

$$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$$

$$\Rightarrow \theta_1 + \theta_2 + \theta_3 = 150$$

$$\Rightarrow 3\theta = 150 \Rightarrow \theta = 50^\circ C$$

20. M grams ofsteam at 100°C is mixed with 200 g ofice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat ofvaporization ofwater is 540 cal/g and heat offusion ofice is 80 cal/g], the value of M is_____ [NA7 Jan. 2020 II]

Sol: (40) Using the principal ofcalorimetry

$$M_{ice}L_f + m_{ice}(40 - 0)C_w$$

$$= m_{stream}L_v + m_{stream}(100 - 40)C_w$$

$$\Rightarrow M(540) + M \times 1 \times (100 - 40)$$

$$= 200 \times 80 + 200 \times 1 \times 40$$

$$\Rightarrow 600M = 24000$$

$$\Rightarrow M = 40g$$

21. When M_1 gram ofice at -10^0C(Specific heat $= 0.5$ cal $g^{10}C^1$) is added to M_2 gram of water at 50^0C, finally no ice is left and the water is at 0^0C. The value oflatent heat of ice, in cal g^1 is: [12 April 2019 I]

(a) $\frac{50M_2}{M_1} - 5$ (b) $\frac{5M_1}{M_2} - 50$ (c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

Sol: (a) $M_1 C_{ice} \times (10) + M_1 L = M_2 C_w (50)$

or $M_1 \times C_{ice} (= 0.5) \times 10 + M_1 L = M_2 \times 1 \times 50$

$$\Rightarrow L = \frac{50 M_2}{M_1} - 5$$

22. A massless spring (K = 800N/m) , attached with a mass (500g) is completely immersed in lkg of water. The spring is stretched by2cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat ofmass = 400 Jlkg K, specific heat ofwater = 4184J/kgK) [9 April 2019 II]

(a) 10<K (b) 10 5K (c) 10 1K (d) 10 3K

Sol: (b) $\frac{1}{2}.kx^2 = mC(\Delta T) + m_{x0} C_{x0} \Delta T$

$$\text{or } \frac{1}{2} \times 800 \times 0.02^2 = 0.5 \times 400 \times \Delta T + 1 \times 4184 \times \Delta T$$

$$\Delta T = 1 \times 10^{-5} K$$

$d\ \theta\ 3d$

23. Two materials having coefficients ofthermal conductivity ‘ $3K$’ and ‘K’ and thickness ‘d’ and ‘ $3d$ ’, respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2’ and θ_1’ respectively, $(\theta_2 > \theta_1)$. The temperature at the interface is: [9 April 2019 II]

(a) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$ (b) $\frac{\theta_2 + \theta_1}{2}$ (c) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (d) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$

Sol: a) $H_1 = H_2 \theta_2 3k\ k\theta_1$

$3k$	k

$$\text{or } (3k)A\left(\frac{\theta_2 - \theta}{d}\right) = kA\left(\frac{\theta - \theta_1}{3d}\right)$$

$$\text{or } \theta = \left(\frac{\theta_1 + 9\theta_2}{10}\right)$$

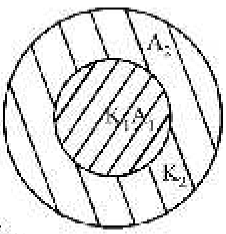
24. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity ofthe material ofthe inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity ofthe system for heat flowing along the length ofthe cylinder is: [12 Jan. 2019 I]

(a) $\frac{K_1 + K_2}{2}$ (b) $K_1 + K_2$ (c) $\frac{2K_1 + 3K_2}{5}$ (d) $\frac{K_1 + 3K_2}{4}$

Sol: (d) Effective thermal conductivity of system

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

$$= \frac{K_1 \pi R^2 + K_2[\pi(2R)^2 - \pi R^2]}{\pi(2R)^2}$$



$$= \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{} = \frac{K_1 + 3K_2}{}$$

$$= \overline{4\pi R^2} = \overline{4}$$

25. Ice at $-20°C$ is added to 50 g ofwater at 40°C, When the temperature ofthe mixture reaches 0°C, it is found that 20 g ofice is still unmelted. The amount of ice added to the water was close to [11 Jan. 2019 I] (Specific heat ofwater $= 4.2 J/g/°C$, Specific heat ofIce $= 2.1 J/g°C$ Heat offusion ofwater at $0°C = 334 J/g$)

(a) 50g (b) 100 g (c) 60 g (d) 40 g

Sol: (d) Let m gram ofice is added.

From principal ofcalorimeter

heat gained (by ice) = heat lost (by water)

$$20 \times 2.1 \times m + (m - 20) \times 334$$
$$= 50 \times 4.2 \times 40$$
$$376m = 8400 + 6680$$
$$m = 40.1$$

26. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature ofthe mixture becomes 90°C. The temperature ofthe mixture, if100 g of liquid A at 100°C is added to 50 g ofliquid B at 50°C, will be: [11 Jan. 2019 II]

(a) 85°C (b) 60°C (c) 80°C (d) 70°C

Sol: (c) Heat loss = Heat gain = $mS\Delta\theta$

$$\text{So, } m_A S_A \Delta\theta_A = m_B S_B \Delta\theta_B$$
$$\Rightarrow 100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$$
$$2S_A = 1.5S_B \Rightarrow S_A = \frac{3}{4}S_B$$
$$\text{Now, } 100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$$
$$2 \times \left(\frac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$$
$$300 - 3\theta = 2\theta - 100$$
$$400 = 5\theta \Rightarrow \theta = 80°C$$

27. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature ofwater and vessel is 30°C. What is the approximate percentage increment in the temperature ofthe water? [Specific Heat Capacities ofwater and metal are, respectively, 4200 Jkg^{-} $1K - 1$ and400 $Jkg^{-1}K^{-1}$] [11 Jan. 2019 II]

(a) 15% (b) 30% (c) 25% (d) 20%

Sol: (d) Assume final temperature = T°C

Heat lass = Heat gain = $ms\Delta T$

$$\Rightarrow m_B s_B \Delta T_B = m_w s_w \Delta T_w$$
$$0.1 \times 400 \times (500 - T)$$
$$= 0.5 \times 4200 \times (T - 30) + 800\,(T - 30)$$

$$\Rightarrow 40(500 - T) = (T - 30)(2100+800)$$

$$\Rightarrow 20000 - 40T = 2900T - 30 \times 2900$$

$$\Rightarrow 20000 + 30 \times 2900 = T(2940)$$

$$T = 30.4°C$$

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100 = 21\%,$$

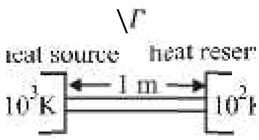
so the closest answer is 20%.

Temp. of Temp. of], }roir

28. A heat source at $T = 10^3 K$ is connected to another heat reservoir at $T = 10^2 K$ by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 $WK^{-1}m^{-1}$, the energy flux through it in the steady state is: [10 Jan. 2019 I]

(a) 90 Wm 2 (b) $120Wm^2$ (c) 65 Wm 2 (d) $200Wm^2$

Sol: (a)



$$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$$

Energy flux, $\frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{k\Delta T}{\ell}$

$$= \frac{(0.1)(900)}{1} = 90W/m^2$$

29. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g ofwater at a temperature of8.4°C. Calculate the specific heat ofthe unknown metal ifwater temperature stablizes at 21.5°C. (Specific heat ofbrass is 394 $Jkg^{-1}K^{-1}$) [10 Jan. 2019 II]

(a) 458 $Jkg^{-1}K^{-1}$ (b) 1232 $Jkg^{-1}K^{-1}$ (c) $916Jkg^{-1}K^{-1}$ (d) $654Jkg^{-1}K^{-1}$

Sol: (c) Let specific heat ofunknown metal be s' According to principle of calorimetry, Heat lost

$= $ Heat gain $m \times s\Delta\theta = m_1 s_{brass}(\Delta\theta_1 + m_2 s_{water} + \Delta\theta_2)$

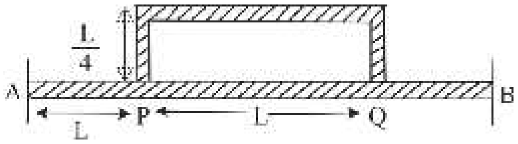
$$\Rightarrow 192 \times S \times (100 - 21.5)$$

$$= 128 \times 394 \times (21.5 - 8.4)$$

Solving we get, $+240 \times 4200 \times (21.5 - 8.4)$

$$S = 916Jkg^{-1}k^{-1}$$

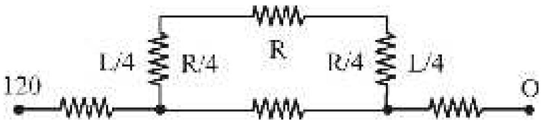
30. Temperature difference of 120°C is maintained between two ends ofa uniform rod AB oflength 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to: [9 Jan. 2019 I]



2

(a) 45°C (b) 75°C (c) 60°C (d) 35°C

Sol: 30. (a) $\frac{\Delta T_{AB}}{R_{AB}} = \frac{120}{\frac{8}{5}R} = \frac{120\times5}{8R}$



A R/2 P Q R/2 B

In steady state temperature difference between P and

Q,

$$\Delta T_{PQ} = \frac{120\times5}{8R}\times\frac{3}{5}R = \frac{360}{8} = 45^\circ C$$

31. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given: room temperature = 30°C, specific heat of copper = 0.1ca1/gm°C [2017]

(a) 1250°C (b)825°C (c) 800°C (d) 885°C

Sol: (d) According to principle ofcalorimetry,

Heat lost = Heat gain

$$100\times0.1(T-75) = 100\times0.1\times45 + 170\times1\times45$$

$$10\,T - 750 = 450 + 7650 = 8100$$

$$\Rightarrow T - 75 = 810$$

$$T = 885^\circ C$$

32. In an experiment a sphere ofaluminium ofmass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature ofthe system is 40°C. The specific heat of aluminium is: (take 4.2 Joule = 1 calorie) [Online April 8, 2017]

(a) 378 J/kg −∘ c (b) 315 J/kg −∘ c

(c) 476 J/kg −∘ c (d) 434 J/kg −∘ c

Sol: (d) According to principle ofcalorimetry,

$$Q_{given} = Q_{used}$$

$$0.2\times S\times(150-40) = 150\times1\times(40-27) + 25\times(40-27)$$

$$0.2\times S\times110 = 150\times13 + 25\times13$$

Specific heat ofaluminium

$$S = \frac{13\times25\times7}{0.2\times110} = 434 J/kg-^\circ C$$

33. An experiment takes 10 minutes to raise the temperature of water in a container from $0^{o}C$ to $100^{o}C$ and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal/g^{o}C, the heat of vapourization according to this experiment will come out to be : [Online April 11, 2015]

(a) 560 cal/g (b) 550 cal/g (c) 540 cal/g (d) 530 cal/g

Sol: (b) As $Pt = mC\Delta T$

So, $P \times 10 \times 60 = mC100$ ... (i)

and $P \times 55 \times 60 = mL$ (ii) Dividing equation (i) by (ii) we get

$$\frac{10}{55} = \frac{C \times 100}{L}$$

$L = 550$ cal. /g.

34. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross- section of each rod = $4cm^{2}$. End ofcopper rod is maintained at $100^{o}C$ where as ends ofbrass and steel are kept at $0^{o}C$. Lengths ofthe copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate ofheat flow through copper rod is:[2014]

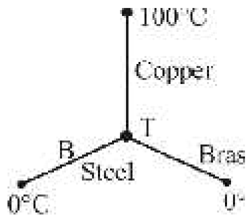
(a) 1.2cal/s (b) 2.4cal/s (c) 4.8cal/s (d) 6.0cal/s

Sol: (c) Rate ofheat flow is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)}{l}$$

Where, K = coefficient ofthermal conductivity

l = length ofrod and A = area ofcross - section ofrod



s

C

Ifthe junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$$\frac{0.92 \times 4(100 - T)}{46}$$

$$= \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times (T - 0)}{12}$$

$$\Rightarrow 200 - 2T = 2T + T$$

$$\Rightarrow T = 40^\circ C$$

$$Q_{Copper} = \frac{0.92 \times 4 \times 60}{46} = 4.8 cal/s$$

35. A black coloured solid sphere ofradius R and mass M is inside a cavitywith vacuum inside. The walls of the cavity are maintained at temperature T_0. The initial temperature ofthe sphere is $3T_0$. Ifthe specific heat ofthe material of the sphere varies as αT^3 per unit mass with the temperature T ofthe sphere, where α is a constant, then the time taken for the sphere to cool down to temperature $2T_0$ will be (0 is Stefan Boltzmann constant) [Online Apri119, 2014]

(a) $\frac{M\alpha}{4\pi R^2 \sigma} \ln\left(\frac{3}{2}\right)$ (b) $\frac{M\alpha}{4\pi R^2 \sigma} \ln\left(\frac{16}{3}\right)$ (c) $\frac{M\alpha}{16\pi R^2 \sigma} \ln\left(\frac{16}{3}\right)$ (d) $\frac{M\alpha}{16\pi R^2 \sigma} \ln\left(\frac{3}{2}\right)$

Sol: (c) In the given problem, fall in temperature ofsphere,

$$dT = (3T_0 - 2T_0) = T_0$$

Temperature ofsurrounding, $T_{surr} = T_0$

Initial temperature ofsphere, $T_{initia1} = 3T_0$

Specific heat ofthe material ofthe sphere varies as,

$c = \alpha T^3$ per unit mass (α = a constant)

Applying formula,

$$\frac{dT}{dt} = \frac{\sigma A}{McJ}(T^4 - T_{surr}^4)$$

$$\Rightarrow \frac{T_0}{dt} = \frac{\sigma 4\pi R^2}{M\alpha(3T_0)^3 J}[(3T_0)^4 - (T_0)^4]$$

$$\Rightarrow dt = \frac{M\alpha 27 T_0^4 J}{\sigma 4\pi R^2 \times 80 T_0^4}$$

Solving we get,

Time taken for the sphere to cool down temperature $2T_0$,

$$t = \frac{M\alpha}{16\pi R^2 \sigma} \ln\left(\frac{16}{3}\right)$$

36. Water of volume 2 L in a closed container is heated with a coil of 1 kW. While water is heated, the container loses energy at a rate of 160 J/s. In how much time will the temperature ofwater rise fiiom27°C to 77°C? (Specific heat ofwater is 4.2kJ/kg and that ofthe container is negligible). [Online April 9, 2014]

(a) 8 min 20s (b) 6 min 2s (c) 7 min (d) 14 min

Sol: (a) From question,

In 1 sec heat gained by water

= 1 KW - 160 J/s

$$= 1000 J/s - 160 J/s$$

$$= 840 J/s$$

Total heat required to raise the temperature ofwater (volume 2L)fiiom27°c to 77°c

$= m_{water} \times sp.\ ht \times \Delta\theta$

$= 2 \times 10^3 \times 4.2 \times 50$ [mass =density × volume]

And, $840 \times t = 2 \times 10^3 \times 4.2 \times 50$

or, $t = \frac{2\times10^3\times4.2\times50}{840}$

$= 500s = 8$ min $20s$

37. Assume that a drop ofliquid evaporates by decrease in its surface energy, so that its temperature remains unchanged.What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is p and L is its latent heat of vaporization. [2013]

(a) pL/T (b) $\sqrt{\frac{T}{pL}}$ (c) T/pL (d) 2T/pL

Sol: (d) When radius is decrease by ΔR,

$$4\pi R^2 \Delta R p L = 4\pi T[R^2 - (R - \Delta R)^2]$$

$$\Rightarrow pR^2 \Delta R L = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$$

$$\Rightarrow pR^2 \Delta R L = T2R\Delta R \text{ [}\Delta R \text{ is very small]}$$

$$\Rightarrow R = \frac{2T}{pL}$$

38. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from30°C to 25°C. Amass of 00g ofanother liquid in an identical vessel with identical surroundings takes the same time to cool from 30°C to 25° C. The specific heat of the liquid is: (The water equivalent ofthe vessel is 30g.) [Online April 25, 2013]

(a) 2.0kcal/kg (b) 7 kcal/kg (c) 3 kcal/kg (d) 0.5 kcal/kg

Sol: d) As the surrounding is identical, vessel is identical time taken to cool both water and liquid(from 30°Cto25°C) is same 2 minutes, therefore

$\left(\frac{dQ}{dt}\right)$ water $= \left(\frac{dQ}{dt}\right)$ liquid

39. 500 g ofwater and 100 g ofice at 0°C are in a calorimeter whose water equivalent is 40 g. 10g ofsteam at 100°C is added to it. Then water in the calorimeter is: (Latent heat of ice = 80cal/g, Latent heat ofsteam = 540cal/g) [Online April 23, 2013]

(a) 580 g (b) 590 g (c) 600 g (d) 610 g

Sol: (b) As lg ofsteam at 100°C melts 8g ofice at 0°C. 10 g of steam will melt 8 × 10g ofice at 0°C

Water in calorimeter = 500 + 80 + 10g = 590g

40. Given that 1 g ofwater in liquid phase has volume 1 cm^3 and in vapour phase 1671 cm^3 at atmospheric pressure and the latent heat ofvaporization ofwater is 2256 J/g; the change in the internal energy injoules for 1 g ofwater at 373 K when it changes from liquid phase to vapour phase at the same temperature is : [Online Apri122, 2013]

(a) 2256 (b) 167 (c) 2089 (d) 1

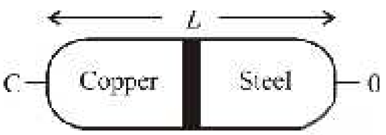
Sol : c

41. A large cylindrical rod oflength L is made byjoining two identical rods of copper and steel of length $\left(\frac{L}{2}\right)$ each. The rods are completely insulated from the surroundings. If the free end of copper rod is maintained at 100°C and that of steel at 0°C then the temperature of junction is (Thermal conductivity of copper is 9 times that of steel) Online May 19, 2012]

(a) 90°C (b) 50°C (c) 10°C (d) 67°C

Sol : (a)

100° ∘ c



← L/2 – L/2 →

Let conductivity of steel $K_{steel} = k$ then from question Conductivity of copper $K_{copper} = 9k$

$$\theta_{copper} = 100°C$$

$$\theta_{steel} = 0°C$$

$$l_{steel} = l_{copper} = \frac{L}{2}$$

From formula temperature ofjunction;

$$\theta = \frac{K_{copper}\theta_{copper}l_{steel} + K_{steel}\theta_{steel}l_{copper}}{K_{copper}l_{steel} + K_{steel}l_{copper}}$$

$$= \frac{9k \times 100 \times \frac{L}{2} + k \times 0 \times \frac{L}{2}}{9k \times \frac{L}{2} + k \times \frac{L}{2}}$$

$$= \frac{\frac{900}{2}kL}{\frac{10kL}{2}} = 90°C$$

42. The heat radiated per unit area in 1 hour by a furnace whose temperature is 3000 K is $(\sigma = 5.7 \times 10^{-8} Wm^{-2}K^{-4})$ [Online May 7, 2012]

(a) 1.7×10^{10}J (b) 1.1×10^{12}J (c) 2.8×10^{8}J (d) 4.6×10^{6}J

Sol : (a) According to Stefan's law

$$E = \sigma T^4$$

Heat radiated per unit area in 1 hour (3600s) is

$$= 5.7 \times 10^{-8} \times (300)^4 \times 3600 = 1.7 \times 10^{10} J$$

43. 100g ofwater is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat ofwater is 4184 J/kg/K): [2011]

(a) 8.4kJ (b) 84 kJ (c) 2.1kJ (d) 4.2kJ

Sol : a) $\Delta U = \Delta Q = mc\Delta T$

$$= \frac{100}{1000} \times 4184(50 - 30) \approx 8.4kJ$$

44. The specific heat capacity ofa metal at low temperature (T) is given as

$$C_p(kJK^{-1}\text{kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C) . The amount of work required to cool the vessel is [2011 RS]

(a) greater than 0.148kJ (b) between 0.148 kJ and 0.028 kJ

(c) less than 0.028 kJ (d) equal to 0.002 kJ

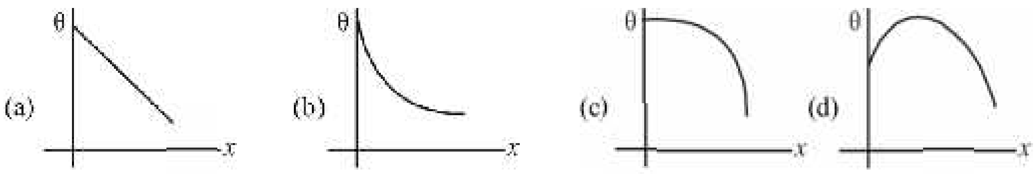
Sol : (d) Required work = energy released

Here, $Q = \int \; mcdT$

$$= \int_{20}^{4} 0.1 \times 32 \times (\quad)^{dT} (\quad) = \int_{20}^{4} \frac{3.2}{64 \times 10^6} T^3 dT$$

$$= 5 \times 10^{-8} \int_{20}^{4} T^3 dT = 0.002kJ$$

45. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x ofthe bar fi;om its hot end is best described by which ofthe following figures?[2009]



Sol: . (a) Let Q be the temperature at a distance x from hot end of bar. Let Q is the temperature ofhot end.

The heat flow rate is given by

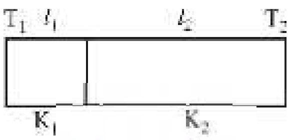
$$\frac{dQ}{dt} = \frac{kA(\theta_1 - \theta)}{x}$$

$$\Rightarrow 9_1 - \theta = \frac{xdQ}{kAdt} \Rightarrow \theta = \theta_1 - \frac{xdQ}{kAdt}$$

Thus, the graph of$Q$ versus x is a straight line with a positive intercept and a negative slope.

The above equation can be graphically represented by option (a).

46. One end ofa thermally insulated rod is kept at atemperature T_1 and the other at T_2. The rod is composed oftwo sections oflength l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two section is [2007]



(a) $\frac{(K_1 l_1 T_1 + K_2 l_2 T_2)}{(K_1 l_1 + K_2 l_2)}$ (b) $\frac{(K_2 l_2 T_1 + K_1 l_1 T_2)}{(K_1 l_1 + K_2 l_2)}$ (c) $\frac{(K_2 l_1 T_1 + K_1 l_2 T_2)}{(K_2 l_1 + K_1 l_2)}$ (d) $\frac{(K_1 l_2 T_1 + K_2 l_1 T_2)}{(K_1 l_2 + K_2 l_1)}$

47. Assuming the Sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant powerd incident ofEarth at a distance $r$fi;om the Sun [2006]

(a) $4\pi r_0^2 R^2 \sigma \frac{T^4}{r^2}$ (b) $\pi r_0^2 R^2 \sigma \frac{T^4}{r^2}$ (c) $r_0^2 R^2 \sigma \frac{T^4}{4\pi r^2}$ (d) $R^2 \sigma \frac{T^4}{r^2}$

where r_0 is the radius ofthe Earth and 0 is Stefan's constant.

Sol : (b) From stefan' s law, totalpower radiated by Sun, $E = \sigma T^4 \times 4\pi R^2$

The intensity ofpower Per unit area incident on earth's surface

$$= \frac{\sigma T^4 \times 4\pi R^2}{4\pi r^2}$$

Total power received by Earth

$$E' = \frac{E}{4\pi r^2} \times \text{Cross- Section area of earth facing the sun} = \frac{\sigma T^4 R^2}{r^2}(\pi r_0^2)$$

48. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box contains one mole ofhelium at temperature $\left(\frac{7}{3}\right) T_0$. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common fmal temperature (ignore the heat capacity of boxes). Then, the fmal temperature of the gases, T_f in terms of T_0 is [2006]

(a) $T_f = \frac{3}{7} T_0$ (b) $T_f = \frac{7}{3} T_0$ (c) $T_f = \frac{3}{2} T_0$ (d) $T_f = \frac{5}{2} T_0$

Sol : (c) When two gases are mixed to gether then Heat lost by He gas = Heat gained by N_2 gas

$$n_1 C_{v_1} \Delta T_1 = n_2 C_{v_2} \Delta T_2$$

$$\frac{3}{2} R \left[\frac{7}{3} T_0 - T_f\right] = \frac{5}{2} R [T_f - T_0]$$

$$7T_0 - 3T_f = 5T_f - 5T_0$$

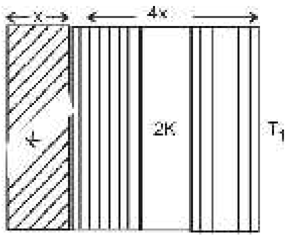
$$\Rightarrow 12T_0 = 8T_f \Rightarrow T_f = \frac{12}{8} T_0$$

$$\Rightarrow T_f = \frac{3}{2} T_0.$$

49. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio ofthe radiant energy received on earth to what it was previously will be [2004]

(a) 32 (b) 16 (c) 4 (d) 64

50. The temperature of the two outer surfaces ofa composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to [2004]



(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

SOLUTION:'

The thermal resistance is given by

$$\frac{x}{KA}+\frac{4x}{2KA}=\frac{x}{KA}+\frac{2x}{KA}=\frac{3x}{KA}$$

Amount of heat flow per second,

$$\frac{dQ}{dt}=\frac{\Delta T}{\frac{3x}{KA}}=\frac{(T_2-T_1)KA}{3x}$$
$$=\frac{1}{3}\left\{\frac{A(T_2-T_1)K}{x}\right\}f=\frac{1}{3}$$

51. The earth radiates in the infia-red region ofthe spectrum. The spectrum is correctly given by [2003]

(a) Rayleigh Jeans law (b) Planck's law of radiation

(c) Stefan's law of radiation (d) Wien's law

Sol : (d) Wein' s law correctly explains the spectrum

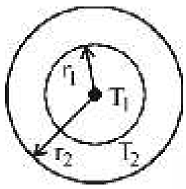
52. Heat given to a bodywhich raises its temperature by 1°C is [2002]

(a) water equivalent (b) thermal capacity

(c) specific heat (d) temperature gradient

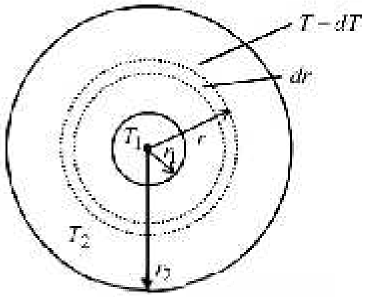
Sol: (b) Heat required for raising the temperature of a body through 1o*Reject* is called its thermal capacity.

53. The figure shows a system of two concentric spheres of radii r_1 and r_2 are kept at temperatures T_1 and T_2, respectively. The radial rate offlow ofheat in a substance between the two concentric spheres is proportional to [2005]



(a) $l_2\left(\frac{r_2}{r_1}\right)$ (b) $\frac{(r_2-r_1)}{(r_1r_2)}$ (c) (r_2-r_1) (d) $\frac{r_1r_2}{(r_2-r_1)}$

(d)



Consider a thin concentric shell ofthickness (dr) and of radius (r) and let the temperature of inner and outer surfaces ofthis shell be T and $(T - dT)$ respectively.

The radial rate of flow of heat through this elementary shell will be

$$\frac{dQ}{dt} = \frac{KA[(T - dT) - T]}{dr} = \frac{-KAdT}{dr}$$

$$= -4\pi K r^2 \frac{dT}{dr} \ (A = 4\pi r^2)$$

Since the area of the surface through which heat will flow is not constant. Integrating both sides between the limits of radii and temperatures of the two shells, we get

$$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2} \frac{1}{r^2} dr = -4\pi K \int_{T_1}^{T_2} dT$$

$$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2} r^{-2} dr = -4\pi K \int_{T_1}^{T_2} dT$$

$$\frac{dQ}{dt}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = -4\pi K[T_2 - T_1]$$

or $\frac{dQ}{dt} = \frac{-4\pi K r_1 r_2 (T_2 - T_1)}{(r_2 - r_1)}$

$$\frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

54. Infrared radiation is detected by [2002]

(a) spectrometer (b) pyrometer

(c) nanometer (d) photometer

Sol : (b) Pyrometer is used to detect infia - red radiation.

55. Which of the following is more close to a black body?[2002]

(a) black board paint (b) green leaves

(c) black holes (d) red roses

Sol: (a) Black body is one which absorb all incident radiation. Black board paint is quite approximately equal to black bodies.

56. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass ofwater should[2002]

(a) increase (b) remain unchanged

(c) decrease (d) first increase then decrease

Sol: (c) When water is cooled at 0°C to form ice, energy is released from water in the form of heat. As energy is equivalent to mass, therefore, when water is cooled to ice, its mass decreases.

57. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio ofthe energy radiated per second by the first sphere to that by the second is [2002]

(a) 1: 1 (b) 16: 1 (c) 4: 1 (d) 1: 9.

Sol: (a) From stefan's law, the energy radiated per second is given by $E = e\sigma T^4 A$

Here, T = temperature of the body

A = surface area of the body

For same material e is same. σ is stefan's constant

Let T_1 and T_2 be the temperature of two spheres. A_1 and A_2 be the area of two spheres.

$$\frac{E_1}{E_2} = \frac{T_1^4 A_1}{T_2^4 A_2} = \frac{T_1^4 4\pi r_1^2}{T_2^4 4\pi r_2^2}$$

$$= \frac{(4000)^4 \times 1^2}{(2000)^4 \times 4^2} = \frac{1}{1}$$

58. A metallic sphere cools from 50°C to 40°C in300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to: [Sep. 03, 2020 (ID]

(a) 31°C (b) 33°C (c) 28°C (d) 35°C

Sol: (b) From Newton's Law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Here, $T_1 = 50°C$, $T_2 = 40°C$

an d$T_0 = 20°C$, $t = 600S = 5$ minutes

$$\Rightarrow \frac{50-40}{5\text{ Min}} = K\left(\frac{50+40}{2} - 20\right) \quad \text{(i)}$$

Let T be the temperature of sphere after next 5 minutes. Then

$\frac{40-T}{5} = K\left(\frac{40+T}{2} - 20\right)$ (ii)

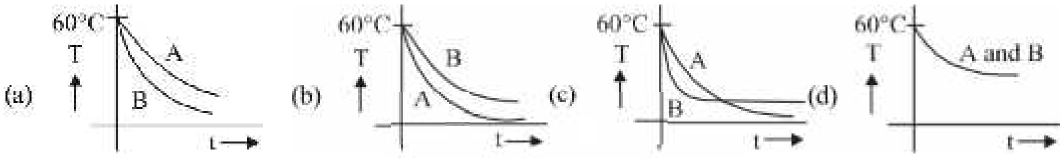
Dividing eqn. (ii) by (i), we get

$$\frac{40 - T}{10} = \frac{40 + T - 40}{50 + 40 - 40} = \frac{T}{50}$$

$$\Rightarrow 40 - T = \frac{T}{5} \Rightarrow 200 - 5T = T$$

$$T = \frac{200}{6} = 33.3°C$$

59. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in Ahas density of$8 \times 10^2 kg/m^3$ and specific heat of 2000 J kg 1K[1] while liquid in B has density of103 kg m[3] and specific heat of4000 J kg 1K[1]. Which ofthe following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same) [8 April 2019 I]



Sol : (b) Rate of Heat loss = $mS\left(\frac{dT}{dt}\right) = e\sigma AT^4$

$$-\frac{dT}{dt} = \frac{e\sigma \times A \times T^4}{\rho \times Vol. \times S} \Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho S}$$

$$\frac{\left(-\frac{dT}{dt}\right)_A}{\left(-\frac{dT}{dt}\right)_B} = \frac{\rho_B}{\rho_A} \times \frac{S_B}{S_A} = \frac{10^3}{8 \times 10^2} \times \frac{4000}{2000}$$

$$\Rightarrow \left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

60. A bodytakes 10 minutes to cool fi;om60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately [Online Apri115, 2018]

(a) 43°C (b) 47°C (c) 41°C (d) 45°C

Sol : . (a) According to Newton's law of cooling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60-50}{10}\right) = K\left(\frac{60+50}{2} - 25\right) \text{..... (i)}$$

and, $\left(\frac{50-\theta}{10}\right) = K\left(\frac{50+\theta}{2} - 25\right)$ ..... (ii) Dividing eq. (i) by (ii),

$$\frac{10}{(50-\theta)} = \frac{60}{\theta} \Rightarrow \theta = 42.85°C \cong 43°C$$

61. Hot water cools fi;om60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is: [Online April 12, 2014]

(a) 25°C (b) 10°C (c) 15°C (d) 20°C

Sol : (b) By Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = -K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

where θ_0 is the temperature of surrounding.

Now, hot water cools from 60°C to 50°C in 10 minutes, $\frac{60-50}{10} = -K\left[\frac{60+50}{2} - \theta_0\right]$ (i)

Again, it cools fi;om50°C to 42°C in next 10 minutes. $\frac{50-42}{10} = -K\left[\frac{50+42}{2} - \theta_0\right]$ (ii)

Dividing equations (i) by (ii) we get

$$\frac{1}{0.8} = \frac{55 - \theta_0}{46 - \theta_0}$$

$$\frac{10}{8}=\frac{55-\theta_0}{46-\theta_0}$$

$$460-10\theta_0=440-8\theta_0$$

$$2\theta_0=20$$

$$\theta_0=10^\circ C$$

62. A hot body, obeying Newton's law of cooling is cooling down from its peak value 80°C to an ambient temperature of30°C. It takes 5 minutes in cooling down from 80°C to 40°C. How much time will it take to cool down from62°C to 32°C?

(Given In 2 = 0.693, In 5 = 1.609) [Online April 11, 2014]

(a) 3.75 minutes (b) 8.6 minutes (c) 9.6 minutes (d) 6.5 minutes

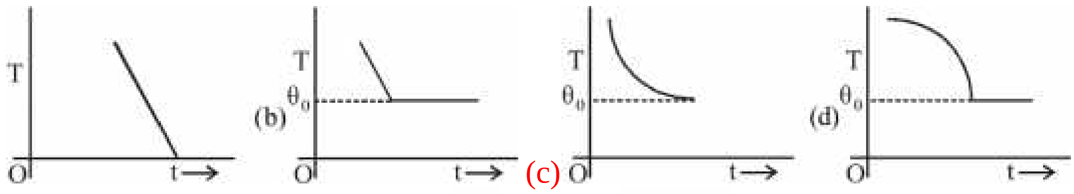
Sol : . (b) From Newton's law of cooling,

$$t=\frac{1}{k}\log_e\left(\frac{\theta_2-\theta_0}{\theta_1-\theta_0}\right)$$

From question and above equation,

63. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T ofthe metal and time t will be closest to [2013]

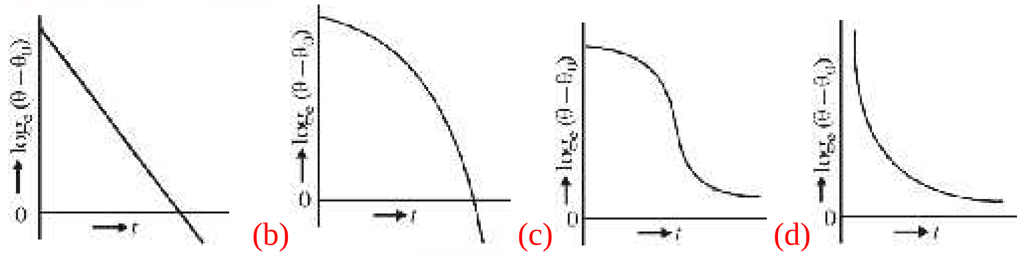
(a) (b) (c) (d)



Sol : (c) According to Newton's law of cooling, the temperature goes on decreasing with time non - linearly.

64. A liquid in a beaker hastemperature $\theta(t)$ at time $t$andθ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta-\theta_0)$ and t is: [2012]

(a) (b) (c) (d)



Sol: . (a) According to newton's law of cooling

$$\frac{d\theta}{dt}=-k(\theta-\theta_0)$$

$$5=\frac{1}{k}\log_e\frac{(40-30)}{(80-30)}\quad (1)$$

$$\Rightarrow \frac{d\theta}{(\theta-\theta_0)}=-kdt$$

And, $t = \frac{1}{k}\log_e \frac{(32-30)}{(62-30)}$ (2) $\Rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{(\theta-\theta_0)} = -k\int_{\theta}^{t} d\,t$

Dividing equation (2) by (1),

$\Rightarrow \log(\theta - \theta_0) = -kt + c$

Which represents an equation of straight line. $\frac{t}{5} = \frac{k(62-30)}{\frac{1}{k}\log_e\frac{(40-30)}{(80-30)}}$ Thus the option (a) is correct.

65. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to [2003]

(a) two (b) three (c) four (d) one

Sol : (d) From Newton's law of cooling $-\frac{dQ}{dt} \propto (\Delta\theta)$ On solving we get, time taken to cool down from62°C to 32°C, $t = 8.6$ minutes.

Thermodynamics

Thermodynamics is a branch of science which deals with exchange of heat energy between bodies and conversion of the heat energy into mechanical energy and vice-versa.

Some Definitions

(1) **Thermodynamic system**

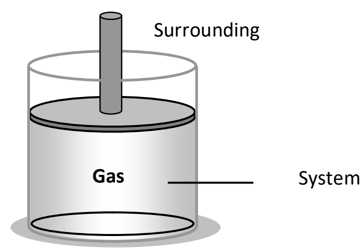


Fig. 14.1

- It is a collection of an extremely large number of atoms or molecules
- It is confined with in certain boundaries.
- Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings.

Thermodynamic system may be of three types

Open system :

It exchange both energy and matter with the surrounding.

Closed system :

It exchange only energy (not matter) with the surroundings.

Isolated system :

It exchange neither energy nor matter with the surrounding.

(2) **Thermodynamic variables and equation of state :**

A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For m moles of an ideal gas, equation of state is $PV = mRT$ and for 1 mole of an it ideal gas is

$$PV = RT$$

(3) Thermodynamic equilibrium :

In steady state thermodynamic variables are independent of time and the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.

- **Mechanical equilibrium :** There is no unbalanced force between the system and its surroundings.
- **Thermal equilibrium :** There is a uniform temperature in all parts of the system and is same as that of surrounding.
- **Chemical equilibrium :** There is a uniform chemical composition through out the system and the surrounding.

(4) Thermodynamic process :

The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system. The process is known as thermodynamic process. Some important processes are

- **Isothermal process :** Temperature remain constant
- **Adiabatic process :** No transfer of heat
- **Isobaric process :** Pressure remains constant
- **Isochoric (isovolumic process :** Volume remains constant
- **Cyclic and non-cyclic process :** Incyclic process Initial and final states are same while in non-cyclic process these states are different.
- **Reversible and irreversible process :**

(5) **Indicator diagram :**

Whenever the state of a gas (P, V, T) is changed, we say the gaseous system is undergone a thermodynamic process. The graphical representation of the change in state of a gas by a thermodynamic process is called indicator diagram. Indicator diagram is plotted generally in pressure and volume of gas.

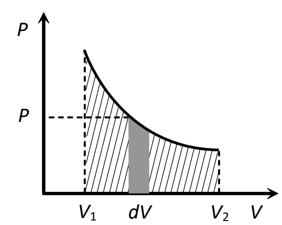


Fig. 14.6

Zeroth law of thermodynamics:

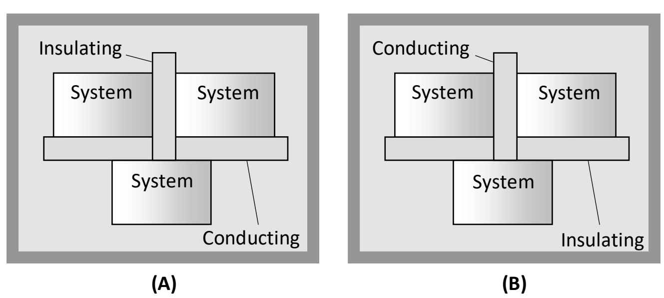


Fig. 14.3

If two isolated bodies A and B are in thermal equilibrium independently with a third body C, then the bodies A and B will also be in thermal equilibrium with each other.

- The zeroth law leads to the concept of temperature. All bodies in thermal equilibrium must have a common property which has the same value for all of them. This property is called the temperature.

- The zeroth law came to light long after the first and seconds laws of thermodynamics had been discovered and numbered. It is so named because it logically precedes the first and second laws of thermodynamics.

- Zeroth law of thermodynamics leads to the concept of temperature(T).
- Temperature is the measure of degree of hotness or coldness of a body.
- Temperature determines the direction of flow of heat when two bodies are placed in thermal contact.
- Heat always flows from the body at higher temperature(hot body) to the body at lower temperature(cold body).

Relation between work and heat (Joule's law):

The amount of heat produced is directly proportional to the amount of mechanical work done.

$$H \propto W$$

$$W = JH.$$

$J = \frac{W}{H}$ where J= Mechanical equivalent of heat.

Mechanical equivalent of heat (J):

- It is the amount of work necessary to produce unit amount of heat energy.
- J is not a physical quantity. It is simply a conversion factor between mechanical work and its equivalent heat energy.

Values of 'J':

- The value of J depends on the units of work W and heat H.
- When W is in Joules, H is in Cal, then $J = 4.186 J/cal \approx 4.2 J/cal$
- When W and H both are expressed in joules, J=1.

Applications of Joule's law :

⇒ A metal ball falls freely on the ground from a height 'h_1' and bounces to height 'h_2'. If the ball absorbs all the heat energy generated, then raise in temperature of the ball is

$$\Delta\theta = \frac{g(h_1 - h_2)}{JS}$$

where 'S' is specific heat of water

⇒ The raise in temperature of water when it falls from a height h to the ground is,

$$\Delta\theta = \frac{gh}{JS};$$

where 'S' is specific heat of water

⇒ The height from which ice is to be dropped to melt it completely is

$$h = \frac{JL}{g};$$

where L= Latent heat of ice.

⇒ When a bullet of mass m moving with a velocity v is stopped abruptly by a target and all of its heat energy liberated is retained by bullet , then the increase in temperature is.

$$\Delta\theta = \frac{v^2}{2JS}$$

If the bullet absorbs $x\%$ of heat liberated, then rise in its temperature is

$$\Delta\theta = \frac{x}{100}\left(\frac{v^2}{2JS}\right)$$

⇒ If a bullet at a temperature lesser than its melting point just melts when abruptly stopped by an obstacle and if all the heat produced is absorbed by the bullet then

$$J\left(mS\Delta\theta + mL\right) = \frac{1}{2}mv^2$$

Where L= Latent heat of fusion the material of the bullet ,

S= Specific heat

$\Delta\theta =$ rise in temperature before it melts.

⇒ When a block of ice of mass M is dragged with constant velocity 'v' on a rough horizontal surface of coefficient of friction μ, through a distance d, then the mass of ice melted is,

$$m = \frac{\mu Mgd}{JL};$$

Where m=mass of ice melted.

In order to melt all the ice completely,

the block should be dragged through a distance $d = \frac{JL}{\mu g}$

Now, the time taken to melt completely is given by $t = \frac{d}{v} = \frac{JL}{\mu gv}$

⇒ When a block is dragged on a rough horizontal surface of coefficient of friction μ, then the rise in temperature of block is,

$$\Delta\theta = \frac{\mu gd}{JS}$$

⇒ When a body rotating with angular speed ω is suddenly stopped, if it absorbs all the heat generated, then rise in temperature of body is

$$\Delta\theta = \frac{I\omega^2}{2JmS}$$

$$\left(\therefore W = \tau\theta = \tau\omega t\right)$$

Where I=Moment of inertia of the given body.

⇒ A drilling machine drills a hole to a metal plate in a time t. The machine is operated by a torque τ with constant angular speed ω. If the heat generated is completely absorbed by the plate, then the raise in temperature of the plate is

$$\Delta\theta = \frac{\tau\omega t}{JmS} \qquad \left(\because W = \tau\theta = \tau\omega t\right)$$

Where, m= mass of the plate

S= specific heat of the material of the given plate

PROBLEMS

1. The height of the Niagara falls is 50m. If $J = 4.2\times10^7\, erg/cal$. Then the difference of temperature of water at the top and bottom of the falls is

SOLUTION:

P.E. is converted into heat.

$$mgh = JmS\Delta t$$

$$\Delta t = \frac{gh}{JS} = \frac{980\times5000}{4.2\times10^7} = 0.117^0 C$$

2. A bullet moving with a uniform velocity *v*, stops suddenly after hitting the target and the whole mass melts be *m*, specific heat *S*, initial temperature 25°*C*, melting point 475°*C* and the latent heat *L*. Then *v* is given by

(a) $mL = mS\,(475-25) + \frac{1}{2}\cdot\frac{mv^2}{J}$ (b) $mS\,(475-25) + mL = \frac{mv^2}{2J}$

(c) $mS\,(475-25) + mL = \frac{mv^2}{J}$ (d) $mS\,(475-25) - mL = \frac{mv^2}{2J}$

SOLUTION:

Firstly the temperature of bullet rises up to melting point, then it melts.

Hence according to JOULES LAW

$$W = JQ \,.$$

$$\frac{1}{2}mv^2 = J.[m.c.\Delta\theta + mL] = J[m\,S\,(475-25) + mL]$$

$$mS\,(475-25) + mL = \frac{mv^2}{2J}$$

3. A lead bullet of mass 21 g travelling at a speed of 100 ms^{-1} comes to rest in a wooden block. If no heat is taken away by the wood, then find the raise in temperature of the wood. (Specific heat of lead = 0.03 calorie/g °C.)

SOLUTION:

kinetic energy of the bullet = heat gained by the bullet,

$$\frac{1}{2}mv^2 = mS\mathrm{D}t$$

$$\mathrm{D}t = \frac{v^2}{2S} = \frac{(100)^2}{2´\ 0.03´\ 4.2´\ 1000} =$$

$$= 39.68^0\mathrm{C}$$

4. If two balls of same metal weighing 5 *gm* and 10 *gm* strike with a target with the same velocity. The heat energy so developed is used for raising their temperature alone, then the temperature will be higher

(a) For bigger ball (b) For smaller ball

(c) Equal for both the balls (d) None is correct from the above three

SOLUTION:

$$\text{Energy} = \frac{1}{2}mv^2 = mc\,\mathrm{D}q;$$

$$\Delta\theta \propto v^2$$

Temperature does not depend upon the mass of the balls.

5. A water fall is 84 *metres* high. If half of the potential energy of the falling water gets converted to heat, the rise in temperature of water will be

(a) $0.098°C$ (b) $0.98°C$ (c) $9.8°C$ (d) $0.0098°C$

SOLUTION:

As $W = JQ$

$$\frac{1}{2}(mgh) = J \times mc\Delta\theta$$

$$\Delta\theta = \frac{gh}{2Jc}$$

$$\Delta\theta = \frac{9.8 \times 84}{2 \times 4.2 \times 1000} = 0.098\,°C$$

$$(\because c_{\text{water}} = 1000\ \frac{cal}{kg \times °C})$$

6. A 10kw drilling machine is used for 5 minutes to bore a hole in an aluminium block of mass 10×10^3 kg. If 40% of the work done is utilised to raise the temperature of the block, then find the raise in temperature of the aluminium block ? (Specific heat of Aluminium = 0.9 $Jkg^{-1}k^{-1}$)

SOLUTION:

Work done by the drilling machine in 5 min

W= power x time

$$W = 10 \times 10^3 \times 5 \times 60 = 3 \times 10^6 J$$

The energy utilised to raise the temperature of the block = 40% of W

$$= 3 \times 10^6 \times \frac{40}{100} = 12 \times 10^5 J$$

Heat gained by aluminium block = mass × specific heat × increase in temperature.

$$12 \times 10^5 = (10 \times 10^3) \times 0.9 \times \Delta t$$

$$\therefore \Delta t = \frac{12 \times 10^5}{0.9 \times 10^4} = 133.3°C$$

7. A lead bullet of 10 *g* travelling at 300 *m*/*s* strikes against a block of wood and comes to rest. Assuming 50% of heat is absorbed by the bullet, the increase in its temperature is (Specific heat of lead = 150*J*/*kg*, *K*)

(a) 100°*C* (b)125°*C* (c) 150°*C* (d) 200°*C*

SOLUTION:

Since specific heat of lead is given in *Joules*,

hence use $W = Q$ instead of $W = JQ$.

$$\frac{1}{2} \times \left(\frac{1}{2} mv^2\right) = m.c.\Delta\theta$$

$$\Delta\theta = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150\,°C\,.$$

8. A body of mass 5 *kg* falls from a height of 30 *metre*. If its all mechanical energy is changed into heat, then heat produced will be

(a) 350 *cal* (b) 150 *cal* (c) 60 *cal* (d) 6 *cal*

SOLUTION:

$$W = JQ$$

$$mgh = J \times Q$$

$$Q = \frac{mgh}{J} = \frac{5 \times 9.8 \times 30}{4.2} = 350\ cal$$

9. Hailstone at $0°C$ falls from a height of 1 *km* on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt $(g = 10\ m/s^2)$

(a) $\frac{1}{33}$ (b) $\frac{1}{8}$ (c) $\frac{1}{33} \times 10^{-4}$ (d) All of it will melt

SOLUTION:

Suppose *m¢ kg* ice melts out of *m kg*

then by using $W = JQ$

$$mgh = J(m'L).$$

Hence fraction of ice melts

$$= \frac{m'}{m} = \frac{gh}{JL} = \frac{9.8 \times 1000}{4.18 \times 80} = \frac{1}{33}$$

10. Hailstones fall from a certain height. If they melt completely on reaching the ground, find the height from which they fall. (g=10 ms-2,L = 80 calorie/g and J = 4.2 J/calorie.)

SOLUTION:

On reaching the ground, a hailstone of mass M losses potential energy which is converted into heat energy required to melt it.

potential energy lost = heat energy required for melting the hailstone

$$Mgh = ML$$

$$gh = L$$

$$h = \frac{L}{g},$$

$$h = \frac{80\times 4.2\times 1000}{10}$$

$$h = 33.6\times 1000\text{m} = 33.6\,\text{km}.$$

11. A girl weighing 42 kg eats bananas whose energy is 980 calories. If this energy is used to go to height h find the value of h.(J=4.2J/ calorie)

SOLUTION:

Energy gained by the girl in eating bananas = 980 calories

$= 980\times 4.2$ J.

W=H (in S.I.)

$$980 \times 4.2 = mgh$$

$$980\times4.2 = 42\times9.8\times h$$

$$\Rightarrow h = \frac{980\times 4.2}{42\times 9.8} = 10m$$

12. A piece of ice at 0^0C falls from rest into a lake of water which is also at 0^0C and 0.5% of ice melts. Find the minimum height from which the ice falls.

SOLUTION:

Let a mass m of ice falls from height h.

Loss in potential energy = mgh,

$$\text{Heat produced} = H = \frac{0.5}{100} mL_{ice}$$

since, W = JH;

$$mgh = J\times\frac{0.5}{100} mL_{ice}$$

$$h = \frac{0.5L_{ice}}{100 \times g}$$

$$= \frac{0.5 \times (4.2J/cal)(80 \times 10^3\ cal/kg)}{100 \times 9.8\, m/s^2} = 171.43m$$

13. An ice block is projected vertically up with a velocity 20 ms^{-1}. The amount of ice that melt when it reaches the ground if the mass of ice block is 4.2 kg.

1) 2.5 gm 2) 2.5 kg 3) 0.25 kg 4) 0.25 gm

SOLUTION:

. Maximum height attained, $h = \frac{u^2}{2g}$

$$W = JH \Rightarrow mgh = J\left(xL_{ice}\right)$$

14. How much will the temperature of 100g of water be raised by doing 4200 J of work in stirring the water?

1) 0.01^{0}c 2) 0.1^{0}c 3) 1^{0}c 4) 10^{0}c

SOLUTION:

$$W = JH$$

$$W = JmS\,\Delta\theta$$

Internal energy (U) :

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and that due to molecular configuration is called internal potential energy U_P

i.e. Total internal energy $U = U_K + U_P$

♦ For an ideal gas, as there is no molecular attraction $U_p = 0$

i.e. internal energy of an ideal gas is totally kinetic and is given by $U = U_K = \frac{3}{2}\mu RT$

change in internal energy $\Delta U = \frac{3}{2}\mu R\,\Delta T$

♦ In case of gases whatever be the process

$$\Delta U = \mu \frac{f}{2} R\Delta T$$

$$= \mu C_V \Delta T$$

$$= \mu \frac{R}{(\gamma - 1)} \Delta T = \frac{\mu R(T_f - T_i)}{\gamma - 1}$$

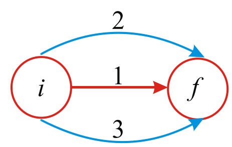
$$= \frac{\mu R T_f - \mu R T_i}{\gamma - 1}$$

$$= \frac{(P_f V_f - P_i V_i)}{\gamma - 1}$$

♦ Change in internal energy does not depend on the path of the process. So it is called *a* point function

i.e. it depends only on the initial and final states of the system,

$$i.e.\ \Delta U = U_f - U_i$$



$\Delta U_1 = \Delta U_2 = \Delta U_3$

♦ The internal energy of ideal gas depends only on its temperature T. When T increases U also increases and vice versa.

♦ Internal energy of real gases depends upon temperature, pressure and volume.

♦ Real gases consists of both kinetic energy and potential energy due to intermolecular forces.

Work (d*W*) :

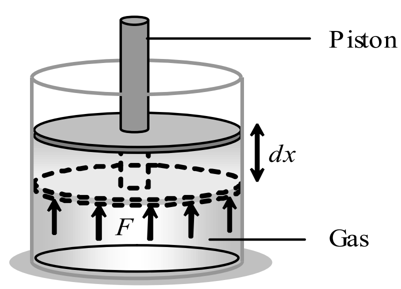
Suppose a gas is confined in a cylinder that has a movable piston at one end.

If P be the pressure of the gas in the cylinder, then force exerted by the gas on the piston of the cylinder

$F = PA$

(A = Area of cross-section of piston)

⇒



When the piston is pushed outward an infinitesimal distance *dx*,

the work done by the gas $dW = F.dx = P(A\,dx) = P\,dV$

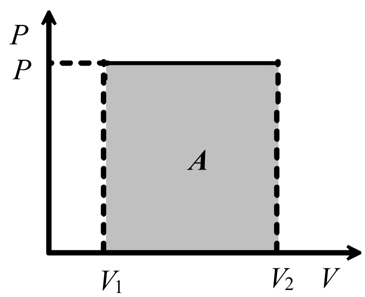
For a finite change in volume from V_i to V_f

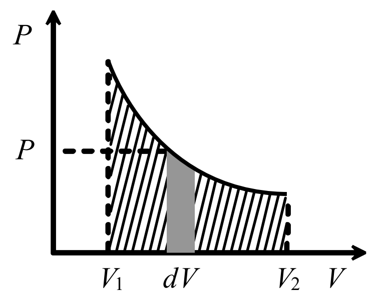
Total amount of work done $W = \int_{Vi}^{V_f} P\,dV = P(V_f - V_i)$

Work from PV graph :

♦ If we draw indicator diagram, the area bounded by *PV*-graph and volume axis represents the work done

⇒

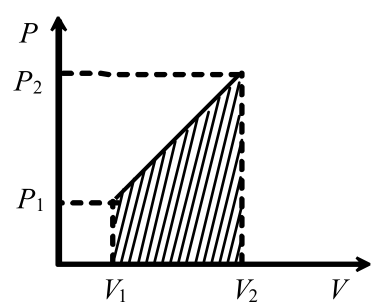




1) Work = Area = $P(V_2 - V_1)$

2) Work $= \int_{V_1}^{V_2} PdV = P(V_2 - V_1)$

⇒



Work = Area of the shown trapezium

$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

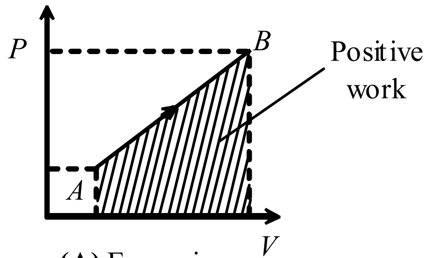
♦ From $\Delta W = P\Delta V = P(V_f - V_i)$

⇒ If system expands against some external force then $V_f > V_i$

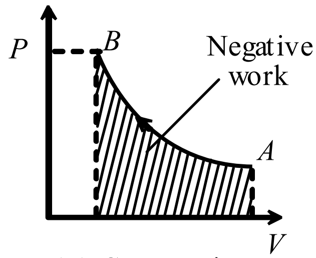
$$dW = \text{positive}$$

⇒ If system contracts because of external force then $V_f < V_i$

$$dW = \text{negative}$$

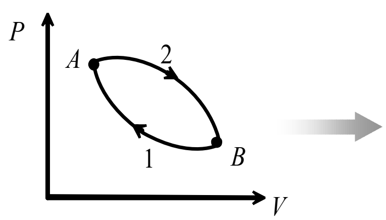


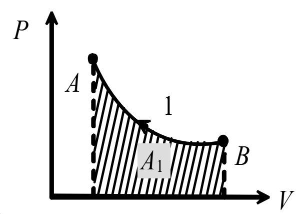
(A) Expansion



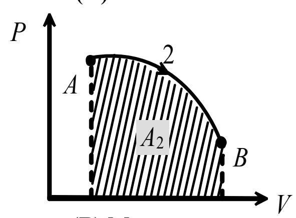
(B) Compression

♦ Like heat, work done is also depends upon initial and final state of the system and path adopted for the process





(A) Less area

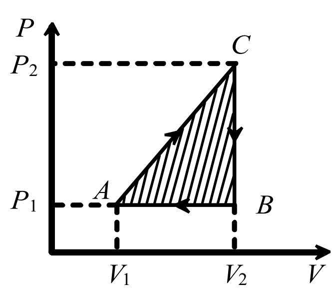


(B) More area

$\because A_1 < A_2$ and $W_1 < W_2$

♦ In cyclic process, work done is equal to the area of closed curve.

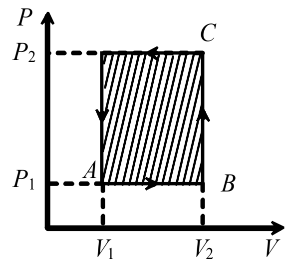
⇒ **It is positive if the cycle is clockwise**



$$\text{Work} = \text{Area of triangle } ABC$$

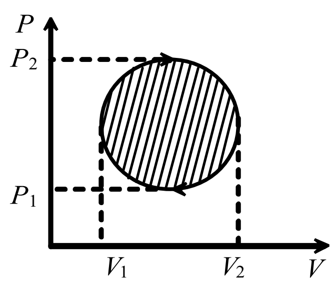
$$= \frac{1}{2} \times (V_2 - V_1) \times (P_2 - P_1)$$

$\Rightarrow$ **It is negative if the cycle is anticlockwise**.



$$\text{Work} = \text{Area of rectangle } ABCD$$
$$= AB \times AD$$
$$= (V_2 - V_1)\,(P_2 - P_1)$$

$\Rightarrow$ **SPECIAL CASE**



$$\text{Work} = \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$$

First Law of Thermodynamics (FLOT)

- It is a statement of conservation of energy in thermodynamical process.
- According to it heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the work done (ΔW) by the system against the surroundings.

The differential form of first law of thermodynamics is

dQ=dU+dW,

where dQ = heat added,

dU = Increase in internal energy

dW= work done = PdV

$$\therefore dQ = dU + PdV$$

for bulk changes $\Delta Q = \Delta U + \int PdV$

Sign convention :

- When heat is added (flows into) to the system

  dQ is + ve(+dQ)

- When heat is taken (flows out) from the system

  dQ is -ve (-dQ)

- When gas expands work is done by the gas, dw is positive

  (+ dW)

- When gas compresses work is done on the gas, then work done by the gas dW is negative

  (-dW)

- When internal energy of system increases

  dU is +ve (+dU)

- When internal energy of system decreases

  dU is -ve(-dU)

NOTE :

- It makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased either by adding heat to it or doing work on it or both.
- ΔQ and ΔW are the path functions but ΔU is the point function.
- In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in *Joule* or in *calorie*.
- The first law introduces the concept of internal energy.

Significance and limitations of first law:

- It is a consequence of law of conservation of energy.
- This law is applicable to any process in nature.
- This law is applicable to all the three phases of matter.
- First law of thermodynamics does not indicate the direction of heat transfer. It does not tell any thing about the conditions under which heat can be transformed into work.

PROBLEMS

1. In changing the state of thermodynamics from *A* to *B* state, the heat required is *Q* and the work done by the system is W. The change in its internal energy is

(a) $Q + W$ (b) $Q - W$ (c) Q (d) $\frac{Q - W}{2}$

SOLUTION:

$\Delta Q = \Delta U + \Delta W$

$\Delta U = \Delta Q - \Delta W = Q - W$ (using proper sign)

2. Heat given to a system is 35 joules and work done by the system is 15 *joules*. The change in the internal energy of the system will be

(a) $-50\ J$ (b) $20\ J$ (c) $30\ J$ (d) $50\ J$

SOLUTION:

$\Delta U = \Delta Q - W = 35 - 15 = 20\ J$

3. When heat energy of 1500J is supplied to a gas the external workdone by the gas is 525J what is the increase in its internal energy

SOLUTION:

Heat energy supplied $\Delta Q = 1500J$

External workdone $\Delta W = 525J$

By 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \Delta U = \Delta Q - \Delta W$$

$$= 1500 - 525$$

$$= 975J.$$

4. Consider the vaporization of 1g of water at 100^{0}C to steam at 100^{0}C at one atmospheric pressure. Compute the work done by the water system in the vaporization and change in internal energy of the system.

SOLUTION:

To change a system of mass m of liquid to vapour, heat required is $Q = mL_v$

The process takes place at constant pressure,

the work done by the system is the work in an isobaric process. $W = P\Delta V$

Where $\Delta V = \left(V_{vapour} - V_{liquid}\right)$

From first law of thermodynamics

$$\Delta U = Q - W = mL_v - P\left(V_{vapour} - V_{liquid}\right)$$

Latent heat of vaporization of water

$$L_v = 22.57 \times 10^5 J/kg$$

$$Q = \left(1.00 \times 10^{-3}\right)\left(22.57 \times 10^5\right) = 2.26 \times 10^3\ J$$

$\because$ No. of moles = weight / gram molecular weight

$$\therefore \text{Moles of water in } 1g = \frac{1}{18} = 0.0556\ mole$$

$$V_{vapour} = \frac{nRT}{P} = \frac{(0.0556)(8.315)(373)}{1.013 \times 10^5} = 1.70 \times 10^{-3} m^3$$

The density of water is $1.00 \times 10^3\ kg/m^3 = 1.00\ g/cm^3$

$$V_{liquid} = 1.00 \times 10^{-6}\ m^3$$

Thus the work done by the water system in vaporization is

$$W = P\left(V_{vapour} - V_{liquid}\right)$$

$$= \left(1.013 \times 10^5\right)\left(1.70 \times 10^{-3} - 1.00 \times 10^{-6}\right) = 172\ J$$

The work done by the system is positive since the volume of the system has increased.

From first law,

$$\Delta U = Q - W \Rightarrow \Delta U = 2.26\times10^3 - 172 = 2.09\times10^3 J$$

5. A thermodynamic system goes from states (i) P_1, V to $2P_1, V$ (ii) P, V to $P, 2V$. Then work done in the two cases is

(a) Zero, Zero (b) Zero, PV_1 (c) PV_1, ,Zero (d) PV_1, P_1V_1

SOLUTION:

Case 1 $\rightarrow$ Volume = constant

$$\Rightarrow \int PdV = 0$$

Case 2 $\rightarrow P =$ constant

$$\Rightarrow \int_{V_1}^{2V_1} PdV = P\int_{V_1}^{2V_1} dV = PV_1$$

6 Find the change in internal energy of the system when a system absorbs 2 *kilocalorie* of heat and at the same time does 500 *joule* of work

(a) 7900 J (b) 8200 J (c) 5600 J (d) 6400 J

SOLUTION:

$$\Delta Q = 2k\,cal = 2\times10^3 \times 4.2J$$

$$= 8400\,J$$

$$\Delta W = 500\,J.$$

Hence from $\Delta Q = \Delta U + \Delta W$,

$$\Delta W = \Delta Q - \Delta U$$

$$= 8400 - 500$$

$$= 7900\ J$$

7. When 1g of water at 100°C is converted into steam at 100°C, it occupies a volume of 1671cc at normal atmospheric pressure. Find the increase in internal energy of the molecules of steam.

SOLUTION:

1 atmosphere = 1.013×10^5 Nm^{-2} ;

volume of 1gm of water, V_1 = 1cc = $10^{-6}m^3$;

Volume of steam = 1671 cc = 1671×10^{-6} m^3

External work done $dW = P(V_2 - V_1)$

$= 1.013 \times 10^5 (1671 \times 10^{-6} - 1 \times 10^{-6})$

$= 1.013 \times 10^5 \times 1670 \times 10^{-6}$

$= 1.013 \times 167 = 169.2$ J.

Latent heat of vaporisation of steam= 540 cal/g

So, heat supplied to convert 1g of water into steam,

$$DQ = 540 \times 4.2J = 2268J$$

By first law of thermodynamics

$$DU = DQ - DW = 2268 - 169.2 = 2098.8J$$

8. Calculate the external workdone by the system in KCal, when 40 KCal of heat is supplied to the system and internal energy raises by 8400 J.

SOLUTION:

$$dQ = dU + dW$$

$$dU = 8400\ J$$

$$= \frac{8400}{4200}\ KCal$$

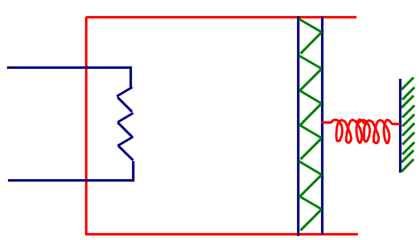
$$= 2\ KCal.$$

$$\therefore\ 40\ KCal = 2\ KCal + \text{External work done}$$

$$\text{The external work done} = 40 - 2 = 38\ KCal$$

9. An ideal monatomic gas is confined in a cylindrical by a spring loaded piston of cross section 8.0 x 10-3m2. Initially the gas is at 300K and occupies a volume of 2.4 x 10-3m3 and the springs is in its relaxed state as shown in figure. The gas is heated by a small heater untill the piston moves out slowly by 0.1 m. The cylinder and the piston are thermally insulated. The piston and spring are massless and there is no friction between the position and the cylinder. The final temperature of th gass will be

(Neglect the heat loss through the load wires of the heater. The heat capacity of the heater coil is also negligible)



1) 300K 2) 800K 3) 500K 4) 1000K

SOLUTION:

$$\text{Final pressure of the gas is } P_f = P_0 + \frac{Kx}{A}$$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{p_i V_i}$$

10. In a thermodynamic process pressure of a fixed mass of a gas is changed in such a manner that the gas releases 20J of heat and 8J of work is done on the gas. If initial internal energy of the gas was

30J, what will be the final internal energy?

SOLUTION:

We know that, dQ = dU+dW

Since heat is released by the system, dQ = -20J.

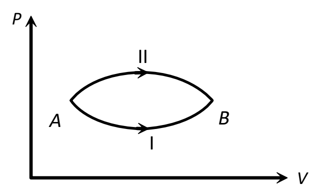
and work is done on the gas, dW = -8J

dU = -20 - (-8)

= -20 + 8 = -12J

$$\Rightarrow U_f - U_i = -12J$$
$$U_f = U_i - 12 = 30 - 12 = 18J$$

11. A system goes from *A* to *B* via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_{II} > \Delta U_I$ (b) $\Delta U_{II} < \Delta U_I$

(c) $\Delta U_I = \Delta U_{II}$ (d) Relation between ΔU_I and ΔU_{II} can not be determined

SOLUTION:

As internal energy is a point function therefore change in internal energy does not depends upon the path followed

i.e. $\Delta U_I = \Delta U_{II}$

12. Consider the melting of 1g of ice at 0^0C to water at 0^0C atmospheric pressure. Then the change in internal energy of the system (density of ice is 920kg/m^3)

1) 334 J 2) 420 J 3) 540 J 4) 680 J

SOLUTION:

Heat required to change the phase of solid $\Delta Q = mL_f$

Work done by system at constant pressure, $\Delta W = P(V_{liquid} - V_{solid})$

From first law of thermodynamics, $\Delta U = \Delta Q - \Delta W$

$$= mL_f - P(V_{liquid} - V_{solid})$$

Latent heat of fusion of water,

$$L_f = 3.335 \times 10^5 \ J/Kg$$

$$\Delta Q = \left(1\times10^{-3}\right)\left(3.335\times10^5\right) = 334\ J$$

The density of ice is 920 kg / m^3.

$$V_{solid} = \frac{1\times10^{-3}}{920} = 1.09\times10^{-6}\ m^3$$

$$V_{liquid} = 1\times10^{-6}\ m^3$$

Thus work done by the system in melting is

$$\Delta W = P\left(V_{liquid} - V_{solid}\right)$$

$$= \left(1.013\times10^5\right)\left(1\times10^{-6} - 1.09\times10^{-6}\right) = -9\times10^{-3}\ J$$

Work done by the system is negative because the system decreased in volume.

$$\Delta U = \Delta Q - \Delta W = 334 - \left(-9\times10^{-3}\right) = 334\ J$$

13. The equation of state for a gas is given by PV=nRT+α V, where n is the number of moles and α is a positive constant. The intial temperature and pressure of one mole of the gas contained in a cylinder are T_0 and P_0 respectively. The work done by the gas when its temperature doubles isobarically will be (Mains 2014)

1) $\frac{P_0T_0R}{P_0-\alpha}$ 2) $\frac{P_0T_0R}{P_0+\alpha}$ 3) $P_0T_0R\text{In}2$ 4) P_0T_0R

SOLUTION:

work done W = $(V_f - V_i)$ P

$$V_i = \frac{nRT_0}{P_0 - \propto}$$

at constant pressure as temp doubles volume double so

$$V_f = \frac{2nRT_0}{P_0 - \propto}$$

Specific heats of a gas:

- Gases have two types of specific heats.
  - Specific heat at constant volume (c_v)
  - Specific heat at constant pressure(c_p)
- Specific heat of all substances is zero at 0K.

Specific heat of gas at constant pressure $\left(c_p\right)$

It is the heat required to rise the temperature of 1g of a gas by $1^o C$ at constant pressure

$$\therefore c_p = \frac{1}{m}\left(\frac{dQ}{dT}\right)_p$$

$$; dQ_p = mc_p dT$$

$\therefore \Delta Q_p = mc_p \Delta T$, if c_p is constant.

$\Delta Q_p = m\int c_p dT$, if c_p depends on temperature.

Specific heat of gas at constant volume (C_V)

It is the heat required to rise the temperature of 1g of a gas by $1^o C$ at constant volume

$$\therefore c_v = \frac{1}{m}\left(\frac{dQ}{dT}\right)_v$$

$$dQ_v = mc_v dT$$

$$\therefore \Delta Q_v = mc_v \Delta T,$$

$$\Delta Q_v = m\int c_v dT,$$

♦ SI unit of both c_p, c_v is J/Kg-K

CGS unit is cal/g-$^o C$

Molar specific heats (C_p,C_V) of a gas

When the above specific heats c_p, c_v are defined per 1mole of gas, then they are said to be molar specific heats and represented by C_P, C_V.

These are, $C_p = \frac{1}{n}\left(\frac{dQ}{dT}\right)_p$

$$C_v = \frac{1}{n}\left(\frac{dQ}{dT}\right)_v = \frac{1}{n}\left(\frac{dU}{dT}\right), (\because \Delta W = 0)$$

SI unit of both molar specific heat is J/ mol-k

NOTE :

- ♦ C_p is greater than C_V
- ♦ $\frac{C_P}{C_V} = \gamma$ (C_p,C_V are molar specific heats)
- ♦ C_p - C_V= R,
  where R is universal gas constant
  R= 8.314 J/ mol-k $\approx 2 cal/mol-K$

$$C_P = \frac{\gamma R}{\gamma - 1} \; and \quad C_v = \frac{R}{\gamma - 1}$$

- $c_p - c_v = r = \frac{R}{M} \quad (but\ \ C = Mc)$

  $\Rightarrow M\left(c_p - c_v\right) = R$

  Where r is specific gas constant

  c_p, c_v are expressed in J/Kg-K.

- For a gas having f degrees of freedom,

  $C_v = \left(\frac{f}{2}\right)R, C_p = \left(1 + \frac{f}{2}\right)R$

  $$\gamma = 1 + \frac{2}{f}$$

C_P, C_V and values of different gases:

S.No	Atomicity of gas	C_P	C_V	$\gamma = \frac{C_P}{C_V}$
1.	Monoatomic	$\frac{5}{2}R$	$\frac{3}{2}R$	$\frac{5}{3} = 1.67$
2.	Diatomic	$\frac{7}{2}R$	$\frac{5}{2}R$	$\frac{7}{5} = 1.4$
3.	Tri non-linear& and poly atomic	4R	3R	$\frac{4}{3} = 1.33$
4.	Tri linear	$\frac{9}{2}R$	$\frac{7}{2}R$	$\frac{9}{7} = 1.29$

γ Of mixture of gases:

- When n_1 moles of a gas is mixed with n_2 moles of another gas. then,

$$(C_v)_{mixture} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

$$(C_p)_{mixture} = (C_v)_{mixture} + R = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

$$\gamma_{mixture} = \frac{C_{p(mixture)}}{C_{v(mixture)}}$$

Also $\frac{n_1 + n_2}{\gamma_{mixture} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

- At constant pressure, fraction of heat absorbed that is converted into internal energy is

$$\frac{dU}{dQ}=\frac{C_v}{C_p}=\frac{1}{\gamma}$$

- At constant pressure, fraction of heat absorbed that is converted into external workdone is

$$\frac{dW}{dQ}=\frac{nRdT}{nC_p dT}=\frac{R}{C_p}=1-\frac{1}{\gamma}$$

PROBLEMS

1. Four moles of a perfect gas is heated to increase its temperature by 2^0C absorbs heat of 40 cal at constant volume. If the same gas is heated at constant pressure find the amount of heat supplied.

SOLUTION:

At constant volume $dQ=nC_v dT=dU=40$

At constant pressure $dQ= dU+nRdT= 40+(4\times 2\times 2)= 56\,cal$

2. A cylinder of fixed capacity 67.2 litres contains helium gas at S.T.P. The amount of heat required to raise the temperature of the gas by 15^0C is (R =8.31 J $mol^{-1}k^{-1}$)

1) 520 J 2) 560.9 J 3) 620 J 4) 621.2 J

SOLUTION:

$dQ = nC_V dT$

$\therefore \Delta Q=\Delta U=nC_V\Delta T$

$\left(C_V=\frac{R}{\gamma-1}\right)$

3. 14 g of N_2 gas is heated in a closed rigid container to increase its temperature from 23^0C to 43^0C. The amount of heat supplied to the gas is

1) 25 cal 2) 50 cal 3) 100 cal 4) 30 cal

SOLUTION:

In a closed container, $\Delta V=0$

4. The specific heat capacity of a metal at low temperature(T) is given as $C_p(kJK^{-1}kg^{-1})=32\left(\frac{T}{400}\right)^3$. A 100 g vessel of this metal is to be cooled from 20K to 4 K by a special refrigerator operating at room temperature $\left(27^0 C\right)$. The amount of work required to cool the vessel is

SOLUTION:

Heat required to change the temperature of vessel by a small amount dT $dQ=mC_p dT$

Total heat required $Q = m\int_{20}^{4} 32\left(\frac{T}{400}\right)^3 dT = 0.001996\,kJ$

work done required to maintain the temperature of sink to T_2

$$W = Q_1 - Q_2 = \left(\frac{Q_1 - Q_2}{Q_2}\right)Q_2$$

$$For\ T_2 = 20K;\ W_1 = \left(\frac{300-20}{20}\right)0.001996 = 0.028\,kJ$$

$$For\quad T_2 = 4K$$

$$W_2 = \left(\frac{300-4}{4}\right)0.001996 = 0.148\,kJ$$

$\therefore$ The work required to cool the vessel from 20K to 4K is

W_2-W_1=0.148-0.028=0.12KJ

As temperature is changing from 20 K to 4 K work done required will be more than W_1 but less than W_2.

5. 70 cal of heat is required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. What is the amount of heat required to rise the temperature of same gas through the same range at constant volume?(R = 2cal mole^{-1}K^{-1})

1) 28 J 2) 50 Cal 3) 75 J 4) Zero

SOLUTION:

$$\Delta Q_P = nC_P\Delta T;$$

$$C_P - C_V = R \Rightarrow C_V = C_P - R$$

$$\Delta U = nC_V\Delta T$$

6. Calculate the difference between the two specific heats of nitrogen, given that the density of nitrogen at N.T.P is 1.25 g/litre and J= 4200 J/KCal., (in KCal/kg-K)

SOLUTION:

$$PV = mrT$$

The difference in specific heats, $r = \frac{P}{\rho T}$.

AT NTP P = 1.013 × 10^5 N/m^2;

T = 273 K;

$$r = 1.25' \frac{10^{-3}\text{kg}}{10^3\text{cm}^3} = \frac{1.25' 10^{-3}\text{kg}}{10^3' 10^{-6}\text{m}^3} = 1.25\text{kg/m}^3$$

$$\therefore r = \frac{1.013 \times 10^5}{1.25 \times 273}$$

$$= 296.8 \text{J/kg K}$$

$\therefore$ The difference of specific heats = 0.0768 KCal/kg-K.

7. When an ideal diatomic gas is heated at constant pressure fraction of the heat energy supplied which increases the internal energy of the gas is

SOLUTION:

Heat used in increasing the internal energy is $Q_1 = C_v dT$;

Heat absorbed at constant pressure to increase the temperature by dT is $Q_2 = C_p dT$

$$\therefore \frac{Q_1}{Q_2} = \frac{C_v}{C_p} = \frac{1}{C_p / C_v} = \frac{1}{\gamma}$$

for diatomic gas, $\gamma = 7/5$; $\therefore \frac{Q_1}{Q_2} = \frac{5}{7}$

8. The relation between internal energy U, pressure P and volume V of a gas in an adiabatic process is : $U = a + bPV$ Where 'a' and 'b' are constants. What is the value of the ratio of the specific heats?

1) $\frac{a}{b}$ 2) $\frac{b+1}{b}$ 3) $\frac{a+1}{a}$ 4) $\frac{b}{a}$

SOLUTION:

. $U = a + bPV$;

But $PV = RT$

$C_V = \frac{dU}{dT}$;

$C_P = C_V + R$;

$\gamma = \frac{C_P}{C_V}$

9. The ratio of specific heats of a gas is γ. The change in internal energy of one mole of gas when the volume changes from V to 2V at constant pressure "P" is

1) $\frac{PV}{\gamma - 1}$ 2) PV 3) $\gamma - 1$ 4) $\frac{PV}{\gamma}$

SOLUTION:

$\Delta U = nC_V \Delta T$

But $C_V = \frac{R}{\gamma - 1}$

and $PdV = RdT$

10. A quantity of heat Q is supplied to a monoatomic ideal gas which expands at constant pressure. The fraction of heat that goes into work done by the gas is

SOLUTION:

$$C_p dT = C_v dT + dW; \; \therefore dW = (C_p - C_v) dT$$

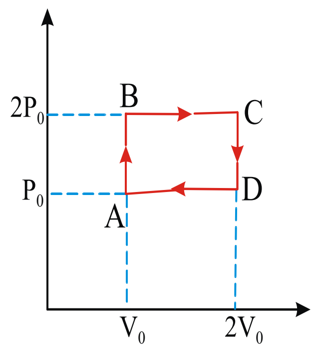
Fraction of heat converted into work

$$\frac{dW}{dQ} = \frac{(C_p - C_v)dT}{C_p dT} = 1 - \frac{C_v}{C_p} = 1 - \frac{1}{\gamma}$$

For monoatomic gas, $\gamma = 5/3$

$$\therefore \frac{dW}{dQ} = 1 - \frac{1}{\gamma} = 1 - \frac{3}{5} = \frac{2}{5}.$$

11. The P-V diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is



SOLUTION:

$$P_0 V_0 = nRT_A \,;$$

$$2P_0 V_0 = nRT_B;$$

$$2P_0 . 2V_0 = nRT_C$$

Heat supplied $H = nC_V \Delta T_{AB} + nC_P \Delta T_{BC}$

$$= nC_V\left(\frac{2P_0V_0}{nR} - \frac{P_0V_0}{nR}\right) + nC_P\left(\frac{4P_0V_0}{nR} - \frac{2P_0V_0}{nR}\right)$$

$$= n\left(\frac{3}{2}R\right)\left(\frac{P_0V_0}{nR}\right) + n\frac{5}{2}R\left(\frac{2P_0V_0}{nR}\right)$$

$$= \frac{13}{2}P_0V_0$$

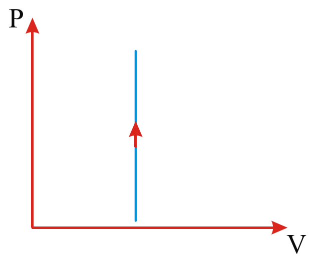
Thermodynamic processes:

Quasi-static process: A quasi-static process can be defined as an infinitesimally slow process in which the system remains in thermal and mechanical equilibrium with the surroundings at each and every intermediate stage. i.e., temperature, pressure are almost constant during infinitesimal small change in the state of gas. It is an ideal process. In practice it does not occur.

Isochoric process (or) Isometric process

- It is a process in which the volume of the system remains constant.

  i.e., $\Delta V = 0$ for such process $\Delta W = 0$
- In this process, the increase in internal energy is maximum where as the work done is zero.
- In this process $\Delta Q = \Delta U$
- It obeys Gay-Lussac's law, $\frac{P}{T}$ = constant
- Indicator diagram



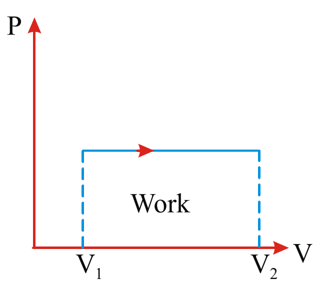
- Slope of isometric curve, $\frac{dP}{dV} = \infty$
- Specific heat is $C_V = \frac{f}{2}R$
- Bulk modulus of elasticity $K = \infty$

Isobaric process:

- It is a process in which the pressure of the system remains constant.

  i.e., $\Delta P = 0$
- It obeys Charles law ,

  $\frac{V}{T}$ = constant
- Indicator diagram



- Slope of isobaric curve, $\frac{dP}{dV} = 0$

- Specific heat is $C_p = \left(\frac{f}{2}+1\right)R$
- Bulk modulus of elasticity K=0.
- Work done in isobaric process is given by

  $\Delta W = P(V_2 - V_1)$

  $= nR(T_2 - T_1) = nR\Delta T$
- Eg:- Boiling of water into steam.
- At constant pressure, dQ:dU:dW= $nC_P\Delta T : nC_V\Delta T : n\left(C_P - C_V\right)\Delta T$

$$= C_P : C_V : R = \frac{\gamma R}{\gamma-1} : \frac{R}{\gamma-1} : R$$

$$\therefore\ \Delta Q : \Delta U : \Delta W = \gamma : 1 : (\gamma - 1)$$

$$= (f+2) : f : 2 \qquad \left(\because \gamma = 1 + \frac{2}{f}\right)$$

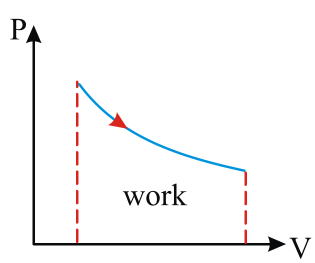
- For monoatomic gas $= 5:3:2$
- For diatomic gas $= 7:5:2$
- For non-linear poly atomic gas $= 4:3:1$

Isothermal process:

In this process, the pressure and volume of gas change, but temperature remains constant. Hence internal energy is also constant.

i.e., $dT = 0; dU = 0$

- The system is in thermal equilibrium with the surroundings.
- It takes place in a thermally conducting vessel. Hence heat exchanges between system and surroundings.
- In this process $dQ = dW$
- It is a slow process.
- It obeys the Boyle's law i.e. PV=Constant
- Specific heat is infinity.
- Indicator diagram



- Slope of isothermal curve, $\tan\theta = \frac{dP}{dV} = -\frac{P}{V}$

- Isothermal bulk modulus $-\frac{dP}{dV/V} = P$
- The workdone during the isothermal change at temperature T for n moles of gas is

$$W = 2.303nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$=2.303 \text{ nRT} \log_{10}\left(\frac{P_1}{P_2}\right)$$

- Isothermal process is ideal. In nature, no process is perfectly isothermal. But we can say melting of ice, boiling of water are approximately isothermal. In these two processes internal energy increases even temperature is constant.

Adiabatic process:

The pressure, volume and temperature of a gas change but total heat remains constant

i.e., dQ=0 (Q=constant).

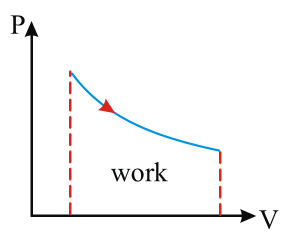
- There should not be any exchange of heat between the system and surroundings. All the walls of the container and the piston must be perfectly insulating.
- It is a quick process.
- The internal energy changes certainly as temperature changes.
- In the adiabatic process P, V & T are related as

(i) PV^{γ} = constant

(ii) $TV^{\gamma-1}$ =constant

(iii) $P^{1-\gamma}T^{\gamma}$=constant

- In this process specific heat is zero.
- Indicator diagram is



- Slope of adiabatic curve, $\tan\theta = \frac{dP}{dV} = -\gamma\frac{P}{V}$

The slope of adiabatic curve is γ times to that of the isothermal curve.

- The adiabatic bulk modulus of gas is γ p

i.e. γ times isothermal bulk modulus.

- The workdone by the system during the adiabatic expansion is

$$W = \frac{nR}{\gamma-1}(T_1 - T_2)$$

$$= nC_v(T_1 - T_2)$$

$$= -nC_V\Delta T$$

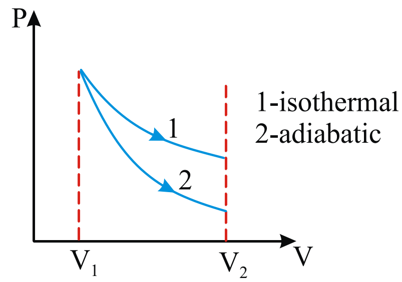
$$= n\frac{C_p}{\gamma}(T_1 - T_2)$$

$$= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

- It takes place in a non conducting vessel. Hence no exchange of heat takes place between system and surroundings.
- Adiabatic expansion causes cooling and compression causes heating.
- Eg:- Sudden bursting of tube of bicycle tyre, Propagation of sound in gases
- In this process $dU = -dW$

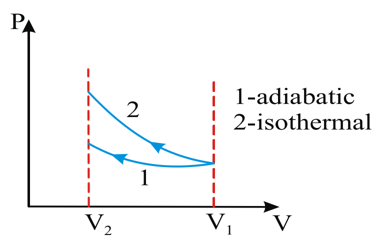
Comparison between isothermal and adiabatic curves :

- When expanded to the same volume from the same initial state.



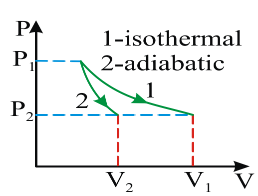
- Work done : $W_1 > W_2$
- Final pressure: $P_1 > P_2$
- Final temperature: $T_1 > T_2$

- When compressed to the same volume from the same initial state.



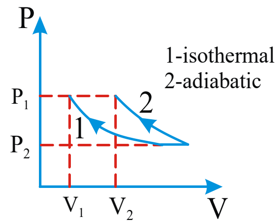
- Work done: $W_1 < W_2$
- Final pressure : $P_1 > P_2$
- Final temperature: $T_1 > T_2$

- When expanded to the same pressure from the same initial state.



- Work done : $W_1 > W_2$
- Final volume : $V_1 > V_2$
- Final temperature : $T_1 > T_2$

- When compressed to the same pressure from the same initial state.



♦ Workdone : $W_1 < W_2$

♦ Final volume : $V_1 < V_2$

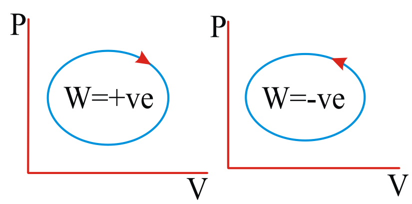
♦ Final temperature : $T_1 < T_2$

Cyclic process :

♦ If a system after undergoing through a series of changes comes back to its initial state, the process is called cyclic.

♦ In a cyclic process (the system finally reaches the same initial state), workdone is equal to the area enclosed by the cycle.

It is +ve if the cycle is clockwise. It is -ve if the cycle is anticlockwise.



In the cyclic process as $U_f = U_i$,

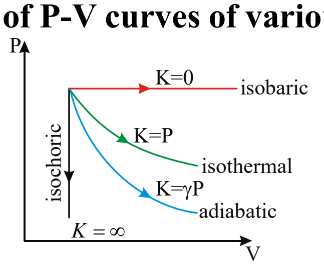
$$\Delta U = U_f - U_i = 0$$

the first law implies $\Delta Q = 0 + \Delta W$,

i.e $\Delta Q = \Delta W$,

heat supplied is equal to the work done (area of the cycle)

Comparison of P-V curves of various processes :



Free expansion :

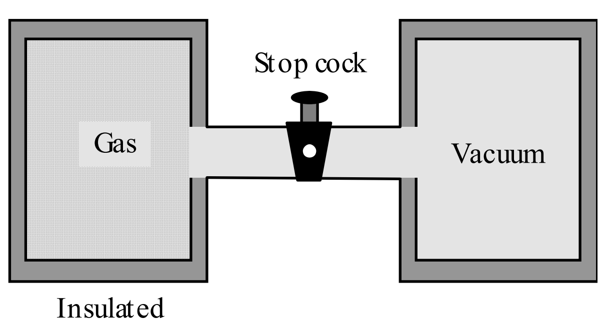


Figure shows an insulated cylinder divided into two parts by a thin massless fixed piston. Volume of left compartment is filled with an ideal gas and the right compartment is vacuum. If we release the piston, gas fills the whole space of the cylinder rapidly.

In this expansion. No heat is supplied to the gas as walls are insulated. $\therefore \Delta Q = 0$.

As the piston is fixed no work is done by the gas, $\Delta W = 0$

hence internal energy remains constant. $\therefore \Delta U = 0$,

T is constant

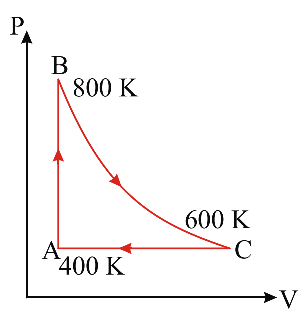
Such an expansion is called "free expansion".

Polytrophic process:

- In this process the gas obeys an additional law in the form of $PV^x =$ constant, (where x is $+ve\ or\ -ve$ constant) along with ideal gas equation $PV = nRT$
- In this process external Work done is W=$\frac{-nR\Delta T}{x-1}$
- In this process Specific heat, C=$\frac{R}{\gamma-1} - \frac{R}{x-1}$

PROBLEMS

1. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperature at A,B and C are 400 k, 800k and 600 k, respectively. Choose the correct statement. [MAIN 2014]



1) The change in internal energy in whole cyclic process is 250 R

2) The change in internal energy in the process CA is 700 R

3) The change in internal energy in the process AB is -350 R

4) The change in internal energy in the process BC is -500R

SOLUTION:

$$\text{Process } A \to B \quad \Delta U = nC_V \Delta T = 1 \times \frac{5R}{2}(800-400) = 1000R$$

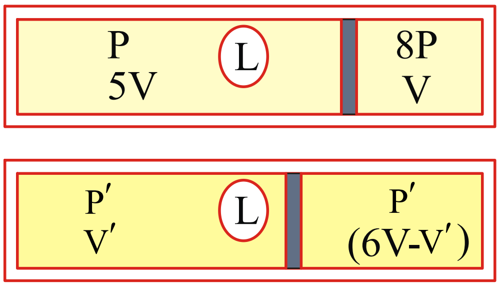
$$C \to A; \Delta U = nC_V \Delta T = 1 \times \frac{5R}{2}(400-600) = -500R$$

$$\Delta U_{cycle} = 0, \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$$

$$1000R + \Delta U_{BC} - 500R = 0; \quad \therefore \Delta U_{BC} = -500R$$

2. A piston divides a closed gas cylinder into two parts. Initially the piston is kept pressed such that one part has a pressure P and volume 5V and the other part has pressure 8P and volume V, the piston is now left free. Find the new pressure and volume for the isothermal and adiabatic process. $(\gamma = 1.5)$

SOLUTION:



Final pressure will be same on both sides. Let it be P', with volume V', in the left side and $(6V - V')$ in the right side

(A) if the change is isothermal.

For the gas enclosed in the left chamber,

$$P \times 5V = P'V' ....(i)$$

While for the gas in the right chamber

$$8P \times V = P'(6V - V').........(ii)$$

Solving these for V' and P', We get

$$V' = \frac{30}{13}V \text{ and } P' = \frac{13}{6}P \text{ and } 6V - V' = \frac{48}{13}V$$

(B) If the change is adiabatic. For the gas in the left chamber,

$$P(5V)^\gamma = P'(V')^\gamma \text{ ....(iii)}$$

and for the gas in the right chamber

$$8P(V)^\gamma = P'(6V - V')^\gamma \text{ ....(iv)}$$

dividing (iv) by (iii)

$$\left(\frac{6V - V'}{V'}\right)^{3/2} = \frac{8}{5^{3/2}} \text{ or } \frac{6V}{V'} = 1 + \frac{4}{5}$$

i.e $V' = \frac{10}{3}V$

Substituting it in Equation (iii)

$$P' = P\left(\frac{5V \times 3}{10V}\right)^{3/2} = \frac{3\sqrt{3}}{2\sqrt{2}}P = 1.84P$$

So $P' = 1.84P; V' = \frac{10}{3}V$ and $6V - V' = \frac{8}{3}V$

3. A gas undergoes a change of state during which 100 J of heat is supplied to it and it does 20 J of work. The system is brought back to its original state through a process during which 20 J of heat is released by the gas. What is the work done by the gas in the second process ?

SOLUTION:

$$dQ_1 = dU_1 + dW_1$$

$$100 = dU_1 + 20 \Rightarrow dU_1 = 80J$$

$$dQ_2 = dU_2 + dW_2 \quad (\therefore dU_1 = -dU_2)$$

$$-20 = -80 + dW_2$$

$$dW_2 = 60J$$

4. During an adiabatic process, if the pressure of an ideal gas is proportional to the cube of its temperature, find γ.

SOLUTION:

For an ideal gas of one mole PV = RT

During an adiabatic process $P \propto T^3$

$P = kT^3$; where k is a constant

$$P = k\left(\frac{PV}{R}\right)^3 \Rightarrow P = \left(\frac{k}{R^3}\right)P^3V^3$$

$$P^2V^3 = \text{constant};$$

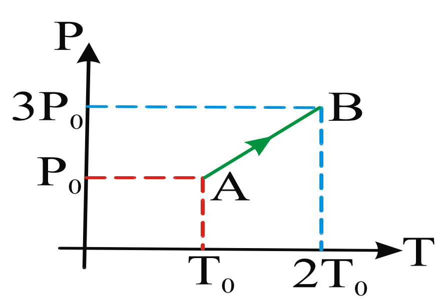
$$PV^{3/2} = \text{constant}$$

Comparing it with the equation

$PV^{\gamma} =$ constant for an adiabatic process,

we get $\gamma = 3/2$

5. Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0. Density at B will be



SOLUTION:

$$\rho = \frac{PM}{RT} \quad or \quad \rho \propto \frac{P}{T}$$

$$\left(\frac{P}{T}\right)_A = \frac{P_0}{T_0} \; and \; \left(\frac{P}{T}\right)_B = \left(\frac{3}{2}\right)\frac{P_0}{T_0}$$

$$\left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P}{T}\right) \quad \therefore \rho_B = \frac{3}{2}\rho_A = \frac{3}{2}\rho_0$$

6. When 5 moles of an ideal gas is compressed isothermally, its volume decreases from 5 litre to 1 litre. If the gas is at 27°C, find the work done on the gas $\left(\log_{10}\left(\frac{1}{5}\right) = -0.6990\right)$.

SOLUTION:

In the case of 'n' moles, work done on the gas

$$W = nRT \log_e\left(\frac{V_2}{V_1}\right)$$

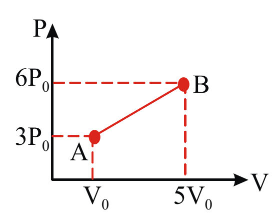
$$W = nRT \times 2.3026 \times \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$\therefore W = 5 \times 8.314 \times 300 \times 2.3026 \times \log_{10}\left(\frac{1}{5}\right)$$

$$= 5 \times 8.314 \times 300 \times 2.3026 \times (-0.6990)$$

$$= -2.007 \times 10^4 J$$

7. One mole of a monoatomicidealgas undergoes the process $A \rightarrow B$ in the given P - V diagram. What is the specific heat for this process ?



SOLUTION:

$$\text{Specific heat } C = \frac{\Delta Q}{\Delta T} = \frac{1}{\Delta T}(\Delta U + W) = C_v + \frac{W}{\Delta T}$$

For the given process

$$W = 4V_0\frac{9P_0}{2} = 18P_0V_0 \quad (\because \text{W= area of P} - \text{V graph})$$

$$\text{Also, } \Delta T = T_2 - T_1$$

$$= \frac{(6P_0)(5V_0)}{R} - \frac{(3P_0)V_0}{R} = \frac{27P_0V_0}{R}$$

and $C_v = \frac{3}{2}R$

$$\therefore C = C_v + \frac{W}{\Delta T} = \frac{3R}{2} + \frac{18P_0V_0}{\left(\frac{27P_0V_0}{R}\right)}$$

$$= \frac{3R}{2} + \frac{2R}{3} = \frac{13R}{6}$$

8. Temperature of 1 mole of an ideal gas is increased from 300 K to 310 K under isochoric process. Heat supplied to the gas in this process is Q = 25 R, where R = universal gas constant. What amount of work has to be done by the gas if temperature of the gas decreases from 310 K to 300 K adiabatically?

SOLUTION:

$$\Delta Q = nC_V\Delta T$$

$$\therefore 25R = (1)(C_V)(310-300)$$

$$C_V = \frac{5}{2}R$$

As the gas is diatomic $\gamma = 1.4$

Now work done in adiabatic process

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{(1)(R)(310 - 300)}{1.4 - 1} = 25R$$

9. Three samples of the same gas A, B and C ($\gamma = 3/2$) have initially equal volume. Now the volume of each sample is doubled. The process is adiabatic for A, isobaric for B and isothermal for C. If the final pressures are equal for all three samples, find the ratio of their initial pressures

SOLUTION:

Let the initial pressure of the three samples be P_A, P_B and P_C

then $P_A(V)^{\frac{3}{2}} = (2V)^{\frac{3}{2}}P$

$P_B = P$;

$$P_C(V) = P(2V)$$

$$\therefore \quad P_A : P_B : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$$

10. Work done to increase the temperature of one mole of an ideal gas by 30°C, if it is expanding under the condition $V \alpha T^{\frac{2}{3}}$ is $\left(R = 8.314 J/mol/K\right)$ (EAM-2012)

SOLUTION:

We have, $V \alpha T^{2/3}$;

But PV = RT

$PV \alpha T$; $PV \alpha V^{\frac{3}{2}}$

$$\Rightarrow P \alpha \sqrt{V}$$

$$\therefore P = k\sqrt{V} \Rightarrow P = kV^{1/2}$$

work done $W = \int PdV = k\int V^{1/2}dV = \frac{2k}{3}V^{3/2}$

11. Find the molar heat capacity in a process of a diatomic gas if it does a work of Q/4 when a heat of Q is supplied to it

SOLUTION:

$$dU = C_v dT = \left(\frac{5}{2}R\right)dT$$

$$dT = \frac{2(dU)}{5R}$$

From first law of thermodynamics

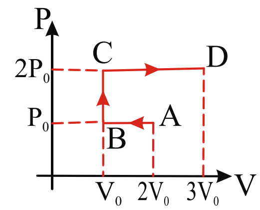
$$dU = dQ - dW = Q - \frac{Q}{4} = \frac{3Q}{4}$$

Now molar heat capacity

$$C = \frac{dQ}{dT} = \frac{Q' \, 5R}{2(dU)}$$

$$= \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

12. P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process ABCD is: (EAM-MED-2011)



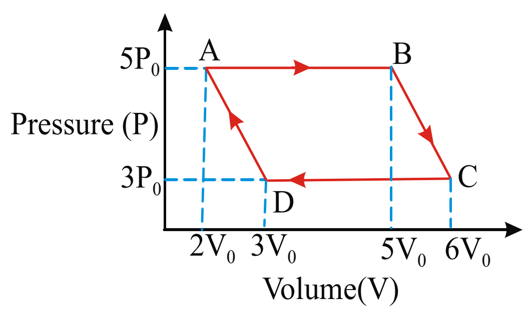
SOLUTION:

$$W_{AB} = -P_0V_0;$$

$$W_{BC} = 0 \;\; and \;\; W_{CD} = 4P_0V_0$$

$$W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$$

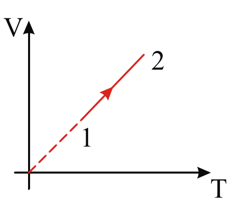
13. An ideal monoatomic gas is taken round the cycle ABCDA as shown in the P-V diagram. Compute the work done in this process.



SOLUTION:

$$\text{Total work done} = \text{Area under P-V curve (parallelogram)}$$
$$= \text{Base x Height}$$
$$= (6V_0 - 3V_0)\,(5P_0 - 3P_0)$$
$$= (3V_0)(2P_0) = 6P_0V_0 \text{ units}$$

14. Volume versus temperature graph of two moles of helium gas is as shown in figure. The ratio of heat absorbed and the work done by the gas in process 1-2 is :



SOLUTION:

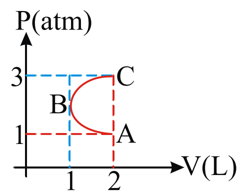
V - T graph is a straight line passing through origin. Hence,

$V \propto T$ or P = constant

$$\frac{dQ}{dW} = \frac{nC_p\Delta T}{nC_p\Delta T - nC_V\Delta T}$$

$$= \frac{nC_p \Delta T}{nR\Delta T} = \frac{5R}{2R} = \frac{5}{2}$$

15. In the P-V diagram shown in figure ABC is a semicircle. The work done in the process ABC is :



SOLUTION:

W_{AB} is negative (volume is decreasing)

W_{BC} is positive (volume is increasing) and since $|W_{BC}| > |W_{AB}|$

$\therefore$ Net work done is positive

$\because$ Workdone=area between semicircle

$$= \pi(\text{pressure radius})(\text{volume radius})$$

$$= \pi(1\text{atm})\left(\frac{1}{2}\textit{litre}\right)$$

$$= \frac{\pi}{2}\text{atm}(\textit{litre})$$

16. The volume of one mole of the gas is changed from V to 2V at constant pressure p. If γ is the ratio of specific heats of the gas, change in internal energy of the gas is (EAM 2014)

SOLUTION:

$$dU = nC_v dT$$

$$= n\frac{R}{\gamma - 1}dT$$

$$= n\frac{PdV}{\gamma - 1}$$

$$= n\frac{P(2V - V)}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

17. An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3}$=constant. What is the temperature inside the balloon

SOLUTION:

$$PV^{2/3} = \text{constant}$$

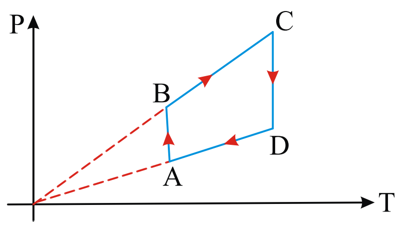
$$\Rightarrow\left(\frac{nRT}{V}\right)\left(V^{2/3}\right)=\text{constant}$$

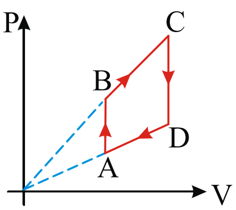
$$T.V^{-1/3}=\text{constant}$$

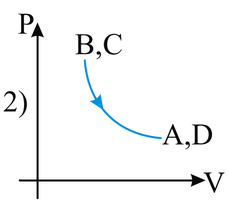
$$\Rightarrow V\propto T^{3}$$

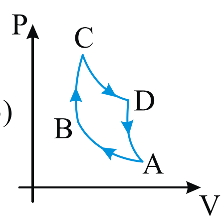
Temperature increases with increase in volume.

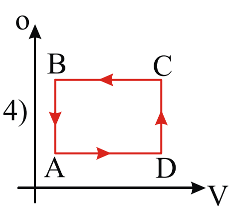
18. Pressure versus temperature graph of an ideal gas is as shown in figure corresponding density (ρ) versus volume(V) graph will be:



1) 

2) 

3) 

4) 

SOLUTION:

Along process AB, CD temperature is constant (isothermal process)

i.e., $P\propto\frac{1}{V}$,

$$\rho\propto\frac{1}{V}$$

$\rho-V$ graph will be a rectangular hyperbola.

Along BC and DA,

V is constant

ρ is constant

19. A tyre pumped to a pressure of 6 atmosphere suddenly bursts. Room temperature is 25°C. Calculate the temperature of escaping air. ($\gamma=1.4$.)

SOLUTION:

From $P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$

Here, $P_1 = 6$ atm ;

$P_2 = 1$ atm ;

$T_1 = 273+25 = 298K$;

$\gamma = 1.4$

$$(6)^{(1-1.4)}(298)^{1.4} = (1)^{(1-1.4)} T_2^{1.4}$$

$$T_2^{1.4} = (298)^{1.4} 6^{-0.4} = \frac{(298)^{1.4}}{6^{0.4}}$$

$$T_2 = \left[\frac{(298)^{1.4}}{6^{0.4}}\right]^{\frac{1}{1.4}} = \frac{(298)^{\frac{1.4}{1.4}}}{(6)^{\frac{0.4}{1.4}}} = \frac{298}{(6)^{\frac{2}{7}}}$$

(or) $\log T_2 = 2.4742 - \frac{2}{7}(0.7782)$

$= 2.4742 - 0.2209 = 2.2533.$

Anti log of 2.2533 = 178.7

$\therefore T_2 = 178.7K$

$t_2 = 178.7 - 273 = - 94.3^{O}C.$

20. 100 g of water is heated from 30^{0}C to 50^{0}C ignoring slight expansion of the water, the change in its internal energy is (specific heat of water is 4180J/Kg/K) (JEE MAIN-11)

SOLUTION:

$$dQ = dU + dW;$$

dW= 0 ($\because dV$ is neglected)

$$\therefore dQ = dU = mS\Delta\theta$$

$$= (100)(10^{-3})(4180)(20) = 8360J \approx 8.4KJ$$

21. P - V diagram of a diatomic gas is a straight line passing through origin. What is the molar heat capacity of the gas in the process

SOLUTION:

P - V diagram of the gas is a straight line passing through origin. Hence,

$$P \propto V$$

$$PV^{-1} = \text{constant} \Rightarrow x = -1$$

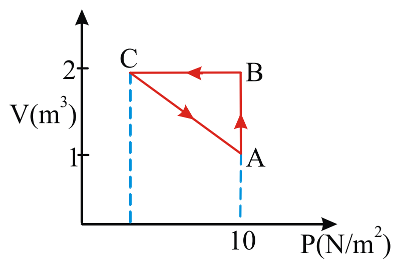
Molar heat capacity in the process

$$C = \frac{R}{g - 1} + \frac{R}{1 - x}$$

Here, $\gamma = 1.4$ (for diatomic gas)

$$\therefore \; C = \frac{R}{1.4 - 1} + \frac{R}{1+1} or \; C = 3R$$

22. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, what is the work done by the gas in the process $C \rightarrow A$



SOLUTION:

$$\Delta W_{AB} = P\Delta V = (10)(2-1) = 10J$$

$$\Delta W_{BC} = 0 \;\; (\text{as } V = \text{constant})$$

From first law of thermodynamics

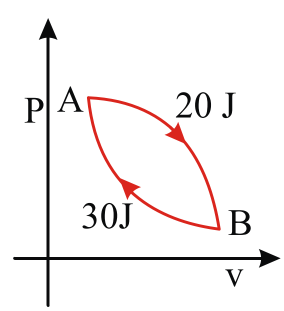
$$\Delta Q = \Delta W + \Delta U;$$

$\Delta U = 0$ (process ABCA is cyclic)

$$\therefore \;\; \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$

$$\therefore \Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC} = 5 - 10 - 0 = -5J$$

23. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B to A, the work done on the gas during the process $B \rightarrow A$ is 30 J, when the gas is taken from $A \rightarrow B$ the heat absorbed by the gas is 20J. What is the change in internal energy of the gas in the process $A \rightarrow B$.



SOLUTION:

$$W_{BA} = -30J, Q_{BA} = 0$$

$$;\Delta U_{BA} = -W_{BA} = 30J$$

Now, $DU_{AB} = - DU_{BA} = - 30J$

24. The relation between U, P and V for an ideal gas is U = 2 + 3PV. What is the atomicity of gas.

SOLUTION:

For an adiabatic process $dQ = 0 = dU + dW$

or $0 = dU + PdV$ —— (1)

From the given equation $dU = 3\,(PdV + VdP)$

Substituting dU from (1),

$-PdV = 3\,(PdV + VdP)$

or $4P\,(dV) + 3V\,(dP) = 0$

$$4\left(\frac{dV}{V}\right) = -3\left(\frac{dP}{P}\right)$$

On integrating, we get

$\text{In}\,(V^4) + \text{In}\,(P^3) = \text{constant},$

$\text{In}\,(V^4P^3) = \text{constant}$

$V^4P^3 = \text{constant}$

$PV^{4/3} = \text{constant}$

$$\text{i.e., } \gamma = \frac{4}{3}$$

i.e., gas is polyatomic.

25. If c_P and c_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then(AIEEE-2007)

1) $c_P - c_V = \frac{R}{28}$ 2) $c_P - c_V = \frac{R}{14}$ 3) $c_P - c_V = R$ 4) $c_P - c_V = 28R$

SOLUTION:

Mayer formula $c_P - c_V = \frac{R}{M} = \frac{R}{28}$

26. Five moles of hydrogen initially at STP is compressed adiabatically so that its temperature becomes 673 K. The increase in internal energy of the gas, in kilo joule is (R = 8.3 J/mol-K; $\gamma = 1.4$ for diatomic gas) (EAM-2014)

SOLUTION:

Work done by an ideal gas in adiabatic expansion

$$dU = n\frac{R}{\gamma - 1}dT = 5\left(\frac{8.3}{1.4-1}\right)(300) = 41500J$$

27. A gas is expanded to double its volume by two different processes. One is isobaric and the other is isothermal. Let W_1 and W_2 be the respective work done, then find W_1 and W_2

SOLUTION:

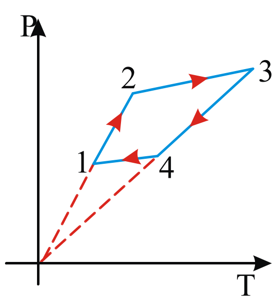
$$W_1 = P_i\left(V_f - V_i\right) = P_iV_i\left(\frac{V_f}{V_i} - 1\right)$$

$$= nRT(2-1) = nRT$$

$$W_2 = nRT\log_e\left(\frac{V_f}{V_i}\right) = nRT\ \log_e(2) = W_1\ \log_e(2)$$

28. Three moles of an ideal monoatomic gas performs a cycle $1 \to 2 \to 3 \to 4 \to 1$ as shown. The gas temperatures in different states are, $T_1 = 400K, T_2 = 800K$,

$T_3 = 2400K$ *and* $T_4 = 1200K$. **The work done by the gas during the cycle is**



SOLUTION:

$1 \to 2$ *and* $3 \to 4$ are isochoric process.

Therefore, work done is zero.

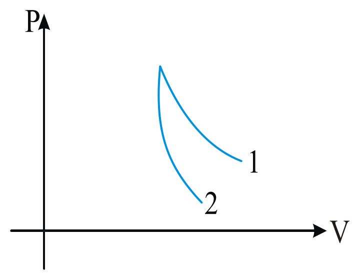
$$\therefore W_{total} = W_{23} + W_{41} = P_2(V_3 - V_2) + P_4(V_1 - V_4)$$

$$= nR(T_3 - T_2) + nR(T_1 - T_4)$$

$$= nR(T_3 - T_2 + T_1 - T_4)$$

$$= 800nR = 2400R$$

29. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to:



SOLUTION:

In adiabatic process Slope of P-V graph,

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

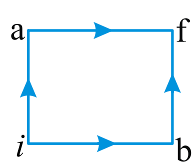
Slope $\propto \gamma$ (with negative sign)

From the given graph, $(\text{slope})_2 > (\text{slope})_1$

$$\therefore \gamma_2 > \gamma_1$$

Therefore, 1 should correspond to O_2 $(\gamma = 1.4)$ and 2 should correspond to He $(\gamma = 1.67)$.

30. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path' ibf Q = 36 cal. W along the path ibf is (AIEEE-2007)



SOLUTION:

From first law of thermodynamics,

$$dQ = dU + dW$$

For path iaf, $50 = \Delta U + 20$

$$\Rightarrow \Delta U = 30\,cal$$

For path ibf, $dW = dQ - dU = 36 - 30 = 6\,cal$

31. A monoatomic gas undergoes a process given by 2dU + 3dW = 0, then what is the process

SOLUTION:

$$dQ = dU + dW$$

$$\Rightarrow dQ = dU - \frac{2dU}{3} = \frac{dU}{3} = \frac{1}{3} nC_v dT$$

$$= \frac{1}{3} n.\frac{3}{2} RdT = \frac{nRdT}{2}$$

$$C = \frac{1}{n}\frac{dQ}{dT} = \frac{R}{2}$$

;It is not isobaric as C is not equal to $\frac{5R}{2}$

; It is not adiabatic as $C \neq 0$

It is not isothermal as $C \neq \infty$

so it is a polytrophic process.

32. thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heat γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings. its temperature increases by (in Kelvin) (JEE MAIN-2011)

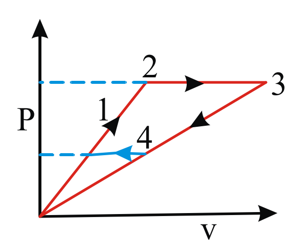
SOLUTION:

.

$$\frac{1}{2} mv^2 = du = nC_V dT\,;$$

$$\frac{1}{2} mv^2 = \frac{m}{M}\frac{R}{\gamma - 1}\Delta T$$

33. Three moles of an ideal monoatomic gas undergoes a cyclic process as shown in the figure. The temperature of the gas in different states marked as 1, 2, 3 and 4 are 400K, 700K, 2500K and 1100K respectively. The work done by the gas during the process 1-2-3-4-1 is (universal gas constant is R) (EAM-2013)



SOLUTION:

Process $1 \to 2$ and $3 \to 4$ are polytrophic and process $2 \to 3$ and $4 \to 1$ are isobaric.

From the graph

$$P \propto V \Rightarrow \frac{P}{V} = K \Rightarrow PV^{-1} = K \Rightarrow x = -1$$

Work done W= $W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$

$$= \frac{nR}{x-1}[T_1 - T_2] + P_2(V_3 - V_2) + \frac{nR}{x-1}[T_3 - T_4] + P_1[V_1 - V_4]$$

$$= \frac{nR}{x-1}[T_1 - T_2] + nR(T_3 - T_2) + \frac{nR}{x-1}[T_3 - T_4] + nR[T_1 - T_4]$$

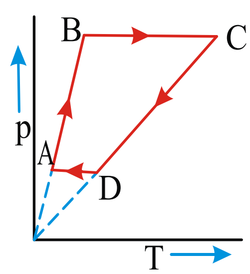
$$= \frac{nR}{x-1}[T_1 - T_2 + T_3 - T_4] + nR(T_3 - T_2 + T_1 - T_4)$$

$$= \frac{3R}{-1-1}[400 - 700 + 2500 - 1100] \quad + 3R(2500 - 700 + 400 - 1100)$$

$$= \frac{3R}{-1-1}[1100] + 3R(1100)$$

$$= 3R(1100)\left(1 - \frac{1}{2}\right) = 1650R$$

34. moles of an ideal monoatomic gas performs ABCDA cyclic process as shown in figure below. The gas temperatures are $T_A = 400K, T_B = 800K, T_C = 2400K$ and $T_D = 1200K$. The work done by the gas is (approximately) $(R = 8.314\, J/mol\, K)$ (EAMCET-2010)



SOLUTION:

Processes A to B and C to D are parts of straight line graphs of form y=mx

and $P = \frac{nR}{V}T \quad (n = 3)$ **i.e** $P \alpha T$

So, volume remains constant for the graphs AB and CD.

So, no work is done during processes for A to B and C to D.

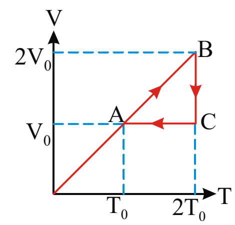
$$W_{AB} = W_{CD} = 0 \text{ and } W_{BC} = P_2(V_C - V_B)$$
$$= nR(T_C - T_B) = 3R(2400 - 800) = 4800R$$

$$W_{DA} = P_1(V_A - V_D) = nR(T_A - T_D)$$
$$= 3R(400 - 1200) = -2400R$$

Work done in the complete cycle

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= 0 + 4800R + 0 + (-2400)R$$
$$= 2400\ R = 19953.6 J \approx 20\ kJ$$

35. An ideal monoatomic gas undergoes a cyclic process ABCA as shown in the figure.The ratio of heat absorbed during AB to the work done on the gas during BC is:



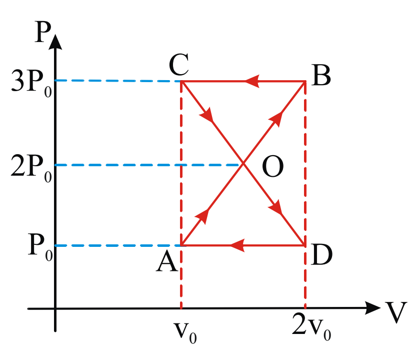
SOLUTION:

$$Q_{AB} = nC_P\Delta T = n\frac{5}{2}R(2T_0 - T_0) = \frac{5nRT_0}{2}$$

$$Q_{BC} = W_{BC} = nR2T_0\, ln\left(\frac{1}{2}\right) = -nR2T_0 \ln(2)$$

$$|Q_{BC}| = nR2T_0 \ln(2) \Rightarrow \frac{Q_{AB}}{W_{BC}} = \frac{5}{4\ln(2)}$$

36. A thermodynamic system undergoes cyclic process ABCDA as shown in figure. The work done by the system is :



SOLUTION:

$$W_{BCOB} = -\text{ Area of triangle } BCO = -\frac{P_0V_0}{2}$$

$$W_{AODA} = +\text{ Area of triangle } AOD = +\frac{P_0V_0}{2}$$

$$\therefore W_{net} = 0$$

Second law of thermodynamics :

Clausius statement: It is impossible for a self acting machine unaided by any external agency to transfer heat from a cold reservoir to a hot reservoir. In other words heat can't flow by itself from a colder to a hotter body.

Kelvin-Planck Statement: It is impossible for any heat engine to convert all the heat absorbed from a reservoir completely into useful work. In other words 100% conversion of heat into work is impossible.

These two statements of the second law are equivalent to each other. Because, if one is violated, the other is also automatically.

Reversible process:

A process which can be retraced back in such a way that the system passes through the states as in direct process and finally the system acquires the initial conditions, leaving no change anywhere else, is called reversible process. Any quasi-static process can be reversible.

Conditions for a process to be reversible:

(a) There should be no loss of energy due to conduction, convection or dissipation of energy against any resistance, like friction, viscosity etc.

(b) No heat should be converted into magnetic or electric energy.

(c) The system must always be in thermal, mechanical and chemical equilibrium with the surroundings.(i.e the process must be quasi-static)

Examples : In practice, there is no
reversible process. But approximately we can give the following examples.

i) The process of change of state from ice into water is a reversible process.

ii) The process of change of state from water to steam.

iii) The gradual extension and compression of an elastic spring is approximately reversible.

iV) The electrolysis process is reversible if internal resistance is negligibly small.

v) Slow compression and expansion of an ideal gas at constant temperature.

Irreversible process:

In this process the system does not pass through the same intermediate states as in the direct process. All the processes occurring in nature are irreversible.

Examples : 1) Diffusion of gas

2) Dissolving of salt in water

3) Sudden expansion or compression of gas

Heat engine :

The device, used to convert heat energy into mechanical energy is called a heat engine.

For conversion of heat into work with the help of a heat engine the following conditions required.

i) There should be a reservoir at constant higher temperature 'T_1' from which heat is extracted. It is called the source.

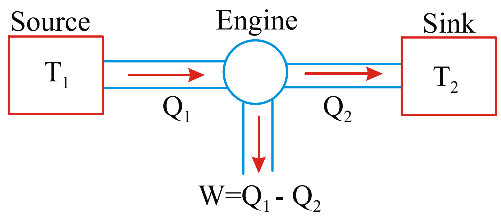
ii) Working substance which undergoes thermodynamic cyclic changes(ex: ideal gas).

iii) There should be a reservoir at constant lower temperature 'T_2' to which heat can be rejected. This is called the sink.

- The source and sink should have very high thermal capacity.

Working of heat engine :

a) Schematic diagram of heat engine



b) Engine derives an amount 'Q_1' of heat from the source.

c) A part of this heat is converted into work 'W'.

d) Remaining heat 'Q_2' is rejected to the sink.

Thus $Q_1 = W + Q_2$ or the work done by the engine is given by $W = Q_1 - Q_2$

e)The energy Q_2 is unavailable in the universe, which causes increase in entropy of universe.

Efficiency of heat engine :

- Efficiency of heat engine (η) is defined as the fraction of total heat, supplied to the engine which is converted into work.

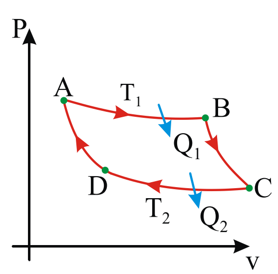
Mathematically $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

- According to this , efficiency is 100% if $Q_2 = 0$, that is no heat is rejected to the cold reservoir or sink. That is the entire heat absorbed must be converted to mechanical work , which is impossible according to Second law of Thermodynamics.

Carnot or Reversible or Ideal heat engine:

- When the working substance is an ideal gas and it is subjected to cyclic process consisting of isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression, then such heat engine is called Carnot engine. The cyclic process is called Carnot cycle.

  Carnot Cycle : Carnot cycle consists of the following four stages (i) Isothermal expansion (process AB), (ii) Adiabatic expansion (process BC), (iii) Isothermal compression (process CD), and (iv) Adiabatic compression (process DA).



The P-V diagram of the cycle is shown in the figure.

In process AB heat Q_1 is taken by the working substance at constant temperature T_1 and in process CD heat Q_2 is liberated by the working substance at constant temperature T_2. The net work done is area enclosed by the cycle ABCDA. After doing the calculations for different processes we can show that : $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

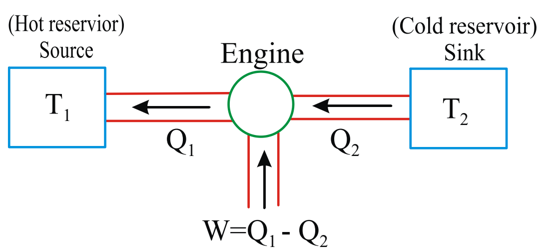
Therefore, efficiency of the Carnot engine is

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

As T_2 is always less than T_1, $\eta < 1$. i.e., the value of η can never be equal or greater than 1. When the temperature of sink T_2 =0 K, then η can be 1 or 100%. But it is impossible.

- For Carnot engine η is independent of the nature of working substance. It depends on only the temperatures of source and sink.
- The efficiency of an irreversible engine is always less than or equal to that of reversible engine when operated between the same temperature limits. $\therefore always\ \eta_{ir} \le \eta_r$

Refrigerator:



The refrigerator is just the reverse of heat engine In refrigerator the working substance extracts an amount of heat Q_2 from the cold reservoir (Sink)

at a lower temperature T_2. An amount of external work W is done on the working substance and finally an amount of heat Q_1 is rejected to the hot reservoir at a higher temperature T_1.

- Coefficient of performance of a refrigerator

$$b = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad [\because\ W = Q_1 - Q_2]$$

- For Carnot refrigerator $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ .Thus $\beta = \frac{T_2}{T_1 - T_2}$
- The relation between efficiency of a heat engine (η) and coefficient of performance of a refrigerator (b) working between the same temperature limits is $b = \frac{1-\eta}{\eta}$
- Let η_1 and η_2 are the efficiencies of heat engines working between temperature limits (T_1, T_2) and (T_2, T_3) respectively then the efficiency of heat engine working between temperature limits T_1 and T_3 is

$$\eta_1 = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \eta_1$$

$$\eta_2 = 1 - \frac{T_3}{T_2} \Rightarrow \frac{T_3}{T_2} = 1 - \eta_2$$

$$(1-\eta_1)(1-\eta_2) = \frac{T_2}{T_1} \times \frac{T_3}{T_2} = \frac{T_3}{T_1}$$

But $\eta = 1 - \frac{T_3}{T_1} \Rightarrow \frac{T_3}{T_1} = 1 - \eta$

$$\therefore \ \eta = 1 - (1-\eta_1)(1-\eta_2)$$

PROBLEMS

1. Efficiency of a heat engine whose sink is at temperature of 300 K is 40%. To increase the efficiency to 60%, keeping the sink temperature constant, the source temperature must increased by

SOLUTION:

$$\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{3}{5}$$

$$\Rightarrow T_1 = \frac{5}{3} T_2 \ \Rightarrow T_1 = \frac{5}{3} \times 300 = 500K$$

New efficiency $\eta' = 60\%$

$$\frac{T_2}{T_1'} = 1 - \eta' = 1 - \frac{60}{100} = \frac{2}{5}$$

$$T_1' = \frac{5}{2} \times 300 = 750K \ ; \Delta T = 750 - 500 = 250K$$

2. A refrigerator, whose coefficient of performance β is 5, extracts heat from the cooling compartment at the rate of 250 J per cycle.

(a) How much work per cycle is required to operate the refrigerator?

(b) How much heat per cycle is discharged to the room which acts as the high temperature reservoir?

SOLUTION:

a) As coefficient of performance of a refrigerator is defined as $\beta = Q_L / W$,So $W = \frac{Q_L}{\beta} = \frac{250}{5} = 50J$

(b)As $Q_H = Q_L + W$; so $Q_H = 250 + 50 = 300\,J$

3. A Carnot engine operating between temperatures T_1 and T_2 has efficiency 1/6. When T_2 is lowered by 62K, its efficiency increases to 1/3. Then T_1 and T_2 are, respectively : **(JEE MAIN-2011)**

SOLUTION:

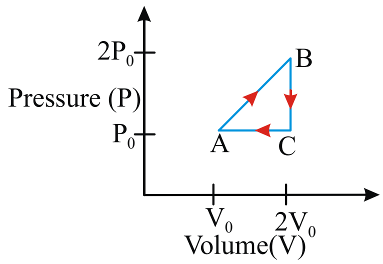
$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{6} = 1 - \frac{T_2}{T_1} \ \Rightarrow \frac{T_2}{T_1} = \frac{5}{6} \ .....(1)$$

$$\eta_2 = 1 - \frac{T_2 - 62}{T_1} \Rightarrow \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} \ ....(2)$$

On solving Equation (1) and (2)

$T_1 = 372\ K$ and $T_2 = 310\ K$

4. Find the efficiency of the thermodynamic cycle shown in figure for an ideal diatomic gas.



SOLUTION:

Sol. Let n be the number of moles of the gas and the temperature be T_0 in the state A.

Now, work done during the cycle

$$W = \frac{1}{2} \times (2V_0 - V_0)(2P_0 - P_0) = \frac{1}{2} P_0 V_0$$

For the heat (ΔQ_1) given during the process $A \to B$, we have $\Delta Q_1 = \Delta W_{AB} + \Delta U_{AB}$

$\Delta W_{AB} =$ area under the straight line AB

$$= \frac{1}{2}(P_0 + 2P_0)(2V_0 - V_0) = \frac{3P_0V_0}{2}$$

Applying equation of state for the gas in the state A & B.

$$\frac{P_0V_0}{T_0} = \frac{(2P_0)(2V_0)}{T_B} \Rightarrow T_B = 4T_0$$

$$\therefore U_{AB} = nC_V\Delta T = n\left(\frac{5R}{2}\right)(4T_0 - T_0) = \frac{15nRT_0}{2} = \frac{15P_0V_0}{2}$$

$$\therefore \Delta Q_1 = \frac{3}{2}P_0V_0 + \frac{15}{2}P_0V_0 = 9P_0V_0$$

Obviously, the processes $B \to C$ and $C \to A$ involve the abstraction of heat from the gas.

$$\text{Efficiency} = \frac{\text{Work done per cycle}}{\text{Total heat supplied per cycle}}$$

i.e., $\eta = \dfrac{\frac{1}{2}P_0V_0}{9P_0V_0} = \dfrac{1}{18}$

5. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

SOLUTION: **(JEE MAIN-2007)**

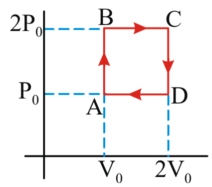
Sol. Coefficient of performance of refrigerator

$$\beta = \frac{1-\eta}{\eta} = \frac{1-1/10}{1/10} = 9 = \frac{Heat\ extracted}{workdone}$$

$\therefore$ Heat extracted $= 9 \times 10 = 90J$

6. Helium gas undergoes through a cycle ABCD (consisting of two isochoric and isobaric line) as shown in figure. Efficiency of this cycle is nearly : (Assume the gas to be close to ideal gas)

(JEE MAIN-2012)



SOLUTION:

Sol. Efficiency $= \dfrac{work\ done\ in\ cycle}{heat\ absorbed} \times 100$

$$= \frac{Area\ under\ P-V\ diagram}{\Delta Q_{AB} + \Delta Q_{BC}}$$

$$\therefore \eta = \frac{P_0 V_0}{nC_V \Delta T_1 + nC_p \Delta T_2}$$

$$= \frac{P_0 V_0}{\frac{3}{2} nR(T_B - T_A) + \frac{5}{2} nR(T_C - T_B)} \qquad = \frac{P_0 V_0}{\frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V_0)}$$

$$= \frac{P_0 V_0}{\frac{3}{2} P_0 V_0 + \frac{5}{2} 2P_0 V_0} = \frac{1}{6.5} = 15.4\%$$

7. A Carnot engine, whose efficiency is 40% takes heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then , the intake temperature for the same exhaust (sink) temperature must be (JEE MAIN-12)

SOLUTION:

Sol. $\eta = 1 - \dfrac{T_2}{T_1}; 0.4 = 1 - \dfrac{T_2}{500} \Rightarrow T_2 = 300K$

$$0.6 = 1 - \frac{T_2}{T_1^1} = 1 - \frac{300}{T_1^1} \Rightarrow T_1^1 = \frac{300}{0.4} = 750K$$

8. A diatomic ideal gas is used in a car engine as the working substance. Volume of the gas increases from V to 32V during the adiabatic expansion part of the cycle. The efficiency of the engine is (JEE MAIN-2010)

SOLUTION:

Sol. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_1 V_1^{1.4-1} = T_2 (32V_1)^{1.4-1}$ $\qquad \Rightarrow \dfrac{T_2}{T_1} = \dfrac{1}{4} \Rightarrow \eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

Entropy(s):

A ♦ The thermodynamic coordinate or parameter that gives the measure of disorder is called entropy. We cannot measure entropy, but we can measure change in entropy during thermodynamic change. If 'ds' is the small change in entropy at temperature T, then $ds = \frac{dQ}{T}$

Where dQ is exchange of heat between system and surroundings at temperature T.

Now the total change in entropy is $\Delta s = \int \frac{dQ}{T}$

- ♦ Change in entropy during an isothermal change is $\Delta s = \frac{\Delta Q}{T} = \frac{\Delta W}{T} = 2.303nR\log_{10}\left(\frac{V_2}{V_1}\right)$
- ♦ Change in entropy during phase change is $\Delta s = \frac{mL}{T}$
- ♦ Change in entropy during temperature change is

$$\Delta s = m\int \frac{sdT}{T}, \quad \text{(if s is temperature dependent)}$$

- ♦ In a reversible process entropy increases if heat is absorbed and vice - versa.
- ♦ Entropy of the universe always increases if system undergoes an irreversible process.
- ♦ Entropy of universe can never be zero.
- ♦ At absolute zero temperature(0K), entropy becomes zero. But it does not occur.

THEORY BITS

1. **Which of the following does not characterise the thermodynamic state of matter**

   1) volume 2) temperature 3) pressure 4) work

KEY : 4

2. **The thermal motion means**

   1) motion due to heat engine 2) disorderly motion of the body as a whole

   3) motion of the body that generates heat 4) random motion of molecules

KEY : 4

3. **Heat required to raise the temperature of one gram of water through $1^0 c$**

   1) 0.001 K cal 2) 0.01 K cal 3) 0.1 K cal 4) 1.0 K cal

KEY : 1

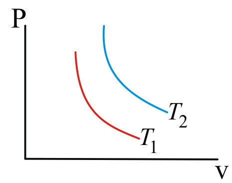
4. **Heat capacity of a substance is infinite. It means**

   1) heat is given out 2) heat is taken in

   3) no change in temperature whether heat is taken in (or) given out 4) all of the above

KEY : 3

5. **For a certain mass of a gas Isothermal relation between 'P' and 'V' are shown by the graphs at two different temperatures T_1 and T_2 then**



   1) $T_1 = T_2$ 2) $T_1 > T_2$ 3) $T_1 < T_2$ 4) $T_1 \geq T_2$

KEY : 3

6. **The temperature range in the definition of standard calorie is**

   1) 14.5^0C to 15.5^0C 2) 15.5^0C to 16.5^0C 3) 1^0C to 2^0C 4) 13.5^0C to 14.5^0C

KEY : 1

7. **The pressure p of a gas is plotted against its absolute temperature T for two different constant volumes V_1 and V_2, where $V_1 > V_2$. If p is taken on y–axis and T on x–axis.**

   1) The curve for V_1 has greater slope than the curve for V_2

   2) The curve for V_2 has greater slope than the curve for V_1

   3) The curves must intersect at some point other than T = 0

   4) The curves have the same slope and do not intersect

KEY : 2

8. **The internal energy of a perfect monoatomic gas is**

   1) Complete kinetic 2) Complete potential

   3) Sum of potential and kinetic energy of the molecules

   4) Difference of kinetic and potential energies of the molecules

9. **dU+dW=0 is valid for**

   1) adiabatic process 2) isothermal process 3) isobaric process 4) isochoric process

KEY : 1

10. A piece of ice at 0^0C is dropped into water at 0^0C. Then ice will

1) melt 2) be converted into water 3) not melt 4) partially melt

KEY : 3

11. The temperature determines the direction of net change of

1) gross kinetic energy 2) intermolecular kinetic energy

3) gross potential energy 4) intermolecular potential energy

KEY : 2

12. A closed vessel contains some gas at a given tmperature and pressure. If the vessel is given a very high velocity, then the temperature of the gas

1) increases 2) decreases

2) may increase or decrease depending upon the nature of the gas 4) does not change

KEY : 4

13. Unit mass of liquid of volume V_1 completely turns into a gas of volume V_2 at constant atmospheric pressure P and temperature T. The latent heat of vaporization is "L". Then the change in internal energy of the gas is

1) L 2) $L+P(V_2 - V_1)$ 3) $L - P(V_2-V_1)$ 4) Zero

KEY : 3

14. Zeroth law of thermodynamics gives the concept of

1) Pressure 2) Volume 3) Temperature 4)Heat

KEY : 3

15. We need mechanical equivalent of heat because

1) it converts work into heat

2) in C.G.S system, heat is not measured in the units of work

3) in SI system, heat is measured in the units of work

4) of some reason other than those mentioned above

KEY :2

16. When we switch on the fan in a closed room. The temperature of the air molecules

1) Increases 2) Decreases

3) Remains unchanged

4) May increase or decrease depending on the speed of rotation of the fan.

KEY : 1

17. Which type of molecular motion does contribute towards internal energy for an ideal monoatomic gas

1) Translational 2) Rotational 3) Vibrational 4) All the above

KEY : 1

18. In which of the following processes the internal energy of the system remains constant ?

1) Adiabatic 2) Isochoric 3) Isobaric 4) Isothermal

KEY : 4

19. In a given process $dW=0$, $dQ<0$ then for a gas

1)Temperature–increases 2)Volume–decreases 3)Pressure–decreases 4)Pressure–increases

KEY : 3

20. Which of the following is constant in an isochoric process

1) Pressure 2) Volume 3) Temperature 4) Mass

KEY : 2

21. How does the internal energy change when the ice and wax melt at their normal melting points?

1) Increases for ice and decreases for wax 2) Decreases for ice and increases for wax

3) Decreases both for ice and wax 4) Increases both for ice and wax

KEY : 25)1

22. On compressing a gas suddenly, its temperature

1) increases 2) decreases 3) remains constant 4) all the above

KEY : 1

23. Certain amount of heat supplied to an ideal gas under isothermal conditions will result in

1) raise in temperature 2) doing external work and a change in temperature

3) doing external work 4) an increase in the internal energy of the gas

KEY : 3

24. When heat is added to a system at constant temperature, which of the following is possible.

1) internal energy of system increases 2) work is done by the system

3) neither internal energy increases nor work done by the system

4) internal energy increases and work is done by the system

KEY : 2

25. The first law of thermodynamics is based on the law of conservation of

1) energy 2) mass 3) momentum 4) pressure

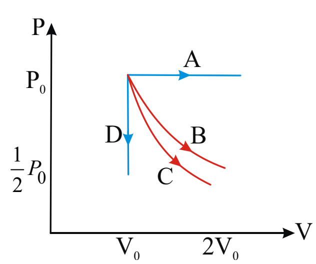
KEY : 1

26. A liquid in a thermos flask is vigorously shaken. Then the temperature of the liquid

1) Is not altered 2) Increases 3) Decreases 4) None

KEY :4

27. The PV diagram shows four different possible paths of a reversible processes performed on a monoatomic ideal gas. Path A is isobaric, path B is isothermal , path C is adiabatic and path D is isochoric . For which process does the temperature of the gas decrease?

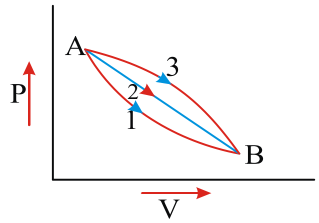


1) Process A only 2) Process C only 3) Processes C & D 4) Processes B, C & D

KEY :3

28. A given mass of a gas expands from the state A to the state B by three paths 1,2 and 3 as shown in

the figure. If W_1, W_2 and W_3 respectively be the work done by the gas along the three paths then



1) $W_1 > W_2 > W_3$ 2) $W_1 < W_2 < W_3$ 3) $W_1 = W_2 = W_3$ 4) $W_1 < W_2 = W_3$

KEY :2

29. A given system undergoes a change in which the work done by the system equals to the decrease in its internal energy. The system must have undergone an

1) Isothermal change 2) Adiabatic change 3) Isobaric change 4) Isochoric change

KEY :2

30. When two blocks of ice are pressed against each other then they stick together because

1) cooling is produced 2) heat is produced

3) increase in pressure will increase in melting point

4) increase in pressure will decrease in melting point

KEY :4

31. A cubical box containing a gas with internal energy U is given velocity V, then the new internal energy of gas

1) less than U 2)more than U 3) U 4) zero

KEY :3

32. In an isobaric (constant pressure) process. the correct ratio is

1) DQ : DU = 1 : 1 2) DQ : DU = 1 : g-1 3) DQ : DU = g-1: 1 4) DQ : DU = g : 1

KEY :4

33. In an isobaric process, the correct ratio is

1) DQ : DW = 1 : 1 2) DQ : DW = g : g-1 3) DQ : DW = g-1: g 4) DQ : DW = g : 1

KEY :2

34. Air in a thermally conducting cylinder is suddenly compressed by a piston, which is then maintained at the same position. With the passage of time :

1) the pressure decreases 2) the pressure increases 3) the pressure remains the same

4) the pressure may increase or decrease depending upon the nature of the gas

KEY :1

35. Which of the following states of matter have two specific heats ?

1) solid 2) gas 3) liquid 4) Plasma

KEY :2

36. The specific heat of a gas in an isothermal process is

1) infinity 2) zero 3) negative 4) remains constant

KEY :1

37. Why the specific heat at a constant pressure is more than that at constant volume

1) there is greater inter molecular attraction at constant pressure

2) at constant pressure molecular oscillations are more violent

3) external work need to be done for allowing expansion of gas at constant pressure

4) due to more reasons other than those mentioned in the above

KEY :3

38. Two identical samples of gases are allowed to expand to the same final volume (i) isothermally (ii) adiabatically. Work done is

1. more in the isothermal process   2. more in the adiabatic process

3. equivalent in both processes   4. equal in all process

KEY :1

39. Which of the following is true in the case of a reversible process

1) There will be energy loss due to friction

2) System and surroundings will not be in thermo dynamic equilibrium

3) Both system and surroundings retains their initial states

4) 1 and 3

KEY :3

40. The ratio of the relative raise in pressure for adiabatic compression to that for isothermal compression is

1) γ 2) $\frac{1}{\gamma}$ 3) $1-\gamma$ 4) $\frac{1}{1-\gamma}$

KEY :1

41. Ratio of isothermal elasticity of gas to the adiabatic elasticity is

1) γ 2) $\frac{1}{\gamma}$ 3) $1-\gamma$ 4) $\frac{1}{1-\gamma}$

KEY :2

42. The conversion of water into ice is an

1) isothermal process   2) isochoric process   3) isobaric process   4) entropy process

KEY :47

43. For the Boyle's law to hold good, the necessary condition is

1) Isobaric   2) Isothermal   3) Isochoric   4) Adiabatic

KEY :2

44. An isothermal process is

1)slow process   2)quick process   3) very quick process   4) both 1 & 2

KEY :1

45. The temperature of the system decreases in the process of

1) free expansion   2) isothermal expansion

3) adiabatic expansion   4) isothermal compression

KEY :3

46. The pressure P and volume V of an ideal gas both increase in a process

1) It is not possible to have such a process   2) The workdone by the system is positive

3) The temperature of the system increases   4) 2 and 3

KEY :2

47. Two samples of gas A and B, initially at same temperature and pressure, are compressed to half of

their initial volume, A isothermally and B adiabatically. The final pressure in

1) A and B will be same 2) A will be more than in B

3) A will be less than in B 4)A will be double that in B

KEY :3

48. In which of the following processes all three thermodynamic variables, that is pressure volume and temperature can change

1) Isobaric 2) Isothermal 3) Isochoric 4) Adiabatic

KEY :4

49. Two steam engines 'A' and 'B', have their sources respectively at 700 K and 650 K and their sinks at 350 K and 300K. Then

1) 'A' is more efficient than 'B' 2) 'B' is more efficient than 'A'

3) both are equally efficient 4) depends on fuels used in A and B

KEY :2

50. During adiabatic expansion the increase in volume is associated with

1) increase in pressure and temperature 2) decrease in pressure and temperature

3) increase in pressure and decrease in temperature

4)Decrese in pressure and increase in temperature

KEY :2

51. A gas is being compressed adiabatically. The specific heat of the gas during compression is

1) zero 2) infinite 3) finite but non zero 4) undefined

KEY :1

52. During adiabatic compression of a gas, its temperature

1) falls 2) raises 3) remains constant 4) becomes zero

KEY :2

53. The work done on the system in an adiabatic compression depends on

1) the increase in internal energy of the system 2) the decrease in internal energy

3) the change in volume of the system 4) all the above

KEY :1

54. The ratio of slopes of adiabatic and isothermal curves is

1) γ 2) $1/\gamma$ 3) γ^2 4) γ^3

KEY :1

55. If the temperature of the sink is decreased, then the efficiency of heat engine

1) first increases then decreases 2) increases

3) decreases 4) remains unchanged

KEY : 2

56. An ideal heat engine can be 100% efficient if its sink is at

1) 0K 2) 273K 3) 0^0C 4) 0^0F

KEY :1

57. If the temperature of a source increases, then the efficiency of a heat engine

1) increases 2) decreases 3) remains unchanged 4) none of these

KEY :1

58. By opening the door of a refrigerator inside a closed room:

1) you can cool the room to a certain degree
2) you can cool it to the temperature inside the refrigerator
3) you can ultimately warm the room slightly
4) you can neither cool nor warm the room

KEY :3

59. Which of the following will extinguish the fire quickly

1) water at 100^0C 2) steam at 100^0C 3) water at 0^0C 4) ice at 0^0C

KEY :71)1

60. Which of the following is true in the case of molecules, when ice melts

1) K.E is gained 2) K.E. is lost 3) P.E is gained 4) P.E. is lost

KEY :3

61. When heat is added to a system then the following is not possible?

1) Internal energy of the system increases
2) Work is done by the system
3) Neither internal energy increases nor work is done by the system
4) Internal energy increases and also work is done by the system

KEY :3

62. A sink, that is the system where heat is rejected, is essential for the conversion of heat into work. From which law the above inference follows?

1) Zeroth 2) First 3) Second 4) Both 1 & 2

KEY :3

63. The gas law $\left[\frac{PV}{T}\right]$ = constant is true for

1) isothermal change only
2) adiabatic change only
3) Both isothermal & adiabatic changes
4) neither isothermal nor adiabatic change

KEY :3

64. The efficiency of a heat engine:

1) is independent of the temperature of the source and the sink
2) is independent of the working substance
3) can be 100%
4) is not affected by the thermal capacity of the source or the sink

KEY :2

65. An ideal heat engine working between temperatures T_H and T_L has efficiency η. If both the temperatures are raised by 100K each, then the new efficiency of the heat engine will be:

1) equal to η 2) greater than η 3) less than η
4) greater or less than η depending upon the nature of the working substance

KEY :3

66. The efficiency of the reversible heat engine is η_r, and that of irreversible heat engine is η_I. Which of the following relation is correct?

1) $\eta_r > \eta_I$ 2) $\eta_r < \eta_I$ 3) $\eta_r \geq \eta_I$ 4) $\eta_r > 1$ and $\eta_I < 1$

KEY :3

67. In a heat engine, the temperature of the working substance at the end of the cycle is

1) equal to that at the beginning 2) more than that at the beginning

3) less than that at the beginning 4) determined by the amount of heat rejected to the sink

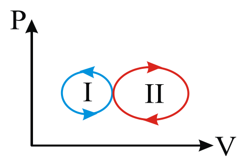
KEY :1

68. The adiabatic and isothermal elasticities B_ϕ and B_θ are related as :

1) $\frac{B_\phi}{B_\theta}=\gamma$ 2) $\frac{B_\theta}{B_\phi}=\gamma$ 3) $B_\phi - B_\theta = \gamma$ 4) $B_\theta - B_\phi = \gamma$

KEY :1

69. For the indicator diagram given below, select wrong statement.



1) Cycle - II is heat engine cycle 2) Net work is done on the gas in cycle - I

3) Workdone is positive for cycle - I 4) Workdone is positive for cycle - II

KEY :3

70. A cubical box containing a gas is moving with some velocity. If it is suddenly stopped, then the internal energy of gas

1) decreases 2) Increases 3) remains constant

4) may increases or decreases depending on the time interval during which box comes to rest.

KEY :2

71. The ratio [C_p / C_v] of the specific heats at a constant pressure and at a constant volume of any perfect gas

1) can't be greater than 5/4 2) can't be greater than 3/2

3) can't be greater than 5/3 4) can have any value

KEY :3

72. Which of the following formula is wrong ?

1) $C_v = \frac{R}{\gamma - 1}$ 2) $\frac{C_p}{C_V}=\gamma$ 3) $C_p = \frac{\gamma R}{\gamma - 1}$ 4) $C_p - C_v = 2R$

KEY :4

73. A common salt is first dissolved in water and extracted again from the water. In this process,

1) entropy decreases 2) entropy increases

3) entropy becomes zero 4) entropy remains constant.

KEY :2

74. A large block of ice is placed on a table where the surroundings are at 0^0C

1) ice melts at the sides 2) ice melts at the top

3) ice melts at the bottom 4) ice does not melt at all

KEY :3

75. Which of the following substance at 100^0C produces most severe burns ?

1) hot air 2) water 3) steam 4) oil

KEY :3

76. What energy transformation takes place when ice is converted into water

1) heat energy to kinetic energy 2) kinetic energy to heat energy
3) heat energy to latent heat 4) heat energy to potential energy

KEY :4

77. Which of the following laws of thermodynamics leads to the interference that it is difficult to convert whole of heat into work

1) zeroth 2)second 3) first 4)both 1 & 2

KEY :2

78. The direction of flow of heat between two gases is determined by

1) Average kinetic energy 2) total energy 3) internal energy 4) potential energy

KEY : 1

79. Heat is absorbed by a body . But its temperature does not raised. Which of the following statement explains the phenomena ?

1) only K.E. of vibration increases 2) only P.E. of inter molecular force changes
3) no increase in internal energy takes place 4) increase in K.E. is balanced by decrease in P.E.

KEY : 2

80. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 T h e amount of work done by the gas is greatest when the expansion is

1) isothermal 2) isobaric 3) adiabatic 4) equal in all cases

KEY :2

81. The second law of thermodynamics implies:

1) whole of heat can be converted into mechanical energy
2) no heat engine can be 100% efficient
3) every heat engine has an efficiency of 100%
4) a refrigerator can reduce the temperature to absolute zero

KEY :2

82. In the free expansion of a gas, its internal energy

1) remains constant 2) increases 3) decreases
4) sometimes increases , sometimes decreases

KEY :1

83. The internal energy of an ideal gas depends upon

1) only its pressure 2) only its volume 3) only its temperature 4) its pressure and volume

KEY :3

84. In the adiabatic compression the decrease in volume is associated with

1)increase in temperature and decrease in pressure
2) decrease in temperature and increase in pressure
3) decrease in temperature and decrease in pressure
4) increase in temperature and increase in pressure

KEY : 4

85. Which of the following is true in the case of an adiabatic process where $\gamma = C_P / C_V$?

1) $P^{1-\gamma}T^{\gamma} =$ constant 2) $P^{\gamma}T^{1-\gamma} =$ constant 3) $PT^{\gamma} =$ constant 4) $P^{\gamma}T =$ constant

KEY :1

86. If an ideal gas is isothermally expanded its internal energy will

1) Increase 2) Decrease 3) Remains same

4) Decrease or increase depending on nature of the gas

KEY :3

87. Heat engine rejects some heat to the sink. This heat

1)converts into electrical energy. 2)converts into light energy.

3)converts into electromagnetic energy 4)is unavailable in the universe.

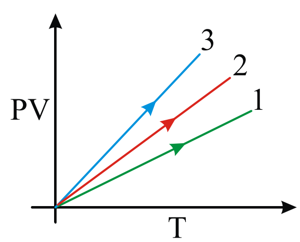
KEY :4

88. For an adiabatic change in a gas, if P, V,T denotes pressure, volume and absolute temperature of gas at any time and γ is the ratio of specific heats of a gas, then which of the following equation is true?

1) $T^{\gamma}P^{1-\gamma} = \text{const.}$ 2) $T^{1-\gamma}P^{\gamma} = \text{const.}$ 3) $T^{\gamma-1}V^{\gamma} = \text{const.}$ 4) $T^{\gamma}V^{\gamma} = \text{const.}$

KEY :1

89. PV versus T graph of equal masses of H_2, He and CO_2 is shown in figure. Choose the correct alternative



(1) 3 corresponds to H_2, 2 to He and 1 to CO_2 (2) 1 corresponds to He, 2 to H_2 and 3 to CO_2

(3) 1 corresponds to He, 3 to H_2 and 2 to CO_2 (4) 1 corresponds to CO_2,2 to H_2 and 3 to He

KEY :1

90. If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, then the change in internal energy of the mass of gas, when the volume changes from V to 2V at constant pressure P, is

1) $R/(\gamma-1)$ 2) PV 3) $PV/(\gamma-1)$ 4) $\gamma PV/(\gamma-1)$

KEY :3

91. Heat is added to an ideal gas and the gas expands. In such a process the temperature

1) must always increase 2) will remain the same if the work done is equal to the heat added

3) must always decrease

4) will remain the same if change in internal energy is equal to the heat added

KEY :2

92. First law of thermodynamics states that

1) system can do work 2) system has temperature

3) system has pressure 4) heat is form of energy

KEY :1

93. The material that has largest specific heat is

1) mercury 2) water 3) hydrogen 4) diamond

KEY :3

94. The law obeyed by isothermal process is

1) Gay-Lussac's law   2) Charles law   3) Boyle's law   4) Dalton's law

KEY :3

95. Which one of the following is wrong statement.

1) During free expansion, temperature of ideal gas does not change.
2) During free expansion, temperature of real gas decreases.
3) During free expansion of real gas temperature does not change.
4) Free expansion is conducted in adiabatic manner.

KEY :3

96. Which law defines entropy in thermodynamics

1) zeroth law   2) First law   3) second law   4) Stefan's law

KEY :3

97. For the conversion of liquid into a solid

1) orderliness decreases and entropy decreases   2) orderliness increases and entropy increases
3) both are not related   4) orderliness increases and entropy decreases

KEY :1

98. Among the following the irreversible process is

1) free expansion of gas   2)extension or compression of spring very slowly
3)motion of an object on a perfectly frictionless surface   4) all of them

KEY :1

99. Water is used in car radiators as coolant because

1) its density is more   2) high specific heat   3) high thermal conductivity 4) free availability

KEY : 2

100. Of the following specific heat is maximum for

1) Mercury   2) Copper   3) Water   4) Silver

KEY : 3

101. Heat is

1) kinetic energy of molecules   2) potential and kinetic energy of molecules
3) energy in transits   4) work done on the system

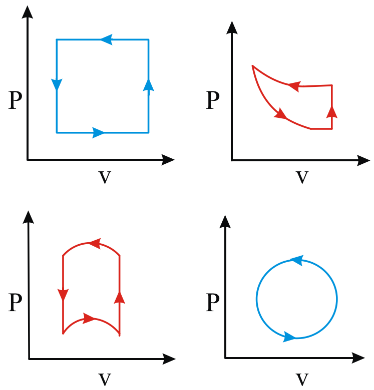
KEY : 3

102. Gas is taken through a cyclic process completely once. Change in the internal energy of the gas is

1) infinity   2) zero   3) Small   4) Large

KEY :2

103. What will be the nature of change in internal energy in case of processes shown below ?



1) + ve in all cases 2) – ve in all cases

3) – ve in 1 and 3 and + ve in 2 and 44) zero in all cases

KEY :4

104. Which of the following is incorrect regarding the first law of thermodynamics ?

1) It introduces the concept of internal energy

2) It introduces the concept of entropy

3) It is applicable to any process

4) It is a restatement of principle of conservation of energy.

KEY :2

105. The heat capacity of material depends upon

1) the structure of a matter 2) temperature of matter

3) density of matter 4) specific heat of matter

KEY : 4

106. Heat cannot flow by itself from a body at lower temperature to a body at higher temperature is a statement or consequence of

1) I^{st} law of thermodynamics 2) II^{nd} law of thermodynamics

3) conservation of momentum 4) conservation of mass

KEY :4

107. For an isothermal process

1) $dQ = dW$ 2) $dQ = dU$ 3) $dW = dU$ 4) $dQ = dU + dW$

KEY :1

108. When thermodynamic system returns to its original state, which of the following is NOT possible?

1) The work done is Zero 2) The work done is positive

3) The work done is negative 4) The work done is independent of the path followed

KEY :2

109. Two completely identical samples of the same ideal gas are in equal volume containers with the same pressure and temperature in containers labeled A and B. The gas in container A performs non-zero positive work W on the surroundings during an isobaric process before the pressure is reduced isochorically to 1/2 its initial amount. The gas in container B has its pressure reduced isochorically to 1/2 its initial value and then the gas performs same non-zero positive work W on

the surroundings during an isobaric process. After the processes are performed on the gases in containers A and B, which is at the higher temperature?

1)The gas in container A   2)The gas in container B

3) The gases have equal temperature.

4) The value of the work W is necessary to answer this question.

KEY :2

110. For an adiabatic process the relation between V and T is given by

1) $TV^{\gamma}=$constant   2) $T^{\gamma}V=$constant 3) $TV^{1-\gamma}=$constant   4) $TV^{\gamma-1}=$ constant

KEY :4

88. The temperature of the system decreases in the process of

1) Free expansion   2) Adiabatic expansion

3) Isothermal expansion   4)Isothermal compression

KEY :2

111. Which of the following conditions of the Carnot ideal heat engine can be realised in practice?

1) infinite thermal capacity of the source   2) infinite thermal capacity of the sink

3) perfectly non conducting stand   4) Less than 100% efficiency

KEY :4

113. A heat engine works between a source and a sink maintained at constant temperatures T_1 and T_2. For the efficiency to be greatest

1) T_1 and T_2 should be high   2) T_1 and T_2 should be low

3) T_1 should be high and T_2 should be low   4) T_1 should be low and T_2 should be high

KEY :3

114. The heat engine would operate by taking heat at a particular temperature and

1) Converting it all into work

2) Converting some of it into work and rejecting the rest at lower temperature

3) Converting some of it into work and rejecting the rest at same temperature

4) Converting some of it into work and rejecting the rest at a higher temperature .

KEY :2

115. Which of the following processes are nearly reversible

a. Heat conduction   b. Electrolysis

c. Diffusion   d. Change of state

1) Only a   2)Both b and d   3) Only c   4) All of the above

KEY :2

PRACTICEBITS

1. A piece of lead falls from a height of 100m on a fixed non-conducting slab which brings it to rest. If the specific heat of lead is 30.6 cal/kg °C, the increase in temperature of the slab immediately after collision

1) 6.72°C   2) 7.62°C 3) 5.62°C   4) 8.72°C

KEY : 1) 2

HINT:

$$mgh = JmS\Delta\theta \Rightarrow \Delta\theta = \frac{gh}{JS}$$

2. Hailstones fall from a certain height. If only 1% of the hailstones melt on reaching the ground, find the height from which they fall.
($g = 10$ ms^{-2}. $L = 80$ calorie/g and $J = 4.2$J/calorie)

1) 336 m 2) 236 m 3) 436 m 4) 536 m

KEY :2) 1

HINT:

$$mgh = \frac{JmL_{ice}}{100}$$

3. From what minimum height a block of ice has to be dropped in order that it may melt completely on hitting the ground

1) mgh 2) $\frac{mgh}{J}$ 3) $\frac{JL}{g}$ 4) $\frac{J}{Lg}$

KEY:3) 3

HINT:

W= JH $\Rightarrow$ mgh = JmL_{ice}

4. Two spheres A and B with masses in the ratio 2 : 3 and specific heat 2 : 3 fall freely from rest. If the rise in their temperatures on reaching the ground are in the ratio 1 : 2 the ratio of their heights of fall is

1) 3 : 1 2) 1 : 3 3) 4 : 3 4) 3 : 4

KEY: 4) 2

HINT:

$$mgh = mS\Delta\theta \Rightarrow h\alpha S\Delta\theta$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{S_1}{S_2} \times \frac{\Delta\theta_1}{\Delta\theta_2}$$

5. From what height a block of ice must fall into a well so that $\frac{1}{100}$th of its mass may be melted ($g = 10$ m/s^2)

1) 300 m 2) 336 m 3) 660 m 4) none

KEY: 5) 2

HINT:

$$mgh = \frac{m}{100} L_{ice}$$

6. Two identical balls 'A' and 'B' are moving with same velocity. If velocity of 'A' is reduced to half

and of 'B' to zero, then the raise in temperatures of 'A' to that of 'B' is

1) 3 : 4 2) 4 : 1 3) 2 : 1 4) 1 : 1

KEY:1

HINT:

$$mS\Delta\theta = J\frac{1}{2}m\left(v_2^2 - v_1^2\right) \Rightarrow \frac{\theta_1}{\theta_2} = \frac{v_2^2 - v_1^2}{v_2^{1^2} - v_1^{1^2}}$$

7. A 50kg man is running at a speed of $18kmh^{-1}$. If all the kinetic energy of the man can be used to increase the temperature of water from 20^0C to 30^0C, how much water can be heated with this energy?

1) 15 g 2) 20 g 3) 30 g 4) 40 g

KEY:1

HINT:

W= JH $\Rightarrow \frac{1}{2}mv^2 = JmS\Delta\theta$

8. A man of 60 kg gains 1000 cal of heat by eating 5 mangoes. His efficiency is 56%. To what height he can jump by using this energy?

1) 4m 2) 20 m 3) 28 m 4) 0.2 m

KEY: 1

HINT:

. mgh=JH

9. How much work to be done in decreasing the voume of an ideal gas by an amount of 2. x 10^{-4} m^3 at constant normal pressure of 1 x 10^5 N/m^2

1) 28 joule 2) 27 joule 3) 24 joule 4)25joule

KEY: 3

HINT:

$dW = PdV$

10. Find the external work done by the system in kcal, wen 20 kcal of heat is supplied to thesystem and the increase in the internal energy is 8400 J (J=4200J/kcal)

1) 16 kcal 2) 18 kcal3) 20 kcal 4)19 kcal

KEY:2

HINT:

$dQ = dU + dW \Rightarrow dW = dQ - dU$

11 Heat of 30 kcal is supplied to a system and 400 J of external work is done on the system so that its volume decreases at constant pessure. What is the change in its internal energy. (J = 4200 J/kcal)

1) 1.302 x 10^5 J 2) 2.302 x 10^5 J

3) 3.302 x 10^5 J 4) 4.302 x 10^5 J

KEY: 1

HINT:

$$dQ = dU + dW$$

12. Air expands from 5 litres to 10 litres at 2 atm pressure. External workdone is

1) 10J 2) 1000J 3) 3000 J 4) 300 J

KEY:2

HINT:

$$W = P(V_2 - V_1)$$

13. Heat given to a system is 35 joules and work done by the system is 15 joules. The change in the internal energy of the system will be

1) – 50 J 2) 20 J 3) 30 J 4) 50 J

KEY:2

HINT:

$$dU = dQ - dW$$

14. A gas is compressed at a constant pressure of 50 N/m^2 from a volume 10m^3 to a volume of 4m^3. 100J of heat is added to the gas then its internal energy

1) Increases by 400J 2) Increases by 200J 3) Decreases by 400J 4) Decreases by 200J

KEY:1

HINT:

$$dU = dQ - P(V_2 - V_1)$$

15. Find the change in internal energy in joule. When 10g of air is heated from 30°C to 40°C (c_v = 0.172 kcal/kg K, J = 4200 J/kcal)

1) 62.24 J 2)72.24 J 3)52.24 J 4)82.24 J

KEY:2

HINT:

$$dU_V = mc_v dT$$

16. The temperature of 5 moles of a gas at constant volume is changed from 100^{0}C to 120^{0}C. The change in internal energy is 80J. The total heat capacity of the gas at constant volume will be in joule/kelvin is

1) 8 2) 4 3) 0.8 4) 0.4

KEY:2

HINT:

$$dQ = dU + PdV = dU + P(0) = dU$$

$$\left(\frac{dQ}{dT}\right)_V = \frac{dU}{dT}$$

17. When an ideal diatomic gas is heated at constant pressure, the fraction of heat energy supplied which is used in doing work to maintain pressure constant is

1) 5/7 2) 7/2 3) 2/7 4) 2/5

KEY:3

HINT:

$$\frac{dW}{dQ} = 1 - \frac{1}{\gamma}$$

18. When a monoatomic gas expands at constant pressure, the percentage of heat supplied that increases temperature of the gas and in doing external work in expansion at constant pressure is

1) 100%, 0 2) 60%, 40% 3) 40%, 60% 4) 75%, 25%

KEY:2

HINT:

$$\frac{dU}{dQ} = \frac{1}{\gamma}; \quad \frac{dW}{dQ} = 1 - \frac{1}{\gamma}$$

19. For a gas, the difference between the two specific heats is 4150J Kg^{-1} K^{-1} and the ratio of specific heats is 1.4. What is the specific heat of the gas at constant volume in J Kg^{-1} K^{-1}?

1)8475 2) 5186 3)1660 4) 10375

KEY:4

HINT:

$$C_P - C_V = R \Rightarrow C_V = \frac{R}{\gamma - 1}$$

20. The specific heat of air at constant pressure is 1.005 kJ/kg K and the specific heat of air at constant volume is 0.718 kJ/kgK. Find the specific gas constant.

1) 0.287 kJ/kg K 2) 0.21 kJ/kg K 3) 0.34 kJ/kg K 4) 0.19 kJ/kg K

KEY:1

HINT:

$$c_P - c_V = r$$

21. The specific heat of Argon at constant volume is 0.3122kJ/kg K. Find the specific heat of Argon at constant pressure if R = 8.314 kJ/k mole K. (Molecular weight of argon = 39.95)

1) 5203 2) 5302 3) 2305 4) 3025

KEY: 1

HINT:

$$c_P - c_V = \frac{R}{M}$$

22. If the ratio of the specific heats of steam is 1.33 and R = 8312J/k mole K find the molar heat capacities of steam at constant pressure and constant volume.

1) 33.5 kJ/k mole, 25.19 kJ /kg K 2) 25.19 kJ/k mole, 33.5 kJ/kg K

3) 18.82 kJ/k mole, 10.82 kJ/k mole 4) 24.12 kJ /k mole, 16.12 kJ/k mole

KEY:1

HINT:

$$C_P = \frac{\gamma R}{\gamma - 1}; \quad C_V = \frac{R}{\gamma - 1}$$

23. One mole of an ideal gas undergoes an isothermal change at temperature 'T' so that its volume V is doubled. R is the molar gas constant. Work done by the gas during this change is (2008 M)

1) RT log4 2) RT log2 3) RT log1 4) RT log3

KEY:2

HINT:

$$W = nRT \log_e\left(\frac{V_2}{V_1}\right)$$

24. One mole of O_2 gas having a volume equal to 22.4 litres at 0^0C and 1 atmospheric pressure is compressed isothermally so that its volume reduces to 11.2 litres. The work done in this process is

1)1672.5J 2)1728J 3) –1728J 4) –1572.5J

KEY:4

HINT:

$$W = 2.303nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

25. The isothermal Bulk modulus of an ideal gas at pressure P is

1) P 2) γP 3) P/2 4) P/γ

KEY:1

HINT:

Isothermal process PdV+VdP=0

$$\frac{dP}{P} = -\frac{dV}{V}; \quad (K) = \frac{dP}{-\left(\frac{dV}{V}\right)} = \frac{dP}{\left(\frac{dP}{P}\right)} = P$$

26. Diatomi pressure becomes 8 times the initial pressure, then the final temperature is $(\gamma = 3/2)$

1) 627^0 C 2) 527^0C 3) 427^0C 4) 327^0C

KEY:3

HINT:

$$PV^{\gamma} = K \Rightarrow \frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$$

27. The volume of a gas is reduced adiabatically to $\frac{1}{4}$ of its volume at 27^0C, if the value of $\gamma = 1.4$, then the new temperature will be

1) 350 x $4^{0.4}$K 2) 300 x $4^{0.4}$K 3) 150 x $4^{0.4}$K 4) None of these

KEY:2

HINT:

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

28. Two moles of an ideal monoatomic gas at 27^0C occupies a volume of V. If the gas is expanded adiabatically to the volume 2V, then the work done by the gas will be $(\gamma = 5/3)$

1) –2767.23J 2) 2767.23J 3) 2500J 4) –2500J

KEY:2

HINT:

$$W = \frac{nR}{\gamma - 1}(T_1 - T_2)$$

29. A container of volume $1m^3$ is divided into two equal compartments, one of which contains an ideal gas at 300 K. The other compartment is vacuum. The whole system is thermally isolated from its surroundings. The partition is removed and the gas expands to occupy the whole volume of the container. Its temperature now would be

1) 300 K 2) 250 K 3) 200 K 4) 100 K

KEY:1

HINT:

For free expansion, $dU = 0 \Rightarrow dT = 0 \Rightarrow T$ is constant.

30. A gas at 10^oC temperature and 1.013×10^5 Pa pressure is compressed adiabatically to half of its volume. If the ratio of specific heats of the gas is 1.4, what is its final temperature?

1) 103^oC 2) 123^oC 3) 93^oC 4) 146^oC

KEY:1

HINT:

. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

31. Find the work done by a gas when it expands isothermally at 37^oC to four times its initial volume.

1) 3753J 2) 3573J 3) 7533J 4) 5375J

KEY:2

HINT:

$$W = 2.303nRT\ \log_{10}\left(\frac{V_2}{V_1}\right)$$

32. The efficiency of a heat engine if the temperature of source 227^oC and that of sink is 27^oC nearly

1) 0.4 2) 0.5 3) 0.6 4) 0.7

KEY: 1

HINT:

$$\eta = 1 - \frac{T_2}{T_1}$$

33. A Carnot engine takes 3×10^6 cal. of heat from a reservoir at 627°C, and gives it to a sink at 27°c. The work done by the engine is

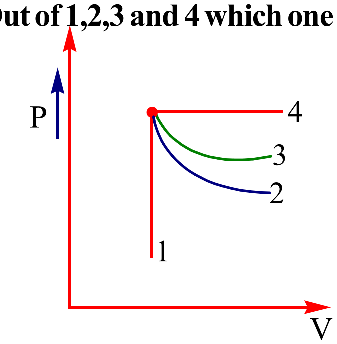
1) 4.2×10^6 J 2) 8.4×10^6 J 3) 16.8×10^6 J 4) zero

KEY: 2

HINT:

$$\eta = \frac{W}{Q} = 1 - \frac{T_2}{T_1} \Rightarrow W = \left(1 - \frac{T_2}{T_1}\right)Q$$

34. An ideal gas undergoes four different processes from the same initial state. Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1,2,3 and 4 which one is adiabatic.

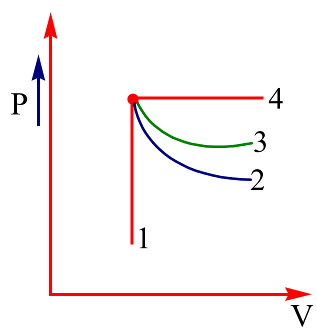


a) 4 b) 3 c) 2 d) 1

KEY: C

HINT:

For the curve 4 pressure is constant, so this is an isobaric process.



For the curve 1, volume is constant, so it is isochoric process. Between curves 3 and 2, curve 2 is steeper, so it is adiabatic and 3 is isothermal.

35. If an average person jogs, hse produces 14.5 x 10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580 x 10^3 cal for evaparation) is

(a) 0.25 kg (b) 2.25 kg (c) 0.05 kg (d) 0.20 kg

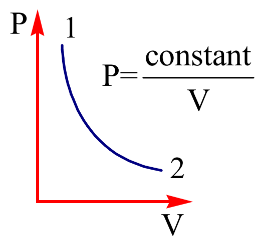
KEY: A

HINT:

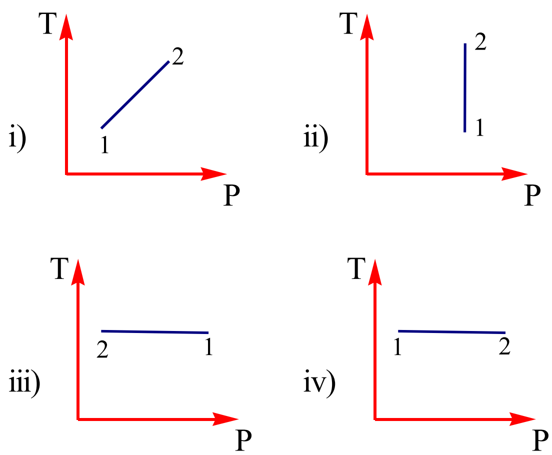
$$\text{Amount of sweat evaporated/minute} = \frac{\text{sweat produced / minute}}{\text{Number of calories required for evaporation / kg}}$$

$$= \frac{\text{Amount of heat produced per minute in jogging}}{\text{Latent heat (in cal/kg)}} = \frac{14.5\times10^3}{580\times10^3} = \frac{145}{580} = 0.25\text{kg}$$

36. Consider P-Vdiagramforan ideal gas shown in figure.



Out of the following diagrams, which represents the T-P diagram?



a) (iv) b) (ii) c) (iii) d) (i)

KEY: C

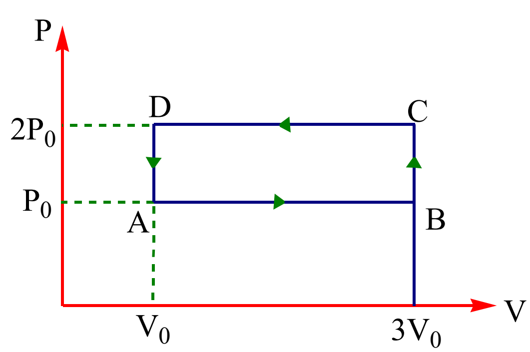
HINT:

According to the question given that pV= constant Hence, we can say that the gas is going through an isothermal process.

Clearly, from the graph that between process 1 and 2 temperature is constant and the gas expands and pressure decreases i.e., $p_2 < p_1$ which corresponds

to diagram (iii)

38. An ideal gas undergoes cyclic process ABCDA as shown in given P-Vdiagram. The amount of work done by the gas is



a) $6P_0V_0$ b)-2 PoVo c) $+ 2 P_0V_0$ d) $+ 4 P_0V_0$

KEY: B

HINT:

Consider the p-V diagram given in the question. Work done in the process ABCD = area of rectangle ABCDA

$= (AB) \times BC = (3V_0 - V_0) \times (2p_0 - p_0)$

$= 2V_0 \times p_0 = 2p_0V_0$

As the process is going anti-clockwise, hence there is a net compression in the gas. So, work done by the gas $= -2p_0V_0$.

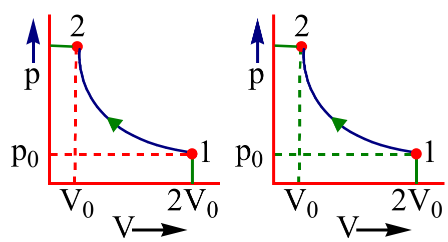
39. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isother- mally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is

a) $2^{\gamma-1}$ b) $\left(\frac{1}{2}\right)^{\gamma-1}$ c) $\left(\frac{1}{1-\gamma}\right)^2$ d) $\left(\frac{1}{\gamma-1}\right)^2$

KEY: A

HINT:

Consider the p-V diagram shown for the container A (isothermal) and for container B (adiabatic).



container A (Isothermal) container B (Adiabatic)

Both the process involving compression of the gas. For isothermal process (gasA) (during $1 \rightarrow 2$)

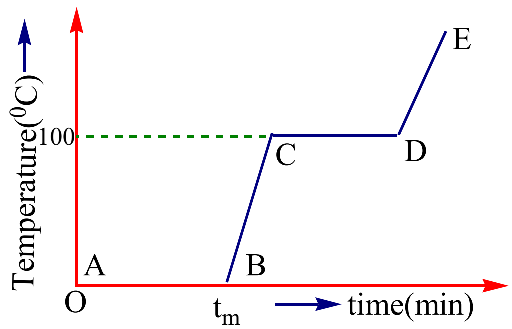
$p_1V_1^{\gamma} = p_2V_2^{\gamma}$

$\Rightarrow p_0(2V_0)^{\gamma} = p_2(V_0)^{\gamma}$ $\Rightarrow p_2 = \left(\frac{2p_0}{V_0}\right)^{\gamma} p_0 = (2)^{\gamma} p_0$

Hence, $\frac{(p_2)_B}{(p_2)_A}$ = Rate of final pressure $\frac{(2)^{\gamma} p_0}{2p_0} = 2^{\gamma-1}$

where, γ is ratio of specific heat capacities for the jas.

40. Refer to the plot of temperature versus time (figure) showing the changes in the state of ice on heating (not to scale). Which of the following is correct ?



a) The region AB represent ice and water in thermal equilibrium

b) At B water starts boiling

c) At C all the water gets converted into steam

d) C to D represents water and steam in equilibrium at boiling point

KEY: A,D

HINT:

During the process AB temperature of the system is 0^0C Hence, it represents phase change that is transformation of ice into water while temperature remains 0^0C.

BC represents rise in temperature of water from 0^0C to 100^0C(atC).

Now, water starts concerting into steam which is represent by CD.

41. A glass full of hot milk is poured on the table. It begins to cool gradually. Which of the following correct ?

a) The rage of cooling is constant till milk attains the temperature of the surrounding

b) The temperature of milk falls off exponentially with time

c) While cooling, there is a flow of heat from milk to the surrounding as well as from surrounding to the milk but the net flow of heat is from milk to the surrounding and that is why it cools

d) All three phenomenon, conduction, convection and radiation are responsible for the loss of heat from milk to the surroundings.

KEY: B,C,D

HINT:

When hot milk spread on the table heat is transferred to the surroundings by conduction, convection and radiation.

According to Newton's law of cooling temperature of the milk falls off exponentially. Heat also will be transferred from surroundings to the milk but will be lesser than that of transferred from milk to the surroundings.

42. Which of the processes described below are irreversible?

a) The increase in temperature of an iron rod by hammering it

b) A gas in a small container at a temperatrue T_1 is brought in contact with a big reservoir at a higher temperature T_2 which increases the temperature of the gas

c) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston

d) An ideal gas is enclosed in a piston cylinder ar-rangement with adiabatic walls. A weight w is added to the piston, resulting in compression of gas.

KEY: A,B,D

HINT:

a) When the rod is hammered, the external work is done on the rod which increases its temperature. The process cannot be retraced itself

b) In this process energy in the form of heat is trans-ferred to the gas in the small container by big reservoir at temperature T_2.

d) As the widht is added to the cylinder arangemer t in the form of external pressure hence, it cannot be reversed back itself.

43. An ideal gas undergoes isothermal process from some initial state i to final state f. Choose the correct alternatives.

a) $dU = 0$ b) $dQ = 0$ c) $dQ = dU$ d) $dQ = dW$

KEY: AD

HINT:

For an isothermal process change in temperature of the system $dT = 0 \Rightarrow T = $ constnat.

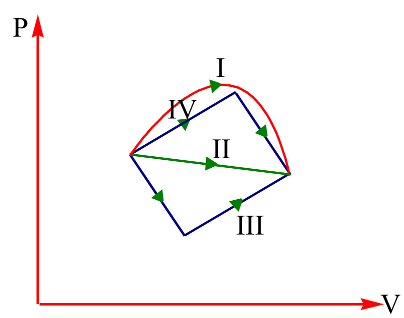
We know that for an ideal gas dU = change in

internal energy $= nC_v\, dT = 0$

[where, n is number of moles and C_v is specific heat capacity at constant volume] From first law of thermodynamics,

$dQ = dU + dW = 0 + dW \Rightarrow dQ = dW$

44. Figure shows theP-V diagram of an ideal gas undergoing a change of state from Ato B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state.



(a) Change in internal energy is same in IV and III cases, but not in I and II.

(b) Change in internal energy is same in all the four cases.

(c) Work done is maximum in case I

(d) Work done is minimum in case II.

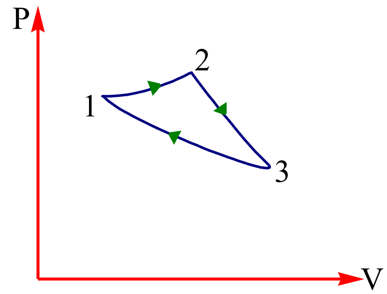
KEY: B,C

HINT:

Change in internal energy for the process A to B

$dU_{A \to B} = nC_v dT = nCv(T_B\text{-}T_A)$ which depends only on temperatures at A and B. Work done for A to B, $dW_{A \to B}$ = ARea under the p-V curve which is maximum for the path I.

45. Consider a cycle followed by an engine (figure.) 1 to 2 is isothermal 2 to 3 is adiabatic 3 to 1 is adiabatic Such a process does not exist because



(a) heat is completely converted to mechanical energy in such a process, which is not possible.

(b) mechanical energy is completely converted to heat in this process,which is not possible.

(c) curves representing two adiabatic processes don't intersect.

(d) curves representing an adiabatic process and an isothermal process don't intersect.

KEY:A,C

HINT:

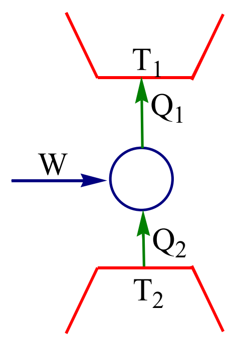
a) The given process is a cyclic process i.e., it returns to the original state 1.

Hence, change in internal energy dU = 0

Hence, total heat supplied is converted to work done by the gas (mechanical energy) which is not possible by second law of thermodynamics,

c) When the gas expands adiabatically from 2 to 3. It is not possible to return to the same state without being heat supplied, hence the process 3 to 1 cannot be adiabatic

46. Consider a heat engine as shown in figure. Q_1 and Q_2 are heat added bath to T_1 and heat taken from T_2 in one cycle of engine. W is the mechanical work done on the engine.



If W> 0, then possibilities are:

(a) $Q_1 > Q_2 > 0$ (b) $Q_2 > Q_1 > 0$ (c) $Q_2 < Q_1 < 0$ (d) $Q_1 < 0, Q2 > 0$

KEY: A,C

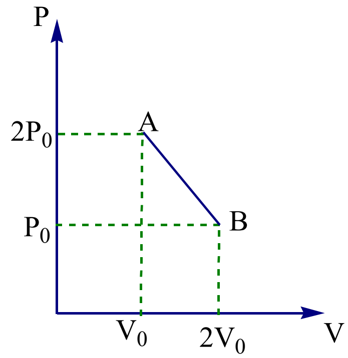
HINT:

Consider the figure we can write $Q_1 = W + Q_2$

$\Rightarrow W = Q_1 - Q2 > 0$ (By question)

$\Rightarrow$ $Q_1 > Q_2 > 0$ (If both Q_1 and Q_2 are positive) We can also, write $Q_2 < Q_1 < 0$ (If both Q_1 and Q_2 are negative).

47. 'n' moles of an ideal gas undergoes a process AB as shown in the figure. The maximum temperature of the gas during the process will be(Mains 2016)



1) $\frac{9P_0V_0}{4nR}$ 2) $\frac{3P_0V_0}{2nR}$ 3) $\frac{9P_0V_0}{2nR}$ 4) $\frac{9P_0V_0}{nR}$

KEY: 1

HINT:

Line equation for given graph (y = mx + c)

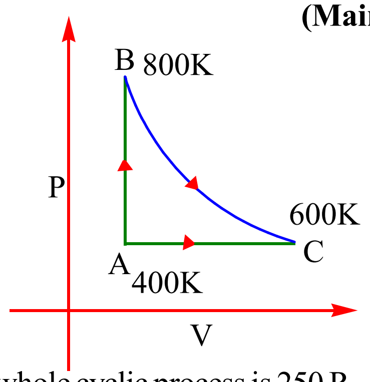
$$P = -\frac{P_0}{V_0}V + 3P_0 = \frac{nRT}{V}$$

$$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + 3P_0V\right] \rightarrow (1)$$

To get max temp $\frac{dT}{dV} = 0$

sub in eq(1) for T_{maxz}.

48. One mole diatomic ideal gas undergoes a cyclic process ABC as shown in the figure. The process BC is adiabilitic. The temperature at A, B and C are 400 K, 800K and 600K respectively. Choose the correct statement (Mains 2014)



1) The change in internal energy in whole cyclic process is 250 R
2) The change in internal energy in the process CA is 700 R
3) The change in internal energy in the process AB is -350 R
4) The change in internal energy in the process BC is -500 R

KEY: 4

HINT:

$$\Delta U_{AB} = nC_V\left(T_B - T_A\right)$$

$$\Delta U_{BC} = nC_V\left(T_C - T_B\right)$$

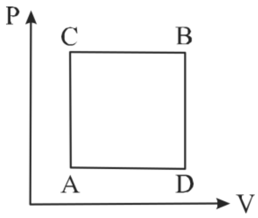
$$\Delta U_{total} = 0$$

$$\Delta U_{CA} = nC_V\left(T_A - T_C\right)$$

Previous JEE Mains Questions And Solutions

Thermodynamics

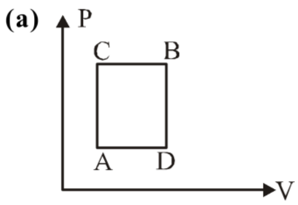
1. A gas can be taken from A to B via two different processes ACB and ADB.



When path ACB is used 60 J ofheat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is: [9 Jan. 2019 I]

(a) 40 J (b) 80 J (c) 100 J (d) 20 J

SOLUTION : (a)



ΔU remains same for both paths ACB and ADB

$$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$$
$$\Rightarrow 60J = 30J + \Delta U_{ACB}$$
$$\Rightarrow U_{ACB} = 30J$$
$$\Delta U_{ADB} = \Delta U_{ACB} = 30J$$
$$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$$
$$= 10J + 30J = 40J$$

2. 200gwater isheated fi$_{\text{i}}$om40°Cto60°C. Ignoringtheslight expansion of water, the change in its internal energy is close to (Given specific heat ofwater = 4184J/kgK): [Online April 9, 2016]

(a) 167.4kJ (b) 8.4kJ (c) 4.2kJ (d) 16.7kJ

SOLUTION : (d)

Volume ofwater does not change, no work is done on or by the system $(W = 0)$

According to first law ofthermodynamics

$$Q = \triangle U + W$$

For Isochoric process $Q = \triangle U$

$$\Delta U = \mu c dT = 2 \times 4184 \times 20 = 16.7kJ.$$

3. A gas is compressed fi$_{\text{i}}$om a volume of$2m^3$ to a volume of $1m^3$ at a constant pressure of 100 N/m^2. Then it is heated at constant volume by supplying 150 J ofenergy. As a result, the internal energy ofthe gas: [Online Apri119, 2014]

(a) increases by 250 J (b) decreases by 250 J

(c) increases by50 J (d) decreases by 50 J

SOLUTION : (a)

As we know,

$$\Delta Q = \Delta u + \Delta w \text{ (Ist law ofthermodynamics)}$$

$$\Rightarrow \Delta Q = \Delta u + P\Delta v$$

$$\text{or } 150 = \Delta u + 100(1-2)$$

$$= \Delta u - 100$$

$$\Delta u = 150 + 100 = 250\text{J}$$

Thus the internal energy ofthe gas increases by250 J

4. An insulated container ofgas has two chambers separated by an insulating partition. One ofthe chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. Ifthe partition is removed without doing any work on the gas, the final equilibrium temperature ofthe gas in the container willbe [2008]

(a) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (b) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$

(c) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$ (d) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

SOLUTION : (a)

Here $Q = 0$ and $W = 0$.

Therefore from first law of thermodynamics $\Delta U = Q + W = 0$

Internal energy offirst vessle + Internal energy ofsecond vessel = Internal energy of combined vessel

$$n_1C_vT_1 + n_2C_vT_2 = (n_1+n_2)C_vT$$

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

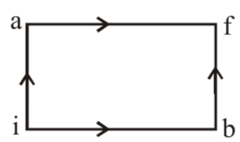
For first vessel $n_1 = \frac{P_1V_1}{RT_1}$ and for second vessle

$$n_2 = \frac{P_2V_2}{RT_2}$$

$$T = \frac{\frac{P_1V_1}{RT_1} \times T_1 + \frac{P_2V_2}{RT_2} \times T_2}{\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}}$$

$$= \frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

5. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf $Q = 36$ cal. W along the path ibf is [2007]



(a) 14 cal **(b) 6 cal** **(c) 16 cal** **(d) 66 cal**

SOLUTION : (b)

For path iaf, $Q_1 = 50$ cal, $W_1 = 20$ cal

By first law ofthermodynamics,

$$\Delta U = Q_1 - W_1 = 50 - 20 = 30 \text{ cal.}$$

For path ibf $Q_2 = 36$ cal $W_2 = ?$

$$\Delta U_{ibf} = Q_2 - W_2$$

Since, the change in intemal energy does not depend on the path, therefore $\Delta U_{iaf} = \Delta U_{ibf}$

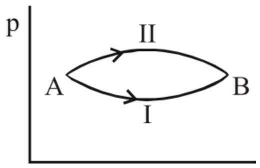
$$\Delta U_{iaf} = \Delta U_{ibf}$$

$$\Rightarrow 30 = Q_2 - W_2$$

$$\Rightarrow W_2 = 36 - 30 = 6 \text{ cal.}$$

6. A system goes from A to B via two processes I and II as shown in figure. IfΔU_1 and ΔU_2 are the changes in internal energiesinthe T 1TT tively, then [2005]

processes I and II respect



v

(a) relation between ΔU_1 and ΔU_2 can not be determined **(b) $\Delta U_1 = \Delta U_2$**

(c) $\Delta U_2 < \Delta U_1$ **(d) $\Delta U_2 > \Delta U_1$**

SOLUTION : (b)

Change in intemal energy is independent ofpath taken bythe process.

It only depends on initial and final states

i. e.,

$$\Delta U_1 = \Delta U_2$$

7. Which ofthe following is incorrect regarding the first law ofthermodynamics? [2005]

(a) It is a restatement of the principle of conservation of energy

(b) It is not applicable to any cyclic process

(c) It does not introduces the concept of the entropy

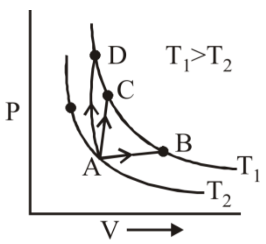
(d) It introduces the concept ofthe internal energy

SOLUTION : (**b**, **c**)

First law is applicable to a cyclic process.

Concept of entropy is introduced bythe second law ofthermodynamics.

8. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are lebelled as $A \rightarrow B$, $A \rightarrow C$ and $A \rightarrow D$. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are: [5 Sep. 2020 (I)]



(a) $E_{AB} = E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} < 0$

(b) $E_{AB} = E_{AC} = E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} > 0$

(c) $E_{AB} < E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} > W_{AD}$

(d) $E_{AB} > E_{AC} > E_{AD}, W_{AB} < W_{AC} < W_{AD}$

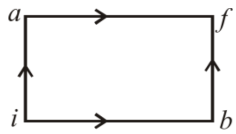
SOLUTION : (b)

Temperature change ΔT is same for all three processes $A \to B; A \to C$ and $A \to D$

$$\Delta U = nC_v \Delta T = \text{same}$$

$$E_{AB} = E_{AC} = E_{AD}$$

Work done, $W = P \times \Delta V$



$$AB \to \text{volume is increasing} \Rightarrow W_{AB} > 0$$

$$AD \to \text{volume is decreasing} \Rightarrow W_{AD} < 0$$

$$AC \to \text{volume is constant} \Rightarrow W_{AC} = 0$$

9. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure ofthe gas is found to be n times the initial pressure. The value of n is:

[5 Sep. 2020 (II)]

(a) 32 (b) 326 (c) 128 (d) $\frac{1}{32}$

SOLUTION : (c)

In adiabatic process

$$Py^{Y} = \text{constant}$$

$$P\left(\frac{m}{p}\right)^{Y} = \text{constant} \left(\because V = \frac{m}{p}\right)$$

As mass is constant $P \propto p^{Y}$

If P_i and P_f be the initial and final pressure ofthe gas and p_i and P_f be the initial and fmal densityofthe gas.

$$\text{Then } \frac{P_f}{P_i} = (\quad)(\quad)^{Y} = (32)^{7/5}$$

$$\Rightarrow \frac{nP_i}{P_i} = (2^5)^{7/5} = 2^7$$

$$\Rightarrow n = 2^7 = 128.$$

10. Match the thermodynamic processes taking place in a system with the correct conditions. In the table: ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy ofthe system. **[4 Sep. 2020 (II)]**

Process	Condition
(D Adiabatic	(A) $\Delta W = 0$
(Π) Isothermal	(B) $\Delta Q = 0$
(III) Isochoric	(C) $\Delta U \neq 0, \Delta W \neq 0, \ \Delta Q \neq 0\omega$
(IV)Isobaric	(D) $\Delta U = 0$

(a) (I) – (A), (II) – (B), (III) – (D), (IV) – (D)
(b) (I) – (B), (II) – (A), (III) – (D), (IV) – (C)
(c) (I) – (A), (II) – (A), (III) – (B), (IV) – (C)
(d) (I) – (B), (II) – (D), (III) – (A), (IV) – (C)

SOLUTION : (d)

(*D* Adiabatic process :

No exchange of heat takes place with surroundings.

$$\Rightarrow \Delta Q = 0$$

(*II*) Isothermal process:

Temperature remains constant

$$\Delta T = 0 \Rightarrow \Delta U = \frac{f}{2} nR\Delta T \Rightarrow \Delta U = 0$$

No change in internal energy $[\Delta U = 0]$.

(IID Isochoric process:

volume remains constant

$$\Delta V = 0 \Rightarrow W = \int P \cdot dV = 0$$

Hence work done is zero.

(4) In isobaric process :

pressure remains constant.

$$W = P \cdot \Delta V \neq 0$$

$$\Delta U = \frac{f}{2} nR\Delta T = \frac{f}{2}[P\Delta V] \neq 0$$

$$\Delta Q = nC_p \Delta T \neq 0$$

11. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as : [3 Sep. 2020 (I)]

(a) irreversible isothermal (b) irreversible adiabatic
(c) reversible adiabatic (d) reversible isotherm7al

SOLUTION : (b)

Bursting ofhelium balloon is irreversible and in this process

$\Delta Q = 0$, so adiabatic.

12. An engine takes in 5 mole of air at 20°C and 1 atm, and compresses it adiabaticaly to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up ofrigid molecules, the change in its internal energy during this process comes out to be XkJ. The value of$X$ to the nearest integer is [NA 2 Sep. 2020 (I)]

SOLUTION : (46)

For adiabatic process, $TV^{\Upsilon-1}$ = constant

or, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_1 = 20^\circ\text{C} + 273 = 293\text{K}\ ,\ V_2 = \frac{V_1}{10} \text{ and } \gamma = \frac{7}{5}$$

$$T_1 (V_1)^{\Upsilon-1} = T_2 \left(\frac{V_1}{10}\right)^{\Upsilon-1}$$

$$\Rightarrow 293 = T_2\left(\frac{1}{10}\right)^{2/5} \Rightarrow T_2 = 293(10)^{2/5} = 736\text{K}$$

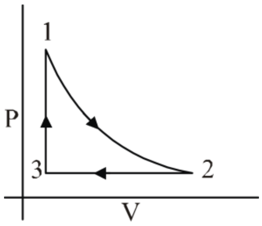
$$\Delta T = 736 - 293 = 443\text{K}$$

During the process, change in internal energy

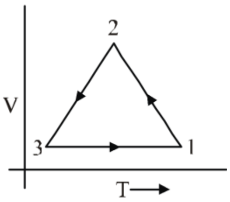
$$\Delta U = NC_V\Delta T = 5 \times \frac{5}{2} \times 8.3 \times 443 = 46 \times 10^3\text{J} = X\text{kJ}$$

$$X = 46.$$

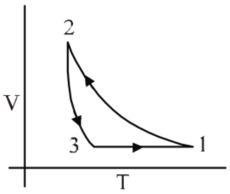
13. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure? where, 1 → 2 is adiabatic.
(Graphs are schematic and are not to scale) [9 Jan. 2020 I]



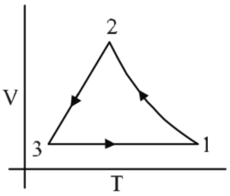
(a)



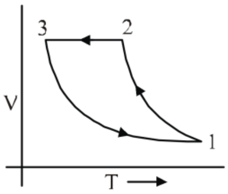
(b)



(c)



(d)



SOLUTION : (c)

For process 3 → 1 volume is constant

Graph given in option (d) is wrong.

And process 1 → 2 is adiabatic graph in option (1) is wrong

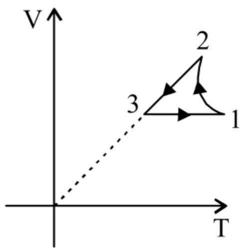
v = constant

$P\uparrow, T\uparrow$

For Process 2 → 3 Pressure constant *i.e.*, P = constant

$V\downarrow T\downarrow$

Hence graph (c) is the correct $V - T$ graph of given $P - V$ graph



14. **Starting at temperature 300K, one mole ofan ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically fi$_i$om volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. Ifall the processes are the quasi-static then the final temperature ofthe gas (in oK) is (to the nearest integer) .** **‘ [9 Jan. 2020 II]**

SOLUTION : . (1818)

For an adiabatic process,

$$TV^{\gamma 1} = \text{constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow T_2 = (300) \times \left(\frac{V_1}{\frac{V_1}{16}}\right)^{1.4-1}$$

$$\Rightarrow T_2 = 300 \times (16)^{04}$$

Ideal gas equation, $PV = nRT$

$$V = \frac{nRT}{P}$$

$\Rightarrow V = kT$ (since pressure is constant for isobaric process)

So, during isobaric process

$$V_2 = kT_2 \text{ (i)}$$

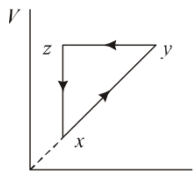
$$2V_2 = kT_f \text{ (ii)}$$

Dividing (i) by (ii)

$$\frac{1}{2} = \frac{T_2}{\tau_f}$$

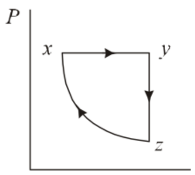
$$T_f = 2T_2 = 300 \times 2 \times (16)^{04} = 1818K$$

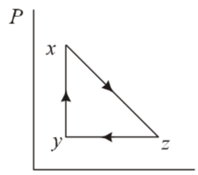
15. **Athermodynamic cycle *xyzx* is shown on a *V-T* diagram.**

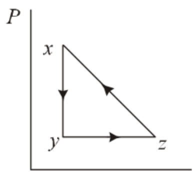


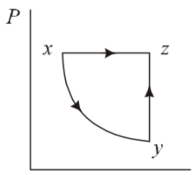
τ

The *P-V* diagram that best describes this cycle is: (Diagrams are schematic and not to scale) **[8 Jan. 2020 I]**

(a)


(b)


(c)


(d)


SOLUTION : (a)

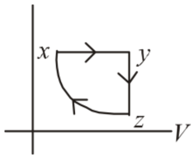
From the corresponding V - T graph given in question,

Process xy →Isobaric expansion,

Process yz →Isochoric (Pressure decreases)

Process zx →Isothermal compression

Therefore, correspon$ng PV graph is as shown in figure



16. A litre ofdry air at STP expands a diabatically to a volume of3 litres. If $\gamma = 1.40$, the work done by air is: $(3^{14} = 4.6555)$ [Take air to be an ideal gas] [7 Jan. 2020 I]

(a) 60.7J (b) 90.5J (c) 100.8J (d) 48 J

SOLUTION : (b)

$$\text{Given, } V_1 = 1\text{litre}, P_1 = 1 \text{ atm}$$

$$V_2 = 3 \text{ litre}, \gamma = 1.40,$$

$$\text{Using, } PV^r = \text{constant} \Rightarrow P_1V_1^\Upsilon = P_2V_2^\Upsilon$$

$$\Rightarrow P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555} atm$$

$$\text{Work done, } W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{\left(1\times 1 - \frac{1}{4.6555}\times 3\right)1.01325\times 10^5 \times 10^{-3}}{0.4} = 90.1J$$

$$\text{Closest value of } W = 90.5J$$

17. Under an adiabatic process, the volume ofan ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes fi¡omτ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for is given by: [7 Jan. 2020 I]

(a) 2 (b) $\frac{1}{2}$ (c) $\left(\frac{1}{2}\right)^\gamma$ (d) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

SOLUTION : (Bonus)

$$\text{We know that Relaxation time, } T \propto \frac{V}{\sqrt{T}} \text{ (i)}$$

$$\text{Equation of adiabatic process is } TV^{/}1 = \text{constant}$$

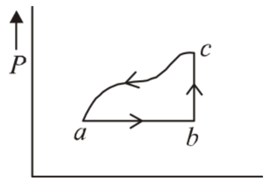
$$\Rightarrow T \propto \frac{1}{V^{\Upsilon - 1}}$$

$$\Rightarrow T \propto V^{1 + \frac{\Upsilon - 1}{2}} \text{ using (i)}$$

$$\Rightarrow T \propto V^{\frac{1+\Upsilon}{2}}$$

$$\Rightarrow \frac{\tau_f}{T_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}} = (2)^{\frac{1+\Upsilon}{2}}$$

18. A sample ofan ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is—180 J, The gas absorbs 250 J ofheat along the path ab and 60 J along the path bc. The work down by the gas along the path abc is: [12 Apr. 2019 I]



$V \rightarrow$

(a) 120J (b) 130J (c) 100J (d) 140J

SOLUTION : (b)

$$\Delta U_{ac} = -(\Delta U_{ca}) = -(-180) = 180\text{J}$$

$$Q = 250 + 60 = 310\text{J}$$

$$\text{Now } Q = \Delta U + W \text{ or } 310 = 180 + W \text{ or } W = 130\text{J}$$

19. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature ofthe gas by $20^{o}\mathrm{C}$ is: [Given that $\mathrm{R} = 8.31\mathrm{J}$ mol 1K1] [10Apr. 2019 I]

(a) 350 J (b) 374 J (c) 748 J (d) 700 J

SOLUTION : . (c)

As the process is isochoric so,

$$\mathrm{Q} = \mathrm{nc_v}\Delta\mathrm{T} = \frac{67.2}{22.4} \times \frac{3\mathrm{R}}{2} \times 20 = 90\mathrm{R} = 90 \times 8.31 = 748\mathrm{j}.$$

20. n moles ofan ideal gas with constant volume heat capacity $\mathrm{C_v}$ undergo an isobaric expansion by certain volume. The ratio ofthe work done in the process, to the heat supplied is: [10 Apr. 2019 I]

(a) $\frac{\mathrm{nR}}{\mathrm{C_v+nR}}$ (b) $\frac{\mathrm{nR}}{\mathrm{C_v-nR}}$ (c) $\frac{4\mathrm{nR}}{\mathrm{C_v-nR}}$ (d) $\frac{4\mathrm{nR}}{\mathrm{C_v+nR}}$

SOLUTION : (a)

At constant volume

$$\text{Work done (W)} = \mathrm{nR}\Delta\mathrm{T}$$

$$\text{Heat given } \mathrm{Q} = \mathrm{C_v}\Delta\mathrm{T} + \mathrm{nR}\Delta\mathrm{T}$$

$$\text{So, } \frac{\mathrm{W}}{\mathrm{Q}} = \frac{\mathrm{nR}\Delta\mathrm{T}}{\mathrm{C_v}\Delta\mathrm{T}+\mathrm{nR}\Delta\mathrm{T}} = \frac{\mathrm{nR}}{\mathrm{C_V+nR}}$$

21. One mole ofan ideal gas passes through a process where pressure and volume obeythe relation $P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$ Here $\mathrm{P_o}$ and $\mathrm{V_o}$ are constants. Calculate the charge in the temperature ofthe gas ifits volume changes from $\mathrm{V_o}$ to $2\mathrm{V_o}$. [10 Apr. 2019 II]

(a) $\frac{1}{2}\frac{\mathrm{P_oV_o}}{\mathrm{R}}$ (b) $\frac{5}{4}\frac{\mathrm{P_oV_o}}{\mathrm{R}}$ (c) $\frac{3}{4}\frac{\mathrm{P_oV_o}}{\mathrm{R}}$ (d) $\frac{1}{4}\frac{\mathrm{P_oV_o}}{\mathrm{R}}$

SOLUTION : (b)

$$\text{We have given, } P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$$

$$\text{When } V_1 = V_0 \Rightarrow P_1 = P_0\left\{\frac{1}{2} - \frac{1}{\ }\right\} = \frac{P_0}{2}$$

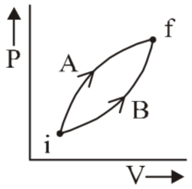
$$\text{When } V_2 = 2V_0 \Rightarrow P_2 = P_0\left[1 - \frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$$

$$\Delta T = T_2 - T_1 = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right| \left[\because T = \frac{PV}{nR}\right]$$

$$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right| = \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$$

$$= \frac{5P_0V_0}{4nR} = \frac{5P_0V_0}{4R}\ (n = 1)$$

22. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then: [9 April 2019 I]



(a) $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$ (b) $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$

(c) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$ (d) $\Delta Q_A = \Delta Q_B; \Delta U_A = \Delta U_B$

SOLUTION : . (c)

Internal energy depends only on initial and final state So,

$$\Delta U_A = \Delta U_B$$

$$\text{Also } \Delta Q = \Delta U + W$$

$$\text{As } W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$$

23. A thermally insulted vessel contains 150 g ofwater at $0^\circ C$. Then the air fi$_i$om the vessel is pumped out adiabatically. A fiiaction ofwater turns into ice and the rest evaporates at $0^\circ C$ itself. The mass ofevaporated water will be closed to: (Latent heat of vaporization ofwater $= 2.10 \times 10^6$J kg 1 and Latent heat ofFusion ofwater $= 3.36 \times 10^5$J kg 1) [8 April 2019 I]

(a) 150g (b) 20 g (c) 130g (d) 35 g

SOLUTION : . (b)

Suppose amount of water evaporated be M gram.

Then (150 - M) gram water converted into ice.

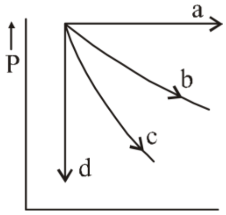
so, heat consumed in evoporation = Heat released in fusion

$$M \times L_v = (150 - M) \times L$$

$$M \times 2.1 \times 10^6 = (150 - M) \times 3.36 \times 10^5 s$$

$$\Rightarrow M - 20g$$

24. The given diagram shows four processes *i.e.*, isochoric, isobaric, isothermal and adiabatic. The correct assignment ofthe processes, in the same order is given by: [8 Apr. 2019 II]



V →

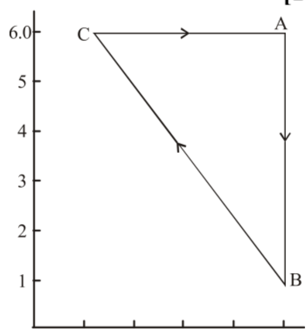
(a) adbc (b) dacb (c) adcb (d) dabc

SOLUTION : . (d)

a → Isobasic,
b → Isothermal,
c →Adiabatic,
d →Isochoric

25. For the given cyclic process CAB as shown for gas, the work doneis: 2 Jan. 2019 I]

[1



1 2 3 4 5 $V(m^3)$

(a) 30 J (b) 10 J (c) 1 J (d) 5 J

SOLUTION : . (b)

Total work done by the gas during the cycle is equal to area oftriangle ABC.

$$\Delta W = \frac{1}{2} \times 4 \times 5 = 10J$$

26. Arigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x =constant, then x is: [11 Jan. 2019 I]

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$

SOLUTION : (b)

Equation ofadiabatic change is$TV^{\gamma-1}$ = constant

Put $= \frac{7}{5}$, we get: $\gamma - 1 = \frac{7}{5} - 1$

$$x = \frac{2}{5}$$

27. **Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm fi$_{i}$om20°C to 90°C. Work done by gas is close to: (Gas constant R = 8.31 J/mol-K)** **[10 Jan. 2019 II]**

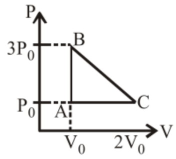
(a) 581 J **(b) 291 J** **(c) 146 J** **(d) 73 J**

SOLUTION : (b)

$$\text{Work done,} \mathbf{W = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70 = 291J}$$

28. **One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by**

[Online April 16, 2018]



(a) $\frac{25P_0V_0}{8R}$ **(b)** $\frac{25P_0V_0}{4R}$ **(c)** $\frac{25P_0V_0}{16R}$ **(d)** $\frac{5P_0V_0}{8R}$

SOLUTION : (a)

$$\text{Equation ofthe BC} \quad \mathbf{P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)}$$

$$\text{using } \mathbf{PV = nRT}$$

$$\text{Temperature, } \mathbf{T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R}} \quad (\mathbf{n = 1} \text{ mole given})$$

$$\mathbf{T = \frac{P_0}{\mathit{\Gamma}}\left\{5V - \frac{2V^2}{V_0}\right\}}$$

$$\mathbf{\frac{dT}{dV} = 0 \Rightarrow 5 - \frac{4V}{V_0} = 0 \Rightarrow V = \frac{5}{4}V_0}$$

$$\mathbf{T = \frac{P_0}{R}\left[5 \times \frac{5V_0}{4} - \frac{2}{V_0} \times \frac{25}{16}V_0^2\right] = \frac{25P_0V_0}{8R}}$$

29. **One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27°C. The work done on the gas will be:**

[Online Apri115, 2018]

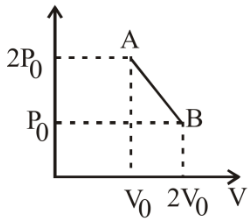
(a) 300R ln 6 **(b) 3α)R** **(c) 300R ln 7** **(d) 3α)Rh2**

SOLUTION : (d)

$$\text{Work done on gas} = \mathbf{nRT\ell n\left(\frac{p_f}{p_1}\right) = R(300)\ell n(2)}$$

$$\mathbf{= 300\ Rln2} \qquad \left(\frac{P_f}{p_i} = 2\text{given}\right)$$

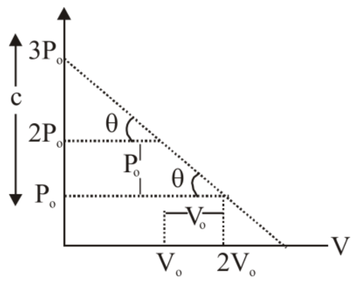
30. **n′ moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature ofthe gas during the process will be:** **[2016]**



(a) $\frac{9P_0V_0}{2nR}$ **(b)** $\frac{9P_0V_0}{nR}$ **(c)** $\frac{9P_0V_0}{4nR}$ **(d)** $\frac{3P_0V_0}{2nR}$

SOLUTION : . (c)

The equation for the line is



$$P = \frac{-P_0}{V_0}V + 3P\ [= \frac{-P_0}{V_0},$$

$$PV_0 + P_0V = 3P_0V_0 \quad \text{(i)}$$

But $pV = nRT$

$P = \frac{nRT}{V}$ (ii) From(i) &(ii) $\frac{nRT}{V}V_0 + P_0V = 3P_0V_0$

From (i) & (ii) $\frac{nRT}{V}V_0 + P_0V = 3P_0V_0 nRTV_0 + P_0V^2 = 3P_0V_0V$ (iii)

For temperature to be maximum $\frac{dT}{dV} = 0$ Differentiating e.q. (iii) by V' we get

$$nRV_0\frac{dT}{dV} + P_0(2V) = 3P_0V_0$$

$$nRV_0\frac{dT}{dV} = 3P_0V_0 - 2P_0V$$

$$\frac{dT}{dV} = \frac{3P_0V_0 - 2P_0V}{nRV_0} = 0$$

$V = \frac{3V_0}{2}$ $P = \frac{3P_0}{2}$ [From (i)] $T_{max} = \frac{9P_0V_0}{4nR}$ [From (iii)]

31. **The pressure ofan ideal gas varies with volume as $P = \alpha V$, where α is a constant. One mole of the gas is allowed to undergo expansion such that its volume becomes'm' times its initial volume. The work done by the gas in the process is** **[Online May 19, 2012]**

(a) $\frac{\alpha V}{2}(m^2 - 1)$ **(b)** $\frac{\alpha^2V^2}{2}(m^2 - 1)$ **(c)** $\frac{\alpha}{2}(m^2 - 1)$ **(d)** $\frac{\alpha V^2}{2}(m^2 - 1)$

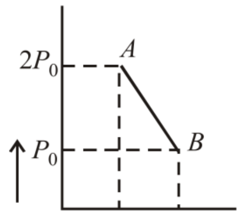
SOLUTION : (d)

$$\text{Given } P = \alpha V$$

$$\text{Work done, } w = \int_V^{mV} P\, dV$$

$$= \int_V^{mV} \alpha V dV = \frac{\alpha V^2}{2}(m^2 - 1)\ .$$

32. n moles ofan ideal gas undergo a process $A \rightarrow B$ as shown in the figure. Maximum temperature ofthe gas during the process is [Online May 12, 2012]



$P\ V_0\ 2V_0$

$V \rightarrow$

(a) $\frac{9P_0V_0}{nR}$ (b) $\frac{3P_0V_0}{2nR}$ (c) $\frac{9P_0V_0}{2nR}$ (d) $\frac{9P_0V_0}{4nR}$

SOLUTION : (b)

Work done during the process $A \rightarrow B$

= Area oftrapezium (= area bounded by indicator diagram with V - axis)

$$= \frac{1}{2}(2P_0 + P_0)(2V_0 - V_0) = \frac{3}{2}P_0V_0$$

Ideal gas eqn: $PV = nRT$

$$\Rightarrow T = \frac{PV}{nR} = \frac{3P_0V_0}{2nR}$$

33. This question has Statement 1 and Statement2. Ofthe four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1:

In an adiabatic process, change in internal energy ofa gas is equal to work done on/by the gas in the process.

Statement 2:

The temperature ofa gas remains constant in an adiabatic process.

[Online May 7, 2012]

(a) Statement 1 is true, Statement2 is true, Statement2 is a correct explanation of Statement 1.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is false, Statement2 is true.

(d) Statement 1 is false, Statement2 is true, Statement2 is not a correct explanation of Statement 1.

SOLUTION : (b)

In an adiabatic process, $6\text{H} = 0$

And according to first law ofthermodynamics

$$6H = 6U + W$$

$$W = -6U$$

34. Acontainer with insulating walls is divided into equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completly evacuated. If the valve is suddenly opened, the pressure and temperature ofthe gas willbe: [2011 RS]

(a) $\frac{P}{2}, \frac{T}{2}$ (b) PT (c) $P, \frac{T}{2}$ (d) $\frac{P}{2}, T$

SOLUTION : (d)

It is the free expansion

So, T remains constant

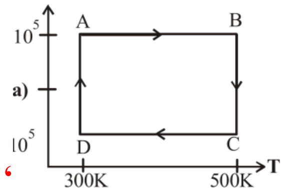
$$\Rightarrow P_1V_1 = P_2V_2$$

$$\Rightarrow P\frac{V}{2} = P_2(V)$$

$$P_2 = \left(\frac{P}{2}\right)$$

Directions for questions 35 to 37 : ***Questions are based on the following paragraph.***

Two moles ofhelium gas are taken over the cycle $ABCDA$, as shown in the P-T diagram. [2009]



35. Assuming the gas to be ideal the work done on the gas in taking it fi¡om$A$ to B is

(a) 300 R (b) 400 R (c) 500 R (d) 200 R

SOLUTION : (b)

The process $A \rightarrow B$ is isobaric.

work done $W_{AB} = nR(T_2 - T_1) = 2R(500 - 300) = 400R$

36. The work done on the gas in taking it from D to A is

(a) +414R (b) −690R (c) + 690R (d) −414R

SOLUTION : (a)

The process D to A is isothermal as temperature is constant.

Work done, $W_{DA} = 2.303nRT\log_{10\frac{P_D}{P_A}} = 2.303 \times 2R \times 300$

$$\log_{10}\frac{1 \times 10^5}{2 \times 10^5} - 414R.$$

Therefore, work done on the gas is $+414R$.

37. The net work done on the gas in the cycle $ABCDA$ is

(a) 279 R (b) 1076 R (c) 1904 R (d) zero

SOLUTION : (a)

The net work in the cycle $ABCDA$ is

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 4\alpha)R + 2.303nRT \log \frac{P_B}{P_C} + (\lhd\, 00R) - 414R$$

$$= 2.303 \times 2R \times 500 \log \frac{2 \times 10^5}{1 \times 10^5} - 414R$$

$$= 693.2R - 414R = 279.2R$$

38. **The work of146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature ofthe gas increases by 7°C. The gas is ($R = 8.3$ Jmol 1K^1)** **[2006]**

(a) diatomic **(b) triatomic**

(c) a mixture ofmonoatomic and diatomic **(d) monoatomic**

SOLUTION : (a)

Work done in adiabatic compression is given by

$$W = \frac{nR\Delta T}{1-\gamma}$$

$$\Rightarrow -146000 = \frac{1000 \times 8.3 \times 7}{1-\gamma}$$

$$\text{or } 1-\gamma = -\frac{58.1}{146} \Rightarrow \gamma = 1 + \frac{58.1}{146} = 1.4$$

Hence the gas is diatomic.

39. **Which ofthe following parameters does not characterize the thermodynamic state ofmatter?** **[2003]**

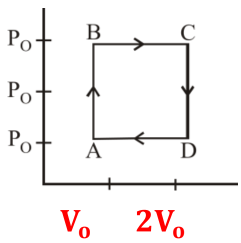
(a) Temperature **(b) Pressure** **(c) Work** **(d) Volume**

SOLUTION : (c)

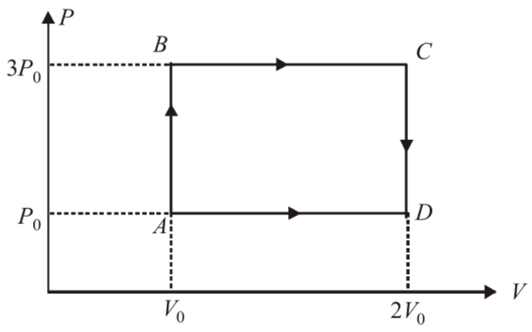
Work is not a state function.

The remaining three parameters are state function.

40. **An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close is .** **[NA 6 Sep. 2020 (II)]**



SOLUTION : (19)



From the figure, Work, $W=2P_0V_0$

Heat given, $Q_{\text{in}}=W_{AB}+W_{BC}=n\cdot C_V\Delta T_{AB}+nC_P\Delta T_{BC}$

$$=n\frac{3R}{2}(T_B-T_A)+\frac{n5R}{2}(T_C-T_B)\qquad \left(\because C_v=\frac{3R}{2}\text{ and }C_P=\frac{5R}{2}\right)$$

$$=\frac{3}{2}(P_BV_B-P_AV_A)+\frac{5}{2}(P_CV_C-P_BV_B)$$

$$=\frac{3}{2}\times[3P_0V_0-P_0V_0]+\frac{5}{2}[6P_0V_0-3P_0V_0]$$

$$=3P_0V_0+\frac{15}{2}P_0V_0=\frac{21}{2}P_0V_0$$

$$\text{EfFiciency, }\eta=\frac{W}{Q_{\text{in}}}=\frac{2P_0V_0}{\frac{21}{2}P_0V_0}=\frac{4}{21}$$

$$\eta\%=\frac{400}{21}\approx 19.$$

41. **If minimum possible work is done by a refiigerator in converting 100 grams ofwater at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat ofice $=80\text{Cal/gram}$) to the nearest integer?** **[NA 3 Sep. 2020 (II)]**

SOLUTION : (8791)

Given, Heat absorbed, $Q_2=mL=80\times100=8000$ Cal

Temperature ofice, $T_2=273\text{K}$

Temperature of surrounding,

$T_1=273+27=300\text{K}$

$$\text{Efficiency}=\frac{W}{Q_2}=\frac{Q_1-Q_2}{Q_2}=\frac{T_1-T_2}{T_2}=\frac{300-273}{273}$$

$$\Rightarrow\frac{Q_1-8000}{8000}=\frac{27}{273}\Rightarrow Q_1=8791\text{ Cal}$$

42. **A heat engine is involved with exchange ofheat of 1915 , -40J, $+125$Jand $-Q$J, during one cycle achieving an efficiency of 50.0%. The value of Q is:** **[2 Sep. 2020 (II)]**

(a) 640 J **(b) 40 J** **(c) 980 J** **(d) 400 J +**

SOLUTION : (c)

$$\text{EfFiciency, }\eta=\frac{\text{Workdone}}{\text{Heatabsorbed}}=\frac{W}{\Sigma Q}$$

$$=\frac{Q_1+Q_2+Q_3+Q_4}{Q_1+Q_3}=0.5$$

$$\text{Here, } Q_1 = 1915\text{J}, Q_2 = -40\text{J and } Q_3 = 125\text{J}$$

$$\frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$$

$$\Rightarrow 1915 - 40 + 125 + Q_4 = 1020$$

$$\Rightarrow Q_4 = 1020 - 2000$$

$$\Rightarrow Q_4 = -Q = -980\text{J}$$

$$\Rightarrow Q = 980\text{J}$$

43. ACarnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. Ifthe work done on the refrigerator is 10 J, the amount ofheat absorbed from the reservoir at lower temperature is: [8 Jan. 2020 II]

(a) 99 J (b) 100 J (c) 1 J (d) 90 J

SOLUTION : . (d)

For carnot refiigerator Efficiency $= \frac{Q_1 - Q_2}{Q_1}$

Where, Q_1 = heat lost from sorrounding

Q_2 = heat absorbed from reservoir at low temperature.

$$\text{Also, } \frac{Q_1 - Q_2}{Q_1} = \frac{w}{Q_1}$$

$$\Rightarrow \frac{1}{10} = \frac{\text{w}}{\text{Q}_1}$$

$$\Rightarrow Q_1 = w \times 10 = 100J$$

$$\text{So, } \text{Q}_1 - Q_2 = w$$

$$\Rightarrow Q_2 = Q_1 - w$$

$$\Rightarrow 100 - 10 = Q_2 = 90J$$

44. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J ofwork per cycle. The heat energy(in J) delivered by the engine to the low temperature reservoir, in a cycle, is [NA 7 Jan. 2020 I]

SOLUTION : (600.00)

Given; $T_1 = 900K, T_2 = 300K, W = 1200J$

$$\text{Using, } 1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$$

$$\Rightarrow 1 - \frac{300}{900} = \frac{1200}{Q_1}$$

$$\Rightarrow \frac{2}{3} = \frac{1200}{Q_1} \Rightarrow Q_1 = 1800$$

Therefore heat energy delivered by the engine to the low temperature reservoir,

$$\text{Q}_2 = \text{Q}_1 - \text{W} = 1800 - 1200 = 600.00J$$

45. **Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir ofthe first engine is T_1 and the temperature ofthe cold reservoir ofthe second engine is T_2. T is temperature ofthe sink offirst engine which is also the source for the second engine. How is T related to T_1 and T_2, ifboth the engines perform equal amount ofwork?**

[7 Jan. 2020 II]

(a) $T = \frac{2T_1T_2}{T_1+T_2}$ **(b)** $T = \frac{T_1+T_2}{2}$ **(c)** $T = \sqrt{T_1T_2}$ **(d)** $T = 0$

SOLUTION : . (b)

Let Q_H = Heat taken by first engine

Q_L = Heat rejected by first engine

Q_2 = Heat rejected by second engine

Work done by 1st engine = work done by 2^{nd} engine

$$W = Q_H - Q_L = Q_L - Q_2 \Rightarrow 2Q_L = Q_H + Q_2$$

$$2 = \frac{\theta_H}{\theta_L} + \frac{\theta_2}{\theta_L}$$

Let T be the temperature of cold reservoir offirst engine. Then in carnot engine.

$$\frac{Q_H}{Q_L} = \frac{T_1}{T} \text{ and } \frac{Q_L}{Q_2} = \frac{T}{T_2}$$

$$\Rightarrow 2 = \frac{T_1}{T} + \frac{T_2}{T} \text{ using (i)}$$

$$\Rightarrow 2T = T_1 + T_2 \Rightarrow T = \frac{T_1+T_2}{2}$$

46. **A Carnot engine has an efficiency of 1/6. When the temperature ofthe sink is reduced by 62^oC, its efficiency is doubled. The temperatures of the source and the sink are, respectively.**

[12 Apr. 2019 II]

(a) $62^oC, 124^oC$ **(b)** $99^oC, 37^o$ **(c)** $124^oC, 62^oC$ **(d)** $37^oC, 99^oC$

SOLUTION : (b)

$$\text{Using, } n = 1 - \frac{T_2}{T_1}$$

$$n = \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\text{and } \frac{T}{3} = 1 - \frac{T_2 - 62}{T_1}$$

On solving, we get $T_1 = 99°C$ and $T_2 = 37°C$

47. **Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T > T(4)$] Thethree engines are equally efficient if: (10Jan. 2019I**

(a) $T_2 = (T_1T_4)^{1/2}; T_3 = (T_1^2T_4)^{1/3}$ **(b)** $T_2 = (T_1^2T_4)^{1/3}; T_3 = (T_1T_4^2)^{1/3}$

(c) $T_2 = (T_1T_4^2)^{1/3}; T_3 = (T_1^2T_4)^{1/3}$ **(d)** $T_2 = (T_1^3T_4)^{1/4}; T_3 = (T_1T_4^3)^{1/4}$

SOLUTION : (b)

According to question, $\eta_1 = \eta_2 = \eta_3$

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

[Three engines are equally efficient]

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2 = \sqrt{T_1 T_3} \text{ (i) } T_3 = \sqrt{T_2 T_4} \text{ (ii) From(i) and(ii)}$$

$$T_2 = (T_1^{2}T_4)^{1/3}$$

$$T_3 = (T_1 T_4^2)^{1/3}$$

48. Two Carnot engines A and B are operated in series. The first one, A receives heat at $T_1 (= 600K)$ and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at $T_3 (= 400K)$. Calculate the temperature T_2 if the work outputs of the two engines are equal: [9 Jan. 2019 II]

(a) 600 K (b) 400 K (c) 300 K (d) 500 K

SOLUTION : (d)

$$\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1} \text{and, } \eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$$

According to question, $W_A = W_B$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2} = 500K$$

49. A Carnot↑s engine works as a refiigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount ofwork done in each cycle to operate the refrigerator is: [Online Apri115, 2018]

(a) 420 J (b) 2100 J (c) 772 J (d) 2520 J

SOLUTION : . (a)

Given: Temperature of cold body, $T_2 = 250K$

temperature ofhot body; $T_1 = 300K$

Heat received, $Q_2 = 500$ cal work done, $W = ?$

$$\text{Efficiency} = 1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W} \Rightarrow 1 - \frac{250}{300} = \frac{W}{Q_2 + W}$$

$$W = \frac{Q_2}{5} = \frac{500 \times 4.2}{5} J = 420J$$

50. Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100K. If the efficiencies ofthe two engines A and B are represented by η_A and η_B respectively, then what is the value of $\frac{\eta_A}{\eta_B}$

[Online Apri115, 2018]

(a) $\frac{12}{7}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

SOLUTION : . (d)

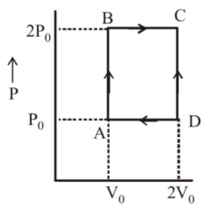
Efficiency ofengine $A, n_A = \frac{T_1 - T_2}{T_1}$ and $n_B = \frac{T_2 - T_3}{T_2}; T_2 = \frac{T_1 + T_3}{2} = 350K$

600−350

$$\text{or } \frac{n_A}{n_B} = \frac{600\ 350-100}{350} = \frac{7}{12}$$

51. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency ofthe engine is: (Take $C_v = 1.5R$, where R is gas constant)

[Online April 8, 2017]



(a) 0.24 (b) 0.15 (c) 0.32 (d) 0.08

SOLUTION : . (b)

$$\text{Work - done } (W) = P_0V_0$$

$$\text{According to principle ofcalorimetry Heat given} = Q_{AB} = Q_{BC}$$

$$= nC_VdT_{AB} + nC_PdT_{BC}$$

$$= \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$$

$$= \frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V) = \frac{13}{2}P_0V_0$$

$$\text{Thermal efficiency of engine } (\eta) = \frac{W}{Q_{given}} = \frac{2}{13} = 0.15$$

52. A Carnot fieezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is $336 \times 10^3 Jkg^{-1}$. If 5 kg of water at 0°C is converted into ice at 0°C by the fieezer, then the energy consumed by the fieezer is close to: [Online Apri110, 2016]

(a) $1.51 \times 10^5 J$ (b) $1.68 \times 10^6 J$ (c) $1.71 \times 10^7 J$ (d) $1.67 \times 10^5 J$

SOLUTION : (d)

$$\Delta H = mL = 5 \times 336 \times 10^3 = Q_{smk}$$

$$\frac{Q_{sink}}{Q_{source}} = \frac{T_{sink}}{T_{source}}$$

$$\therefore Q_{source} = \frac{T_{source}}{T_{sink}} \times Q_{sink}$$

Energy consumed by freezer

$$\therefore w_{output} = Q_{source} - Q_{sink} = Q_{sink}\left(\frac{T_{source}}{T_{sink}} - 1\right)$$

$$\text{Given: } T_{source} = 27^\circ C + 273 = 300K,$$

$$T_{sink} = 0^\circ C + 273 = 273k$$

$$W_{output} = 5 \times 336 \times 10^3 \left(\frac{300}{273} - 1\right) = 1.67 \times 10^5 J$$

53. A solid body ofconstant heat capacity 1 $J/^oC$ is being heated by keeping it in contact with reservoirs in two ways :

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount ofheat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount ofheat.

In both the cases body is brought from initial temperature $100^\circ C$ to fmal temperature $200^\circ C$. Entropy change ofthe body in the two cases respectively is : [2015]

(a) $ln2, 2ln2$ (b) $2ln2, 8ln2$ (c) $ln2, 4ln2$ (d) $ln2, ln2$

SOLUTION : (d)

The entropy change of the body in the two cases is same as entropy is a state function.

54. A Carnot engine absorbs 1000 J of heat energy from a reservoir at $127^\circ C$ and rejects 600 J ofheat energy during each cycle. The efficiency of engine and temperature of sink will be: [Online April 12, 2014]

(a) 20% and $-43^\circ C$ (b) 40% and $-33^\circ C$ (c) 50% and $-20^\circ C$ (d) 70% and $-10^\circ C$

SOLUTION : (b)

$$\text{Given: } Q_1 = 1000J \ , Q_2 = 600J$$

$$T_1 = 127^\circ C = 400K \ , T_2 = ? \ \eta = ?$$

$$\text{Efficiency ofcarnot engine, } \eta = \frac{W}{Q_1} \times 100\%$$

$$\text{or, } \eta = \frac{Q_2 - Q_1}{Q_1} \times 100\% \ \text{ or, } \eta = \frac{1000-600}{1000} \times 100\%$$

$$\eta = 40\%$$

$$\text{Now, for carnot cycle } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

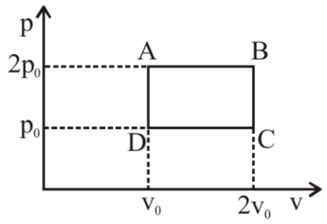
$$\frac{600}{1000} = \frac{T_2}{400}$$

$$T_2 = \frac{600\times400}{1000} = 240K = 240 - 273$$

$$T_2 = -33°C$$

55. The above p-v diagramrepresents the thermodynamic cycle ofan engine, operating with an ideal monatomic gas. The amount ofheat, extracted from the source in a single cycle is [2013]

67.



(a) p_0v_0 (b) $\left(\frac{13}{2}\right)p_0v_0$ (c) $\left(\frac{11}{2}\right)p_0v_0$ (d) $4p_0v_0$

SOLUTION : (b)

Heat is extracted from the source in path DA andAB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right)$$

$$\Rightarrow \frac{3}{2}P_0V_0 + \frac{5}{2}2P_0V_0$$

$$= \left(\frac{13}{2}\right)P_0V_0$$

56. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be: [2012]

(a) efficiency of Camot engine caimot *k* made larger than 50%

(b) 1200K (c) 750K (d) 600K

SOLUTION : (c)

The efficiency ofthe carnot's heat engine is given as $\eta = \left(1 - \frac{T_2}{T_1}\right)\times 100$

When efficiency is 40%,

$$T_1 = 500K; \eta = 40$$

$$40 = \left(1 - \frac{T_2}{500}\right)\times 100$$

$$\Rightarrow \frac{40}{100} = 1 - \frac{T_2}{500}$$

$$\Rightarrow \frac{T_2}{500} = \frac{60}{100} \Rightarrow T_2 = 300K$$

When efficiency is 60%, then

$$\frac{60}{100} = \left(1 - \frac{300}{T_2}\right) \Rightarrow \frac{300}{T_2} = \frac{40}{100}$$

$$\Rightarrow T_2 = \frac{100\times300}{40} \Rightarrow T_2 = 750K$$

57. The door of a working refrigerator is left open in a well insulated room. The temperature of air in the room will [Online May 26, 2012]

(a) decrease (b) increase in winters and decrease in summers

(c) remain the same (d) increase

SOLUTION : (d)

In a refrigerator,

the heat dissipated in the atmosphere is more than that taken from the cooling chamber, therefore the room is heated. If the door of a refrigerator is kept open.

58. This question has Statement 1 and Statement2. Ofthe four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1:

An inventor claims to have constructed an engine that has an efficiency of 30% when operated between the boiling and fieezing points of water. This is not possible.

Statement 2:

The efficiency of a real engine is always less than the efficiency of a Carnot engine operating between the same two temperatures. [Online May 19, 2012]

(a) Statement 1 is true, Statement2 is true, Statement2 is not the correct explanation of Statement 1.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is false, Statement2 is true.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

SOLUTION : (d)

According to Camot' s theorem

- no heat engine working between two given temperatures of source and sink can be more efficient than a perfectly reversible engine

i.e. Carnot engine working between the same two temperatures.

Efficiency of Carnot' s engine, $n=1-\frac{T_2}{T_1}$

where, T_1 = temperature of source

T_2 = temperature of sink

59. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by62 K its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively: [2011]

(a) 372Kand310K (b) 330Kand268K (c) 310Kand248K (d) 372Kand310K

SOLUTION : (d)

$$\text{Efficiency ofengine } \eta_1=1-\frac{T_2}{T_1}=\frac{1}{6}$$

$$\Rightarrow \frac{T_2}{T_1}=\frac{5}{6} \quad \text{(i)}$$

When T_2 is lowered by 62K, then

$$\text{Again, } \eta_2 = 1 - \frac{T_2 - 62}{T_1} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1} = \frac{1}{3} \text{ (ii)}$$

Solving (i) and (ii), we get,

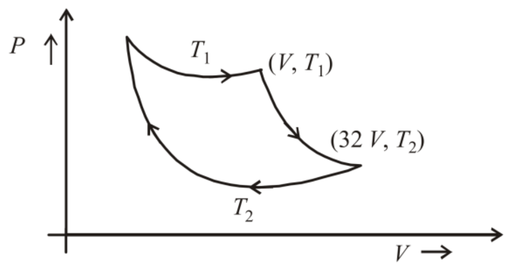
$$T_1 = 372\text{K and } T_2 = \frac{5}{6} \times 372 = 310\text{K}$$

60. **A diatomic ideal gas is used in a Carnot engine as the working substance. Ifduring the adiabatic expansion part ofthe cycle the volume ofthe gas increases from V to 32 V, the efficiency ofthe engine is** **[2010]**

(a) 0.5 (b) 0.75 (c) 0.99 (d) 0.25

SOLUTION : **(b)**

72. (b)



$$\text{For adiabatic expansion } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow T_1 V^{g-1} = T_2 (32V)^{g-1}$$

$$\Rightarrow \frac{T_1}{T_2} = (32)^{\gamma-1}$$

$$\text{For diatomic gas, } \gamma = \frac{7}{5}$$

$$\gamma - 1 = \frac{2}{5}$$

$$\frac{T_1}{T_2} = (32)^{\frac{2}{5}} \Rightarrow T_1 = 4T_2$$

$$\text{Now, efficiency} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{4T_2} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75.$$

61. **ACarnot engine, having an efficiency of $\eta = 1/10$asheat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is** **[2007]**

(a) 100J (b) 99 J (c) 90 J (d) 1 J

SOLUTION : (c)

The efficiency (η) of a Carnot engine and the coefficient ofperformance (β) of a refrigerator are

$$\text{related as } \beta = \frac{1-\eta}{\eta}$$

$$\text{Also, } \beta = \frac{Q_2}{W}$$

$$\beta = \frac{1-n}{n} = \frac{Q_2}{W}$$

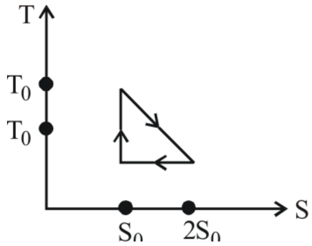
$$\beta = \frac{1 - \frac{1}{10}}{\left(\frac{1}{10}\right)} = \frac{Q_2}{W}.$$

is independent of path taken by the process.

$$\Rightarrow 9 = \frac{Q_2}{10}$$

$$\Rightarrow Q_2 = 90 \text{ J.}$$

62. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is [2005]



(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

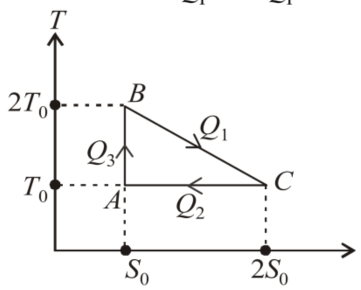
SOLUTION : (d)

$$Q_1 = \text{area under BC} = T_0S_0 + \frac{1}{2}T_0S_0$$

$$Q_2 = \text{area under AC} = T_0(2S_0 - S_0) = T_0S_0$$

$$\text{and } Q_3 = 0$$

$$\text{Efficiency, } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$



$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_0S_0}{\frac{3}{2}T_0S_0} = \frac{1}{3}$$

63. Which of the following statements is correct for any thermodynamic system *Reject* [2004]

(a) The change in entropy can never be zero

(b) Internal energy and entropy are state functions

(c) The internal energy changes in all processes

(d) The work done in an adiabatic process is always zero.

SOLUTION : (b)

Intemal energy and entropy are state ffinction,

they are independent ofpath taken.

64. "Heat cannot by itselfflow fi$_{\text{iO}}$m a body at lower temperature to a body at higher temperature" is a statement or consequence of [2003]

(a) second law ofthermodynamics (b) conservation of momentum

(c) conservation of mass (d) first law ofthermodynamics

SOLUTION : (a)

This is a consequence of second law of thermodynamics

65. A Carnot engine takes 3×10^6cal ofheat fi$_{iom}$ areservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is [2003]

(a) 4.2×10^6J (b) 8.4×10^6J (c) 16.8×10^6J (d) zero

SOLUTION : (b)

$$\text{Here, } T_1 = 627+273 = 900\text{K}$$

$$T_2 = 27+273 = 300\text{K}$$

$$\text{EfFiciency } \eta = 1-\frac{T_2}{T_1} = 1-\frac{300}{900} = 1-\frac{1}{3} = \frac{2}{3}$$

$$\text{But } \eta = \frac{W}{Q}$$

$$\frac{W}{Q} = \frac{2}{3} \Rightarrow W = \frac{2}{3}\times Q = \frac{2}{3}\times 3\times10^6 = 2\times10^6 \text{ cal}$$

$$= 2\times10^6\times4.2\text{J} = 8.4\times10^6\text{J}$$

66. Which statement is incorrectQ [2002]

(a) All reversible cycles have same efficiency

(b) Reversible cycle has more efficiency than an irreversible one

(c) Carnot cycle is a reversible one

(d) Carnot cycle has the maximum efficiency in all cycles

SOLUTION : (a)

All reversible engines have same efficiencies if they are working for the same temperature ofsource and sink.

If the temperatures are different, the efficiency is different.

67. Even Carnot engine cannot give 100% efficiency because we cannot [2002]

(a) prevent radiation (b) fmd ideal sources

(c) reach absolute zero temperature (d) eliminate fiction

SOLUTION : (c)

In Carnot' s cycle we assume frictionless piston, absolute insulation and ideal source and sink(reservoirs).

$$\text{The efficiency of carnot' s cycle } \eta = 1-\frac{T_2}{T_1}$$

The efficiency of carnot engine will be 100% when its sink (T_2) is at 0 K.

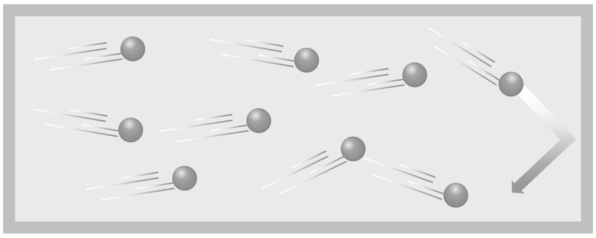
The temperature of0K (absolute zero) cannot be realised in practice

so, efficiency is never 100%.

Kinetic Theory of Gases

Gas

In gases the intermolecular forces are very weak and its molecule may fly apart in all directions. So the gas is characterized by the following properties.

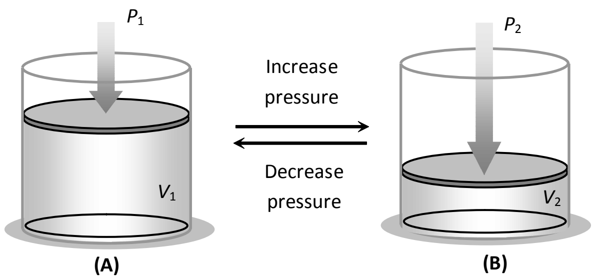


- In gases, molecules are far apart from each other and mutual attractions between them are negligible.
- They can easily compressed and expand.
- It has no shape and size and can be obtained in a vessel of any shape or size.
- It expands indefinitely and uniformly to fill the available space.
- It exerts pressure on its surroundings.

GAS LAWS :

BOYLE'S LAW:

At constant temperature, the volume of a given mass of gas is inversely proportional to its pressure.



Let us consider an ideal gas in a container with piston , it has initial volume V_1 at pressure P_1. if piston pushed inword at constant temperature , pressure increased to P_2 then its volume decreases to V_2. therefore volume is inversely proportional to pressure at constant temperature

$$i.e.\quad V \propto \frac{1}{P}$$

$$P = \frac{constant}{V}$$

$$PV = \text{constant}$$

$$P_1V_1 = P_2V_2$$

NOTE :

$$From\ Density\ \ \rho = \frac{mass}{volume} = \frac{m}{V}$$

$$\text{volume V} = \frac{m}{\rho(\text{Density of the gas})}$$

$$Then\ \ PV = \text{constant}$$

$$PV = P\left(\frac{m}{\rho}\right) = \text{constant}$$

$Here\ mass\ m = \text{constant}$

$$\therefore\ \frac{P}{\rho} = \text{constant}$$

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

NOTE :

♦ In PV = K, the value of the constant 'K' depends on

1. Mass of gas 2. Temperature of gas 3. System of units

♦ Boyle's law generally holds good only at low pressure and high temperatures.A gas which obey Boyle's law under all conditions of temperature and pressure is called ideal gas.

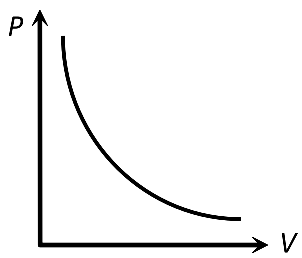
♦ Boyle's law can be experimentally verified by Quill's tube (or) Boyle's law apparatus.

♦ The graphs drawn between P & V at constant temperture of a gas are called isotherms

Graphical representation :

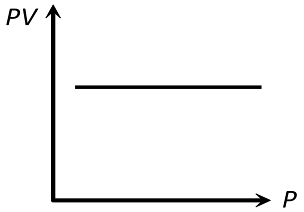
If m and T are constant

P - V Graph :

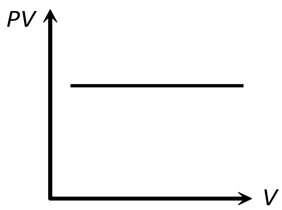


(A)

PV - P and PV - V Graph :



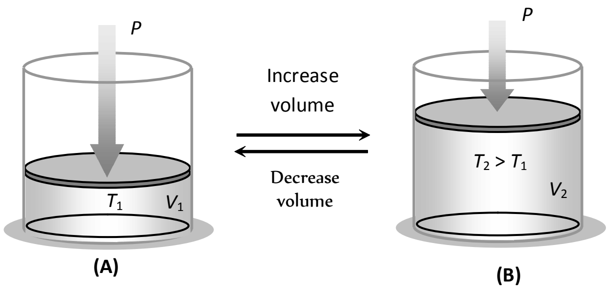
(B)



(C)

Charle's law :

If the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its absolute temperature.



Let us consider an ideal gas in a container with piston , it has initial volume V_1 at absolute temperature T_1. . At constant pressure , temerature of gas increased to T_2 then its volume increases to V_2. therefore volume of gas is directly proportional to its absolute temperature at constant pressure.

$$i.e., \quad V \propto T$$

$$V = constant(T)$$

$$\frac{V}{T} = \text{constant} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

NOTE :

$$From\ Density \quad \rho = \frac{mass}{volume} = \frac{m}{V}$$

$$\text{volume V} = \frac{m}{\rho(\text{Density of the gas})}$$

$$V = \frac{m}{\rho}$$

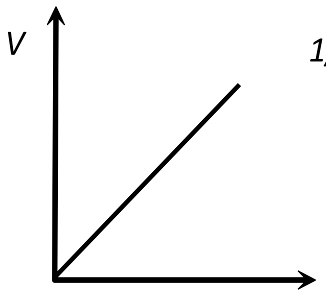
Then $\frac{V}{T} = \text{constant}$

$$\frac{V}{T} = \frac{m}{\rho T} = \text{constant}$$

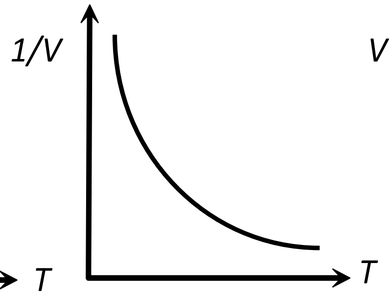
$$\rho T = \text{constant}$$

$$\rho_1 T_1 = \rho_2 T_2$$

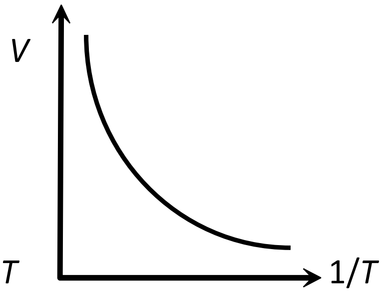
Graphical representation: If m and P are constant



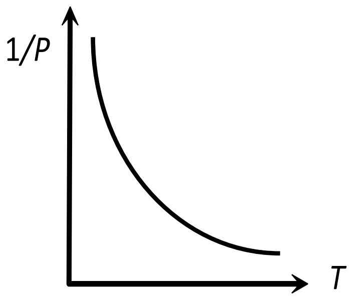
(A)



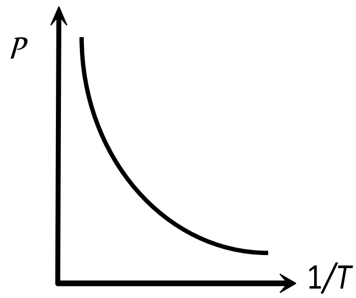
(B)



(C)



(D)



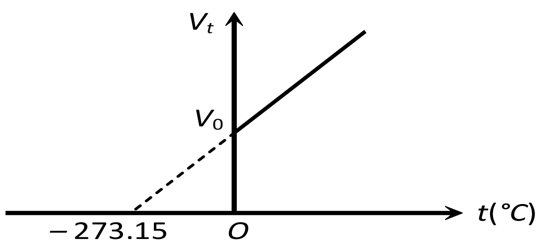
(E)

NOTE :

If the pressure remains constant, the volume of the given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at $0°C$ for each $1°C$ rise or fall in temperature.

$$V_t = V_0\left(1 + \frac{1}{273.15}t\right).$$

This is Charle's law for centigrade scale.



Gay-Lussac's law or pressure law :

The volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.

Let us consider an ideal gas in a container with piston , it has initial pressure P_1 at absolute temperature T_1. At constant volume , temerature of gas increased to T_2 then its pressure increases to P_2. therefore PRESSURE of gas is directly proportional to its absolute temperature at constant volume .

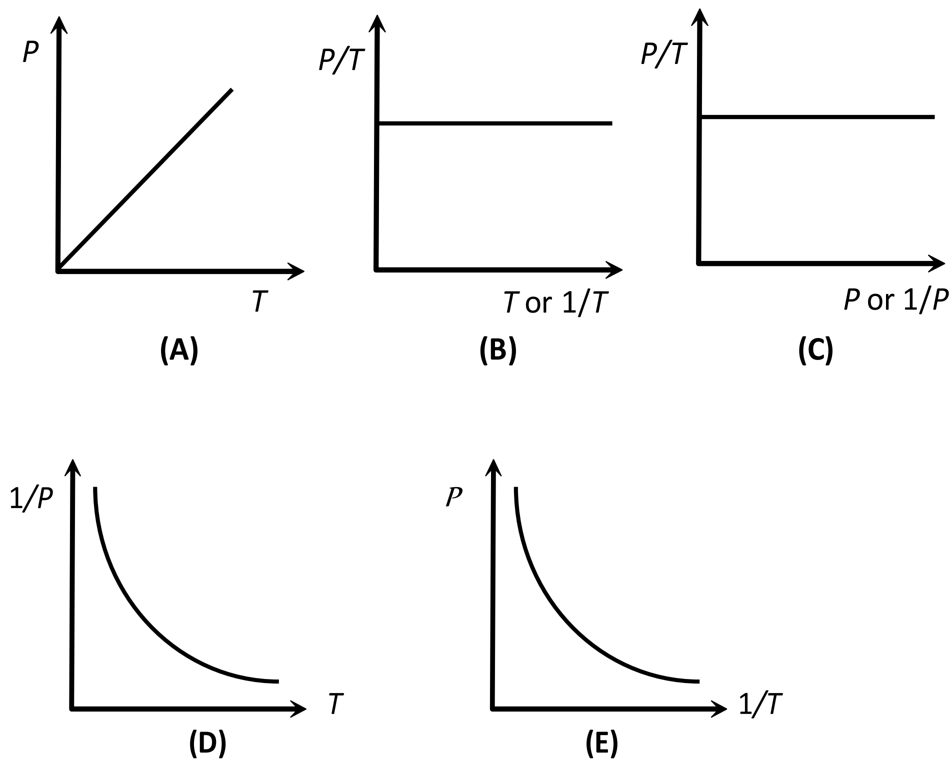
$$P \propto T$$

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Graphical representation :

If m and V are constants



Grahm's law of diffusion :

When two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas

i.e. rate of diffusion $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$

$\rho \rightarrow$ is the density of the gas

M $\rightarrow$ is the molecular weight of the gas

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$$

If V is the volume of gas diffused in t sec then

rate of diffusion $r = \frac{V}{t}$

$$\frac{r_1}{r_2} = \frac{V_1}{V_2} \times \frac{t_2}{t_1}$$

Dalton's law of partial pressure :

The total pressure exerted by a mixture of non-reacting gases occupying a vessel is equal to the sum of the individual pressures which each gases exert if it alone occupied the same volume at a given temperature.

For n gases

$$P = P_1 + P_2 + P_3 + ... P_n$$

where

P = Pressure exerted by mixture

$P_1, P_2, P_3, ... P_n$ = Partial pressure of component gases.

Ideal Gas :

A gas which obey Boyle's law under all conditions of temperature and pressure is called ideal gas.

- ♦ Real gases obey gas laws only at low pressure and high temperatures. All Gases are real gases only.
- ♦ Attraction between the molecules of perfect gas is zero.
- ♦ Ideal or perfect gas obey gas laws at all temperatures and pressures without any limitations.
- ♦ Hydrogen or Helium behaves closely as perfect gas. Hence it is preferred in constant volume gas thermometers.

Assumption of Ideal Gases (or Kinetic Theory of Gases):

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature *etc.*) to the microscopic properties of the gas molecules (such as speed, momentum, kinetic energy of molecule *etc.*)

Actually it attempts to develop a model of the molecular behaviour which should result in the observed behaviour of an ideal gas. It is based on following assumptions :

- ♦ Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those of another gas.
- ♦ The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- ♦ Their size is negligible in comparison to intermolecular distance (10^{-9} *m*)
- ♦ The volume of molecules is negligible in comparison to the volume of gas. (The volume of molecules is only 0.014% of the volume of the gas).

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity
- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collisions are perfectly elastic.
- The time spent in a collision between two molecules is negligible in comparison to time between two successive collisions.
- The number of collisions per unit volume in a gas remains constant.
- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
- Molecules constantly collide with the walls of container due to which their momentum changes. The change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.
- The density of gas is constant at all points of the container.

Equation of State or Ideal Gas Equation :

The equation which relates the pressure (P) volume (V) and temperature (T) of the given state of an ideal gas is known as ideal gas equation or equation of state.

From Boyle's Law

$$V \propto \frac{1}{P} \quad ....(1)$$

From charles Law

$$V \propto T \quad ....(2)$$

From equation (1) and (2)

$$V \alpha \frac{T}{P}$$

$$V = K\frac{T}{P}$$

$$\frac{PV}{T} = K(cons\tan t)$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For 1 mole of gas $K = R$ (universal gas constant.)

$$\frac{PV}{T} = R \text{ (constant)}$$

$$PV = RT$$

where R = universal gas constant.

For n moles of gas $PV = nRT$

Universal gas constant (R) :

A gas constant per one mole of gas is called as Universal gas constant

→ it is same for all gases

→ The value of "R" does not depend on the mass of gas or its chemical formula.

→ The fact that R is a constant for all gases is constant with Avagadro's hypothesis that "equal volumes of all gases under same conditions of temperature and pressure contains equal number of molecules".

$$R = \frac{PV}{nT} = \frac{\text{Pressure} \times \text{Volume}}{n \times \text{Temperature}} = \frac{\text{Work done}}{\text{n} \times \text{Temperature}}$$

At S.T.P. the value of universal gas constant is same for all gases

$$R = 8.31 \frac{J}{mole \times kelvin} = 1.98 \frac{cal}{mole \times kelvin} \simeq 2 \frac{cal}{mol \times kelvin} = 0.8221 \frac{litre \times atm}{mole \times kelvin}.$$

It's unit is $\dfrac{Joule}{mole \times kelvin}$

Dimension : $[ML^2T^{-2}\theta^{-1}]$

Specific gas constant (r) :

A gas constant per unit mass of gas is called as specific gas constant

→ it is different for different gases

It is represented by per gram gas constant

i.e., $r = \dfrac{R}{M}$.

Since the value of M is different for different gases. Hence the value of r is different for different gases.

e.g. It is maximum for hydrogen $r_{H_2} = \dfrac{R}{2}$

It's unit is $\dfrac{Joule}{kg \times kelvin}$

Dimension : $[L^2T^{-2}\theta^{-1}]$

ideal gas equation $PV = nRT$

But No of moles $n = \dfrac{m}{M}$

$$PV = \frac{m}{M}RT$$

$$PV = mrT$$

Boltzman's constant (k) :

A gas constant per one molecule of gas is called as Boltzman's constant .

$\rightarrow$ it is same for all gases

$$i.e., \; k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} \; J/K$$

unit is $\frac{J}{kelvin}$

dimension : $[ML^2T^{-2}\theta^{-1}]$

ideal gas equation $PV = nRT$

But No of moles $\mu = \frac{\text{no.of molecules}}{\text{avagadro's number}} = \frac{N}{N_A}$

$$n = \frac{N}{N_A}$$

$$PV = \frac{N}{N_A} RT$$

$$PV = Nk_BT$$

Different forms of gas equation

Quantity of gas	**Equation**	**Constant**
1 mole gas	$PV = RT$	***R = universal gas constant***
n mole gas	$PV = n\,RT$	***R = universal gas constant***
1 molecule of gas	$PV = Nk_BT$	k_B ***= Boltzmann's constant***
1 gm gas	$PV = mrT$	***r = specific gas constant***

Real Gases :

The gases actually found in nature are called real gases. They do not obeys gas Laws.

For exactly one mole of an ideal gas

$$\frac{PV}{RT} = 1 \qquad V = K\frac{T}{P}$$

$$\frac{PV}{T} = K(cons\tan t)$$

$$K = R$$

$$PV = nRT$$

Plotting the experimentally determined value of $\frac{PV}{RT}$ for exactly one mole of various real gases as a function of pressure P, shows a deviation from identity.

$\rightarrow$ The quantity $\frac{PV}{RT}$ is called the compressibility factor and should be unit for an ideal gas.

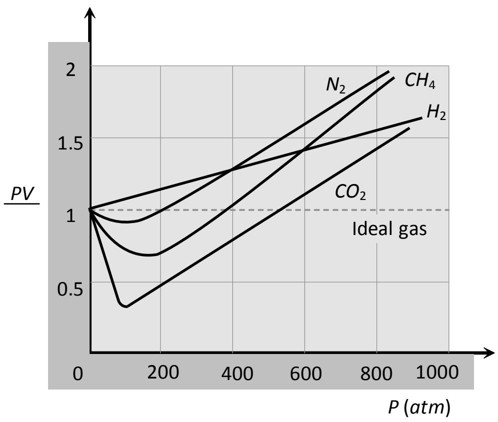


Fig. 13.7

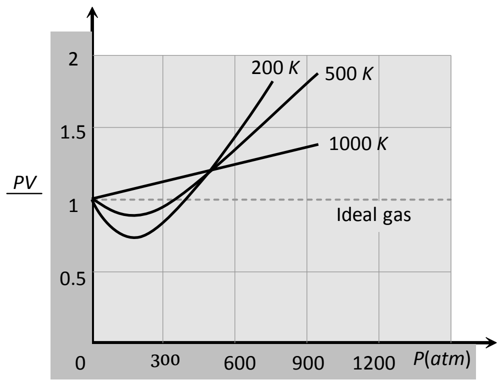


Fig. 13.8

$\rightarrow$ A real gas behaves as ideal gas most closely at low pressure and high temperature. Also can actual gas can be liquefied most easily which deviates most from ideal gas behaviour at low temperature and high pressure.

Equation of state for real gases :

It is given by Vander Waal's with two correction in ideal gas equation. The it know as Vander Waal's gas equation.

(i) **Volume correction :**

Due to finite size of molecule, a certain portion of volume of a gas is covered by the molecules themselves. Therefore the space available for the free motion of molecules of gas will be slightly less than the volume V of a gas.

Hence the effective volume becomes $(V-b)$.

(ii) **Pressure correction :**

Due to intermolecular force in real gases, molecule do not exert that force on the wall which they would have exerted in the absence of intermolecular force. Therefore the observed pressure P of the gas will be less than that present in the absence of intermolecular force.

Hence the effective pressure becomes $\left(P+\frac{a}{V^2}\right)$.

(iii) **Vander Waal's gas equations**

For 1 *mole* of gas $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

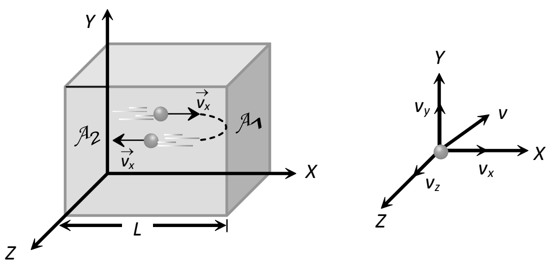
For n *moles* of gas $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$

Here a and b are constant called Vander Waal's constant.

Dimension : $[a]=[ML^5T^{-2}]$ and $[b]=[L^3]$

Units : $a = N \times m^4$ and $b = m^3$.

Pressure of an Ideal Gas :



Consider an ideal gas (consisting of N molecules each of mass m) enclosed in a cubical box of side L.

Let Any molecule of gas moves with velocity $\vec{v}$ in any direction and collide with wall A_1 and rebounds

where $\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$

$$v=\sqrt{v_x^2+v_y^2+v_z^2}\ .$$

Due to random motion of molecule $v_x=v_y=v_z$

$$v^2=3v_x^2=3v_y^2=3v_z^2$$

Time between two successive collision with the wall A_1

$$\Delta t = \frac{\text{Distance travelled by molecule between two successive collision}}{\text{Velocity of molecule}}$$

$$\Delta t = \frac{2L}{v_x}$$

Then the number of collision per second. $n = \frac{1}{\Delta t} = \frac{v_x}{2L}$

This molecule collides with the shaded wall (A_1) with velocity v_x and rebounds with velocity $-v_x$.

The change in momentum of the molecule $\Delta p = (-mv_x) - (mv_x) = -2mv_x$

As the momentum remains conserved in a collision,

the change in momentum of the wall A_1 is $\Delta p = 2mv_x$ $\Delta p = 2mv_x$

After rebound this molecule travel toward opposite wall A_2 with velocity $-v_x$, collide to it and again rebound with velocity v_x towards wall A_1.

Force exerted by a single molecule on shaded wall is equal to rate at which the momentum is transferred to the wall by this molecule.

$$i.e.\ F_{\text{Single molecule}} = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{(2L/v_x)} = \frac{mv_x^2}{L}$$

The total force on the wall A_1 due to all the molecules $F_x = \frac{m}{L}\sum v_x^2$

$$F_X = \frac{m}{M}(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + ...) = \frac{mN}{L}\overline{v_x^2}$$

$\overline{v_x^2}$ = mean square of x component of the velocity.

Now pressure is defined as force per unit area, hence pressure on shaded wall

$$P_x = \frac{F_x}{A} = \frac{mN}{AL}\overline{v_x^2} = \frac{mN}{V}\overline{v_x^2}$$

For any molecule, the mean square velocity $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$

by symmetry $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{\overline{v^2}}{3}$$

Total pressure inside the container $P = \frac{1}{3}\frac{mN}{V}\overline{v^2} = \frac{1}{3}\frac{m\,N}{V}v_{rms}^2$ (where $v_{rms} = \sqrt{\overline{v^2}}$)

Relation between pressure and kinetic energy :

As we know $P = \frac{1}{3}\frac{m\,N}{V}v_{rms}^2$

but $M = mN$ = Total mass of the gas

$$P = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

Density of gas $\rho = \frac{M}{V}$

Pressure $P = \frac{1}{3}\rho v_{rms}^2$ ... (i)

Kinetic Energy K.E $= \frac{1}{2}Mv_{rms}^2$

$\therefore$ K.E. per unit volume $\frac{K.E}{V} = E = \frac{1}{2}\left(\frac{M}{V}\right)v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$ ...(ii)

From (i) and (ii), we get $P = \frac{2}{3}E$

i.e. the pressure exerted by an ideal gas is numerically equal to the two third of the mean kinetic energy of translation per unit volume of the gas.

Effect of mass on pressure :

Pressure $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

$P \propto \frac{(mN)T}{V}$ [As $v_{rms}^2 \propto T$]

If volume and temperature of a gas are constant

$P \propto mN$

Pressure $\propto$ (Mass of gas).

i.e. if mass of gas is increased, number of molecules and hence number of collision per second increases . *so* pressure will increase.

Effect of on pressure :

$P \propto \frac{(mN)T}{V}$

If mass and temperature of a gas are constant.

$P \propto (1/V)$,

i.e., if volume decreases, number of collisions per second will increase due to lesser effective distance between the walls resulting in greater pressure.

Effect of on pressure :

$$P \propto \frac{(mN)T}{V}$$

If mass and volume of gas are constant,

$P \propto (v_{rms})^2 \propto T$

i.e., if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure.

Average translational kinetic energy of a gas:

Let M be the molecular mass and V be the molar volume of a gas. Let m be the mass of each molecule. Then

- Mean K.E. per mole of a gas, $E = \frac{3}{2}PV = \frac{3}{2}RT = \frac{3}{2}k_B N\, T$
- Mean K.E. per molecule of a gas $\overline{E} = \frac{3}{2}k_B T$
- K.E of 1 gram of gas $= \frac{3}{2}\frac{RT}{M}$

Note :

- Kinetic energy per molecule of gas does not depends upon the mass of the molecule but only depends upon the temperature of the gas.

  It is given as $E = \frac{3}{2}k_B T \;\; or \;\; E \propto T$
- Molecules of different gases say He, H_2 and O_2 etc. at same temperature will have same translational kinetic energy though their rms speeds are different
- Kinetic energy per mole of gas depends only upon the temperature of gas.
- Kinetic energy per gram of gas depend upon the temperature as well as molecular weight (or masso f one molecule) of the gas

  $$KE_{gram} = \frac{3}{2}\frac{kT}{m} \Rightarrow KE_{gram} \propto \frac{T}{m}$$
- From the above expression it is clear that higher the temperature of the gas, more will be the gas, more will be the average kinetic energy possessed by the gas molecules. At T=0, E=0, i.e. at absolute zero the molecular motion ceases.
- The kinetic interpretation of temperature gives the relation between the average kinetic energy of a gas molecule and absolute temperature.

  $$i.e \quad E = (3/2)NK_B T \Rightarrow E \propto T$$
- Average kinetic energy is independent of pressure, volume and the nature of the ideal gas

PROBLEMS

1. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take R = $\frac{25}{3}$ J/mol–K and k = 1.38×10^{-23} J/K.

(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

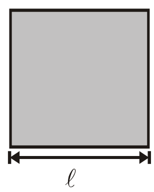
SOLUTION :

Volume of the box = 1m^3,

Pressure of the gas = 100 N/m^2.

Let T be the temperature of the gas

(a) Time between two consecutive collisions with one wall = $\frac{1}{500}$ sec



This time should be equal to $\frac{2I}{v_{rms}}$, where ℓ is the side of the cube.

$$2\ell v_{rms} = \frac{1}{500}$$

$$v_{rms} = 1000 \text{ m/s}$$

$$\therefore \sqrt{\frac{3RT}{M}} = 1000$$

$$T = \frac{(1000)^2 M}{3R} = \frac{(10)^6 (3 \times 10^{-3})}{3\left(\frac{25}{3}\right)} = 160K$$

(b) Average kinetic energy per atom = $\frac{3}{2}kT = \frac{3}{2}$ [(1.38 × 10^{-23}) 160J $= 3.312 \times^{-21}$ J

(c) From $PV = nRT = \frac{m}{M}RT$,

Mass of helium gas in the box $m = \frac{PVM}{RT}$

Substituting the values, $m = \frac{(100)(1)(4 \times 10^{-3})}{\left(\frac{25}{3}\right)(160)} = 3.0 \times 10^{-4}$ kg

2. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

SOLUTION :

Total energy of molecules of first gas $= \frac{3}{2} n_1 k T_1$,

Total energy of molecules of second gas $= \frac{3}{2} n_2 k T_2$

Let temperature of mixture be T

then total energy of molecules of mixture $= \frac{3}{2} k(n_1 + n_2)T$

$$\frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k(n_1 T_1 + n_2 T_2)$$

$$T = \frac{(n_1 T_1 + n_2 T_2)}{(n_1 + n_2)}$$

3. 1 kg of diatomic gas is at a pressure of $8 \times 10^4 N/m^2$. The density of the gas is $4 kg/m^3$. The energy of the gas due to its thermal motion is

SOLUTION :

Energy of diatomic gas due to its thermal motion is $= \frac{5}{2} PV$

$$= \frac{5}{2} P\left(\frac{m}{\rho}\right)$$

4. Gas at a pressure P_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds are doubled, the resulting pressure P will be equal to

1) $4P_0$ 2) $2P_0$ 3) p_0 4) $\frac{p_0}{2}$

SOLUTION :

Pressure $P = \frac{1}{3}\rho(v_{rms})^2$

$$P \propto m(v_{rms})^2$$

$$\frac{P_2}{P_1} = \frac{m_2}{m_1}\left[\frac{v_2}{v_1}\right]^2$$

$$\frac{P_2}{P_1} = \frac{m/2}{m}\left[\frac{2v}{v}\right]^2$$

$$P_2 = 2P_0$$

Various Speeds of Gas Molecules:

The motion of molecules in a gas is characterised by any of the following three speeds.

Root mean square speed :

It is defined as the square root of mean of squares of the speed of different molecules

$$i.e.\ v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + ...}{N}} = \sqrt{\overline{v^2}}$$

From the expression of pressure

$$P = \frac{1}{3}\rho v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

where $\rho = \frac{\text{Mass of gas}}{V} = \text{Density of the gas}$,

$M = \mu \times$ (mass of gas),

$pV = \mu RT$,

$R = kN_A$,

$k =$ Boltzmann's constant,

$m = \frac{M}{N_A} =$ mass of each molecule.

♦ With rise in temperature *rms* speed of gas molecules increases as

$$v_{rms} \propto \sqrt{T}\ .$$

♦ With increase in molecular weight *rms* speed of gas molecule decreases as

$$v_{rms} \propto \frac{1}{\sqrt{M}}\ .$$

e.g., *rms* speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.

♦ *rms* speed of gas molecules is of the order of *km/s*

e.g., at NTP for hydrogen gas

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 273}{2 \times 10^3}} = 1840\ m/s.$$

♦ *rms* speed of gas molecules is $\sqrt{\frac{3}{\gamma}}$ times that of speed of sound in gas, as

rms speed of gas $v_{rms} = \sqrt{\frac{3RT}{M}}$

speed of sound in gas $v_s = \sqrt{\frac{\gamma RT}{M}}$

$$v_{rms} = \sqrt{\frac{3}{\gamma}} v_s$$

♦ *rms* speed of gas molecules does not depends on the pressure of gas (if temperature remains constant)

because $P \alpha\ \rho$ (Boyle's law)

if pressure is increased *n* times then density will also increases by *n* times but v_{rms} remains constant.

♦ Moon has no atmosphere because v_{rms} of gas molecules is more than escape velocity (v_{e}).

A planet or satellite will have atmosphere only if $v_{rms} < v_e$

♦ The molecules of gases will escape out from a planet. If the temperature of planet

$$T \geq \frac{M v_e^2}{3R} \quad (\because v_{rms} \geq v_e); \quad v_e = \text{escape velocity of planet}$$

M = molecular mass of gas

♦ At $T = 0$; $v_{rms} = 0$

i.e. the *rms* speed of molecules of a gas is zero at 0 *K*. This temperature is called absolute zero.

(2) **Most probable speed :**

The particles of a gas have a range of speeds. This is defined as the speed which is possessed by maximum fraction of total number of molecules of the gas. *e.g.*, if speeds of 10 molecules of a gas are 1, 2, 2, 3, 3, 3, 4, 5, 6, 6 *km/s*, then the most probable speed is 3 *km/s*, as maximum fraction of total molecules possess this speed.

$$\text{Most probable speed } v_{mp} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{\text{mass of gas}}} = \sqrt{\frac{2P}{\rho}}$$

(3) **Average speed :** It is the arithmetic mean of the speeds of molecules in a gas at given temperature.

$$v_{av} = \frac{v_1 + v_2 + v_3 + v_4 + ...}{N}$$

and according to kinetic theory of gases

Average speed $v_{avg} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi(mass\,of\,gas)}} = \sqrt{\frac{8P}{\pi\rho}}$.

Relation between $\upsilon_{av}, \upsilon_{rms}$ and υ_{mp} :

Average speed $v_{avg} = 0.92 v_{rms}$

Most probable speed $v_{mp} = 0.816 v_{rms}$

$v_{rms} : v_{avg} : v_{mp} = 1.73 : 1.60 : 1.41$

Clearly, $v_{rms} > v_{avg} > v_{mp}$

PROBLEMS

1. The root-mean-square (rms) speed of oxygen molecules (O_2) at a certain absolute temperature is υ. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, the rms speed would be

1) υ 2) $\sqrt{2}\upsilon$ 3) 2υ 4) $2\sqrt{2}\upsilon$

SOLUTION :

rms speed of gas $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$V \alpha \sqrt{T/M}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}\frac{M_1}{M_2}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{2T}{T}\frac{2M}{M}}$$

$$\frac{V_2}{V_1} = 2$$

$$V_2 = 2V$$

2. Calculate the temperature at which the oxygen moleules will have the same rm,s velocity as the hydrogen molecules at $150^0 C$. Molecular weight of oxygen is 32 and that of hydrogen is 2

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$As\, V_{1rms} = V_{2\,rms}, therefore\; T \infty M$$

$$\frac{T_1}{T_2} = \frac{M_1}{M_2}; T_2 = T_1\left(\frac{M_2}{M_1}\right)$$

$$T_2 = 423\left[\frac{32}{2}\right], T_2 = 6768K$$

$$t_2 = 6768 - 273 = 6495^0 C$$

3.Two moles of an ideal gas X occupying a volume V exerts a pressure P. The same pressure is exerted by one of another gas Y occupying a volume 2V. if the molecular weight of Y is 16 times the molecular weight of X, find the ratio of the rms speeds of the molecules of X and Y.

SOLUTION :

$$V_{rms} = \sqrt{\frac{3PV}{nM}}\left(\frac{m}{M} = n\right)$$

$$\frac{V_{1rms}}{V_{2\,rms}} = \sqrt{\frac{V_1}{V_2}\frac{n_2}{n_1}\frac{M_2}{M_1}}; \frac{V_{1\,rms}}{V_{2\,rms}} = \sqrt{\frac{V}{2V}\left(\frac{1}{2}\right)\frac{16}{1}}$$

$$\frac{V_{1\,rms}}{V_{2\,rms}} = 2$$

4. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at $-20^0 C$? Atomic mass of argon $= 39.9u$ and that of helium =4.0u

SOLUTION :

Here, atomic mass of argon, $M_1 = 39.9u$

atomic mass of helium, $M_2 = 4.0u$

Suppose that the $r.m.s$ speed of argon gas atoms at temperature T is equal to the $r.m.s$. speed of Helium gas atoms at $-20^0 C$ i.e., at $T = -20 + 273 = 253K$.

Suppose that $u_{r.m.s}$ and $u_{r.m.s}$ are $u_{r.m.s}$ speeds of argon and helium gas atoms at temperatures T and T' respectively

Now. $u_{r.m.s} = \sqrt{\frac{3RT}{M}}$

$$\therefore u_{r.m.s} = \sqrt{\frac{3RT}{39.9}} \text{ and } u_{r.m.s} = \sqrt{\frac{3R \times 253}{4.0}}$$

Since $u_{r.m.s} = u_{r.m.s}$, we have

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4.0}}$$

$$\text{(or)} \quad T = \frac{253 \times 39.9}{4.0} = 2{,}523.7K$$

5. Your are given the following group of particles, n_i represents the number of molecules with speed v_i

n_i	2	4	8	6	3
$v_i\left(ms^{-1}\right)$	1.0	2.0	3.0	4.0	5.0

calculate (i) average speed (ii) rms speed

SOLUTION : (i) average speed

$$v_{ang} = \frac{n_1v_1+n_2v_2+n_3v_3+n_4v_4+n_5v_5}{\left(n_1+n_2+n_3+n_4+n_5\right)}$$

$$= \frac{(2\times1)+(4\times2)+(8\times3)+(6\times4)+(3\times5)}{(2+4+8+6+3)} = \frac{73}{23} = 3.17ms^{-1}$$

(ii) root mean square speed is

$$v_{rms} = \left(\frac{n_1v_1^2+n_2v_2^2+n_3v_3^2+n_4v_4^2+n_5v_5^2}{n_1+n_2+n_3+n_4+n_5}\right)$$

$$= \left(\frac{2\times1+4\times4+8\times9+6\times16+3\times25}{2+4+8+6+3}\right)^{1/2} = 3.36ms^{-1}$$

6.. If the molecular weight of two gases are M_1 and M_2 then at a temperature the ratio of root mean square velocity v_1 and v_2 will be

(a) $\sqrt{\frac{M_1}{M_2}}$ (b) $\sqrt{\frac{M_2}{M_1}}$ (c) $\sqrt{\frac{M_1+M_2}{M_1-M_2}}$ (d) $\sqrt{\frac{M_1-M_2}{M_1+M_2}}$

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \alpha \sqrt{\frac{1}{M}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{M_2}{M_1}}$$

7. The *r.m.s.* velocity of a gas at a certain temperature is $\sqrt{2}$ times than that of the oxygen molecules at that temperature. The gas can be

(a) H_2 (b) *He* (c) CH_4 (d) SO_2

SOLUTION :

$$v_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\therefore \frac{1}{\sqrt{2}} = \sqrt{\frac{M_2}{32}} \Rightarrow M_2 = 16.$$

Hence the gas is CH_4.

8. At what temperature will the rms speed of oxygen molecule will be sufficient for escaping from earth? (Ve= 11.2km/s, m=2.76x10^{-26}kg and k=1.38x10^{-23}J/K)

1) $T = -91.2$ 2) $9.36\times10^4 K$

3) $0.36\times10^4 K$ 4) $5.36\times10^4 K$

SOLUTION :

If the temperature is T,

according to kinetic theory of gases translational $KE = \frac{3}{2}k_B T$

The oxygen molecule will escape from earth

$$\text{if } \frac{3}{2}kT > \frac{1}{2}mv_e^2$$

$$\text{i.e. } T > \frac{mv_e^2}{3k}$$

$$T > \frac{2.76\times10^{-26}\times\left(11.2\times10^3\right)^2}{3\times1.38\times10^{-23}}$$

$$T > 8.36\times10^4 K$$

9. If the density of hydrogen at STP is $0.09 kgm^{-3}$, calculate the rms velocity of its molecules at $0^0 C$.

1) $2.84\times10^3 ms^{-1}$ 2) $1.84\times10^3 ms^{-1}$

3) $0.84\times10^3 ms^{-1}$ 4) $2\times10^3 ms^{-1}$

SOLUTION :

$$P = \frac{1}{3}\rho\left(\text{v}_{\text{rms}}\right)^2$$

$$\therefore v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Here P=76cm of Hg $= 1.013 \times 10^5\, Nm^{-2}$

$$\rho = 0.09 kg\, m^{-3}$$

$$; v_{rms} = \sqrt{\frac{(3)(1.013\times10^5)}{0.09}} = 1.84\times10^3\, ms^{-1}$$

10. At what temperature is the root mean square speed of oxygen molecules equal to the r.m.s speed of carbon dioxide molecules at -23^{0}C, molecular weight of oxygen =32 and that of carbon dioxide = 44.

1) +91.2^{0}C 2) -91.2^{0}C 3) +112.2^{0}C 4) -112.2^{0}C

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V'_{rms} = \sqrt{\frac{3RT'}{M}} = \sqrt{\frac{3R\times250}{44}}$$

$$V_{rms} = V'_{rms}$$

$$\sqrt{\frac{3RT}{32}} = \sqrt{\frac{3R\times250}{44}}$$

$$T = -91.2\ {}^0\mathrm{C}$$

11. Your are given the following group of particles, n_i represents the number of molecules with speed v_i

n_i	2	4	8	6	3
$v_i\,(ms^{-1})$	1.0	2.0	3.0	4.0	5.0

most probable speed is .....

SOLUTION :

By definition,velocity belongs to more molecules is most probable speed=3.0 ms^{-1}.

12. If the density of hydrogen at STP is $0.09 kgm^{-3}$, calculate the rms velocity of its molecules at $0^0 C$.

SOLUTION :

$$P = \frac{1}{3}\rho(v_{rms})^2$$

$\therefore v_{rms} = \sqrt{\frac{3P}{\rho}}$

Here P=76cm of Hg $= 1.013 \times 10^5\ Nm^{-2}$

$\rho = 0.09 kg\, m^{-3}$;

$$v_{rms} = \sqrt{\frac{(3)(1.013\times10^5)}{0.09}}$$

$$v_{rms} = 1.84\times10^3 ms^{-1}$$

13. When the temperature of a gas is raised from $27°C$ to $90°C$, the percentage increase in the *r.m.s.* velocity of the molecules will be

(a) 10% (b) 15% (c) 20% (d) 17.5%

SOLUTION :

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{(273+90)}{(273+30)}} = 1.1$$

$$\%\ \text{increase} = \left(\frac{v_2}{v_1} - 1\right)\times100 = 0.1\times100 = 10\%$$

14. The r.m.s speed of oxygen molecule (O_2) at a certain temperature T is V. If on increasing the temperature of the oxygen gas to 2T, the oxygen molecules dissociate into atomic oxygen, find the speed of the oxygen atom

1) 2V 2) V 3) V/2 4) 3V

SOLUTION :

$$V_{rms} = \sqrt{\frac{3KT}{m}}$$

$$V'_{rms} = \sqrt{\frac{3K(2T)}{m/2}}$$

$$V'_{rms} = \sqrt{\frac{3K4T}{m}}$$

$$V'_{rms} = 2V_{rms}$$

15. Calculate the temperature at which root mean square velocity of SO_2 gas molecules is same as that of O_2 **molecules at 127^0C. Molecular weights of O_2 and SO_2 are 32 and 64 respectively**

1) 527^0C 2) 800^0C 3) 500^0c 4) 627^0c

SOLUTION :

root mean square velocity of SO_2 $V_{rms} = \sqrt{\frac{3RT}{M}}$

root mean square velocity of O_2 $V_{rms} = \sqrt{\frac{3RT'}{M'}}$

$$V_{rms} = V'_{rms}$$

$$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT'}{M'}}$$

$$T' = 527^0C$$

16. If three molecules have velocities 0.5, 1 and 2 kms^{-1} respectively, calculate the relation between the root mean square speed and average speed

1) $V_{rms} = V_{avg}/2$ 2) $V_{rms} = V_{avg}$ 3) $V_{rms} = 1.134\ V_{avg}$ 4) $V_{rms} = 2V_{avg}$

SOLUTION :

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2}{3}} = 1.323$$

$$V_{avg} = \frac{V_1 + V_2 + V_3}{3} = 1.67 km/s$$

$$V_{rms} = 1.134\ V_{avg}$$

17. If the *r.m.s.* velocity of a gas at a given temperature (Kelvin scale) is 300 *m/sec*. What will be the *r.m.s.* velocity of a gas having twice the molecular weight and half the temperature on Kelvin scale =

(a) 300 *m/sec* (b) 600 *m/sec* (c) 75 *m/sec* (d) 150 *m/sec*

SOLUTION :

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{\frac{T}{M}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{M_1}{M_2} \times \frac{T_2}{T_1}}$$

$$= \sqrt{\frac{1}{2} \times \frac{1}{2}}$$

$$v_2 = \frac{v_1}{2} = \frac{300}{2} = 150 m/sec$$

18. At what temperature is the rms speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20^0C? Atomic mass of argon $= 39.9u$ and that of helium =4.0u

SOLUTION :

Given that $(v_{rms})_{Ar} = (v_{rms})_{He}$ $\sqrt{\frac{3RT_{ar}}{M_{ar}}} = \sqrt{\frac{3RT_{He}}{M_{He}}}$

$$;\ \sqrt{\frac{3RT_{ar}}{40}}=\sqrt{\frac{3R(253)}{4}}$$

$$\frac{T}{40}=\frac{253}{4}$$

$$T=2530K$$

$$T=2530-273$$

$$T=2257^{0}C$$

19. Two vessels having equal volume contain molecular hydrogen at one atmosphere, and helium at two atmospheres respectively. What is the ratio of rms speeds of hydrogen molecule to that if helium molecule, if both the samples are at the same temperature

SOLUTION :

According to kinetic theory of gases,

$$v_{rms}=\sqrt{\frac{3RT}{M}}$$

$$\frac{(v_{rms})_H}{(v_{rms})_{He}}=\sqrt{\frac{M_{He}}{M_H}}$$

$$v_H=\sqrt{(4/2)}\ v_{He}$$

$$v_H=\sqrt{2}\left(v_{He}\right)$$

20. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic)

(a) Do the vessels contain equal number of respective molecules?

(b) Is the root mean square speed of molecules the same in the three cases ? If not, in which case is v_{rms} largest?

SOLUTION :

(a) Yes, the vessels contain equal number of respective molecules. It is because, equal volume of all the gases under same temperature and pressure contain equal number of molecules (Avogadro's hypothesis)

(b) The root mean square speed is given by $v_{rms}=\sqrt{\frac{3kT}{m}}\Rightarrow v_{rms}\propto\frac{1}{\sqrt{m}}$

Since v_{rms} .depends upon mass of the molecule of the gas, it will not be same in the three cases.

Since v_{rms} is inversely proportional to the square root of the mass of the molecules of the gas, it will be largest for the gas, whose molecules are lightest. Therefore v_{rms} .will be largest for neon gas.

DEGREE OF FREEDOM (f)

- The number of possible ways in which a system can possess internal energy is called degrees of freedom.

OR

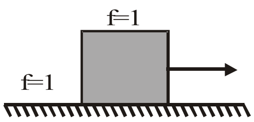
- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.

OR

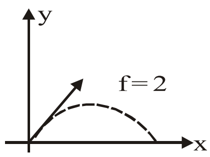
- The number of independent coordinates required to specify the dynamical state of a system is called it's degrees of freedom.

For example

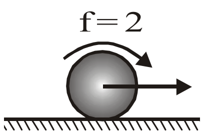
(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one transistional degree of freedom.



(b) The projectile has two degrees of freedom becomes it is confined to move in a plane and so it has two translational degrees of freedom.



(c) The sphere has two degrees of freedom one rotational and another translational. Similarly a particle free to move in space will have three translational degrees of freedom.



Note : In pure rolling sphere has one degree of freedom as KE = $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = \frac{7}{10}mv^2$

- The degrees of freedom are of three types :

(i) Translational degrees of freedom :

The maximum number of translational degrees of freedom can be three.

These are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2.$$

(ii) Rotational degrees of freedom :

The maximum number of rotational degrees of freedom can be three. The number of degrees of freedom in this case depends on the structure of the molecule.

These are $\frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2, \frac{1}{2}I_z\omega_z^2$

(iii) Vibrational degrees of freedom:

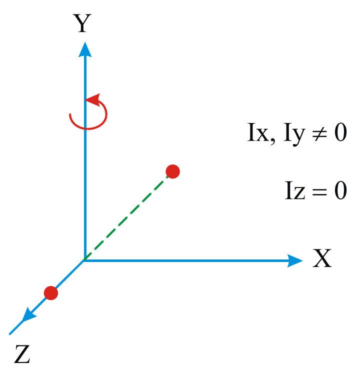
Their numbers depend on atoms in the molecule and their arrangement. These degrees of freedom are considered at a very high temperature.

Note : At room temperature only translational and rotational degrees of freedom are taken into account.

1. **Monoatomic gas:** The degrees of freedom of monoatomic gas molecules are due to three independent translational motions along x, y and z axis. The degrees of freedom are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2.$$

2. **Diatomic (or) Linear polyatomic gas:**



The molecule has three degrees of freedom of translational and two degrees of freedom of rotational these are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2, \frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2.$$

Note : If vibrational degrees of freedom are taken into account, then total number of degrees of freedom of diatomic molecule becomes 7 at high temperature. These are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2, \frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2, \frac{1}{2}\mu v^2, \frac{1}{2}kr^2.$$

Here $\frac{1}{2}\mu v^2$ corresponds to kinetic energy of vibration (μ is the reduced mass) and $\frac{1}{2}kr^2$ corresponds to potential energy of vibration (k is the force constant, r is the separation between the atoms)

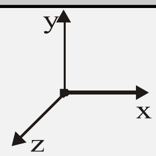
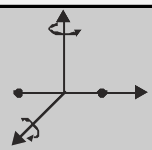
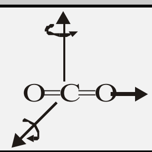
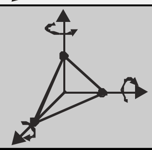
3. Triatomic or non linear polyatomic gas:

The molecule of polyatomic gases like CO_2, H_2O, NH_3, CH_4, etc. has more than two atoms.

It has 3 translational and 3 rotational degrees of freedom as shown in figure Apart from this, if such a molecule has v vibrational modes, it will have additional 2v vibrational degrees of freedom, each vibrational mode contributing 2 vibrational degrees of freedom.

Thus, total number of degrees of freedom of a polyatomic gas molecule,

i.e., $f = 3 + 3 + 2v = (6 + 2v)$

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	
Triatomic (linear) **Ex.** CO_2, C_2H_2	3	2	5	
Triatomic (Non–linear) or Polyatomic **Ex.** H_2O, NH_3, CH_4	3	3	6	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Law of equipartition of energy :

According to this law, for a system in thermal equilibrium, the total energy of a dynamic system is equally distributed among its various degrees of freedom.

The energy associated with each degree of freedom is $\frac{1}{2}K_BT$ per molecule

or $\frac{1}{2}RT$ per mole.

For a molecule with f degrees of freedom

Energy related with each degree of freedom = $\frac{1}{2}$kT   Energy related with all degree of freedom = $\frac{f}{2}$kT

Energy per mole $U = f \times \frac{RT}{2} = \frac{fRT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{fR}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{fR}{2} + R = R\left(\frac{f}{2}+1\right)$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \left(1+\frac{2}{f}\right)$

Note: $\gamma_{poly} = \frac{4+f_{vib}}{3+f_{vib}}$,

f_{vib} = number of modes of vibrations (at high temperatures)

C_v, C_p and γ for different gases:

Monoatomic gas: Degrees of freedom f = 3

Kinetic energy per mole $E = U = 3\times\frac{RT}{2} = \frac{3RT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{3R}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{5R}{2}$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{5R/2}{3R/2} = \frac{5}{3} = 1.66$

Diatomic Gas : Degrees of freedom f = 5

Kinetic energy per mole $U = 5 \times \frac{RT}{2} = \frac{5RT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{5R}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{7R}{2}$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{7R/2}{5R/2} = \frac{7}{5} = 1.4$

Triatomic (or) polyatomic Gas:

Degrees of freedom f = 6

Kinetic energy per mole $U = 6 \times \frac{RT}{2} = 3RT$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = 3R$

Molar specific heat at constant pressure $C_p = C_v + R = 4R$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = \frac{4}{3} = 1.33$

Specific heat capacity of solids

A solid consists of a regular array of atoms in which each of the atoms has a fixed equilibrium position. As such in a solid, an atom has no translational or rotational degrees of freedom. It has only three vibrational modes along three mutually perpendicular directions. Since each vibrational mode contributes two vibrational degrees of freedom, an atom in a solid has six vibrational degrees of freedom. Further, as

energy associated per degree of freedom per atom is $\frac{1}{2}k_BT$,

total inernal energy per mole (N_A atoms) of solid, i.e.,

$$U = 6\left(\frac{1}{2}k_BT\right)N_A = 3RT \quad \left(as\, k_B = \frac{R}{N_A}\right)$$

Thus, $C = \frac{dU}{dT} = \frac{d}{dT}(3RT)$ or

$C = 3R \approx 6cal/mol\,C^o \approx 25J/mol\ K$

The specific heat capacity at high temperatures (usually above 300K) is the same for all solids and is approximately equal to 3R or 6 cal/mol C° or 25 J/mol K.

Specific heat capacity of Water

A water molecule (H_2O) consists of 2 hydrogen atoms and 1 oxygen atom. Treating water like a solid, each atom in its molecule has 6 degrees of freedom and as such its each molecule has $3 \times 6 = 18$ degrees of freedom. According to the law of equipartition of energy,

energy associated per molecule per degree of freedom $= \frac{1}{2}k_BT$

internal energy associated per mole of water, $U = 18\left(\frac{1}{2}k_BT\right)N_A = 9RT$

Tus, $C = \frac{dU}{dT} = \frac{d}{dT}(9RT) = 9R$

PROBLEMS

01. Each molecule of a gas has *f* degrees of freedom. The ratio $\frac{C_p}{C_V} = \gamma$ for the gas is

1) $1+\frac{f}{2}$ 2) $1+\frac{1}{f}$ 3) $1+\frac{2}{f}$ 4) $1+\frac{(f-1)}{3}$

SOLUTION :

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{fR}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{fR}{2} + R = R\left(\frac{f}{2}+1\right)$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \left(1+\frac{2}{f}\right)$

2 A cylinder of fixed capacity 44.8 litres contains helium gas at standard temperature and pressure. What is the amount of heat needed to raise the temperature of the gas in the cylinder by 15.0 °C ? (R = 8.31 J mo1^{-1} K^{-1}).

SOLUTION :

1 mole of any gas occupies 22.4 litres

so 44.8 litres of helium contains 2 moles.

Since the volume of cylinder is fixed , the heat required is C_V

Here Helium is monoatomic gas $n = 3/2$

$\therefore C_V = \frac{3}{2}R$

so heat required = $nC_V dT = 2\times 1.5R \times 15 = 45R = 3745$

3.. The ratio of two specific heats $\frac{C_P}{C_V}$ of CO is

(a) 1.33 (b) 1.40 (c) 1.29 (d) 1.66

SOLUTION :

Co is diatomic gas, for diatomic gas

$C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$

$\gamma_{di} = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = 1.4$

4. Considering the gases to be ideal, the value of $\gamma = \frac{C_P}{C_V}$ for a gaseous mixture consisting of = 3 moles of carbon dioxide and 2 moles of oxygen will be $(\gamma_{O_2} = 1.4, \gamma_{CO_2} = 1.3)$

(a) 1.37 (b) 1.34 (c) 1.55 (d) 1.63

SOLUTION :

$$\gamma_{\text{mix}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}} = \frac{\frac{3\times 1.3}{(1.3-1)} + \frac{2\times 1.4}{(1.4-1)}}{\frac{3}{(1.3-1)} + \frac{2}{(1.4-1)}} = 1.33$$

5. One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is

(a) 8 (b) $\frac{3}{2}R$ (c) $2R$ (d) $2.5\,R$

SOLUTION :

$$(C_V)_{mix} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2}$$

$$= \frac{1\times\frac{3}{2}R + 1\times\frac{5}{2}R}{1+1} = 2R$$

$$\left((C_V)_{mono} = \frac{3}{2}R,\ (C_V)_{di} = \frac{5}{2}R\right)$$

6. A gas has molar heat capacity $C = 37.55\,\text{J mole}^{-1}\text{K}^{-1}$, in the process PT=constant. Find the number of degrees of freedom of the molecules of the gas.

SOLUTION :

PT = K(constant) ...... (i)

But $PV = RT \Rightarrow P = \frac{RT}{V}$ $\therefore \frac{RT}{V} \times T = K$

$$\frac{dV}{dT} = \frac{2R}{P} \left(\because PT = K\right) \text{ ...... (ii)}$$

From first law of thermodynamics

$$dQ = dU + dW$$

$$;\ CdT = C_v dT + PdV$$

$$C = C_v + P\left(\frac{dV}{dT}\right) \text{ as, } C = C_v + P\frac{dV}{dT}$$

$$C = C_v + P\left(\frac{2R}{P}\right) \Rightarrow C - C_v = 2R$$

$\Rightarrow C_v = C - 2R$ .... (iii)

But $C_v = \frac{fR}{2}$ .... (iv) from (iii) and (iv)

$$\frac{fR}{2} = C - 2R \Rightarrow f = \frac{2(C-2R)}{R}$$

$$= \frac{2(37.55 - 2\times 8.3)}{8.3} = 5$$

7. What is the total kinetic energy of 2g of Nitrogen gas at temperature 300 K.

SOLUTION :

Given, mass of Nitrogen gas= 2 g

molecular waight $M_W = 28$, gm

Temperature $T = 300K$,

$R = 8.31\times 10^7 erg\ mol^{-1}K^{-1}$

Kinetic energy of 2g of $N_2 = 2\times\frac{3}{2}\frac{RT}{M_W} = \frac{3RT}{M_W}$

$$= \frac{3\times 8.31\times 10^7\times 300}{28} erg = 267\times 10^7 erg$$

8. Calculate the total kinetic energy of one kilo mole of Oxygen gas at $27^0 C$

SOLUTION :

oxygen is a diatomic gas, its molecules have 5 degrees of freedom.

Therefore, the total kinetic energy of a molecule of Oxygen is $\frac{5}{2}k_B T$.

As, 1kg-mole of Oxygen has N molecules,
the total kinetic energy of one kg-mole of Oxygen at temperature T is

$$\frac{5}{2}RT = \frac{5}{2}\times 8.31\times 10^3\times 300$$

$$= 6.23\times 10^6 Joule/Kg - mole$$

9. Estimate the average thermal energy of a helium at (i) room temperature $(27^0 C)$ (ii) the temperature on the surface of the sun (6,000 K) and (iii) the temperature of $10^7 K$. Given, $k = 1.38\times 10^{-23} J K^{-1}$

SOLUTION :

Here, $k = 1.38 \times 10^{-23} JK^{-1}$

(i) $T = 27 + 273 = 300K$

$\therefore \overline{K.E} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$,

$= 6.21 \times 10^{-21} J$

(ii) T=6,000 K,

$\therefore \overline{K.E.} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6,000$, $= 1.242 \times 10^{-19} J$

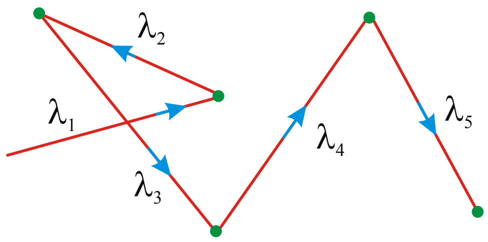
(iii) $T = 10^7 K$

$\therefore \overline{K.E.} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6,000$,

$= 2.07 \times 10^{-16} J$

Mean free path (λ):

→ The mean free path of a gas molecule may be defined as the average distance travelled by the molecule between two successive collisions.



Let $\lambda_1, \lambda_2, ...\lambda_n$ be the distance travelled by a gas molecule during n collisions, then the mean free path of gas molecule is given by

$$\lambda = \frac{\lambda_1 + \lambda_2 + ... + \lambda_n}{n}$$

During the collision, a molecule of a gas moves in a straight line with constant velocity. The statistical study of heat gives the mean free path as following

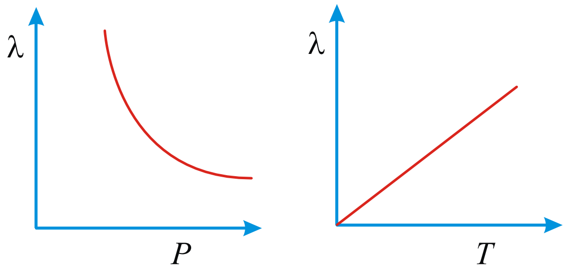
$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

Where d is the diameter of molecule, n is the number of molecules per unit volume.

Collision frequency $f = \frac{V_{rms}}{\lambda}$

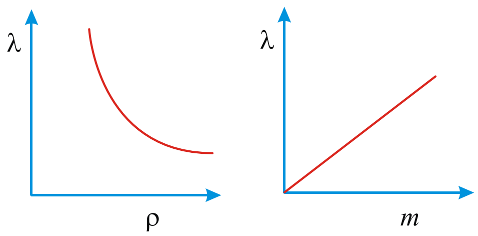
→ Mean free path depends on nature of molecule and with increase in n (number density) it decreases.

→ Here $n = \frac{N}{V} = \frac{P}{K_B T}$ Hence, $\lambda = \frac{1}{\sqrt{2}} \frac{K_B T}{\pi d^2 P}$



$$\rightarrow \quad \lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{m}{\sqrt{2}\pi (m.n) d^2} = \frac{m}{\sqrt{2}\pi d^2 \rho}$$

$\rightarrow$ As $\lambda \propto \frac{1}{\rho}$ and $\lambda \propto m$,



Brownian motion :

It provides a direct evidence for the existence of molecules and their motion. The zig-zag motion of gas molecules is Brownian motion because it occurs due to random collision of molecules. But this motion cannot be seen. However, the zig-zag motion of pollen grains $\left(\approx 10^{-15} m\right)$ can be seen under a microscope

PROBLEMS

1. Estimate the mean free path for a water molecule in water vapour at 373K. The diameter of the molecule is $2 \times 10^{-10} m$, and at STP number of molecules per unit volume is $2.7 \times 10^{25} m^{-3}$

SOLUTION :

The number density(n) is inversely proportional to absolute temperature

$$\therefore n \alpha \frac{1}{T}$$

$$\Rightarrow \frac{n_{373}}{n_{273}} = \frac{273}{373}$$

$$n_{373} = n_{273} \times \frac{273}{373}$$

$$n_{373} = 2.7 \times 10^{25} \times \frac{273}{373} = 2 \times 10^{25} m^{-3}$$

Given d= $2 \times 10^{-10} m$

Hence , mean free path $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$$= \frac{1}{\sqrt{2} \times 3.14 \times \left(2 \times 10^{-10}\right)^2 \times 2 \times 10^{25}} = 4 \times 10^{-7} m$$

2. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2 atm and temperature $17^0 C$. Take the radius of nitrogen molecule to be $1A^0$. (Molecular mass of nitrogen = 28u)

SOLUTION :

Here, $P = 2 \times 1.013 \times 10^5 = 2.026 \times 10^5 \, Nm^{-2}$;

$T = 17 + 273 = 290K; M = 28g = 28 \times 10^{-3} kg$

molecular diameter

. $d = 1 \times 2 = 2A^0 = 2 \times 10^{-10} m;$

Now , mass of a nitrogen molecule,

$$m = \frac{M}{N} = \frac{28 \times 10^{-3}}{6.02 \times 10^{23}} = 4.65 \times 10^{-26} kg$$

Also, volume occupuied by the nitrogen gas

$$V = \frac{RT}{P} = \frac{8.31 \times 290}{2.026 \times 10^5} = 1.19 \times 10^{-2} m^3$$

Therefore, density of the nitrogen gas,

$$\rho = \frac{M}{V} = \frac{28 \times 10^{-1}}{1.19 \times 10^{-2}} = 2.353 kgm^{-1}$$

Now, $\vec{\lambda} = \frac{m}{\sqrt{2}\pi d^2 \rho}$

$$= \frac{4.65 \times 10^{-26}}{\sqrt{2}\pi \times \left(2 \times 10^{-10}\right)^2 \times 2.353}$$

$$= 1.11 \times 10^{-7} m$$

Now, $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 2.026 \times 10^5}{2.353}}$

$$= 508.24 ms^{-1}$$

Therefore, collision frequency,

$$f=\frac{v_{rms}}{\lambda}=\frac{508.24}{1.11\times10^{-7}}=4.58\times10^{9}S^{-1}$$

3. The collision frequency of nitrogen molecule in a cylinder containing nitrogen molecule in a cylinder containing at 2.0 atm pressure and temperature $17^{0}C$. (Take radius of a nitrogen molecule is $1.0A^{0}$)

SOLUTION :

mean free path $\lambda=\frac{1}{\sqrt{2}\pi d^{2}\lambda}$

$$\lambda=\frac{K_BT}{\sqrt{2}\pi d^{2}p}\left(p=nK_BT\right)$$

$$\lambda=\frac{\left(1.38\times10^{-23}\right)(290)}{(1.414)(3.14)\left(2\times10^{-10}\right)\left(2.026\times10^{5}\right)}=1.1\times10-7$$

$$V_{rms}=\sqrt{\frac{3K_BT}{m}}=\sqrt{\frac{3\times1.38^{-23}\times290}{28\times1.66\times10^{-27}}}=5.1\times10^{2}m/s$$

$\therefore$ collision frequency

$$f=\frac{V_{rms}}{\lambda}=\frac{5.1\times10^{2}}{1.1\times10^{-7}}=4.6\times10^{9}s^{-1}$$

4. Estimate the mean free path for a water molecule in water vapour at 373K. The diameter of the molecule is $2\times10^{-10}m$, and at STP number of molecules per unit volume is $2.7\times10^{25}m^{-3}$

SOLUTION :

The number density(n) is inversely proportional to absolute temperature

$$\therefore n\alpha\frac{1}{T}$$

$$\Rightarrow\frac{n_{373}}{n_{273}}=\frac{273}{373}$$

$$\Rightarrow n_{373}=n_{273}\times\frac{273}{373}$$

$$n_{373}=2.7\times10^{25}\times\frac{273}{373}=2\times10^{25}m^{-3}$$

Given d= $2\times10^{-10}m$;

Hence, mean free path $\lambda=\frac{1}{\sqrt{2}\pi d^{2}n}=2.81\times10^{-7}m$

5. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of system is

1) 5RT 2) 11RT 3) 3RT 4) 7RT

SOLUTION :

Total internal energy of system

$$= U_{oxygen} + U_{argon} = \mu_1 \frac{f_1}{2} RT + \mu_2 \frac{f_2}{2} RT$$

$$= 2\frac{5}{2}RT + 4\frac{3}{2}RT = 5RT + 6RT = 11RT$$

[As $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon)]

6. A container has 1 mole of nitrogen at 27^0C. The pressure in side the container is 2 atmosphere. Assuming the molecules move with root mean square speed. Then find The number of collisions per second which the molecules make with unit area of the container wall.

SOLUTION :

a) The number of molecules n present per unit volume at pressure P and temperature T.

$$n = \frac{P}{kT} = \frac{2\times 1.05\times 10^5}{1.38\times 10^{-23}\times 300} = 5.07\times 10^{25}$$

$$\upsilon_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_A kT}{M}}$$

$$= \sqrt{\frac{3\times 6.02\times 10^{23}\times 1.38\times 10^{-23}\times 300}{28\times 10^{-3}}} = 516.75 ms^{-1}$$

Number of collisions/m^2 = $\frac{1}{6}\times n\times \upsilon_{rms}$

$$= \frac{1}{6}\times 5.07\times 10^{25}\times 516.75 = 4.37\times 10^{27}$$

7. Each molecule of nitrogen gas heated in a vessel to a temperature of 5000 K has an aver age energy E_1. Some molecules of the gas escape into atmosphere at 300 K. Due to collision with air molecules, average kinetic energy of the nitrogen molecule changes to E_2.
Find the ratio E_1/E_2.

1) 7/6 2) 3/7 3) 6/7 4) 7/3

SOLUTION :

At T = 5000 K (HIGH TEMPERATURE)

f = 7

$$E_1 = f \times \frac{1}{2} KT = 7 \times \frac{1}{2} K \times 5000$$

$= 17{,}500$ K

At T = 300 K (LOW TEMPERATURE)

f = 5

$$E_2 = f \times \frac{1}{2} KT == 5 \times \frac{1}{2} K \times 300$$

$= 750$ K

$E_1 / E_2 = 7/3$

8. A vessel contains two non-reactive gases neon and oxygen . The ratio of their partial pressures is 3:2. Estimate the ratio of number of molecules Atomic mass of Ne=20.2 u, molecular mass of $O_2 = 32.0u$

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{3}{5}$ 4) $\frac{5}{2}$

SOLUTION :

Since V and T are CONSTANTS

we have $P_1V = \mu_1 RT$ *and* $P_2V = \mu_2 RT$

$\therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2}$ where 1 and 2 refer to neon and oxygen respectively

Given $\frac{P_1}{P_2} = \frac{3}{2}$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

By definition $\mu_1 = \frac{N_1}{N_A}$ *and* $\mu_2 = \frac{N_2}{N_A}$

where N_1 and N_2 are the number of molecules of 1 and 2

$\therefore$ The ratio of number of molecules

$$\frac{N_1}{N_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

9. A vessel contains two non-reactive gases neon and oxygen . The ratio of their partial pressures is 3:2. Find mass density of neon and oxygen in the vessel. Atomic mass of Ne=20.2 u, molecular mass of $O_2 = 32.0u$

SOLUTION :

Since V and T are CONSTANTS

we have $P_1V = \mu_1 RT \; and \; P_2V = \mu_2 RT$

$$\therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2}$$

where 1 and 2 refer to neon and oxygen respectively

Given $\frac{P_1}{P_2} = \frac{3}{2}$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

If $\rho_1 \; and \; \rho_2$ are mass densities of 1 and 2 respectively,

we have $\frac{\rho_1}{\rho_2} = \frac{m_1/V}{m_2/V} = \frac{m_1}{m_2}$

But we can also write $\mu_1 = \frac{m_1}{M_1} \; and \; \mu_2 = \frac{m_2}{M_2}$

$$\therefore \frac{m_1}{m_2} = \frac{\mu_1}{\mu_2} \times \frac{M_1}{M_2} = \frac{3}{2} \times \frac{20.2}{32.0} = 0.947$$

10: A vessel is filled with a gas at a pressure of 76cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at same temperature. Find out the resultant pressure of the gas.

SOLUTION :

According kinetic theory of gases,

pressure $P = \frac{1}{3}\frac{m}{V}\left(v_{rms}\right)^2$.

Given, T= constant

so v_{rms} = constant.

also V=onstant

$\therefore P \propto m$

$$\frac{P_2}{P_1} = \frac{m_2}{m_1}$$

; $\frac{P_2}{76} = \frac{\left(m_1 + 50m_1\right)}{m_1} = \frac{3}{2}$

$P_2 = \frac{3}{2} \times 76 = 114cm$ of Hg

11. The total number of air molecules in a room of capacity 20 m^3 at a temperature of 27^0C and

1 atm pressure is.

SOLUTION :

Given ;

Volume (V) = 20 m^3

Temperature (T) = 27+273=300K

Pressure (P) = 1 atm = $1.01\times10^5\,Pa$

Total number of air molecules $N = \frac{PV}{K_B T}$

$$N = \frac{1.01\times10^5\times20}{1.38\times10^{-23}\times300}$$

$$N = 4.87\times10^{26}$$

12. At what temperature, the mean kinetic energy of O_2 will be the same for H_2 molecules at $-73°C$

(a) $127°C$ (b) $527°C$ (c) $-73°C$ (d) $-173°C$

SOLUTION :

Mean kinetic energy of molecule depends upon temperature only. For O_2 it is same as that of H_2 at the same temperature of $-73°C$.

13. A cylinder of fixed capacity 22.4 litres contains helium gas at standard temperature and pressure. What is the amount of heat needed to raise the temperature of the gas in the cylinder by 30 °C ? (R = 8.31 J mo1⁻¹ K⁻¹).

SOLUTION :

1 mole of any gas occupies 22.4 litres.

Since the volume of cylinder is fixed , the heat required $= nC_V dT$

Here Helium is monoatomic gas so $C_V = \frac{3}{2}R$

Heat required $= 1\times1.5R\times30 = 45R = 374.5\,J$

14. A pressure cooker contains air at 1 *atm* and 30°*C*. If the safety value of the cooler blows when the inside pressure ≥ 3 *atm*, then the maximum temperature of the air, inside the cooker can be

(a) $90°C$ (b) $636°C$ (c) $909°C$ (d) $363°C$

SOLUTION :

Since volume is constant,Hence $\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$$\frac{1}{3} = \frac{(273+30)}{T_2}$$

$$T_2 = 909\,K = 636\,°C$$

15. **A container has 1 mole of nitrogen at 27^{0}C. The pressure in side the container is 2 atmosphere. Assuming the molecules move with root mean square speed. If the container is made thermally insulated and moves with constant speed υ_0. If it stops suddenly, the process results in the rise of the temperature of the gas by 1^{0}C. Calculate the speed υ_0.**

SOLUTION :

Kinetic energy $= \frac{1}{2}m\upsilon_0^2$

Heat energy gained $= C_v\Delta T = C_v \times 1 = C_v$ and $C_P - C_v = R$

$$\frac{C_P}{V_v} - 1 = \frac{R}{C_v}$$

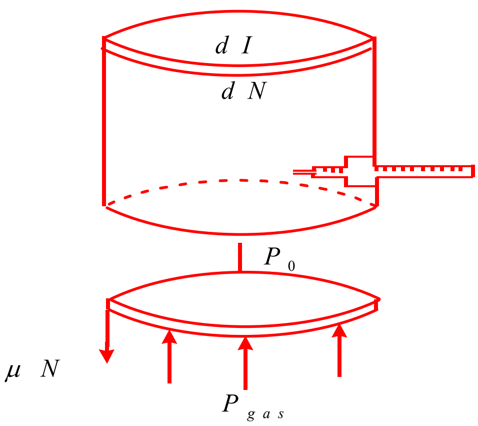
$$\gamma - 1 = \frac{R}{C_v} \quad i.e.\ C_v = \frac{R}{\gamma - 1}$$

$$C_v = \frac{1}{2}mv_0^2 \qquad \frac{R}{\gamma - 1} = \frac{1}{2}mv_0^2$$

which gives $V_0 = \sqrt{\frac{2R}{m(\gamma - 1)}}$

$$= \sqrt{\frac{2 \times 8.31}{28 \times 10^{-3} \times (1.4 - 1)}} = 38.5ms^{-1}$$

16. **Figure shows a cylindrical tube of radius r and length l, fitted with a cork. The friction coefficient between the cork and the tube is μ. The tube contains an ideal gas at temperature T, and atmosphereic pressure P$_0$. The tube is slowly heated; the cork pipe out when temperature is doubled. What is normal force per unit length exerted by the cork on the periphery of tube ? Assume uniform temperature throughout gas at any instant.**



SOLUTION :

Since volume of the gas is constant

$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = 2P_i = 2P_0$$

The forces acting on the cork are shown in the figure, in equilibrium.

$$P_0 \times A + \mu N = 2P_0 A$$

$$N = \frac{P_0 A}{\mu}$$

N is the total normal force exerted by the tube on the cork; hence contact force per unit length is

$$\frac{dN}{dl} = \frac{N}{2\pi r} = \frac{P_0 A}{2\pi \mu r}$$

17. One mole of an ideal monatomic gas requires 210 J heat to raise the temperature by 10 K, when heated at constant temperature. If the same gas is heated at constant volume to raise the temperature by 10 K then heat required is

(a) 238 J (b) 126 J (c) 210 J (d) 350 J

SOLUTION :

$(\Delta Q)_P = \mu C_P \Delta T$ and $(\Delta Q)_V = \mu C_V \Delta T$

$$\frac{(\Delta Q)_V}{(\Delta Q)_P} = \frac{C_V}{C_P} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$$

$$\left[\because (C_V)_{mono} = \frac{3}{2}R, (C_P)_{mono} = \frac{5}{2}R\right]$$

$$(\Delta Q)_V = \frac{3}{5} \times (\Delta Q)_P = \frac{3}{5} \times 210 = 126\ J$$

18. A nitrogen molecule at the surface of earth happens to have rms speed for that gas at 0°C. If it were to go straight up without colliding with other molecules, how high would it rise? (Mass of nitrogen molecule ,m = 4.65 $\times$ 10^{-26} kg, R = 8.3 J/mol/K)

SOLUTION :

The molecule goes to a height h till its entire K.E is converted into P.E

$$\therefore mgh = \frac{1}{2}m\left(v_{rms}\right)^2 \ ;$$

$$h = \frac{\left(v_{rms}\right)^2}{2g} = \frac{3RT}{M.2g}$$

$$h = \frac{3 \times 8.31 \times 273}{28 \times 10^{-3} \times 2 \times 9.8}$$

19. Certain amount of an ideal gas are contained in a closed vessel. The vessel is moving with a constant velocity *v*. The molecular mass of gas is *M*. The rise in temperature of the gas when the vessel is suddenly stopped is $(\gamma = C_P / C_V)$

(a) $\frac{Mv^2}{2R(\gamma+1)}$ (b) $\frac{Mv^2(\gamma-1)}{2R}$ (c) $\frac{Mv^2}{2R(\gamma+1)}$ (d) $\frac{Mv^2}{2R(\gamma+1)}$

SOLUTION :

If *m* is the total mass of the gas then its kinetic energy $= \frac{1}{2}mv^2$

When the vessel is suddenly stopped then total kinetic energy will increase the temperature of the gas.

Hence $\frac{1}{2}mv^2 = \mu C_v \Delta T = \frac{m}{M} C_v \Delta T$ [As $C_v = \frac{R}{\gamma - 1}$]

$$\frac{m}{M}\frac{R}{\gamma-1}\Delta T = \frac{1}{2}mv^2$$

$$\Delta T = \frac{Mv^2(\gamma-1)}{2R}.$$

20. The energy of a *gas/litre* is 300 joules, then its pressure will be

(a) $3\times10^5\ N/m^2$ (b) $6\times10^5\ N/m^2$

(c) $10^5\ N/m^2$ (d) $2\times10^5\ N/m^2$

SOLUTION :

Energy $= 300\ J/litre = 300\times10^3\ J/m^3$

$$P = \frac{2}{3}E = \frac{2\times300\times10^3}{3} = 2\times10^5\ N/m^2$$

21. Two thermally insulated bulbs, filled with air and connected by a short tube containing a valve, initially closed. The pressures, volumes and temperatures in the two vessels are P_1,V_1,T_1 and P_2,V_2,T_2 respectively. Find the P, T values after opening the valve.

SOLUTION :

When the valve is opened, the air flows from the bulb at higher pressure to the bulb at lower pressure.

In equilibrium both the vessels have the same pressure.=P

Total volume of the air, $V = V_1 + V_2$

After mixing of air, total number of moles, $n = n_1 + n_2$

After mixing of air, total number of moles, $n = n_1 + n_2$ Let the common temperature attained be T.

Hence $P(V_1+V_2)=(n_1+n_2)RT$

$$P = \frac{(n_1+n_2)RT}{(V_1+V_2)} \quad ....(1)$$

The combined system is thermally insulated; hence Q = 0;

system does no mechanical work, since dV = 0, W = pdV = 0.

From first law of thermodynamics, Q = dU + W

Hence dU = 0.

There is no change in internal energy.

The internal energy U of an ideal gas is given by

$$U = nC_{\upsilon}T = \frac{nRT}{(\gamma - 1)} = \frac{PV}{(\gamma - 1)}$$

$$U_{initial} = \frac{n_1 RT_1}{(\gamma - 1)} + \frac{n_2 RT_2}{(\gamma - 1)}$$

$$= \left(\frac{R}{\gamma - 1}\right)(n_1 T_1 + n_2 T_2) \text{.........(2)}$$

$$U_{final} = \frac{(n_1 + n_2)RT}{(\gamma - 1)} \text{..........(3)}$$

$$U_{initial} = U_{final}$$

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2)T\,]$$

$$T = \left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right) \text{.......(4)}$$

$$= \frac{P_1 V_1 + P_2 V_2}{\left[\left(\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}\right)\right]} = \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

From eqn. (1)

$$P = \frac{(n_1 + n_2)RT}{(V_1 + V_2)} = \frac{R}{V_1 + V_2}(n_1 + n_2)T$$

From eqn. (4)

$$(n_1 + n_2)T = n_1 T_1 + n_2 T_2$$

$$P = \frac{R}{(V_1 + V_2)}(n_1 T_1 + n_2 T_2) = \frac{(P_1 V_1 + P_2 V_2)}{(V_1 + V_2)}$$

as $P_1 V_1 = n_1 RT_1$

and $P_2 V_2 = n_2 RT_2$

Maxwell's Law (The distribution of molecular speeds).

a. The v_{rms} gives us a general idea of molecular speeds in a gas at a given temperature. This doesn't mean that the speed of each molecule is v_{rms}.

b. Maxwell derived an equation giving the distribution of molecules in different speeds as follows:

$$dN = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}} dv$$

Where dN = Number of molecules with speeds between v and v+dv,

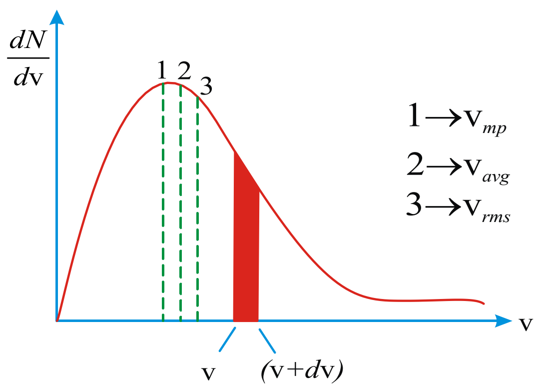
m = mass of the gas molecule

k_B = Boltzmann's constant N = total number of molecules

c. Graph between $\frac{dN}{dv}$ (number of molecules at particular speed) and v (speed of these molecules).

From the graph it is seen that $\frac{dN}{dv}$ is maximum at most probable speed.

$$\Rightarrow \sqrt{\frac{3RT}{M}} > \sqrt{\frac{8RT}{\pi M}} > \sqrt{\frac{2RT}{M}}$$



From the graph

1) The area under the graph represents total number of molecules.

2) The shape of the curve is such that area (shown shaded enclosed by its portion on right side of v_{mp} is more than the area on the left side of v_{mp}. Thus, the number of molecules having speeds less than v_{mp} is less than the number of molecules having speeds more than v_{mp}.

Note: 1.If μ_1 moles of C_{V_1}, μ_2 moles of C_{V_2} ..... are mixed, then by conservation of energy

$$U_1 + U_2 + \ldots\ldots = U$$

$$\mu_1 C_{V_1}\Delta T + \mu_2 C_{V_2}\Delta T + ... = (\mu_1 + \mu_2 + ...)C_{V_{mix}}\Delta T$$

$$\therefore C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2} + ...}{n_1 + n_2 + ...} \text{ and } C_{P_{mix}} = C_{V_{mix}} + R$$

2. If μ_1 moles of γ_1 and μ_2 moles of γ_2 .... are mixed then by conservation of energy we have

$$U_1 + U_2 + ... = U$$

$$\mu_1 C_{V_1}\Delta T + \mu_2 C_{V_2}\Delta T + ... = (\mu_1 + \mu_2 + ...)C_{V_{mix}}\Delta T$$

$$\mu_1\left(\frac{R}{\gamma_1 - 1}\right) + \mu_2\left(\frac{R}{\gamma_2 - 1}\right) + .. = (\mu_1 + \mu_2 + ..)\frac{R}{\gamma_{mix} - 1}$$

$$\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1} + ... = \frac{(\mu_1 + \mu_2 + ...)}{\gamma_{mix} - 1}$$

For two gases $\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1} = \frac{\mu_1 + \mu_2}{\gamma_{mix} - 1}$

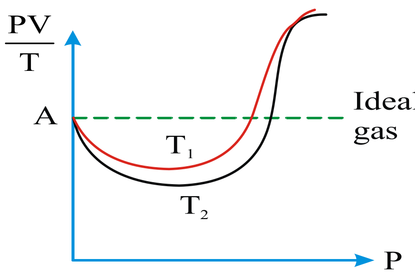
PRACTICE BITS

1. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and . If the average speed of the molecules in vessel A is , that of the molecules in vessel B is , the average speed of the molecules in vessel C is

(a) $(V_1 + V_2)/2$ (b) V_1 (c) $(V_1 V_2)^{1/2}$ (d) $\sqrt{3kT/M}$

KEY:b

2. **Given graph gives variation of PV/T with P for 1gm of oxygen at two different temperatures T_1 and T_2. If density of oxygen is $1.427\, kg/m^3$. The value of PV/T at point A and relation b/w T_1 & T_2 is**



1) 0.256J/K and $T_1 < T_2$ 2) 8.314 J/K and $T_1 > T_2$

3) 8.314 J/K and $T_1 < T_2$ 4) 0.256J/K and $T_1 > T_2$

KEY:4

HINT : $PV = \mu RT = \frac{m}{M}RT \quad \Rightarrow \frac{PV}{T} = \frac{m}{M}R$

Real gas behave as ideal gases at low pressure and high temperature, so, $T_1 > T_2$

03. A gas has volume V and pressure p. The total translational kinetic energy of all the molecules of the gas is

1) $\frac{3}{2}pV$ only if the gas is monoatomic 2) $\frac{3}{2}pV$ only if the gas is diatomic

3) $> \frac{3}{2}pV$ if the gas is diatomic 4) $\frac{3}{2}pV$ **in all cases**

KEY:4

04. Three closed vessels A, B are C are at the same temperature. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is υ_1, that of N_2 molecules in vessel B is υ_2 , the average speed of O_2 molecules is vessel C is

1) $\frac{1}{2}(\upsilon_1 + \upsilon_2)$ 2) υ_1 3) $\sqrt{\upsilon_1 \upsilon_2}$ 4) $\sqrt{\frac{3kT}{M}}$

KEY:2

05. If the pressure in a closed vessel is reduced by drawing out some of the gas, the mean free path of the two molecules

1) Increases 2) decreases 3) Remains unchanged

4) Increases of decrease according to the nature of the gas

KEY:2

6. The temperature of the gas consisting of rigid diatomic molecules is T = 300K. Calculate the angular root mean square velocity of a rotating molecule if its moment of inertia is equal to $I = 2.1 \times 10^{-39}\ g\,cm^2$.

1) $6.3 \times 10^{12}\ rad/\sec$ 2) $6.8 \times 10^{12}\ rad/\sec$

3) $3.6 \times 10^{12}\ rad/\sec$ 4) $3.2 \times 10^{12}\ rad/\sec$

KEY:1

HINT : $\frac{1}{2}I\omega^2 = 2.\frac{1}{2}kT$

KEY:1

07. The temperature of a gas is -68°c. To what temp should it be heated so that the root mean square velocity of the molecules is doubled.

1) 547°c 2) 820°c 3) 1092 K 4) 547 K

KEY:1

8. N Molecules each of mass m of gas A and 2N molecules each of mass 2m of gas B are contained in the same vessel, which are maintained at a temperature T. The mean square velocity of molecules of B type is denoted by v^2 and the mean square of the 'X' component of the velocity of A type is denoted by ω^2, then $\frac{\omega^2}{v^2}$ is

1) 2 2) 1 3) $1/3$ 4) $2/3$

KEY:4

HINT :

Mean kinetic energy of two types of molecules should be equal.

$$\text{so, } \frac{1}{2}m\left(3\omega^2\right)=\frac{1}{2}(2m)\text{v}^2 \Rightarrow \frac{\omega^2}{\text{v}^2}=\frac{2}{3}$$

09. The pressure exerted on the walls of the container by a gas is due to the fact that gas molecules are

1) Losing their kinetic energy at walls 2) Stricking to the energy

3) Changing their moment due to collision with the walls

4) Getting accelerated towards the wall

KEY:3

10. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :

1) 4 RT 2) 15 RT 3) 9 RT 4) 11 RT

KEY:4

11. The figure given below shows the plot of versus P for oxygen gas at two different temperatures. Read the following statements concerning the curves given below

i) The dotted line corresponds to the 'ideal' gas behaviour
ii) $T_1 > T_2$
iii) The value of at the point, where the curves meet on the y-axis is the same for all gases.
Which of the above statement is true ?
1) i only 2) i and ii 3) all of these 4) none of these

KEY:3

12. Three closed vessels A, B and C at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only contains only contains a

mixture of equal quantities of. If the average speed of the molecules in vessel A is and that of the molecules in vessel B is, the average speed of the molecules in vessel C is

1) **2)** **3)** **4)**

KEY:2

13.1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K in figure. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time

1) The pressure on EFGH would be zero
2) The pressure on all the faces will be equal
3) The pressure of EFGH would be double the pressure on ABCD
4) The pressure on EFGH would be half that on ABCD.

KEY:4

14. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

1) remains the same because is very much smaller than of the gas
2) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls
3) will increase by a factor equal to where was the original mean square velocity of the gas
4) will be different on the top wall and bottom wall of the vessel

15. A vessel of volume V = 5.0 litre contains 1.4 gm of nitrogen at temperature T=1800 K. Find the pressure of the gas if 30% of its molecule are dissociated into atom at this temperature.

1) 0.54×10^5 N/m^2 2) 1.94×10^5 N/m^2

3) 2.62×10^5 N/m^2 4) 3.75×10^5 N/m^2

KEY:2

HINT : $P = \frac{\mu}{M}\frac{RT}{V}$, $\frac{P_1}{P_2} = \frac{\mu_1}{\mu_1 + \mu_2}$

16. The pressure of an ideal gas varies according to the law $P = P_0 - AV^2$ where P_0 and A are positive constants. What is the highest temperature that can be attained by the gas.

1) $\frac{P_0}{nR}\sqrt{\frac{P_0}{A}}$ 2) $\frac{P_0}{nR}\sqrt{\frac{P_0}{2A}}$

3) $\frac{2P_0}{nR}\sqrt{\frac{P_0}{2A}}$ 4) $\frac{2P_0}{3nR}\sqrt{\frac{P_0}{3A}}$

KEY:4

HINT : $P = P_0 - AV^2$

$$\Rightarrow \quad \frac{nRT}{V} = P_0 - AV^2$$ at max

temperature $\frac{dT}{dV} = 0 \Rightarrow P_0 = 3AV^2$

17. The mass 15 gram of Nitrogen is enclosed in a vessel at 300K. What heat must be supplied to it to double the rms velocity of its molecules

1) 10J 2) 10KJ 3) 10^3J 4) 10^2J

KEY:2

HINT :since $v^2 \propto T \Rightarrow T_2 = 1200K$

$$Heat(H) = nc_v dT = \frac{m}{M}\frac{R}{(\gamma - 1)}(T_2 - T_1)$$

18 At what absolute temperature T, is rms speed of a hydrogen molecule equal to its escape velocity from the surface of the moon? The radius of moon is R, g is the acceleration due to gravity on moon's surface, m is the mass of a hydrogen molecule and k is the Boltzmann constant.

1) $\frac{mgR}{2k}$ 2) $\frac{2mgR}{k}$ 3) $\frac{3mgR}{2k}$ 4) $\frac{2mgR}{3k}$

KEY:4

HINT : $v_{rms} = \sqrt{\frac{3KT}{m}}$ $v_e = \sqrt{2gR}$

19 Find the number of degrees of freedom of molecules in a gas whose molar heat capacity at constant pressure is equal to C_p=29 J/(mol.K)

1) 3 2) 4 3) 5 4) 6

KEY:3

HINT : $C_p = \frac{\gamma R}{\gamma - 1}; \gamma = \left(1 + \frac{2}{f}\right)$

20 Determine the gas temperature at which the root mean square velocity of hydrogen molecules exceeds their most probable velocity by $\Delta v = 400\, m/s$

1) 384K 2) 342K 3) 300K 4) 280K

KEY:1

HINT : $v_{rms} - v_p = \left(\sqrt{3} - \sqrt{2}\right)\sqrt{\frac{RT}{M}} = \Delta v$

$$\therefore T = \frac{M}{R}\left(\frac{\Delta v}{\sqrt{3} - \sqrt{2}}\right)^2 = 384k$$

21 **Two chambers one containing m_1 *gram* of a gas at pressure P_1 and other containing m_2 *gram* of same gas at pressure P_2 are put in communication with each other. If temperature remains constant, the common pressure reached will be,**

1) $\dfrac{P_1P_2(m_1+m_2)}{P_1m_2+P_2m_1}$ 2) $\dfrac{m_1m_2(P_1+P_2)}{P_1m_2+P_2m_1}$

3) $\dfrac{P_1P_2m_1}{P_1m_2+P_2m_1}$ 4) $\dfrac{m_1m_2P_2}{P_1m_2+P_2m_1}$

KEY:1

HINT : From Boyle's law, $\dfrac{P}{\rho} = Const = K$

$\rho_1 = \dfrac{P_1}{K}$, (or) $V_1 = \dfrac{m_1}{\rho_1} = \dfrac{m_1K}{P_1}$

Illy, $V_2 = \dfrac{m_2K}{P_2}$

$V = V_1 + V_2 = K\left[\dfrac{m_1}{P_1} + \dfrac{m_2}{P_2}\right]$; $\rho = \dfrac{m_1+m_2}{V_1+V_2}$

$P = K\rho = \dfrac{P_1P_2(m_1+m_2)}{P_1m_2+P_2m_1}$

22. **How many degrees of freedom does a gas molecule have under standard conditions if gas has density 1.3 kg/m³ and the velocity of sound propagation in the gas is v=330m/s?**

1) 2 2) 3 3) 4 4) 5

KEY:4

HINT : $v = \sqrt{\dfrac{\gamma p}{\rho}}$

$\gamma = 1 + \dfrac{2}{f}$ *where f is* degrees of freedom

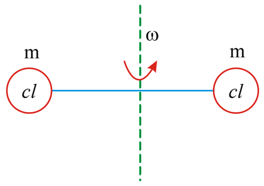
23 **In crude model of a rotating diatomic molecule of chlorine (Cl_2), the two Cl atoms are $2.0\times10^{-10}\,m$ apart and rotate about their centre of mass with angular speed $\omega = 2.0\times10^{12}\,rad/s$. What is the rotational kinetic energy of one molecule of Cl_2, which has a molar mass of 70.0g/mol?**

1) $2.32\times10^{-21}\,J$ 2) $2.32\times10^{21}\,J$

3) $2.32\times10^{-21}\,erg$ 4) $2.32\times10^{21}\,erg$

KEY:1

HINT :



$m = \dfrac{70\times10^{-3}}{2\times6.02\times10^{23}} = 5.81\times10^{-26}kg$ and

$r = \dfrac{2.0\times10^{-10}}{2} = 10^{-10}m$;

$M.I = 2(mr^2) = 2mr^2$

$I = 2\left(5.81\times10^{-26}\right)\left(10^{-10}\right)^2$

$\therefore K_R = \dfrac{1}{2}I\omega^2 = 2.32\times10^{-21}J$

24. An ideal gas undergoes a process in which PV^{-a} = constant, where V is the volume occupied by the gas initially at pressure P. At the end of the process, rms speed of gas molecules has become $a^{1/2}$ times of its initial value. What will be the value of C_V so that energy transferred in the form of heat to the gas is 'a' times of the initial energy.

1) $\dfrac{(a^2+1)R}{a^2-1}$ 2) $\dfrac{(a^2+1)R}{(a^2+1)}$ 3) $\dfrac{(a+1)R}{(a-1)}$ 4) $\dfrac{(a-1)R}{(a+1)}$

KEY:4

HINT : $\Delta Q = aU_1$

$nC(T_2 - T_1) = a\dfrac{f}{2}nRT_1$ -----(1)

$but\ v_{rms} \alpha \sqrt{T} \Rightarrow T_2 = aT_1$

(as rms speed became $\sqrt{a}$ times).

From (1) $\therefore C(a-1)T_1 = \dfrac{af}{2}RT_1$

$C = \dfrac{afR}{2(a-1)}\left(here, f = \dfrac{2C_v}{R}\right)$ $C = C_v + \dfrac{R}{1-n}$ (for polytrophic process)

where n = -a

THEORY BITS

1. **Which of the following statements are true regarding the kinetic theory of gases ?**
   1) The pressure of the gas is directly proportional to the average speed of the molecules
   2) The root mean square speed of the molecules is directly proportional to the pressure
   3) The rate of diffusion is directly proportional to average speed of the molecules
   4) The average kinetic energy per molecule is inversely proportional to the absolute temperature

KEY:3

2. **At a given temperature if V_{rms} is the root mean square velocity of the molecules of a gas and V_s be the velocity of sound in it, then these are related as $\left(\gamma = \frac{C_p}{C_v}\right)$**

   1) $v_{rms} = v_s$ 2) $v_{rms} = \sqrt{\frac{3}{\gamma}} \times v_s$ 3) $v_{rms} = \sqrt{\frac{\gamma}{3}} \times v_s$ 4) $v_{rms} = \left(\frac{3}{\gamma}\right) \times v_s$

KEY:2

3.The temperature of an ideal gas is increased from 120K to 480K. If at 120K, the root mean square velocity of the gas molecules is v then at 480K, it becomes

1) 4v 2) 2v 3) v/2 4) v/4

KEY:2

4. **At a given volume and temperature the pressure of a gas:**
   1) varies inversely as its mass 2) varies inversely as the square of its mass
   3) varies linearly as its mass 4) is independent of its mass

KEY:3

5. **The root mean square speed of the molecules of a gas at absolute temperature T is proportional to**

   1) $1/T$ (2) $\sqrt{T}$ (3) T (4) T^2

KEY:2

6. **If gas molecules undergo inelastic collision with the wall of the container:**
   1) the temperature of the gas will decrease 2) the pressure of the gas will increase
   3) neither the temperature nor the pressure will change 4) the temperature of the gas will increase

KEY:3

7. **Which of the following methods will enable the volume of an ideal gas to be made four times greater? Consider absolute temperature**
   1) Quarter the pressure at constant temperature
   2) Quarter the temperature at constant pressure

3) Half the temperature, double the pressure

4) Double the temperature, double the pressure

KEY:1

8. If k is the Boltzmann constant, the average kinetic energy of a gas molecule at absolute temperature T is

1) $kT/2$ (2) $3kT/4$ (3) kT 4) $3kT/2$

KEY: 4

9. A gas has volume V and pressure P. The total translational kinetic energy of all the molecules of the gas is

1) 3/2 PV only if the gas is monoatomic 2) 3/2 PV only if the gas is diatomic

3) > 3/2 PV if the gas is diatomic 4) 3/2 PV in all cases

KEY:4

10. The root mean square speed of the molecules of an enclosed gas is v. what will be the root mean square speed if the pressure is doubled, the temperature remaining the same?

1) v/2 (2) v (3) 2v (4) 4v

KEY:2

11. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300K. The ratio of the rotational kinetic energy per O_2 **molecule to that of per N_2 molecule is**

1) 1:1 2) 1:2 3) 2:1

4) depends on the moment of inertia of the two molecules

KEY:1

12. On any planet, the presence of atmosphere implies (C_{rms} = root mean square velocity of molecules and v_e = escape velocity)

1) $C_{rms} < v_e$ 2) $C_{rms} > v_e$ 3) $C_{rms} = v_e$ 4) $C_{rms} = 0$

KEY:1

13. Choose the correct statement. When the temperature of a gas is increased

1) the kinetic energy of its molecules increases

2) the potential energy of its molecules increases

3) the potential energy decreases and the kinetic energy increases; the total energy remaining unchanged

4) the potential energy increases and the kinetic energy decreases; the total energy remaining unchanged

KEY:1

14. The number of molecules per unit volume (n) of a gas is given by

1) $\frac{P}{kT}$ 2) $\frac{kT}{P}$ 3) $\frac{P}{RT}$ 4) $\frac{RT}{P}$

KEY:1

15. The number of molecules of N_2 and O_2 in a vessel are same. If a fine hole is made in the vessel then which gas escapes out more rapidly?

1) N_2 2) O_2 3) both 4) sometimes N_2 and sometimes O_2

KEY:1

16. The following four gases are at the same temperature . In which gas do the molecules have the maximum root mean square speed?

1) Hydrogen 2) Oxygen 3) Nitrogen 4) Carbon dioxide

KEY:1

17. Two vessels having equal volume contain molecular hydrogen at one atmosphere and helium at two atmospheres respectively. If both samples are at the same temperature, the rms velocity of hydrogen molecules is:

1)equal to that of helium 2) twice that of helium 3) half that of helium 4) $\sqrt{2}$ times that of helium

KEY:4

18. E_0 *and* E_h **respectively represent the average kinetic energy of a molecule of oxygen and hydrogen. If the two gases are at the same temperature, which of the following statements is true?**

1) $E_0 > E_h$ (2) $E_0 = E_h$ (3) $E_0 < E_h$

4) Nothing can be said about the magnitude of E_0 and E_h as the information given is insufficient.

KEY:2

19. If the Avogadro's number was to tend to infinity; the phenomenon of Brownian motion would:

1) remain completely unaffected

2) become more vigorous than that observed with present finite values of Avogadro's number, for all sizes of the Brownian particles

3) become more vigorous than that observed with the present finite value of Avogadro's number, only for relatively large Brownian particles

4) become practically unobservable as the molecular impact would tend to balance one another, for practically all sizes of Brownian particles

KEY:4

20. The root mean square speed of a group of N gas molecules, having speeds $v_1, v_2, ..., v_N$ **is:**

1) $\frac{1}{N}\sqrt{\left(v_1+v_2+.......+v_N\right)^2}$ b) $\frac{1}{N}\sqrt{\left(v_1^2+v_2^2+.......+v_N^2\right)}$

3) $\sqrt{\frac{1}{N}\left(v_1^2+v_2^2+.......+v_N^2\right)}$ 4) $\sqrt{\left(v_1+v_2+.......+v_N\right)^2}$

KEY:3

21. The average kinetic energy of a molecule of a gas at absolute temperature T is proportional to

1) $1/T$ (2) $\sqrt{T}$ (3) T (4) T^2

KEY:3

22. The relation between rms velocity, v_{rms} and the most probable velocity, v_{mp}, of a gas is:

1) $v_{rms} = v_{mp}$ 2) $v_{rms} = \sqrt{\frac{3}{2}} v_{mp}$ 3) $v_{rms} = \sqrt{\frac{2}{3}} v_{mp}$ 4) $v_{rms} = \frac{2}{3} v_{mp}$

KEY:2

23. Some gas at 300K is enclosed in a container. Now the container is placed on a fast moving train. While the train is in motion, the temperature of the gas:

1) rises above 300K 2) falls below 300K

3) remains unchanged 4) becomes unsteady

KEY:3

24. The average energy for molecules in one degree of freedom is :

1) $\frac{3}{2}kT$ (2) $\frac{kT}{2}$ (3) $\frac{3}{4}kT$ (4) kT

KEY:2

25. Two balloons are filled, one with pure He gas and other by air, respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is:

1) more in the He filled balloon 2) same in both balloons

3) more in air filled balloon (4) in the ratio of 1:4

KEY:2

26. Which of the following phenomena gives evidence of the molecular motion?

1) Brownian movement 2) Diffusion 3) Evaporation 4) All the above

KEY:4

27. On colliding in a closed container the gas molecules:

1) transfer momentum to the walls 2) momentum becomes zero

3) move in opposite directions 4) perform Brownian motion

KEY:4

28. The root mean square velocity, v_{rms}, the average velocity v_{av} and the most probable velocity, v_{mp} of the molecules of the gas are in the order:

1) $v_{mp} > v_{avg} > v_{rms}$ 2) $v_{rms} > v_{avg} > v_{mp}$

3) $v_{avg} > v_{mp} > v_{rms}$ 4) $v_{mp} > v_{rms} > v_{avg}$

KEY:2

29. The temperature of a gas is raised while its volume remains constant, the pressure exerted by the gas on the walls of the container increases because its molecules.

(1) Lose more kinetic energy to the wall

(2) Are in contact with the wall for a shorter time

(3)Strike the wall more often with higher velocities

(4)Collide with each other with less frequency.

KEY:3

30. At upper atmosphere, an astronaut feels:

1) extremely hot 2) slightly hotter

3) extremely cool 4) slightly cooler

KEY:4

31. The average distance travelled by a molecule of gas at temperature T between two successive collisions is called its mean free path which can be expressed by (P is pressure of gas, K is Boltzmann constant, d diameter of molecule)

(1) $\frac{1}{\sqrt{2}\pi d^2 P}$ (2) $\frac{P}{\sqrt{2}\pi d^2 T}$ (3) $\frac{KT}{\sqrt{2}\pi d^2 P}$ (4) $\frac{KP}{\sqrt{2}\pi d^2 T}$

KEY:3

32. Which of the following statements about kinetic theory of gases is wrong

1)The molecules of a gas are in continuous random motion

2) The molecule continuously undergo inelastic collisions

3) The molecules do not interact with each other except during collisions

4) The collisions amongst the molecules are of short duration

KEY:2

33. The root mean square speed of gas molecules

1) is same for all gases at the same temperature

2) depends on the mass of the gas molecule and its temperature

3) is independent of the density and pressure of the gas

4) depends only on the temperature and volume of the gas

KEY:2

34. Consider a gas with density ρ and $\bar{c}$ as the root mean square velocity of its molecules contained in a volume. If the system moves as whole with velocity v, then the pressure exerted by the gas is

1) $\frac{1}{3}\rho \bar{c}^2$ 2) $\frac{1}{3}\rho\left(\bar{c}+\text{v}\right)^2$ 3) $\frac{1}{3}\rho\left(\bar{c}-\text{v}\right)^2$ 4) $\frac{1}{3}\rho\left(\bar{c}^2-\text{v}\right)$

KEY:1

35. Choose the only correct statement from the following:

1) The pressure of a gas is equal to the total kinetic energy of the molecules in a unit volume of the gas.

2) The product of pressure and volume of a gas is always constant.

3) The average kinetic energy of molecules of a gas is proportional to its absolute temperature.

4) The average kinetic energy of molecules of a gas is proportional to the square root of its absolute temperature.

KEY:3

36. If the pressure in a closed vessel is reduced by drawing out some gas, the mean free path of the molecules

(1) Decreases (2) increases (3) Remains unchanged

(4) Increases or decreases according to the nature of the gas

KEY:2

37. If P is the pressure of the gas then the KE per unit volume of the gas is

1) $\frac{P}{2}$ 2) P 3) $\frac{3P}{2}$ 4) $2P$

KEY:3

38. At absolute zero temperature, the kinetic energy of the molecules:

1) becomes zero (2) becomes maximum 3) becomes minimum (4) remains constant

KEY:1

39. Choose the correct statement from the following:

1) The average kinetic energy of a molecule of any gas is the same at the same temperature.

2) The average kinetic energy of a molecule of a gas is independent of its temperature.

3) The average kinetic energy of 1g of any gas is the same at the same temperature.

4) The average kinetic energy of 1g of a gas is independent of its temperature.

KEY:1

40. Two gases of equal mass are in thermal equilibrium. If P_a, P_b and V_a and V_b are their respective pressures and volumes, then which relation is true

(a) $P_a \neq P_b; V_a = V_b$ (b) $P_a = P_b; V_a \neq V_b$

(c) $\frac{P_a}{V_a} = \frac{P_b}{V_b}$ (d) $P_a V_a = P_b V_b$ KEY:d

PREVIOUS JEE MAINS QUESTIONS AND SOLUTIONS

1. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is –. [NA Sep. 06, 2020 (I)]

Sol : (5) Using ideal gas equation, $PV = nRT$

$$\Rightarrow P_1V_1 = nR \times 250 \; [\because T_1 = 250\text{K}] \quad \text{(i)}$$

$P_2(2V_1 = \frac{5n}{4}R \times 2000 \; [\because T_2 = 2000\text{K}]$ (ii) Dividing eq. (i) by(ii),

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000} \Rightarrow \frac{P_1}{P_2} = \frac{1}{5}$$

$$\frac{P_2}{P_1} = 5.$$

2. The change in the magnitude ofthe volume ofan ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure ofthe gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$, then value of C in (K/atm.) is[NA Sep. 04, 2020(II)]

Sol : (150) In first case,

From ideal gas equation

$$PV = nRT$$

$P\Delta V + y\Delta P = 0$ (As temperature is constant) $\Delta V = -\frac{\Delta P}{P}y$ (i)

In second case, using ideal gas equation again

$$P\Delta V = -nR\Delta T$$

$\Delta V = -\frac{nR\Delta T}{P}$ (ii)

Equating (i) and (ii), we get

$$\frac{nR\Delta T}{P} = -\frac{\Delta P}{P}V \Rightarrow \Delta T = \Delta P\frac{V}{nR}$$

Comparing the above equation with $|\Delta T| = C|\Delta P|$, we have

$$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300\text{K}}{2\text{atm}} = 150\text{K/atm}$$

3. The number density of molecules of a gas depends on their distance r from the origin as , $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to:[12 April 2019 II]

(a) $n_0\alpha^{-3/4}$ (b) $\sqrt{n_0}\alpha^{1/2}$ (c) $n_0\alpha^{1/4}$ (d) $n_0\alpha^{-3}$

Sol :. (a) $N = \int \; p\,(dv)$

$$= \int_0^r n_0 e^{-\alpha r^4} \times 4\pi r^2 dr_{=4pn_0} \int_0^r r^2 \left(e^{-\alpha r^4}\right)dr$$

$$\propto n_0 a^{-3/4}$$

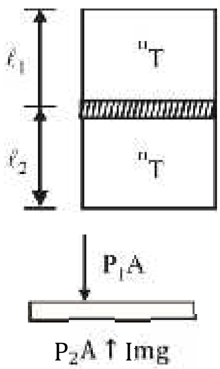
4. A vertical closed cylinder is separated into two parts by a frictionless piston ofmass m and ofnegligible thickness. The piston is free to move along the length ofthe cylinder. The length ofthe cylinder above the piston is l_1 , and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles ofan ideal gas at equal temperature T. Ifthe piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity) [12 Jan. 2019 II]

(a) $\frac{RT}{ng}\left[\frac{l_1-3l_2}{l_1 l_2}\right]$ (b) $\frac{RT}{g}\left[\frac{2l_1+l_2}{l_1 l_2}\right]$ (c) $\frac{nRT}{g}\left[\frac{1}{l_2}+\frac{1}{l_1}\right]$ (d) $\frac{nRT}{g}\left[\frac{l_1-l_2}{l_1 l_2}\right]$

Sol : (d) Clearly from figure,

$$P_2A = P_1A + mg$$

$$\text{or, } \frac{nRT\cdot A}{A\ell_2} = \frac{nRT\cdot A}{A\ell_1} + mg$$



$$\Rightarrow nRT\left(\frac{1}{l_2}-\frac{1}{\ell_1}\right) = mg$$

$$m = \frac{nRT}{g}\left(\frac{l_1 - p_2}{\ell_1 \cdot \ell_2}\right)$$

5. Thetemperamre ofan open room ofvolume $30m^3$ increases from 17°C to 27°C due to sunshine. The ahnospheric pressure in the room remains 1×10^5 Pa. Ifn_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : [2017]

(a) 2.5×10^{25} (b) -2.5×10^{25} (c) -1.61×10^{23} (d) 1.38×10^{23}

Sol : (b) Given: Temperature $T_i = 17 + 273 = 290K$

Temperature $T_f = 27 + 273 = 300K$

Atmospheric pressure, $P_0 = 1 \times 10^5\ Pa$

Volume of room, $V_0 = 30m^3$

Difference in number of molecules, $n_f - n_i$ =?

Using ideal gas equation, $= nRT(N_0)$,

N_0 = Avogadro's number

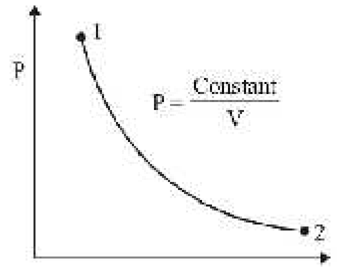
$$\Rightarrow n = \frac{Py}{RT}(N_0)$$

$$n_f - n_i = \frac{P_0 V_0}{R}(\ ) N_0(\ )$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23} \left(\frac{1}{300} - \frac{1}{290}\right)$$

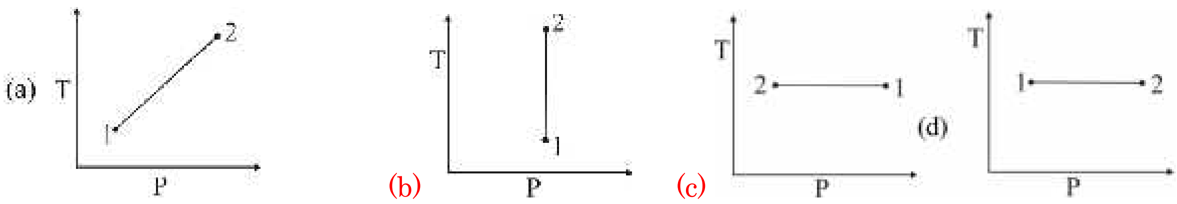
$$= -2.5 \times 10^{25}$$

6. For the P - V diagram given for an ideal gas,



v

out of the following which one correctly represents the T - P diagram? [Online April 9, 2017]



Sol : (c) From P - V graph,

$P \propto \frac{1}{V}$, T = constant and Pressure is increasing from 2 to 1 so option (3) represents correct T - P graph.

7. There are two identical chambers, completely thermally insulated from surroundings. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and Compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement - 1: No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature ofreal gas goes down (cooling) when it expands in vacuum.

Statement - 2: The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. [Online April 9, 2013]

(a) Statement - 1 is false and Statement - 2 is true.

(b) Statement - 1 and Statement - 2 both are true. Statement - 2 is the correct explanation of Statement - 1.

(c) Statement - 1 is true and Statement - 2 is false.

(d) Statement - 1 and Statement - 2 both are true. Statement - 2 is not correct explanation of Statement - 1.

Sol : a) In ideal gases the molecules are considered as point particles and for point particles, there is no internal excitation, no vibration and no rotation. For an ideal gas the internal energy can only be translational kinetic energy and for real gas both kinetic as well as potential energy

8. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will [2002]

(a) increase (b) decrease (c) remain same

(d) decrease for some, while increase for others

Sol : ∴ 8. (c) The centre ofmass ofgas molecules also moves with lorry with uniform speed. As there is no relative motion of gas molecule. So, kinetic energy and hence temperature remain same

9. Number of molecules in a volume of $4\ cm^3$ of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, $g = 980 cm/s^2$, density of mercury $= 13.6 g/cm^3$) [Sep. 05, 2020 (I)]

(a) 4.0×10^{18} (b) 4.0×10^{16} (c) 5.8×10^{16} (d) 5.8×10^{18}

Sol : (c) Given: K.E. mean $= \frac{3}{2}kT = 4 \times 10^{-14}$

$P = 2$ cm ofHg, $V = 4cm^3$

$$N = \frac{PV}{KT} = \frac{P\rho g^{y} 2 \times 13.6 \times 980 \times 4}{KT\frac{8}{3} \times 10^{-14}} = 4 \times 10^{18}$$

10. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed ofa H_2 molecule would be equal to the rms speed of a nitrogen molecule, is . (Molar mass ofN_2 gas 28 g); [NA Sep. 05, 2020 (II)]

Sol : Room mean square speed is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Here, M = Molar mass ofgas molecule T = temperature ofthe gas molecule We have given $v_{N_2} = v_{H_2}$

$$\sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\square\ \frac{T_{H_2}}{2} = \frac{573}{28}\ \square\ T_{H_2} = 41K$$

11. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed ofthe molecules will be: [9 April 2019 I]

(a) 100m/s (b) $80\sqrt{5}$m/s (c) $100\sqrt{5}$m/s (d) 80 m/s

Sol : (c) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2}=\sqrt{\frac{T_1}{T_2}}=\frac{(273+127)}{(273+237)}=\sqrt{\frac{400}{500}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}$$

$$v_2=\frac{\sqrt{5}}{2}v_1=\frac{\sqrt{5}}{2}\times 200=100\sqrt{5}\text{m/s.}$$

12. If 10^{22} gas molecules each ofmass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of: [8 April 2019 I]

(a) $10^4 N/m^2$ (b) $10^8 N/m^2$ (c) $10^3 N/m^2$ (d) $10^{16} N/m^2$

Sol : (Bouns) Rate ofchange ofmomentum during collision

$$=\frac{mv-(-mv)}{\Delta t}=\frac{2mv}{\Delta t}N$$

so pressure $p=\frac{N\times(2mv)}{\Delta t\times A}$

$$=\frac{10^{22}\times 2\times 10^{-26}\times 10^4}{1\times 1}=2N/m^2$$

13. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to: [8April 2019II] [Boltzmann Constant $k_B=1.38\times 10^{-23}$ J/K Avogadro Number $N_A=6.02\times 10^{26}$/kg Radius ofEarth: 6.4×10^6m Gravitational acceleration on Earth = $10ms^{-2}$]

(a) 800K (b) 3×10^5K (c) 10^4K (d) 650K

Sol : (c) $v_{rs}=v_e$

$$\sqrt{\frac{3RT}{M}}=11.2\times 10^3$$

or $\sqrt{\frac{3kT}{m}}=11.2\times 10^3$

or $\sqrt{\frac{3\times 1.38\times 10^{-23}T}{2\times 10^{-3}}}=11.2\times 10^3$ $v=10^4$K

14. Amixmre of2 moles ofhelium gas (atomic mass = 4u), and 1 mole ofargon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio oftheir rms speeds

$\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to: [9 Jan. 2019 I]

(a) 3.16 (b) 0.32 (c) 0.45 (d) 2.24

Sol : (a) Using $\frac{V_{1rms}}{V_{2rms}}=\sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

15. Nmoles ofa diatomic gas in a cylinder are at atemperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles ofthe diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy ofthe gas? [Online April 9, 2017]

(a) $\frac{1}{2}$nRT (b) 0 (c) $\frac{3}{2}$nRT (d) $\frac{5}{2}$nRT

Sol : (a) Energy associated with N moles ofdiatomic gas,

$U_i = N\frac{5}{2}RT$

Energy associated with n moles ofmonoatomic gas

$= n\frac{3}{2}RT$

Total energywhen n moles ofdiatomic gas converted into monoatomic $(U_f) = 2n\frac{3}{2}RT + (N - n)\frac{5}{2}RT$

$= \frac{1}{2}nRT + \frac{5}{2}NRT$

Now, change in total kinetic energy ofthe gas

$$\Delta U = Q = \frac{1}{2}nRT$$

16. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is: [Online April 10, 2015]

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

Sol : (c) Pressure, $P = \frac{1}{3}\frac{mN}{V}V_{rms}^2$

or, $P = \frac{(mN)T}{V}$

If the gas mass and temperature are constant then

$$P \propto (V_{rms})^2 \propto T$$

So, force □ $(V_{rms})^2 \propto T$

i. e., Value of $q = 1$

17. A gas molecule of mass M at the surface ofthe Earth has kinetic energy equivalent to 0°C. If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than radius ofthe earth. (k_B is Boltzmann constant). [Online Apri119, 2014]

(a) 0 (b) $\frac{273k_B}{2Mg}$ (c) $\frac{546k_B}{3Mg}$ (d) $\frac{819k_B}{2Mg}$

Sol : d) Kinetic energy ofeach molecule,

K.E. = $\frac{3}{2}K_BT$

In the given problem,

Temperature, T = 0°C = 273K

Height attained by the gas molecule, h =?

K.E. = $\frac{3}{2}K_B(273) = \frac{819K_B}{2}$

K.E. = P.E. $\Rightarrow \frac{819K_B}{2} =$ Mgh

or h = $\frac{819K_B}{2Mg}$

18. At room temperature a diatomic gas is found to have an r.m.s. speed of1930 ms^{-1}. The gas is: [Online Apri112, 2014]

(a) H_2 (b) $C1_2$ (c) O_2 (d) F_2

Sol : (a) $C = \sqrt{\frac{3RT}{M}}$

$(1930)2 = \frac{3\times8.314\times300}{M}$

$$M = \frac{3 \times 8.314 \times 300}{1930 \times 1930} \approx 2 \times 10^{-3}kg$$

The gas is H_2.

19. In the isothermal expansion of 10g of gas fiom volume V to 2V the work done by the gas is 575J. Mat is the root mean square speed of the molecules of the gas at that temperature? [Online April 25, 2013]

(a) 398m/s (b) 520m/s (c) 499m/s (d) 532m/s

Sol : (c) $v_{rms} = \sqrt{\frac{3pv}{massofthegas}}$

20. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The final temperature of the gas will be, close to [Online May 7, 2012]

(a) 327°C (b) $2\alpha)$°C (c) 54°C (d) $3\alpha)$°C

Sol : (a) Given, $V_1 = V$

$$V_2 = 2V$$

$$T_1 = 27^{\circ} + 273 = 300K$$

$$T_2 = ?$$

From charle' s law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ (Pressure is constant) or, } \frac{V}{300} = \frac{2V}{T_2}$$

$$T_2 = 600K = 600 - 273 = 327°C$$

21. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it' s suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by: [2011]

(a) $\frac{(\gamma-1)}{2\gamma R} Mv^2K$ (b) $\frac{\gamma M^2 v}{2R} K$ (c) $\frac{(\gamma-1)}{2R} Mv^2K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R} Mv^2K$

Sol : (c) As, work done is zero.

So, loss in kinetic energy = heat gain by the gas

$$\frac{1}{2}mv^2 = nC_v\Delta T = n\frac{R}{\Upsilon - 1}\Delta T$$

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\Upsilon - 1}\Delta T$$

22. Three perfect gases at absolute temperatures T_1 , T_2 and T_3 are mixed. The masses ofmolecules are m_1 , m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : [2011]

(a) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$ (b) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (c) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (d) $\frac{(T_1+T_2+T_3)}{3}$

Sol : $P_1V_1 + P_2V_2 + P_3V_3 = PV$

$$\frac{n_1}{N_A}RT_1 + \frac{n_2}{N_A}RT_2 + \frac{n_3}{N_A}RT_3$$

$$= \frac{n_1 + n_2 + n_3}{N_A}RT_{mix}$$

$$T_{mix} = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

23. One kg of a diatomic gas is at a pressure of $8 \times 10^4 N/m^2$. The density of the gas is $4 kg/m^3$. What is the energy of the gas due to its thermal motion?[2009]

(a) $5 \times 10^4 J$ (b) $6 \times 10^4 J$ (c) $7 \times 10^4 J$ (d) $3 \times 10^4 J$

Sol :. (a) Given, mass = 1 kg

Density = $4\ kg\ m^{-3}$

Volume = $\frac{mass}{density} = \frac{1}{4} m^3$

Internal energy ofthe diatomic gas

$$= \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 J$$

Alternatively:

K.E = $\frac{5}{2}nRT = \frac{5}{2}\frac{m}{M}RT = \frac{5}{2}\frac{m}{M} \times \frac{PM}{d}$ [$\because PM = dRT$]

$$= \frac{5}{2}\frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4 J$$

24. The speed ofsound in oxygen (O_2) at a certain temperature is $460\ ms^{-1}$. The speed ofsound in helium (He) at the same temperature will be (assume both gases to be ideal) [2008]

(a) $1421 ms^{-1}$ (b) $500\ ms^{-1}$ (c) $650\ ms^{-1}$ (d) $330\ ms^{-1}$

Sol : (a) The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma RT}{M}}$ $v \propto \sqrt{\frac{\gamma}{M}}$ [As R and T is constant]

$$\frac{v_{O_2}}{v_{He}} = \sqrt{\frac{\gamma_{O_2}}{M_{O_2}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{14}{32} \times \frac{4}{167}} = 0.3237$$

$$v_{He} = \frac{v_{O_2}}{0.3237} = \frac{460}{0.3237} = 1421 \text{m/s}$$

25. At what temperature is the r.m.s velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C? [2002]

(a) 80 K (b) −73K (c) 3 K (d) 20 K

Sol : (d) RMS velocity ofa gas molecule is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Let T be the temperature at which the velocity ofhydrogen molecule is equal to the velocity ofoxygen molecule.

$$\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R \times (273 + 47)}{32}}$$

$$\therefore T = 20\text{K}$$

26. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T.

The total internal energy, U ofa mole ofthis gas, and the value of $\gamma \left(= \frac{C_p}{C_v}\right)$ are given, respectively, by: [Sep. 06, 2020 (D]

(a) U = $\frac{5}{2}$ RT and $\gamma = \frac{6}{5}$ (b) U = 5RT and $\gamma = \frac{7}{5}$

(c) U = $\frac{5}{2}$ RT and $\gamma = \frac{7}{5}$ (d) U = 5RT and $\gamma = \frac{6}{5}$

Sol :. (c) Total degree offreedom $f = 3 + 2 = 5$

Total energy, $u = \frac{nfRT}{2} = \frac{5RT}{2}$

And $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

27. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T is: [Sep. 06, 2020 (II)]

(a) T (b) $\frac{1}{\sqrt{T}}$ (c) $\frac{1}{T}$ (d) $\sqrt{T}$

Sol :. (b) Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

where, d = diameter ofthe molecule

n = number ofmolecules per unit volume But, mean time ofcollision, $\Gamma = \frac{\lambda}{v_{rms}}$

But $v_{rms} = \sqrt{\frac{3kT}{R}}$

$$t = \frac{\lambda}{\sqrt{\frac{3kT}{m}}} \Rightarrow t \propto \frac{1}{\sqrt{T}}$$

28. Match the C_p/C_v ratio for ideal gases with different type of molecules : [Sep. 04, 2020 (I)]

Column - I C	olumn - II
Molecule	Type C_p/C_v
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non - rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(a) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(b) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(c) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

(d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Sol :. (c) As we know,

$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$, where f = degree offreedom

(A) Monatomic, $f = 3$

$$\gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) Diatomic rigid molecules, $f = 5$

$$\gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) Diatomic non - rigid molecules, $f = 7$

$$\gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) Triatomic rigid molecules, $f = 6$

$$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

29. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to [NA Sep. 04, 2020 (I)]

Sol : (266.67) Here work done on gas and heat supplied to the gas are zero.

Let Tbe the fmal equilibrium temperature ofthe gas in the vessel.

Total internal energyofgases remain same.

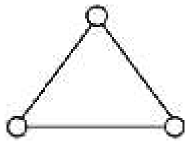
i.e., $u_1 + u_2 = u_1 + u_2$

or, $n_1 C_v \Delta T_1 + n_2 C_v \Delta T_2 = (n_1 + n_2) C_v T$

$$\square\ (0.1) C_v (200) + (0.05) C_v (400) = (0.15) C_v T$$

$$T = \frac{800}{3} = 266.67\text{K}$$

30.



Consider a gas oftriatomic molecules. The molecules are assumed to be triangular and made Of mass less rigid rods whose vertices are occupied by atoms. The internal energy ofa mole of the gas at temperature T is: [Sep. 03, 2020 (I)]

(a) $\frac{5}{2}RT$ (b) $\frac{3}{2}RT$ (c) $\frac{9}{2}RT$ (d) $3RT$

Sol : (d) Here degree of fiieedom, $f = 3 + 3 = 6$ for triatomic nonlinear molecule.

Internal energy ofa mole ofthe gas at temperature T,

$$U = \frac{f}{2} nRT = \frac{6}{2} RT = 3RT$$

31. To raise the temperature ofa certain mass ofgas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass ofgas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [Sep. 03, 2020 (II)]

(a) 5 (b) 6 (c) 3 (d) 7

Sol :b) Let C_p and C_v be the specific heat capacity ofthe gas at constant pressure and volume.

At constant pressure, heat required

$$\Delta Q_1 = nC_p \Delta T$$

$\square$ $160 = nC_p \cdot 50$ (i)

At constant volume, heat required

$$\Delta Q_2 = nC_v \Delta T$$

$\square$ $240 = nC_v \cdot 100$ (ii)

Dividing (i) by(ii), we get

$$\frac{160}{240} = \frac{C_p}{C_v} \cdot \frac{50}{100}\ \square\ \frac{C_p}{C_v} = \frac{4}{3}$$

$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f}$ (Here, f = degree of fiieedom)

$$\square\ f = 6.$$

32. A gas mixture consists of3 moles ofoxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) ofthe

mixture is: [Sep. 02, 2020]

(a) 15 (b) 13 (c) 20 (d) 11

Sol : (a) Total energy ofthe gas mixture,

$$E_{mix} = \frac{f_1 n_1 RT_1}{2} + \frac{f_2 n_2 RT_2}{2}$$

$$= 3 \times \frac{5}{2}RT + \frac{5}{2} \times 3RT = 15RT$$

33. An ideal gas in a closed container is slowly heated. As its temperature increases, which ofthe following statements are true? [Sep. 02, 2020 (II)]

(1) The mean free path of the molecules decreases

(2) The mean collision time between the molecules decreases

(3) The mean free path remains unchanged

(4) The mean collision time remains unchanged

(a) (2) and (3) (b) (1) and (2) (c) (3) and (4) (d) (1) and(4)

Sol : (a) As we know mean f : ee path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

Here, N = no. ofmolecule

V = volume ofcontainer

d = diameter ofmolecule

But $PV = nRT = nNKT$

$$\Rightarrow \frac{N}{V} = \frac{P}{KT} = n$$

$$\lambda = \frac{1KT}{\sqrt{2}\pi d^2 P}$$

For constant volume and hence constant number density n of gas molecules $\frac{P}{T}$ is constant.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases

34. Consider two ideal diatomic gases A and B at some temperature T. Molecules ofthe gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) ofgas A and B, respectively is: [9 Jan 2020 I]

(a) 7: 9 (b) 5: 9 (c) 3: 5 (d) 5: 7

Sol : (d) Specific heat of gas at constant volume

$C_v = \frac{1}{2}fR$; f = degree of freedom

For gas A (diatomic)

f = 5(3translational +2 rotational)

$$C_V^A = \frac{5}{2}R$$

For gas B (diatomic) in addition to (3translational +2 rotational) 2 vibrational degree of freedom.

$C_v^B = \frac{7}{2}R$ Hence $\frac{C_v^A}{C_v^B} = \frac{\frac{5}{2}R}{\frac{7}{2}R} = \frac{5}{7}$

35. Two gases - argon (atomic radius 0.07nm, atomic weight 40) and xenon (atomic radius 0.1nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to: [9 Jan 2020 II]

(a) 3.67 (b) 1.83 (c) 2.3 (d) 4.67

Sol : (Bonus) Mean free path of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Here, n = number of collisions per unit volume

d = diameter of the molecule

If average speed of molecule is v then

Mean free time, $\Gamma = \frac{\lambda}{v}$

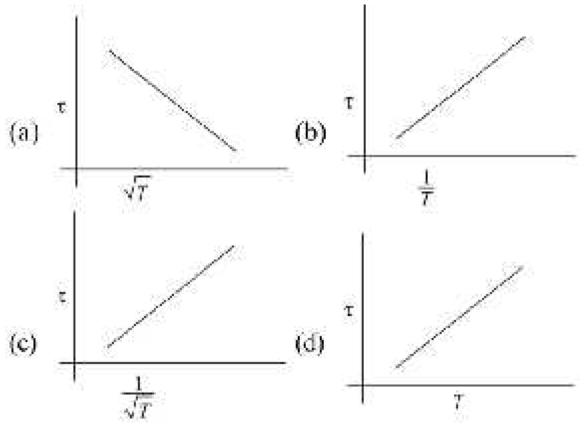
$$\square\ \Gamma = \frac{1}{\sqrt{2}\pi n d^2 v} = \frac{1}{\sqrt{2}\pi n d^2}\sqrt{\frac{M}{3RT}}$$

$$\left(\left(v = \sqrt{\frac{3RT}{M}}\right)\right)$$

$$\Gamma \propto \frac{\sqrt{M}}{d^2}\ \frac{\tau_1}{\tau_2} = \frac{\sqrt{M_1}}{d_1^2} \times \frac{d_2^2}{\sqrt{M_2}}$$

$$= \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = 1.09$$

36. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale) [8 Jan. 2020 I]



Sol :. (c) Relaxation time (Γ) □ $\frac{\text{meanfreepath}}{\text{speed}}$ □ $\Gamma \propto \frac{1}{v}$

and, $v \propto \sqrt{T}$

$$\Gamma \propto \frac{1}{\sqrt{T}}$$

Hence graph between Γ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

37. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be: [8 Jan. 2020 II]

(a) 19/13 (b) 67/45 (c) 40/27 (d) 23/15

Sol : (a) Helium is a monoatomic gas and Oxygen is a diatomic gas.

For helium, $C_{V_1} = \frac{3}{2}R$ and $C_{P_1} = \frac{5}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$ and $C_{P_2} = \frac{7}{2}R$

$$\gamma = \frac{N_1 C_{P_1} + N_2 C_{P_2}}{N_1 C_{V_1} + N_2 C_{V_2}}$$

$$\Rightarrow \gamma = \frac{n.\frac{5}{2}R + 2n.\frac{7}{2}R}{n.\frac{3}{2}R + 2n.\frac{5}{2}R} = \frac{19nR \times 2}{2(13nR)}$$

□. ( )( ) $mixture = \frac{19}{13}$

38. Two moles ofan ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of for the mixture is: [7 Jan. 2020 I]

(a) 1.45 (b) 1.50 (c) 1.47 (d) 1.42

Sol :d) Using, $\gamma_{\text{mixture}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

$$□ \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1}$$

$$□ \frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$□ \frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1} \; □ \; \gamma_m - 1 = \frac{5}{12}$$

$$□ \; \gamma_m = \frac{17}{12} = 1.42$$

39. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat ofmixture at constant volume? (R = 8.3J/mo1K) [12 April 2019 I]

(a) 19.7 J/mo1L (b) 15.7 J/mo1K (c) 17.4J/mo1K (d) 21.6J/mo1K

Sol : (c) $[C_v]_{min} = \frac{n_1[C_{v_1}]+n_2[C_{v_2}]}{n_1+n_2}$

$$= \left[\frac{2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}}{2+3}\right]$$

$$= 2.1R = 2.1 \times 8.3 = 17.4 J/mol - k$$

40. A diatomic gas with rigid molecules does 10 J ofwork when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process? [12 April 2019 II]

(a) 25 J (b) 35 J (c) 30 J (d) 40 J

Sol : (b) $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

or $\frac{W}{Q} = 1 - \frac{5}{7} = \frac{2}{7}$

or $Q = \frac{7}{2}W = \frac{7 \times 10}{2} = 35J$

41. A$25 \times 10^{-3} m^3$ volume cylinder is filled with 1 molofO_2 gas at room temperature (300K) . The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? [10 April 2019 I]

(a) -10^{12} (b) □ 10^{11} (c) □ 10^{10} (d) □ 10^{13}

Sol : (c) $V = 25 \times 10^3 m^3, N = 1$mole of$O_2$

$$T = 300K$$

$$V_r = 200 m/s$$

$$\lambda = \frac{1}{\sqrt{2} N \pi r^2}$$

Average time $\Gamma \underline{1} = \frac{\langle V \rangle}{\lambda} = 200. N\pi r^2 . \sqrt{2}$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} . \pi \times 10^{-18} \times 0.09$$

The closest value in the given option is $= 10^{10}$

42. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is: [10 April 2019 II]

(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$ (c) $\frac{7}{5}Q$ (d) $\frac{3}{2}Q$

Sol :. (c) Amount ofheat required (Q) to raise the temperature at constant volume

$Q = nC_v \Delta T$ (i)

Amount ofheat required (Q_1) at constant pressure

$Q_1 = nC_P \Delta T$ (ii)

Dividing equation (ii) by(i), we get

$$\frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\square\ Q_1 = (Q)\left(\frac{7}{5}\right)\left(\left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)\right)$$

43. An HC1 molecule has rotational, translational and vibrational motions. Ifthe rms velocity ofHCl molecules in its gaseous phase is $\overline{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be:[9 April 2019 I]

(a) $\overline{6k_B}$ (b) $\overline{3k_B}$ (c) $\frac{m\overline{v}^2}{7k_B}$ (d) $\frac{m\overline{v}^2}{5k_B}$

Sol : (a) $\frac{1}{2}m\overline{v}^2 = 3k_BT$

$$m\overline{v}^2$$

or $T = \overline{6k_B}$

44. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of $Jmol^{-1}k^{-1}$) by29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. Ifthey are treated as ideal gases, then: [9April 2019II]

(a) A is rigid but B has a vibrational mode.

(b) A has a vibrational mode but B has none.

(c) A has one vibrational mode and B has two.

(d) Both A and B have a vibrational mode each.

Sol : (b) $\gamma_A = \frac{C_P}{C_v} = \frac{29}{22} = 1.32 < 1.4$ (diatomic) and $\gamma_B = \frac{30}{21} = \frac{10}{7} = 1.43 > 1.4$

Gas A has more than 5 - degrees of fiieedom.

45. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy ofthe gas: [12 Jan. 2019 I]

(a) 9×10^6 J (b) 6×10^4 J (c) 10^8 J (d) 3×10^2 J

Sol : (a) Energy ofthe gas, E

$= \frac{f}{2}nRT = \frac{f}{2}PV$

$$= \frac{f}{2}(3 \times 10^6)(2) = f \times 3 \times 10^6$$

Considering gas is monoatomic i. e., f = 3

Energy, $E = 9 \times 10^6$ J

46. An ideal gas is enclosed in a cylinder at pressure of2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. Ifthe pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions wiil be close to: [12 Jan. 2019 II]

(a) 2×10^{-7} s (b) 4×10^{-8} s (c) 0.5×10^{-8} s (d) 3×10^{-6} s

Sol : (b) Using, $\Gamma = \frac{1}{2n\pi d^2 V_{avg}}$

$$t \propto \frac{\sqrt{T}}{P}\left[\because n = \frac{\text{no. of mo1ecules}}{\text{Vo1ume}}\right]$$

or, $\frac{t_1}{6\times10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}} \approx 4 \times 10^{-8}$

47. A gas mixture consists of3 moles ofoxygen and 5 moles ofargon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is : [11 Jan. 2019 I]

(a) 15 RT (b) 12 RT (c) 4 RT (d) 20 RT

Sol : (a) $U = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$

Considering translational and rotational modes, degrees offreedom $f_1 = 5$ and $f_2 = 3$

$$u = \frac{5}{2}(3RT) + \frac{3}{2} \times 5RT$$

$$U = 15RT$$

48. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature ofthe gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant): [11 Jan. 2019 II]

(a) $\frac{1}{2}R\Delta T$ (b) $\frac{1}{2}KR\Delta T$ (c) $\frac{3}{2}R\Delta T$ (d) $\frac{2K}{3}\Delta T$

Sol : (a) According to question VT = K

we also know that PV = nRT

$$\Rightarrow T = \left(\frac{PV}{nR}\right)$$

$$\therefore V\left(\frac{PV}{nR}\right) = k \Rightarrow PV^2 = K$$

$C = \frac{R}{1-x} + C_v$ (For polytropic process)

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$$\Delta Q = nC\Delta T$$

$= \frac{R}{2} \times \Delta T$ [here, n = 1 mole]

49. Two kg ofa monoatomic gas is at a pressure of $4\times 10^4 N/m^2$ The density of the gas is 8 kg/m^3 What is the order of energy of the gas due to its thermal motion? [10 Jan 2019 II]

(a) 10^3 J (b) 10^5 J (c) 10^4 J (d) 10^6 J

Sol : (c) Thermal energy ofN molecule

$$= N\left(\frac{3}{2}kT\right)$$

$= \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT) = \frac{3}{2}PV$

$$= \frac{3}{2}P\left(\frac{m}{p}\right) = \frac{3}{2}P\left(\frac{2}{8}\right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8} = 1.5 \times 10^4 J$$

therefore, order = $10^4 J$

50. A 15g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3J/K mole] [9 Jan. 2019 II]

(a) 0.9kJ (b) 6 kJ\ (c) 10 kJ (d) 14 kJ

Sol :c) Heat transferred,

$Q = nC_v\Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times i. e., $T' = 4T$ as $v_{rms} = \sqrt{3RT/M}$

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$$\left[\because \frac{CP}{CV} = \gamma_{diatomic} = \frac{7}{5} \& C_p - C_v = R\right]$$

or, Q = 10000J = 10kJ

51. Two moles ofan ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabaticallyto a volume 2 V. Calculate(1) the final temperature ofthe gas and(2) change in its internal energy. [2018]

(a) (1) 189 K (2) 2.7kJ

(b) (1) 195 K (2) −2.7kJ

(c) (1) 189 K (2) −2.7kJ

(d) (1) 195 K (2) 2.7kJ

Sol : (c) In an adiabatic process

$TV^{\gamma-1}$ = Constant or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$(300)\ V^{2/3} = T_2(2V)^{2/3}$ □ $T_2 = \frac{300}{(2)^{2/3}}$

T_2 = 189K(final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\Delta T$

$$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7kJ$$

52. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value ofn is [Online Apri116, 2018]

(a) 3/2 (b) 2 (c) 1 (d) 3

Sol : (b) Using formula,

$$\gamma_{mixture} = (\ )_{mix}(\ ) = \frac{\frac{n_1\gamma_1}{\gamma_1-1} + \frac{n_2\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}}$$

Putting the value of $n_1 = 2, n_2 = n, (\ )_{mix}(\ ) = \frac{3}{2}$

$\gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$ and solving we get, $n = 2$

53. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that $C_p - C_v = a$ for hydrogen gas $C - C_v = b$ for nitrogen gas T1i correct relation between a and b is: [2017]

(a) a = 14b (b) a = 28b (c) $a = \frac{1}{14}b$ (d) a = b

Sol : (a) As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \frac{R}{M}$

For hydrogen $(M = 2) C_p - C_v = a = \frac{R}{2}$

For nitrogen $(M = 28) C_p - C_v = b = \frac{R}{28}$

$\frac{a}{b} = 14$ or, $a = 14b$

54. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is: [Online April 8, 2017]

(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

Sol : (d) The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$$\frac{C_P}{C_V} = \gamma = \left(1 + \frac{2}{f}\right)$$

where fis degree offreedom

$$\frac{C_P}{C_V} = \left(1 + \frac{2}{5}\right) = \frac{7}{5}$$

55. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation ofpressure P and volume V is given by PV^n =constant, then n is given by(Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) : [2016]

(a) $n = \frac{C_P - C}{C - C_V}$ (b) $n = \frac{C - C_V}{C - C_P}$ (c) $n = \frac{C_P}{C_V}$ (d) $n = \frac{C - C_P}{C - C_V}$

Sol : . (d) For a polytropic process

$$C = C_v + \frac{R}{1-n} \quad C - C_v = \frac{R}{1-n}$$

$$1 - n = \frac{R}{C - C_v} \quad 1 - \frac{R}{C - C_v} = n$$

$$n = \frac{C - C_v - R}{C - C_v} = \frac{C - C_v - C_p + C_v}{C - C_v}$$

$$= \frac{C - C_p}{C - C_v}(C_p - C_{v=R})$$

56. Using equipartition of energy, the specific heat (in $Jkg^{-1}K^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27) [Online April 11, 2015]

(a) 410 (b) 25 (c) 1850 (d) 925

Sol : (d) Using equipartition of energy, we have

$$\frac{6}{2}KT = mCT$$

$$C = \frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{23}}{27 \times 10^{-3}}$$

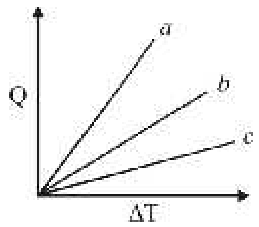
$$C = 925 J/kgK$$

57. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking $R = 8.0 JK^{-1} mole^{-1}$, 1 atm = 10^5 Pa and $N_{Avogadro} = 6 \times 10^{23} mole^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of. [Online April 9, 2014]

(a) 0.2μm (b) 02mm (c) 0.2 cm (d) 0.2nm

Sol :. (b)

58. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic(P) gas. The initial state ofall the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to : [Online April 9, 2013]



(a) P, M and D (b) M, D and P (c) P, D and M (d) D, M and P

Sol : (b) On giving same amount of heat at constant pressure, there is no change in temperature for mono, dia and polyatomic.

$$(\Delta Q)_P = \mu C_p \Delta T \left(\mu = \frac{\text{No. ofmo1ecu1e. s}}{\text{Avogedro'sno}}\right)$$

or $\Delta T \propto \frac{1}{\text{no.ofmolecules}}$

59. A given ideal gas with $\gamma = \frac{C_p}{C_v} = 1.5$ at a temperature T. If the gas is compressed adiabatically to

one - fourth of its initial volume, the final temperature will be [Online May 12, 2012]

(a) $2\sqrt{2}T$ (b) $4T$ (c) $2T$ (d) $8T$

Sol : (c) $TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow T(V)^{1/2} = T_2\left(\frac{V}{4}\right)^{1/2}$$

[$\because \gamma = 1.5, T_1 = T, V_1 = V$ and $V_2 = \frac{V}{4}$]

$$T_2 = \left(\frac{4V}{V}\right)^{1/2} T = 2T$$

60. If C_p and C_V denote the specific heats ofnitrogen per unit mass at constant pressure and constant volume respectively, then [2007]

(a) $C_p - C_V = 28R$ (b) $C_p - C_V = R/28$ (c) $C_p - C_V = R/14$ (d) $C_p - C_V = R$

Sol : (b) According to Mayer's relationship

$C_p - C_V = R$, as per the question $(C_p - C_V)M = R$

$$\therefore C_p - C_V = R/28$$

Here M = 28 = mass ofl unit ofN_2

61. A gaseous mixmre consists of16 g ofhelium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ ofthe mixture is [2005]

(a) 1.62 (b) 1.59 (c) 1.54 (d) 1.4

Sol : (a) For mixture ofgas specific heat at constant volume

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2}$$

62. One mole ofideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$) . What is γ for the mixture? Y Denotes the ratio of specific heat at constant pressure, to that at constant volume [2004]

(a) 35/23 (b) 23/15 (c) 3/2 (d) 4/3

Sol :No. ofmoles ofhelium,

$$n_1 = \frac{m_{He}}{M_{He}} = \frac{16}{4} = 4$$

Number ofmoles ofoxygen,

$$n_2 = \frac{16}{32} = \frac{1}{2}$$

$$C_v = \frac{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}{4 + \frac{1}{2}} = \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$ and

Specific heat at constant pressure

$$C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{(n_1 + n_2)} = -4 \times 5\left(\frac{4+\frac{1}{}}{2}\right) 2R_{+\frac{1}{2}\times\frac{7R}{2}}$$

$$= \frac{10R + \frac{7}{4}R}{} \frac{9}{2} = \frac{47R}{18}$$

$$\frac{C_p}{C_v} = \frac{47R}{18} \times \frac{18}{29R} = 1.62$$

63. During an adiabatic process, the pressure ofa gas is found to be proportional to the cube ofits absolute temperature. The ratio C_p/C_v for the gas is [2003]

(a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$

Sol : (d) $P \propto T^3 \Rightarrow PT^{-3}$ = constant ....(i)

But for an adiabatic process, the pressure temperature relationship is given by

$P^{1-\gamma}T^{\gamma}$ = constant

□ $PT^{\frac{\gamma}{1-\gamma}}$ =constt. (ii)

From (i) and(ii) $\frac{\gamma}{1-\gamma} = -3$ □ $\gamma = -3 + 3\gamma \Rightarrow \gamma = \frac{3}{2}$